# Algebra Summary

## Mensuration

| | | | |
|---|---|---|---|
| Area of a trapezium | $\frac{1}{2}(a+b)h$ | Volume of a prism | $Ah$ |
| Curved Surface of a cylinder | $2\pi rh$ | Volume of a cylinder | $\pi r^2 h$ |
| Area of a triangle | $\frac{1}{2}ab\sin(C)$ | Volume of a pyramid | $\frac{1}{3}Ah$ |
| Surface area of a prism | $2A+Ph$ | Volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| Surface area of a pyramid | $A+\frac{1}{2}Ph$ | Volume of a sphere | $\frac{4}{3}\pi r^3$ |

## Functions

$f: D \to R, f(x) = \cdots$
Name: domain → co-domain (not range), rule

Function must be one-to-one or many-to-one (vertical line test).

## Inverse functions and solving equations

The rule of the inverse can be found by swapping the $x$ and $y$ in the equation.
This can also be seen as a reflection in the line $y = x$.

$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$
There may be restrictions on the inside function and there may be more solutions to consider.

Inverse <u>function</u> must be one-to-one and must be written as $f^{-1}: D \to R, f^{-1}(x) = \cdots$

| Function | Inverse | Composition to solve | |
|---|---|---|---|
| $f(x) = x^n$ | $f^{-1}(x) = \sqrt[n]{x}$ | $\sqrt[n]{x^n} = x$ | $(\sqrt[n]{x})^n = x$ |
| $f(x) = x^{\frac{p}{q}}$ | $f^{-1}(x) = x^{\frac{q}{p}}$ | $\left(x^{\frac{p}{q}}\right)^{\frac{q}{p}} = x$ | $\left(x^{\frac{q}{p}}\right)^{\frac{p}{q}} = x$ |
| $f(x) = e^x$ | $f^{-1}(x) = \log_e(x)$ | $\log_e(e^x) = x$ | $e^{\log_e(x)} = x$ |
| $f(x) = \sin(x)$ | $f^{-1}(x) = \sin^{-1}(x)$ | $\sin^{-1}(\sin(x)) = x$ | |
| $f(x) = \cos(x)$ | $f^{-1}(x) = \cos^{-1}(x)$ | $\cos^{1}(\cos(x)) = x$ | |
| $f(x) = \tan(x)$ | $f^{-1}(x) = \tan^{-1}(x)$ | $\tan^{-1}(\tan(x)) = x$ | |

## Polynomials

### Remainder and factor theorems

The remainder of a polynomial $P(x)$ divided by $(x-a)$ is equal to $P(a)$.
A polynomial $P(x)$ has a factor $(x-a)$ if and only if $P(a) = 0$.

### Complete the square

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

### Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Discriminant

$$\Delta = b^2 - 4ac$$

## Exponential and logarithmic functions

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

$a^0 = 1$

$(a^m)^n = a^{m \times n}$

$(a \times b)^m = a^m \times b^m$

$a^{-1} = \frac{1}{a}, \qquad a^{-m} = \frac{1}{a^m}$

$a^{\frac{1}{n}} = \sqrt[n]{a}, \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.71828 \ldots$

$\log_a(x) + \log_a(y) = \log_a(xy)$

$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$

$\log_a(1) = 0$

$\log_a(x^n) = n \log_a(x)$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$\log_a\left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a(x)$

$\log_a\left(\sqrt[n]{a^m}\right) = \log_a\left(x^{\frac{m}{n}}\right) = \frac{m}{n}$

$\log_e(x) = \ln(x)$

## Circular functions and trigonometry

Sine is the $y$ value on the unit circle. Cosine is the $x$ value on the unit circle.
Tangent is the length of the tangent to the $x$-axis. Check the sign of the gradient of the radius.

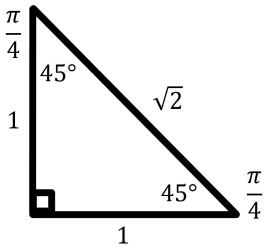
$\sin(\theta) = \frac{O}{H} \qquad \cos(\theta) = \frac{A}{H} \qquad \tan(\theta) = \frac{O}{A}$

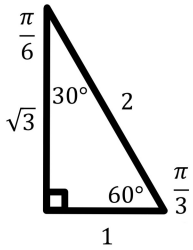
| | |
|---|---|
| Period of sine and cosine | $2\pi$ |
| Period of tangent | $\pi$ |

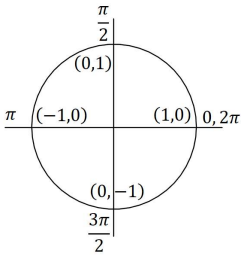
$\cos^2(x) + \sin^2(x) = 1$

$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$




## Linear simultaneous equations

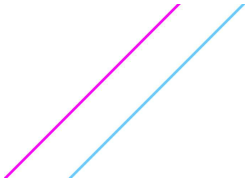
### Unique Solution

Non-parallel lines

$m_1 \neq m_2$

### No Solutions

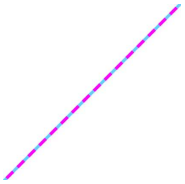
Parallel lines that do not meet



$m_1 = m_2$, and $c_1 \neq c_2$

### Infinite Solutions

Parallel lines that completely overlap each other



$m_1 = m_2$, and $c_1 = c_2$