Exponent and Logarithm Laws

Product of Powers is the Sum of Exponents / Logarithms

$a^m \times a^n = a^{m+n}$

$\log_a(x) + \log_a(y) = \log_a(xy)$

Example

$5^2 \times 5^3 = 5^{2+3} = 5^5 = 3125$

Example

$\log_6(4) + \log_6(9) = \log_6(4 \times 9) = \log_6(36) = 2$

Example

$\log_4(48) = \log_4(16) + \log_4(3) = 2 + \log_4(3)$

Quotient of Powers is the Difference of Exponents / Logarithms

$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$

Example

$\frac{7^8}{7^5} = 7^{8-5} = 7^3 = 343$

Example

$\log_5(500) - \log_5(4) = \log_5\left(\frac{500}{4}\right) = \log_5(125) = 3$

Example

$\log_3\left(\frac{9}{4}\right) = \log_3(9) - \log_3(4) = 2 - \log_3(4)$

Exponent / Logarithm is Zero - Power is One

$a^0 = 1$

$\log_a(1) = 0$

Example

$120^0 = 1$

Example

$\log_{77}(1) = 0$

Power with Another Exponent is the Product of Exponents / Logarithms

$(a^m)^n = a^{m \times n}$

$\log_a(x^n) = n \log_a(x)$

Example

$(2^2)^3 = 2^{2\times 3} = 2^6 = 64$

Example

$\log_7(9) = \log_7(3^2) = 2\log_7(3)$

Example

$8^5 = (2^3)^5 = 2^{3\times 5} = 2^{15} = 32768$

Example

$\log_2(64) = \log_2(2^6) = 6\log_2(2) = 6$

Example

$\frac{16^2}{4^4} = \frac{(4^2)^2}{4^4} = \frac{4^4}{4^4} = 1$

Example

$2\log_9(3) = \log_9(3^2) = \log_9(9) = 1$

Products and Quotients with Exponents

$(a \times b)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example

$(3 \times 5)^3 = 3^3 \times 5^3$
$= 27 \times 125 = 3375$

Example

$22^3 = (2 \times 11)^3 = 2^3 \times 11^3$
$= 8 \times 1331 = 10648$

Example

$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

Example

$\frac{24^4}{12^4} = \left(\frac{24}{12}\right)^4 = 2^4 = 16$

Negative Exponents / Logarithms are the Reciprocal of the Power

$a^{-1} = \frac{1}{a}$

$a^{-m} = \frac{1}{a^m}$

$\frac{1}{a^{-m}} = a^m$

$\log_a\left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a(x)$

$\log_{\frac{1}{a}}(x) = -\log_a(x)$

Example

$4^{-1} = \left(\frac{4}{1}\right)^{-1} = \frac{1}{4}$

Example

$\left(\frac{1}{4}\right)^{-1} = \frac{4}{1} = 4$

Example

$\frac{3}{4^{-1}} = \frac{3}{\frac{1}{4}} = 3 \times 4 = 12$

Example

$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

Example

$\log_8\left(\frac{1}{5}\right) = -\log_8(5)$

Example

$-3\log_{11}(4) = \log_{11}(4^{-3}) = \log_{11}\left(\frac{1}{64}\right)$

Example

$-\log_2\left(\frac{16}{9}\right) = \log_2\left(\frac{9}{16}\right)$

Fractional Exponents / Logarithms are the Roots of the Power

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$\log_a\left(\sqrt[n]{a^m}\right) = \log_a\left(x^{\frac{m}{n}}\right) = \frac{m}{n}$

$\log_{a^n}(a) = \frac{1}{n}$

Example

$9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3$

Example

$8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2$

Example

$8^{\frac{5}{3}} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

Example

$\log_3\left(\sqrt{27}\right) = \log_3\left(3^{\frac{3}{2}}\right) = \frac{3}{2}$

Example

$\log_9(3) = \frac{1}{2}$

Change of Base

There are times where a particular base of an exponential or logarithm is needed.
You can change the base by manipulating the exponent and logarithm laws.
Specifically, changing the logarithm base to 10 allows logarithms of any base to be evaluated on a scientific calculator.

$a^x = b^{(\log_b(a))x}$

$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Example

$2^{13} = 10^{13\log_{10}(2)} \approx 10^{3.913}$

Example

$\log_2(8192) = \frac{\log_{10}(8192)}{\log_{10}(2)}$

Example

$\log_a(b) \times \log_b(c) \times \log_c(a) = \frac{\log_{10}(b)}{\log_{10}(a)} \times \frac{\log_{10}(c)}{\log_{10}(b)} \times \frac{\log_{10}(a)}{\log_{10}(c)} = 1$