Answers

Chapter 1

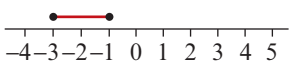
Exercise 1A

1 a $\{8, 11\}$ **b** $\{8, 11\}$
c $\{1, 3, 8, 11, 18, 22, 23, 24, 25, 30\}$
d $\{3, 8, 11, 18, 22, 23, 24, 25, 30, 32\}$
e $\{1, 3, 8, 11, 18, 22, 23, 24, 25, 30, 32\}$
f $\{1, 8, 11, 25, 30\}$

2 a $\{3, 18, 22, 23, 24\}$ **b** $\{25, 30, 32\}$
c $\{3, 18, 22, 23, 24\}$ **d** $\{1, 25, 30\}$

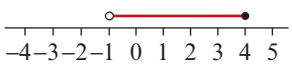
3 a

b

c


d

e

f


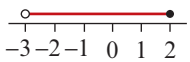
4 a $\{7, 9\}$ **b** $\{7, 9\}$
c $\{2, 3, 5, 7, 9, 11, 15, 19, 23\}$ **d** $\{2, 3, 5, 11\}$
e $\{2\}$ **f** $\{2, 7, 9\}$ **g** $\{2, 3, 5, 7\}$
h $\{7\}$ **i** $\{7, 9, 15, 19, 23\}$ **j** $(3, \infty)$

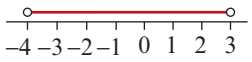
5 a $\{a, e\}$ **b** $\{a, b, c, d, e, i, o, u\}$
c $\{b, c, d\}$ **d** $\{i, o, u\}$

6 a $\{6\}$ **b** $\{2, 4, 8, 10\}$ **c** $\{1, 3, 5, 7, 9\}$
d $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$
e $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$ **f** $\{5, 7\}$
g $\{5, 7\}$ **h** $\{6\}$

7 a $[-3, 1)$ **b** $(-4, 5]$ **c** $(-\sqrt{2}, 0)$
d $\left(-\frac{1}{\sqrt{2}}, \sqrt{3}\right)$ **e** $(-\infty, -3)$ **f** $(0, \infty)$
g $(-\infty, 0)$ **h** $[-2, \infty)$

8 a $(-2, 3)$ **b** $[-4, 1)$ **c** $[-1, 5]$ **d** $(-3, 2]$

9 a


b


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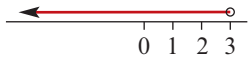
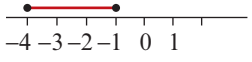
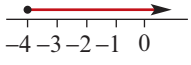
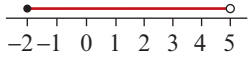


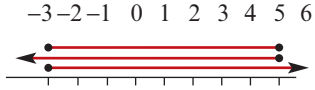
d


e


f


10 a

b

c


d

Exercise 1B

1 a Domain $= \mathbb{R}$ Range $= [-2, \infty)$
b Domain $= (-\infty, 2]$ Range $= \mathbb{R}$
c Domain $= (-2, 3)$ Range $= [0, 9)$
d Domain $= (-3, 1)$ Range $= (-6, 2)$
e Domain $= [-4, 0]$ Range $= [0, 4]$
f Domain $= \mathbb{R}$ Range $= (-\infty, 2)$

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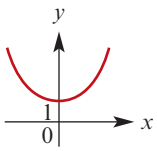
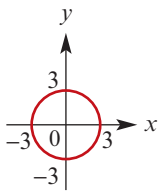
2 a



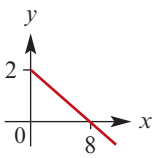
Domain = $\mathbb{R}$
Range = $[1, \infty)$

b



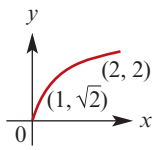
Domain = $[-3, 3]$
Range = $[-3, 3]$

c



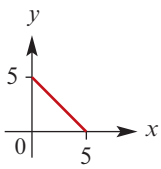
Domain = $[0, \infty)$
Range = $(-\infty, 2]$

d



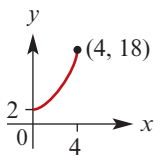
Domain = $[0, \infty)$
Range = $[0, \infty)$

e



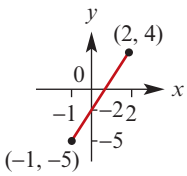
Domain = $[0, 5]$
Range = $[0, 5]$

f



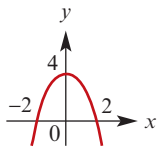
Domain = $[0, 4]$
Range = $[2, 18]$

g



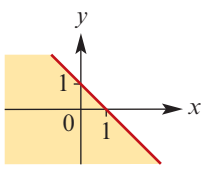
Domain = $[-1, 2]$
Range = $[-5, 4]$

h



Domain = $\mathbb{R}$
Range = $(-\infty, 4]$

i



Domain = $\mathbb{R}$
Range = $\mathbb{R}$

3 a Not a function; Domain = $\{-1, 1, 2, 3\}$; Range = $\{1, 2, 3, 4\}$

b A function; Domain = $\{-2, -1, 0, 1, 2\}$; Range = $\{-4, -1, 0, 3, 5\}$

c Not a function; Domain = $\{-2, -1, 2, 4\}$; Range = $\{-2, 1, 2, 4, 6\}$

d A function; Domain = $\{-1, 0, 1, 2, 3\}$; Range = $\{4\}$

4 a A function; Domain = $\mathbb{R}$; Range = $\{4\}$

b Not a function; Domain = $\{2\}$; Range = $\mathbb{Z}$

c A function; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

d Not a function; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

e Not a function; Domain = $[-4, 4]$; Range = $[-4, 4]$

5 a $f(-1) = -2$, $f(2) = 16$, $f(-3) = 6$, $f(2a) = 8a^2 + 8a$

b $g(-1) = -10$, $g(2) = 14$, $g(3) = 54$, $g(a - 1) = 2a^3 - 6a^2 + 8a - 10$

6 a $g(-2) = 10$, $g(4) = 46$

b i $12x^2 - 2$ **ii** $3x^2 - 12x + 10$ **iii** $3x^2 + 12x + 10$ **iv** $3x^4 - 2$

7 a 3 **b** 7 **c** $-\frac{3}{2}$ **d** $(3, \infty)$

8 a $x = -3$ **b** $x > -3$ **c** $x = \frac{2}{3}$

9 a $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2x + 3$

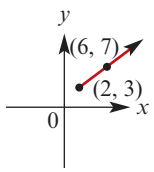
b $f: \mathbb{R} \to \mathbb{R}$, $f(x) = -\frac{4}{3}x + 4$

c $f: [0, \infty) \to \mathbb{R}$, $f(x) = 2x - 3$

d $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 - 9$

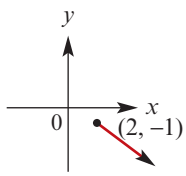
e $f: [0, 2] \to \mathbb{R}$, $f(x) = 5x - 3$

10 a



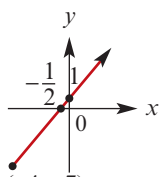
Range = $[3, \infty)$

b



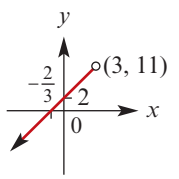
Range = $(-\infty, -1]$

c



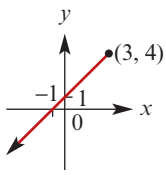
Range = $[-7, \infty)$

d



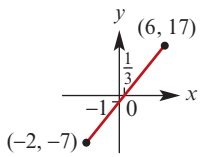
Range = $(-\infty, 11)$

e



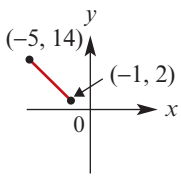
Range = $(-\infty, 4]$

f



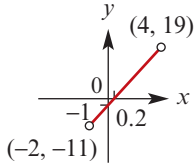
Range = $[-7, 17]$

g



Range = $[2, 14]$

h



Range = $(-11, 19)$

11 a $f(2) = -3$, $f(-3) = 37$, $f(-2) = 21$

b $g(-2) = 7$, $g(1) = 1$, $g(-3) = 9$

c i $f(a) = 2a^2 - 6a + 1$

ii $f(a + 2) = 2a^2 + 2a - 3$

iii $g(-a) = 3 + 2a$ **iv** $g(2a) = 3 - 4a$

v $f(5 - a) = 21 - 14a + 2a^2$

vi $f(2a) = 8a^2 - 12a + 1$

vii $g(a) + f(a) = 2a^2 - 8a + 4$

viii $g(a) - f(a) = 2 + 4a - 2a^2$

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12 a $\{\frac{2}{3}, -1\}$ **b** $\left\{-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right\}$ **c** $\{0, -\frac{1}{3}\}$
d $(-\infty, -1) \cup (\frac{2}{3}, \infty)$
e $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$ **f** $[-\frac{1}{3}, 0]$

13 a $f(-2) = 2$ **b** $f(2) = 6$
c $f(-a) = a^2 - a$ **d** $f(a) + f(-a) = 2a^2$
e $f(a) - f(-a) = 2a$ **f** $f(a^2) = a^4 + a^2$

14 a $\{2\}$ **b** $\{x : x > 2\}$ **c** $\left\{\frac{a+2}{3}\right\}$
d $\left\{-\frac{8}{3}\right\}$ **e** $\{1\}$ **f** $\left\{\frac{13}{18}\right\}$

15 a $\frac{4}{3}$ **b** 6 **c** $-\frac{7}{3}$ **d** 9 **e** $\frac{1}{3}$ **f** $-\frac{2}{9}$

16 a $\frac{6}{5}$ **b** $\frac{1}{5}$ **c** $\pm\frac{1}{3}$ **d** 1 **e** $-1, 2$

Exercise 1C

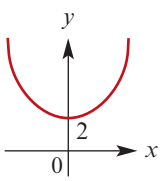
1 One-to-one functions: b, c

2 One-to-one functions: b, d, f

3 a Functions: i, iii, iv, vi, vii, viii
b One-to-one functions: iii, vii

4 $y = \sqrt{x+2},\ x \geq -2$; Range $= \mathbb{R}^+ \cup \{0\}$
$y = -\sqrt{x+2},\ x \geq -2$; Range $= \mathbb{R}^- \cup \{0\}$

5 a



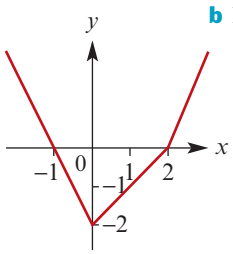
b $g_1(x) = x^2 + 2,\ x \geq 0$
$g_2(x) = x^2 + 2,\ x < 0$

6 a Domain $= \mathbb{R}$ Range $= \mathbb{R}$
b Domain $= [0, \infty)$ Range $= [0, \infty)$
c Domain $= \mathbb{R}$ Range $= [-2, \infty)$
d Domain $= [-4, 4]$ Range $= [0, 4]$
e Domain $= \mathbb{R} \setminus \{0\}$ Range $= \mathbb{R} \setminus \{0\}$
f Domain $= \mathbb{R}$ Range $= (-\infty, 4]$
g Domain $= [3, \infty)$ Range $= [0, \infty)$

7 a Domain $= \mathbb{R}$ Range $= \mathbb{R}$
b Domain $= \mathbb{R}$ Range $= [-2, \infty)$
c Domain $= [-3, 3]$ Range $= [0, 3]$
d Domain $= \mathbb{R} \setminus \{1\}$ Range $= \mathbb{R} \setminus \{0\}$

8 a $\mathbb{R} \setminus \{3\}$ **b** $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$
c $\mathbb{R}$ **d** $[4, 11]$ **e** $\mathbb{R} \setminus \{-1\}$
f $(-\infty, -1] \cup [2, \infty)$ **g** $\mathbb{R} \setminus \{-1, 2\}$
h $(-\infty, -2) \cup [1, \infty)$ **i** $\left[0, \frac{1}{3}\right]$
j $[-5, 5]$ **k** $[3, 12]$

9 a

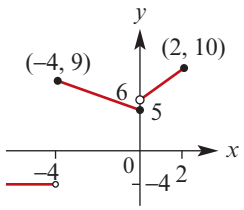


b Range $= [-2, \infty)$

10 Domain $= (-3, 0] \cup [1, 3)$; Range $= [-2, 3)$

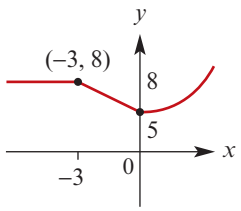
11 Domain $= [-5, 4]$; Range $= [-4, 0) \cup [2, 5]$

12 a



b Domain $= (-\infty, 2]$; Range $= [5, 10] \cup \{-4\}$

13 a



b Range $= [5, \infty)$

14 a $f(-4) = -8$ **b** $f(0) = 0$ **c** $f(4) = \frac{1}{4}$

d $f(a+3) = \begin{cases} \frac{1}{a+3}, & a > 0 \\ 2(a+3), & a \leq 0 \end{cases}$

e $f(2a) = \begin{cases} \frac{1}{2a}, & a > \frac{3}{2} \\ 4a, & a \leq \frac{3}{2} \end{cases}$

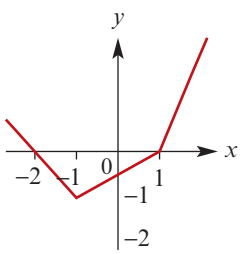
f $f(a-3) = \begin{cases} \frac{1}{a-3}, & a > 6 \\ 2(a-3), & a \leq 6 \end{cases}$

15 a $f(0) = 4$ **b** $f(3) = \sqrt{2}$ **c** $f(8) = \sqrt{7}$

d $f(a+1) = \begin{cases} \sqrt{a}, & a \geq 0 \\ 4, & a < 0 \end{cases}$

e $f(a-1) = \begin{cases} \sqrt{a-2}, & a \geq 2 \\ 4, & a < 2 \end{cases}$

16



17 $y = \begin{cases} -x - 4, & x < -2 \\ \frac{1}{2}x - 1, & -2 \leq x \leq 3 \\ -\frac{1}{2}x + 2, & x > 3 \end{cases}$

18 a Even **b** Odd **c** Neither
d Even **e** Odd **f** Neither

19 a Even **b** Even **c** Odd
d Odd **e** Neither **f** Even
g Neither **h** Neither **i** Even

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Exercise 1D

1 a $(f+g)(x) = 4x+2$ **b** $(f+g)(x) = 1$

$(fg)(x) = 3x^2 + 6x$ $(fg)(x) = x^2 - x^4$

dom $= \mathbb{R}$ dom $= (0, 2]$

c $(f+g)(x) = \dfrac{x+1}{\sqrt{x}}$

$(fg)(x) = 1$

dom $= [1, \infty)$

d $(f+g)(x) = x^2 + \sqrt{4-x}$

$(fg)(x) = x^2\sqrt{4-x}$

dom $= [0, 4]$

2 a i Even **ii** Odd **iii** Even **iv** Odd

b $(f+h)(x) = x^2 + 1 + \dfrac{1}{x^2}$, even;

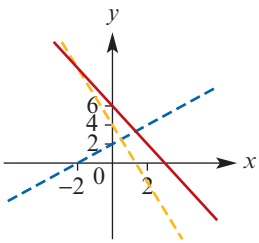
$(gk)(x) = 1$, even; $(fh)(x) = 1 + \dfrac{1}{x^2}$, even;

$(f+g)(x) = x^2 + x + 1$, neither;

$(g+k)(x) = x + \dfrac{1}{x}$, odd;

$(fg)(x) = x^3 + x$, odd

3



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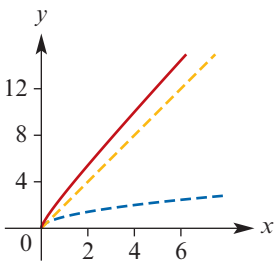

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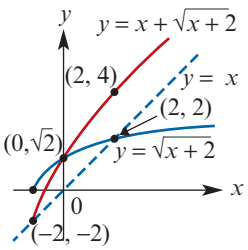

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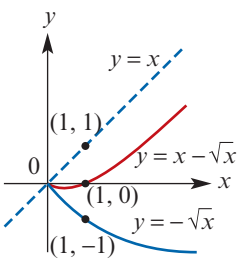

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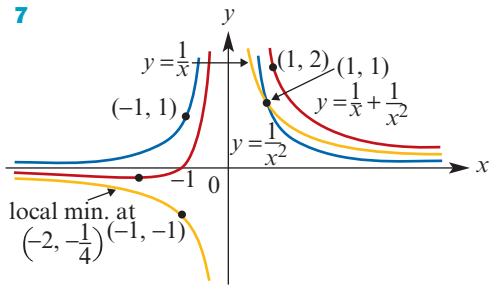

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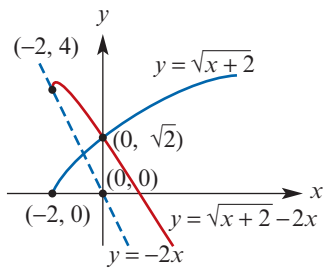

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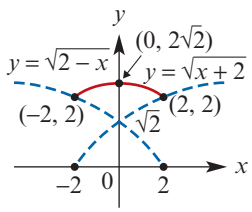

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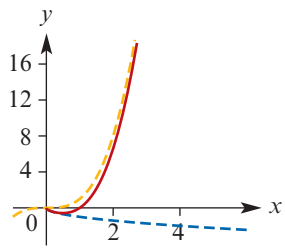

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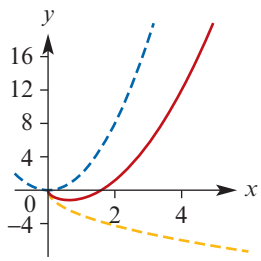

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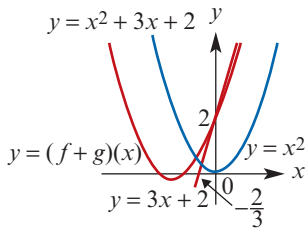

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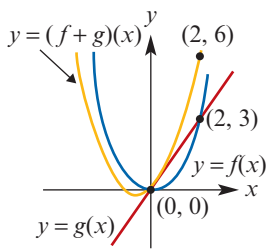
12 a



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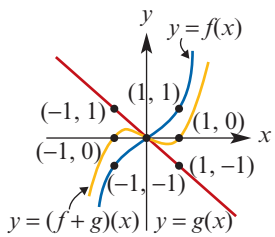

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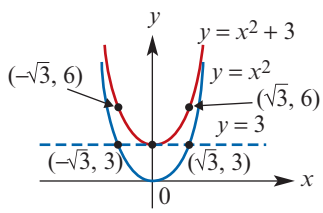

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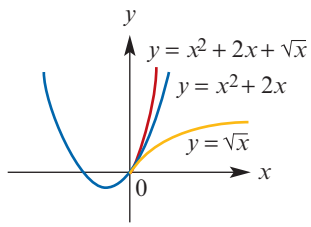

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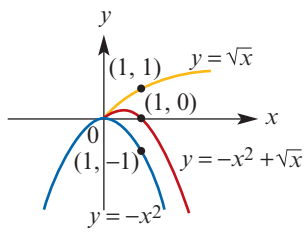


Exercise 1E

1 a $f(g(x)) = 4x - 1$, $g(f(x)) = 4x - 2$
b $f(g(x)) = 8x + 5$, $g(f(x)) = 8x + 3$
c $f(g(x)) = 4x - 7$, $g(f(x)) = 4x - 5$
d $f(g(x)) = 2x^2 - 1$, $g(f(x)) = (2x - 1)^2$
e $f(g(x)) = 2(x - 5)^2 + 1$, $g(f(x)) = 2x^2 - 4$
f $f(g(x)) = 2x^2 + 1$, $g(f(x)) = (2x + 1)^2$

2 a $f(h(x)) = 6x + 3$ **b** $h(f(x)) = 6x - 1$
c 15 **d** 11 **e** 21 **f** −7 **g** 3

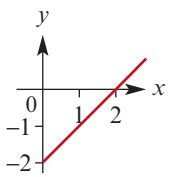
3 a $9x^2 + 12x + 3$ **b** $3x^2 + 6x + 1$
c 120 **d** 46 **e** 3 **f** 1

4 a $h(g(x)) = \dfrac{1}{(3x + 2)^2}$, $\text{dom}(h \circ g) = \mathbb{R}^+$
b $g(h(x)) = \dfrac{3}{x^2} + 2$, $\text{dom}(g \circ h) = \mathbb{R} \setminus \{0\}$
c $\dfrac{1}{25}$ **d** 5

5 a $\text{ran } f = [-4, \infty)$, $\text{ran } g = [0, \infty)$
b $f \circ g(x) = x - 4$, $\text{ran}(f \circ g) = [-4, \infty)$
c $\text{ran } f \not\subseteq \text{dom } g$

6 a $f \circ g(x) = x$, $\text{dom} = \mathbb{R} \setminus \{\frac{1}{2}\}$, $\text{ran} = \mathbb{R} \setminus \{\frac{1}{2}\}$
b $g \circ f(x) = x$, $\text{dom} = \mathbb{R} \setminus \{0\}$, $\text{ran} = \mathbb{R} \setminus \{0\}$

7 a $\text{ran } f = [-2, \infty) \not\subseteq \text{dom } g = \mathbb{R}^+ \cup \{0\}$
b $f \circ g(x) = x - 2$, $x \geq 0$



8 a $\text{ran } g = [-1, \infty) \not\subseteq \text{dom } f = (-\infty, 3]$
b $g^*: [-2, 2] \to \mathbb{R}$, $g^*(x) = x^2 - 1$
$f \circ g^*: [-2, 2] \to \mathbb{R}$, $f \circ g^*(x) = 4 - x^2$

9 a $\text{ran } g = \mathbb{R} \not\subseteq \text{dom } f = \mathbb{R}^+$
b $g_1: (-\infty, 3) \to \mathbb{R}$, $g_1(x) = 3 - x$

10

	Domain	Range
f	$\mathbb{R}$	$[0, \infty)$
g	$(-\infty, 3]$	$[0, \infty)$

a $\text{ran } g \subseteq \text{dom } f$, so $f \circ g$ exists
b $\text{ran } f \not\subseteq \text{dom } g$, so $g \circ f$ does not exist

11 a $S = [-2, 2]$
b $\text{ran } f = [0, 2]$, $\text{ran } g = [1, \infty)$
c $\text{ran } f \subseteq \text{dom } g$, so $g \circ f$ is defined
$\text{ran } g \not\subseteq \text{dom } f$, so $f \circ g$ is not defined

12 $a \in [2, 3]$

Exercise 1F

1 a $f^{-1}(x) = \dfrac{x - 3}{2}$ **b** $f^{-1}(x) = \dfrac{4 - x}{3}$
c $f^{-1}(x) = \dfrac{x - 3}{4}$

2 a $f^{-1}(x) = x + 4$ **b** $f^{-1}(x) = \dfrac{x}{2}$
c $f^{-1}(x) = \dfrac{4x}{3}$ **d** $f^{-1}(x) = \dfrac{4x + 2}{3}$

3 a $f^{-1}(x) = \dfrac{1}{2}(x + 4)$
$\text{dom} = [-8, 8]$
$\text{ran} = [-2, 6]$

b $g^{-1}(x) = 9 - \dfrac{1}{x}$
$\text{dom} = (-\infty, 0)$
$\text{ran} = (9, \infty)$

c $h^{-1}(x) = \sqrt{x - 2}$
$\text{dom} = [2, \infty)$
$\text{ran} = \mathbb{R}^+ \cup \{0\}$

d $f^{-1}(x) = \dfrac{1}{5}(x + 2)$
$\text{dom} = [-17, 28]$
$\text{ran} = [-3, 6]$

e $g^{-1}(x) = \sqrt{x + 1}$
$\text{dom} = (0, \infty)$
$\text{ran} = (1, \infty)$

f $h^{-1}(x) = x^2$
$\text{dom} = (0, \infty)$
$\text{ran} = (0, \infty)$

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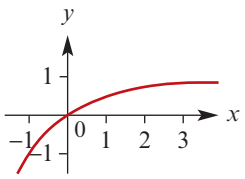
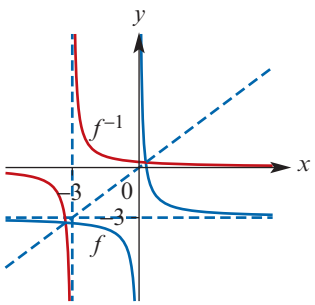
4 a $g^{-1}(x) = \sqrt{x+1} - 1$
$\text{dom}\, g^{-1} = [-1, \infty)$, $\text{ran}\, g^{-1} = [-1, \infty)$

b



5 $f^{-1} : \mathbb{R} \setminus \{-3\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{1}{x+3}$



Intersection points: $\left(\dfrac{-3+\sqrt{13}}{2}, \dfrac{-3+\sqrt{13}}{2}\right)$
$\left(\dfrac{-3-\sqrt{13}}{2}, \dfrac{-3-\sqrt{13}}{2}\right)$

6 $f^{-1}(2) = \dfrac{1}{2}$, $\text{dom}\, f^{-1} = [-3, 3]$

7 a $f^{-1}(x) = \dfrac{x}{2}$,
$\text{dom}\, f^{-1} = [-2, 6]$, $\text{ran}\, f^{-1} = [-1, 3]$

b $f^{-1}(x) = \sqrt{\dfrac{x+4}{2}}$,
$\text{dom}\, f^{-1} = [-4, \infty)$, $\text{ran}\, f^{-1} = [0, \infty)$

c $\{(6, 1), (4, 2), (8, 3), (11, 5)\}$,
$\text{dom} = \{6, 4, 8, 11\}$, $\text{ran} = \{1, 2, 3, 5\}$

d $h^{-1}(x) = -x^2$, $\text{dom}\, h^{-1} = \mathbb{R}^+$, $\text{ran}\, h^{-1} = \mathbb{R}^-$

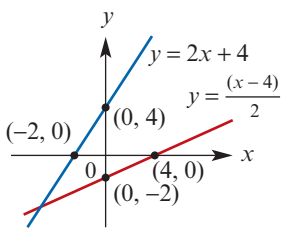
e $f^{-1}(x) = \sqrt[3]{x-1}$, $\text{dom}\, f^{-1} = \mathbb{R}$, $\text{ran}\, f^{-1} = \mathbb{R}$

f $g^{-1}(x) = -1 + \sqrt{x}$,
$\text{dom}\, g^{-1} = (0, 16)$, $\text{ran}\, g^{-1} = (-1, 3)$

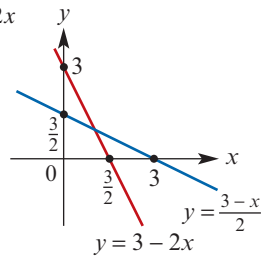
g $g^{-1}(x) = x^2 + 1$,
$\text{dom}\, g^{-1} = \mathbb{R}^+ \cup \{0\}$, $\text{ran}\, g^{-1} = [1, \infty)$

h $h^{-1}(x) = \sqrt{4 - x^2}$,
$\text{dom}\, h^{-1} = [0, 2]$, $\text{ran}\, h^{-1} = [0, 2]$

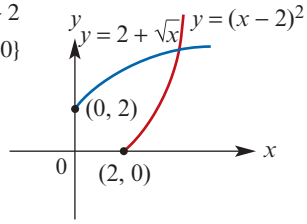
8 a $y = \dfrac{x-4}{2}$
$\text{dom} = \mathbb{R}$
$\text{ran} = \mathbb{R}$



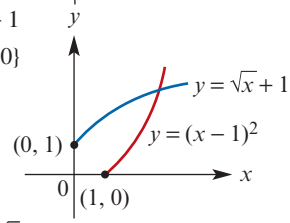
b $f^{-1}(x) = 3 - 2x$
$\text{dom} = \mathbb{R}$
$\text{ran} = \mathbb{R}$



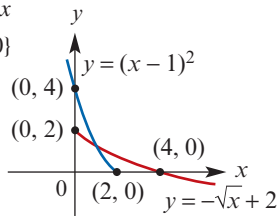
c
$f^{-1}(x) = \sqrt{x} + 2$
$\text{dom} = \mathbb{R}^+ \cup \{0\}$
$\text{ran} = [2, \infty)$



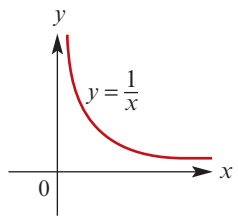
d $f^{-1}(x) = \sqrt{x} + 1$
$\text{dom} = \mathbb{R}^+ \cup \{0\}$
$\text{ran} = [1, \infty)$



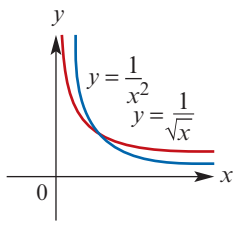
e $f^{-1}(x) = 2 - \sqrt{x}$
$\text{dom} = \mathbb{R}^+ \cup \{0\}$
$\text{ran} = (-\infty, 2]$



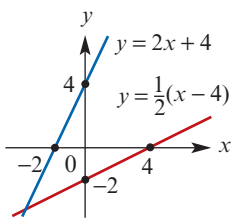
f $f^{-1}(x) = \dfrac{1}{x}$
$\text{dom} = \mathbb{R}^+$
$\text{ran} = \mathbb{R}^+$



g $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$
$\text{dom} = \mathbb{R}^+$
$\text{ran} = \mathbb{R}^+$



h $h^{-1}(x) = 2x + 4$
$\text{dom} = \mathbb{R}$
$\text{ran} = \mathbb{R}$



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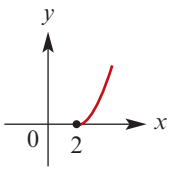
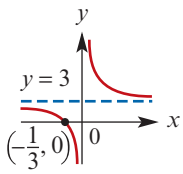
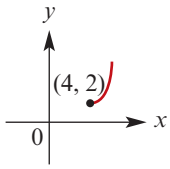
9 a $f^{-1}: [2, \infty) \to \mathbb{R}$, $f^{-1}(x) = (x-2)^2$



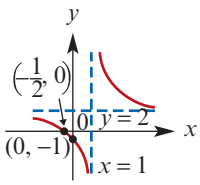
b $f^{-1}: \mathbb{R} \setminus \{0\} \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{x} + 3$



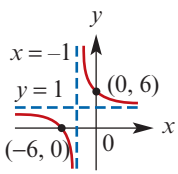
c $f^{-1}: [4, \infty) \to \mathbb{R}$, $f^{-1}(x) = (x-4)^2 + 2$



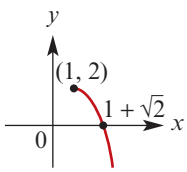
d $f^{-1}: \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f^{-1}(x) = \frac{3}{x-1} + 2$



e $f^{-1}: \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $f^{-1}(x) = \frac{5}{x+1} + 1$



f $f^{-1}: [1, \infty) \to \mathbb{R}$, $f^{-1}(x) = 2 - (x-1)^2$

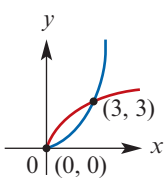


10 a $f^{-1}: \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f^{-1}(x) = \frac{x+1}{x-1}$

b $f^{-1}: \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$, $f^{-1}(x) = x^2 + 2$

c $f^{-1}: \mathbb{R} \setminus \{\frac{2}{3}\} \to \mathbb{R}$ $f^{-1}(x) = \frac{2x+3}{3x-2}$

11 a



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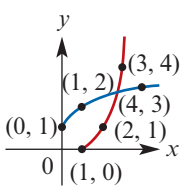

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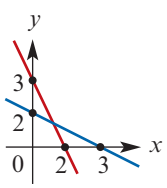

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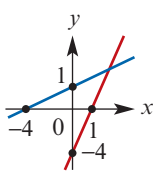
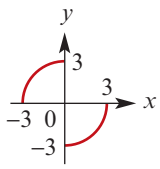


e



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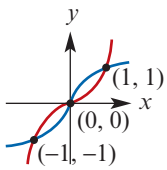
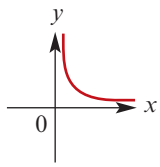
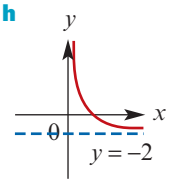


g



h



12 a C **b** B **c** D **d** A

13 a $A = (-\infty, 3]$

b $b = 0$, $g^{-1}(x) = \sqrt{1-x}$, $x \in [-3, 1]$

14 $b = -2$, $g^{-1}(x) = -2 + \sqrt{x+4}$

15 $a = 3$, $f^{-1}(x) = 3 - \sqrt{x+9}$

16 a $y = \frac{3}{x}$ domain $= \mathbb{R} \setminus \{0\}$

b $y = (x+4)^3 - 2$ domain $= \mathbb{R}$

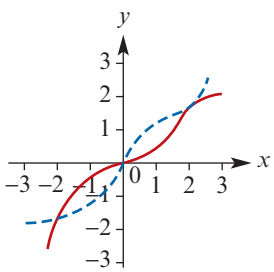
c $y = (2-x)^2$ domain $= (-\infty, 2]$

d $y = \frac{3}{x-1}$ domain $= \mathbb{R} \setminus \{1\}$

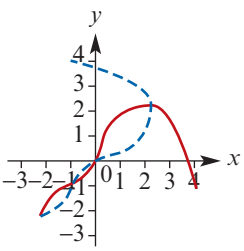
e $y = \sqrt[3]{\frac{2}{5-x}} + 6$ domain $= \mathbb{R} \setminus \{5\}$

f $y = \frac{1}{(x-2)^{\frac{4}{3}}} + 1$ domain $= (2, \infty)$

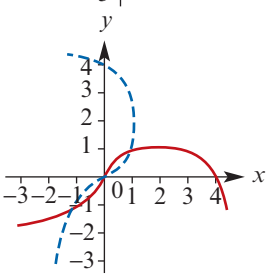
17 a Inverse is a function



b Inverse is not a function



c Inverse is not a function



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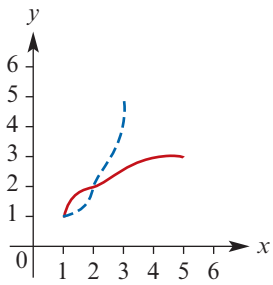
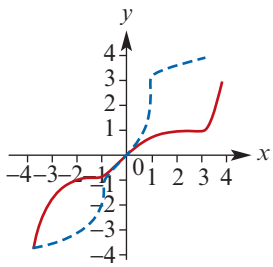
d Inverse is a function

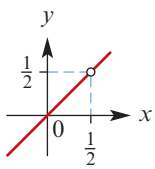


e Inverse is not a function



18 a dom $f = \mathbb{R} \setminus \{\frac{1}{2}\}$, ran $f = \mathbb{R} \setminus \{\frac{1}{2}\}$, $f \circ f$ is defined as ran $f \subseteq$ dom f

b $f \circ f(x) = x$, $x \in \mathbb{R} \setminus \{\frac{1}{2}\}$



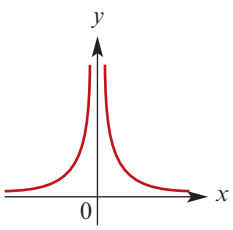
c $f^{-1} : \mathbb{R} \setminus \{\frac{1}{2}\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{x+3}{2x-1}$

Exercise 1G

1 a Maximal domain $= \mathbb{R} \setminus \{0\}$; Range $= \mathbb{R}^+$

b **i** $\frac{1}{16}$ **ii** $\frac{1}{16}$ **iii** 16 **iv** 16

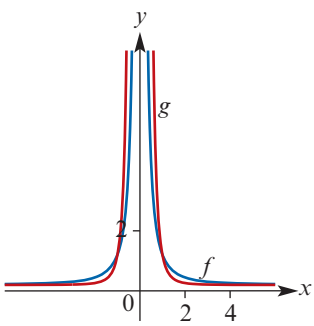
c



2 a Odd **b** Even **c** Odd
d Odd **e** Even **f** Odd

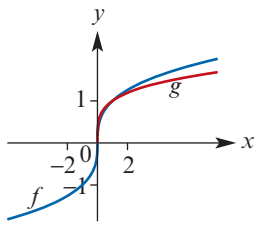
3 a $x = 1$ or $x = -1$

b



4 a $x = 1$ or $x = 0$

b



5 a $f^{-1} : \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = x^{\frac{1}{7}}$

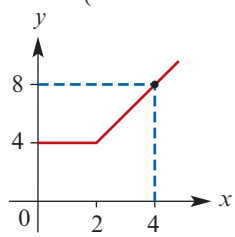
b $f^{-1} : [0, \infty) \to \mathbb{R}$, $f^{-1}(x) = -x^{\frac{1}{6}}$

c $f^{-1} : \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{3}x^{\frac{1}{3}}$

d $f^{-1} : (16, \infty) \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{2}x^{\frac{1}{4}}$

Exercise 1H

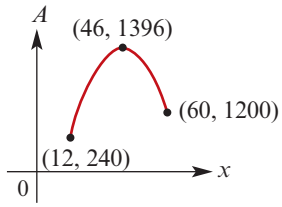
1 $f(x) = \begin{cases} 4 & \text{if } 0 \le x \le 2 \\ 2x & \text{if } x > 2 \end{cases}$



2 $V(x) = 4x(10 - x)(18 - x)$, domain $= [0, 10]$

3 a $A(x) = -x^2 + 92x - 720$ **b** $12 \le x \le 60$

c



d Maximum area 1396 m^2 occurs when $x = 46$ and $y = 34$

4 a **i** $S = 2x^2 + 6xh$ **ii** $S = 2x^2 + \dfrac{3V}{x}$

b Maximal domain $= (0, \infty)$

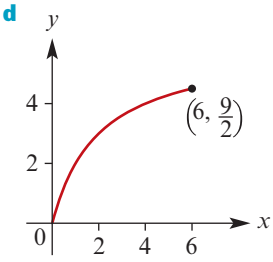
c Maximum value of $S = 1508$ m^2

5 Area $= x\sqrt{4a^2 - x^2}$, domain $= [0, 2a]$

6 a $A = \dfrac{6a}{a+2}$

b Domain $= (0, 6]$; Range $= \left(0, \frac{9}{2}\right]$

c $\frac{9}{2}$ **d**



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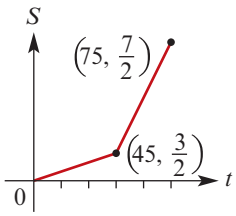
7 a $a = \frac{1}{30}$, $b = \frac{1}{15}$, $c = 45$, $d = -\frac{3}{2}$, $e = 75$

b



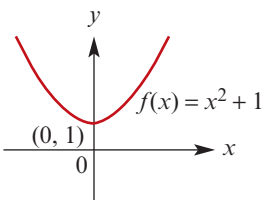
c Range $= \left[0, \frac{7}{2}\right]$

Chapter 1 review

Technology-free questions

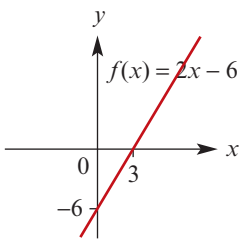
1 a Domain $= \mathbb{R}$

Range $= [1, \infty)$



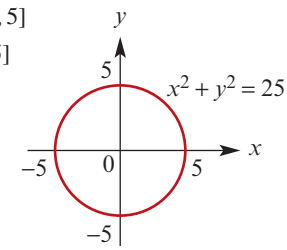
b Domain $= \mathbb{R}$

Range $= \mathbb{R}$



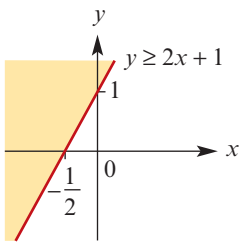
c Domain $= [-5, 5]$

Range $= [-5, 5]$



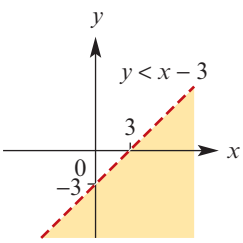
d Domain $= \mathbb{R}$

Range $= \mathbb{R}$

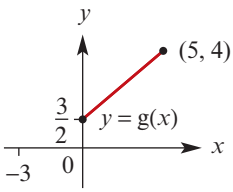


e Domain $= \mathbb{R}$

Range $= \mathbb{R}$



2 a



b ran $g = \left[\frac{3}{2}, 4\right]$

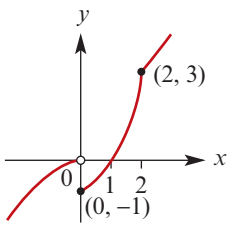
c $g^{-1}: \left[\frac{3}{2}, 4\right] \to \mathbb{R}$, $g^{-1}(x) = 2x - 3$

dom $g^{-1} = \left[\frac{3}{2}, 4\right]$, ran $g^{-1} = [0, 5]$

d $\{5\}$ **e** $\left\{\frac{7}{2}\right\}$

3 a $\left\{\frac{1}{5}\right\}$ **b** $\{11\}$ **c** $\left\{-\frac{1}{10}\right\}$

4



5 a $\mathbb{R} \setminus \{3\}$ **b** $\mathbb{R} \setminus [-\sqrt{5}, \sqrt{5}]$ **c** $\mathbb{R} \setminus \{1, -2\}$
d $[-5, 5]$ **e** $[5, 15]$ **f** $\mathbb{R} \setminus \{2\}$

6 $(f + g)(x) = x^2 + 5x + 1$,
$(fg)(x) = (x - 3)(x + 2)^2$

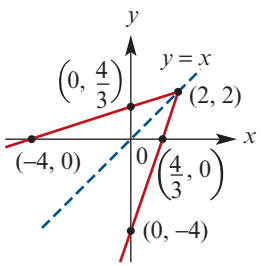
7 $(f + g): [1, 5] \to \mathbb{R}$, $(f + g)(x) = x^2 + 1$
$(fg): [1, 5] \to \mathbb{R}$, $(fg)(x) = 2x(x - 1)^2$

8 $f^{-1}: [8, \infty) \to \mathbb{R}$, $f^{-1}(x) = \sqrt{x + 1}$

9 a $(f + g)(x) = -x^2 + 2x + 3$

b $(fg)(x) = -x^2(2x + 3)$ **c** $\{-1, 3\}$

10



11 a $f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{2}x^{\frac{1}{3}}$

b $f^{-1}: (-\infty, 0] \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{2}x^{\frac{1}{5}}$

c $f^{-1}: [0, \infty) \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{2}x^{\frac{1}{6}}$

d $f^{-1}: (10\,000, \infty) \to \mathbb{R}$, $f^{-1}(x) = \frac{1}{10}x^{\frac{1}{4}}$

12 a $f \circ g(x) = -2x^3 + 3$

b $g \circ f(x) = -(2x + 3)^3$

c $g \circ g(x) = x^9$ **d** $f \circ f(x) = 4x + 9$

e $f \circ (f + g)(x) = -2x^3 + 4x + 9$

f $f \circ (f - g)(x) = 2x^3 + 4x + 9$

g $f \circ (f \cdot g)(x) = -4x^4 - 6x^3 + 3$

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13 $x \geq -1$ or $x \leq -9$

14 $h^{-1}(x) = \left(\dfrac{x-64}{2}\right)^{\frac{1}{5}}$

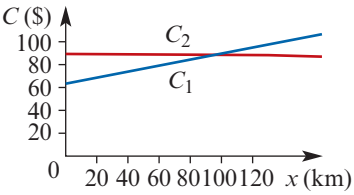
Multiple-choice questions

1 E	**2** B	**3** E	**4** C	**5** E	**6** C
7 D	**8** B	**9** B	**10** C	**11** B	**12** E
13 C	**14** C	**15** A	**16** B	**17** A	**18** D
19 B	**20** B	**21** C	**22** A	**23** D	

Extended-response questions

1 a $C_1 = 64 + 0.25x$, $C_2 = 89$

b

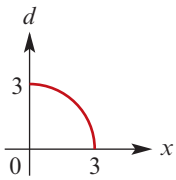


c $x > 100$ km

2 a $S = 6x^2$ **b** $S = 6V^{\frac{2}{3}}$

3 a $A = \dfrac{\sqrt{3}s^2}{4}$ **b** $A = \dfrac{\sqrt{3}h^2}{3}$

4 a $d(x) = \sqrt{9 - x^2}$ **b** dom $= [0, 3]$, ran $= [0, 3]$



5 $S(x) = \dfrac{160x}{x + 80}$

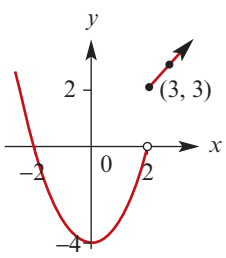
6 a $V_1 : (0, 12) \to \mathbb{R}$, $V_1(h) = \pi h\left(36 - \dfrac{h^2}{4}\right)$

b $V_2 : (0, 6) \to \mathbb{R}$, $V_2(r) = 2\pi r^2\sqrt{36 - r^2}$

7 a ran $f = \mathbb{R} =$ dom g, and so $g \circ f$ exists;
$g \circ f(x) = 2 + (1 + x)^3$

b $g \circ f$ is one-to-one and so $(g \circ f)^{-1}$ exists;
$(g \circ f)^{-1}(10) = 1$

8 a



b i -3 **ii** 3

c $S = (-\infty, 0]$

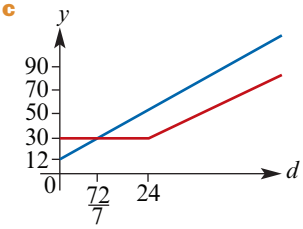
d $f(h(x)) = \begin{cases} 4x^2 - 4 & \text{if } x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$

$h(f(x)) = \begin{cases} 2x^2 - 8 & \text{if } x < 2 \\ 2x & \text{if } x \geq 2 \end{cases}$

9 $A(t) = \begin{cases} \dfrac{3t^2}{2}, & 0 < t \leq 1 \\ 3t - \dfrac{3}{2}, & t > 1 \end{cases}$

Domain $= (0, \infty)$; Range $= (0, \infty)$

10 c



d i \$41.75 **ii** \$30

e Thrifty Taxi

f Greater than $\dfrac{72}{7}$ km

11 a $f^{-1} : \mathbb{R} \setminus \left\{\dfrac{a}{c}\right\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{b - dx}{cx - a}$

b i $f^{-1} : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{2 - x}{3x - 3}$

ii $f^{-1} : \mathbb{R} \setminus \left\{\dfrac{3}{2}\right\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{3x + 2}{2x - 3}$

iii $f^{-1} : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{1 - x}{x + 1}$

iv $f^{-1} : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $f^{-1}(x) = \dfrac{1 - x}{x + 1}$

c For $a, b, c, d \in \mathbb{R} \setminus \{0\}$, $f = f^{-1}$ when $a = -d$

12 a i $YB = r$ **ii** $ZB = r$

iii $AZ = x - r$ **iv** $CY = 3 - r$

b $r = \dfrac{x + 3 - \sqrt{x^2 + 9}}{2}$

c i $r = 1$ **ii** $x = 1.25$

13 b $f(x) = \dfrac{q}{x}$

c i $f^{-1}(x) = \dfrac{3x + 8}{x - 3} = f(x)$

ii $x = 3 \pm \sqrt{17}$

14 a i $f(2) = 3$, $f(f(2)) = 2$, $f(f(f(2))) = 3$

ii $f(f(x)) = x$

b $f(f(x)) = \dfrac{-x - 3}{x - 1}$,
$f(f(f(x))) = x$, i.e. $f(f(x)) = f^{-1}(x)$

Chapter 2

Exercise 2A

1 a 10 **b** 1 **c** 4 **d** 28 **e** $8\frac{1}{2}$

f $\dfrac{17}{9}$ **g** $\dfrac{7}{5}$ **h** 21 **i** 2 **j** $\dfrac{7}{2}$

2 a $x = 12, y = 8$ **b** $x = 5, y = -8$

c $x = 3, y = 1$ **d** $x = 2, y = 1$

e $x = 17, y = -19$ **f** $x = 10, y = 6$

3 Width $= 6$ cm, length $= 10$ cm

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4 John scored 4, David 8

5 a $w = 20n + 800$ **b** \$1400 **c** 41 units

6 a $V = 15t + 250$ **b** 1150 litres
c 5 hours, 16 minutes and 40 seconds

7 a $V = 10\,000 - 10t$ **b** 9400 litres
c 16 hours and 40 minutes

8 80 km

9 96 km

10 a $C = 25t + 100$
b i \$150 **ii** \$162.50
c i 11 hours **ii** 12 hours

Exercise 2B

1 a $x = \frac{m-n}{a}$ **b** $x = \frac{b}{b-a}$ **c** $x = -\frac{bc}{a}$
d $x = \frac{5}{p-q}$ **e** $x = \frac{m+n}{n-m}$ **f** $x = \frac{ab}{1-b}$
g $x = 3a$ **h** $x = -mn$ **i** $x = \frac{a^2-b^2}{2ab}$
j $x = \frac{p-q}{p+q}$ **k** $x = \frac{3ab}{b-a}$ **l** $x = \frac{1}{3a-b}$
m $x = \frac{p^2+p^2t+t^2}{q(p+t)}$ **n** $x = -\frac{5a}{3}$

2 a $x = \frac{d-bc}{1-ab}, y = \frac{c-ad}{1-ab}$
b $x = \frac{a^2+ab+b^2}{a+b}, y = \frac{ab}{a+b}$
c $x = \frac{t+s}{2a}, y = \frac{t-s}{2b}$
d $x = a+b, y = a-b$
e $x = c, y = -a$
f $x = a+1, y = a-1$

3 a $s = a(2a+1)$ **b** $s = \frac{2a^2}{1-a}$
c $s = \frac{a^2+a+1}{a(a+1)}$ **d** $s = \frac{a}{(a-1)^2}$
e $s = 3a^3(3a+1)$ **f** $s = \frac{3a}{a+2}$
g $s = 2a^2 - 1 + \frac{1}{a^2}$ **h** $s = \frac{5a^2}{a^2+6}$

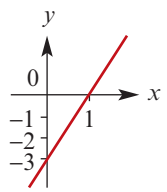
Exercise 2C

1 a $\sqrt{205}$ **b** $\left(1, -\frac{1}{2}\right)$ **c** $-\frac{13}{6}$
d $13x + 6y = 10$ **e** $13x + 6y = 43$
f $13y - 6x = -\frac{25}{2}$

2 a $(3, 7\frac{1}{2})$ **b** $\left(-\frac{5}{2}, -2\right)$ **c** $\left(\frac{3}{2}, \frac{1}{2}\right)$

3 a $(4, 7)$ **b** $(5, -2)$ **c** $(2, 19)$ **d** $(-2, -9)$

4 a



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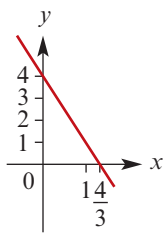

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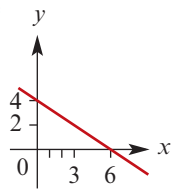

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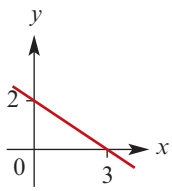

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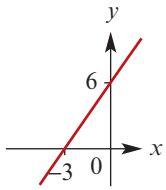

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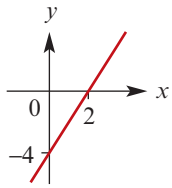
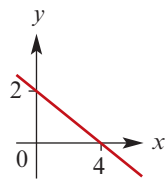
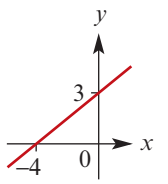


