Exponential Equations

Equations Involving Exponentials

To solve an exponential equation, we can attempt to write both sides of the equation as an exponential expression with the same base to then equate the exponents. Alternatively, and more consistently, we can take the logarithm with the same base of both sides which, as the inverse of the exponential, leaves the exponent and writes the other side as a logarithmic expression which may be simplified.

Express with Same Base

$a^x = b$

$a^x = a^{\log_a(b)}$

Equal powers with equal bases have equal exponents

$x = \log_a(b)$

Inverse: $\log_a(a^x) = x$

$a^x = b$

Take the log base a of both sides

$\log_a(a^x) = \log_a(b)$

$x = \log_a(b)$

Example

Solve the equation $2 \times 5^x = 50$ for x.

$2 \times 5^x = 50$

$5^x = 25$

$5^x = 5^2$

Equal powers with equal bases have equal exponents

$x = 2$

$2 \times 5^x = 50$

$5^x = 25$

Take the log base 5 of both sides

$\log_5(5^x) = \log_5(25) = \log_5(5^2)$

$x = 2$

$2 \times 5^x = 50$

Take the log base 5 of both sides

$\log_5(2 \times 5^x) = \log_5(50)$

$\log_5(2) + \log_5(5^x) = \log_5(50)$

$\log_5(5^x) = \log_5(50) - \log_5(2)$

$\log_5(5^x) = \log_5(50/2) = \log_5(25)$

$x = 2$

Exponent Laws and Equations

Where possible express the equation using a single exponential expression on both sides of the equation using exponent laws. Remember $a^x > 0$.

Example VCAA 2013 Exam 1 Question 5b

Solve the equation $3^{-4x} = 9^{6-x}$ for x.

$3^{-4x} = 9^{6-x}$

$3^{-4x} = 3^{2(6-x)}$

Equal powers with equal bases have equal exponents or take the log of both sides:

$\log_3(3^{-4x}) = \log_3\left(3^{2(6-x)}\right)$

$-4x = 2(6 - x)$

$-4x = 12 - 2x$

$-2x = 12$

$x = -6$

$3^{-4x} = 9^{6-x}$

$3^{-4x} \times 3^{4x} = 9^{6-x} \times 3^{4x}$

$1 = 9^{6-x} \times 3^{4x}$

$1 = 3^{2(6-x)} \times 3^{4x}$

$1 = 3^{12-2x+4x}$

$1 = 3^{12+2x}$

The exponent of 3 that gives 1, is 0 or take the log of both sides: $\log_3(1) = \log_3(3^{12+2x})$

$0 = 12 + 2x$

$2x = -12$

$x = -6$

Example VCAA 2011 Exam 1 Question 2b

Solve the equation $4^x - 15 \times 2^x = 16$ for x.

$(2^2)^x - 15 \times 2^x - 16 = 0$

$(2^x)^2 - 15 \times 2^x - 16 = 0$

Let $a = 2^x$

$a^2 - 15a - 16 = 0$

$(a - 16)(a + 1) = 0$

$a = 16, \qquad a = -1$

$2^x = 16, \qquad 2^x = -1$

$2^x = 2^4, \qquad 2^x > 0 \therefore$ no real solution

$x = 4$

$\therefore x = 4$

Example VCAA 2016 Sample Exam 1 Question 6b / Example VCAA 2015 Exam 1 Question 7b

Solve $3e^t = 5 + 8e^{-t}$ for t.

$3e^t = 5 + 8e^{-t}$

$3e^{2t} = 5e^t + 8$

$3e^{2t} - 5e^t - 8 = 0$

Let $a = e^t$

$3a^2 - 5a - 8 = 0$

$(3a + 3)(3a - 8) = 0$

$a = -1 \Rightarrow e^x = -1, \qquad a = \frac{8}{3} \Rightarrow e^x = \frac{8}{3}$

$e^x > 0 \therefore$ no real solution, $\qquad x = \log_e(8/3)$