Power Equations

Linear Equations

The most basic form of a linear equation is $ax + b = 0$, which has a solution of $x = -\frac{b}{a}$.
Linear equations with one instance of the variable, can be solved by using inverse operations. That is undoing the operations that were applied to the variable, in the reverse order that they were applied.

Linear equations can look more complicated such as $2(4x + 7) = 5(6 - 3x)$ or $\frac{8x - 3}{5} = \frac{5x + 7}{2}$.

These equations are best solved by making one side of the equation equal 0 or a constant. This allows the equation to be solved like the basic form above. This will often involve multiplying or dividing or expanding to remove brackets and fractions. Alternatively, put all the of x terms on one side of the equation and the constant are on the other side to reduce the number of steps.

Example

$$2(4x + 7) = 5(6 - 3x)$$

$$2 \times 4x + 2 \times 7 = 5 \times 6 + 5 \times -3x$$

$$8x + 14 = 30 - 15x$$

$$8x + 14 + 15x - 14 = 30 - 15x + 15x - 14$$

$$23x = 16$$

$$x = \frac{16}{23}$$

Example

$$\frac{8x - 3}{5} = \frac{5x + 7}{2}$$

$$\frac{8x - 3}{5} \times 5 \times 2 = \frac{5x + 7}{2} \times 5 \times 2$$

$$8x \times 2 - 3 \times 2 = 5x \times 5 + 7 \times 5$$

$$16x - 6 = 25x + 35$$

$$16x - 6 - 16x - 35 = 25x + 35 - 16x - 35$$

$$-41 = 9x$$

$$x = -\frac{41}{9}$$

Remember that for an equation to stay balanced, the same operation must be applied to both sides. It is a common mistake (especially when multiplying or dividing an equation that involves a sum to not multiply or divide each term. Every term must be multiplied or divided to correctly scale the equation.

Example

$10 + 5 = 15$
$10 + 5 + 5 = 15 + 5$
$10 + 5 + 5 = 20$

Example

$10 + 5 = 15$
$2 \times (10 + 5) = 2 \times 15$
$2 \times 10 + 2 \times 5 = 30$
$20 + 10 = 30$

Example

$10 + 5 = 15$
$(10 + 5) \div 5 = 15 \div 5$
$10 \div 5 + 5 \div 5 = 3$
$2 + 1 = 3$

Example

$10 \times 5 = 50$
$2 \times (10 \times 5) = 2 \times 50$
$2 \times 10 \times 5 = 100$

Example

$10 \div 5 = 2$
$2 \times (10 \div 5) = 2 \times 2$
$(2 \times 10) \div 5 = 4$

Power Equations

Equations involving a power of x where there is only one x in the equation (or only one x after simplifying or changing forms) can be solved by doing inverse operations much like a linear equation. Keep in mind, even powers have two solutions when taking the root, but the value must be positive.

Natural Powers

$ax^n + b = 0$

Example

$$2(x-3)^2 - 8 = 0$$
$$2(x-3)^2 = 8$$
$$(x-3)^2 = 4$$
$$x - 3 = \pm\sqrt{4} = \pm 2$$
$$x = 3 \pm 2 = 1,5$$

Example

$$5x^3 + 135 = 0$$
$$5x^3 = -135$$
$$x^3 = -27$$
$$x = \sqrt[3]{-27} = -3$$

Example

$$x^2 + 5 = 0$$
$$x^2 = -5$$
$$x = \pm\sqrt{-5}$$

No real solution

Negative Powers

$ax^{-n} + b = 0, \quad \frac{a}{x^n} + b = 0$

Example

$$\frac{10}{x^2} - 250 = 0$$
$$\frac{10}{x^2} = 250$$
$$\frac{x^2}{10} = \frac{1}{250}$$
$$x^2 = \frac{1}{25}$$
$$x = \pm\sqrt{\frac{1}{25}} = \pm\frac{1}{5}$$

Example

$$\frac{10}{x^2} - 250 = 0$$
$$\frac{10}{x^2} = 250$$
$$10 = 250x^2$$
$$x^2 = \frac{1}{25}$$
$$x = \pm\sqrt{\frac{1}{25}} = \pm\frac{1}{5}$$

Example

$$\frac{7}{x+1} - 11 = 0$$
$$\frac{7}{x+1} = 11$$
$$\frac{x+1}{7} = \frac{1}{11}$$
$$x + 1 = \frac{7}{11}$$
$$x = -\frac{4}{11}$$

Example

$$\frac{7}{x+1} - 11 = 0$$
$$\frac{7}{x+1} = 11$$
$$7 = 11(x+1)$$
$$x + 1 = \frac{7}{11}$$
$$x = -\frac{4}{11}$$

Rational Powers

$ax^{\frac{p}{q}} + b = 0, \quad a\left(\sqrt[q]{x}\right)^p + b = 0, \quad a\sqrt[q]{x^p} + b = 0$

Example

$$4\sqrt{x} - 1 = 0$$
$$4\sqrt{x} = 1$$
$$\sqrt{x} = \frac{1}{4}$$
$$x = \frac{1}{16}$$

Example

$$6x^{\frac{7}{3}} - 768 = 0$$
$$6x^{\frac{7}{3}} = 768$$
$$x^{\frac{7}{3}} = 128$$
$$x = \left(\sqrt[7]{128}\right)^3$$
$$x = 2^3 = 8$$

Example

$$\sqrt[3]{2x+1} + 5 = 0$$
$$\sqrt[3]{2x+1} = -5$$
$$2x + 1 = -125$$
$$2x = -126$$
$$x = -63$$

Example

$$x^{\frac{3}{2}} + 8 = 0$$
$$x^{\frac{3}{2}} = -8$$
$$x = \left(\sqrt[3]{-8}\right)^2$$
$$x = (-2)^2 = 4$$

Example

$$\frac{1}{3}x^{\frac{2}{3}} - 27 = 0$$
$$\frac{1}{3}x^{\frac{2}{3}} = 27$$
$$x^{\frac{2}{3}} = 81$$
$$x = \left(\sqrt{81}\right)^3$$
$$x = 9^3 = 729$$