5 a $2x - 6 = y$ **b** $-3x - 5 = y$
c $5 = 3y - 4x$ **d** $y = 2x + 1$

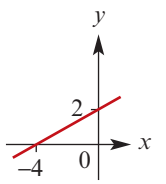
6 a $\frac{y}{2} - \frac{x}{3} = 1$ **b** $\frac{x}{4} + \frac{y}{6} = 1$
c $-\frac{x}{4} - \frac{y}{3} = 1$ **d** $\frac{x}{6} - \frac{y}{2} = 1$

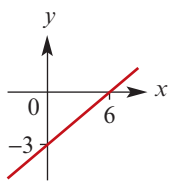
7 a $\frac{x}{4} + \frac{y}{2} = 1$ **b** $\frac{y}{3} - \frac{x}{4} = 1$





c $\frac{y}{2} - \frac{x}{4} = 1$ **d** $\frac{x}{6} - \frac{y}{3} = 1$





8 $C = \frac{11}{200}n + 2$, \$57

9 a $C = 5n + 175$ **b** Yes **c** \$175

10 a $\sqrt{5} \approx 2.236$ **b** $\sqrt{2} \approx 1.414$
c $\sqrt{29} \approx 5.385$ **d** $2\sqrt{82} \approx 18.111$
e $\sqrt{20} \approx 4.472$ **f** 5

11 a i $y = 2x + 4$ **ii** $2y + x = 13$
b i $y = -2x + 7$ **ii** $2y - x = 4$

12 $y = 2x - 3$ **13** $y = -5$ or $y = 3$
14 $y = 12$ or $y = 0$ **15** $y = 32$ or $y = -16$

16 a i $5y + 2x = 13$ **ii** $5y + 4x = 11$
b i $2y - 5x = 11$ **ii** $4y - 5x = 17$

17 a 32.01° **b** 153.43° **c** 56.31° **d** 120.96°

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18 45° **19** $a = -12$ or $a = 8$

20 **a** $y = 3x - 6$ **b** $(2, 0)$

21 $k = 5$ and $h = 4$, or $k = -2$ and $h = -3$

22 **a** $a + 2$ **b** $\frac{4}{5}$

23 **a** $m = \frac{1}{2}$ **b** $(5, 7)$

c $AB = \sqrt{13}$, $AC = 2\sqrt{13}$

24 **a** $3y - x = 22$ **b** $(14, 12)$

c $(16, 6)$ **d** 80 square units

25 **a** $(2, 3)$

b $y + 5x = 13$

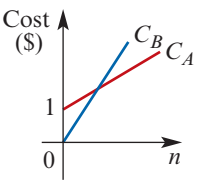
c **i** $2y = 3x - 13$ **ii** $(3, -2)$ **iii** $(1, 8)$

Exercise 2D

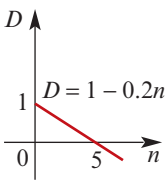
1 **a** $C_A = 0.4n + 1$

$C_B = 0.6n$

b



c $n = 5$ **d**



The difference in charges against kilometres travelled

2 **a** $4 - T$

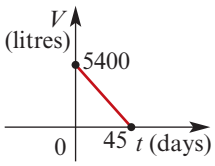
b **i** $90T$ **ii** $70(4 - T)$

c **i** $T = 1$

ii 90 km freeway; 210 km country roads

3 **a** $L = -120t + 5400$ **b** 5400 litres

c



d $[0, 45]$

e 45 days **f** 120 litres per day

4 **a** $y = \frac{9}{4}x$ **b** 24 622 m

c $y = -\frac{27}{26}x + \frac{855}{26}$ **d** $\frac{5393}{108}$

5 **a** **i** -4 **ii** $\frac{4}{9}$

b **i** $y = \frac{4}{9}x + \frac{10}{3}$ **ii** $y = -4x + 30$

c AC: $y = x$; BD: $y = 4$

d $(4, 4)$

6 **a** $M(7, 5)$, $N(11, 5)$

b **i** $y = \frac{5}{2}(x - 5)$ **ii** $y = -\frac{5}{2}(x - 13)$ **iii** $y = 5$

c $y - 5 = -\frac{2}{5}(x - 7)$ and $y - 5 = \frac{2}{5}(x - 11)$

Intersection point $\left(9, \frac{21}{5}\right)$

Exercise 2E

1 **a** $x = 4$, $y = -3$ **b** $x = \frac{-3}{2}$, $y = \frac{1}{2}$

c $x = \frac{51}{38}$, $y = \frac{-31}{38}$ **d** $x = \frac{37}{10}$, $y = \frac{7}{5}$

2 **a** one solution **b** infinitely many solutions

c no solutions

3 Their graphs are parallel straight lines that do not coincide

4 $x = t + 6$, $y = t$, where $t \in \mathbb{R}$

5 **a** $m = -5$ **b** $m = 3$

6 $m = 9$

7 **a** **i** $m = -2$ **ii** $m = 4$

b $x = \frac{4}{m + 2}$, $y = \frac{2(m + 4)}{m + 2}$

8 **a** $x = 2$, $y = 0$, $k \neq \frac{-3}{2}$ **b** $k = \frac{-3}{2}$

9 **a** $b \in \mathbb{R} \setminus \{10\}$ **b** $b = 10$, $c = 8$

c $b = 10$, $c \neq 8$

Exercise 2F

1 **a** $x = 2, y = 3, z = 1$ **b** $x = -3, y = 5, z = 2$

c $x = 5, y = 0, z = 7$ **d** $x = 6, y = 5, z = 1$

2 **a** $y = 4z - 2$

b $x = 8 - 5\lambda$, $y = 4\lambda - 2$, $z = \lambda$

3 **a** $-y + 5z = 15$, $-y + 5z = 15$

b The two equations are the same

c $y = 5\lambda - 15$ **d** $x = 43 - 13\lambda$

4 **a** $x = \lambda - 1$, $y = \lambda$, $z = 5$

b $x = \lambda + 3$, $y = 3\lambda$, $z = \lambda$

c $x = \frac{14 - 3\lambda}{6}$, $y = \frac{10 - 3\lambda}{6}$, $z = \lambda$

5 $z = t$, $y = \frac{-3(t + 2)}{4}$, $x = \frac{26 - 3t}{4}$, $w = \frac{t - 2}{2}$,

where $t \in \mathbb{R}$; $w = 6$, $x = -4$, $y = -12$, $z = 14$

6 **a** $x = 1$, $y = 2$, $z = 3$

b $x = \frac{-5}{3}$, $y = \frac{-(3\lambda + 5)}{3}$, $z = \lambda$

c $z = t$, $y = -2(t - 1)$, $x = \frac{2 - 3t}{2}$

Chapter 2 review

Technology-free questions

1 **a** -8 **b** $\frac{7}{5}$ **c** 30 **d** 7

2 **a** $x = -2$, $y = 2$ **b** $x = -44$, $y = -39$

3 **a** $\frac{n + m}{b}$ **b** $\frac{b}{c + b}$ **c** d

d $\frac{6}{q - p}$ **e** $\frac{m + n}{m - n}$ **f** $\frac{a^2}{a - 1}$

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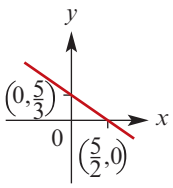
4 a



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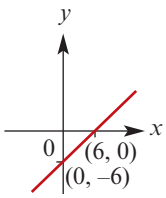

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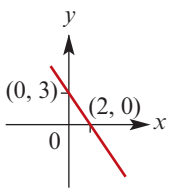


5 a $y = -2x + 5$ **b** $y = 2x + 2$

c $y = \frac{1}{2}x + \frac{1}{2}$ **d** $y = -2x + 3$

6 $\sqrt{13}$

7 $(1, 7)$

8 a $(22, 4)$ **b** $(5, -12)$

9 $y = 24$ or $y = 0$

10 a $m = -2$ **b** $m \in \mathbb{R} \setminus \{-8, -2\}$

11 a $x = \frac{3\lambda - 1}{2}$, $y = \lambda$, $z = 7$, where $\lambda \in \mathbb{R}$

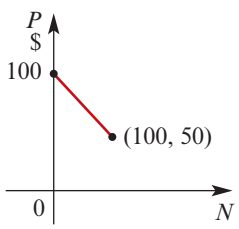
b $x = \frac{4 - \lambda}{2}$, $y = \frac{3\lambda + 8}{2}$, $z = \lambda$, where $\lambda \in \mathbb{R}$

Multiple-choice questions

1 E **2** E **3** D **4** C **5** B
6 A **7** A **8** C **9** B **10** C
11 D **12** A **13** D **14** C

Extended-response questions

1 a



b $P = -\frac{1}{2}N + 100$

c i \$56 **ii** $N = 80$

2 a $y = \frac{5}{3}x - 4$ **b** $\left(\frac{66}{7}, \frac{82}{7}\right)$

c $\frac{5}{3}$ **d** 15 **e** $\frac{629}{14}$ square units

3 a $y = \frac{4}{7}x + \frac{31}{14}$ **b** $\frac{59}{14}$ **c** $\sqrt{65}$

d $\frac{65}{28}$ square units

4 a $(1, -\frac{1}{2})$ **b** $\sqrt{269}$ **c** $y = -\frac{13}{10}x + \frac{4}{5}$

d $y = \frac{10}{13}x - \frac{33}{26}$ **e** $\left(\frac{7}{2}, -\frac{15}{4}\right)$

f $(26, -33)$

5 a 125 litres **b** $x = 291\frac{2}{3}$, $y = 208\frac{1}{3}$

Chapter 3

Exercise 3A

1 a $(-1, 3)$ **b** $(-5, 10)$ **c** $(-3, -1)$

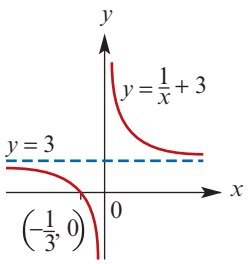
d $(-5, 7)$ **e** $(-3, 6)$

2 a $y = \frac{1}{x - 2} - 3$ **b** $y = \frac{1}{x + 2} + 3$

c $y = \frac{1}{x - \frac{1}{2}} + 4 = \frac{2}{2x - 1} + 4$

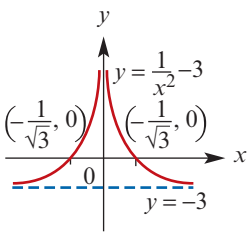
3 a Domain $= \mathbb{R} \setminus \{0\}$

Range $= \mathbb{R} \setminus \{3\}$



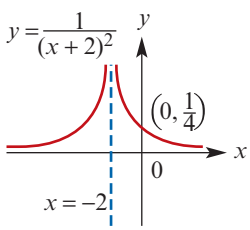
b Domain $= \mathbb{R} \setminus \{0\}$

Range $= (-3, \infty)$



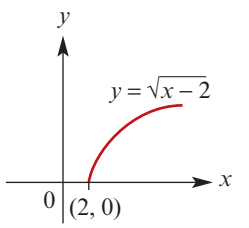
c Domain $= \mathbb{R} \setminus \{-2\}$

Range $= \mathbb{R}^+$



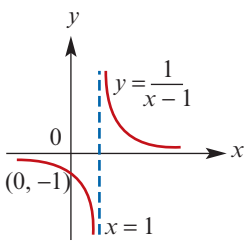
d Domain $= [2, \infty)$

Range $= \mathbb{R}^+ \cup \{0\}$



e Domain $= \mathbb{R} \setminus \{1\}$

Range $= \mathbb{R} \setminus \{0\}$



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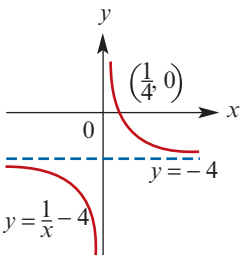
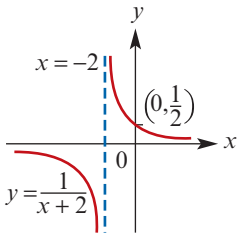
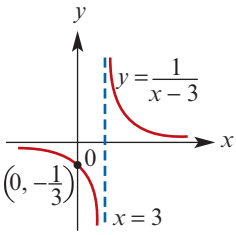
f Domain = $\mathbb{R} \setminus \{0\}$
Range = $\mathbb{R} \setminus \{-4\}$



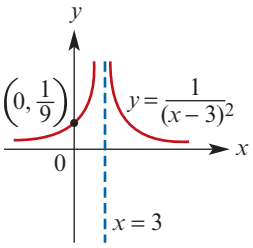
g Domain = $\mathbb{R} \setminus \{-2\}$
Range = $\mathbb{R} \setminus \{0\}$



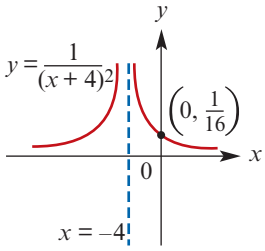
h Domain = $\mathbb{R} \setminus \{3\}$
Range = $\mathbb{R} \setminus \{0\}$



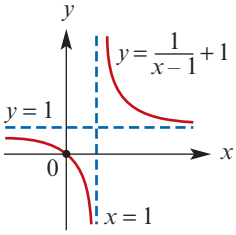
i Domain = $\mathbb{R} \setminus \{3\}$
Range = $\mathbb{R}^+$



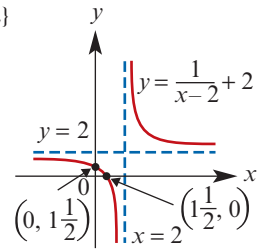
j Domain = $\mathbb{R} \setminus \{-4\}$
Range = $\mathbb{R}^+$



k Domain = $\mathbb{R} \setminus \{1\}$
Range = $\mathbb{R} \setminus \{1\}$



l Domain = $\mathbb{R} \setminus \{2\}$
Range = $\mathbb{R} \setminus \{2\}$



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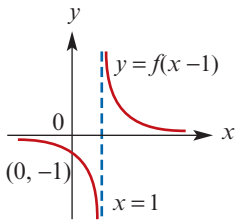

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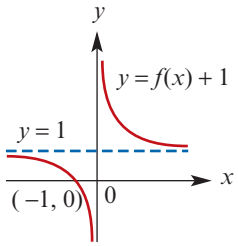

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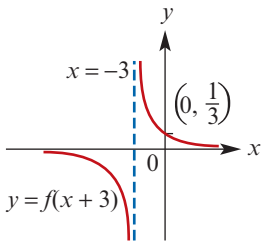

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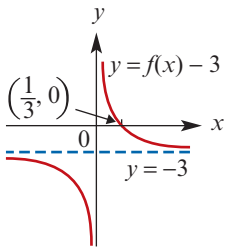

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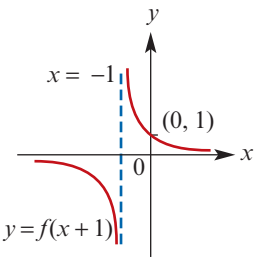

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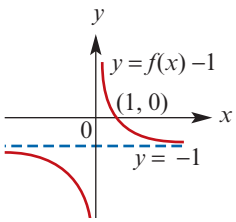
f



5 a Translation $(x, y) \to (x - 5, y)$
b Translation $(x, y) \to (x, y + 2)$
c Translation $(x, y) \to (x, y + 4)$
d Translation $(x, y) \to (x, y + 3)$
e Translation $(x, y) \to (x - 3, y)$

6 a i $y = (x - 7)^{\frac{1}{4}} + 1$ **ii** $y = (x + 2)^{\frac{1}{4}} - 6$
iii $y = (x - 2)^{\frac{1}{4}} - 3$ **iv** $y = (x + 1)^{\frac{1}{4}} + 4$
b i $y = \sqrt[3]{x - 7} + 1$ **ii** $y = \sqrt[3]{x + 2} - 6$
iii $y = \sqrt[3]{x - 2} - 3$ **iv** $y = \sqrt[3]{x + 1} + 4$
c i $y = \dfrac{1}{(x - 7)^3} + 1$ **ii** $y = \dfrac{1}{(x + 2)^3} - 6$
iii $y = \dfrac{1}{(x - 2)^3} - 3$ **iv** $y = \dfrac{1}{(x + 1)^3} + 4$
d i $y = \dfrac{1}{(x - 7)^4} + 1$ **ii** $y = \dfrac{1}{(x + 2)^4} - 6$
iii $y = \dfrac{1}{(x - 2)^4} - 3$ **iv** $y = \dfrac{1}{(x + 1)^4} + 4$

7 a $y = (x + 1)^2 + 5$ **b** $y = 2x^2$
c $y = \dfrac{1}{(x - 6)^2} + 1$ **d** $y = (x + 3)^2 + 2$
e $y = \sqrt[3]{x - 2} + 3$

8 a $(x, y) \to (x + 2, y + 3)$
b $(x, y) \to (x - 2, y - 3)$
c $(x, y) \to (x - 4, y + 2)$

Exercise 3B

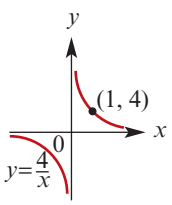
1 a $y = \dfrac{3}{x}$ **b** $y = \dfrac{3}{x}$

2 a $y = \dfrac{2}{x^2}$ **b** $y = \dfrac{4}{x^2}$

3 a $y = 2\sqrt{x}$ **b** $y = \sqrt{\dfrac{x}{2}}$

4 a $y = 2x^3$ **b** $y = \dfrac{x^3}{8}$

5 a



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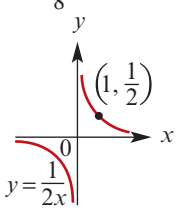

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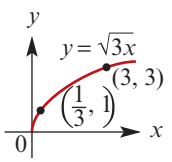

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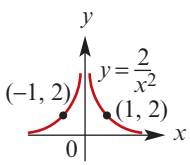

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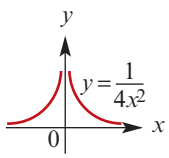

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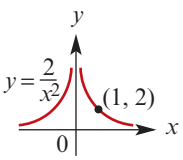

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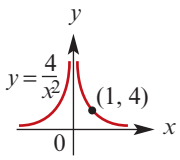

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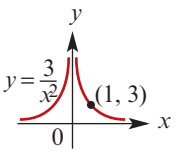

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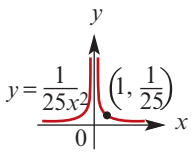

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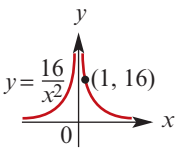

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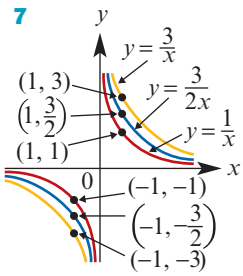

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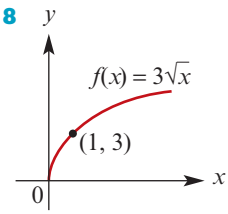

Range = $\mathbb{R}^+$

9 a $\dfrac{1}{5}$ **b** $\sqrt{5}$

10 a Dilation of factor 5 from the x-axis
b Dilation of factor 4 from the x-axis
c Dilation of factor $\dfrac{1}{5}$ from the y-axis
d Dilation of factor $\dfrac{1}{3}$ from the y-axis
e Dilation of factor 2 from the y-axis

11 a i $y = 4x^2$ **ii** $y = \dfrac{2}{3}x^2$
iii $y = 4x^2$ **iv** $y = \dfrac{1}{25}x^2$
b i $y = \dfrac{4}{x^2}$ **ii** $y = \dfrac{2}{3x^2}$
iii $y = \dfrac{1}{4x^2}$ **iv** $y = \dfrac{25}{x^2}$
c i $y = 4\sqrt[3]{x}$ **ii** $y = \dfrac{2}{3} \times \sqrt[3]{x}$
iii $y = \sqrt[3]{2x}$ **iv** $y = \sqrt[3]{\dfrac{x}{5}}$
d i $y = \dfrac{4}{x^3}$ **ii** $y = \dfrac{2}{3x^3}$
iii $y = \dfrac{1}{8x^3}$ **iv** $y = \dfrac{125}{x^3}$
e i $y = \dfrac{4}{x^4}$ **ii** $y = \dfrac{2}{3x^4}$
iii $y = \dfrac{1}{16x^4}$ **iv** $y = \dfrac{625}{x^4}$

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f i $y = 4\sqrt[4]{x}$ **ii** $y = \frac{2}{3} \times \sqrt[4]{x}$

iii $y = \sqrt[4]{2x}$ **iv** $y = \sqrt[4]{\frac{x}{5}}$

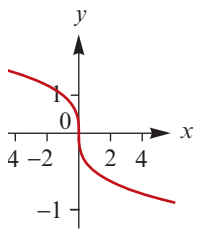
g i $y = 4x^{\frac{1}{5}}$ **ii** $y = \frac{2}{3}x^{\frac{1}{5}}$

iii $y = (2x)^{\frac{1}{5}}$ **iv** $y = \left(\frac{x}{5}\right)^{\frac{1}{5}}$

Exercise 3C

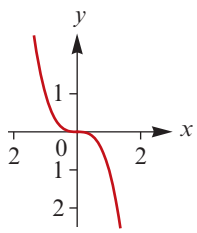
1 a $y = -(x-1)^2$ **b** $y = (x+1)^2$

2 a



Domain = $\mathbb{R}$

b



Domain = $\mathbb{R}$

3 Reflection in the y-axis

4 a i $y = -x^3$ **ii** $y = -x^3$

b i $y = -\sqrt[3]{x}$ **ii** $y = -\sqrt[3]{x}$

c i $y = \frac{-1}{x^3}$ **ii** $y = \frac{-1}{x^3}$

d i $y = \frac{-1}{x^4}$ **ii** $y = \frac{1}{x^4}$

e i $y = -x^{\frac{1}{3}}$ **ii** $y = -x^{\frac{1}{3}}$

f i $y = -x^{\frac{1}{5}}$ **ii** $y = -x^{\frac{1}{5}}$

g i $y = -x^{\frac{1}{4}}$ **ii** $y = (-x)^{\frac{1}{4}}$

Exercise 3D

1 a i $y = 2(x-2)^2 - 3$ **ii** $y = \left(\frac{x+2}{3}\right)^2 - 4$

iii $y = 2x^2$

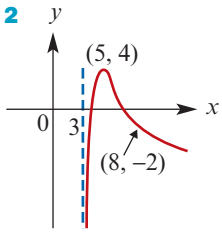
b i $y = 2\sqrt[3]{x-2} - 3$ **ii** $y = \sqrt[3]{\frac{x+2}{3}} - 4$

iii $y = -2\sqrt[3]{x}$

c i $y = \frac{2}{(x-2)^2} - 3$ **ii** $y = \frac{9}{(x+2)^2} - 4$

iii $y = \frac{2}{x^2}$

2



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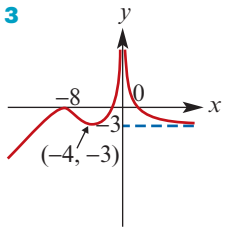

4 a i $y = -2(x-3)^2 - 4$

ii $y = -2(x-3)^2 + 4$

iii $y = -2(x-3)^2 - 4$

iv $y = -2(x-3)^2 - 8$

v $y = -2(x-3)^2 + 8$

vi $y = -2(x-3)^2 + 8$

b i $y = -2\sqrt[3]{x-3} - 4$

ii $y = -2\sqrt[3]{x-3} + 4$

iii $y = -2\sqrt[3]{x-3} - 4$

iv $y = -2\sqrt[3]{x-3} - 8$

v $y = -2\sqrt[3]{x-3} + 8$

vi $y = -2\sqrt[3]{x-3} + 8$

c i $y = \frac{-2}{(x-3)} - 4$ **ii** $y = \frac{-2}{(x-3)} + 4$

iii $y = \frac{-2}{(x-3)} - 4$ **iv** $y = \frac{-2}{(x-3)} - 8$

v $y = \frac{-2}{(x-3)} + 8$ **vi** $y = \frac{-2}{(x-3)} + 8$

d i $y = -2(x-3)^4 - 4$

ii $y = -2(x-3)^4 + 4$

iii $y = -2(x-3)^4 - 4$

iv $y = -2(x-3)^4 - 8$

v $y = -2(x-3)^4 + 8$

vi $y = -2(x-3)^4 + 8$

e i $y = \frac{-2}{(x-3)^3} - 4$ **ii** $y = \frac{-2}{(x-3)^3} + 4$

iii $y = \frac{-2}{(x-3)^3} - 4$ **iv** $y = \frac{-2}{(x-3)^3} - 8$

v $y = \frac{-2}{(x-3)^3} + 8$ **vi** $y = \frac{-2}{(x-3)^3} + 8$

f i $y = \frac{-2}{(x-3)^4} - 4$ **ii** $y = \frac{-2}{(x-3)^4} + 4$

iii $y = \frac{-2}{(x-3)^4} - 4$ **iv** $y = \frac{-2}{(x-3)^4} - 8$

v $y = \frac{-2}{(x-3)^4} + 8$ **vi** $y = \frac{-2}{(x-3)^4} + 8$

g i $y = \frac{-2}{(x-3)^2} - 4$ **ii** $y = \frac{-2}{(x-3)^2} + 4$

iii $y = \frac{-2}{(x-3)^2} - 4$ **iv** $y = \frac{-2}{(x-3)^2} - 8$

v $y = \frac{-2}{(x-3)^2} + 8$ **vi** $y = \frac{-2}{(x-3)^2} + 8$

5 $y = -\sqrt{\frac{x+12}{3}}$

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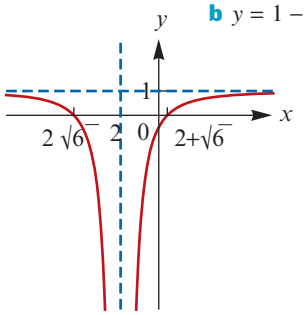
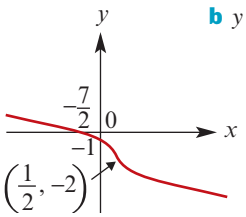
6 a



b $y = 1 - \dfrac{6}{(x+2)^2}$

7 a



b $y = (1-2x)^{\frac{1}{3}} - 2$

Exercise 3E

1 a i Dilation of factor 2 from the x-axis, then a translation 1 unit to the right and 3 units up

ii Reflection in the x-axis, then translation 1 unit to the left and 2 units up

iii Dilation of factor $\frac{1}{2}$ from the y-axis, then translation $\frac{1}{2}$ unit to the left and 2 units down

b i Dilation of factor 2 from the x-axis, then translation 3 units to the left

ii Translation 3 units to the left and 2 units up

iii Translation 3 units to the right and 2 units down

c i Translation 3 units to the left and 2 units up

ii Dilation of factor $\frac{1}{3}$ from the y-axis and dilation of factor 2 from the x-axis

iii Reflection in the x-axis, then translation 2 units up

2 a Translation 1 unit to the left and 6 units down

b Dilation of factor $\frac{1}{2}$ from the x-axis, then translation $\frac{3}{2}$ units up and 1 unit to the left

c Translation 1 unit to the left and 6 units up

d Dilation of factor $\frac{1}{2}$ from the x-axis, then translation $\frac{5}{2}$ units up and 1 unit to the left

e Dilation of factor 2 from the y-axis, then translation of 1 unit to the left and 6 units down

3 a Dilation of factor $\frac{1}{5}$ from the x-axis, then translation $\frac{7}{5}$ units up and 3 units to the left

b Dilation of factor 3 from the y-axis, then translation 2 units to the right and 5 units down

c Reflection in the x-axis, dilation of factor $\frac{1}{3}$ from the x-axis, translation $\frac{7}{3}$ units up, dilation of factor 3 from the y-axis, translation 1 unit to the right

d Reflection in the y-axis, translation 4 units to the right, dilation of factor $\frac{1}{2}$ from the x-axis

e Reflection in the y-axis, translation 4 units to the right, reflection in the x-axis, dilation of factor $\frac{1}{2}$ from the x-axis, translation $\frac{15}{2}$ units up

4 a Dilation of factor 2 from the x-axis, then translation 1 unit to the right and 3 units up

b Dilation of factor 2 from the x-axis, then translation 4 units to the left and 7 units down

c Reflection in the y-axis and dilation of factor 4 from the x-axis (in either order), then translation 1 unit to the right and 5 units down

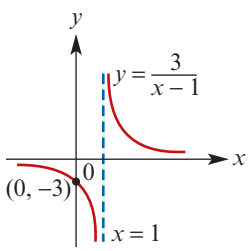
d Reflection in the x-axis, then translation 1 unit to the left and 2 units up

e Reflection in the y-axis and dilation of factor 2 from the x-axis (in either order), then translation 3 units up

f Translation 3 units to the left and 4 units down, then reflection in either axis and dilation of factor $\frac{1}{2}$ from the x-axis (in either order)

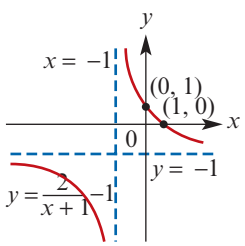
Exercise 3F

1 a



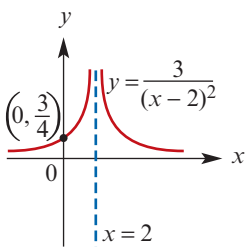
Range $= \mathbb{R} \setminus \{0\}$

b



Range $= \mathbb{R} \setminus \{-1\}$

c



Range $= \mathbb{R}^+$

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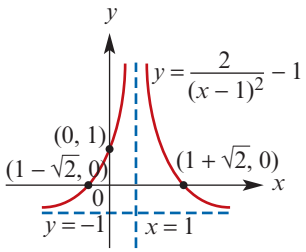
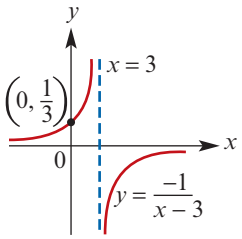
d



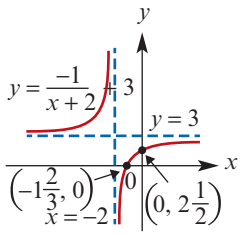
Range = $(-1, \infty)$

e



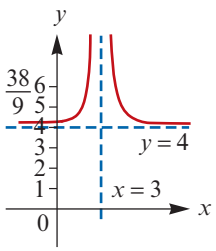
Range = $\mathbb{R} \setminus \{0\}$

f



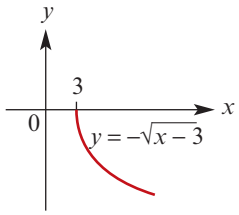
Range = $\mathbb{R} \setminus \{3\}$

g



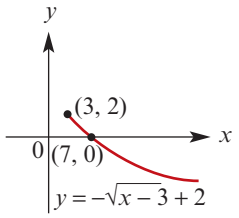
Range = $(4, \infty)$

2 a



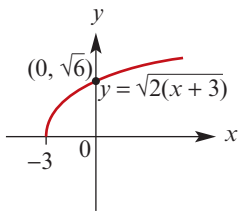
Range = $\mathbb{R}^- \cup \{0\}$

b



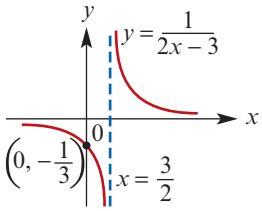
Range = $(-\infty, 2]$

c



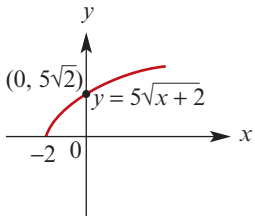
Range = $\mathbb{R}^+ \cup \{0\}$

d



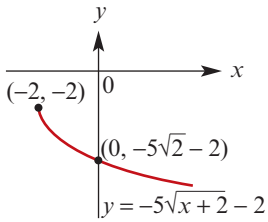
Range = $\mathbb{R} \setminus \{0\}$

e



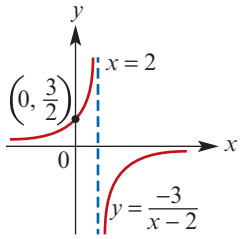
Range = $\mathbb{R}^+ \cup \{0\}$

f



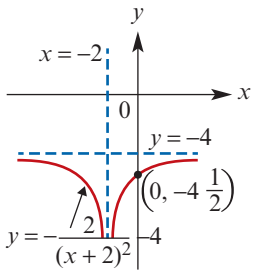
Range = $(-\infty, -2]$

g



Range = $\mathbb{R} \setminus \{0\}$

h



Range = $(-\infty, -4)$

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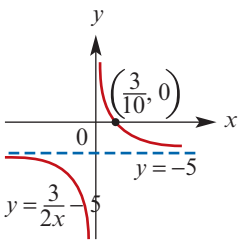
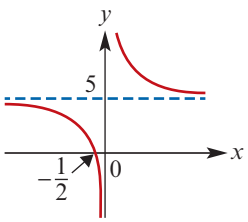
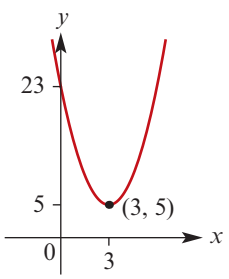
i Range = $\mathbb{R} \setminus \{-5\}$



j Range = $\mathbb{R} \setminus \{5\}$



k Range = $[5, \infty)$



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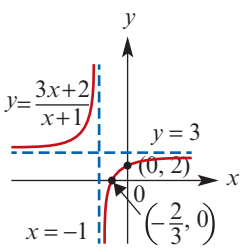

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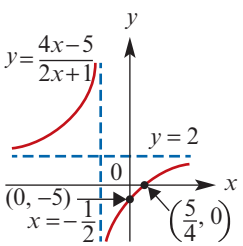
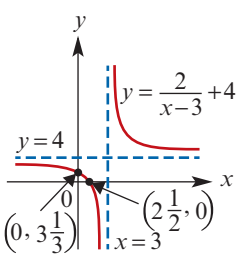


5 a Range = $\mathbb{R} \setminus \{4\}$



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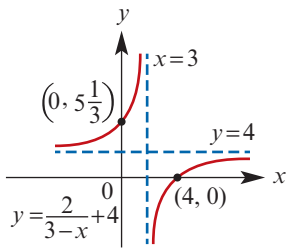
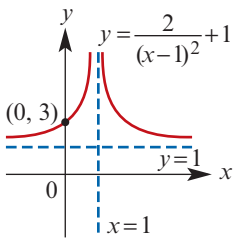
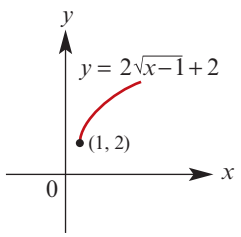


Range = $\mathbb{R} \setminus \{4\}$

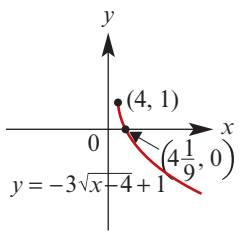
c Range = $(1, \infty)$



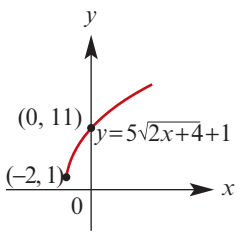
d Range = $[2, \infty)$



e Range = $(-\infty, 1]$



f Range = $[1, \infty)$



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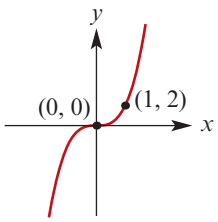
Exercise 3G

1 a



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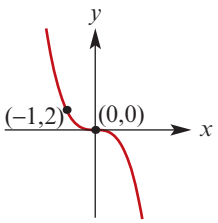

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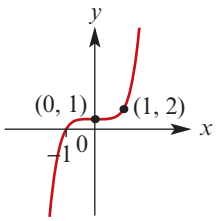

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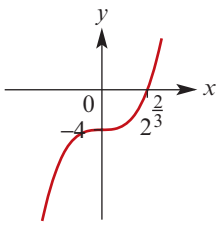
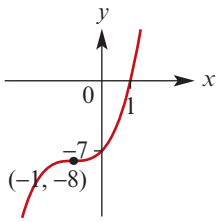


e



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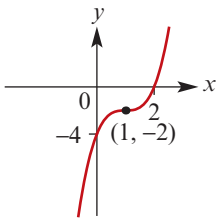

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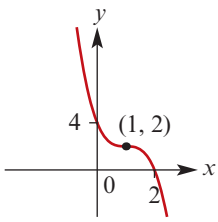

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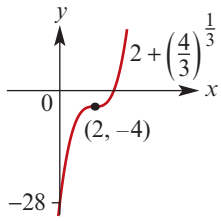

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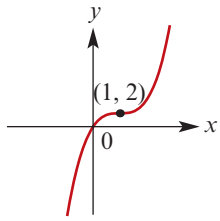

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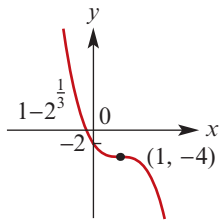

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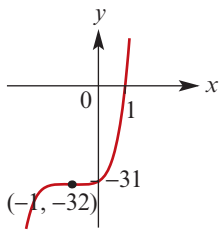

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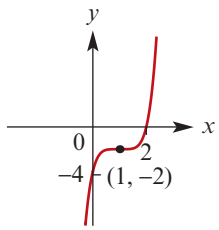

2 $a = -3,\ h = 0,\ k = 4$

3 a $y = 3x^3$ **b** $y = (x + 1)^3 + 1$

c $y = -(x - 2)^3 - 3$ **d** $y = 2(x + 1)^3 - 2$

e $y = \frac{x^3}{27}$

4 a $y = \frac{(3 - x)^3}{27} + 1$

b Dilation of factor 3 from the x-axis, reflection in the x-axis, then translation 1 unit to the left and 4 units up

5 $y = \frac{(x + 2)^4}{16} - 1$

6 Dilation of factor 3 from the x-axis, reflection in the x-axis, then translation 1 unit to the left and 5 units up

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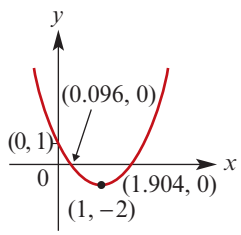
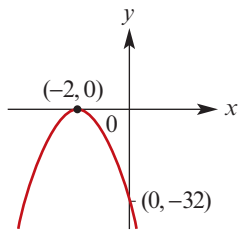
7 a



b



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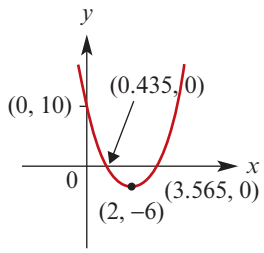

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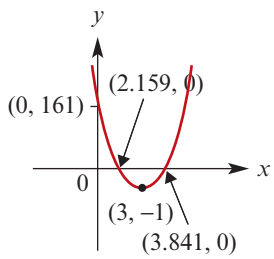

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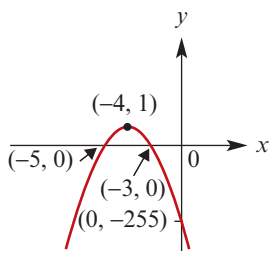

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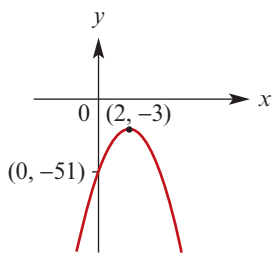

8 $a = -\frac{9}{16},\ h = -2,\ k = 3$

9 $a = 16,\ h = 1,\ k = 7$

Exercise 3H

1 $a = \frac{9}{2}, b = -\frac{1}{2}$ **2** $A = 1, b = -1, B = 2$

3 $a = \frac{5}{2}, b = -\frac{3}{2}$ **4** $A = 2, B = 3$

5 $A = 2, B = -1$ **6** $A = 8, b = 2, B = -3$

7 $a = -2, b = 1$ **8** $a = -6, b = -2$

Exercise 3I

1 a **i** $(-4, 13)$ **ii** $(2, 7)$

b $y = 8 \times 2^{x+2} + 3$

2 $a = -11, b = -\frac{9}{10}, h = -18, k = \frac{33}{10}$

3 a $(x, y) \to (-2x + 2, -2y)$

b $(x, y) \to (-2x + 4, -2y - 9)$

4 $T^{-1}(x, y) = \left(-\frac{1}{2}(x - 2), y + 3\right)$

5 $\{(x, y) : x \in [10, 16], y \in [-6, 4]\}$

6 a $[0, 16]$

b $y = \frac{x^2}{2} + 4$,
Domain $= [-8, 0]$. Range $= [4, 36]$

7 a $(x, y) \to \left(-\frac{1}{2}x, y\right)$

b $(x, y) \to \left(-\frac{1}{2}x, y\right)$

c $(x, y) \to (-x, y - 6)$

d $(x, y) \to (-x, y - 6)$

e $(x, y) \to (2x, y)$

f $(x, y) \to (2x, y)$

8 a $(x, y) \to (-x + 2, -y - 3)$

b $(x, y) \to (-x + 2, -y - 3)$

c $(x, y) \to \left(-\frac{1}{3}(x + 2), 6 - y\right)$

d $(x, y) \to \left(\frac{1}{2}(-x + 3), 4 - y\right)$

9 a $[-1, 8]$

b $y = 2(x - 3)^3 + 4$,
Domain $= [1, 4]$, Range $= [-12, 6]$

10 a **i** $(x, y) \to \left(-5 - x, \frac{1}{2}y + 2\right)$

ii $(x, y) \to \left(5 - x, \frac{1}{2}y + 1\right)$

iii $(x, y) \to (x - 10, y + 4)$

b **i**
- a reflection in the y-axis.
- a dilation of factor $\frac{1}{2}$ from the x-axis.
- a translation of 5 units in the negative direction of the x-axis.
- a translation of 2 units in the positive direction of the y-axis.

ii
- a reflection in the y-axis.
- a dilation of factor $\frac{1}{2}$ from the x-axis.
- a translation of 5 units in the positive direction of the x-axis.

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- a translation of 1 units in the positive direction of the y-axis.

iii
- a translation of 10 units in the negative direction of the x-axis.
- a translation of 4 units in the positive direction of the y-axis.

11 a i $(x, y) \to (-3x + 9, 2y - 4)$

ii $(x, y) \to (-3x + 3, 2y - 2)$

iii $(x, y) \to (-3x - 9, 2y - 4)$

b i
- a reflection in the y-axis.
- a dilation of factor 3 from the y-axis.
- a dilation of factor 2 from the x-axis.
- a translation of 9 units in the positive direction of the x-axis.
- a translation of 4 units in the negative direction of the y-axis.

ii
- a reflection in the y-axis.
- a dilation of factor 3 from the y-axis.
- a dilation of factor 2 from the x-axis.
- a translation of 3 units in the positive direction of the x-axis.
- a translation of 2 units in the negative direction of the y-axis.

iii
- a reflection in the y-axis.
- a dilation of factor 3 from the y-axis.
- a dilation of factor 2 from the x-axis.
- a translation of 9 units in the negative direction of the x-axis.
- a translation of 4 units in the negative direction of the y-axis.

12 $a = \frac{1}{2}, h = \frac{5}{2}, b = -3, k = 6, c = 3, d = 11$

13 a $f^{-1} : \mathbb{R}\backslash\{0\} \to \mathbb{R}\ f(x) = \frac{1}{5x} - \frac{2}{5}$

b $(x, y) \to \left(x + \frac{2}{5}, y - \frac{2}{5}\right)$

14 a $T_1 \circ T_2(x, y) = (a_1a_2x + a_1h_2 + h_1, b_1b_2y + b_1k_2 + k_1)$

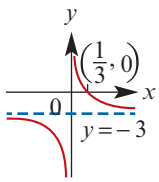
$(T_1 \circ T_2)^{-1}(x, y) = \left(\frac{1}{a_1a_2}(x - (a_1h_2 + h_1)), \frac{1}{b_1b_2}(y - (b_1k_2 + k_1))\right)$

b $T_2^{-1} \circ T_1^{-1}(x, y) = \left(\frac{1}{a_1a_2}(x - (a_1h_2 + h_1)), \frac{1}{b_1b_2}(y - (b_1k_2 + k_1))\right)$

Chapter 3 review

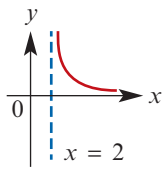
Technology-free questions

1 a



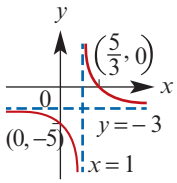
Range = $\mathbb{R} \setminus \{-3\}$

b



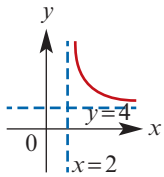
Range = $(0, \infty)$

c



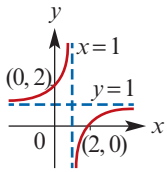
Range = $\mathbb{R} \setminus \{-3\}$

d



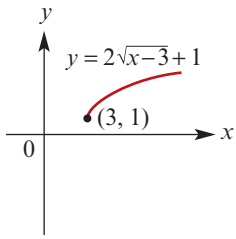
Range = $(4, \infty)$

e



Range = $\mathbb{R} \setminus \{1\}$

2 a



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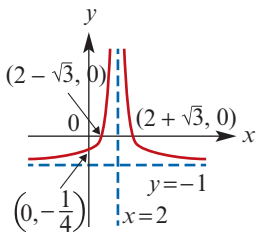

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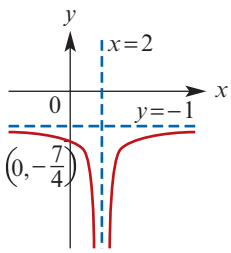
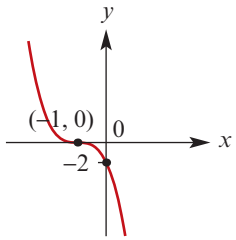


3 a



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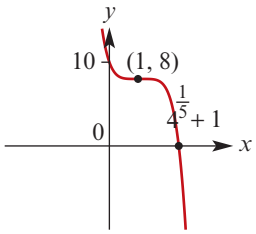
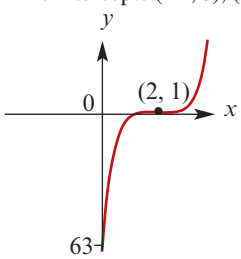
Point of zero gradient $(-1, 0)$;
Axis intercepts $(-1, 0)$, $(0, -2)$

b



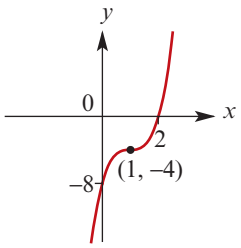
Point of zero gradient $(1, 8)$;
Axis intercepts $(4^{\frac{1}{5}}, 0)$, $(0, 10)$

c



Point of zero gradient $(2, 1)$;
Axis intercepts $(-\frac{1}{2})^{\frac{1}{5}} + 2, 0)$, $(0, -63)$

d



Point of zero gradient $(1, -4)$;
Axis intercepts $(2, 0)$, $(0, -8)$

4 $a = 2, b = 4$

5 $y = -2\sqrt{x+4} - 1$

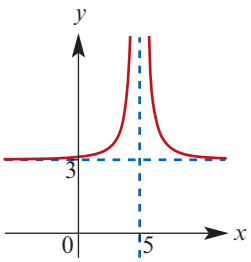
6 $y = -\frac{7}{2} - 2\sqrt{\frac{x-8}{3}}$

7 $a = -6, b = 9$

8 a $y = 6 - \frac{(x-4)^2}{4}$

b Reflection in the x-axis, dilation of factor 4 from the x-axis, then translate 1 unit to the left and 6 units up.

