# Composite Function Equations

## Composite Function Equations

Equations involving composite functions can be made easier by making a substitution of the inside function or by using an inverse operations to remove the outside function.

### Example

$(2x+1)^2 + 5(2x+1) + 6 = 0$

Let $y = 2x+1$

$$y = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2} = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2}$$

$y = 2x+1 = -\frac{4}{2} = -2,$ $\quad$ $y = 2x+1 = -\frac{6}{2} = -3$

$2x = -3,$ $\quad$ $2x = -4$

$x = -\frac{3}{2},$ $\quad$ $x = -2$

### Example

$x^4 + x^2 - 2 = 0$

Let $a = x^2$

$a^2 + a - 2 = 0$

$$a = \frac{-1 \pm 3}{2} = -2, 1$$

$a = x^2 = -2, 1$

$x = \pm\sqrt{-2}, \pm\sqrt{1}$

reject $\sqrt{-2}$, $\quad x = \pm 1$

### Example VCAA 2011 Exam 1 Question 2b

Solve the equation $4^x - 15 \times 2^x = 16$ for $x$.

$(2^2)^x - 15 \times 2^x - 16 = 0$

$(2^x)^2 - 15 \times 2^x - 16 = 0$

Let $a = 2^x$

$a^2 - 15a - 16 = 0$

$(a-16)(a+1) = 0$

$a = 16, \quad a = -1$

$2^x = 16, \quad 2^x = -1$

$2^x = 2^4, \quad 2^x > 0 \therefore$ no real solution

$x = 4$

$\therefore x = 4$

### Example VCAA 2016 Sample Exam 1 Question 6b / Example VCAA 2015 Exam 1 Question 7b

Solve $3e^t = 5 + 8e^{-t}$ for $t$.

$3e^t = 5 + 8e^{-t}$

$3e^{2t} = 5e^t + 8$

$3e^{2t} - 5e^t - 8 = 0$

Let $a = e^t$

$3a^2 - 5a - 8 = 0$

$(3a+3)(3a-8) = 0$

$a = -1 \Rightarrow e^x = -1, \quad a = \frac{8}{3} \Rightarrow e^x = \frac{8}{3}$

$e^x > 0 \therefore$ no real solution, $\quad x = \log_e(8/3)$

### Example VCAA 2017 NHT Exam 1 Question 3b

Solve $2\sin^2(x) + 3\sin(x) - 2 = 0$, where $0 \le x \le 2\pi$.

$2\sin^2(x) + 3\sin(x) - 2 = 0$

Let $a = \sin(x)$

$2a^2 + 3a - 2 = 0$

$$a = \frac{-3 \pm \sqrt{3^2 - 4(2)(-2)}}{2(2)} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$$

$a = \frac{-3+5}{4} = \frac{2}{4} = \frac{1}{2}, \quad a = \frac{-3-5}{4} = -\frac{8}{4} = -2$

$\sin(x) = \frac{1}{2}, \quad \sin(x) = -2 \Rightarrow$ has no real solutions

Sine is positive in the first and second quadrants

$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}, \quad x = \pi - \sin^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

### Example

Solve $x - 3 = \sqrt{2x-3}$ for $x$.

$x - 3 = \sqrt{2x-3}$

$2x - 6 = 2\sqrt{2x-3}$

$2x - 3 - 3 = 2\sqrt{2x-3}$

$(2x-3) - 2\sqrt{2x-3} - 3 = 0$

Let $u = \sqrt{2x-3}$

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3$ or $u = -1$

$\sqrt{2x-3} = 3, \quad \sqrt{2x-3} = -1$

$x = 6,$ $\quad$ no real solution