9 Dilation of factor 3 from the x-axis, then translation 5 units to the right and 3 units up



Asymptotes $x = 5$, $y = 3$; Intercept $\left(0, \frac{78}{25}\right)$

10 Dilation of factor $\frac{1}{2}$ from the x-axis, then translation $\frac{3}{2}$ units up

11 Dilation of factor $\frac{1}{2}$ from the x-axis, then translation 3 units to the left and 2 units down

Multiple-choice questions

1 B	**2** B	**3** B	**4** E	**5** D
6 A	**7** D	**8** A	**9** A	**10** A

Extended-response questions

1 a $\mathbb{R} \setminus \{-2\}$

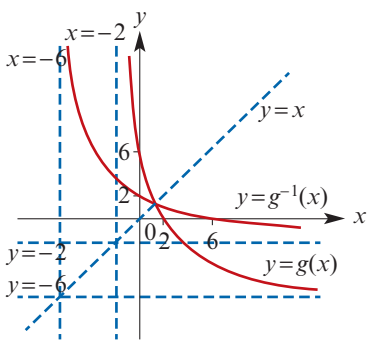
b Dilation of factor 24 from the x-axis, then translation 2 units to the left and 6 units down

c $(0, 6)$, $(2, 0)$

d $g^{-1}(x) = \frac{24}{x+6} - 2$

e Domain of g^{-1} = range of $g = (-6, \infty)$

f



g $x = -4 + 2\sqrt{7}$

2 a $[-3, \infty)$

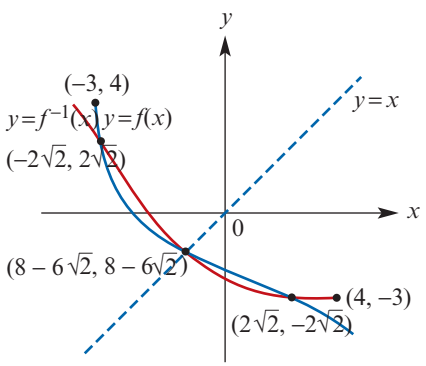
b Dilation of factor $\frac{1}{2}$ from the y-axis, dilation of factor 2 from the x-axis, reflection in the x-axis, then translation 3 units to the left and 4 units up

c $(0, 4 - 2\sqrt{6})$, $(-1, 0)$

d $f^{-1}(x) = \frac{(x-4)^2}{8} - 3$

e Domain of f^{-1} = range of $f = (-\infty, 4]$

f



g $x = 8 - 6\sqrt{2}$ or $x = 2\sqrt{2}$ or $x = -2\sqrt{2}$

3 a i $\frac{3}{125}$ **ii** $(x, y) \to (x, -y)$

iii $(x, y) \to (x + 25, y + 15)$

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iv $(x, y) \to \left(x + 25, \frac{-3}{125}y + 15\right)$

b i $y = \frac{-3}{125}(x - 25)^2 + 15$

ii $(x, y) \to (x + 50, y)$

iii $y = \frac{-3}{125}(x - 75)^2 + 15$

c i $(x, y) \to \left(x + \frac{m}{2}, -\frac{4n}{m^2}y + n\right)$

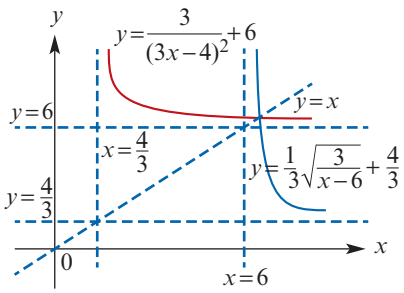
ii $y = -\frac{4n}{m^2}\left(x - \frac{m}{2}\right)^2 + n$

iii $y = -\frac{4n}{m^2}\left(x - \frac{3m}{2}\right)^2 + n$

4 a $\mathbb{R} \setminus \left\{\frac{4}{3}\right\}$ **b** $\frac{4}{3}$

c $f^{-1}(x) = \frac{4}{3} + \frac{1}{3}\sqrt{\frac{3}{x - 6}}$ **d** $x = 6.015$

e



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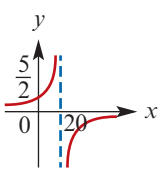

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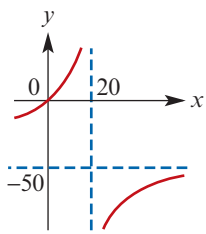


c $g^{-1}(x) = \frac{20x}{50 + x}$

6 a i $y = f^{-1}(x - 5) + 3$

ii $y = f^{-1}(x - 3) + 5$

iii $y = 5f^{-1}\left(\frac{x}{3}\right)$ **iv** $y = 3f^{-1}\left(\frac{x}{5}\right)$

b $y = cf^{-1}\left(\frac{x - b}{a}\right) + d$

Reflection in the line $y = x$, then dilation of factor c from the x-axis and factor a from the y-axis, and a translation b units to the right and d units up

7 a $[-9, 6]$

b i $k \in (-\infty, -9) \cup (6, \infty)$

ii $k = -9$ or $k = 6$

iii $k \in \left(\frac{7}{2}, 6\right) \cup \left(-9, -\frac{14}{3}\right)$

iv $k \in \left(0, \frac{7}{2}\right) \cup \left(-\frac{14}{3}, -\frac{9}{2},\right)$

v $k = 0$ or $k = -\frac{9}{2}$ **vi** $k \in \left(-\frac{9}{2}, 0\right)$

c

$$g(x) = \begin{cases} -\frac{1}{2}(x^2 + 12x + 27) & \text{if } -10 \le x < -3 \\ x^2 - 9 & \text{if } -3 \le x < 3 \\ -\frac{2}{3}(x^2 - 12x + 27) & \text{if } 3 \le x \le 10 \end{cases}$$

d $[-12, 18]$

e Domain = $[-8, 12]$, Range = $[-8, 22]$

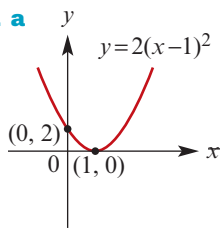
f i $(x, y) \to (x + 3, -2y + 2)$

ii $\begin{cases} -x^2 - 6x + 2 & \text{if } -7 \le x < 0 \\ -2x^2 + 12x + 2 & \text{if } 0 \le x < 6 \\ \frac{4}{3}x^2 + 24x + 98 & \text{if } 6 \le x \le 13 \end{cases}$

Chapter 4

Exercise 4A

1 a



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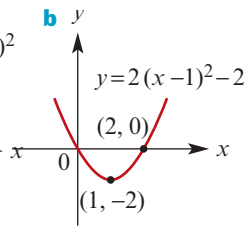

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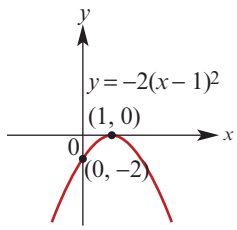

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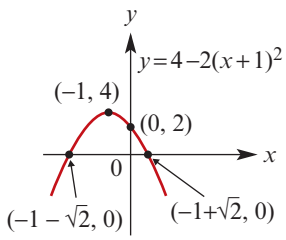

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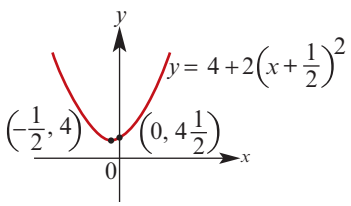

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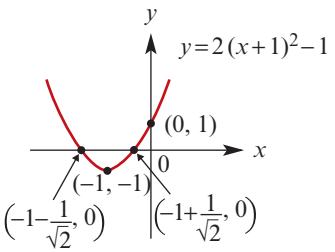

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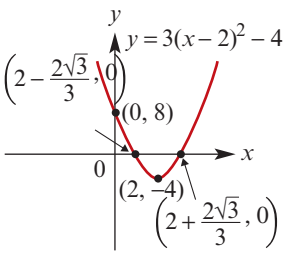

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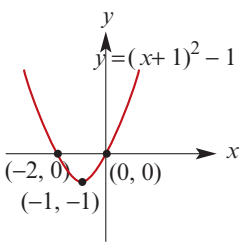

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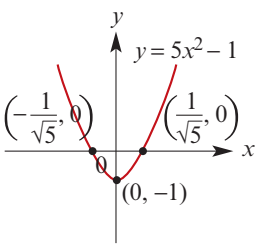

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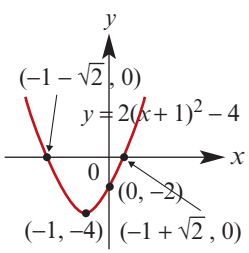


2 a $f(x) = (x + \frac{3}{2})^2 - 4\frac{1}{4}$
Minimum $= -4\frac{1}{4}$; Range $= [-4\frac{1}{4}, \infty)$

b $f(x) = (x - 3)^2 - 1$
Minimum $= -1$; Range $= [-1, \infty)$

c $f(x) = 2(x + 2)^2 - 14$
Minimum $= -14$; Range $= [-14, \infty)$

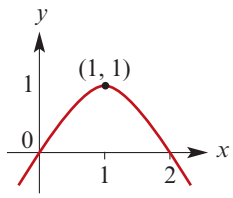
d $f(x) = 4(x + 1)^2 - 11$
Minimum $= -11$; Range $= [-11, \infty)$

e $f(x) = 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8}$
Minimum $= -\frac{25}{8}$; Range $= \left[-\frac{25}{8}, \infty\right)$

f $f(x) = -3\left(x + \frac{1}{3}\right)^2 + \frac{22}{3}$
Maximum $= \frac{22}{3}$; Range $= \left(-\infty, \frac{22}{3}\right]$

g $f(x) = -2\left(x - \frac{9}{4}\right)^2 + \frac{169}{8}$
Maximum $= \frac{169}{8}$; Range $= \left(-\infty, \frac{169}{8}\right]$

3 a



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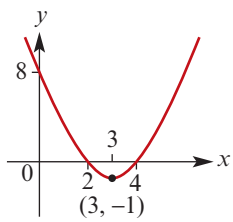

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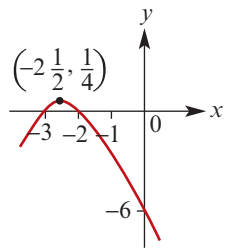

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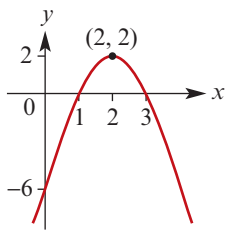

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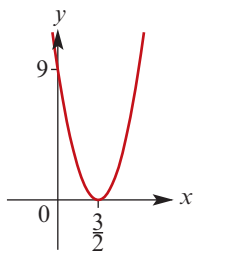

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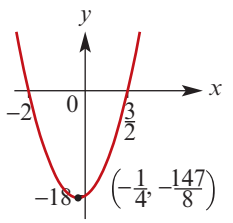

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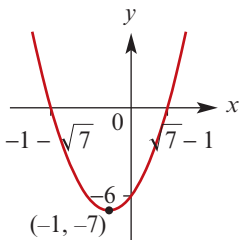
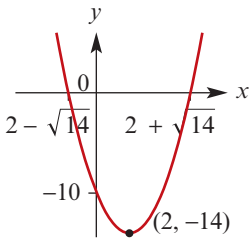
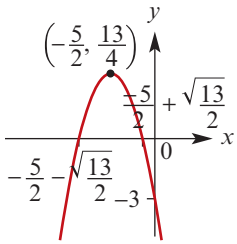
4 a $y = (x + 1)^2 - 7$



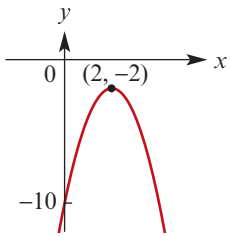
b $y = (x - 2)^2 - 14$



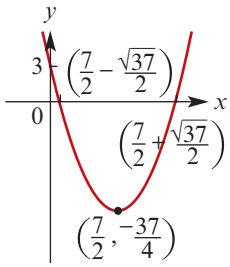
c $y = \frac{13}{4} - \left(x + \frac{5}{2}\right)^2$



d $y = -2(x - 2)^2 - 2$



e $y = \left(x - \frac{7}{2}\right)^2 - \frac{37}{4}$



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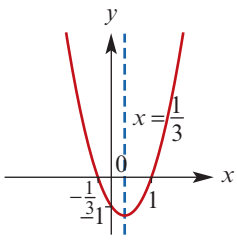

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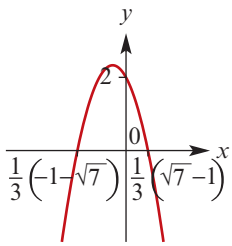
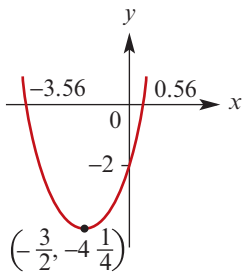


7 a



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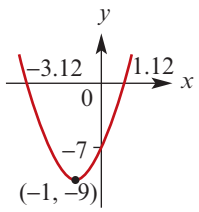

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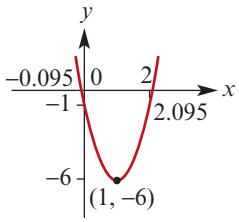

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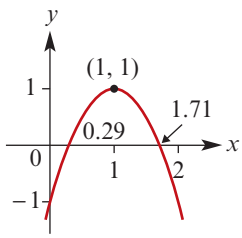

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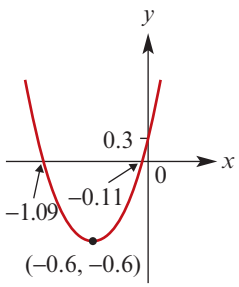

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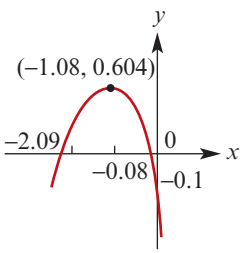

8 **a** B **b** D

9 **a** C **b** B **c** D **d** A

10 **a** Crosses the x-axis
b Does not cross the x-axis
c Just touches the x-axis
d Crosses the x-axis
e Does not cross the x-axis
f Does not cross the x-axis

11 **a** $m > 3$ or $m < 0$ **b** $m = 3$

12 $m = 2$ or $m = -\frac{2}{9}$

13 $a < -6$

14 Show that $\Delta > 0$ for all a

15 Show that $\Delta \geq 0$ for all k

16 **a** $k < -5$ or $k > 0$ **b** $k = -5$

17 **a** $k > -6$ **b** $k = -6$

18 Show that $\Delta \geq 0$ for all a, b

Exercise 4B

1 $y = -2(x + 3)(x + 2)$ **2** $y = (x + 3)(2x + 3)$

3 $y = \frac{3}{2}(x + 2)^2 + 4$ **4** $y = -2(x + 2)^2 - 3$

5 $y = -5x^2 + 6x + 18$ **6** $y = -2x^2 - 8x + 10$

7 **a** $y = 4 - \frac{4}{25}x^2$ **b** $y = -x^2$

c $y = x^2 + 2x$ **d** $y = 2x - x^2$

e $y = x^2 - 5x + 4$ **f** $y = x^2 - 4x - 5$

g $y = x^2 - 2x - 1$ **h** $y = x^2 - 4x + 6$

8 $y = -\frac{1}{8}x^2 + x + 1$, $y = \frac{1}{8}x^2 + x - 5$

9 $A = 1, b = 2, B = 4$

Exercise 4C

1 **a** 3 **b** −5 **c** 7 **d** −21 **e** $\frac{17}{8}$ **f** $-\frac{9}{8}$

2 **a** 6 **b** 6 **c** 18 **d** 12

e $a^3 + 3a^2 - 4a + 6$ **f** $8a^3 + 12a^2 - 8a + 6$

3 **a** $a = -5$ **b** $a = \frac{40}{9}$ **c** $c = 8$

d $a = -23,\ b = -4$ **e** $a = -17,\ b = 42$

4 **a** $2x^3 - x^2 + 2x + 2$ **b** $2x^3 + 5x$

c $2x^3 - x^2 + 4x - 2$ **d** $6x^3 - 3x^2 + 9x$

e $-2x^4 + 5x^3 - 5x^2 + 6x$ **f** $4x - x^3$

g $2x^3 + 4x + 2$ **h** $2x^5 + 3x^4 + x^3 + 6x^2$

5 **a** $x^3 - 5x^2 + 10x - 8$ **b** $x^3 - 7x^2 + 13x - 15$

c $2x^3 - x^2 - 7x - 4$

d $x^3 + (b + 2)x^2 + (2b + c)x + 2c$

e $2x^3 - 9x^2 - 2x + 3$

6 **a** $x^3 + (b + 1)x^2 + (c + b)x + c$
b $b = -2$ and $c = -4$
c $(x + 1)(x + \sqrt{5} - 1)(x - \sqrt{5} - 1)$

7 **a** $a = 2$ and $b = 5$
b $a = -2,\ b = -2$ and $c = -3$

8 $A = 1,\ B = 3$

9 **a** $A = 1,\ B = -2,\ C = 6$
b $A = 4,\ B = -\frac{3}{2},\ C = 5$
c $A = 1,\ B = -3,\ C = 5$

Exercise 4D

1 **a** $x^2 - 5x + 6$ **b** $2x^2 + 7x - 4$

2 **a** $x^2 - 4x - 3 + \frac{34}{x + 3}$

b $2x^2 + 6x + 14 + \frac{54}{x - 3}$

3 **a** $x^2 - \frac{5}{2}x - \frac{15}{4} + \frac{145}{4(2x + 3)}$

b $2x^2 + 6x + 7 + \frac{33}{2x - 3}$

4 **a** $2x^2 - x + 12 + \frac{33}{x - 3}$

b $5x^4 + 8x^3 - 8x^2 + 6x - 6$

5 **a** $x^2 - 9x + 27 - \frac{26(x - 2)}{x^2 - 2}$ **b** $x^2 + x + 2$

6 **a** −16 **b** $a = 4$

7 **a** 28 **b** 0 **c** $(x + 2)(3x + 1)(2x - 3)$

8 **b** $k = \frac{11}{2}$

9 **a** $a = 3,\ b = 8$ **b** $2x - 1,\ x - 1$

10 $a = \frac{-92}{9},\ b = 9$

11 81

12 **b** $6x - 4$

13 $x - 3,\ 2x - 1$

14 **b** $x^2 - 3,\ x^2 + x + 2$

15 **a** $(2a + 3b)(4a^2 - 6ab + 9b^2)$
b $(4 - a)(a^2 + 4a + 16)$
c $(5x + 4y)(25x^2 - 20xy + 16y^2)$
d $2a(a^2 + 3b^2)$

16 **a** $(2x - 1)(2x + 3)(3x + 2)$
b $(2x - 1)(2x^2 + 3)$

17 **a** $(2x - 3)(2x^2 + 3x + 6)$
b $(2x - 3)(2x - 1)(2x + 1)$

18 **a** $-4, 2, 3$ **b** $0, 2$ **c** $\frac{1}{2}, 2$ **d** $-2, 2$

e $0, -2, 2$ **f** $0, -3, 3$

g $1, -2, -\frac{1}{4}, \frac{1}{3}$ **h** $1, -2$ **i** $1, -2, \frac{1}{3}, \frac{3}{2}$

19 **a** $(-1, 0),\ (0, 0),\ (2, 0)$
b $(-2, 0),\ (0, 6),\ (1, 0),\ (3, 0)$
c $(-1, 0),\ (0, 6),\ (2, 0),\ (3, 0)$
d $\left(-\frac{1}{2}, 0\right),\ (0, 2),\ (1, 0),\ (2, 0)$

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e $(-2, 0),\ (-1, 0),\ (0, -2),\ (1, 0)$

f $(-1, 0),\ \left(-\frac{2}{3}, 0\right),\ (0, -6),\ (3, 0)$

g $(-4, 0),\ (0, -16),\ \left(-\frac{2}{5}, 0\right),\ (2, 0)$

h $\left(-\frac{1}{2}, 0\right),\ (0, 1),\ \left(\frac{1}{3}, 0\right),\ (1, 0)$

i $(-2, 0),\ \left(-\frac{3}{2}, 0\right),\ (0, -30),\ (5, 0)$

20 $p = 1,\ q = -6$

21 -33

22 a $(x - 9)(x - 13)(x + 11)$

b $(x + 11)(x - 9)(x - 11)$

c $(x + 11)(2x - 9)(x - 11)$

d $(x + 11)(2x - 13)(2x - 9)$

23 a $(x - 1)(x + 1)(x - 7)(x + 6)$

b $(x - 3)(x + 4)(x^2 + 3x + 9)$

24 a $(x - 9)(x - 5)(2x^2 + 3x + 9)$

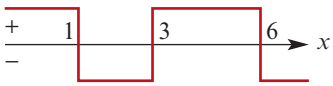
b $(x + 5)(x + 9)(x^2 - x + 9)$

c $(x - 3)(x + 5)(x^2 + x + 9)$

d $(x - 4)(x - 3)(x + 5)(x + 6)$

Exercise 4E

1 a



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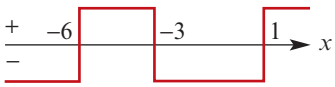

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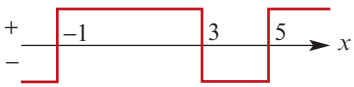

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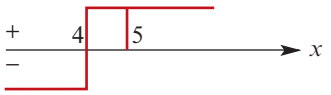

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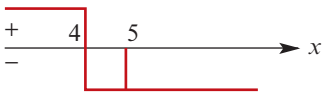

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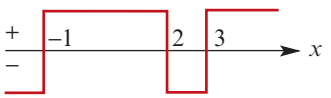

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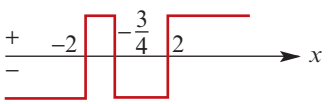

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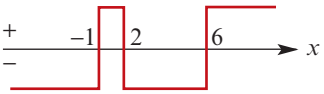

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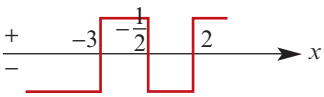
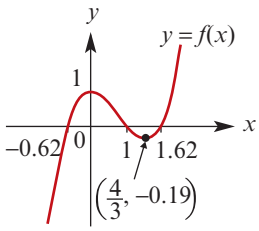


3 a



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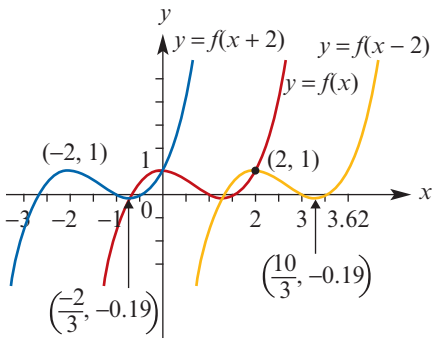
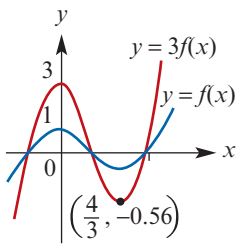
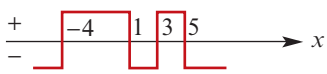


For clarity the graph of $y = 3f(x)$ is shown on separate axes:



Exercise 4F

1 a



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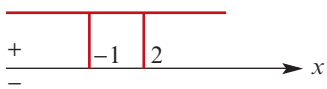
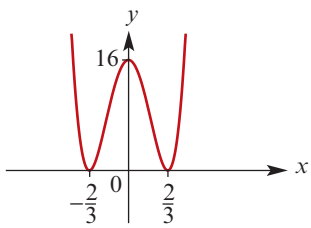


2 $(0, 16)$

$\left(\frac{2}{3}, 0\right)$

$\left(-\frac{2}{3}, 0\right)$



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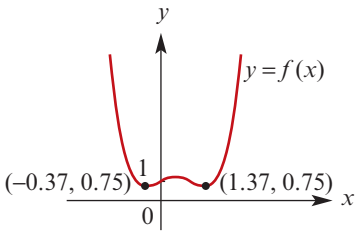
3 a



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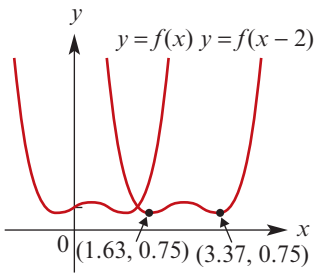
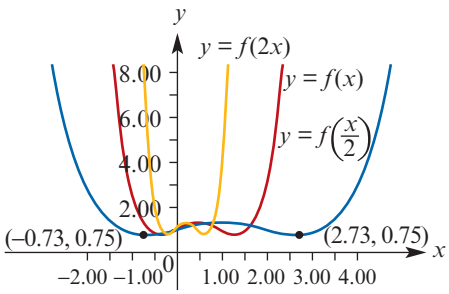
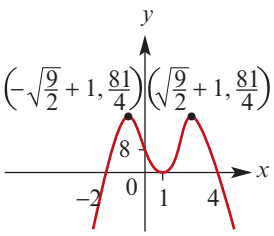


Graphs of dilations shown on separate axes for clarity:



Turning points for $y = f(2x)$ are at $(-0.18, 0.75)$ and $(0.68, 0.75)$

4 a



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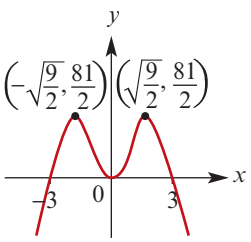

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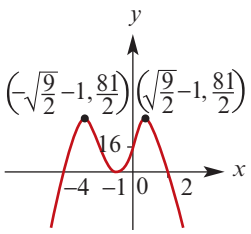

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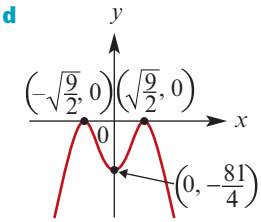

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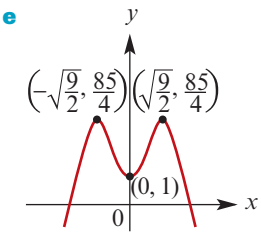

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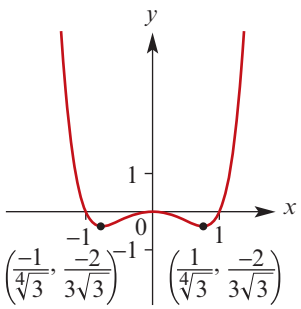

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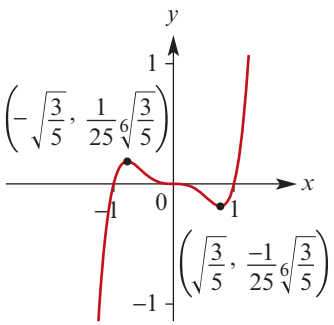

Exercise 4G

1 a $a = -2$ **b** $a = 3$ **c** $a = \frac{10}{3}$, $b = -\frac{70}{3}$

2 $a = -3$, $b = 2$, $c = -4$, $d = 5$

3 $y = \frac{11}{60}(x + 5)(x + 2)(x - 6)$

4 $y = \frac{5}{9}(x + 1)(x - 3)^2$

5 a $y = x^3 + x + 1$ **b** $y = x^3 - x + 1$
c $y = 2x^3 - x^2 + x - 2$

6 a $y = (2x + 1)(x - 1)(x - 2)$
b $y = \frac{1}{4}x(x^2 + 2)$ **c** $y = x^2(x + 1)$
d $y = x^3 + 2x^2 - x - 2$
e $y = (x + 2)(x - 3)^2$

7 a $y = -2x^3 - 25x^2 + 48x + 135$
b $y = 2x^3 - 30x^2 + 40x + 13$

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8 a $y = -2x^4 + 22x^3 - 10x^2 - 37x + 40$
b $y = x^4 - x^3 + x^2 + 2x + 8$
c $y = \frac{31}{36}x^4 + \frac{5}{4}x^3 - \frac{157}{36}x^2 - \frac{5}{4}x + \frac{11}{2}$

Exercise 4H

1 a $x = \frac{-1 \pm \sqrt{1 - 4k^2}}{2k}$, for $k \in [-\frac{1}{2}, \frac{1}{2}] \setminus \{0\}$
b $x = 4a, 3a$ or 0 **c** $x = 0$ or $x = a^{\frac{1}{3}}$
d $x = \frac{k \pm \sqrt{k^2 - 4k}}{2}$, for $k \geq 4$ or $k \leq 0$
e $x = 0$ or $x = \pm\sqrt{a}$ if $a > 0$ **f** $x = \pm a$
g $x = a$ or $x = b$
h $x = a$ or $x = a^{\frac{1}{3}}$ or $x = \pm\sqrt{a}$ if $a \geq 0$

2 a $x = \sqrt[3]{\frac{2c - b}{a}}$ **b** $x = \pm\sqrt{\frac{c + b}{a}}$
c $x = \pm\sqrt{\frac{a - c}{b}}$ **d** $x = a^3$
e $x = (a - c)^n$ **f** $x = 2b + \sqrt[3]{\frac{c}{a}}$
g $x = \left(\frac{b}{a}\right)^3$ **h** $x = (c + d)^{\frac{1}{3}}$

3 a $(0, 0),\ (1, 1)$ **b** $(\frac{1}{2}, \frac{1}{2}),\ (0, 0)$
c $\left(\frac{3 + \sqrt{13}}{2}, \sqrt{13} + 4\right),\ \left(\frac{3 - \sqrt{13}}{2}, 4 - \sqrt{13}\right)$

4 a $(13, 3),\ (3, 13)$ **b** $(10, 5),\ (5, 10)$
c $(11, 8),\ (-8, -11)$ **d** $(9, 4),\ (4, 9)$
e $(9, 5),\ (-5, -9)$

5 a $(17, 11),\ (11, 17)$ **b** $(37, 14),\ (14, 37)$
c $(14, 9),\ (-9, -14)$

6 $(2, 4),\ (0, 0)$

7 $\left(\frac{\sqrt{5} + 5}{2}, \frac{\sqrt{5} + 5}{2}\right),\ \left(\frac{5 - \sqrt{5}}{2}, \frac{5 - \sqrt{5}}{2}\right)$

8 $\left(\frac{-130 - 80\sqrt{2}}{41}, \frac{60 - 64\sqrt{2}}{41}\right)$,
$\left(\frac{80\sqrt{2} - 130}{41}, \frac{64\sqrt{2} + 60}{41}\right)$

9 $\left(\frac{1 + \sqrt{21}}{2}, \frac{-1 + \sqrt{21}}{2}\right),\ \left(\frac{1 - \sqrt{21}}{2}, \frac{-1 + \sqrt{21}}{2}\right)$

10 $\left(\frac{4}{9}, 2\right)$ **11** $\left(\frac{-6\sqrt{5}}{5}, \frac{3\sqrt{5}}{5}\right)$

12 $\left(-2, \frac{1}{2}\right)$ **13** $(3, 2),\ (0, -1)$

14 a $(3, 2),\ \left(\frac{-8}{5}, \frac{-15}{4}\right)$ **b** $\left(\frac{27}{2}, \frac{10}{3}\right),\ (5, 9)$
c $(6, 4),\ \left(\frac{-12}{5}, -10\right)$

15 $c^2 - ac + b = 0$

16 $(-1 - \sqrt{161}, 1 - \sqrt{161}),\ (\sqrt{161} - 1, \sqrt{161} + 1)$

17 $y = -7x + 14,\ y = 5x + 2$

18 $m < -7$ or $m > 1$

19 $c = -8$ or $c = 4$

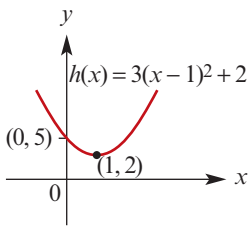
20 a $x = \frac{5 \pm \sqrt{4m + 25}}{2m}$ **b** $m = \frac{-25}{4},\ \left(-\frac{2}{5}, \frac{5}{2}\right)$
c $m < \frac{-25}{4}$ and $m \neq 0$

21 $y = 3x + 3,\ y = -x + 3$

Chapter 4 review

Technology-free questions

1 a



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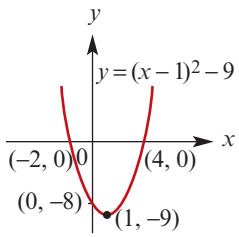

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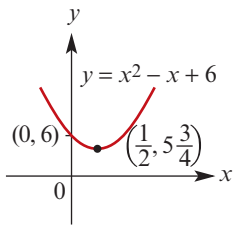

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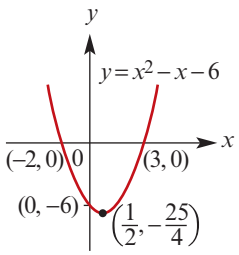

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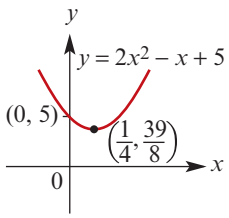

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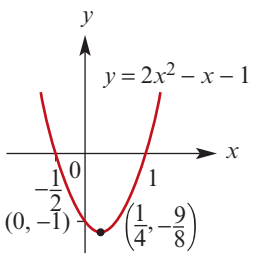
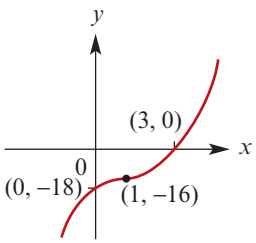


2 $y = \frac{4}{3}x^2 - \frac{1}{3}$; $a = \frac{4}{3}$, $b = -\frac{1}{3}$

3 $\frac{1}{3}(1 \pm \sqrt{31})$

4 a



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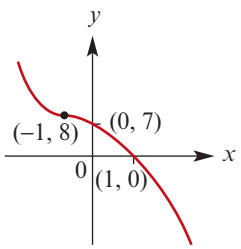

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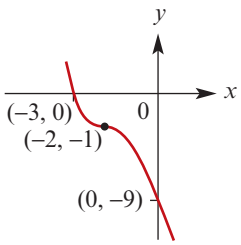

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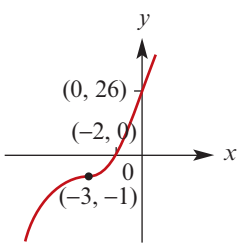

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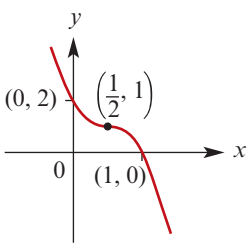
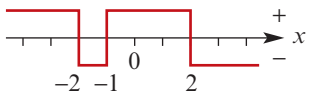


5 a $(x + 2)^2 - 4$ **b** $3(x + 1)^2 - 3$

c $(x - 2)^2 + 2$ **d** $2\left(x - \frac{3}{2}\right)^2 - \frac{17}{2}$

e $2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$ **f** $-\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$

6 a



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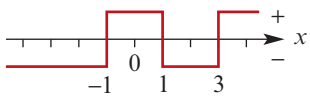

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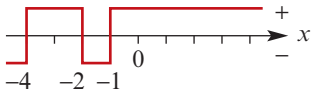

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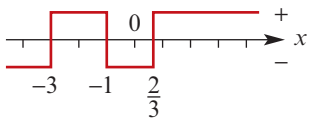


7 a 8 **b** 0 **c** 0

8 $y = (x - 7)(x + 3)(x + 2)$

9 a $(x - 2)(x + 1)(x + 3)$

b $(x - 1)(x + 1)(x - 3)$

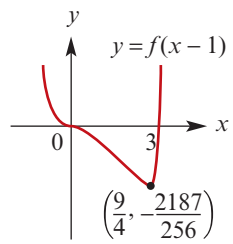
c $(x - 1)(x + 1)(x - 3)(x + 2)$

d $\frac{1}{4}(x - 1)(2x + 3 + \sqrt{13})(2x + 3 - \sqrt{13})$

10 $x^2 + 4 = 1 \times (x^2 - 2x + 2) + 2x + 2$

11 $a = -6$

12 a



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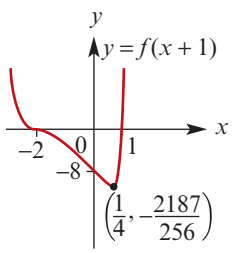

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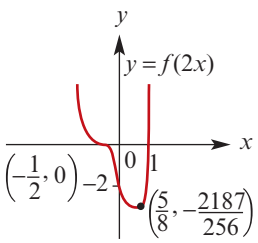

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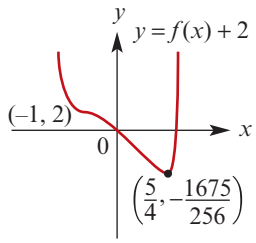
d



13 $k = \pm 8$

14 $(-4, -5),\ (3, 9)$

15 $a = 3,\ b = -\frac{5}{6},\ c = -\frac{13}{12}$

16 $64x^3 + 144x^2 + 108x + 27$

17 $a = 1,\ b = -1,\ c = 4$

18 $-2 < p < 6$

19 $y = -x^3 + 7x^2 - 11x + 6$

Multiple-choice questions

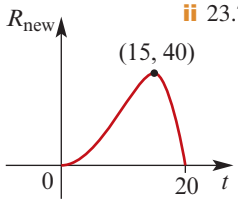
1 E **2** D **3** E **4** C **5** E **6** A
7 C **8** E **9** C **10** C **11** C **12** B

Extended-response questions

1 a $k = \frac{4}{3375}$

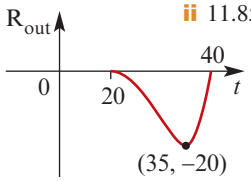
b 11.852 mL/min

c i



ii 23.704 mL/min

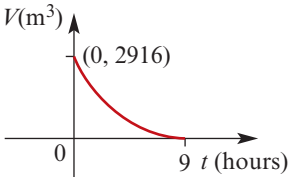
d i



ii 11.852 mL/min out

2 a i 2916 m^3 **ii** 0 m^3

b

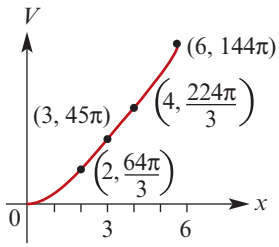


c 3.96 hours

3 a i $\frac{64\pi}{3}$ cm^3 **ii** 45π cm^3 **iii** $\frac{224\pi}{3}$ cm^3

b 144π cm^3

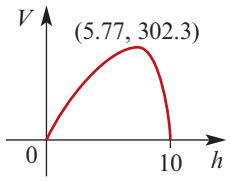
c



d $x = 5$; depth is 5 cm

4 a $r = \sqrt{25 - \frac{h^2}{4}}$

b



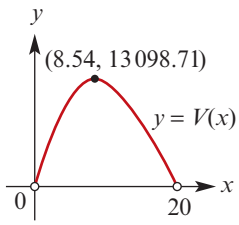
c $V = 96\pi$ cm^3

d $h = 2,\ r = 2\sqrt{6}$, i.e. height = 2 cm and radius = $2\sqrt{6}$ cm, or $h = 8.85$ and $r = 2.33$

5 a $V = (84 - 2x)(40 - 2x)x$

b $(0, 20)$

c



d i $x = 2,\ V = 5760$
ii $x = 6,\ V = 12\,096$
iii $x = 8,\ V = 13\,056$
iv $x = 10,\ V = 12\,800$

e $x = 13.50$ or $x = 4.18$

f 13 098.71 cm^3

6 a i $A = 2x(16 - x^2)$ **ii** $(0, 4)$

b i 42 **ii** $x = 0.82$ or $x = 3.53$

c i $V = 2x^2(16 - x^2)$ **ii** $x = 2.06$ or $x = 3.43$

7 a $A = \frac{\pi}{2}x^2 + yx$

b i $y = 100 - \pi x$ **ii** $A = 100x - \frac{\pi}{2}x^2$

iii $\left(0, \frac{100}{\pi}\right)$

c $x = 12.43$

d i $V = \frac{x^2}{50}\left(100 - \frac{\pi}{2}x\right),\ x \in \left(0, \frac{100}{\pi}\right)$

ii 248.5 m^3 **iii** $x = 18.84$

8 a $y = \frac{1}{12000}x^3 - \frac{1}{200}x^2 + \frac{17}{120}x$

b $x = 20$

d $y = -\frac{1}{6000}x^3 + \frac{29}{3000}x^2 - \frac{1}{20}x$

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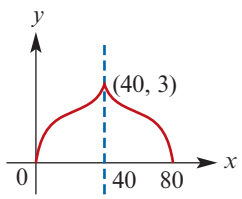
e i

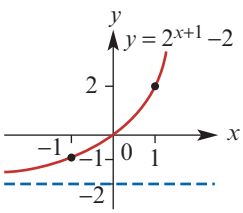


ii Second section of graph is formed reflecting the graph of $y = f(x)$, $x \in [0, 40]$, in the line $x = 40$

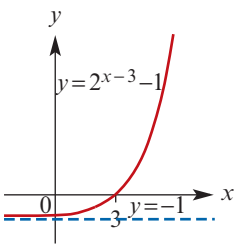
Chapter 5

Exercise 5A

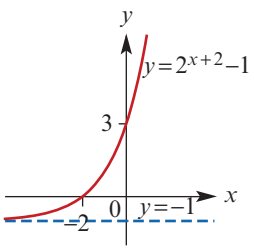
1 a Range = $(-2, \infty)$



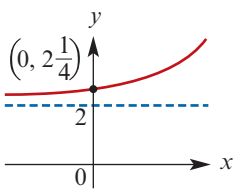
b Range = $(-1, \infty)$



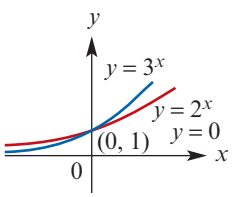
c Range = $(-1, \infty)$



d Range = $(2, \infty)$



2 a



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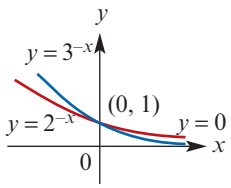

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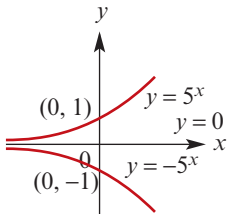

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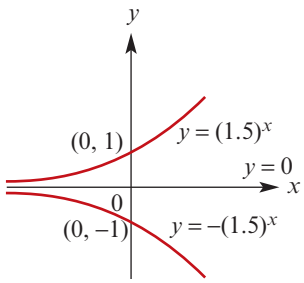
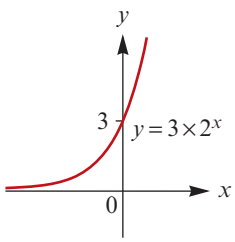
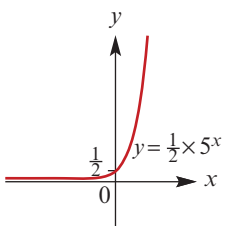


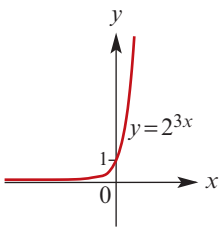
3 a Range = $(0, \infty)$



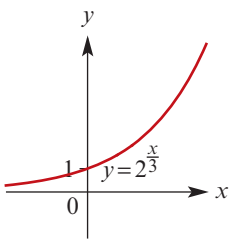
b Range = $(0, \infty)$



c Range = $(0, \infty)$



d Range = $(0, \infty)$



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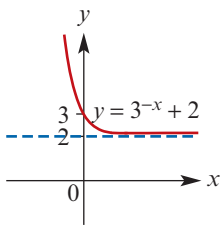
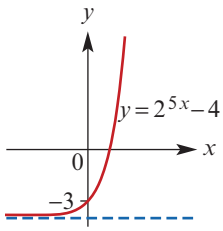
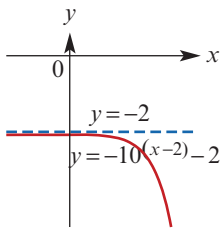
4 a Range = $(2, \infty)$



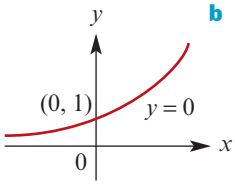
b Range = $(-4, \infty)$



c Range = $(-\infty, -2)$

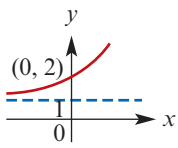


5 a



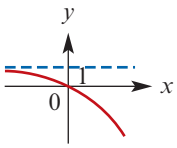
Range = $\mathbb{R}^+$

b



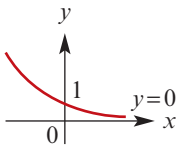
Range = $(1, \infty)$

c



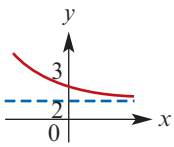
Range = $(-\infty, 1)$

d



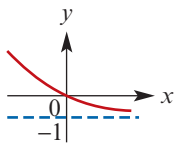
Range = $\mathbb{R}^+$

e



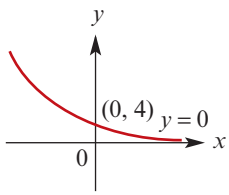
Range = $(2, \infty)$

f

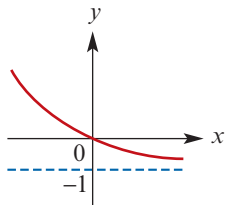


Range = $(-1, \infty)$

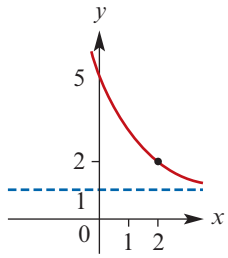
6 a Range = $\mathbb{R}^+$



b Range = $(-1, \infty)$



c Range = $(1, \infty)$



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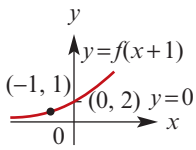

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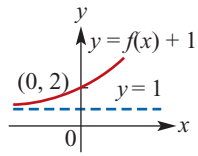

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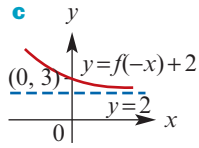

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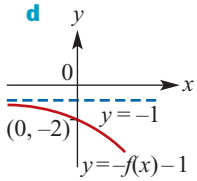

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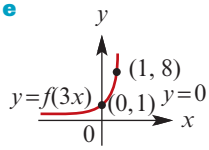

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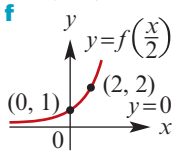

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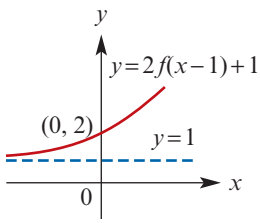

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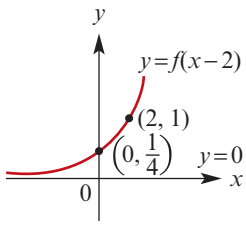
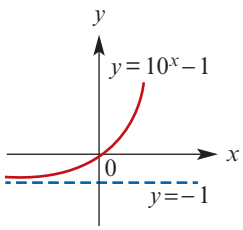


8 a Range = $(-1, \infty)$



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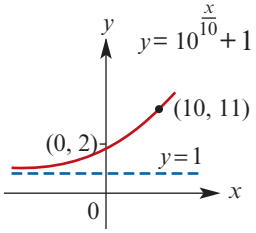
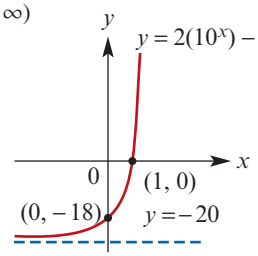
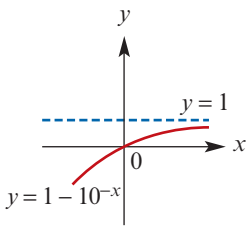
b Range = $(1, \infty)$



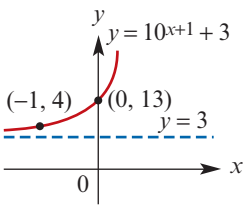
c Range = $(-20, \infty)$



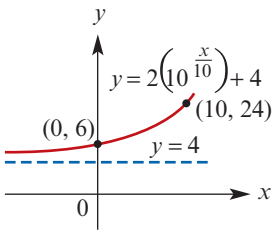
d Range = $(-\infty, 1)$



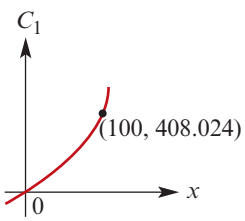
e Range = $(3, \infty)$



f Range = $(4, \infty)$



9 a



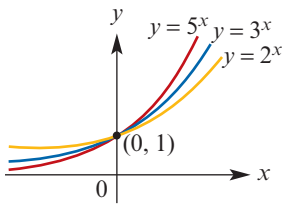
b **i** \$408.02 **ii** \$1274.70

c 239 days

d **ii** 302 days

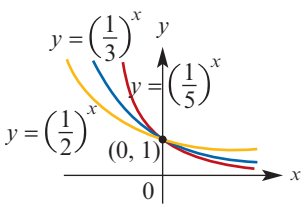
10 36 days

11 a **i**



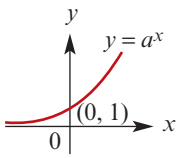
ii $x < 0$ **iii** $x > 0$ **iv** $x = 0$

b **i**

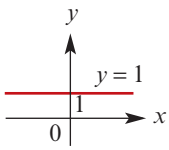


ii $x > 0$ **iii** $x < 0$ **iv** $x = 0$

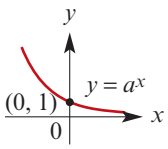
c **i** $a > 1$



ii $a = 1$

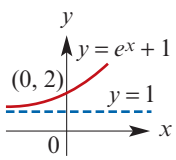


iii $0 < a < 1$



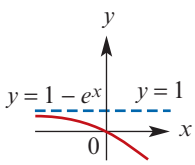
Exercise 5B

1 a



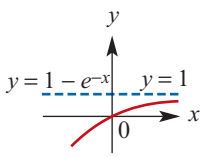
Range = $(1, \infty)$

b



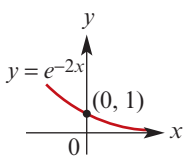
Range = $(-\infty, 1)$

c



Range = $(-\infty, 1)$

d



Range = $(0, \infty)$

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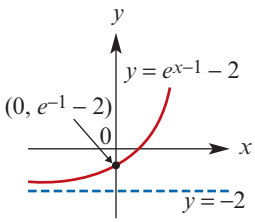
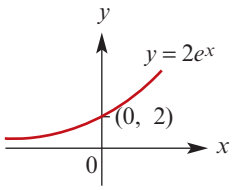
e



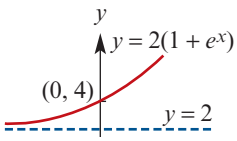
Range = $(-2, \infty)$

f



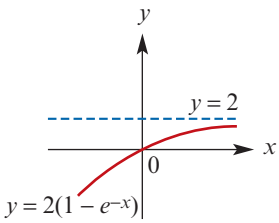
Range = $(0, \infty)$

g



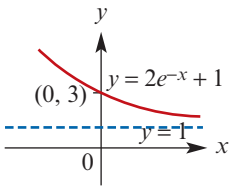
Range = $(2, \infty)$

h



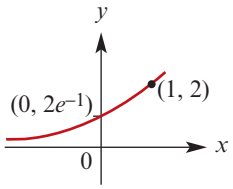
Range = $(-\infty, 2)$

i



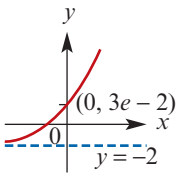
Range = $(1, \infty)$

j



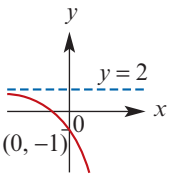
Range = $(0, \infty)$

k



Range = $(-2, \infty)$

l



Range = $(-\infty, 2)$

2 a Translation 2 units to the left and 3 units down

b Dilation of factor 3 from the x-axis, then translation 1 unit to the left and 4 units down

c Dilation of factor 5 from the x-axis and factor $\frac{1}{2}$ from the y-axis, then translation $\frac{1}{2}$ unit to the left

d Reflection in the x-axis, then translation 1 unit to the right and 2 units up

e Dilation of factor 2 from the x-axis, reflection in the x-axis, then translation 2 units to the left and 3 units up

f Dilation of factor 4 from the x-axis and factor $\frac{1}{2}$ from the y-axis, then translation 1 unit down

3 a $y = -2e^{x-3} - 4$ **b** $y = 4 - 2e^{x-3}$

c $y = -2e^{x-3} - 4$ **d** $y = -2e^{x-3} - 8$

e $y = 8 - 2e^{x-3}$ **f** $y = -2e^{x-3} + 8$

4 a Translation 2 units to the right and 3 units up

b Translation 1 unit to the right and 4 units up, then dilation of factor $\frac{1}{3}$ from the x-axis

c Translation $\frac{1}{2}$ unit to the right, then dilation of factor $\frac{1}{5}$ from the x-axis and factor 2 from the y-axis

d Translation 1 unit to the left and 2 units down, then reflection in the x-axis

e Translation 2 units to the right and 3 units down, then dilation of factor $\frac{1}{2}$ from the x-axis and reflection in the x-axis

f Translation 1 unit up, then dilation of factor $\frac{1}{4}$ from the x-axis and factor 2 from the y-axis

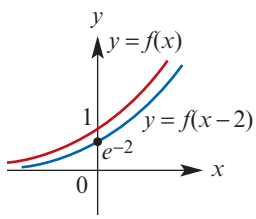
5 a $x = 1.146$ or $x = -1.841$

b $x = -0.443$

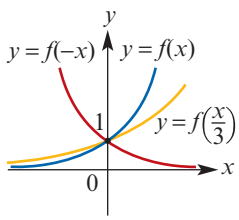
c $x = -0.703$

d $x = 1.857$ or $x = 4.536$

6 a



b



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Exercise 5C

1 a $6x^6y^9$ **b** $3x^6$ **c** $\frac{6y^2}{x^2}$ **d** 8
e 16 **f** $\frac{5x^{28}}{y^6}$ **g** $24x^5y^{10}$ **h** $2xy^2$
i x^2y^2

2 a 4 **b** $\frac{1}{2}$ **c** 8 **d** $\frac{1}{4}$ **e** $\frac{3}{5}$ **f** 3
g $\frac{5}{2}$ **h** 6 **i** 4

3 a 1 **b** 1 **c** $-\frac{3}{2}$ **d** 3 **e** -2 **f** 4
g $-\frac{10}{3}$ **h** $-\frac{3}{2}$ **i** 6 **j** $\frac{3}{5}$ **k** $\pm\frac{1}{2}$

4 a 1 **b** 2 **c** 1 **d** 1, 2 **e** 0, 1
f 2, 4 **g** 0, 1 **h** -1, 2 **i** -1, 0

Exercise 5D

1 a 3 **b** -4 **c** -3 **d** 6 **e** 6 **f** -7

2 a $\log_e 6$ **b** $\log_e 4$
c $\log_e(10^6) = 6\log_e 10$ **d** $\log_e 7$
e $\log_e \frac{1}{60} = -\log_e 60$
f $\log_e(u^3v^6) = 3\log_e(uv^2)$
g $\log_e(x^7) = 7\log_e x$ **h** $\log_e 1 = 0$

3 a $x = 100$ **b** $x = 16$ **c** $x = 6$ **d** $x = 64$
e $x = e^3 - 5 \approx 15.086$ **f** $x = \frac{1}{2}$ **g** $x = -1$
h $x = 10^{-3} = \frac{1}{1000}$ **i** $x = 36$

4 a $x = 15$ **b** $x = 5$ **c** $x = 4$
d $x = 1$ ($x = -\frac{1}{2}$ is not an allowable solution)
e $x = \frac{3}{2}$

5 a $\log_{10} 27$ **b** $\log_2 4 = 2$
c $\frac{1}{2}\log_{10}\left(\frac{a}{b}\right) = \log_{10}\sqrt{\frac{a}{b}}$ **d** $\log_{10}\left(\frac{10a}{b^{\frac{1}{3}}}\right)$
e $\log_{10}\left(\frac{1}{8}\right) = -3\log_{10} 2$

6 a 1 **b** 1 **c** $2\frac{1}{2}$ **d** 3 **e** 0

7 a $-x$ **b** $2\log_2 x$ **c** 0

8 a $x = 4$ **b** $x = \frac{3e}{5 + 2e} \approx 0.7814$

9 a $x = \frac{-1 + \sqrt{1 + 12e}}{6}$, i.e. $x \approx 0.7997$
b $x = \log_e 2 \approx 0.6931$

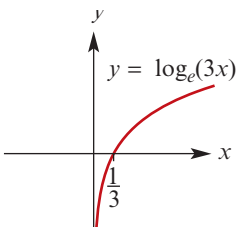
10 a $x = 3$ **b** $x = \frac{1}{2}$

11 $\frac{1}{4}$, 2

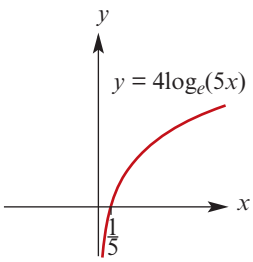
12 $N = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Exercise 5E

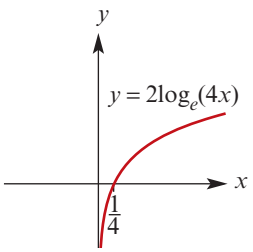
1 a



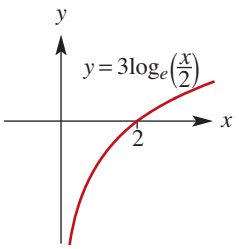
b



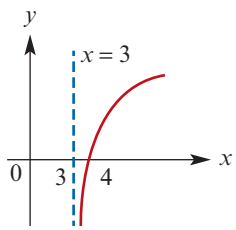
c



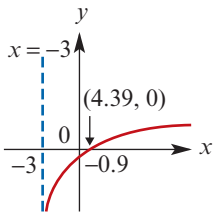
d



2 a Domain = $(3, \infty)$
Range = $\mathbb{R}$



b Domain = $(-3, \infty)$
Range = $\mathbb{R}$



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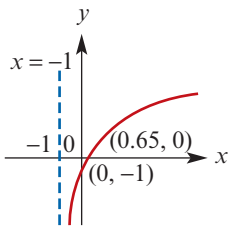
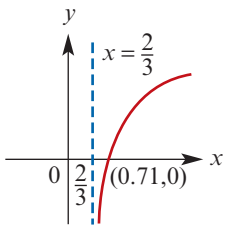
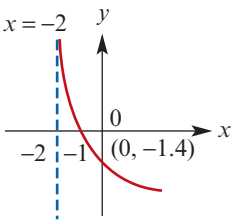
c Domain = $(-1, \infty)$
Range = $\mathbb{R}$



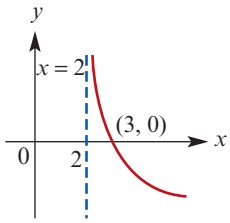
d Domain = $\left(\frac{2}{3}, \infty\right)$
Range = $\mathbb{R}$



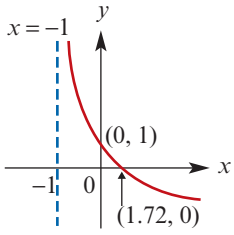
e Domain = $(-2, \infty)$
Range = $\mathbb{R}$



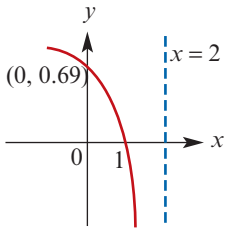
f Domain = $(2, \infty)$
Range = $\mathbb{R}$



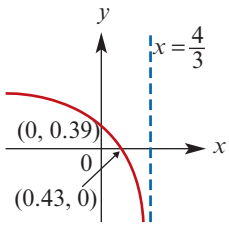
g Domain = $(-1, \infty)$
Range = $\mathbb{R}$



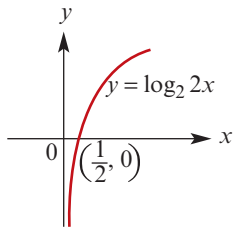
h Domain = $(-\infty, 2)$
Range = $\mathbb{R}$



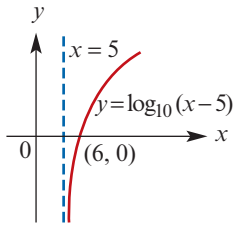
i Domain = $\left(-\infty, \frac{4}{3}\right)$
Range = $\mathbb{R}$



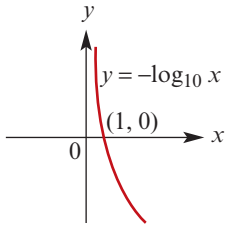
3 a Domain = $\mathbb{R}^+$



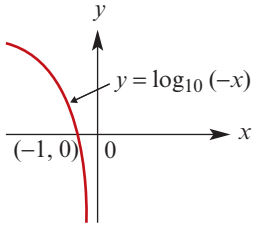
b Domain = $(5, \infty)$



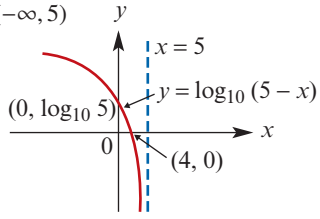
c Domain = $\mathbb{R}^+$



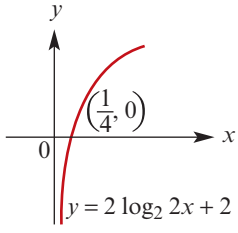
d Domain = $\mathbb{R}^-$



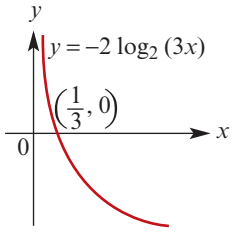
e Domain = $(-\infty, 5)$



f Domain = $\mathbb{R}^+$



g Domain = $\mathbb{R}^+$



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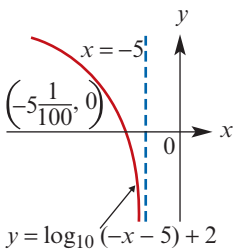
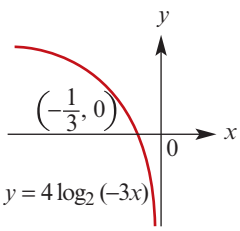
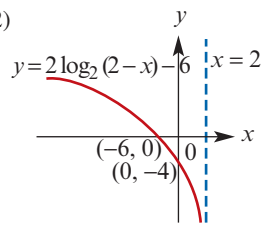
h Domain = $(-\infty, -5)$



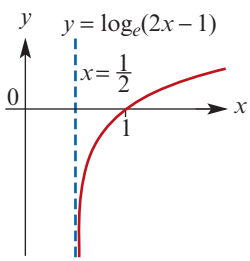
i Domain = $\mathbb{R}^-$



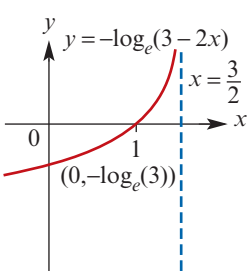
j Domain = $(-\infty, 2)$



k Domain = $(\frac{1}{2}, \infty)$

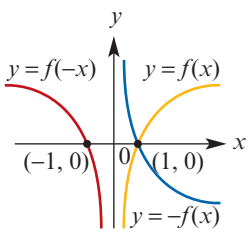


l Domain = $(-\infty, \frac{3}{2})$

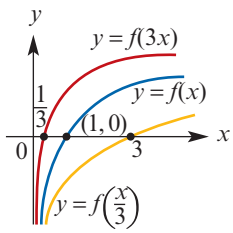


4 a $x = 1.557$
b $x = 1.189$

5 a



b



6 A dilation of factor $\log_e 3$ from the y-axis

7 A dilation of factor $\frac{1}{\log_e 2}$ from the y-axis

Exercise 5F

1 $a = \frac{6}{e^4 - 1}$, $b = \frac{5e^4 - 11}{e^4 - 1}$

2 $a = \frac{2}{\log_e 6}$, $b = -4$ **3** $a = 2$, $b = 4$

4 $a = \frac{14}{e - 1}$, $b = \frac{14}{1 - e}$ $(a \approx 8.148, b \approx -8.148)$

5 $a = 250$, $b = \frac{1}{3}\log_e 5$

6 $a = 200$, $b = 500$ **7** $a = 2$, $b = 4$

8 $a = 3$, $b = 5$ **9** $a = 2$, $b = \frac{1}{3}\log_e 5$

10 $a = 2$, $b = 3$

11 $b = 1$, $a = \frac{2}{\log_e 2}$, $c = 8$ $(a \approx 2.885)$

12 $a = \frac{2}{\log_e 2}$, $b = 4$

Exercise 5G

1 a $k = \frac{1}{\log_2 7}$ **b** $x = \frac{\log_2 7 - 4}{\log_2 7}$

c $x = \frac{\log_e 7 - 1}{\log_e 14}$

2 a 2.58 **b** −0.32 **c** 2.18 **d** 1.16
e −2.32 **f** −0.68 **g** −2.15 **h** −1.38
i 2.89 **j** −1.70 **k** −4.42 **l** 5.76
m −6.21 **n** 2.38 **o** 2.80

3 a $x < 2.81$ **b** $x > 1.63$ **c** $x < -0.68$
d $x \leq 3.89$ **e** $x \geq 0.57$

4 a $\log_2 5$ **b** $\frac{1}{2}(\log_3 8 + 1)$

c $\frac{1}{3}(\log_7 20 - 1)$ **d** $\log_3 7$ **e** $\log_3 6$

f $\log_5 6$ **g** $x = \log_3 8$ or $x = 0$ **h** $x = 1$

5 a $x > \log_7 52$ **b** $x < \frac{1}{2}\log_3 120$

c $x \geq \frac{1}{3}\log_2\left(\frac{5}{4}\right)$ **d** $x \leq \log_3 7290$

e $x < \log_3 106$ **f** $x < \log_5\left(\frac{3}{5}\right)$

6 a 0.544 **b** 549.3

8 a 9^u **b** $u + \frac{1}{2}$ **c** $\frac{2}{u}$

9 625, $\frac{1}{625}$ **10** $\frac{2}{p}$

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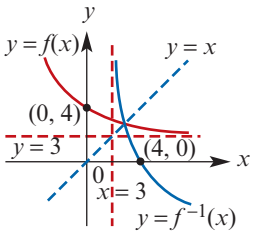
Exercise 5H

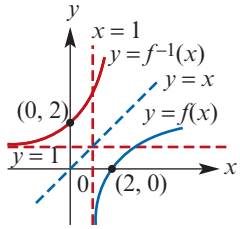
1 $f^{-1}: (-2, \infty) \to \mathbb{R}$, $f^{-1}(x) = \log_e(x + 2)$

2 $f(x) = e^{-x} + 3$, $f^{-1}(x) = -\log_e(x - 3)$



3 $f(x) = \log_e(x - 1)$

$f^{-1}(x) = e^x + 1$



4 $x = e^{\frac{y+4}{3}}$

5 a $f^{-1}(x) = \frac{1}{2}e^x$, dom $= \mathbb{R}$, ran $= \mathbb{R}^+$

b $f^{-1}(x) = \frac{1}{2}e^{\frac{x-1}{3}}$, dom $= \mathbb{R}$, ran $= \mathbb{R}^+$

c $f^{-1}(x) = \log_e(x - 2)$, dom $= (2, \infty)$, ran $= \mathbb{R}$

d $f^{-1}(x) = \log_e(x) - 2$, dom $= \mathbb{R}^+$, ran $= \mathbb{R}$

e $f^{-1}(x) = \frac{1}{2}(e^x - 1)$,

dom $= \mathbb{R}$, ran $= \left(-\frac{1}{2}, \infty\right)$

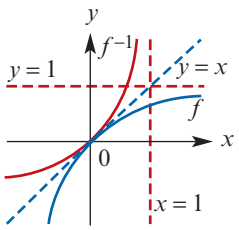
f $f^{-1}(x) = \frac{1}{3}\left(e^{\frac{x}{4}} - 2\right)$,

dom $= \mathbb{R}$, ran $= \left(-\frac{2}{3}, \infty\right)$

g $f^{-1}(x) = 10^x - 1$, dom $= \mathbb{R}$, ran $= (-1, \infty)$

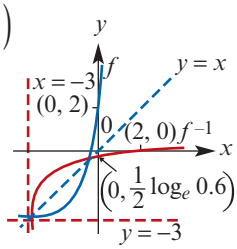
h $f^{-1}(x) = \log_e\left(\frac{x}{2}\right) + 1$, dom $= \mathbb{R}^+$, ran $= \mathbb{R}$

6 $f^{-1}(x) = -\log_e(1 - x)$, dom $f^{-1} = (-\infty, 1)$



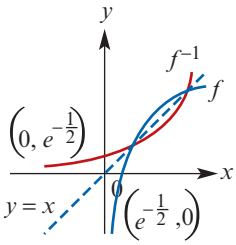
7 $f^{-1}(x) = \frac{1}{2}\log_e\left(\frac{x+3}{5}\right)$

dom $f^{-1} = (-3, \infty)$



8 $f^{-1}(x) = e^{\frac{x-1}{2}}$

ran $f^{-1} = \mathbb{R}^+$



9 $t = \frac{-1}{k}\log_e\left(\frac{P-b}{A}\right)$

10 a $x = e^{\frac{y-5}{2}}$ **b** $x = -\frac{1}{6}\log_e\left(\frac{P}{A}\right)$

c $n = \frac{\log_e\left(\frac{y}{a}\right)}{\log_e x}$ **d** $x = \log_{10}\left(\frac{y}{5}\right)$

e $x = \frac{1}{2}e^{\frac{5-y}{3}}$ **f** $n = \frac{1}{2}\left(\frac{\log_e\left(\frac{y}{6}\right)}{\log_e x}\right)$

g $x = \frac{1}{2}(e^y + 1)$ **h** $x = \log_e\left(\frac{5}{5-y}\right)$

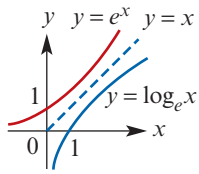
11 a $f^{-1}(x) = \log_e\left(\frac{x+4}{2}\right)$

b (0.895, 0.895), (−3.962, −3.962)

12 a $f^{-1}(x) = e^{\frac{x-4}{3}} - 3$

b (8.964, 8.964), (−2.969, −2.969)

13 a i



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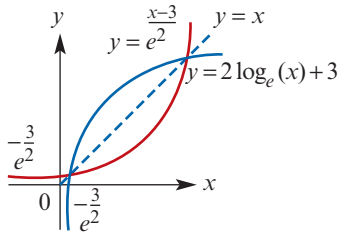
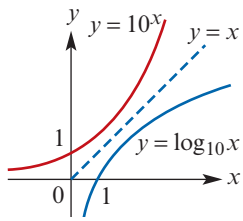


iii



b f and g are inverse functions

Exercise 5I

1 a $N = 1000 \times 2^{\frac{t}{15}}$ **b** 50 minutes

2 $d_0 = 52\left(\frac{13}{20}\right)^{\frac{1}{2}}$, $m = \frac{1}{2}\log_{10}\left(\frac{20}{13}\right)$

3 a i $N_0 = 20\,000$ **ii** −0.223

b 6.2 years

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4 a $M_0 = 10,\ k = 4.95 \times 10^{-3}$
b 7.07 grams **c** 325 days

5 a $k = \frac{1}{1690}\log_e 2$ **b** 3924 years

6 55 726 years

7 7575 years

8 a 16 600 **b** 33 years on from 2002

9 18.4 years

10 a 607 millibars **b** 6.389 km

11 21.82 hours

12 6.4°C

13 $k = 0.349,\ N_0 = 50.25$

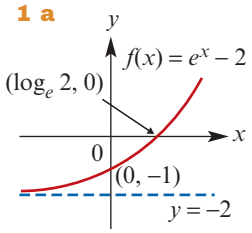
14 a $k = \log_e\left(\frac{5}{4}\right)$ **b** 7.21 hours

15 a $a = 1000,\ b = 15^{\frac{1}{5}}$ **b** 3 hours **c** 13 hours
d 664 690

Chapter 5 review

Technology-free questions

1 a



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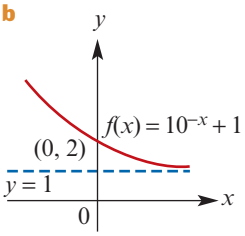

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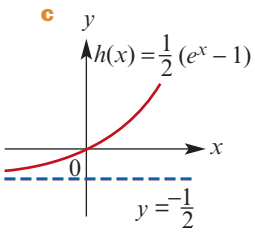

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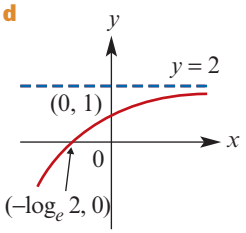

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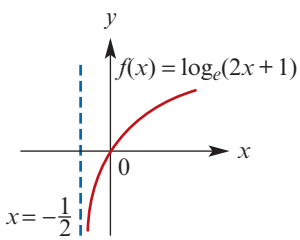

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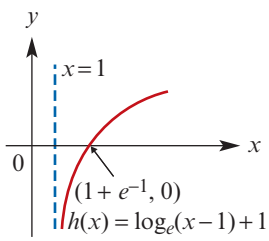

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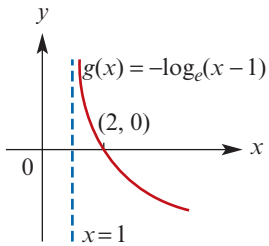

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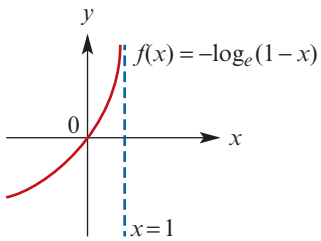

2 a $f^{-1}: (-1, \infty) \to \mathbb{R},\ f^{-1}(x) = \frac{1}{2}\log_e(x + 1)$
b $f^{-1}: \mathbb{R} \to \mathbb{R},\ f^{-1}(x) = e^{\frac{x}{3}} + 2$
c $f^{-1}: \mathbb{R} \to \mathbb{R},\ f^{-1}(x) = 10^x - 1$
d $f^{-1}: (2, \infty) \to \mathbb{R},\ f^{-1}(x) = \log_2(x - 1)$

3 a $y = e^2x$ **b** $y = 10x$ **c** $y = 16x^3$
d $y = \frac{x^5}{10}$ **e** $y = \frac{e^3}{x}$ **f** $y = e^{2x-3}$

4 a $x = \frac{\log_e 11}{\log_e 3}$ **b** $x = \frac{\log_e 0.8}{\log_e 2}$
c $x = \frac{\log_e 3}{\log_e(\frac{2}{3})}$

5 a $x = 1$ **b** $x = \frac{2}{3}$ **c** $x = \frac{1}{20}$
d $x = \log_{10} 3$ or $x = \log_{10} 4$ **e** $x = \log_e\left(\frac{2}{3}\right)$
f $x = \frac{10}{7}$

6 $a = 2^{-\frac{2}{3}} - 1,\ b = 2,$ **7** $10^{\frac{6}{5}} - 1$

8 $\frac{1}{3}\log_e\left(\frac{287}{4}\right)$ **9** $2a$ **11** 3

12 a $k = \frac{1}{7}$ **b** $q = \frac{3}{2}$

13 $y = e^a x^b$

14 $\mathbb{R}$

15 $a = \log_e 5,\ b = 5,\ k = 2$

16 $k = \log_e 3$

17 a $f^{-1}(x) = \frac{1}{3}\log_e(x + 4),\ \text{dom}\, f^{-1} = (-4, \infty)$
b $\frac{1}{3x + 4} - 4$

18 $k = 9$

19 a $-2, 1, 4$ **b** $0, \log_e 4$

20 a $f(g(x)) = \log_e(2x^2 + 4)$
Domain $= \mathbb{R}$, Range $= [\log_e 4, \infty)$
b $h^{-1}(x) = -\sqrt{\frac{1}{2}(e^x - 4)}$,
Domain $= [\log_e 4, \infty)$, Range $= \mathbb{R}^- \cup \{0\}$

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21 a $x = 2$ or $x = 3$ **b** $x = 8$ or $x = 4$
c $x = 16$ or $x = 256$

22 $a = -1 + \sqrt{5}$

24 $x = \frac{1}{2}\log_e 2$ or $x = -\frac{1}{2}\log_e 2$

25 a $c = -2,\ d = -2$ **b** $\left(-\log_3(4), \frac{1}{4}\right)$
c $f^{-1} : (-2, \infty) \to \mathbb{R},\ f^{-1}(x) = \log_3(x + 2) - 2$

26 a $f(-x) = f(x)$ **b** $2(e^u + e^{-u})$
c 0 **d** $e^{2u} + e^{-2u}$
e $g(-x) = -g(x)$ **f** $2e^x,\ 2e^{-x},\ e^{2x} - e^{-2x}$

Multiple-choice questions

1 C **2** D **3** B **4** E **5** A **6** C
7 B **8** A **9** C **10** D **11** A **12** C
13 C **14** D **15** B **16** D

Extended-response questions

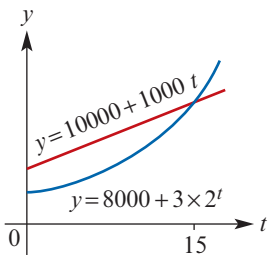
1 a 73.5366°C **b** 59.5946

2 a 770 **b** 1840

3 a $k = 22\,497,\ \lambda = 0.22$ **b** \$11 612

4 a $A = 65\,000,\ p = 0.064$ **b** \$47 200

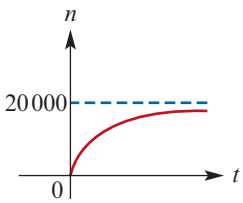
5 a



b i (12.210, 22 209.62)
ii $t = 12.21$ **iii** 22 210
c ii (12.21, 12.21)
d $c = 0.52$

6 a iii $a = \frac{1}{2}$ or $a = 1$
iv If $a = 1$, then $e^{-2B} = 1$, and so $B = 0$;
If $a = \frac{1}{2}$, then $B = \frac{1}{2}\log_e 2$
v $A = 20\,000$

b

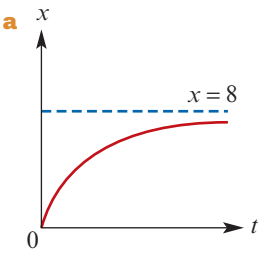


c $\frac{\log_e 0.1}{\frac{1}{2}\log_e(\frac{1}{2})} = \frac{2\log_e 10}{\log_e 2} \approx 6.644$
After 6.65 hours, the population is 18 000

7 a 75 **b** 2.37 **c** 0.646

8 $k = -0.579,\ A_0 = 108.3$

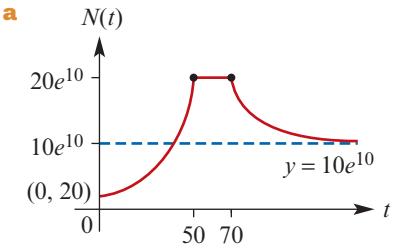
9 a



b i 0 grams **ii** 2.64 grams **iii** 6.92 grams
c 10.4 minutes

10 a $k = 0.235$ **b** 22.7°C **c** 7.18 minutes

11 a



b i $N(10) = 147.78$
ii $N(40) = 59\,619.16$
iii $N(60) = 20e^{10} = 440\,529.32$
iv $N(80) = 220\,274.66$
c i 25 days **ii** 35 days

Chapter 6

Exercise 6A

1 a $\frac{5\pi}{18}$ **b** $\frac{34\pi}{45}$ **c** $\frac{25\pi}{18}$
d $\frac{17\pi}{9}$ **e** $\frac{7\pi}{3}$ **f** $\frac{49\pi}{18}$

2 a 60° **b** 150° **c** 240°
d 140° **e** 630° **f** 252°

3 a 45.84° **b** 93.97° **c** 143.24°
d 226.89° **e** 239.50° **f** 340.91°

4 a 0.65 **b** 1.29 **c** 2.01
d 2.13 **e** 5.93 **f** 2.31

Exercise 6B

1 a 0 **b** 0 **c** −1 **d** −1 **e** 0
f 0 **g** 0 **h** 0 **i** −1 **j** 0
k −1 **l** 0 **m** 0 **n** 1

2 a 0.99 **b** 0.52 **c** −0.87 **d** 0.92
e −0.67 **f** −0.23 **g** −0.99 **h** 0.44
i −34.23 **j** −2.57 **k** 0.95 **l** 0.75

3 a $\frac{1}{\sqrt{2}}$ **b** $-\frac{1}{2}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $\frac{1}{2}$
e $-\frac{1}{2}$ **f** $-\frac{1}{\sqrt{2}}$ **g** $-\frac{1}{\sqrt{2}}$ **h** $\frac{1}{2}$
i $\frac{1}{2}$ **j** $\frac{\sqrt{3}}{2}$ **k** $\frac{1}{2}$ **l** $\frac{\sqrt{3}}{2}$

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m $\frac{1}{\sqrt{2}}$ **n** $-\frac{1}{2}$ **o** $\frac{1}{\sqrt{2}}$ **p** $-\frac{1}{2}$

q $-\sqrt{3}$ **r** $-\sqrt{3}$ **s** $\frac{1}{\sqrt{3}}$ **t** $\sqrt{3}$

u -1 **v** $-\frac{1}{\sqrt{3}}$

4 a 0.52 **b** −0.68 **c** 0.52 **d** 0.4
e −0.52 **f** 0.68 **g** −0.4 **h** −0.68
i −0.52 **j** 0.68 **k** −0.4

5 a 0.4 **b** −0.7 **c** 0.4 **d** 1.2
e −0.4 **f** 0.7 **g** −1.2 **h** −0.7
i −0.4 **j** 0.7 **k** −1.2

6 a $\frac{1}{2}, -\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$ **b** $-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 1$
c $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$ **d** $-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \sqrt{3}$
e $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$ **f** $-\frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$

Exercise 6C

1 a 0.6 **b** 0.6 **c** −0.7 **d** 0.3
e −0.3 **f** $\frac{10}{7}$ **g** −0.3 **h** 0.6
i −0.6 **j** −0.3 **k** $\frac{10}{7}$ **l** 0.3

2 a $-\frac{4}{5}, -\frac{4}{3}$ **b** $-\frac{12}{13}, -\frac{5}{12}$
c $-\frac{2\sqrt{6}}{5}, -2\sqrt{6}$ **d** $-\frac{5}{13}, \frac{12}{5}$
e $-\frac{3}{5}, -\frac{3}{4}$ **f** $-\frac{12}{13}, \frac{5}{12}$ **g** $-\frac{3}{5}, -\frac{3}{4}$

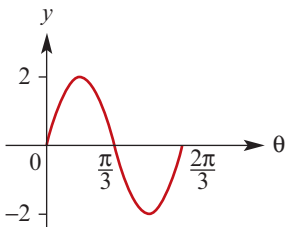
Exercise 6D

1 a $2\pi, 3$ **b** $\frac{2\pi}{3}, 5$ **c** $\pi, \frac{1}{2}$ **d** $6\pi, 2$
e $\frac{\pi}{2}, 3$ **f** $2\pi, \frac{1}{2}$ **g** $4\pi, 3$ **h** $3\pi, 2$

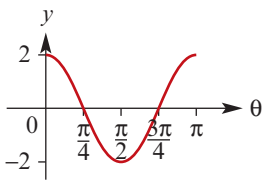
2 a Dilation of factor 4 from the x-axis, dilation of factor $\frac{1}{3}$ from the y-axis;
Amplitude = 4; Period = $\frac{2\pi}{3}$
b Dilation of factor 5 from the x-axis, dilation of factor 3 from the y-axis;
Amplitude = 5; Period = 6π
c Dilation of factor 6 from the x-axis, dilation of factor 2 from the y-axis;
Amplitude = 6; Period = 4π
d Dilation of factor 4 from the x-axis, dilation of factor $\frac{1}{5}$ from the y-axis;
Amplitude = 4; Period = $\frac{2\pi}{5}$

3 a Dilation of factor 2 from the x-axis, dilation of factor $\frac{1}{3}$ from the y-axis;
Amplitude = 2; Period = $\frac{2\pi}{3}$
b Dilation of factor 3 from the x-axis, dilation of factor 4 from the y-axis;
Amplitude = 3; Period = 8π
c Dilation of factor 6 from the x-axis, dilation of factor 5 from the y-axis;
Amplitude = 6; Period = 10π
d Dilation of factor 3 from the x-axis, dilation of factor $\frac{1}{7}$ from the y-axis;
Amplitude = 3; Period = $\frac{2\pi}{7}$

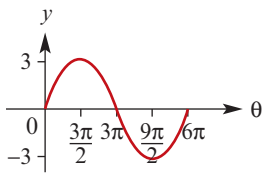
4 a Amplitude = 2
Period = $\frac{2\pi}{3}$



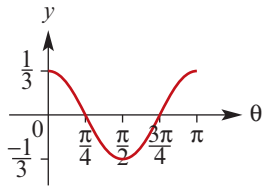
b Amplitude = 2
Period = π



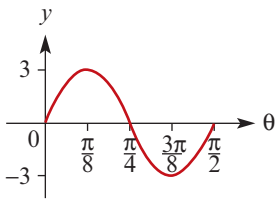
c Amplitude = 3
Period = 6π



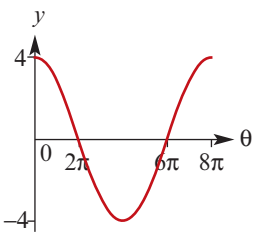
d Period = π
Amplitude = $\frac{1}{3}$



e Amplitude = 3
Period = $\frac{\pi}{2}$



f Amplitude = 4
Period = 8π



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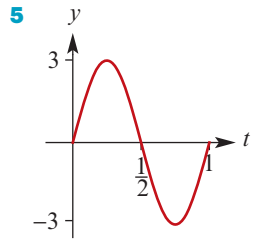

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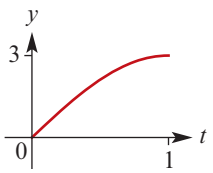

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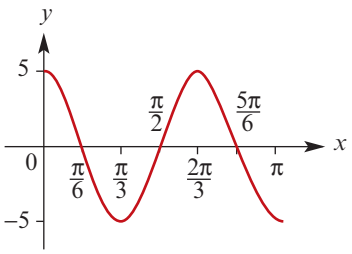

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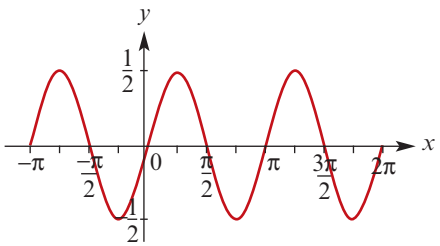

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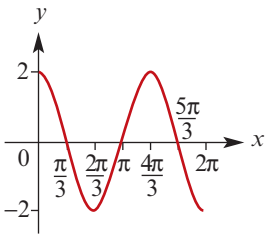

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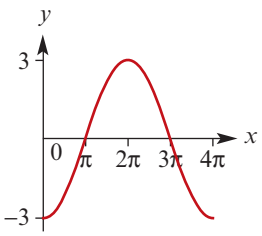

11 $y = 2\sin\left(\frac{x}{3}\right)$ **12** $y = \frac{1}{2}\cos\left(\frac{x}{3}\right)$

13 $y = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

Exercise 6E

1 a $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ **b** $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$
c $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$ **d** $\frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$
e $\frac{\pi}{2}, \frac{5\pi}{2}$ **f** $\pi, 3\pi$

2 a $-\frac{5\pi}{6}, -\frac{\pi}{6}$ **b** $-\frac{\pi}{6}, \frac{\pi}{6}$ **c** $-\frac{5\pi}{6}, \frac{5\pi}{6}$

3 a $\frac{\pi}{4}, \frac{3\pi}{4}$ **b** $\frac{3\pi}{4}, \frac{5\pi}{4}$ **c** $\frac{5\pi}{6}, \frac{7\pi}{6}$
d $\frac{7\pi}{6}, \frac{11\pi}{6}$ **e** $\frac{\pi}{4}, \frac{7\pi}{4}$ **f** $\frac{2\pi}{3}, \frac{4\pi}{3}$

4 a 0.643501, 2.49809 **b** 0.643501, 5.63968
c 3.60836, 5.81642 **d** 1.77215, 4.51103

5 a 17.46°, 162.54° **b** 66.42°, 293.58°
c 233.13°, 306.87° **d** 120°, 240°

6 a 60°, 300° **b** 60°, 120° **c** 225°, 315°
d 120°, 240° **e** 60°, 120° **f** 150°, 210°

7 a $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$
b $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$
c $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
d $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{23\pi}{12}$
e $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$ **f** $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

8 a $\frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$
b $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$
c $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$
d $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$
e $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$
f $\frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$
g $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{23\pi}{12}$
h $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
i $\frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

9 a 2.03444, 2.67795, 5.17604, 5.81954
b 1.89255, 2.81984, 5.03414, 5.96143
c 0.57964, 2.56195, 3.72123, 5.70355
d 0.309098, 1.7853, 2.40349, 3.87969, 4.49789, 5.97409

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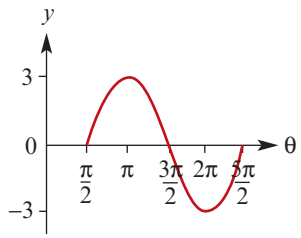
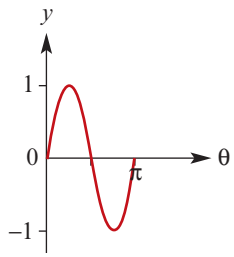
Exercise 6F

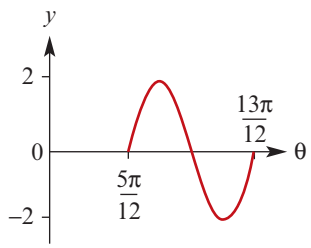
1 a Period = 2π; Amplitude = 3; $y = \pm 3$



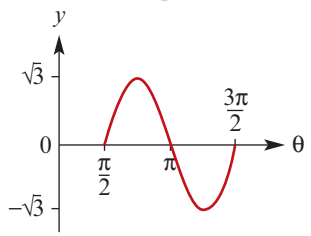
b Period = π; Amplitude = 1; $y = \pm 1$



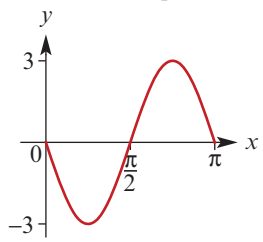
c Period = $\frac{2\pi}{3}$; Amplitude = 2; $y = \pm 2$



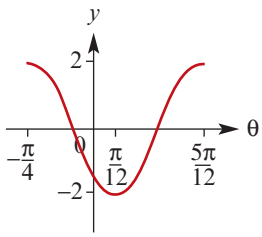
d Period = π; Amplitude = $\sqrt{3}$; $y = \pm\sqrt{3}$



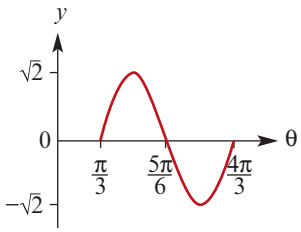
e Period = π; Amplitude = 3; $y = \pm 3$



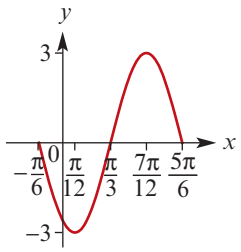
f Period = $\frac{2\pi}{3}$; Amplitude = 2; $y = \pm 2$



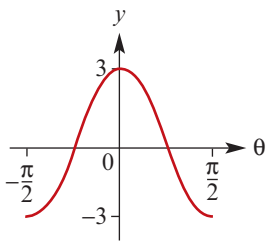
g Period = π; Amplitude = $\sqrt{2}$; $y = \pm\sqrt{2}$



h Period = π; Amplitude = 3; $y = \pm 3$

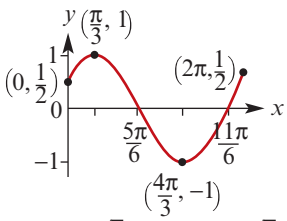


i Period = π; Amplitude = 3; $y = \pm 3$



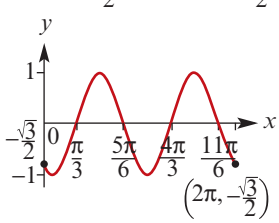
2 a $f(0) = \frac{1}{2}$, $f(2\pi) = \frac{1}{2}$

b



3 a $f(0) = -\frac{\sqrt{3}}{2}$, $f(2\pi) = -\frac{\sqrt{3}}{2}$

b



4 a $f(-\pi) = -\frac{1}{\sqrt{2}}$, $f(\pi) = -\frac{1}{\sqrt{2}}$

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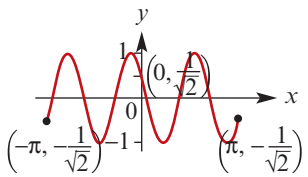
b



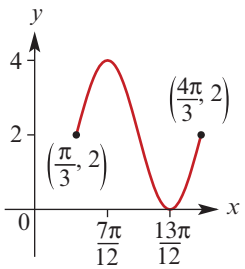
5 a $y = 3\sin\left(\frac{x}{2}\right)$ **b** $y = 3\sin(2x)$

c $y = 2\sin\left(\frac{x}{3}\right)$ **d** $y = \sin 2\left(x - \frac{\pi}{3}\right)$

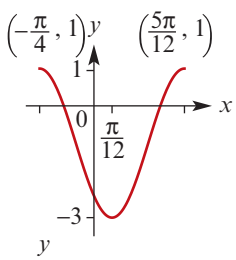
e $y = \sin\frac{1}{2}\left(x + \frac{\pi}{3}\right)$

Exercise 6G

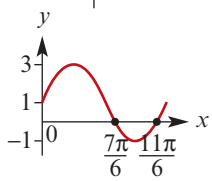
1 a



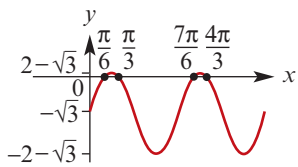
b



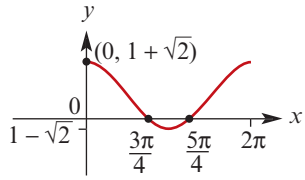
2 a



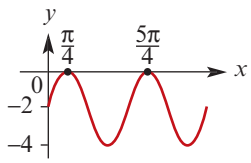
b



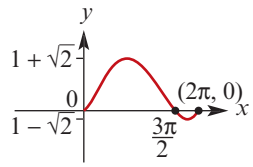
c



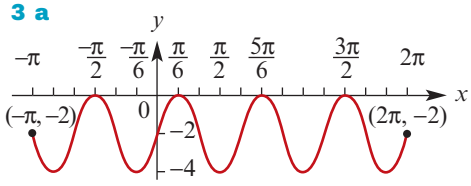
d



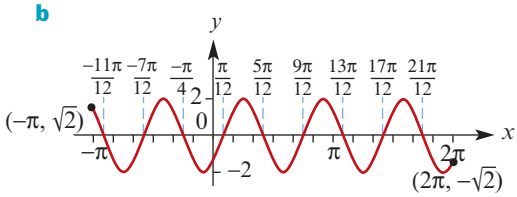
e



3 a



b



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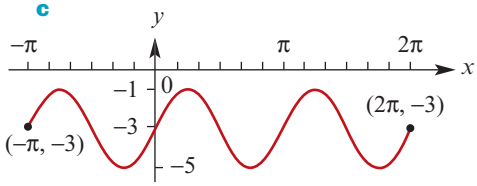

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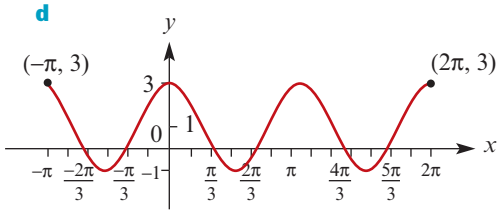
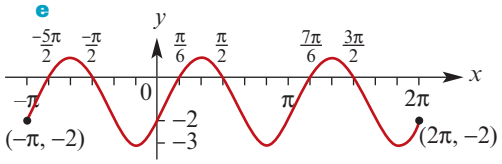
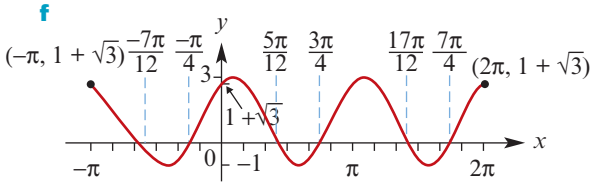


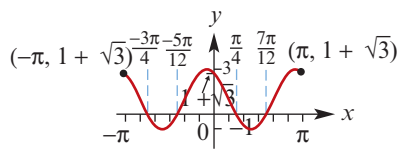
e



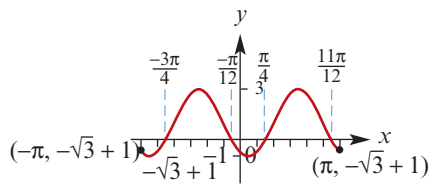
f



4 a



b



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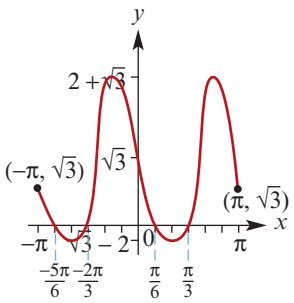
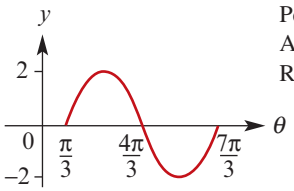
c

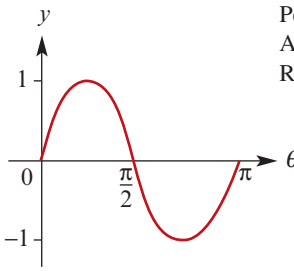


5 a



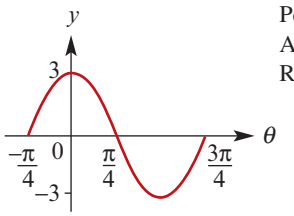
Period = 2π
Amplitude = 2
Range = $[-2, 2]$

b



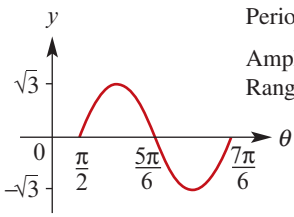
Period = π
Amplitude = 1
Range = $[-1, 1]$

c



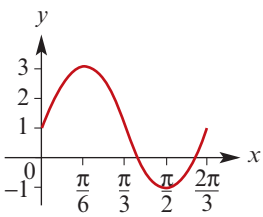
Period = π
Amplitude = 3
Range = $[-3, 3]$

d



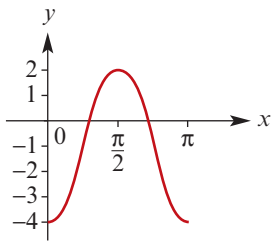
Period = $\frac{2\pi}{3}$
Amplitude = $\sqrt{3}$
Range = $[-\sqrt{3}, \sqrt{3}]$

e



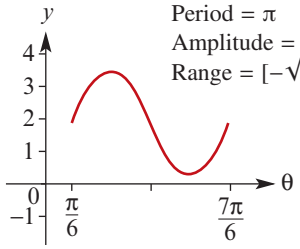
Period = $\frac{2\pi}{3}$
Amplitude = 2
Range = $[-1, 3]$

f



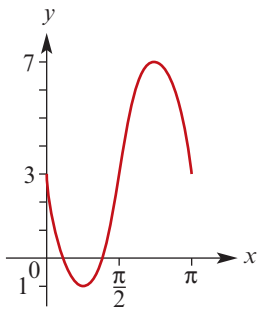
Period = π
Amplitude = 3
Range = $[-4, 2]$

g



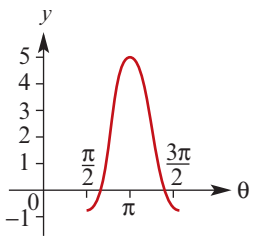
Period = π
Amplitude = $\sqrt{2}$
Range = $[-\sqrt{2}+2, 2+\sqrt{2}]$

h



Period = π
Amplitude = 4
Range = $[-1, 7]$

i



Period = π
Amplitude = 3
Range = $[-1, 5]$

6 a $y = \frac{1}{2}\cos\left(\frac{1}{3}\left(x - \frac{\pi}{4}\right)\right)$

b $y = 2\cos\left(x - \frac{\pi}{4}\right)$

c $y = -\frac{1}{3}\cos\left(x - \frac{\pi}{3}\right)$

7 a
- Dilation of factor 3 from the x-axis
- Dilation of factor $\frac{1}{2}$ from the y-axis
- Reflection in the x-axis

b
- Dilation of factor 3 from the x-axis
- Dilation of factor $\frac{1}{2}$ from the y-axis
- Reflection in the x-axis
- Translation $\frac{\pi}{3}$ units to the right

c
- Dilation of factor 3 from the x-axis
- Dilation of factor $\frac{1}{2}$ from the y-axis
- Translation $\frac{\pi}{3}$ units to the right and 2 units up

d
- Dilation of factor 2 from the x-axis
- Dilation of factor $\frac{1}{2}$ from the y-axis

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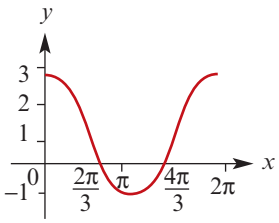
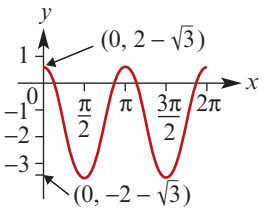
- Reflection in the x-axis
- Translation $\frac{\pi}{3}$ units to the right and 5 units up

8 a



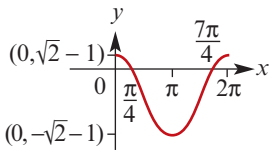
Intercepts: $\left(\frac{2\pi}{3}, 0\right), \left(\frac{4\pi}{3}, 0\right)$

b



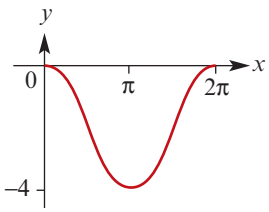
Intercepts: $\left(\frac{\pi}{12}, 0\right), \left(\frac{11\pi}{12}, 0\right), \left(\frac{13\pi}{12}, 0\right), \left(\frac{23\pi}{12}, 0\right)$

c



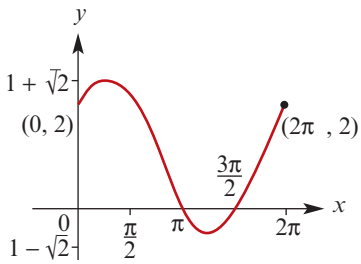
Intercepts: $\left(\frac{\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

d



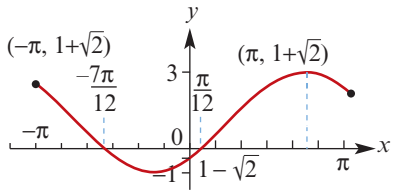
Intercepts: $(0, 0), (2\pi, 0)$

e

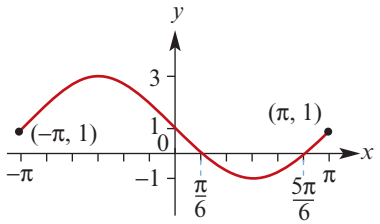


Intercepts: $(\pi, 0), \left(\frac{3\pi}{2}, 0\right)$

9 a



b



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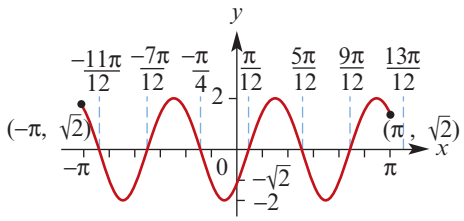
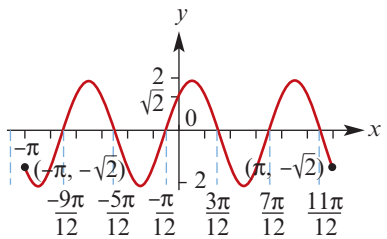
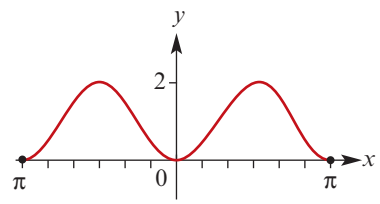


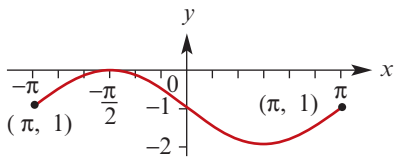
d



e

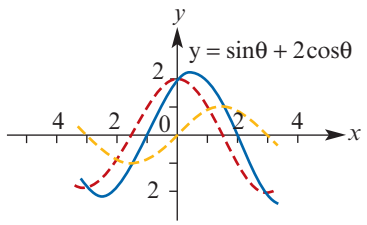


f



Exercise 6H

1 a



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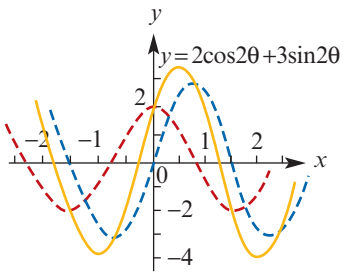
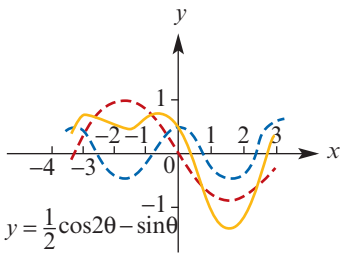
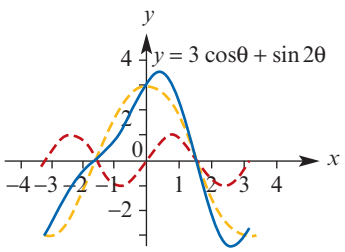
b



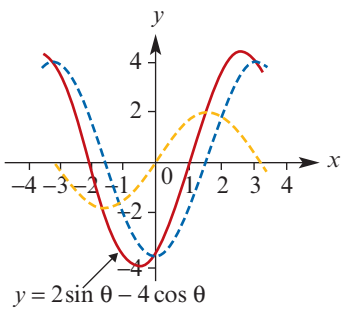
c



d



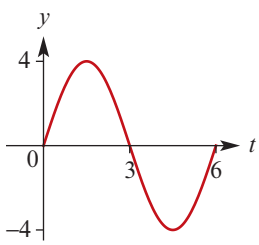
e



Exercise 6I

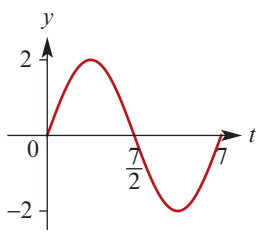
1 a $n = \frac{\pi}{3}$

$A = 4$



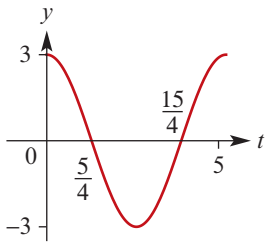
b $n = \frac{2\pi}{7}$

$A = 2$



c $n = \frac{2\pi}{5}$

$A = 3$



2 $A = 3,\ n = \frac{\pi}{4}$

3 $A = -4,\ n = \frac{\pi}{6}$

4 $A = 0.5,\ \varepsilon = \frac{-\pi}{3}$

5 $A = 3,\ n = 3,\ b = 5$

6 $A = 4,\ n = \frac{\pi}{4},\ \varepsilon = \frac{-\pi}{2}$

(*Note:* ε can take infinitely many values)

7 $A = 2,\ n = \frac{\pi}{3},\ \varepsilon = \frac{-\pi}{6}$

(*Note*: ε can take infinitely many values)

8 $A = 4,\ n = \frac{\pi}{4},\ d = 2,\ \varepsilon = \frac{-\pi}{2}$

(*Note*: ε can take infinitely many values)

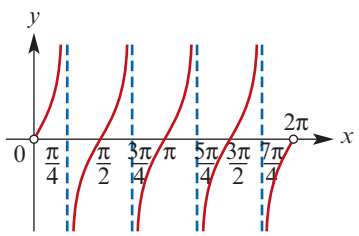
9 $A = 2,\ n = \frac{\pi}{3},\ d = 2,\ \varepsilon = \frac{-\pi}{6}$

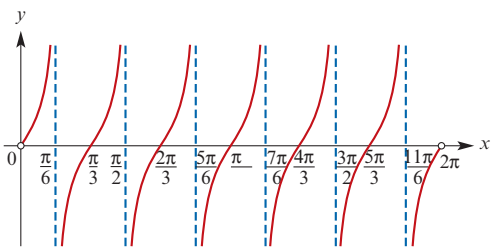
(*Note:* ε can take infinitely many values)

Exercise 6J

1 a $\frac{\pi}{3}$ **b** 2π **c** $\frac{2\pi}{3}$ **d** 1 **e** 2

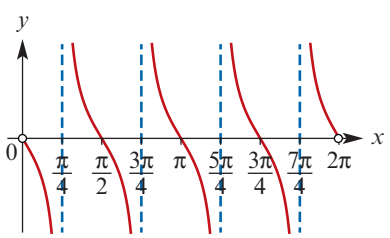
2 a





b

c



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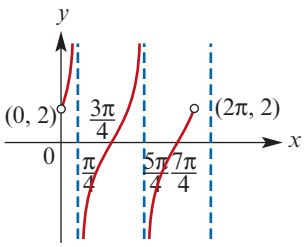
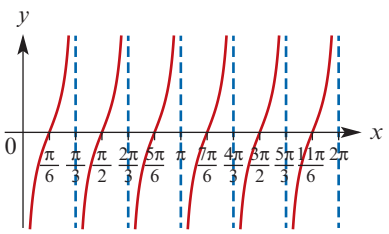
3 a



b



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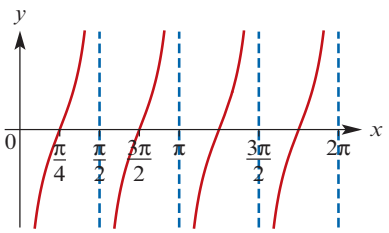


4 a $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ **b** $-\frac{5\pi}{8}, -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$

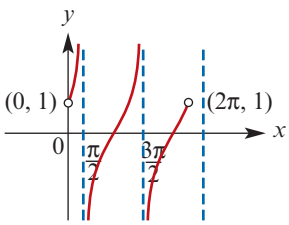
c $-\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}$ **d** $-\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$

e $-\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$

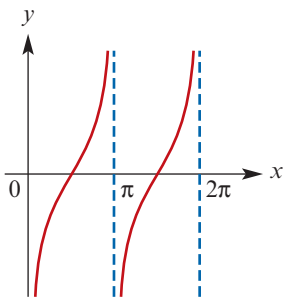
5 $\frac{11\pi}{24}, \frac{23\pi}{24}, \frac{35\pi}{24}, \frac{47\pi}{24}$

6 $\frac{7\pi}{12}, \frac{19\pi}{12}$

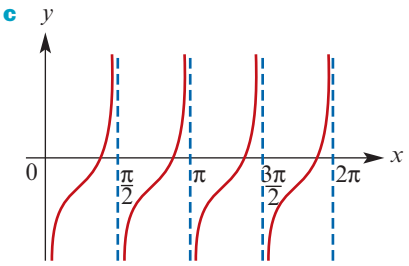
7 a



b



c



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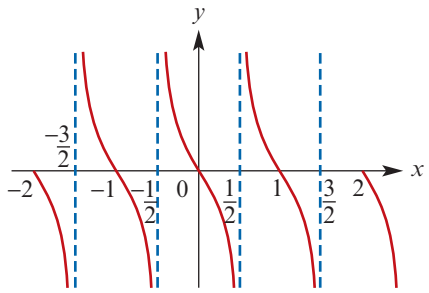

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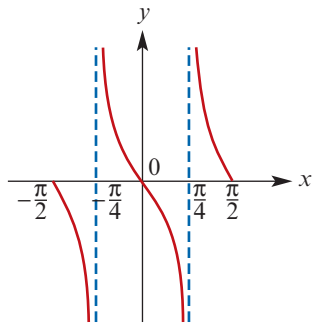
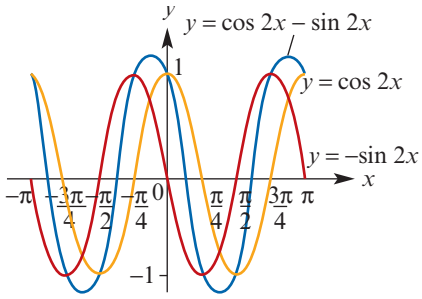


10 a c



b $\left(\frac{-5\pi}{8}, \frac{-1}{\sqrt{2}}\right), \left(\frac{-\pi}{8}, \frac{1}{\sqrt{2}}\right), \left(\frac{3\pi}{8}, \frac{-1}{\sqrt{2}}\right), \left(\frac{7\pi}{8}, \frac{1}{\sqrt{2}}\right)$

11 a $\frac{\pi}{6}, \frac{7\pi}{6}$

b $\frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}$

c $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

d $\frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$

e $\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}$

f 0.4636, 3.6052

g 1.1071, 4.2487

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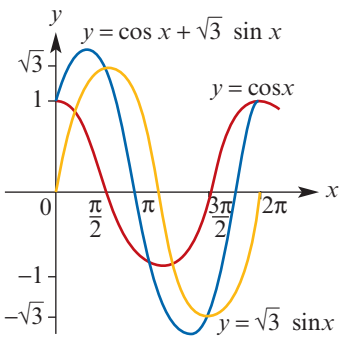
h $\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

i $\frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

j $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$

12 a c



b $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right), \left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2}\right)$

13 a $\frac{7\pi}{24}, \frac{19\pi}{24}, \frac{31\pi}{24}, \frac{43\pi}{24}$

b $\frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$

c $\frac{11\pi}{36}, \frac{23\pi}{36}, \frac{35\pi}{36}, \frac{47\pi}{36}, \frac{59\pi}{36}, \frac{71\pi}{36}$

14 $A = 5,\ n = 3$

15 $A = 6,\ n = \frac{\pi}{2}$

Exercise 6K

1 a i 2π **ii** 4π **iii** -4π

b i $\frac{4\pi}{3}, \frac{8\pi}{3}$ **ii** $\frac{14\pi}{3}, \frac{10\pi}{3}$ **iii** $\frac{-14\pi}{3}, \frac{-10\pi}{3}$

2 a $2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$

b $\frac{2n\pi}{3} + \frac{\pi}{9}$ or $\frac{2n\pi}{3} + \frac{2\pi}{9}, n \in \mathbb{Z}$

c $n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

3 a $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\frac{\pi}{12}, \frac{11\pi}{12}$ **c** $\frac{\pi}{3}, \frac{5\pi}{6}$

4 $\frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

5 $\frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

6 a $x = n\pi - \frac{\pi}{6}$ or $x = n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

b $x = \frac{n\pi}{2} - \frac{\pi}{12}, n \in \mathbb{Z}$

c $x = 2n\pi + \frac{5\pi}{6}$ or $x = 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$

7 $x = \frac{(4n-1)\pi}{4}$ or $x = n\pi, n \in \mathbb{Z}$;
$\left\{\frac{-5\pi}{4}, -\pi, -\frac{\pi}{4}, 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}\right\}$

8 $x = \frac{n\pi}{3}, n \in \mathbb{Z}; \left\{-\pi, \frac{-2\pi}{3}, \frac{-\pi}{3}, 0\right\}$

9 $x = \frac{6n-1}{12}$ or $x = \frac{3n+2}{6}, n \in \mathbb{Z}$;
$\left\{\frac{-2}{3}, \frac{-7}{12}, \frac{-1}{6}, \frac{-1}{12}, \frac{1}{3}, \frac{5}{12}, \frac{5}{6}, \frac{11}{12}\right\}$

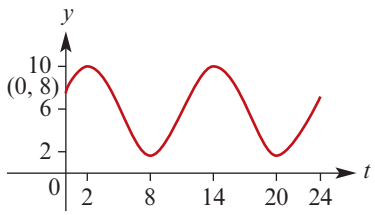
Exercise 6L

1 a i Amplitude $= 1\frac{1}{2}$ **ii** Period $= 12$

iii $d(t) = 3.5 - 1.5\cos\left(\frac{\pi}{6}t\right)$ **iv** 1.5 m

b $[0, 3) \cup (9, 15) \cup (21, 24]$

2 a

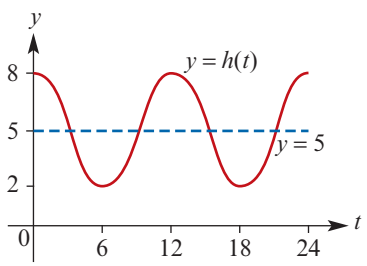


b 2:00 **c** 8:00, 20:00

3 a $A = 3, n = \frac{\pi}{6}, b = 5, \varepsilon = \frac{\pi}{2}$

b 2:21 a.m., 9:39 a.m., 2:21 p.m., 9:39 p.m.

c



4 a 5

b 1

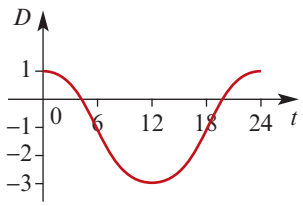
c $t = 0.524$ s, 2.618 s, 4.712 s

d $t = 0$ s, 1.047 s, 2.094 s

e Particle oscillates about the point $x = 3$ from $x = 1$ to $x = 5$

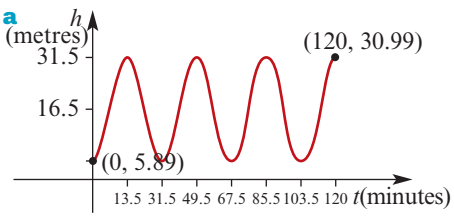
5 a 19.5°C **b** $D = -1 + 2\cos\left(\frac{\pi t}{12}\right)$

c



d $\{t : 4 < t < 20\}$

6 a



b 5.89 m **c** 27.51 s **d** 6 times

e 20 times **f** 4.21 m **g** 13.9 m

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Chapter 6 review

Technology-free questions

1 a $\frac{\pi}{6}, \frac{5\pi}{6}$ **b** $\frac{-2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

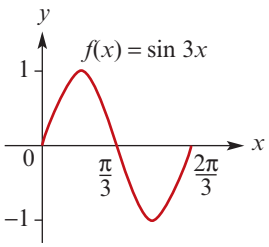
c $\frac{-\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$ **d** $\frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

e $\frac{-\pi}{6}, \frac{-5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **f** $\frac{-\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

g $\frac{-3\pi}{8}, \frac{-5\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

h $\frac{-7\pi}{18}, \frac{-11\pi}{18}, \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

2 a



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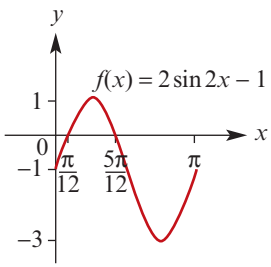

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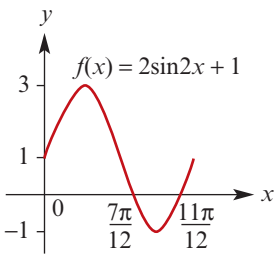
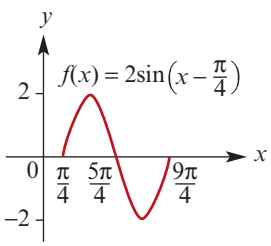
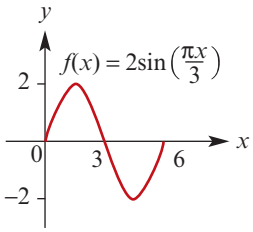


d



e



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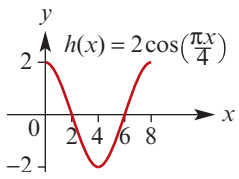
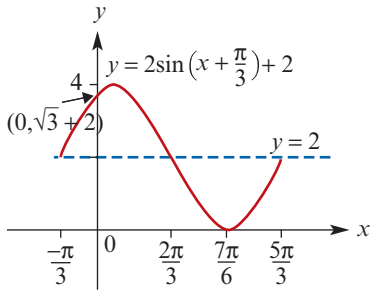


3 a 30, 150 **b** 45, 135, 225, 315

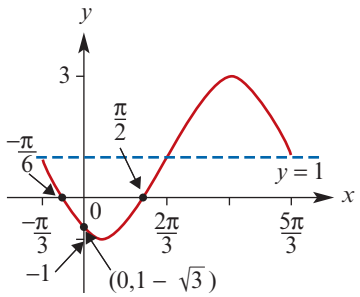
c 240, 300 **d** 90, 120, 270, 300

e 120, 240

4 a



b



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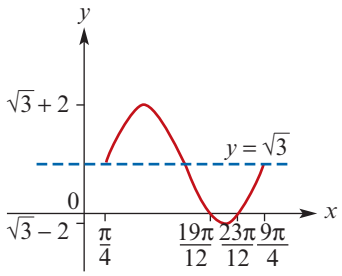
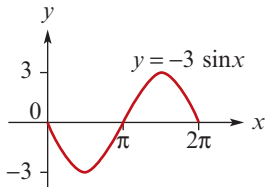
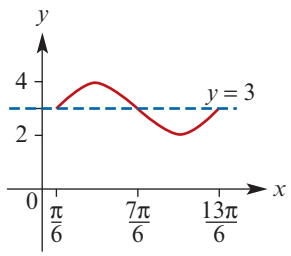


d



e



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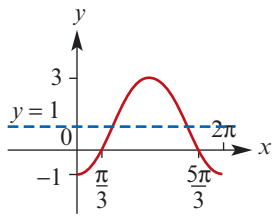
f



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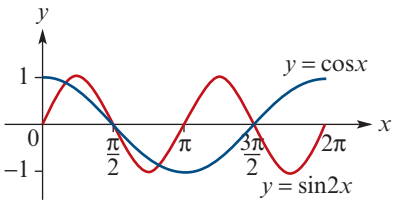
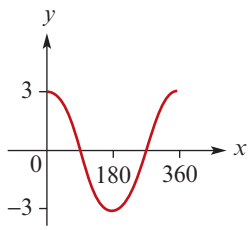
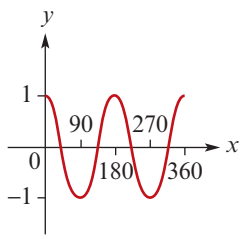


a 4 solutions **b** 4 solutions **c** 2 solutions

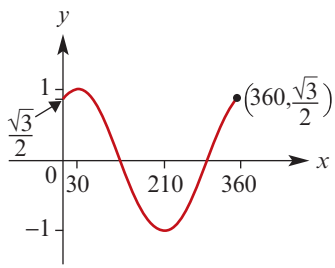
6 a



b



c



7 a $\frac{-2\pi}{3}, \frac{\pi}{3}$ **b** $\frac{-\pi}{4}, \frac{3\pi}{4}$

c $\frac{-5\pi}{8}, \frac{-\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$ **d** $\frac{-2\pi}{3}, \frac{-\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}$

8 $-\frac{2\pi}{3}, \frac{\pi}{3}$

9 a $\frac{1}{\sqrt{3}}$ **b** $-\frac{5\pi}{6}$

10 a $n\pi - \frac{\pi}{4}, n \in \mathbb{Z}$ **b** $\frac{2n\pi}{3}, n \in \mathbb{Z}$

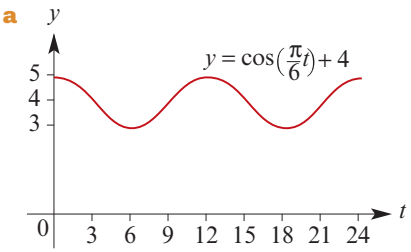
c $-\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

Multiple-choice questions

1 C **2** A **3** E **4** D **5** A **6** C
7 C **8** B **9** C **10** E **11** C **12** B

Extended-response questions

1 a



b 9:00, 15:00 **c** 8:00, 16:00

2 a Maximum = 210 cm; Minimum = 150 cm; Mean = 180 cm

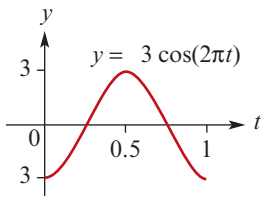
b $A = 30,\ n = \frac{\pi}{6},\ \varepsilon = \frac{-\pi}{2},\ b = 180$

c i 165 cm **ii** $180 - 15\sqrt{3} \approx 154$ cm

d ≈ 4:24, ≈ 7:36

3 a $a = -3,\ n = 2\pi$

b

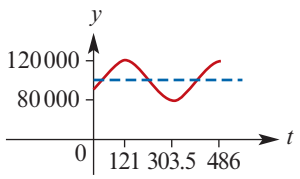


c i $t = \frac{1}{3}$ second **ii** $t = \frac{1}{6}$ second

d $t = 0.196$ seconds

4 a $a = 20\,000,\ b = 100\,000,\ n = \frac{2\pi}{365},\ \varepsilon \approx 5.77$

b



c i $t = 242.7,\ t = 364.3$

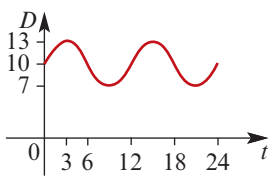
ii $t = 60.2,\ t = 181.8$

d ≈ 117 219 m^3/day

5 a i 1.83×10^{-3} hours **ii** 11.79 hours

b 25 April ($t = 3.856$), 14 August ($t = 7.477$)

6 a

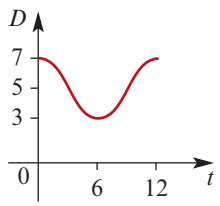


b $\{t : D(t) \geq 8.5\} = [0, 7] \cup [11, 19] \cup [23, 24]$

c 12.898 m

7 a $p = 5$
$q = 2$
$r = 30$

b



c A ship can enter 2 hours after low tide

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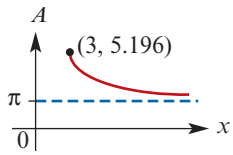
8 a i $25\sqrt{3}$ **ii** 30
b 2.27, 0.53 **d** $b = 8$
e $\theta = 0.927$ or 1.837 **f** $a = 4\sqrt{3}$

9 c

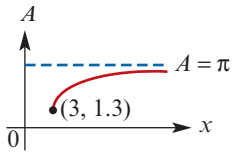


d i 2.055 **ii** 0.858
iii 0.0738 **iv** 0.0041

e $nr\tan\left(\frac{\pi}{n}\right)$

f i $n\sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)$

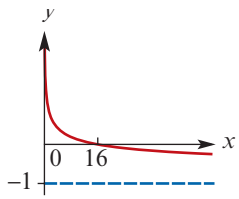
ii



Chapter 7

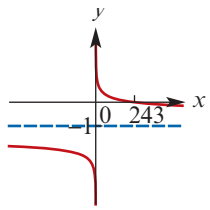
Exercise 7A

1 a



Domain = $\mathbb{R}^+$
Range = $(-1, \infty)$
Neither

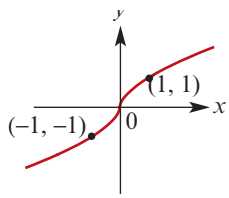
b



Domain = $\mathbb{R} \setminus \{0\}$
Range = $\mathbb{R} \setminus \{-1\}$
Neither

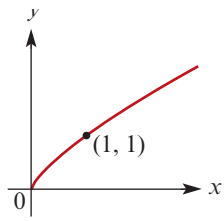
2 a 4 **b** 4 **c** 8 **d** −8 **e** −32 **f** 81

3 a



Domain = $\mathbb{R}$
Range = $\mathbb{R}$
Odd

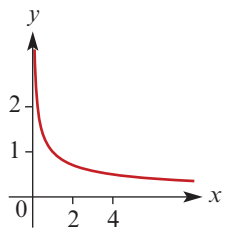
b



Domain = $\mathbb{R}^+ \cup \{0\}$
Range = $\mathbb{R}^+ \cup \{0\}$
Neither

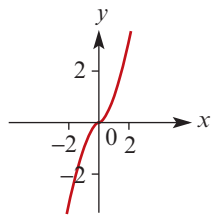
4 a i Domain = $\mathbb{R}^+$; Range = $\mathbb{R}^+$;
Asymptotes: $x = 0$, $y = 0$

ii



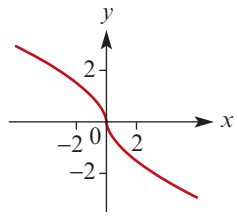
b i Domain = $\mathbb{R}$; Range = $\mathbb{R}$

ii



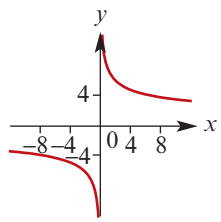
c i Domain = $\mathbb{R}$; Range = $\mathbb{R}$

ii



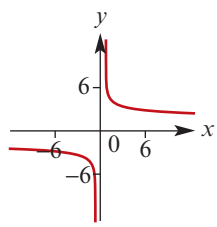
d i Domain = $\mathbb{R} \setminus \{0\}$; Range = $\mathbb{R} \setminus \{0\}$;
Asymptotes: $x = 0$, $y = 0$

ii



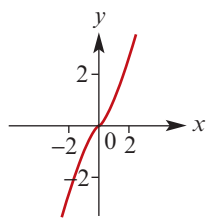
e i Domain = $\mathbb{R} \setminus \{0\}$; Range = $\mathbb{R} \setminus \{0\}$;
Asymptotes: $x = 0$, $y = 0$

ii



f i Domain = $\mathbb{R}$; Range = $\mathbb{R}$

ii



5 a (0, 1) **b** (0, 1)

6 a Odd **b** Even **c** Odd
d Odd **e** Even **f** Odd

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Exercise 7B

1 a $h(x) = f \circ g(x),\ f(x) = e^x,\ g(x) = x^3$
b $h(x) = f \circ g(x),\ f(x) = \sin x,\ g(x) = 2x^2$
c $h(x) = f \circ g(x),\ f(x) = x^n,\ g(x) = x^2 - 2x$
d $h(x) = f \circ g(x),\ f(x) = \cos x,\ g(x) = x^2$
e $h(x) = f \circ g(x),\ f(x) = x^2,\ g(x) = \cos x$
f $h(x) = f \circ g(x),\ f(x) = x^4,\ g(x) = x^2 - 1$
g $h(x) = f \circ g(x),\ f(x) = x^2,\ g(x) = \cos(2x)$
h $h(x) = f \circ g(x),\ f(x) = x^3 - 2x,\ g(x) = x^2 - 2x$

2 a $f^{-1}: (0, \infty) \to \mathbb{R},\ f^{-1}(x) = \frac{1}{3}\log_e\left(\frac{x}{4}\right)$
b $g^{-1}: \mathbb{R} \setminus \{0\} \to \mathbb{R},\ g^{-1}(x) = \frac{8}{x^3}$
c $f \circ g: \mathbb{R} \setminus \{0\} \to \mathbb{R},\ f \circ g(x) = 4e^{\frac{6}{\sqrt[3]{x}}}$
d $g \circ f: \mathbb{R} \to \mathbb{R},\ g \circ f(x) = \frac{2}{\sqrt[3]{4e^{3x}}}$
e $(f \circ g)^{-1}: \mathbb{R}^+ \to \mathbb{R},\ (f \circ g)^{-1}(x) = \left(\frac{6}{\log_e(\frac{x}{4})}\right)^3$
f $(g \circ f)^{-1}: \mathbb{R}^+ \to \mathbb{R},\ (g \circ f)^{-1}(x) = \frac{1}{3}\log_e\left(\frac{2}{x^3}\right)$

3 a $f^{-1}: \mathbb{R}^+ \cup \{0\} \to \mathbb{R},\ f^{-1}(x) = x^{\frac{5}{2}}$
Both f and f^{-1} are strictly increasing
b $f^{-1}: \mathbb{R}^+ \cup \{0\} \to \mathbb{R},\ f^{-1}(x) = -x^{\frac{5}{2}}$
Both f and f^{-1} are strictly decreasing
c $f^{-1}: \mathbb{R}^+ \to \mathbb{R},\ f^{-1}(x) = x^{\frac{2}{5}}$
Both f and f^{-1} are strictly increasing

4 a i $f \circ g(x) = 3\sin(2x^2),\ g \circ f(x) = 9\sin^2(2x)$
ii $\text{ran}(f \circ g) = [-3, 3],\ \text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(g \circ f) = [0, 9],\ \text{dom}(g \circ f) = \mathbb{R}$
b i $f \circ g(x) = -2\cos(2x^2)$, $g \circ f(x) = 4\cos^2(2x)$
ii $\text{ran}(f \circ g) = [-2, 2],\ \text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(g \circ f) = [0, 4],\ \text{dom}(g \circ f) = \mathbb{R}$
c i $f \circ g(x) = e^{x^2},\ g \circ f(x) = e^{2x}$
ii $\text{ran}(f \circ g) = (1, \infty),\ \text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(g \circ f) = (0, \infty),\ \text{dom}(g \circ f) = \mathbb{R}$
d i $f \circ g(x) = e^{2x^2} - 1,\ g \circ f(x) = (e^{2x} - 1)^2$
ii $\text{ran}(f \circ g) = [0, \infty),\ \text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(g \circ f) = [0, \infty),\ \text{dom}(g \circ f) = \mathbb{R}$
e i $f \circ g(x) = -2e^{x^2} - 1,\ g \circ f(x) = (2e^x + 1)^2$
ii $\text{ran}(f \circ g) = (-\infty, -3],\ \text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(g \circ f) = (1, \infty),\ \text{dom}(g \circ f) = \mathbb{R}$
f i $f \circ g(x) = \log_e(2x^2)$, $g \circ f(x) = (\log_e(2x))^2$
ii $\text{ran}(f \circ g) = \mathbb{R},\ \text{dom}(f \circ g) = \mathbb{R} \setminus \{0\}$, $\text{ran}(g \circ f) = [0, \infty),\ \text{dom}(g \circ f) = \mathbb{R}^+$
g i $f \circ g(x) = \log_e(x^2 - 1)$, $g \circ f(x) = (\log_e(x - 1))^2$
ii $\text{ran}(f \circ g) = \mathbb{R},\ \text{dom}(f \circ g) = \mathbb{R} \setminus [-1, 1]$, $\text{ran}(g \circ f) = [0, \infty),\ \text{dom}(g \circ f) = (1, \infty)$
h i $f \circ g(x) = -\log_e(x^2),\ g \circ f(x) = (\log_e x)^2$
ii $\text{ran}(f \circ g) = \mathbb{R},\ \text{dom}(f \circ g) = \mathbb{R} \setminus \{0\}$, $\text{ran}(g \circ f) = [0, \infty),\ \text{dom}(g \circ f) = \mathbb{R}^+$

5 a $g \circ f: \mathbb{R} \to \mathbb{R},\ g \circ f(x) = \sin\left(2x - \frac{\pi}{3}\right)$
b Dilation of factor $\frac{1}{2}$ from the y-axis, then translation $\frac{\pi}{6}$ units to the right

6 a $g \circ f: (\frac{1}{3}, \infty) \to \mathbb{R},\ g \circ f(x) = \log_e(3x - 1)$
b Dilation of factor $\frac{1}{3}$ from the y-axis, then translation $\frac{2}{3}$ units to the right

7 a $g(x) = 3,\ g(x) = 4$ **b** $g(x) = 3x,\ g(x) = 4x$

8 $g(x) = \log_e(2x - 1)$

9 a $2e^{2x}$ **b** $\frac{1}{2}\log_e\left(\frac{x}{2}\right)$ **c** e^{x^2}

10 a $f^{-1}(x) = -\frac{1}{2}\log_e x,\ g^{-1}(x) = (x - 1)^{\frac{1}{3}}$
b $f \circ g(x) = e^{-2(x^3+1)}$, $\text{ran}(f \circ g) = \mathbb{R}^+$, $g \circ f(x) = e^{-6x} + 1$, $\text{ran}(g \circ f) = (1, \infty)$

11 a $f^{-1}(x) = \frac{1}{x} - 1$ **b** $x = \frac{\sqrt{5} - 1}{2}$

12 a $f^{-1}(x) = e^x - 1$, $\text{dom}\, f^{-1} = \mathbb{R}$, $g^{-1}(x) = \sqrt{x + 1} - 1$, $\text{dom}\, g^{-1} = (-1, \infty)$
b $\log_e(x^2 + 2x + 1)$

13 $f \circ g(x) = \log_e\left(\frac{1}{x}\right),\ f(x) + f \circ g(x) = 0$

14 x

15 a $f(g(x)) = (x^2 - 10)(x^2 - 8)$, $g(f(x)) = x^4 - 20x^3 + 148x^2 - 480x + 572$
b $x = 1$

16 $x = \pm\sqrt{6}$ or $x = \pm\sqrt{2}$

18 $a = \frac{1}{6},\ b = -\frac{1}{2}$

20 $b = 0,\ a = 6,\ g(x) = e^{6x}$

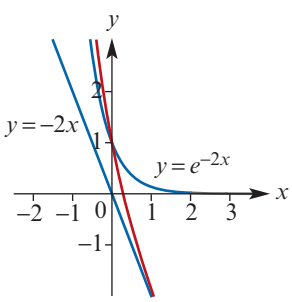
21 a $f^{-1}: [1, \infty) \to \mathbb{R},\ f^{-1}(x) = \log_e(x + \sqrt{x^2 - 1})$
b $g^{-1}: \mathbb{R} \to \mathbb{R},\ g^{-1}(x) = \log_e(x + \sqrt{x^2 + 1})$
c Yes **d** Yes

22 a $x > y \Rightarrow f(x) > f(y) \Rightarrow g(f(x)) > g(f(y))$
b $x > y \Rightarrow f(x) < f(y) \Rightarrow g(f(x)) > g(f(y))$
c $g \circ f$ is strictly decreasing

Exercise 7C

1 a i $e^{2x} - 2x$ **ii** $-2xe^{2x}$
b i $e^{-1} + 1$ **ii** e^{-1}

2



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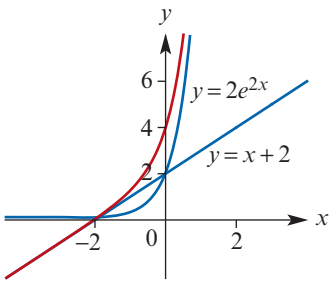
3



4 a i $\sin\left(\frac{\pi x}{2}\right) - 2x$ **ii** $-2x\sin\left(\frac{\pi x}{2}\right)$

b i -1 **ii** -2

5 a i $\cos\left(\frac{\pi x}{2}\right) + e^x$ **ii** $e^x\cos\left(\frac{\pi x}{2}\right)$

b i 2 **ii** 1

6 $f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}$

Exercise 7D

1 a $f(x - y) = 2(x - y) = 2x - 2y = f(x) - f(y)$

b $f(x - y) = x - y - 3 \neq f(x) - f(y)$

2 $f(x - y) = f(x) - f(y)$

3 $f(x + y) = f(x) + f(y) - 3$; $a = -3$

4 $f(x) + f(y) = \frac{3}{x} + \frac{3}{y} = \frac{3(x + y)}{xy} = (x + y)f(xy)$

5 $g(x) = 0$ or $g(x) = 1$

6 $g(x) = \pm 1$

7 $f(2 + 3) = 5^3 = 125 \neq f(2) + f(3)$

8 $f\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \sin\pi = 0 \neq 2 = f\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right)$

9 $f(x) + f(y) = \frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2 + y^2}{x^2y^2} = (x^2 + y^2)f(xy)$

10 a $h(1 + 2) = 3^2 = 9$, $h(1) + h(2) = 1 + 4 = 5$

11 $g(x + y) = 2^{3x+3y} = 2^{3x} \times 2^{3y} = g(x) \times g(y)$

12 $f(xy) = (xy)^n = x^n y^n = f(x)f(y)$

$f\left(\frac{x}{y}\right) = \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} = \frac{f(x)}{f(y)}$

13 Let $x = 2$ and $y = 3$. Then $f(xy) = f(6) = 6a$ and $f(x)f(y) = 2a \cdot 3a = 6a^2$. But $6a = 6a^2$ implies $a = 0$ or $a = 1$.

Exercise 7E

1 a $\frac{4}{m}$ **b** $m \geq 4$ or $m < 0$

c $f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = \frac{x + 4}{m}$

d $\left(\frac{4}{m - 1}, \frac{4}{m - 1}\right)$, where $m \in \mathbb{R} \setminus \{0, 1\}$

e $y = -\frac{1}{m}x - 4$

2 a $\frac{c}{2}$ **b** $c \leq 2$

c $f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = \frac{c - x}{2}$

d $\left(\frac{c}{3}, \frac{c}{3}\right)$ **e** $y = \frac{1}{2}x + c$

3 a 0 and b

b $\left(\frac{b}{2}, \frac{-b^2}{4}\right)$

c i $(0, 0)$ and $(b - 1, 1 - b)$

ii $b = 1$ **iii** $b \in \mathbb{R} \setminus \{1\}$

4 $a = 5 - c$ and $b = -1$, where $y = ax^2 + bx + c$

5 a $-1 \pm 2\sqrt{2}$ **b** $\pm 2\sqrt{2}$ **c** $a = -8$, $b = 16$

6 a $(-\infty, 2a]$

b $\left(\frac{-1 + \sqrt{1 + 8a}}{2}, \frac{-1 + \sqrt{1 + 8a}}{2}\right)$

c $a = 1$ **d** $a = 3$ **e** $a = \frac{c^2 + c}{2}$

7 a $(0, 0)$ and $(a, 0)$ **b** $(0, 0)$ **c** $\frac{a^4}{16}$

d $a = 3$ or $a = -5$

8 a $\frac{1}{b}\log_e\left(\frac{c}{a}\right)$ **b** $e^{\frac{b}{c}} - a$

c $\frac{a + 1}{c}$ **d** $\frac{\log_e(c) - b}{a}$

9 a $x = a$ **b** $(a + 1, 0)$

c $(a + e^{\frac{1}{c}}, 1)$ **d** $c = \frac{1}{\log_e(2 - a)}$

10 a $y = -b$

b $(\log_e(b) + 1, 0)$

c i $b = \frac{1}{e}$ **ii** $0 < b < \frac{1}{e}$

11 $a = \frac{3d + 4}{6}$, $b = 2 - d$ and $c = \frac{-3d - 28}{6}$, where $y = ax^3 + bx^2 + cx + d$

12 a $c = 28 - 8\sqrt{6}$ or $c = 28 + 8\sqrt{6}$

b $c \in (-\infty, 8) \cup (8, 28 - 8\sqrt{6}) \cup (28 + 8\sqrt{6}, \infty)$

13 $a = \frac{5d - 9}{30}$, $b = \frac{41 - 10d}{30}$ and $c = \frac{-25d - 2}{30}$, where $y = ax^3 + bx^2 + cx + d$

14 a $x = \frac{3 - x'}{4}$ and $y = \frac{y' - 2}{k}$

b $y = \frac{4k}{3 - x} + 2$ **c** $k = \frac{-3}{2}$

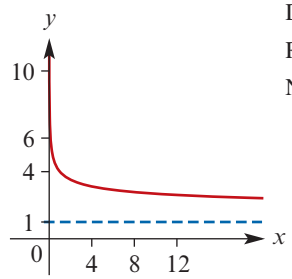
15 a $x = \frac{a - x'}{4}$ and $y = \frac{y' + 2}{2}$

b $y = 2 \times 2^{\frac{a-x}{4}} - 2$ **c** $a = 0$

Chapter 7 review

Technology-free questions

1 a



Domain $= \mathbb{R}^+$

Range $= (1, \infty)$

Neither

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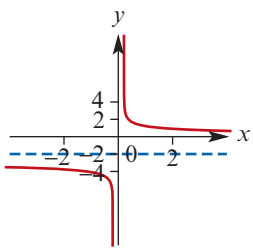
b



Domain = $\mathbb{R} \setminus \{0\}$
Range = $\mathbb{R} \setminus \{-2\}$
Neither

2 a 9 **b** 9 **c** 27
d 9 **e** −243 **f** 625

3 a i $f \circ g(x) = 3\cos(2x^2)$,
$g \circ f(x) = 9\cos^2(2x)$
ii $\text{dom}(f \circ g) = \mathbb{R}$, $\text{ran}(f \circ g) = [-3, 3]$,
$\text{dom}(g \circ f) = \mathbb{R}$, $\text{ran}(g \circ f) = [0, 9]$

b i $f \circ g(x) = \log_e(3x^2)$,
$g \circ f(x) = (\log_e(3x))^2$
ii $\text{dom}(f \circ g) = \mathbb{R} \setminus \{0\}$, $\text{ran}(f \circ g) = \mathbb{R}$,
$\text{dom}(g \circ f) = \mathbb{R}^+$, $\text{ran}(g \circ f) = [0, \infty)$

c i $f \circ g(x) = \log_e(2 - x^2)$,
$g \circ f(x) = (\log_e(2 - x))^2$
ii $\text{dom}(f \circ g) = (-\sqrt{2}, \sqrt{2})$,
$\text{ran}(f \circ g) = (-\infty, \log_e 2)$,
$\text{dom}(g \circ f) = (-\infty, 2)$, $\text{ran}(g \circ f) = [0, \infty)$

d i $f \circ g(x) = -\log_e(2x^2)$,
$g \circ f(x) = (\log_e(2x))^2$
ii $\text{dom}(f \circ g) = \mathbb{R} \setminus \{0\}$, $\text{ran}(f \circ g) = \mathbb{R}$,
$\text{dom}(g \circ f) = (0, \infty)$, $\text{ran}(g \circ f) = [0, \infty)$

4 a $h(x) = f \circ g(x)$, $g(x) = x^2$, $f(x) = \cos x$
(*Note:* answer not unique)

b $h(x) = f \circ g(x)$, $g(x) = x^2 - x$, $f(x) = x^n$
(*Note:* answer not unique)

c $h(x) = f \circ g(x)$, $g(x) = \sin x$, $f(x) = \log_e x$
(*Note:* answer not unique)

d $h(x) = f \circ g(x)$, $g(x) = \sin(2x)$,
$f(x) = -2x^2$ (*Note:* answer not unique)

e $h(x) = f \circ g(x)$, $g(x) = x^2 - 3x$,
$f(x) = x^4 - 2x^2$ (*Note:* answer not unique)

5 a i $(f + g)(x) = 2\cos\left(\frac{\pi x}{2}\right) + e^{-x}$

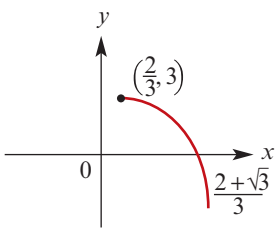
ii $(fg)(x) = 2e^{-x}\cos\left(\frac{\pi x}{2}\right)$

b i $(f + g)(0) = 3$ **ii** $(fg)(0) = 2$

6 a $\frac{2}{3}$

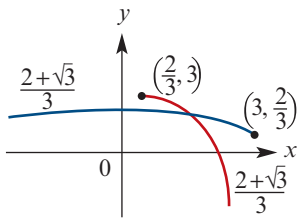
b $(-\infty, 3]$

c



d $f^{-1}(x) = \frac{2 + \sqrt{3 - x}}{3}$, $\text{ran} = [\frac{2}{3}, \infty)$,
$\text{dom} = (-\infty, 3]$

e



7 a $x = a$ **b** $(a + 1, 0)$
c $(e + a, c)$ **d** $f^{-1}(x) = e^{\frac{x}{c}} + a$
e (a, ∞) **f** $c = \frac{1}{\log_e 2}$, $a = 0$

8 $a = \frac{1}{4}$, $b = \frac{3}{2}$

9 a $f^{-1}(x) = \left(\frac{x - 1}{3}\right)^3$ **b** $f^{-1}(x) = \left(\frac{x + 2}{4}\right)^3$

c $f^{-1}(x) = \frac{1}{3}\left((x - 4)^{\frac{1}{3}} + 2\right)$

d $f^{-1}(x) = \left(\frac{3 - x}{2}\right)^{\frac{1}{3}}$

10 $f(g(x)) = a\cos x$,
Domain $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Range $= [0, a]$

Multiple-choice questions

1 B **2** E **3** E **4** D **5** E
6 E **7** B **8** C **9** D **10** C
11 B **12** D

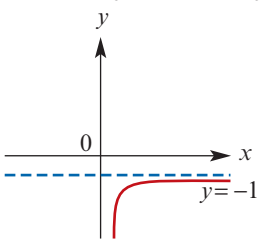
Extended-response questions

1 a $(0, 1)$, $(-\infty, 0)$

b $f^{-1}(x) = -\log_e x$, $g^{-1}(x) = \frac{1}{x} + 1$

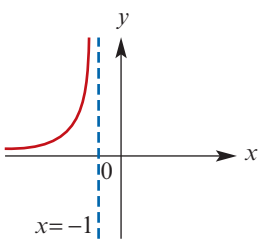
c i $g \circ f(x) = \frac{1}{e^{-x} - 1} = \frac{e^x}{1 - e^x}$

ii



d i $(g \circ f)^{-1}(x) = \log_e\left(\frac{x}{x + 1}\right)$

ii



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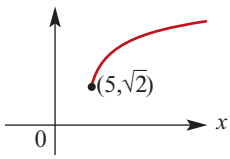
2 a i

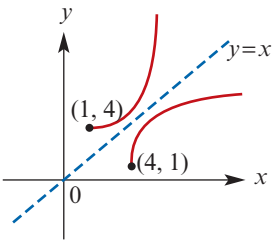


ii $[\sqrt{2}, \infty)$

iii $f^{-1}: [\sqrt{2}, \infty) \to \mathbb{R}, f^{-1}(x) = x^2 + 3$

b i $p = 3$ **ii** $h^{-1}(x) = x^2 + 3$

iii



3 a $(0, 1]$

b $(0, 1]$

c $\text{ran}\, g \subseteq \text{dom}\, f$, $f \circ g(x) = \sin\left(\frac{1}{x}\right)$

d Not defined as $\text{ran}\, f \not\subseteq \text{dom}\, g$

e $g^{-1}(x) = \frac{1}{x}$, $\text{dom}\, g^{-1} = (0, 1]$, $\text{ran}\, g^{-1} = [1, \infty)$

f $\text{ran}\, f = \text{dom}\, g^{-1}$, $g^{-1} \circ f(x) = \frac{1}{\sin x}$, $\text{dom}(g^{-1} \circ f) = (0, \pi)$, $\text{ran}(g^{-1} \circ f) = [1, \infty)$

4 a $a = 2$ **b** $c = 2 - k\log_e(2)$

c $k = \frac{10}{\log_e\left(\frac{d+2}{2}\right)}$ **d** $k = 10$

5 a $b = -3 - \sqrt{13}$

b $f(g(x)) = \sqrt{x^2 + 6x + 5}$
Domain $= (-\infty, -3 - \sqrt{13}]$ Range $= [3, \infty)$

c $h(x) = -3 - \sqrt{x^2 + 4}$
Domain $= [3, \infty)$ Range $= (-\infty, -3 - \sqrt{13}]$

Chapter 8

Technology-free questions

1 a Domain $= \mathbb{R} \setminus \{0\}$; Range $= \mathbb{R} \setminus \{2\}$

b Domain $= \left[\frac{2}{3}, \infty\right)$; Range $= (-\infty, 3]$

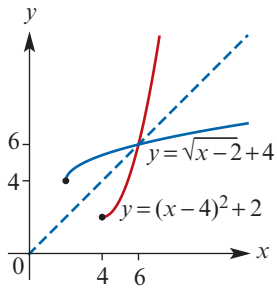
c Domain $= \mathbb{R} \setminus \{2\}$; Range $= (3, \infty)$

d Domain $= \mathbb{R} \setminus \{2\}$; Range $= \mathbb{R} \setminus \{4\}$

e Domain $= [2, \infty)$; Range $= [-5, \infty)$

f Domain$= (-\infty, -4] \cup [2, \infty)$; Range$= [0, \infty)$

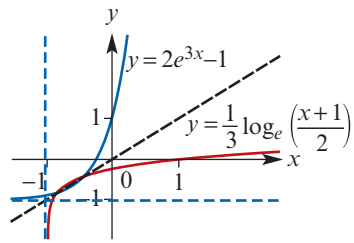
2 $f^{-1}: [4, \infty) \to \mathbb{R}, f^{-1}(x) = (x - 4)^2 + 2$



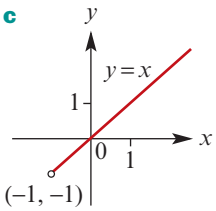
3 $f^{-1}: \mathbb{R} \setminus \{1\} \to \mathbb{R}, f^{-1}(x) = \frac{x + 2}{1 - x}$

4 a $f^{-1}(x) = \frac{1}{3}\log_e\left(\frac{x+1}{2}\right)$, $\text{dom}\, f^{-1} = (-1, \infty)$

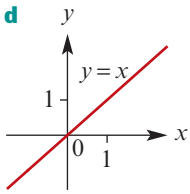
b



c



d



e $y = 2x$

5 1

6 $x = \sqrt[3]{12}a$

7 $x = -9$

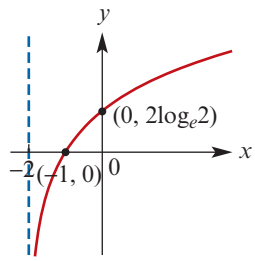
8 $\frac{2e - 12}{e + 1}$

9 $\frac{1}{2}$

10 $x = \log_e\left(\frac{3}{2}\right)$

11 a $a = -1$ and $b = 2\log_e 2$

b



12 $x = 0$ or $x = 1$

13 $x = -\frac{7\pi}{9}$ or $x = \frac{\pi}{9}$ or $x = \frac{5\pi}{9}$

14 a Range $= [2, 8]$; Period $= 6$

b $x = \frac{\pi}{12}$ or $x = \frac{3\pi}{4}$

15 a $m \in \mathbb{R} \setminus \{-1, 2\}$ **b** $m = 2$ **c** $m = -1$

16 $a = -2$ and $b = 1$

17 a $m = \pm 2\sqrt{2}$ **b** $m > 2\sqrt{2}$ or $m < -2\sqrt{2}$

c $-2\sqrt{2} < m < 2\sqrt{2}$

18 $a = -3$ or $a = -1$or $a = 2$

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19 a i $a = -3$ **ii** $a = 5$ or $a = 1$ **iii** $a = -3$

b $5y - 3x + 4 = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

20 a Odd

b $f^{-1}(x) = \sqrt[3]{\frac{x}{2}}$

c i 2 **ii** -1

iii $x = 0$ or $x = 2^{-\frac{1}{2}}$ or $x = -2^{-\frac{1}{2}}$

21 a 4 **b** $\sqrt{5}$ **c** $2 - 2a$ **d** $\sqrt{2a - 5}$

e $x = -8$ **f** $x = \frac{103}{2}$ **g** $x < 1$

22 a i $f \circ g(x) = 4x^2 + 8x - 3$

ii $g \circ f(x) = 16x^2 - 16x + 3$

iii $g \circ f^{-1}(x) = \frac{1}{16}(x^2 + 14x + 33)$

b Dilation of factor $\frac{1}{4}$ from the y-axis, then translation $\frac{3}{4}$ units to the right

c Translation 1 unit to the left and 1 unit down

23 $x = \frac{2\pi}{3}$

24 $x = 2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$

25 $A = \frac{8}{5}$ and $k = \log_e\left(\frac{5}{2}\right)$

26 a $\frac{2\pi}{5}$ **b** 8

c i
- Dilation of factor 8 from the x-axis
- Dilation of factor $\frac{1}{5}$ from the y-axis

ii
- Dilation of factor 8 from the x-axis
- Translation $\frac{\pi}{2}$ units to the right
- Reflection in the y-axis
- Dilation of factor $\frac{1}{5}$ from the y-axis

27 a $a = -6$, $b = 13$

b $P(x) = (x - 1)^2(x - 2)^2$

28 $h(x) = f(5x - 7) + 3$

29 $a = -18$, $b = 30$

30 $n = 3$

31 $x = \log_e 6$ or $x = 0$

32
- A translation of 5π units in the positive direction of the x-axis
- A reflection in the x-axis

33 a $-2 \leq x \leq \frac{1}{2}$ or $x \geq 3$ **b** $x < 0$

34 a $f(g(x)) = e^2x^3$ **b** $k = 8$

Multiple-choice questions

1	D	**2**	A	**3**	B	**4**	E	**5**	E	**6**	C
7	A	**8**	B	**9**	C	**10**	A	**11**	C	**12**	C
13	B	**14**	A	**15**	E	**16**	D	**17**	E	**18**	C
19	D	**20**	D	**21**	B	**22**	C	**23**	A	**24**	B
25	E	**26**	E	**27**	A	**28**	D	**29**	D	**30**	A
31	E	**32**	D	**33**	E	**34**	A	**35**	E	**36**	C
37	D	**38**	C	**39**	E	**40**	B	**41**	B	**42**	C
43	D	**44**	B	**45**	B	**46**	E	**47**	A	**48**	C
49	D	**50**	B	**51**	C	**52**	C	**53**	A	**54**	B
55	A	**56**	B	**57**	D	**58**	D	**59**	A	**60**	A

61 D **62** B **63** D **64** B **65** E **66** C
67 C **68** B **69** C

Extended-response questions

1 a $a = -0.09$, $b = 9$ **b** $DE = 2.79$ m

c Length $= 2\sqrt{30} \approx 10.95$ m

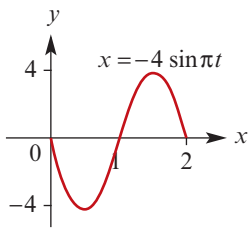
2 a $a = -3$ **b** $x = -1$, $x = -\frac{1}{2}$, $x = 2$

c ii $b = \frac{7}{2}$, $c = \frac{3}{2}$

3 a $a = 5$

b $b = -6$

4 a



b i $x = 0$ **ii** $x = -4$ **iii** $x = 0$

c $t = \frac{7}{6}$

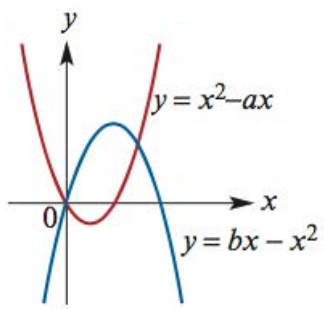
d Period $= \frac{2\pi}{\pi} = 2$ seconds

5 a $h = (a - 1)x - x^2$ **b** $\frac{a - 1}{2}$ **c** $\frac{(a - 1)^2}{4}$

d i 2 **ii** 3 **iii** $1 + 2\sqrt{5}$ **iv** 7 **v** $1 + 2\sqrt{10}$

6 a $(0, 0)$, $\left(\frac{a + b}{2}, \frac{b^2 - a^2}{4}\right)$

b

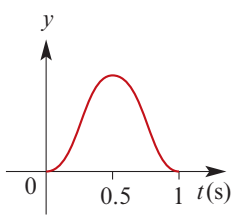


c $(a + b)x - 2x^2$ **d** $\frac{(a + b)^2}{8}$

7 a i 0 **ii** 2.5 **iii** 0

b 1 second

c



d $t = 0.35$ seconds

8 a $k = 0.0292$ **b** 150×10^6

c 6.4494×10^8 **d** 23.738 years

9 a $A = 80$, $k = 0.3466$

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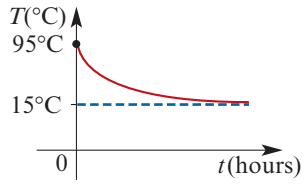
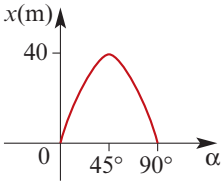
b 17.5°C
c 6 hours 18 minutes and 14 seconds after 2:00 p.m., i.e. 8:18:14
d



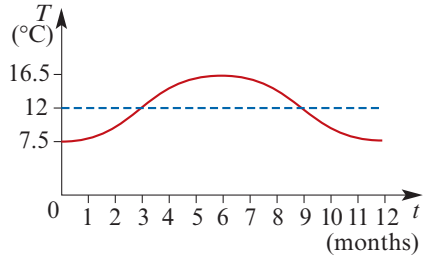
10 a 62.5 metres **b**



c 24.3° or 65.7°

11 a Area $= 0.02(0.92)^{\frac{x}{10}}$ **b** 0.0197 mm^2
c Load $= 0.02(0.92)^{10-2.9x}$ **d** $x < 2.59$ m

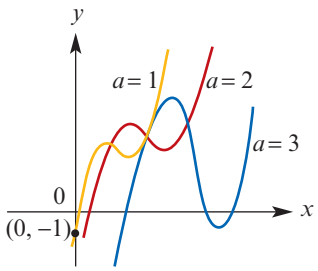
12 a i 12 units **ii** $OQ = h - k$, $OR = h + k$
b



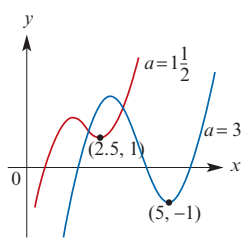
c $h = 12$, $k = 4.5$

13 a Carriage A: $(0.83)^n I$
Carriage B: $0.66(0.89)^n I$
b 6 stations

14 a i $\left(3 + \frac{1}{\sqrt{a}}, 0\right), \left(3 - \frac{1}{\sqrt{a}}, 0\right)$ **ii** $\frac{1}{\sqrt{a}}$
b i



ii $a = \frac{3\sqrt{3}}{2}$ **iii** $a > \frac{3\sqrt{3}}{2}$ **iv** $a = 3$
v $a = 1\frac{1}{2}$ **vi**



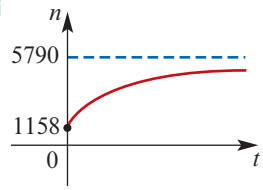
c i $\left(a, \frac{-4}{27}a^3 + a\right)$ **ii** $\frac{4}{27}a^3, \frac{2a}{3}$
iii $A = \frac{4a^4}{81}$ **iv** $a = 3$ **v** $3\sqrt[4]{375}$

15 a $D = 0.05t^2 - 0.25t + 1.8$
b \$3 000 000

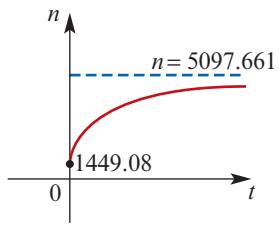
16 $a = \frac{-7}{48}$, $b = \frac{23}{24}$, $c = 7.5$
Rainfall at noon was $\frac{35}{6}$ mm per hour
Rainfall greatest at $\frac{23}{7}$ hours after 4 a.m. (approx 7:17 a.m.)

17 a $a = 45$, $b = 10$ **b** 104 dB
c Power setting 4

18 a i $n = 5790$ **ii** 1158
iii



iv $t = \frac{-100}{3}\log_e\left(\frac{179}{1600}\right)$
b i $a = 2.518$, $b = 0.049$, $c = 5097.661$
ii



19 b Min area $= \frac{147}{2}$ when $x = \frac{7}{2}$, $y = \frac{7}{2}$
c Min area $= \frac{3a^2}{2}$ when $x = \frac{a}{2}$, $y = \frac{a}{2}$

20 a $f(g(x)) = 64x^2 + 32(3a-2)x + 6a^2 - 48a + 16$
b $f(g(x)) = 64\left(x + \frac{3a-2}{4}\right)^2 - 30a^2$
c $[-30a^2, \infty)$
d $f(g^{-1}(x)) = \frac{1}{64}x^2 + \frac{12a+1}{8}x + 6a^2 + 6a + \frac{1}{4}$,
Range $= [-30a^2, \infty)$
e $a = 3$

21 a $S = [-1, 1]$
b $a = -1$, $(g \circ f)^{-1}(x) = -1 + \sqrt{x+1}$
Range = Domain$= [-1, \infty)$

Algorithms and pseudocode

See online solutions

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Chapter 9

Exercise 9A

1 -1

2 -1

3 **a** $h + 9$ **b** 9

4 **a** $x + 1$ **b** $2x^3 + 1$ **c** 40 **d** 0
e 5 **f** 1 **g** $2x + 1$ **h** $3x$
i $3x^3 + x$ **j** $6x$

5 **a** $2 + 3h + h^2$ **b** 2

6 $2x + h,\ 2x$

7 $h + 6,\ 6$

8 **a** $10x$ **b** 3 **c** 0 **d** $6x + 4$
e $15x^2$ **f** $10x - 6$

9 See solutions for comparison.

Exercise 9B

1 **a** $5x^4$ **b** $28x^6$ **c** 6 **d** $10x - 4$
e $12x^2 + 12x + 2$ **f** $20x^3 + 9x^2$
g $-4x + 4$ **h** $18x^2 - 4x + 4$

2 **a** -4 **b** -8 **c** -2 **d** -4

3 **a** -4 **b** -36

4 **a** $3t^2$ **b** $3t^2 - 2t$ **c** $x^3 + 9x^2$

5 **a** -2 **b** 0 **c** $15x^2 - 6x + 2$
d $\dfrac{6x^2 - 8}{5}$ **e** $4x - 5$ **f** $12x - 12$
g $50x^4$ **h** $27x^2 + 3$

6 **a** $4x - 15x^2$ **b** $-4z - 6$ **c** $18z^2 - 8z$
d $-2 - 15x^2$ **e** $-4z - 6$ **f** $-3z^2 - 8z$

7 **a** $(-\frac{1}{2}, 3\frac{1}{2})$ **b** $(2, 32),\ (-2, -32)$
c $(2, 6)$ **d** $(0, 0),\ (2, -4)$

8 **a** $(1, 7)$ **b** $\left(\dfrac{5}{4}, \dfrac{59}{8}\right)$

9 **a** $x = 1$ **b** $x = 0$ **c** $x = \dfrac{1 + \sqrt{3}}{2}$
d $x = \dfrac{3 + \sqrt{3}}{6}$ **e** $x = \dfrac{1 - \sqrt{3}}{2}$

10 **a** 78.69° **b** 0° **c** 45° **d** 135°
e 63.43° **f** 116.57°

11 **a** $8x - 4$ **b** $2x + 2$ **c** $6x^2 - 12x + 18$
d $x^2 - 2x + 1$

12 **a** $(3, 16)$, gradient $= 8$
b $(0, -1)$, gradient $= -1$
c $(-1, 6)$, gradient $= -8$
d $(4, 594)$, gradient $= 393$
e $(1, -28)$, gradient $= -92$
f $(2\frac{1}{2}, 0)$, gradient $= 0$

13 **a** $x = 1$ **b** $x = 1$ **c** $x > 1$ **d** $x < 1$
e $x = 2\frac{2}{3}$ **f** $x = 4$ or $x = -2$

14 **a** $(-\infty, -1) \cup (1, \infty)$ **b** $(-1, 1)$ **c** $\{1, -1\}$

15 **a** $(-1, 0.5) \cup (2, \infty)$ **b** $(-\infty, -1) \cup (0.5, 2)$
c $\{-1, 0.5, 2\}$

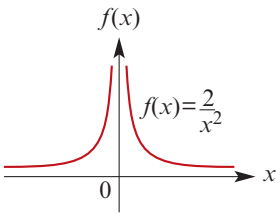
16 **a** $(-\frac{1}{4}, \infty) \setminus \{2\}$ **b** $(-\infty, -\frac{1}{4})$ **c** $\{-\frac{1}{4}, 2\}$

17 **a** $(2, -12)$ **b** $(3, -11)$ **c** $\left(\dfrac{5}{4}, -\dfrac{183}{16}\right)$

21 **a** $(-\infty, -1]$ **b** $[2, \infty)$ **c** $(-\infty, 0]$
d $[\frac{3}{2}, \infty)$

Exercise 9C

1 **a**



b $\dfrac{-2(2 + h)}{(1 + h)^2}$ **c** -4

2 **a** $-\dfrac{1}{(x - 3)^2}$ **b** $\dfrac{-1}{(x + 2)^2}$

3 $-4x^{-5}$

4 **a** $-6x^{-3} - 5x^{-2}$ **b** $12x - \dfrac{15}{x^4}$
c $\dfrac{15}{x^4} - \dfrac{8}{x^3}$ **d** $-18x^{-4} - 6x^{-3}$ **e** $-\dfrac{2}{x^2}$

5 **a** $\dfrac{4}{z^2}$ **b** $\dfrac{-18 - 2z}{z^4}$ **c** $3z^{-4}$
d $\dfrac{-2z^3 + z^2 - 4}{z^2}$ **e** $\dfrac{6 - 12z}{z^4}$
f $-6x - \dfrac{6}{x^2}$

6 **a** $11\frac{3}{4}$ **b** $\dfrac{1}{32}$ **c** -1 **d** 5

7 $f'(x) = 10x^{-6} > 0$ for all $x \in \mathbb{R} \setminus \{0\}$

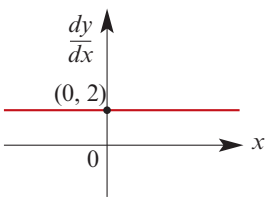
8 $\pm\dfrac{1}{2}$ **9** $a = -1,\ b = 4$

10 $\dfrac{1}{2}$ **11** $a = -9,\ b = 1$

12 $k = 0$ or $k = \dfrac{3}{2}$

Exercise 9D

1 **a**



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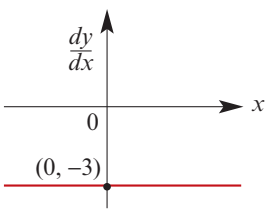

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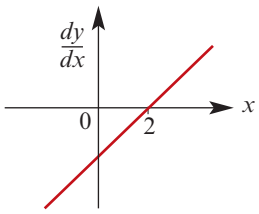
c



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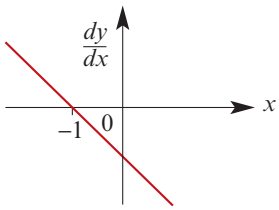

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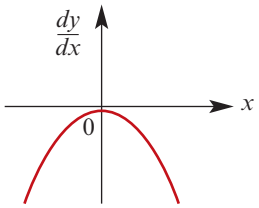

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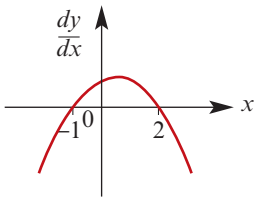

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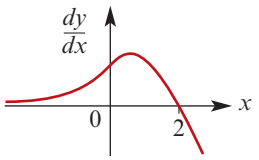

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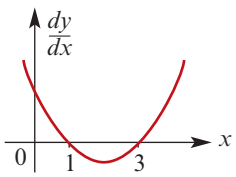

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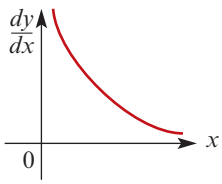

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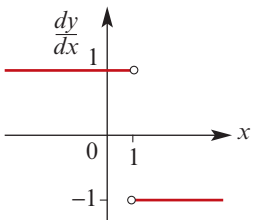

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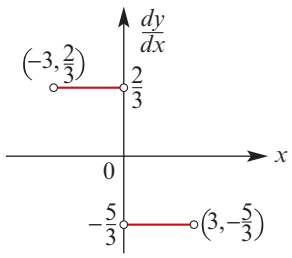

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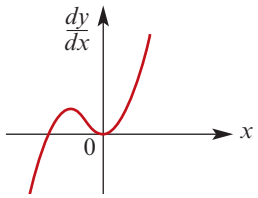

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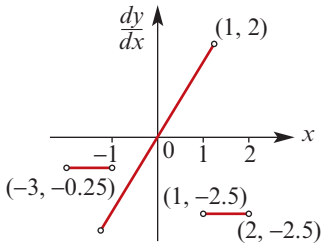

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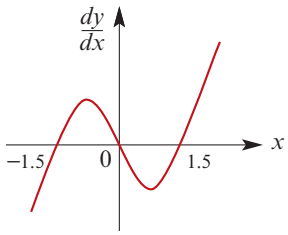

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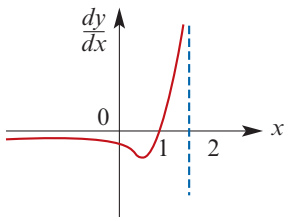

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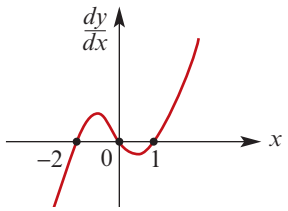

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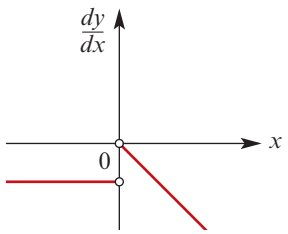

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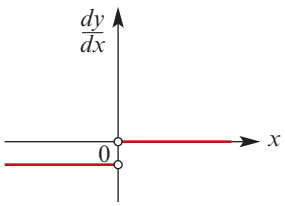
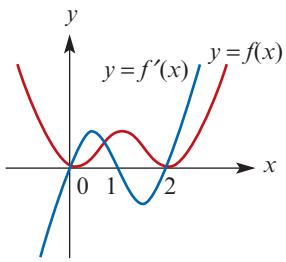
i



3 a D **b** F **c** B **d** C **e** A **f** E

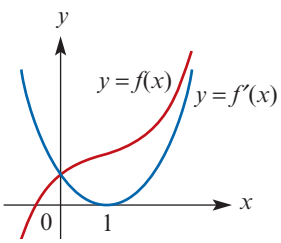
4 a b



c i 0 **ii** 0 **iii** 0 **iv** 96

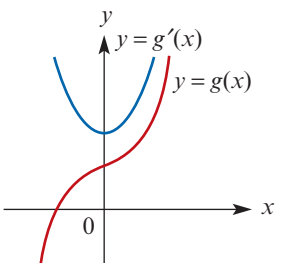
d i 1 **ii** 0.423

5



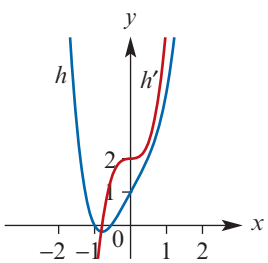
Gradient is 0 at $(1, \frac{4}{3})$;
Gradient is positive for $\mathbb{R} \setminus \{1\}$

6



Gradient is always positive;
Minimum gradient where $x = 0$

7 a



b i $x = -1.4945$ or $x = 0.7976$

ii $x = 0.6300$

Exercise 9E

1 a $8x(x^2 + 1)^3$ **b** $20x(2x^2 - 3)^4$

c $24(6x + 1)^3$ **d** $an(ax + b)^{n-1}$

e $2anx(ax^2 + b)^{n-1}$ **f** $\dfrac{6x}{(1 - x^2)^4}$

g $-3\left(x^2 - \dfrac{1}{x^2}\right)^{-4}\left(2x + \dfrac{2}{x^3}\right)$ **h** $(1 - x)^{-2}$

2 a $6(x + 1)^5$ **b** $4x^3(3x + 1)(x + 1)^7$

c $4\left(6x^3 + \dfrac{2}{x}\right)^3\left(18x^2 - \dfrac{2}{x^2}\right)$ **d** $-4(x + 1)^{-5}$

3 -10 **4** $-\dfrac{1}{2}$ and $\dfrac{1}{2}$ **5** $2x\sqrt{3x^2 + 1}$

6 a $n[f(x)]^{n-1}f'(x)$ **b** $\dfrac{-f'(x)}{[f(x)]^2}$

7 $x = \dfrac{3}{2}$ **8** 48

Exercise 9F

1 $x^{-\frac{1}{2}}$

2 a $\dfrac{x^{-\frac{4}{5}}}{5}$ **b** $\dfrac{5}{2}x^{\frac{3}{2}}$

c $\dfrac{5}{2}x^{\frac{3}{2}} - \dfrac{3}{2}x^{\frac{1}{2}}$ **d** $\dfrac{3}{2}x^{-\frac{1}{2}} - \dfrac{20}{3}x^{\frac{2}{3}}$

e $-\dfrac{6}{7}x^{-\frac{13}{7}}$ **f** $-\dfrac{1}{4}x^{-\frac{5}{4}} + 2x^{-\frac{1}{2}}$

3 a $\dfrac{1}{27}$ **b** $\dfrac{1}{12}$ **c** $\dfrac{2}{9}$ **d** $\dfrac{5}{2}$

4 a $\dfrac{1}{\sqrt{2x + 1}}$ **b** $\dfrac{-3}{2\sqrt{4 - 3x}}$

c $\dfrac{x}{\sqrt{x^2 + 2}}$ **d** $\dfrac{-1}{\sqrt[3]{(4 - 3x)^2}}$

e $\dfrac{3}{2}\sqrt{x} - \dfrac{1}{\sqrt{x^3}}$ **f** $3\sqrt{x}\left(\dfrac{5x + 6}{2}\right)$

7 a $\dfrac{x}{\sqrt{x^2 + 2}}$ **b** $\dfrac{2x - 5}{3\sqrt[3]{(x^2 - 5x)^2}}$

c $\dfrac{2x + 2}{5\sqrt[5]{(x^2 + 2x)^4}}$

Exercise 9G

1 a $5e^{5x}$ **b** $-21e^{-3x}$

c $-12e^{-4x} + e^x - 2x$ **d** $e^x - e^{-x}$

e $e^{-2x}(e^x - 1)$ **f** $2e^{2x} - 2e^{-2x}$

2 a $-6x^2e^{-2x^3}$ **b** $2xe^{x^2} + 3$

c $(2x - 4)e^{x^2-4x} + 3$ **d** $(2x - 2)e^{x^2-2x+3} - 1$

e $-\dfrac{1}{x^2}e^{\frac{1}{x}}$ **f** $\dfrac{1}{2}x^{-\frac{1}{2}}e^{x^{\frac{1}{2}}}$

3 a $\dfrac{9}{2}$ **b** $\dfrac{1}{2}e^{\frac{1}{2}} + 4$

4 a 5 **b** $5e^4 + 2$

5 a $2f'(x)e^{2f(x)}$ **b** $2e^{2x}f'(e^{2x})$

6 a $8e^{2x}(e^{2x} - 1)^3$ **b** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

c $\dfrac{e^x}{2\sqrt{e^x - 1}}$ **d** $\dfrac{2}{3}e^{x^{\frac{2}{3}}}x^{-\frac{1}{3}}$

e $(2x - 3)e^{(x-1)(x-2)}$ **f** e^{e^x+x}

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Exercise 9H

1 a $\frac{2}{x}$ **b** $\frac{2}{x}$ **c** $2x + \frac{3}{x}$
d $\frac{3x-1}{x^2}$ **e** $\frac{3+x}{x}$ **f** $\frac{1}{x+1}$
g $\frac{1}{x+2}$ **h** $\frac{3}{3x-1}$ **i** $\frac{6}{6x-1}$

2 a $\frac{3}{x}$ **b** $\frac{3(\log_e x)^2}{x}$ **c** $\frac{2x+1}{x^2+x-1}$
d $\frac{3x^2+2x}{x^3+x^2}$ **e** $\frac{4}{2x+3}$ **f** $\frac{4}{2x-3}$

3 a $\frac{2x}{x^2+1}$ **b** 1

4 a $(e, 1),\ m = \frac{1}{e}$
b $(e, \log_e(e^2+1)),\ m = \frac{2e}{e^2+1}$
c $(-e, 1),\ m = -\frac{1}{e}$ **d** $(1, 1),\ m = 2$
e $(1, 0),\ m = 0$ **f** $(\frac{3}{2}, \log_e 2),\ m = 1$

5 $\frac{1}{2}$ **6** $\frac{1+2x}{1+x+x^2}$ **7** $\frac{3}{5}$ **8** 2

Exercise 9I

1 a $5\cos(5x)$ **b** $-5\sin(5x)$
c $5\sec^2(5x)$ **d** $2\sin x\cos x$
e $3\sec^2(3x+1)$ **f** $-2x\sin(x^2+1)$
g $2\sin\left(x-\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)$
h $-2\cos\left(x-\frac{\pi}{3}\right)\sin\left(x-\frac{\pi}{3}\right)$
i $6\sin^2\left(2x+\frac{\pi}{6}\right)\cos\left(2x+\frac{\pi}{6}\right)$
j $-6\sin^2\left(2x+\frac{\pi}{4}\right)\cos\left(2x+\frac{\pi}{4}\right)$

2 a $\frac{1}{\sqrt{2}},\ \sqrt{2}$ **b** 1, 0 **c** 2, 0
d 0, 0 **e** 1, 0 **f** 1, 4

3 a $-5\sin(x) - 6\cos(3x)$
b $-\sin x + \cos x$ **c** $\cos x + \sec^2 x$
d $2\tan x\sec^2 x$

4 a $-\frac{\pi}{90}\sin x°$ **b** $\frac{\pi}{60}\cos x°$
c $\frac{\pi}{60}\sec^2(3x)°$

5 a $\tan x$ **b** $\frac{-1}{\sin x\cos x}$

6 a $2\cos(x)\,e^{2\sin x}$ **b** $-2\sin(2x)\,e^{\cos(2x)}$

Exercise 9J

1 a $20x^4 + 36x^2 + 4x$ **b** $9x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$
c $3(2x-1)^2(8x-1)$ **d** $8x(2x^2+1)(6x^2+1)$
e $5(3x+1)^{\frac{1}{2}}(3x+4)$ **f** $\frac{5x^2-8x+1}{\sqrt{2x-4}}$
g $x^2(3x^2+4x+3)(3x^2+2x+1)^{-2}$
h $2x^3(5x^2-2)(2x^2-1)^{-\frac{1}{2}}$
i $2x\sqrt[3]{x^2+2x} + \frac{2x^2(x+1)}{3\sqrt[3]{(x^2+2x)^2}}$
j $\frac{4(5x^2-4)^2(5x^2+2)}{x^3}$
k $\frac{3(x^6-16)}{x^4}$ **l** $\frac{2x^3(9x^2-8)}{5(x(x^2-1))^{4/5}}$

2 a $e^x(x^2+2x+1)$
b $e^{2x}(2x^3+3x^2+6x+5)$
c $2e^{4x+1}(x+1)(2x+3)$ **d** $\frac{-8x-7}{2e^{4x}\sqrt{x+1}}$

3 a $1 + \log_e x$ **b** $2x + 4x\log_e x$
c $e^x\log_e x + \frac{e^x}{x}$ **d** $1 + \log_e(-x)$

4 a $\frac{2x^3(2-x)}{e^{2x}}$ **b** $2e^{2x+3}$
c $\frac{3}{2}(2e^{2x}+1)(e^{2x}+x)^{\frac{1}{2}}$ **d** $\frac{e^x(x-1)}{x^2}$
e $xe^{\frac{1}{2}x^2}$ **f** $-x^2e^{-x}$

5 a $e^x(f'(x)+f(x))$ **b** $\frac{e^x(f(x)-f'(x))}{[f(x)]^2}$
c $f'(x)e^{f(x)}$ **d** $2e^xf'(x)f(x) + [f(x)]^2e^x$

6 a $3x^2\cos(x) - x^3\sin(x)$
b $2x\cos x - (1+x^2)\sin x$
c $-e^{-x}\sin x + e^{-x}\cos x$
d $6\cos x - 6x\sin x$
e $3\cos(3x)\cos(4x) - 4\sin(4x)\sin(3x)$
f $2\sin(2x) + 2\tan(2x)\sec(2x)$
g $12\sin x + 12x\cos x$
h $2xe^{\sin x} + x^2\cos x e^{\sin x}$
i $2x\cos^2 x - 2x^2\cos x\sin x$
j $e^x\tan x + e^x\sec^2 x$

7 a $-e^{\pi}$ **b** 0

8 2

Exercise 9K

1 a $\frac{4}{(x+4)^2}$ **b** $\frac{4x}{(x^2+1)^2}$ **c** $\frac{x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{2(1+x)^2}$
d $\frac{(x+2)^2(x-3)(x-1)}{(x^2+1)^2}$ **e** $\frac{2+2x-x^2}{(x^2+2)^2}$
f $\frac{-4x}{(x^2-1)^2}$ **g** $\frac{x^2+4x+1}{(x^2+x+1)^2}$
h $\frac{-2(4x^3+3x^2+1)}{(2x^3+2x)^2}$

2 a 81, 378 **b** 0, 0 **c** 0, 0
d $\frac{1}{2}$, 0 **e** $\frac{3}{2}, -\frac{1}{2}$

3 a $\frac{2x^2+x+1}{\sqrt{x^2+1}}$ **b** $\frac{x(7x^3+3x+4)}{2\sqrt{x^3+1}}$
c $\frac{5}{(x+3)^2}$

4 a $\frac{3e^x - 2e^{4x}}{(3+e^{3x})^2}$ **b** $-\left(\frac{(x+1)\sin(x) + \cos(x)}{(x+1)^2}\right)$
c $\frac{x - x\log_e(x) + 1}{x(x+1)^2}$

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5 a $\dfrac{1-\log_e x}{x^2}$ **b** $\dfrac{1+x^2-2x^2\log_e x}{x(1+x^2)^2}$

6 a $\dfrac{9e^{3x}}{(3+e^{3x})^2}$ **b** $\dfrac{-2e^x}{(e^x-1)^2}$ **c** $\dfrac{-8e^{2x}}{(e^{2x}-2)^2}$

7 a -2 **b** -6π **c** $-e^{\pi}$ **d** $-\dfrac{1}{\pi}$

Exercise 9L

1 a 17 **b** 3 **c** -4 **d** $\dfrac{1}{8}$

e 3 **f** 4 **g** 2 **h** $2\sqrt{3}$

i -2 **j** 12 **k** $\dfrac{11}{9}$ **l** $\dfrac{1}{4}$

2 a 3, 4 **b** 7

3 a Discontinuity at $x = 0$, as $f(0) = 0$, $\lim\limits_{x\to 0^+} f(x) = 0$, but $\lim\limits_{x\to 0^-} f(x) = 2$

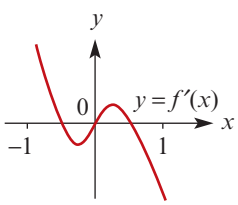
b Discontinuity at $x = 1$ as $f(1) = 3$, $\lim\limits_{x\to 1^+} f(x) = 3$, but $\lim\limits_{x\to 0^-} f(x) = -1$

c Discontinuity at $x = 0$ as $f(0) = 1$, $\lim\limits_{x\to 0^+} f(x) = 1$, but $\lim\limits_{x\to 1^-} f(x) = 0$

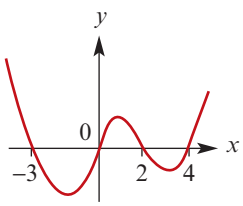
4 $\mathbb{R} \setminus \{1\}$

Exercise 9M

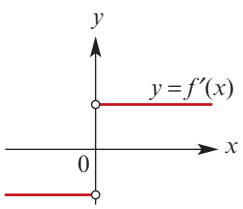
1 a



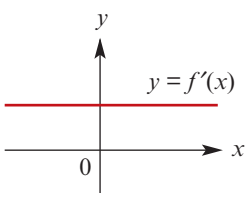
b



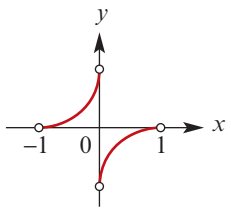
c



d



e



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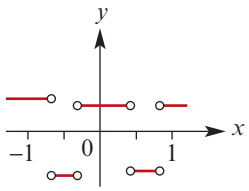

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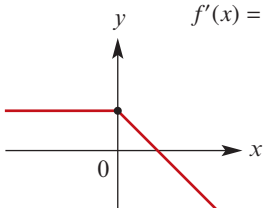
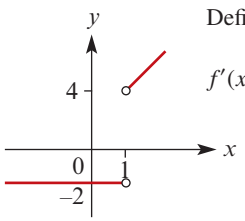


$$f'(x) = \begin{cases} -2x+3 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$$

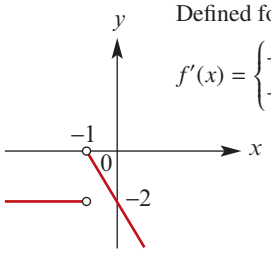
3



Defined for $\mathbb{R} \setminus \{1\}$

$$f'(x) = \begin{cases} 2x+2 & \text{if } x > 1 \\ -2 & \text{if } x < 1 \end{cases}$$

4



Defined for $\mathbb{R} \setminus \{-1\}$

$$f'(x) = \begin{cases} -2x-2 & \text{if } x > -1 \\ -2 & \text{if } x < -1 \end{cases}$$

5 a $\mathbb{R} \setminus \{1\}$, $f'(x) = \dfrac{1}{3}(x-1)^{-\frac{2}{3}}$

b $\mathbb{R} \setminus \{0\}$, $f'(x) = \dfrac{1}{5}(x)^{-\frac{4}{5}}$

c $\mathbb{R} \setminus \{0\}$, $f'(x) = \dfrac{2}{3}x^{-\frac{1}{3}}$

d $\mathbb{R} \setminus \{-2\}$, $f'(x) = \dfrac{2}{5}(x+2)^{-\frac{3}{5}}$

Chapter 9 review

Technology-free questions

1 a 8 **b** -8

2 a $1 - \dfrac{x}{\sqrt{1-x^2}}$ **b** $\dfrac{-4x^2-2x+12}{(x^2+3)^2}$

c $\dfrac{3}{2\sqrt{1+3x}}$ **d** $\dfrac{-2}{x^2} - \dfrac{1}{2}x^{-\frac{3}{2}}$

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e $\dfrac{3x-15}{2\sqrt{x-3}}$ **f** $\dfrac{1+2x^2}{\sqrt{1+x^2}}$

g $\dfrac{4x}{(x^2+1)^2}$ **h** $\dfrac{-x^2+1}{(x^2+1)^2}$

i $\dfrac{10x}{3}(2+5x^2)^{-\frac{2}{3}}$ **j** $\dfrac{-2x^2-2x+4}{(x^2+2)^2}$

k $4x(3x^2+2)^{-\frac{1}{3}}$

3 a -6 **b** 1 **c** 5 **d** $\dfrac{1}{6}$

4 a $\dfrac{1}{x+2}$ **b** $3\cos(3x+2)$

c $-\dfrac{1}{2}\sin\left(\dfrac{x}{2}\right)$ **d** $(2x-2)e^{x^2-2x}$

e $\dfrac{1}{x-3}$ **f** $2\pi\cos(2\pi x)$

g $6\sin(3x+1)\cos(3x+1)$

h $\dfrac{1}{2x\sqrt{\log_e x}}$ **i** $\dfrac{2-2\log_e 2x}{x^2}$

j $2x\sin(2\pi x)+2\pi x^2\cos(2\pi x)$

5 a $e^x\sin(2x)+2e^x\cos(2x)$

b $4x\log_e x+2x$ **c** $\dfrac{1-3\log_e x}{x^4}$

d $2\cos(2x)\cos(3x)-3\sin(2x)\sin(3x)$

e $\dfrac{2}{\cos^2(2x)}=2\sec^2(2x)$

f $-9\cos^2(3x+2)\sin(3x+2)$

g $2x\sin^2(3x)+6x^2\cos(3x)\sin(3x)$

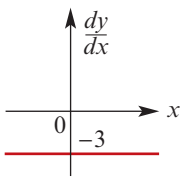
6 a $2e^2\approx 14.78$ **b** 0

c $15e^3+2\approx 303.28$ **d** 1

7 a ae^{ax} **b** ae^{ax+b} **c** $-be^{a-bx}$

d $abe^{ax}-abe^{bx}$ **e** $(a-b)e^{(a-b)x}$

8 a



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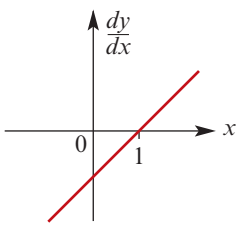

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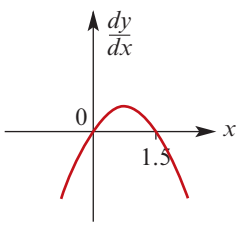

9 $2\left(4-\dfrac{9}{x^2}\right)\left(4x+\dfrac{9}{x}\right)$, $x=\pm\dfrac{3}{2}$

10 b $(\frac{3}{2},\infty)\cap(-1,4)=(\frac{3}{2},4)$

11 a $xf'(x)+f(x)$ **b** $\dfrac{-f'(x)}{[f(x)]^2}$

c $\dfrac{f(x)-xf'(x)}{[f(x)]^2}$ **d** $\dfrac{2xf(x)-2x^2f'(x)}{[f(x)]^3}$

12 a $f\circ g(x)=2\cos^3 x-1$

b $g\circ f(x)=\cos(2x^3-1)$

c $g'\circ f(x)=-\sin(2x^3-1)$

d $(g\circ f)'(x)=-(6x^2)\sin(2x^3-1)$

e $\dfrac{3}{2}$ **f** $-\dfrac{3\sqrt{3}}{4}$

13 $0<x<2$

14 $x=-\dfrac{1}{2}$

15 $b=\dfrac{1}{4}$, $c=1$

16 a $\log_e 18$ **b** $(3\log_e(2),-100)$

c $x>3\log_e(2)$ **d** $\dfrac{50}{\log_e\frac{3}{2}}$

Multiple-choice questions

1 A **2** C **3** A **4** A **5** B
6 C **7** D **8** D **9** A **10** B
11 E **12** C **13** A

Extended-response questions

1 a i -4 **ii** -6 **iii** -18 **iv** -18 **v** 6 **vi** $-\dfrac{1}{6}$

b $a=\dfrac{5}{2}$, $b=1$, $c=-\dfrac{7}{2}$, $d=6$

2 a i -1 and 3 **ii** $x>3$ and $x<-1$

b $(3,6)$ and $(7,1)$ **c** $\left(\dfrac{1}{2},6\right)$ and $\left(\dfrac{5}{2},1\right)$

d $(2,6)$ and $(10,1)$ **e** $(2,18)$ and $(10,3)$

3 a $x=\alpha$ or $x=\beta$

b $(x-\beta)^{m-1}(x-\alpha)^{n-1}((m+n)x-\alpha m-\beta n)$

c $x=\alpha$ or $x=\beta$ or $x=\dfrac{\alpha m+\beta n}{m+n}$

d i $x>\dfrac{\alpha m+\beta n}{m+n}$, $x\neq\beta$

ii $x<\alpha$ or $x>\dfrac{\alpha m+\beta n}{m+n}$

4 b $\dfrac{nx^{n-1}}{(x^n+1)^2}$ **d** $x=0$ **e** $x>0$

Chapter 10

Exercise 10A

1 $y=4x-5$

2 $y=-\dfrac{1}{3}x-1$

3 $y=x-2$ and $y=-x+3$

4 $y=18x+1$, $y=-\dfrac{1}{18}x+1$

5 $\left(\dfrac{3}{2},-\dfrac{11}{4}\right)$, $c=-\dfrac{29}{4}$

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6 a i $y = 2x - 3$ **ii** $y = -\frac{1}{2}x - \frac{1}{2}$

b i $y = -3x - 1$ **ii** $y = \frac{1}{3}x - 1$

c i $y = -x - 2$ **ii** $y = x$

d i $y = 8x + 2$ **ii** $y = -\frac{1}{8}x - \frac{49}{8}$

e i $y = \frac{3}{2}x + 1$ **ii** $y = -\frac{2}{3}x + 1$

f i $y = \frac{1}{2}x + \frac{1}{2}$ **ii** $y = -2x + 3$

g i $y = \frac{2}{3}x + \frac{4}{3}$ **ii** $y = -\frac{3}{2}x + \frac{7}{2}$

h i $y = 4x - 16$ **ii** $y = -\frac{1}{4}x - \frac{15}{2}$

i i $y = -2$ **ii** $x = 2$

j i $y = 4x - 4$ **ii** $y = -\frac{1}{4}x + \frac{1}{4}$

7 $y = 56x - 160$

8 a $y = -1$ **b** $y = \frac{3}{2}x + \frac{1}{2}$

c $y = -2x - 1$ **d** $y = -4x + 5$

9 a $y = 2x$ **b** $y = -1$ **c** $y = 2x - \frac{\pi - 2}{2}$

d $y = 2x$ **e** $y = x$ **f** $y = -x + \frac{\pi - 2}{2}$

10 a $y = 2$ **b** $y = x$

c $y = 4e^2x - 3e^2$ **d** $y = \frac{e}{2}(x + 1)$

e $y = 3xe - 2e$ **f** $y = 4e^{-2}$

11 a $y = x - 1, y = -x + 1$

b $y = 2x - 1$ **c** $y = kx - 1$

12 a $x = 0$ **b** $x = 0$ **c** $x = 4$

d $x = -5$ **e** $x = -\frac{1}{2}$ **f** $x = -5$

13 $\frac{\pi - 2}{2}$ **14** $a = 1$ **15** $a = e$

16 $a = 0$ **17** $a = 0$ or $a = \frac{3}{2}$

Exercise 10B

1 a 21 **b** $3h + 18$ **c** 18

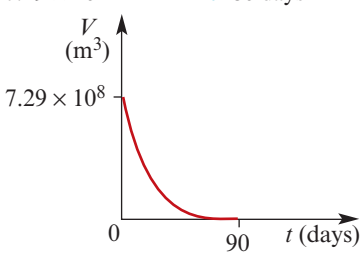
2 a $\frac{dV}{dt}$ **b** $\frac{dS}{dr}$ **c** $\frac{dV}{dx}$ **d** $\frac{dA}{dt}$ **e** $\frac{dV}{dh}$

3 Wanes by 0.006 units per day

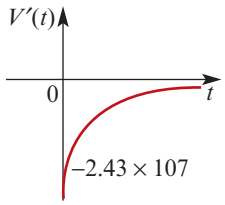
4 a $-3 \times 10^3(90 - t)^2$ **b** 90 days

c 7.29×10^8 m^3 **d** 80 days

e

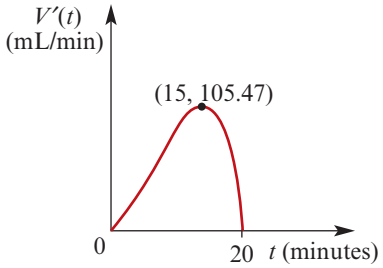


f



5 a $V'(t) = \frac{t^3}{160}(20 - t)$

b



c $t = 15$

6 a $t \approx 100, t \approx 250, t \approx 500$

b $\approx 430\,000$ m^3/day **c** $\approx 270\,000$ m^3/day

d $(100, 250) \cup (500, 600)$

7 a $\lambda = 0.1373, P_0 = 30$

b 9.625 hours

c i 4.120 units/hour **ii** 1.373 units/hour

8 a $-0.3(T - 15)$

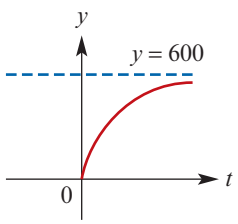
b i −22.5°C/minute **ii** −13.5°C/minute

iii −4.5°C/minute

9 $\frac{dy}{dx} = 3 - 2\sin x$, gradient always positive

10 a 4.197 **b** −0.4

11 a



b 3.33

12 a $-2y$ **b** ky

13 a 0.18 kg

b 3.47 hours

c i 6.93 hours **ii** 10.4 hours

d 0.2 m

Exercise 10C

1 a $(2, -16), (-2, 16)$ **b** $(1, -2)$

c $(0, 0), (1, 1)$ **d** $(4, 48)$

e $(0, 0), \left(\frac{2}{\sqrt{3}}, \frac{16}{3}\right), \left(\frac{-2}{\sqrt{3}}, \frac{16}{3}\right)$ **f** $\left(\frac{1}{3}, \frac{14}{3}\right)$

g $(3, 2)$ **h** $(0, -10), (2, 6)$

2 a $(0, 1)$ **b** $\left(\frac{1}{3e}, -\frac{1}{3e}\right)$

c $(0, 1), (-\pi, 1), \left(-\frac{\pi}{2}, -1\right), \left(\frac{\pi}{2}, -1\right), (\pi, 1)$

d $(-1, -e^{-1})$ **e** $(0, 0), (2, 4e^{-2})$

f $(e^{-1}, -2e^{-1})$

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3 a $a = 6$ **b** $b = 3$

4 $b = -2,\ c = 1,\ d = 3$

5 $a = 2,\ b = -4,\ c = -1$

6 $a = \frac{2}{3},\ b = -2\frac{1}{2},\ c = -3,\ d = 7\frac{1}{2}$

7 a $a = 2$ and $b = 9$ **b** $(-1, -5)$

8 $x = \dfrac{1}{2}$ or $x = \dfrac{1 - 4n}{2n + 2}$

9 $x = \pm 1$ or $x = 0$

10 $(1, \frac{1}{2})$ or $(-1, -\frac{1}{2})$

Exercise 10D

1 a $x = 0$

	0	
+	0	+
↗	—	↘

inflexion

b $x = 2,\ x = -5$

	−5		2	
+	0	−	0	+
↗	—	↘	—	↘

max. min.

c $x = -1,\ x = \dfrac{1}{2}$

	−1		$\frac{1}{2}$	
+	0	−	0	+
↗	—	↘	—	↘

max. min.

d $x = -3,\ x = 4$

	−3		4	
−	0	+	0	−
↘	—	↗	—	↘

min. max.

e $x = -3,\ x = 4$

	−3		4	
+	0	−	0	+
↗	—	↘	—	↘

max. min.

f $x = 0,\ x = \dfrac{27}{5}$

	0		$\frac{27}{5}$	
+	0	−	0	+
↗	—	↘	—	↘

max. min.

g $x = 1,\ x = 3$

	1		3	
+	0	−	0	+
↗	—	↘	—	↘

max. min.

h $x = 1,\ x = 3$

	1		3	
−	0	+	0	−
↘	—	↗	—	↘

min. max.

2 a $x = -2$ (max), $x = 2$ (min)

b $x = 0$ (min), $x = 2$ (max)

c $x = \frac{1}{3}$ (max), $x = 3$ (min)

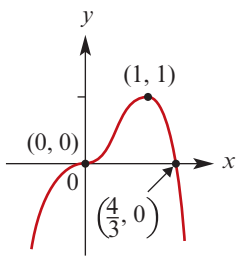
d $x = 0$ (inflection)

e $x = -2$ (inflection), $x = 0$ (min)

f $x = -\dfrac{1}{\sqrt{3}}$ (max), $x = \dfrac{1}{\sqrt{3}}$ (min)

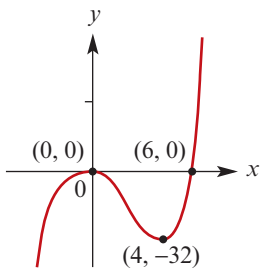
3 a i $(0, 0),\ (\frac{4}{3}, 0)$

ii $(0, 0)$ inflection, $(1, 1)$ max



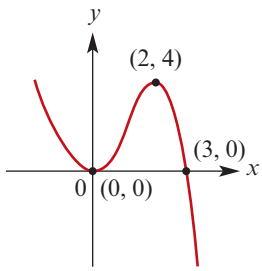
b i $(0, 0),\ (6, 0)$

ii $(0, 0)$ max, $(4, -32)$ min



c i $(0, 0),\ (3, 0)$

ii $(0, 0)$ min, $(2, 4)$ max



d i $(-4, 0),\ (-1, 0),\ (0, 4)$

ii $(-3, 4)$ max, $(-1, 0)$ min

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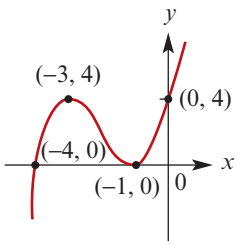
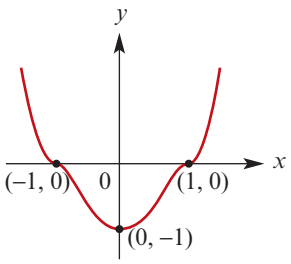
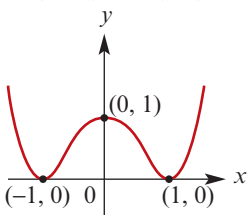


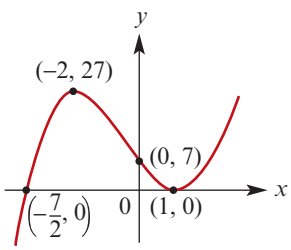
e **i** $(-1,0)$, $(0,-1)$, $(1,0)$
ii $(-1,0)$ infl, $(0,-1)$ min, $(1,0)$ infl



f **i** $(-1,0)$, $(0,1)$, $(1,0)$
ii $(-1,0)$ min, $(0,1)$ max, $(1,0)$ min

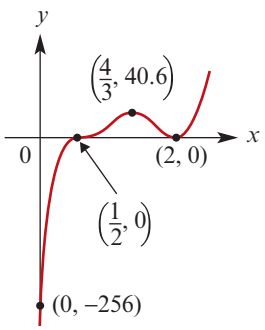


4 **a** $(-2,27)$ max, $(1,0)$ min
b $(1,0)$ is a turning point
c $\left(-\frac{7}{2},0\right)$, $(0,7)$
d

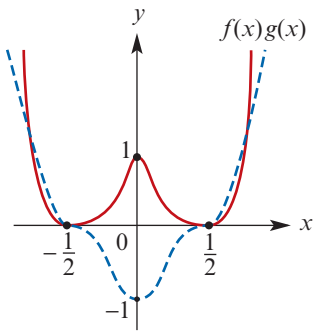


5 **b** $a=3$, $b=2$, $(0,2)$ min, $(-2,6)$ max

6 **a** $(0,-256)$, $(\frac{1}{2},0)$, $(2,0)$
b $\left(\frac{1}{2},0\right)$ inflection, $\left(\frac{4}{3},40.6\right)$ max, $(2,0)$ min



7 **a**

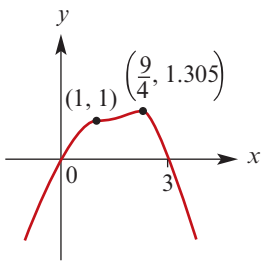


b **i** $\left(-\infty,-\frac{1}{\sqrt{2}}\right)\cup\left(-\frac{1}{2},\frac{1}{2}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)$
ii $\left(\frac{-\sqrt{66}}{12},-\frac{1}{2}\right)\cup\left(-\frac{1}{2},0\right)\cup\left(\frac{\sqrt{66}}{12},\infty\right)$

8 **a** $(-2,0)$ max, $\left(\frac{4}{3},-18\frac{14}{27}\right)$ min
b No stationary points

9 **a** $(0,0)$ stationary point of inflection, $(-1,-1)$ minimum
b $(0,-1)$ stationary point of inflection, $(-1.5,-2.6875)$ minimum
c No stationary points, gradient is always positive

10 **b** $x \le \frac{9}{4}$ **c**



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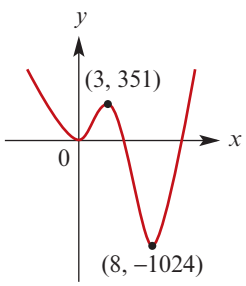

12 **a** $x=-1$ (infl), $x=1$ (min), $x=5$ (max)
b $x=0$ (max), $x=2$ (min)
c $x=-4$ (min), $x=0$ (max)
d $x=-3$ (min), $x=2$ (infl)

13 **a** $(0,0)$ local max;
$(2\sqrt{2},-64)$ and $(-2\sqrt{2},-64)$ local min
b $(0,0)$ local max;
$\left(\pm4\sqrt{\frac{m-1}{m}},-\frac{16^m(m-1)^{m-1}}{m^m}\right)$ local min

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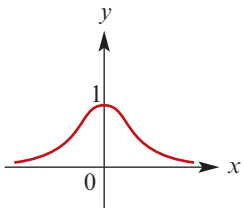
14



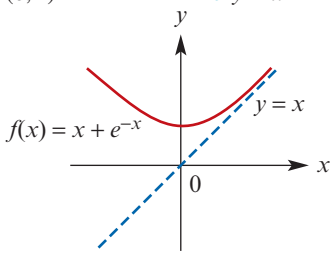
15 $\{x : -2 < x < 0\}$

16 $x < 1$; Max value $= \dfrac{100}{e^4} \approx 1.83$

17 a Min value $= f(0) = 0$

18 a $(0, 1)$ min **b** $y = x$

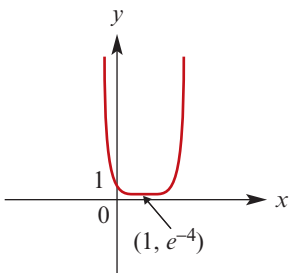
c



19 $p = 1, q = -6, r = 9$

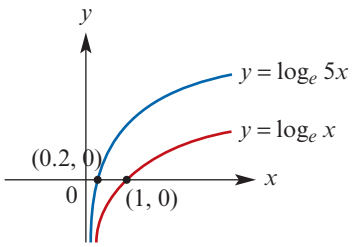
20 a $(8x - 8)e^{4x^2-8x}$ **b** $(1, e^{-4})$ min

c



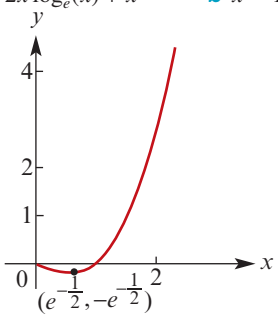
d $y = -\dfrac{1}{8}x + \dfrac{5}{4}$

21 Tangents are parallel for any given value of x.

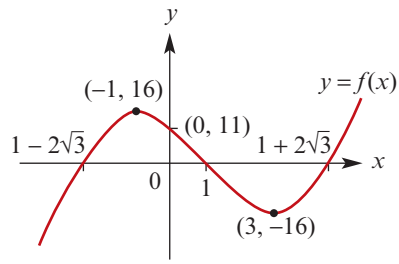


22 a $2x \log_e(x) + x$ **b** $x = 1$ **c** $x = e^{-\frac{1}{2}}$

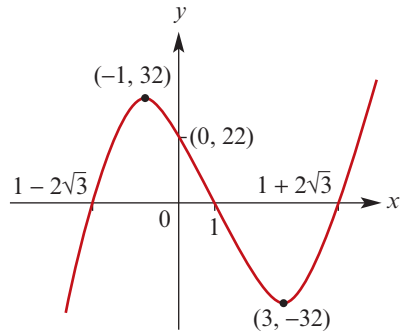
d



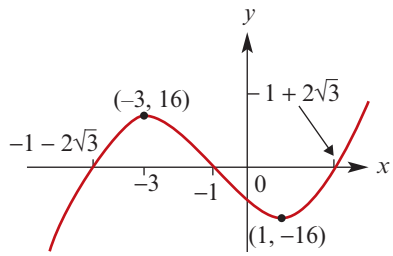
23 a



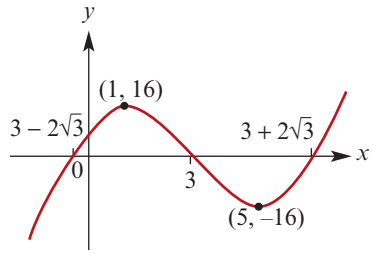
b Dilated by a factor of 2 from the x-axis:



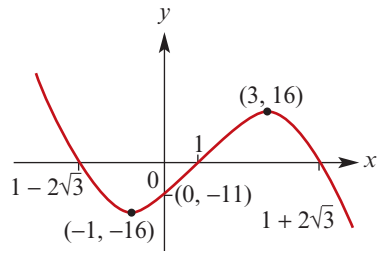
c Translated 2 units to the left:



d Translated 2 units to the right:



e Reflected in the x-axis:



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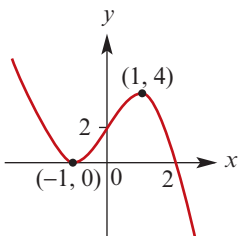
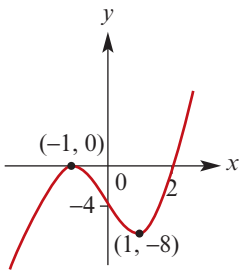
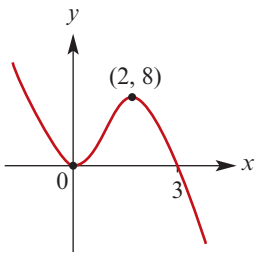
24 a



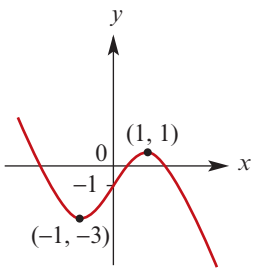
b



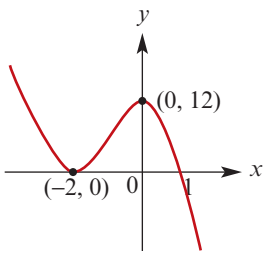
c



d



e



25 a $(a + \ell, 0)$, $(b + \ell, 0)$ **b** $(h + \ell, kp)$

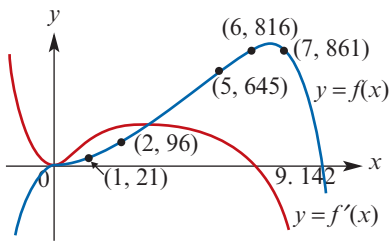
26 a Max $x = \frac{\pi}{3}, \frac{5\pi}{3}$; Min $x = 0, \pi, 2\pi$

b Max $x = \frac{\pi}{6}$; Min $x = \frac{5\pi}{6}$; Infl $x = \frac{3\pi}{2}$

c Max $x = \frac{\pi}{2}, \frac{3\pi}{2}$; Min $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

d Max $x = \frac{\pi}{3}$; Infl $x = \pi$; Min $x = \frac{5\pi}{3}$

27 a b $y = -x^4 + 8x^3 + 10x^2 + 4x$



Local max at (6.761, 867.07); No stationary point of inflection: $\frac{dy}{dx} = 4$ when $x = 0$

c −960

d $x = 4.317$ or $x = 8.404$

Exercise 10E

1 Absolute max = 2; Absolute min = −70

2 Absolute max = 15; Absolute min = −30

3 Absolute max = 0; Absolute min = −20.25

4 Absolute max = 2304; Absolute min = −8

5 b $\frac{dV}{dx} = 30x - 36x^2$

c Local max at $\left(\frac{5}{6}, \frac{125}{36}\right)$

d Absolute max value is 3.456 when $x = 0.8$

e Absolute max value is $\frac{125}{36}$ when $x = \frac{5}{6}$

6 a $25 \leq y \leq 28$

b Absolute max = 125; Absolute min = 56

7 a $\frac{1}{(x-4)^2} - \frac{1}{(x-1)^2}$ **b** $\left(\frac{5}{2}, \frac{4}{3}\right)$

c Absolute max $= \frac{3}{2}$; Absolute min $= \frac{4}{3}$

8 b $\frac{dA}{dx} = \frac{1}{4}(x - 5)$ **c** $x = 5$

d $\frac{25}{4}$ m^2, but only one square is formed

9 Absolute max = 12.1; Absolute min = 4

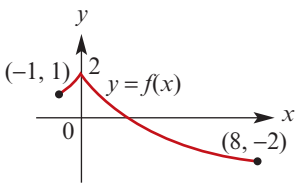
10 a $\frac{1}{(x-4)^2} - \frac{1}{(x+1)^2}$ **b** $\left(\frac{3}{2}, \frac{4}{5}\right)$

c Absolute max $= \frac{5}{4}$; Absolute min $= \frac{4}{5}$

11 Absolute max $= \frac{\sqrt{2}}{2}$; Absolute min = −1

12 Absolute max = 1; Absolute min $= \frac{\sqrt{2}}{2}$

13 Absolute max = 2; Absolute min = −2



14 Absolute max $= \frac{1}{e^2} + 2e^2$;
Absolute min $= 2\sqrt{2}$

15 Absolute max $= 2e^9$; Absolute min = 2

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16 Absolute max $= -\log_e 10$;
Absolute min $= -\dfrac{10}{e}$

Exercise 10F

1 625 m^2

2 First $= \dfrac{4}{3}$; Second $= \dfrac{8}{3}$

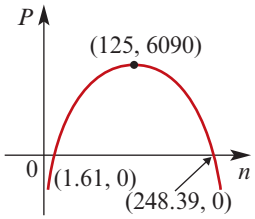
3 Max value of P is 2500

4 Max area is 2 km $\times$ 1 km $= 2$ km^2

5 $p = \dfrac{3}{2}$, $q = \dfrac{8}{3}$

6 **b** $V = \dfrac{75x - x^3}{2}$ **c** 125 cm^3

7 **a** **i** $n = 125$
ii Maximum daily profit is \$6090

b


c $2 \le n \le 248$

d $n = 20$

8 12°C

9 8 mm for maximum; $\dfrac{4}{3}$ mm for minimum

10 **a** $8\cos\theta$

b Area $= 16(1 + \cos\theta)\sin\theta$;
Max area $= 12\sqrt{3}$ square units

11 (1, 1)

12 **a** $\dfrac{75}{\cos\theta}$ seconds

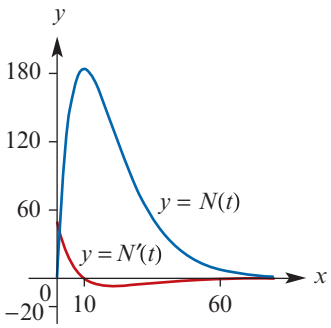
b $220 - 60\tan\theta$ seconds

d $\dfrac{dT}{d\theta} = \dfrac{75\sin\theta - 60}{\cos^2\theta}$

e $\theta = \sin^{-1}\left(\dfrac{4}{5}\right) \approx 53.13°$

f Min time $T = 265$ seconds occurs when distance BP is 400 metres

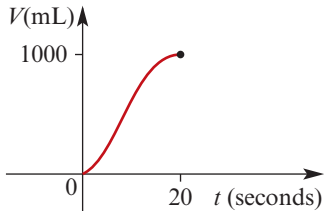
13 Max population $\dfrac{500}{e}$ occurs when $t = 10$

14 **a**


b Max rate of increase is 50, occurs at $t = 0$;
Max rate of decrease is $\dfrac{50}{e^2}$, occurs at $t = 20$

15 **a** **i** $V(0) = 0$ mL **ii** $V(20) = 1000$ mL

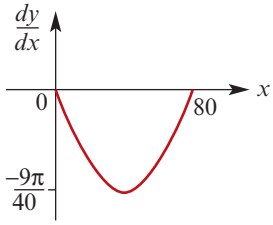
b $V'(t) = \dfrac{3}{4}(20t - t^2)$

c


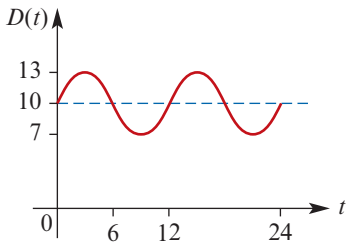
d Check the graph of $v'(t)$ on your calculator

e $t = 10$ s, 75 mL/s

16 **a** $\dfrac{dy}{dx} = \dfrac{-9\pi}{40}\sin\left(\dfrac{\pi x}{80}\right)$

b


c $\left(40, \dfrac{-9\pi}{40}\right)$

17 **a**


b $\{t : D(t) \ge 8.5\} = [0, 7] \cup [11, 19] \cup [23, 24]$

c **i** 0 m/h **ii** $-\dfrac{\pi}{2}$ m/h **iii** $\dfrac{\pi}{2}$ m/h

d **i** $t = 0, 12, 24$ **ii** $t = 6, 18$

Exercise 10G

1 **a** $f'(x) = (x - 1)(3x - 2b - 1)$

b $\left(\dfrac{2b + 1}{3}, \dfrac{-4(b - 1)^3}{27}\right)$ and $(1, 0)$

c $\dfrac{2b + 1}{3} > 1$ since $b > 1$ **d** $b = \dfrac{11}{2}$

2 **a** $(0, 0)$, $(\sqrt{2}, -4)$, $(-\sqrt{2}, -4)$

b (a, b), $(\sqrt{2} + a, -4 + b)$, $(\sqrt{2} + a, -4 + b)$

3 **a** $a = c - 20$, $b = 30 - 2c$ **b** $\dfrac{45}{2}$

4 **a** Increasing $\left[0, \dfrac{2}{3a}\right]$; Decreasing $\left[\dfrac{2}{3a}, \infty\right)$

b $y = \dfrac{-1}{a}x + \dfrac{1}{a^2}$ **c** $y = ax - 1$

d $\left(-\infty, \dfrac{4}{27a^2}\right]$

5 **a** **i** $2(a - 3)$ **ii** $m = 2(a - 3)$

b $(a, (a - 3)^2)$

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c $y = 2(a-3)x - a^2 + 9$ **d** $\frac{3+a}{2}$

6 a $h = 1$ or $h = -3$ **b** $a = 2$

c $a = -48,\ b = -64$

7 a $(a, 0),\ \left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$

b $(a, 0)$ local minimum;
$\left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$ local maximum

c i $y = (a-1)^2x - (a-1)^2$ **ii** $y = 0$

iii $y = -\frac{(a-1)^2}{4}(x-a)$

8 a $2(x-1)(x-b)(2x-b-1)$

b $(1, 0), (b, 0), \left(\frac{b+1}{2}, \frac{(b-1)^4}{16}\right)$ **c** $b = 3$

9 $a = \frac{1}{486},\ b = 0,\ c = \frac{-1}{161},\ d = \frac{1459}{243}$

10 a $a = \frac{4d-1}{4},\ b = \frac{-(2d+3)}{2},\ c = \frac{11-4d}{4}$

b $d = \frac{38}{67}$

Exercise 10H

1 a 2.151 **b** −1.75 **c** 2.554 **d** 1.564

2 1.442

3 1.618

4 $x_{n+1} = \frac{3x_n^4 - 4x_n^3 - 1}{4x_n^3 - 6x_n^2}$

5 $x_{n+1} = \frac{4x_n^5 + 158}{5x_n^4}$; 2.75253

6 a $x_1 = 0.6355,\ x_2 = 0.6412$

b $x_1 = -\frac{22}{e+8}$

7 a $0 < x \leq 4$

b $x_{n+1} = \frac{4x_n(\log_e(x_n) - 1)}{x_n - 4}$

c $y = \frac{(4-e)x}{4e}$ **e** $x_1 = \frac{4}{3}$

g $12\log_e(6) - 12$ **h** 1.43, 8.61

Chapter 10 review

Technology-free questions

1 a $y = -x$ **b** $(0, 0)$

2 $y = 6ax - 3a^2,\ P(0, -3a^2)$

3 a $y = 3x - 3$ **b** $x = \frac{11}{3}$

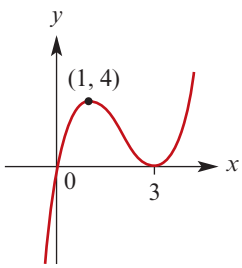
4 a 5π square units/unit **b** 6π square units/unit

5 a $(1, 1)$ max; $(0, 0)$ inflection

b $(-1, 0)$ max; $(1, -4)$ min

c $(-\sqrt{3}, 6\sqrt{3} + 1)$ max; $(\sqrt{3}, -6\sqrt{3} + 1)$ min

6



7 $x = 1$ (inflection); $x = 2$ (minimum)

8 $y = -9x + 7$

9 a $\mathbb{R} \setminus \{1\};\ f'(x) = \frac{4}{5}(x-1)^{-\frac{1}{5}}$

b $y = \frac{4}{5}x - \frac{3}{5}$ and $y = -\frac{4}{5}x + 1$ **c** $\left(1, \frac{1}{5}\right)$

10 a 64π cm^3/cm **b** 64π cm^3/s

11 a $(25e^{100t})$°C/s **b** $(25e^5)$°C/s

12 $y = ex$

13 b 20 cm/year

14 2

15 b $m = -3$

16 a $(16, -16)$

b $x = 64$

c i $y = -\frac{x}{2} - \frac{32}{3}$ **ii** $y = \frac{x}{2} - 32$

d $\left(\frac{64}{3}, -\frac{64}{3}\right)$

17 a $y = \frac{1}{e}x$ **b** $y = \frac{x}{\sqrt{2}} - \frac{\pi}{2\sqrt{2}} + \sqrt{2}$

c $y = x - \frac{3\pi}{2}$ **d** $y = \frac{-2}{\sqrt{e}}x - 1$

Multiple-choice questions

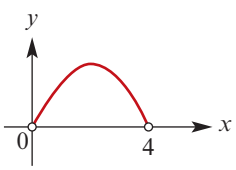
1 A	**2** E	**3** E	**4** B	**5** C
6 B	**7** A	**8** D	**9** E	**10** A
11 E	**12** B	**13** D	**14** C	

Extended-response questions

1 a $y = 4x - x^2$ **b** $0 < x < 4$ **c** $y = 4,\ x = 2$

d Gradient is positive to the left of $x = 2$, and negative to the right

e



f $0 < y < 4$

2 a $A = 4xy$ **b** $y = -\frac{2}{3}x + 8$

c $A = 32x - \frac{8}{3}x^2$ **d** $x = 6,\ y = 4$

e 96 m^2

3 a i \$12.68 **ii** \$12.74

b $C = 12 + 0.008x + \frac{14.40}{x}$

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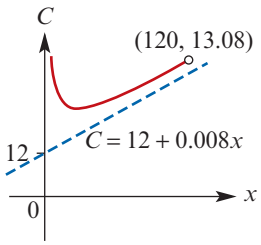
c



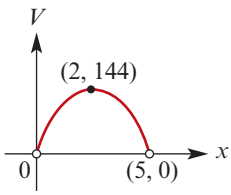
d 42.43 km/h

4 a $V = 4(x^3 - 13x^2 + 40x)$

b $0 < x < 5$ **c** $x = 2$

d 2 cm, 12 cm, 6 cm **e** 144 cm^3

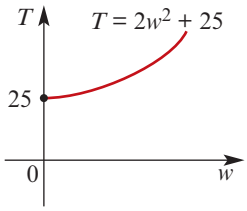
f



5 32

6 a $T = 2w^2 + 25$

b



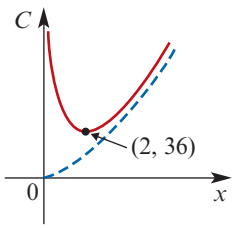
c $A = \dfrac{25}{w} + 2w$

d i $\dfrac{5\sqrt{2}}{2} \approx 3.54$ kg **ii** $10\sqrt{2} \approx 14.14$ s

7 10 m, 10 m, 5 m; Area 300 m^2

8 b $C = 3x^2 + \dfrac{48}{x}$

c



d i $x = 2$, $h = 3$; i.e. 2 m, 2 m, 3 m **ii** 36 m^2

9 a $A = \dfrac{1}{2}a^2\theta$ **b** $A = \dfrac{1}{2}\left(\dfrac{100}{\theta + 2}\right)^2\theta$

c $\theta = 2$ **d** 625 cm^2

10 b i $r = \dfrac{L}{4}$ **ii** $\theta = 2$ **iii** Maximum

11 b $\dfrac{dT}{dx} = \dfrac{x}{\sqrt{x^2 + 900}} - \dfrac{3}{5}$

c i $x = 22.5$ **ii** 71 seconds

d 63 seconds

12 a $y = ex$

b $y = 2ex$

c $y = kex$

e i $k = \dfrac{1}{e}$ or $k \leq 0$ **ii** $k > \dfrac{1}{e}$

13 b $T = \dfrac{20 + 16\sqrt{2}}{15} \approx 2.84$ hours

14 $t = 1.16$, 1.2 km apart

15 b $0 < x < 1$ **c** $x = \dfrac{1}{\sqrt{2}}$, $y = \pm 1$

d $A = 2\sqrt{2}$

16 c ii $\dfrac{dA}{dx} = -3x^2 - 2ax + a^2$

17 $t = 5$, $N(5) = \dfrac{120}{e}$

18 a $b = 5$, $c = 6$

b i 6 weeks **ii** 3.852 weeks

c $190\frac{2}{3}$ cm^2

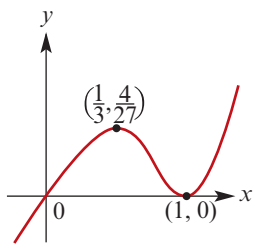
19 a $(1, -6)$ **b** $3(x - 1)^2 + 3$

c $3(x - 1)^2 + 3 > 3$ for all $x \in \mathbb{R} \setminus \{1\}$

20 a $a = 1$, $c = 1$, $b = -2$, $d = 0$

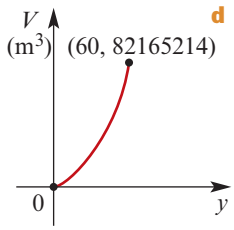
b $\{x : \frac{1}{3} < x < 1\}$

c

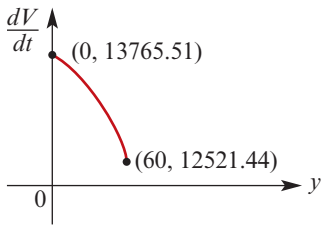


21 a 53 109 671 m^3 **b** $\dfrac{dV}{dy} = \pi(y + 630)^2$

c **d** 82 165 214 m^3



e



22 a i $r = \dfrac{2\pi - \theta}{2\pi}$

ii $h = \sqrt{1 - \left(\dfrac{2\pi - \theta}{2\pi}\right)^2}$

b $\dfrac{49\sqrt{15}\pi}{1536}$

c 0.3281, 2.5271

d i $\theta = 1.153$

ii $V_{max} = 0.403$ m^3

e 0.403 m^3

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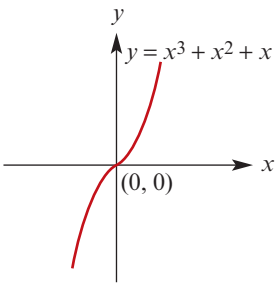
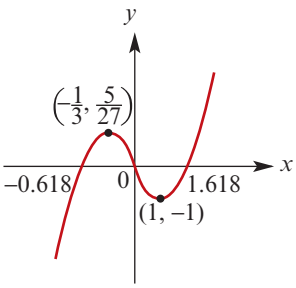
23 a i

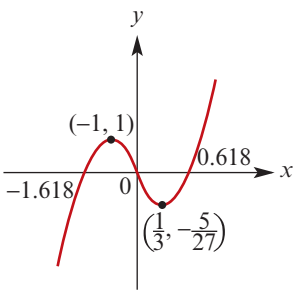


No stationary points

ii



iii



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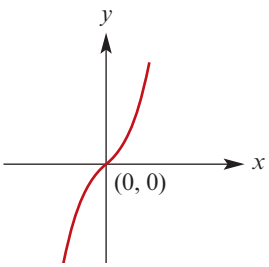


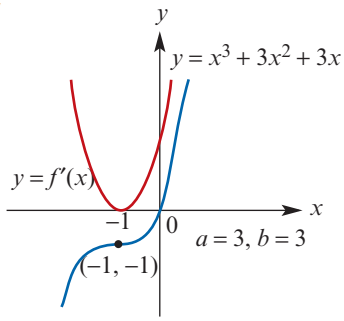
No stationary points

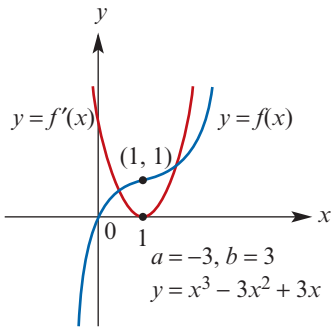
b i $f'(x) = 3x^2 + 2ax + b$

ii $x = \dfrac{-a \pm \sqrt{a^2 - 3b}}{3}$

c ii $a = -3$ or $a = 3$; $(-1, -1)$, $(1, 1)$ stationary points of inflection

iii iv



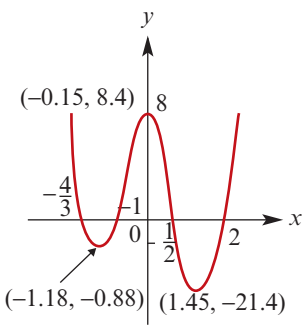


d $a^2 < 3b$

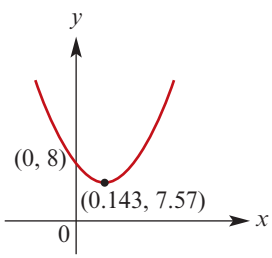
24 $x = e$

25 a i $a = -21$

ii



b i



ii Min at (0.143, 7.57)

iii $g'(x) = 24x^3 - 3x^2 + 42x - 6$

iv 0.1427

v $g'(0) = -6$, $g'(10) = 24\,114$

vi $g''(x) = 72x^2 - 6x + 42$

vii $g''(x) > 0$ for all x; thus $y = g'(x)$ has no turning points and crosses the x-axis only once

26 b i $x = a$ or $x = b$ or $x = \dfrac{b + a}{2}$

ii $x = a$ or $x = b$

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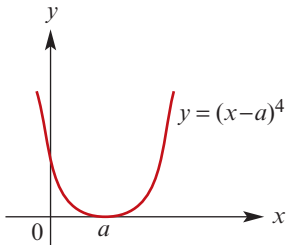
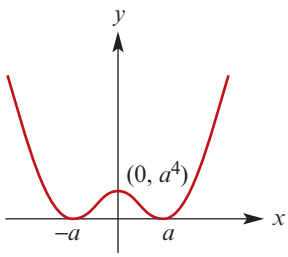
c $(a, 0),\ (b, 0),\ \left(\frac{a+b}{2}, \frac{(a-b)^4}{16}\right)$

e i



ii $(a, 0),\ (-a, 0),\ (0, a^4)$

iii



27 b i $x = a$ or $x = \frac{3b+a}{4}$

ii $x = a$ or $x = b$

c Local min at $\left(\frac{3b+a}{4}, \frac{-27}{256}(b-a)^4\right)$;
Stationary point of inflection at $(a, 0)$

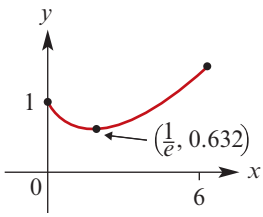
e $\left(-\frac{a}{2}, \frac{-27a^4}{16}\right)$ and $(a, 0)$

f i $b = -\frac{a}{3}$

28 a $f'(x) = \log_e x + 1$

b $x = \frac{1}{e} \approx 0.37$, i.e. during the first month

c

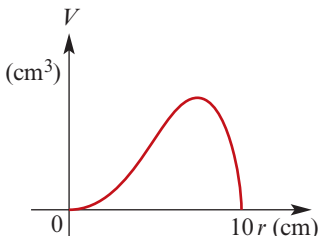


d When $x = 6$

29 a i $y = \sqrt{100 - r^2},\ h = 2\sqrt{100 - r^2}$

ii $V = 2\pi r^2\sqrt{100 - r^2}$

b i



ii $V = 2418.4,\ r = 8.165,\ h = 11.55$

iii $r = 6.456$ or $r = 9.297$

c i $\frac{dV}{dr} = \frac{400\pi r - 6\pi r^3}{\sqrt{100 - r^2}}$

ii $V_{\max} = \frac{4000\pi\sqrt{3}}{9}$ when $r = \frac{10\sqrt{6}}{3}$

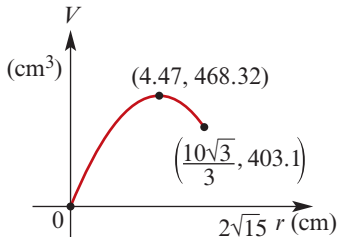
d ii $\frac{dV}{dr} > 0$ for $r \in \left(0, \frac{20\sqrt{6}}{6}\right)$

iii $\frac{dV}{dr}$ is increasing for $r \in (0, 5.21)$

30 a $h = \frac{100 - 3r^2}{2r}$ **b** $V = \frac{\pi r}{6}(300 - 5r^2)$

c $0 < r < \frac{10\sqrt{3}}{3}$ **d** $\frac{dV}{dr} = \frac{\pi}{6}(300 - 15r^2)$

e



31 a i $y = \frac{100}{x^2}$ **ii** $S = \frac{3000}{x} + 60x^2$

b i $\frac{dS}{dx} = -\frac{3000}{x^2} + 120x$ **ii** 1538.99 cm^2

c 585 cm^2/s

32 a $f'(x) = \frac{3\,000\,000e^{-0.3x}}{(1 + 100e^{-0.3x})^2}$

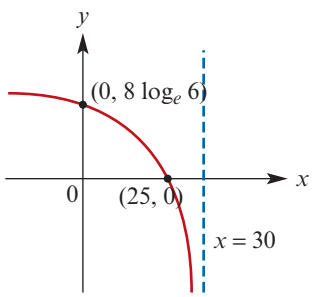
b i 294 kangaroos per year

ii 933 kangaroos per year

33 a $a = 30$ **b** $(0, 8\log_e 6),\ (25, 0)$

c $f'(20) = -0.8$ **d** $f^{-1}(x) = 5\left(6 - e^{\frac{x}{8}}\right)$

e $\mathbb{R}$ **f**



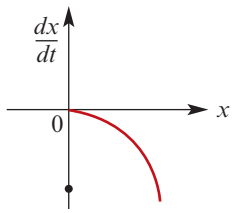
34 b $\left(\frac{\pi}{2}, e\right), \left(\frac{3\pi}{2}, \frac{1}{e}\right)$ **c** $\left[\frac{1}{e}, e\right]$

d Period 2π, since $g(x + 2\pi) = g(x)$

36 a i 30 g **ii** 12.28 g

b $\frac{dx}{dt} = \frac{-300\lambda e^{\lambda t}}{(5e^{\lambda t} - 3)^2}$

c ii



37 b $MP = \frac{2}{\tan\theta}$ **c** $NQ = 8\tan\theta$

d $x = \frac{2}{\tan\theta} + 8\tan\theta + 10$

e $\frac{dx}{d\theta} = -2\operatorname{cosec}^2\theta + 8\sec^2\theta$

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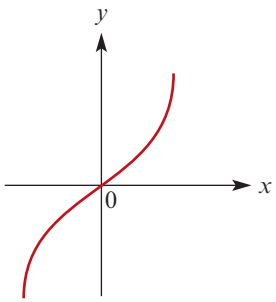
f $x = 18,\ \theta = 26.6°$

38 a $f'(x) = e^x + e^{-x}$ **b** $\{0\}$

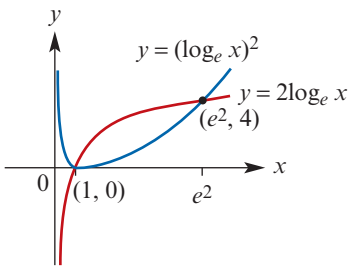
d



39 a $x = 1$ or $x = e^2$

b When $x = 1$, gradient of $y = 2\log_e x$ is 2 and gradient of $y = (\log_e x)^2$ is 0

c



d $\{x : 2\log_e x > (\log_e x)^2\} = (1, e^2)$

40 a $h = a(1 + \cos\theta)$

b $r = a\sin\theta$

d $\dfrac{dV}{d\theta} = \dfrac{\pi a^3}{3}[2\sin\theta\cos\theta(1 + \cos\theta) - \sin^3\theta]$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right) \approx 70.53°$

e $V = \dfrac{32\pi a^3}{81}$ cm^3

41 b $\dfrac{dy}{dt} = \dfrac{bAe^{bt}}{(1 + Ae^{bt})^2}$

e After 7 hours (to the nearest hour)

42 a $f'(x) = \dfrac{xe^x - e^x}{x^2}$

b $x = 1$

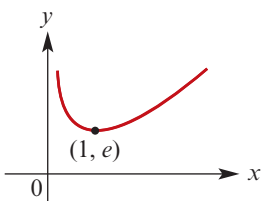
c $(1, e)$ minimum

d i $\dfrac{f'(x)}{f(x)} = \dfrac{x - 1}{x}$

ii $\lim\limits_{x\to\infty} \dfrac{f'(x)}{f(x)} = 1$

i.e. $f'(x) \to f(x)$ as $x \to \infty$

e



f $t = \dfrac{1}{k} \approx 45.27$ years, i.e. during 1945

43 a $A = 1000,\ k = \dfrac{1}{5}\log_e 10 \approx 0.46$

b $\dfrac{dN}{dt} = kAe^{kt}$ **c** $\dfrac{dN}{dt} = kN$

d i $\dfrac{dN}{dt} \approx 2905.7$ **ii** $\dfrac{dN}{dt} \approx 4.61 \times 10^{12}$

44 a i $r = \dfrac{1}{6}$ **ii** $p = 12,\ q = 8$

b $T'(3) = -\dfrac{4\pi}{3}$, i.e. length of night decreasing by $\dfrac{4\pi}{3}$ hours/month; $T'(9) = \dfrac{4\pi}{3}$, i.e. length of night increasing by $\dfrac{4\pi}{3}$ hours/month

c $-\dfrac{8}{3}$ hours/month

d $t = 9$, i.e. after 9 months

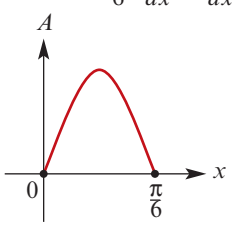
45 a $A = 2x\cos(3x)$

b i $\dfrac{dA}{dx} = 2\cos(3x) - 6x\sin(3x)$

ii When $x = 0$, $\dfrac{dA}{dx} = 2$;

When $x = \dfrac{\pi}{6}$, $\dfrac{dA}{dx} = \dfrac{dA}{dx} = -\pi$

c i



ii $x = 0.105$ or $x = 0.449$

iii Max area 0.374, occurs when $x = 0.287$

d ii (0.287, 1.162)

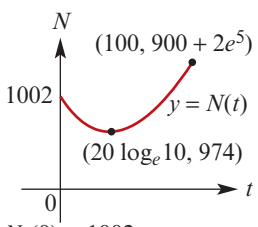
46 a i $N'(t) = -1 + \dfrac{1}{10}e^{\frac{t}{20}}$

ii Minimum population is 974, occurs when $t = 20\log_e 10$

iii $N(0) = 1002$

iv $N(100) = 900 + 2e^5$

v



b i $N_2(0) = 1002$

ii $N_2(100) = 990 + 2e^{\frac{1}{2}}$

iv Minimum population is 974, occurs when $t = (20\log_e 10)^2$

c ii Minimum population is 297, occurs when $t = 100.24$

d i $N_3'(t) = -\dfrac{3}{2}t^{\frac{1}{2}} + \dfrac{1}{10}e^{\frac{t}{20}}$

47 a $a = \dfrac{1}{3}\log_e\left(\dfrac{10}{3}\right)$

b i $x = 0$ and $x = \dfrac{5}{2}$

ii $x = \dfrac{-4 + 5a \pm \sqrt{25a^2 + 16}}{4a}$

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48 See online solutions

Chapter 11

Exercise 11A

1 68

2 $\frac{101}{60}$

3 11.7

4 a $\frac{35}{8}$ **b** $\frac{112}{25}$

5 36.75

6 $\pi \approx 3.13$

7 a 4.371 **b** 1.128

8 109.5 m^2

9 a $\frac{9}{2}$ **b** 9 **c** 4

Exercise 11B

1 a $\frac{x^4}{8} + c$ **b** $\frac{5}{4}x^4 - x^2 + c$

c $\frac{x^4}{5} - x^3 + c$ **d** $2z + \frac{5}{2}z^2 - z^3 + c$

2 a $y = -\frac{1}{2x^2} + c$ **b** $y = 3x^{\frac{4}{3}} + c$

c $y = \frac{4}{5}x^{\frac{5}{4}} + \frac{5}{2}x^{\frac{2}{5}} + c$

3 a $-\frac{3}{x} + c$ **b** $-\frac{2}{3x^3} + 3x^2 + c$

c $-\frac{2}{x} - \frac{3}{x^2} + c$ **d** $\frac{9}{4}x^{\frac{4}{3}} - \frac{20}{9}x^{\frac{9}{4}} + c$

e $\frac{12}{7}x^{\frac{7}{4}} - \frac{14}{3}x^{\frac{3}{2}} + c$ **f** $\frac{5}{2}x^{\frac{8}{5}} + \frac{9}{2}x^{\frac{8}{3}} + c$

4 a $y = x^2 - 3x + 3$ **b** $y = \frac{x^4}{4} + 6$

c $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - \frac{22}{3}$

5 a $\frac{4}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$ **b** $\frac{3z^3 - 4}{2z} + c$

c $\frac{5}{3}x^3 + x^2 + c$ **d** $\frac{4}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}} + c$

e $\frac{2x^3}{3} + \frac{3x^5}{5} + c$ **f** $\frac{3}{7}x^{\frac{7}{3}} + \frac{3}{16}x^{\frac{16}{3}} + c$

6 $f(x) = x^3 + \frac{1}{x} - \frac{17}{2}$

7 $s = \frac{3}{2}t^2 + \frac{8}{t} - 8$

8 a $k = -32$ **b** $f(7) = 201$

Exercise 11C

1 a $\frac{1}{6}(2x - 1)^3 + c$ **b** $-\frac{1}{4}(t - 2)^4 + c$

c $\frac{1}{20}(5x - 2)^4 + c$ **d** $\frac{1}{24 - 16x} + c$

e $\frac{1}{8(6 - 4x)^2} + c$ **f** $\frac{-1}{8(3 + 4x)^2} + c$

g $\frac{2}{9}(3x + 6)^{\frac{3}{2}} + c$ **h** $\frac{2}{3}(3x + 6)^{\frac{1}{2}} + c$

i $\frac{1}{9}(2x - 4)^{\frac{9}{2}} + c$ **j** $\frac{1}{7}(3x + 11)^{\frac{7}{3}} + c$

k $-\frac{2}{9}(2 - 3x)^{\frac{3}{2}} + c$ **l** $-\frac{1}{10}(5 - 2x)^5 + c$

2 a $\frac{1}{2}\log_e(x) + c$ **b** $\frac{1}{3}\log_e(3x + 2) + c$

c $\log_e(1 + 4x) + c$ **d** $\frac{5}{3}\log_e(3x - 2) + c$

e $-\frac{3}{4}\log_e(1 - 4x) + c$ **f** $-6\log_e(x - 4) + c$

3 a $5\log_e|x| + c$ **b** $3\log_e|x - 4| + c$

c $5\log_e|2x + 1| + c$ **d** $-3\log_e|2x - 5| + c$

e $-3\log_e|1 - 2x| + c$ **f** $-\frac{1}{3}\log_e|3x - 4| + c$

4 a $3x + \log_e|x| + c$ **b** $x + \log_e|x| + c$

c $-\frac{1}{x + 1} + c$ **d** $2x + \frac{x^2}{2} + \log_e|x| + c$

e $-\frac{3}{2(x - 1)^2} + c$ **f** $-2x + \log_e|x| + c$

5 a $y = \frac{1}{2}\log_e(x) + 1, x > 0$

b $y = 10 - \log_e(5 - 2x), x < \frac{5}{2}$

6 $y = 10\log_e(x - 5)$

7 a $x - \log_e|x + 1| + c$

b $-2(x + 1) + 3\log_e|x + 1| + c$

c $2(x + 1) - \log_e|x + 1| + c$

8 $y = 3\log_e\left(\frac{2 - x}{2}\right) + 10$

9 $y = \frac{5}{4}\log_e\left(\frac{5}{1 - 2x}\right) + 10$

10 $y = \frac{5}{4}\log_e\left(\frac{1}{2x - 1}\right) + 10$

Exercise 11D

1 a $\frac{1}{6}e^{6x} + c$ **b** $\frac{1}{2}e^{2x} + \frac{3}{2}x^2 + c$

c $-\frac{1}{3}e^{-3x} + x^2 + c$ **d** $-\frac{1}{2}e^{-2x} + \frac{1}{2}e^{2x} + c$

2 a $\frac{1}{2}e^{2x} - 2e^{\frac{x}{2}} + c$ **b** $e^x - e^{-x} + c$

c $\frac{2}{3}e^{3x} + e^{-x} + c$ **d** $15e^{\frac{x}{3}} - 10e^{\frac{x}{5}} + c$

e $\frac{9}{2}e^{\frac{2x}{3}} - \frac{15}{7}e^{\frac{7x}{5}} + c$ **f** $\frac{15}{4}e^{\frac{4x}{3}} - \frac{9}{2}e^{\frac{2x}{3}} + c$

3 a $y = \frac{1}{2}(e^{2x} - x^2 + 9)$ **b** $y = -\frac{3}{e^x} - e^x + 8$

4 $y = 9 - 2e^{-2}$

5 a $k = 2$ **b** $y = \frac{1}{2}e^{2x} + \frac{1}{2}e^2$

6 a $k = 3$ **b** $y = -\frac{1}{3}e^{3x} - \frac{2}{3}e^3$

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Exercise 11E

1 a $\frac{7}{3}$ **b** 20 **c** $-\frac{1}{4}$ **d** 9
e $\frac{1}{2}$ **f** $\frac{140}{3}$ **g** $15\frac{1}{3}$ **h** $343\frac{11}{20}$

2 a 10 **b** 1 **c** $\frac{13}{3}$ **d** $\frac{1}{3}$
e $\frac{10}{441}$ **f** 34 **g** $\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)$
h $2-2^{\frac{1}{2}}$ **i** $\frac{1}{15}$

3 a $\frac{1}{2}(e^2-1)$ **b** $\frac{1}{2}(3-e^{-2})$
c $6e^{\frac{1}{3}}-4$ **d** e^2-e^{-2}

4 a 10 **b** 17 **c** −5 **d** 9 **e** −3

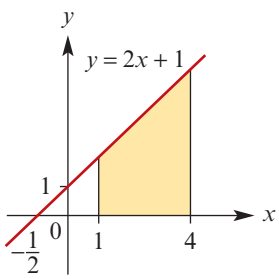
5 a $\log_e\left(\frac{1}{3}\right)$ **b** $\frac{1}{2}\log_e 5$ **c** $\frac{3}{2}\log_e\left(\frac{19}{17}\right)$

Exercise 11F

1 a 3
b 44
c i 8 **ii** 10

2 a $\frac{4}{3}$ **b** $\frac{1}{6}$ **c** $121\frac{1}{2}$ **d** $\frac{1}{6}$ **e** $4\sqrt{3}$ **f** 108

3 a



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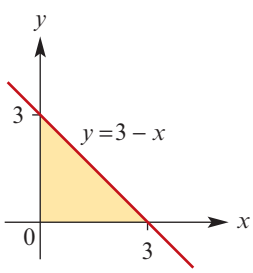

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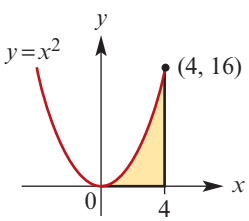

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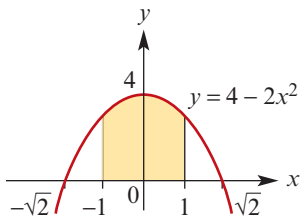

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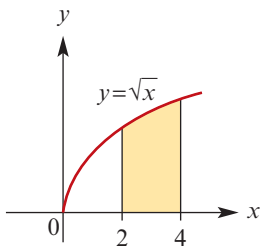
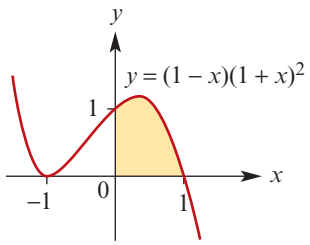
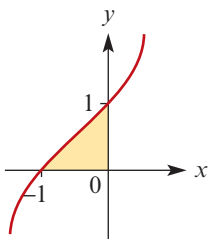


f

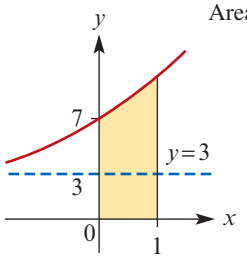


4 $\frac{321}{10}$ square units

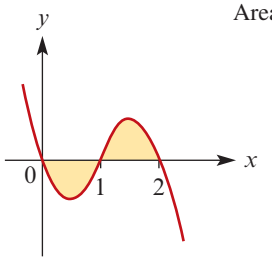
5 Area = $\frac{3}{4}$ square units



6 Area = $2e^2+1$
≈ 15.78 square units



7 Area = 0.5 square units



8 a $\frac{5}{6}$ square units **b** $8\frac{1}{6}$ square units

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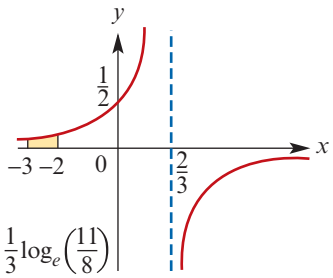
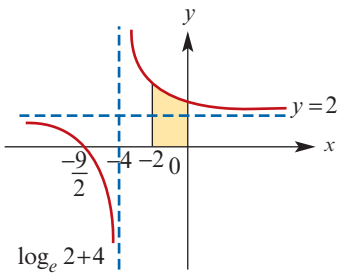
9 **a** $A(0, 3)$, $B(1, 0)$ **b** 2 square units

10



11



12 **b** Derivative: $(\log_e a)e^{x\log_e a}$

Antiderivative: $\dfrac{e^{x\log_e a}}{\log_e a}$

Exercise 11G

1 **a** $\frac{1}{3}\sin(3x)$ **b** $-2\cos\left(\frac{1}{2}x\right)$ **c** $\sin(3x)$

d $-4\cos\left(\frac{1}{2}x\right)$ **e** $-\frac{1}{2}\cos\left(2x - \frac{\pi}{3}\right)$

f $\frac{1}{3}\sin(3x) - \frac{1}{2}\cos(2x)$

g $\frac{1}{4}\sin(4x) + \frac{1}{4}\cos(4x)$

h $\frac{1}{4}\cos(2x) + \frac{1}{3}\sin(3x)$

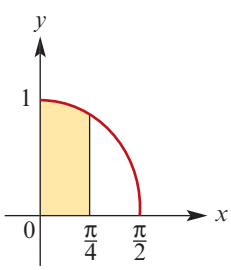
i $-\frac{1}{4}\sin\left(2x + \frac{\pi}{3}\right)$ **j** $-\frac{1}{\pi}\cos(\pi x)$

2 **a** $1 - \frac{1}{\sqrt{2}}$ **b** $\frac{1}{2}$ **c** $1 + \frac{1}{\sqrt{2}}$ **d** 2 **e** 1

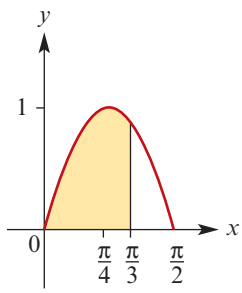
f $\frac{2}{3}$ **g** $-\frac{1}{2}$ **h** 4 **i** $\frac{1-\sqrt{3}}{4}$ **j** -2

3 $-\sqrt{2} + 2$ square units

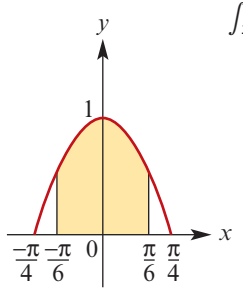
4 **a** $\int_0^{\frac{\pi}{4}} \cos x \, dx = \frac{1}{\sqrt{2}}$



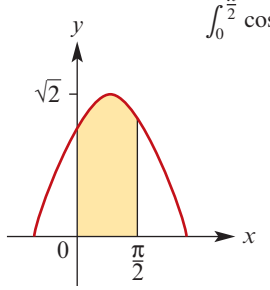
b $\int_0^{\frac{\pi}{3}} \sin(2x) \, dx = \frac{3}{4}$



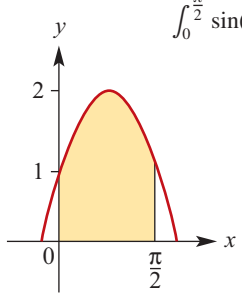
c $\int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} \cos(2x) \, dx = \frac{\sqrt{3}}{2}$



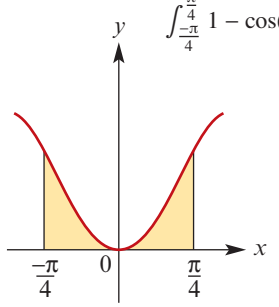
d $\int_0^{\frac{\pi}{2}} \cos\theta + \sin\theta \, d\theta = 2$



e $\int_0^{\frac{\pi}{2}} \sin(2\theta) + 1 \, d\theta = 1 + \frac{\pi}{2}$



f $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} 1 - \cos(2\theta) \, d\theta = \frac{\pi}{2} - 1$



5 **a** $\frac{\sqrt{2}}{2}$ **b** $-\frac{1}{3}$

c $-\frac{\sqrt{3}}{3}$ **d** $-\frac{1}{\sqrt{2}}$

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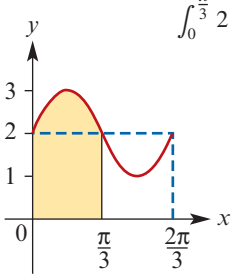
6 $\int_0^{\frac{\pi}{3}} 2 + \sin(3x)\,dx = \frac{2\pi}{3} + \frac{2}{3}$



Exercise 11H

1 a $4\frac{2}{3}$ **b** $2\frac{2}{3}$ **c** 12 **d** $\frac{5\sqrt{3}}{4} - 2$
e $\frac{e^4}{2} + 4\log_e 2 - \frac{e^2}{2}$ **f** $\frac{2}{3}$ **g** 4
h $\frac{5\pi^2}{8} + 1$ **i** $8\log_e 2 + \frac{51}{4}$ **j** $\frac{1}{12}$

2 0.5 square units

3 a $\frac{1}{\cos^2 x}$, $\tan x$ **b** $-\frac{2}{\sin^2(2x)}$, $-\frac{\cos(2x)}{2\sin(2x)}$
c $\frac{6x}{3x^2+7}$, $\frac{1}{6}\log_e\left(\frac{19}{7}\right)$
d $\sin(x) + x\cos(x)$, $-1 + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$

4 a $1 + \log_e(2x)$, $-x + x\log_e(2x)$
b $x + 2x\log_e(2x)$, $\frac{1}{2}x^2\log_e(2x) - \frac{x^2}{4}$
c $1 + \frac{x}{\sqrt{1+x^2}}$, $\log_e(1+\sqrt{2})$

5 $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$, $2e^{\sqrt{2}} - 2e$

6 $6\sin^2(2x)\cos(2x)$, $\frac{1}{6}$

7 a 139.69 **b** 18.50 **c** −0.66
d −23.76 **e** 2.06 **f** 0.43

8 b $5\log_e 3 + 4$

9 b $5 + 6\log_e 2$

10 a $\frac{dy}{dx} = -4\left(1 - \frac{1}{2}x\right)^7$
Hence $\int\left(1 - \frac{1}{2}x\right)^7 dx = -\frac{1}{4}\left(1 - \frac{1}{2}x\right)^8 + c$
b $\frac{dy}{dx} = -\tan x$; Hence $\int_0^{\frac{\pi}{3}} \tan x\,dx = \log_e 2$

11 $f(x) = 1 - 2\cos\left(\frac{1}{2}x\right)$

12 a $f(x) = \frac{1}{2}\sin 2x + 1$ **b** $f(x) = 3\log_e x + 6$
c $f(x) = 2e^{\frac{x}{2}} - 1$

13 $\sin(3x) + 3x\cos(3x)$
Hence $\int_0^{\frac{\pi}{6}} x\cos(3x)\,dx = \frac{\pi}{18} - \frac{1}{9}$

14 $a = 1,\ b = -2$; Area $= \frac{12\sqrt{3} - \pi - 12}{3\pi}$

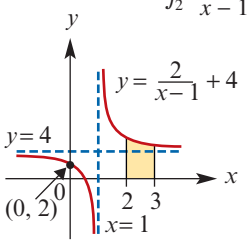
15 a 1.450 square units **b** 1.716 square units

16 0.1345

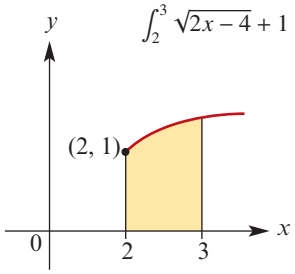
17 $f(x) = \frac{1}{2}\left(x^2 - \cos(2x) + 3\right)$

18 a $(x^2+1)^3 + c$ **b** $\sin(x^2) + c$
c $(x^2+1)^3 + \sin(x^2) + c$
d $-(x^2+1)^3 + c$ **e** $(x^2+1)^3 - 4x + c$
f $3\sin(x^2) + c$

19 $\int_2^3 \frac{2}{x-1} + 4\,dx = 2\log_e 2 + 4$



20 $\int_2^3 \sqrt{2x-4} + 1\,dx = \frac{1}{3} \times 2^{\frac{3}{2}} + 1$

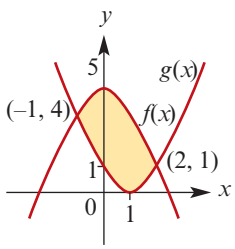


21 a $\frac{4\sqrt{2}}{3} - \frac{2}{3}$ **b** $\frac{2^{\frac{5}{2}}}{3}$ **c** $\frac{1}{3}\log_e 4$
d $\frac{1}{2}\log_e 3 + 3$ **e** $-2\frac{2}{3}$ **f** $2\sqrt{2} - 2$

Exercise 11I

1 36 square units

2 Area = 9 square units

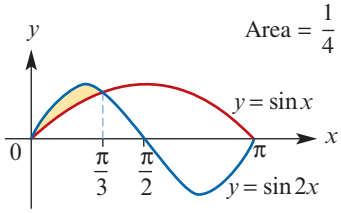


3 a 36 square units **b** $20\frac{5}{6}$ square units
c 4 square units **d** $4\frac{1}{2}$ square units
e $4\frac{1}{2}$ square units

4 a 2 square units
b $e + e^{-1} - 2 \approx 1.086$ square units

5 3.699 square units

6 Area $= \frac{1}{4}$ square units



7 $\frac{1}{2}$ square units

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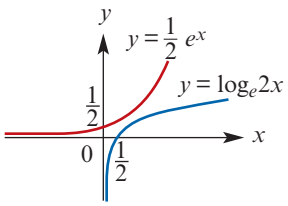
8 $P(\log_e 3, 3)$; Area ≈ 2.197 square units

9 a $f^{-1}(x) = \frac{1}{2}e^x$ **b** $\frac{3}{2}$



c $4\log_e(2) - \frac{3}{2}$

Exercise 11J

1 a $\frac{2}{3}$ **b** $\frac{2}{\pi}$ **c** $\frac{2}{\pi}$ **d** 0 **e** $\frac{1}{2}(e^2 - e^{-2})$

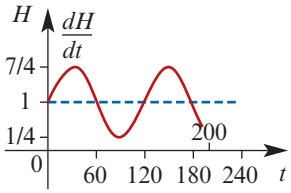
2 $10(e^5 - 1)e^{-5} \approx 9.93°C$

3 $\frac{a^2}{6}$

4 a $3000(2 - 2^{0.9})$ N/m^2

b $1000(4^{0.1} - 1)$ N/m^2

5 a



b $t \in (10, 50) \cup (130, 170)$

c $t = 30$ or $t = 150$

d i 120 kilojoules

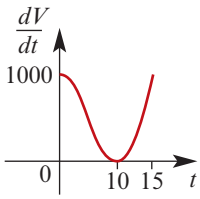
ii 221.48 kilojoules

6 a When $t = 0$, 1000 million litres per hour;
When $t = 2$, 896 million litres per hour

b i $t = 0$ and $t = 15$

ii 1000 million litres per hour

c

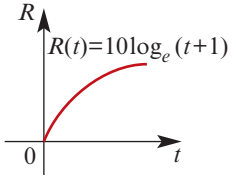


d i 5000

ii 5000 million litres flowed out in the first 10 hours

7 a When $t = 5$, ≈ 17.9 penguins per year;
When $t = 10$, ≈ 23.98 penguins per year;
When $t = 100$, ≈ 46.15 penguins per year

b



c $t = e^{\frac{R}{10}} - 1, R \geq 0$; i.e. $R^{-1}(t) = e^{\frac{t}{10}} - 1$

d i 3661

ii The growth in the size of the penguin population over 100 years (assuming zero death rate)

8 71 466$\frac{2}{3}$ m^3

9 a 465 m^2 **b** 46 500 m^3

10 1.26 m

11 a 6 metres

b 18π m^2

c i $y - 3 + 3\cos\left(\frac{a}{3}\right) = \frac{-1}{\sin\left(\frac{a}{3}\right)}(x - a)$

ii 5.409

12 a i 9 **ii** $\frac{3(\sqrt{2}+2)}{2}$ **iii** 12

b Max value is 12; Min value is 0.834

c $\frac{48(\pi + 1)}{\pi}$ litres

Chapter 11 review

Technology-free questions

1 a $\frac{65}{4}$ **b** 0 **c** $\frac{-5a^{\frac{3}{2}}}{3}$ **d** $-\frac{55}{3}$

e $\frac{1}{2}$ **f** 1 **g** 0 **h** 0

2 $\frac{23}{2}$ **3** 3 **4** 4 **5** 820

6 $\frac{85}{4}$ **7** $\frac{5}{3}$ **8** $\frac{5}{3}$

9 a 6 **b** $\frac{16}{3}$ **c** $-\frac{51}{16}$

10 $\int_a^b f(x) - g(x)\,dx + \int_b^c g(x) - f(x)\,dx + \int_c^d f(x) - g(x)\,dx$

11 a $P(3, 9)$, $Q(7.5, 0)$ **b** 29.25 square units

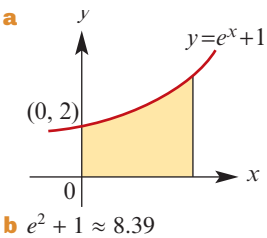
12 a 5 **b** $p = \frac{20}{7}$

13 3.45 square units

14 a $A(0, 6)$, $B(5, 5)$ **b** $15\frac{1}{6}$ square units

c $\frac{125}{6}$ square units

15 a



b $e^2 + 1 \approx 8.39$

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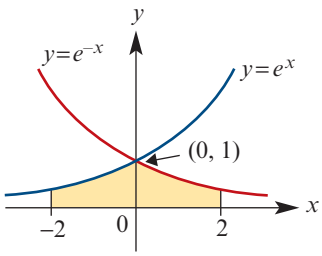
16 a



b $2 - 2e^{-2}$

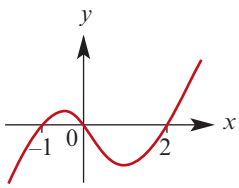
17 a $e - 1 \approx 1.72$

b $2(e - 1) \approx 3.44$ square units

18 $\frac{14}{9}$

19 $2 + e^2 \approx 9.39$ square units

20 $3\frac{1}{12}$ square units



21 $2e^{\frac{5}{2}} + 4\log_e(2) - 14$

22 a $C(-\frac{1}{2}\ln 6, 0)$, $D(10, 0)$

b $3\ln 6 + \frac{45}{2}$

23 a $C\left(\frac{7}{6}, 0\right)$, $D\left(\frac{11}{6}, 0\right)$ **b** $\frac{1}{2} + \frac{3\sqrt{3} + 2}{\pi}$

24 a (6, 2), (2, 6) **b** $16 - 12\ln 3$

25 a $3 - e^{-2}$ **b** $\log_e\left(\frac{2}{3}\right) - \frac{3}{2}$ **c** $\frac{\pi^2}{8} + 1$

d $\frac{1}{2}\log_e\left(\frac{5}{6}\right) - e^{-4} + e^{-5}$

Multiple-choice questions

1 C	**2** D	**3** C	**4** B	**5** A
6 D	**7** C	**8** C	**9** C	**10** C
11 D	**12** D			

Extended-response questions

1 a $4y - 5x = -3$ **b** $\left(\frac{3}{5}, 0\right)$

c (1, 0) **d** $\frac{9}{40}$ **e** 9 : 49

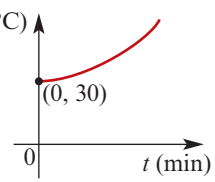
3 a $\frac{1}{3}$ square units

d $1 - \frac{n-1}{n+1} = \frac{2}{n+1}$ square units

e $\frac{9}{11}, \frac{99}{101}, \frac{999}{1001}$

f Area between the curves approaches 1

4 a 968.3° **b**



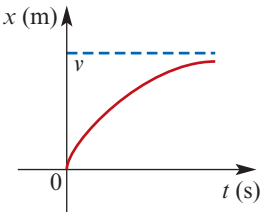
c 2.7 minutes **d** 64.5°C/min

5 a 5×10^4 m/s^2

b Magnitude of velocity becomes very small

c $5 \times 10^4(1 - e^{-20})$ m **d** $x = v(1 - e^{-t})$

e



6 a $\frac{d}{dx}\left(e^{-3x}\sin(2x)\right) = -3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x)$

c $\int e^{-3x}\sin(2x)\,dx = \frac{-1}{13}\left(3e^{-3x}\sin(2x) + 2e^{-3x}\cos(2x)\right) + c$

7 a i $\tan a = \frac{4}{3}$ **ii** $\sin a = \frac{4}{5}$, $\cos a = \frac{3}{5}$

b 2 square units

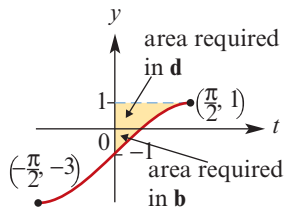
8 a $\frac{dy}{dx} = \log_e x + 1$, $\int_1^e \log_e x\,dx = 1$

b $\frac{dy}{dx} = (\log_e x)^n + n(\log_e x)^{n-1}$

d $\int_1^e (\log_e x)^3\,dx = 6 - 2e$

9 $s = \sqrt[3]{a^2 b}$, $r = \sqrt[3]{ab^2}$

10 a



b $\int_0^{\frac{\pi}{6}} f(x)\,dx = 2 - \sqrt{3} - \frac{\pi}{6}$

c $f^{-1}: [-3, 1] \to \mathbb{R}$, $f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{2}\right)$

d $\int_0^1 f^{-1}(x)\,dx = \frac{\pi}{2} - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(x)\,dx$

$= \frac{5\pi}{6} - \sqrt{3}$

11 a $\frac{dy}{dx} = -\frac{x}{10}e^{\frac{x}{10}}$, $\frac{dy}{dx} = -x(100 - x^2)^{-\frac{1}{2}}$

b When $x = 0$, $\frac{dy}{dx} = 0$ for both functions

c $-e$

d 6.71 square units

e 8.55%

f $(25\pi - 50)$ square units or $(100e - 250)$ square units

g i $10(10e - 20)$

ii $(25\pi - 100e + 200)$ square units

12 a i $R(0) = 0$ **ii** $R(3) = 0$

b $R'(t) = e^{-\frac{t}{10}}\left(\frac{10\pi}{3}\cos\left(\frac{\pi t}{3}\right) - \sin\left(\frac{\pi t}{3}\right)\right)$

c i 1.41, 4.41, 7.41, 10.41

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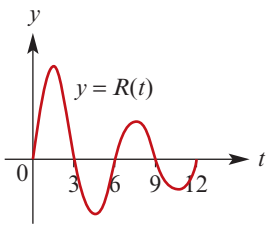
ii Local max: (1.41, 8.65), (7.41, 4.75); Local min: (4.41, −6.41) (10.41, −3.52)

d $t = 0, 3, 6, 9$ or 12

e



f i 16.47 litres **ii** 12.20 litres **iii** 8.27 litres

g 12.99 litres

13 b $1 - \frac{\pi}{4}$

Chapter 12

Technology-free questions

1 a $\frac{-2x^5 + 4x^3 - 2x}{(x^4 - 1)^2}$ **b** $\{0\}$

2 $4(6x - 4)(3x^2 - 4x)^3$ **3** $2x\log_e(2x) + x$

4 a $b = \frac{1}{2}$ **b** $k = (2b - 1)e^{2b+1}$

5 $m = \frac{1}{12}$, $a = -\frac{22}{3}$, $c = -\frac{28}{3}$

6 $\frac{1}{6}\log_e 7$

7 a $\frac{3}{5}\log_e(5x - 2)$ **b** $\frac{3}{10 - 25x}$

8 a -7 **b** -14 **c** -20

9 a $5\frac{1}{2}$ **b** $\frac{1}{8}$

10 $2x\sqrt{3x^2 + 1}$

11 a $4x - 3$ **b** -3 **c** $\{1\}$

12 $\frac{f'(x)}{f(x)}$ **13** $a = \frac{145}{144}$

14 $m = \frac{1}{4}(-3 + \sqrt{105})$

15 a $(0, -4)$ and $(-2, 0)$ **b** 0 **c** 4 **d** $9\frac{1}{2}$

16 $-\frac{3}{25}$ **18** 24π **19** $\left(\frac{5}{18}, \frac{5}{36}\right)$

20 $\frac{2}{3}$ **21** $-\frac{8x}{(x^2 - 2)^2}$ **22** 6

23 $-63(5 - 7x)^8$ **24** $\frac{1}{9}$

25 $\frac{2}{3}$ **26** -70 **27** 0

28 -1

29 a $-\frac{2}{(2x + 1)^2}$ **b** -2

30 a $x = 0$ or $x = -2$ **b** $x > 0$ or $x < -2$

c $-2 < x < 0$

31 a $\frac{1}{(1 - x)^2}$ **b** $(y + 1)^2$

32 $-3x(x^2 + 1)^{-\frac{5}{2}}$

34 $f'(x) = 10x^4 > 0$ for all $x \in \mathbb{R} \setminus \{0\}$. If $b > 0$, then $b^5 > 0$, and if $b < 0$, then $b^5 < 0$.

35 a $4 - 2\sqrt{2}$ **b** $2(e^{\frac{3}{4}} - 1)$ **c** $\frac{1}{2}\log_e 2$

d $-\frac{1}{2}\log_e 2$ **e** $\frac{1}{4}$ **f** $\frac{1}{20}$

36 a $\left(\frac{a^2}{4} - 1, \frac{a^2}{4}\right)$

b i $a = 10$ **ii** $y = -\frac{x}{6} + \frac{179}{6}$

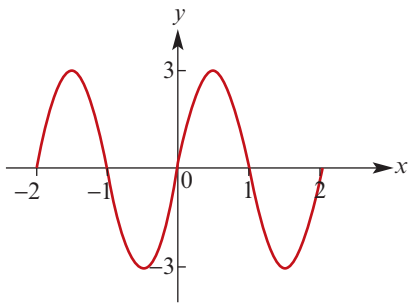
iii $\left(0, \frac{179}{6}\right), (179, 0)$

38 a $\frac{1}{m}\left(2 - \log_e\left(\frac{4}{m}\right)\right)$ **b** $0 < m < 4e^{-2}$

39 a $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right), (0, 0), \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

b $(-\pi, -\pi), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), (0, 0), \left(\frac{\pi}{2}, -\frac{\pi}{2}\right), (\pi, \pi)$

40 a



b $y = 3$ **c** $\frac{3}{\pi} - \frac{1}{32}$

41 $4\log_e 2$ square units

42 $a = 2$, $b = -4$, $c = -2$

43 $\frac{24}{5}$ cm, 10 cm

Multiple-choice questions

1 B	**2** C	**3** D	**4** C	**5** B	**6** E
7 A	**8** D	**9** B	**10** A	**11** E	**12** B
13 C	**14** D	**15** E	**16** B	**17** C	**18** E
19 D	**20** E	**21** A	**22** D	**23** E	**24** A
25 D	**26** D	**27** A	**28** C	**29** C	**30** D
31 B	**32** B	**33** B	**34** E	**35** A	**36** A
37 A	**38** A	**39** D	**40** B	**41** D	**42** C
43 B	**44** A	**45** C	**46** B	**47** C	**48** D
49 B	**50** D	**51** B	**52** E	**53** D	**54** A
55 B	**56** C	**57** C	**58** A	**59** D	**60** A
61 D	**62** E				

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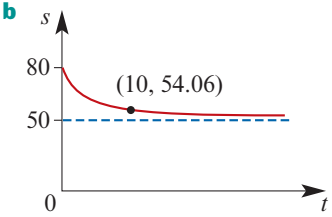
Extended-response questions

1 a 54.06 g **b**

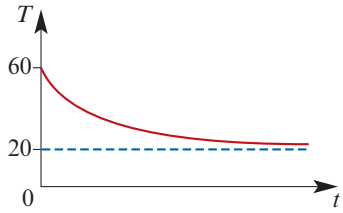


c $\frac{ds}{dt} = -6e^{-\frac{1}{5}t}$ **d** $\frac{ds}{dt} = -\frac{1}{5}(s - 50)$

e 0.8 g/L **f** 17 seconds

2 a 60°C

b

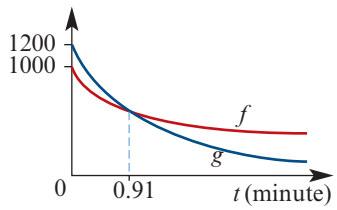


c $\frac{dT}{dt} = -14.4e^{-0.36t}$ **d** $\frac{dT}{dt} = -0.36(T - 20)$

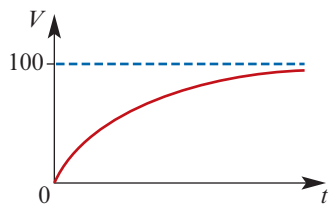
3 a 1.386 minutes **b** 2200, 5.38%

c 66.4 spores/minute **d** 0.9116 minutes

e



4 a



b i $20e^{-0.2t}$ m/s^2 **ii** $20 - \frac{V}{5}$ m/s^2

c 8.05 seconds

5 100

6 b $k = 0.028$ **c** 0.846°C/min

7 a i 0.1155 **ii** 0.2

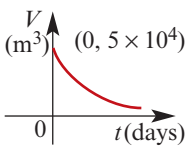
b 13.86 days

8 \$600

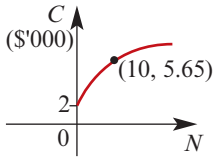
9 a 5×10^4 m^3 **b** $-12\,500$ m^3/day

c -3500 m^3/day **d** After 222.61 days

e



10 a

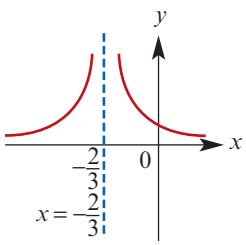


b $\frac{3N^2}{4(N^3 + 16)^{\frac{3}{4}}}$

c Rate of change of cost in \$1000s with respect to the increase in the number of bottle tops produced

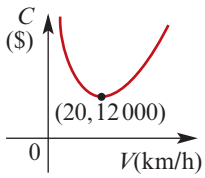
11 $p = 4$; Number of items sold = 50

12 $a = \pm 3$, $b = \pm 2$



13 a \$17 000 **b** $C = \frac{160\,000}{V} + 10V^2$

c



d $V = 20$, $C = 12\,000$ **e** \$12 560

14 a $\frac{60}{\sqrt{39}} \approx 9.61$ km **b** $\frac{60}{\sqrt{39}} \approx 9.61$ km

15 $4 - \sqrt{\frac{12}{7}} \approx 2.7$ m

16 a $\{x : x > 1\}$ **b** $\{x : 0 < x < 2\}$

c $\{x : x > 1\}$, $\left\{x : 0 < x < \frac{3}{2}\right\}$

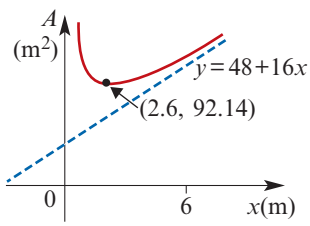
d $\{x : x > 1\}$, $\left\{x : 0 < x < \frac{n+1}{n}\right\}$

17 a $(1, 1), (-1, -1)$ **b** $\left(\pm 2^{\frac{1}{6}}, 2^{-\frac{1}{3}}\right)$

c $\left(\pm n^{\frac{1}{2n+2}}, n^{-\frac{n}{2n+2}}\right)$

18 a $A = 48 + 16x + \frac{108}{x}$

b



c Height = $\frac{3\sqrt{3}}{2}$ m, width = $\frac{4\sqrt{3}}{3}$ m

d 172 m^2

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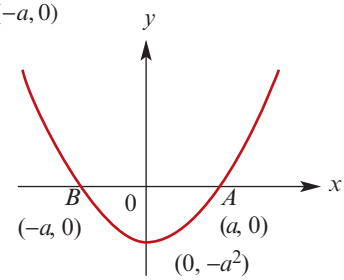
19 a $(a, 0), (-a, 0)$



b $\frac{4a^3}{3}$ **c** $\frac{2a^2}{3}$ **d** 2 : 3

20 a $-5\frac{1}{3}$ **c** $a = 1$ or $a = -2$

21 a i $50e^{-1}$ litres/minute
ii $t = 5$
iii 2 minutes 18 seconds
iv 3 minutes 48 seconds
b 14.74 litres
c 53 seconds

22 a $\frac{3}{\ln 2}$
b i $A_1 = 5$ **ii** $E_1 = 0.67$
c i $A_2 = 4.5$ **ii** $E_2 = 0.17$
d $A_4 = 4.37$, $E_4 = 0.043$;
$A_8 = 4.34$, $E_8 = 0.011$

23 a $f'(x) = 1 - \frac{1}{x}$ **b** $(1, 1)$
c $x = \frac{n}{n-1}$ **d** $a = e$
e $y = (1 - e)x + 2$
f $y = (1 - e^{-n})x + 1 - n$, $(0, 1 - n)$
g $\ln x + 1$, $-x \ln x + \frac{x^2}{2} + x$ **h** $\frac{e^2 - 3}{2}$

24 a $f'(x) = 1 + \cos x$, $f''(x) = -\sin x$
c $(-3\pi, -3\pi), (-\pi, -\pi), (\pi, \pi), (3\pi, 3\pi)$
d $-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$
e $\left(-\frac{4\pi}{3}, \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right), \left(-\frac{2\pi}{3}, -\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right),$
$\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2} + \frac{\pi}{3}\right), \left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right)$

Algorithms and pseudocode

See online solutions

Chapter 13

Exercise 13A

1 1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T

2 HH1, HH2, HH3, HH4, HH5, HH6,
HT1, HT2, HT3, HT4, HT5, HT6,
TH1, TH2, TH3, TH4, TH5, TH6,
TT1, TT2, TT3, TT4, TT5, TT6

3 a $\frac{1}{13}$ **b** $\frac{3}{4}$ **c** $\frac{4}{13}$ **d** $\frac{2}{13}$

4 a $\frac{1}{2}$ **b** $\frac{2}{3}$

5 0.8

6 0.65

7 a 0.067 **b** 0.047

8 5%

9 $\frac{6}{7}$

10 a $\frac{17}{500}$ **b** $\frac{9}{250}$ **c** $\frac{41}{125}$ **d** $\frac{41}{500}$

11 a $\frac{13}{20}$ **b** $\frac{7}{20}$

12 a $\frac{57}{100}$ **b** $\frac{2}{19}$ **c** $\frac{27}{100}$ **d** $\frac{1}{250}$

13 $\frac{9}{25}$

14 a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$

15 a 0.13 **b** 0.32

16 a 0.40 **b** 0.67 **c** 0.18

17 a 0.35 **b** 0.18 **c** 0.12 **d** 0.17

18 a 0.36 **b** 0.06

Exercise 13B

1 a 0.2 **b** 0.675 **c** 0.275

2 a $\frac{1}{6}$ **b** $\frac{1}{3}$

3 a 0.06 **b** $\frac{1}{5}$

4 $\frac{3}{5}$ **5** $\frac{24}{59}$

6 a $\frac{1}{2}$ **b** $\frac{77}{200}$ **c** $\frac{40}{77}$ **d** 0.4

7 a $\frac{65}{224}$ **b** $\frac{115}{448}$ **c** $\frac{21}{65}$ **d** $\frac{61}{246}$

8 a 0.24 **b** 0.86

9 a Yes **b** No **c** No

10 a 0.5 **b** 0.2 **c** 0.7

11 0.39 **12** 0.22 **13** 0.1 **14** $\frac{1}{7}$

15 $\frac{1}{9}$ **16** 0.0479

17 a 0.486 **b** 0.012 **c** 0.138

18 a $\frac{2}{5}$ **b** $\frac{1}{15}$ **c** $\frac{8}{15}$

19 a $\frac{2}{5}$ **b** $\frac{7}{40}$ **c** $\frac{7}{16}$ **d** $\frac{7}{15}$

20 a $\frac{5}{14}$ **b** $\frac{3}{5}$

21 $\frac{3}{44}$

22 a 0.735 **b** 0.453

Exercise 13C

1 a Discrete **b** Not discrete

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c Discrete **d** Discrete

2 a Not discrete **b** Discrete
c Not discrete **d** Discrete

3 a $\{HHH, THH, HTH, HHT, HTT, THT, TTH, TTT\}$

b

x	Outcomes
0	TTT
1	HTT, THT, TTH
2	THH, HTH, HHT
3	HHH

c $\frac{1}{2}$

4 a Yes, as the sum of the probabilities is 1 and $p(x) \geq 0$ for all x
b 0.8

5 a

x	0	1	2	3
$p(x)$	$\frac{125}{729}$	$\frac{300}{729}$	$\frac{240}{729}$	$\frac{64}{729}$

b $\frac{604}{729}$ **c** $\frac{304}{729}$

6 a $\{(1,1),(1,2),(1,3),\ldots,(6,4),(6,5),(6,6)\}$
b $Y = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

		2nd die					
		1	2	3	4	5	6
1st die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

c i $\frac{1}{6}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{5}$ **iv** $\frac{7}{10}$ **v** $\frac{1}{5}$ **vi** $\frac{2}{7}$

7 a

		2nd die					
		1	2	3	4	5	6
1st die	1	1	1	1	1	1	1
	2	1	2	2	2	2	2
	3	1	2	3	3	3	3
	4	1	2	3	4	4	4
	5	1	2	3	4	5	5
	6	1	2	3	4	5	6

b 1, 2, 3, 4, 5, 6 **c** 0.19

8 a 0.288 **b** 0.064 **c** 0.352 **d** 0.182

9 a $\{(1,1),(1,2),(1,3),\ldots,(6,4),(6,5),(6,6)\}$
b $\Pr(A) = \frac{1}{6}$, $\Pr(B) = \frac{1}{6}$, $\Pr(C) = \frac{5}{12}$, $\Pr(D) = \frac{1}{6}$
c $\Pr(A \mid B) = \frac{1}{6}$, $\Pr(A \mid C) = \frac{1}{5}$, $\Pr(A \mid D) = \frac{1}{6}$
d i Independent **ii** Not independent
iii Independent

10 a Yes **b** 0.5

11 a and **c**

12 a

x	0	1	2	3
$p(x)$	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

b

x	0	1	2	3
$p(x)$	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$

13

x	0	1	2
$p(x)$	0.36	0.48	0.16

14 a

x	1	2	3	4	5
$p(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

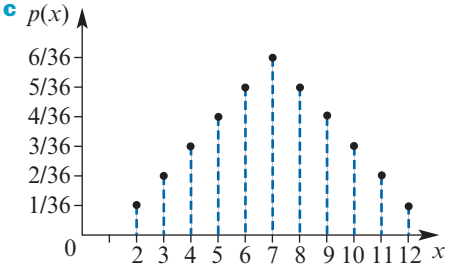
b $\frac{3}{5}$ **c** $\frac{1}{3}$

15 a $\{(1,1),(1,2),(1,3),\ldots,(6,4),(6,5),(6,6)\}$

b

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

c



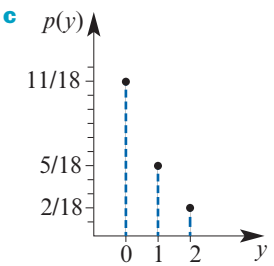
d $\frac{5}{18}$ **e** $\frac{7}{10}$

16 a $\{(1,1),(1,2),(1,3),\ldots,(6,4),(6,5),(6,6)\}$

b

y	0	1	2
$p(y)$	$\frac{11}{18}$	$\frac{5}{18}$	$\frac{2}{18}$

c



17 a

x	0	1	2
$p(x)$	$\frac{1}{3}$	$\frac{8}{15}$	$\frac{2}{15}$

b $\frac{7}{15}$

18 a

x	10	20	100
$p(x)$	$\frac{3}{4}$	$\frac{6}{25}$	$\frac{1}{100}$

b

y	20	30	40	110	120	200
$p(y)$	$\frac{9}{16}$	$\frac{9}{25}$	$\frac{36}{625}$	$\frac{3}{200}$	$\frac{3}{625}$	$\frac{1}{10000}$

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19 a $\frac{1}{4}$

b $\{EENE, ENEE, ENNN, NEEE, NENN, NNEN\}$, $\Pr(X = 4) = \frac{3}{8}$

c $\Pr(X = 5) = \frac{3}{8}$

20

x	−2	1	4
$p(x)$	0.24	0.52	0.24

Exercise 13D

1 \$60

2 a $E(X) = 4.6$ **b** $E(X) = 0.5$

c $E(X) = 2.38$ **d** $E(X) = 0.569$

e $E(X) = 7$ **f** $E(X) = 0$

3 Expected profit = \$3000

4 A loss of 17c

5 1.54

6

x	1	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$E(X) = \frac{49}{12}$

7 a $E(X) = 4.11$ **b** $E(X^3) = 78.57$

c $E(5X - 4) = 16.55$ **d** $E\left(\frac{1}{X}\right) = 0.255$

8 \$5940

9 a $p = \frac{1}{16}$ **b** $E(X) = 2$ **c** $\text{Var}(X) = 3.5$

10 a $k = \frac{1}{21}$ **b** $E(X) = \frac{91}{21}$ **c** $\text{Var}(X) = \frac{20}{9}$

11 a

x	1	2	3	4	6	8	9	12	16
$p(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

b i $\frac{1}{4}$ **ii** $\frac{25}{4}$ **iii** $\frac{275}{16}$

12 a $\frac{21}{4}$ **b** $\frac{7}{12}$ **c** $\frac{497}{48}$

13 a $\text{Var}(2X) = 64$ **b** $\text{Var}(X + 2) = 16$

c $\text{Var}(1 - X) = 16$ **d** $\text{sd}(3X) = 12$

14 a $c = 0.35$ **b** $E(X) = 2.3$

c $\text{Var}(X) = 1.61$, $\text{sd}(X) = 1.27$

15 a $k = \frac{1}{15}$ **b** $E(X) = 3.667$

c $\text{Var}(X) = 1.556$

16 a 7 **b** 5.83

17 a 3 **b** 1.5

Chapter 13 review

Technology-free questions

1 a $\frac{40}{81}$ **b** $\frac{5}{9}$

2 a $\frac{m - q}{m}$

b $\frac{(m - q)(m - q - 1)}{m(m - 1)}$

3 0.4

4 a 0.026 **b** $\frac{9}{13}$

6 a 0.1 **b** 1.3 **c** 2.01

7 a 21.5 **b** 630.75 **c** $\frac{29\sqrt{3}}{2}$

8 a

p	$x - 2$	-2
$\Pr(P = p)$	$\frac{4}{5}$	$\frac{1}{5}$

b $\frac{4}{5}x - 2$

c $x > \$2.50$

9 a 0.47 **b** $\frac{47}{70}$

10 a 21.5% **b** $\frac{7}{43}$

11 a $\frac{1}{24}$ **b** $\frac{17}{24}$ **c** $\frac{5}{6}$ **d** $\frac{11}{18}$

Multiple-choice questions

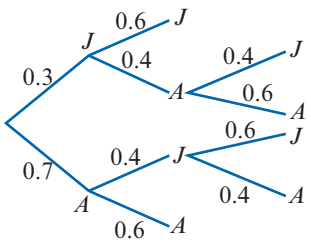
1 A **2** E **3** D **4** C **5** D

6 C **7** E **8** C **9** C **10** B

Extended-response questions

1 a 0.1 **b** 0.2 **c** 4

2 a



b i 0.396 **ii** 0.604

c i

x	2	3
$\Pr(X = x)$	0.6	0.4

ii 2.4

d 0.46

3 \$14

4 a 0.5 **b** 0.05 **c** 0.033 **d** $\frac{25}{33}$

5 a i 1.21

ii $\text{Var}(P) = 1.6659$, $\text{sd}(P) = 1.2907$

iii 0.94

b i

t	1	0.40	0
$p(t)$	0.39	0.27	0.34

ii $E(T) = 0.498 \approx 0.50$ **iii** 1

6 a $E(Y) = 2.002$

b $\text{Var}(Y) = 2.014$, $\text{sd}(Y) = 1.419$

c i

b	0	100	200
$p(b)$	0.677	0.270	0.053

ii $E(B) = \$37.60$

7 a $\mu = 13.5\%$, $\sigma = 16.2\%$ **b** 0.95

c $E(G) = 6.9\%$, $\text{sd}(G) = 9.726\%$

8 Yes

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9 $1.00

10 a i 0.65 **ii** 0.2275 **iii** 0.079625 **iv** 0.042875
b Expected cost: $8 439 375
c Expected profit: $10 703 125

11 a $\frac{1}{3}(304 - 2x)$ **b** $x = 2$ **c** $27\frac{2}{9}$

12 b $x = \frac{1}{2}, \frac{49}{288}$

Chapter 14

Exercise 14A

1 a and **b**

2 0.2734

3 a 0.0256 **b** 0.0016

4 a 0.0778 **b** 0.2304 **c** 0.01024

5 a $\Pr(X = x) = \binom{3}{x}(0.5)^x(0.5)^{3-x}$, $x = 0, 1, 2, 3$
b 0.375

6 a $\Pr(X = x) = \binom{6}{x}(0.48)^x(0.52)^{6-x}$, $x = 0, 1, 2, 3, \ldots, 6$
b 0.2527

7 a 0.0536 **b** 0.0087 **c** 0.0623

8 a $\Pr(X = x) = \binom{10}{x}(0.1)^x(0.9)^{10-x}$, $x = 0, 1, 2, 3, \ldots, 10$
b i 0.3487 **ii** 0.6513

9 a $\Pr(X = x) = \binom{11}{x}(0.2)^x(0.8)^{11-x}$, $x = 0, 1, 2, 3, \ldots, 11$
b i 0.2953 **ii** 0.0859 **iii** 0.9141

10 a $\Pr(X = x) = \binom{7}{x}(0.2)^x(0.8)^{7-x}$, $x = 0, 1, 2, 3, \ldots, 7$
b i 0.000013 **ii** 0.2097 **iii** 0.3899

11 0.624

12 a $\left(\frac{x}{100}\right)^6$ **b** $\frac{6x^5(100 - x)}{100^6}$
c $\frac{x^6}{100^6} + \frac{6x^5(100 - x)}{100^6} + \frac{15x^4(100 - x)^2}{100^6}$

13 0.6836

14 a 0.1156 **b** 0.7986 **c** 0.3170

15 0.6791

16 a 0.1123 **b** 0.5561 **c** 0.000 01
d 0.000 01

17 0.544

18 a $\left(\frac{1}{4}\right)^6 \approx 0.000\,24$ **b** 0.1694

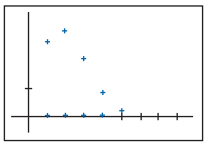
19 a 0.0138 **b** 0.2765 **c** 0.8208 **d** 0.3368

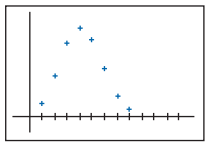
20 a $(0.8)^8 \approx 0.168$ **b** 0.001 23
c 0.0021

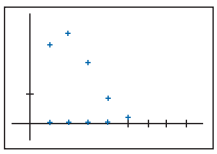
21 a $(0.15)^{10} \approx 0.000\,000\,006$
b $1 - (0.85)^{10} \approx 0.8031$ **c** 0.5674

22 a 0.011 529 **b** 0.002 59 **c** 0.0393

Exercise 14B

1 

2 

3 a b 

c The distribution in part b is the reflection of the distribution in part a in the line $X = 5$.

4 a Mean = 5; Variance = 4
b Mean = 6; Variance = 2.4
c Mean = $\frac{500}{3}$; Variance = $\frac{1000}{9}$
d Mean = 8; Variance = 6.4

5 a 1 **b** 0.2632

6 37.5

7 $n = 48$, $p = \frac{1}{4}$, $\Pr(X = 7) = 0.0339$

8 $n = 100$, $p = \frac{3}{10}$, $\Pr(X = 20) = 0.0076$

9 Mean = 10, sd = $\sqrt{5}$; The probability of obtaining between 6 and 14 heads is 0.95

10 Mean = 120, sd = $4\sqrt{3}$;
The probability that between 107 and 133 students attend a state school is 0.95

Exercise 14C

1 a i $(0.8)^5 \approx 0.3277$ **ii** 0.6723
b 14
c 22

2 a i 0.1937 **ii** $1 - (0.9)^{10} \approx 0.6513$
b 12

3 7 **4** 7 **5** 10 **6** 42 **7** 86

Chapter 14 review

Technology-free questions

1 $\frac{12}{125}$ **2** $\frac{5p(1 - p)^4}{1 - (1 - p)^5}$

3 a $\frac{1}{27}$ **b** $\frac{4}{13}$

4 a 2 **b** $\frac{39 \times 19^{19}}{20^{20}}$

5 a 0.8 **b** 0.5

6 a 0.1 **b** $\frac{5 \times 9^4}{10^5}$ **c** 180

7 a $\frac{1}{27}$ **b** $\frac{27^3 - 26^3}{27^3}$

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Multiple-choice questions

1 D **2** A **3** E **4** B **5** A **6** A
7 B **8** D **9** C **10** C **11** D **12** A
13 E **14** B **15** E **16** A

Extended-response questions

1 a 0.0173 **b** 0.2131

2 a i 0.3874 **ii** 0.4052
b 0.18
c $0 \leq p \leq 0.005$

3 a i 0.0819 **ii** 0.9011
b i $P = 15p^2(1-p)^4$
ii $\frac{dP}{dp} = 30p(1-p)^3(1-3p)$

4 a 2
b $n = 6,\ p = \frac{1}{3}$
c i 0.3292 **ii** 0.4926

5 a 0.9139 **b** 0.041 45 **c** 10.702

6 a 0.0735 **b** 0.5015 **c** 27

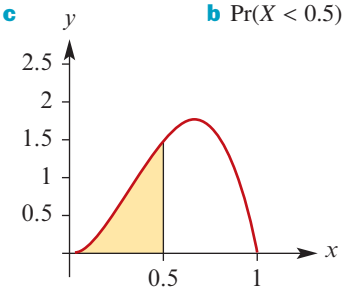
7 $\frac{1}{3} \leq q \leq 1$

Chapter 15

Exercise 15A

2 $k = -\frac{11}{6}$

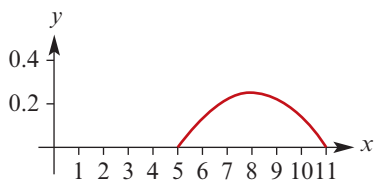
3 a c



b $\Pr(X < 0.5) = \frac{5}{16}$

4 a $k = 1$ **b** 0.865

5 a

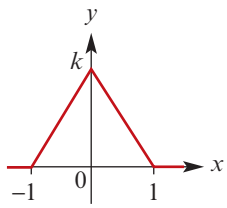


b 0.259 **c** 0.244 **d** 0.28

6 b i 0.024 = 2.4% **ii** 0.155 = 15.5%

7 a $k = 0.005$ **b** 0.007

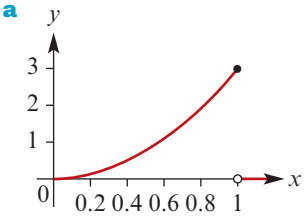
8 a



b $k = 1$

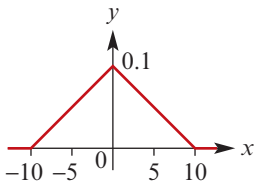
c $\frac{3}{4}$

9 a



b 0.406

10 a



b 0.190

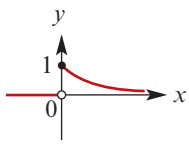
11 a $k = 1000$ **b** 0.5

12 a $\frac{2}{3}$ **b** $\frac{17}{30}$

13 a 0.202 **b** 0.449

14 a 0.45 **b** 0.711

15 a



b i $1 - e^{-\frac{1}{2}}$ **ii** e^{-1} **iii** $e^{-\frac{1}{2}}$

Exercise 15B

1 a $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** Does not exist

2 a 1 **b** 2.097 **c** 1.132 **d** 0.4444

3 a 0.567 **b** 0.458

4 0

5 $A = \frac{2}{9},\ B = 3$

6 a 2 **b** 1.858

7 a 0.632 **b** 0.233 **c** 0.693

8 a 1 **b** 0.5

9 0.0559

10 $4 \log_e 10$ minutes

11 a 1 **b** 1

12 a 0.714 **b** 0.736

13 12

14 a 0.4 **b** $\frac{\sqrt{19}-1}{6}$

15 a $ke^{-kx} - k^2xe^{-kx},\ \frac{-(kx+1)}{k}e^{-kx}$

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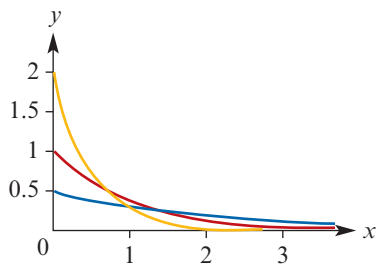
c



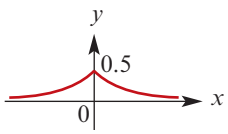
d $y = e^{-x}$ is dilated by factor $\frac{1}{\lambda}$ from the x-axis and by factor λ from the y-axis

Exercise 15C

1 $\text{Var}(X) = \frac{1}{18}$, $\text{sd}(X) = \frac{\sqrt{2}}{6}$

2 a 0.630 **b** 0.909 **c** 0.279

3 a



b 1.386

4 a $\frac{1}{\log_e 9}$ **b** $\text{E}(X) = 3.641$, $\text{Var}(X) = 4.948$

5 a 0.366 **b** $\text{E}(X) = 0.333$, $\text{Var}(X) = 0.056$

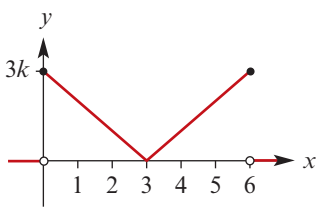
6 0.641

7 a 0.732 **b** $\text{E}(X) = \frac{4}{3}$, $\text{Var}(X) = \frac{2}{9}$

8 a 0.0004 **b** $\frac{16}{3}$ **c** 2.21

9 a $\frac{3}{4a^3}$ **b** $2\sqrt{5}$

10 a



b $k = \frac{1}{9}$ **d** 4.5

Exercise 15D

1 a 1300 **b** 22 500

2 a $\frac{31}{40}$ **b** $\frac{91}{20}$

3 a 45.794 **b** 17.592

4 a 0.708, 0.048 **b** \$98.94, \$0.33

5 a 0, 5.4 **b** 3, 0.6 **c** 1, 5.4

d $g(x) = \begin{cases} \frac{x^2}{18} & \text{if } -3 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$

e $h(x) = \begin{cases} \frac{(x-1)^2}{18} & \text{if } -2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

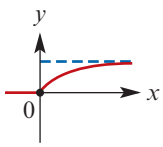
Exercise 15E

1 a $F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ \frac{1}{5}x & \text{if } 0 < x < 5 \\ 1 & \text{if } x \ge 5 \end{cases}$ **b** $\frac{3}{5}$

2 a $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4}x & \text{if } 0 \le x < 1 \\ \frac{1}{20}(x^4 + 4) & \text{if } 1 \le x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$

b 1.565

3 a



b e^{-4} **c** 0.0183

4 a $k = \frac{1}{36}$ **b** $\frac{1}{48}$

5 a $\frac{2}{3}$ **b** 20 **c** $a = \frac{400}{39}b = 400$

6 $f(x) = \begin{cases} 12x^2 - 12x^3 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

7 $f(x) = \begin{cases} 5(1-x)^4 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

8 $f(x) = \begin{cases} 0.5e^x & \text{if } x \le 0 \\ 0.5e^{-x} & \text{if } x > 0 \end{cases}$

Chapter 15 review

Technology-free questions

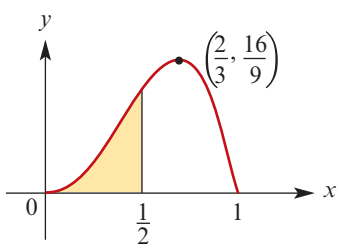
1 a 2 **b** 0.21 **c** 0.44

2 $a = \frac{1}{3}$, $b = 2$

3 $\frac{\pi}{2}$

4 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$

5 a



b $\text{Pr}(X < 0.5) = \frac{5}{16}$

6 a $k = 12$ **b** $\text{Pr}(X < \frac{2}{3}) = \frac{16}{27}$

c $\text{Pr}(X < \frac{1}{3} \mid X < \frac{2}{3}) = \frac{3}{16}$

7 a 0.008 **b** $\frac{8}{27}$

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8 $\frac{2}{3}$

9 a $\frac{7}{3}$ **b** $a = 1$

10 a $c = \frac{3}{4}$ **b** 0

12 a $e^{\frac{1}{2}}$ **b** $e^{\frac{3}{4}}$

13 b $\frac{1}{4}$ **c** $\sqrt{\frac{\pi}{2}}$

Multiple-choice questions

1 B **2** D **3** D **4** A **5** E
6 B **7** C **8** D **9** C **10** A
11 B **12** C

Extended-response questions

1 a 25 **b** $\frac{2}{3}$ **d** $\frac{2}{3}$

2 a $a = -\frac{2}{81}$ **b** 0.1080 **c** 700 hours
d 736.4 hours **e** 0.023

3 a 0.0245 **b** 0.36 mm **c** 6.40 mm
d 9.12 mm **e** 0.040 **f** 0.826

4 a $\frac{19}{30}$ **b** $\frac{61}{88}$ **c** $\frac{7}{3}$ kg
d i 1kg **ii** 0.6083

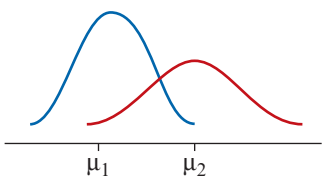
5 $c = \frac{8}{3}$ or $c = 4$

6 a $k = \frac{1}{4}$ **b** $\mu = 2$, $\text{Var}(X) = \frac{2}{3}$ **c** $\frac{3}{4}$
d $\frac{4\sqrt{5}}{5} \approx 1.8$

Chapter 16

Exercise 16A

1

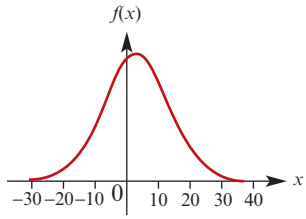


2 c

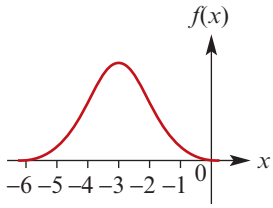
3 a 1
b i $\text{E}(X) = \frac{1}{3\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2} dx$
ii 2
c i $\text{E}(X^2) = \frac{1}{3\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2} dx$
ii 13 **iii** 3

4 a 1
b i $\text{E}(X) = \frac{1}{5\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2}\left(\frac{x+4}{5}\right)^2} dx$
ii −4
c i $\text{E}(X^2) = \frac{1}{5\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{1}{2}\left(\frac{x+4}{5}\right)^2} dx$
ii 41 **iii** 5

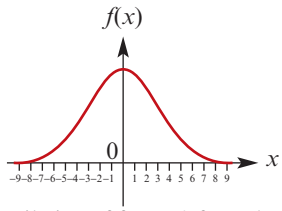
5 a $\mu = 3$, $\sigma = 10$
b



6 a $\mu = -3$, $\sigma = 1$
b



7 a $\mu = 0$, $\sigma = 3$
b



8 a Dilation of factor 2 from the y-axis and dilation of factor $\frac{1}{2}$ from the x-axis, then translation 3 units to the right
b Dilation of factor $\frac{1}{2}$ from the y-axis and dilation of factor 2 from the x-axis, then translation 3 units to the right
c Dilation of factor 2 from the y-axis and dilation of factor $\frac{1}{2}$ from the x-axis, then translation 3 units to the left

9 a Translation 3 units to the left, then dilation of factor $\frac{1}{2}$ from the y-axis and dilation of factor 2 from the x-axis
b Translation 3 units to the left, then dilation of factor 2 from the y-axis and dilation of factor $\frac{1}{2}$ from the x-axis
c Translation 3 units to the right, then dilation of factor $\frac{1}{2}$ from the y-axis and dilation of factor 2 from the x-axis

Exercise 16B

1 a 16% **b** 16% **c** 2.5% **d** 2.5%

2 a $\mu = 135$, $\sigma = 5$ **b** $\mu = 10$, $\sigma = \frac{4}{3}$

3 a 68% **b** 16% **c** 0.15%

4 21.1, 33.5

5 one, 95, 99.7, three

6 2.5%

7 a 16% **b** 16%

8 a 68% **b** 16% **c** 2.5%

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9 **a** 95% **b** 16% **c** 50% **d** 99.7%

10 **a** 0 **b** $-\frac{5}{4}$ **c** 1.5

11 **a** −1.4 **b** 1.1 **c** 3.5

12 Michael 1.4, Cheryl 1.5; Cheryl

13 Biology 1.73, History 0.90; Biology

14 **a**

Student	French	English	Maths
Mary	1	0.875	0
Steve	−0.5	−1	1.25
Sue	0	0.7	−0.2

b **i** Mary **ii** Mary **iii** Steve

c Mary

Exercise 16C

1 **a** 0.9772 **b** 0.9938 **c** 0.9938 **d** 0.9943
e 0.0228 **f** 0.0668 **g** 0.3669 **h** 0.1562

2 **a** 0.9772 **b** 0.6915 **c** 0.9938 **d** 0.9003
e 0.0228 **f** 0.0099 **g** 0.0359 **h** 0.1711

3 **a** 0.6827 **b** 0.9545 **c** 0.9973

4 **a** 0.0214 **b** 0.9270 **c** 0.0441 **d** 0.1311

5 $c = 1.2816$ **6** $c = 0.6745$

7 $c = 1.96$ **8** $c = -1.6449$

9 $c = -0.8416$ **10** $c = -1.2816$

11 $c = -1.9600$

12 **a** 0.9522 **b** 0.7977 **c** 0.0478 **d** 0.1545

13 **a** 0.9452 **b** 0.2119 **c** 0.9452 **d** 0.1571

14 **a** $c = 9.2897$ **b** $k = 8.5631$

15 **a** $c = 10$ **b** $k = 15.88$

16 **a** $a = 0.99$ **b** $b = 1.96$ **c** $c = 2.97$

17 **a** 0.7161 **b** 0.0966 **c** 0.5207
d $c = 33.5143$ **e** $k = 13.02913$
f $c_1 = 8.28$, $c_2 = 35.72$

18 **a** 0.9772 **b** 0.9772 **c** $c = 10.822$
d $k = 9.5792$ **e** $c_1 = 9.02$, $c_2 = 10.98$

Exercise 16D

1 **a** **i** 0.2525 **ii** 0.0478 **iii** 0.0903
b 124.7

2 **a** **i** 0.7340 **ii** 0.8944 **iii** 0.5530
b 170.25 cm
c 153.267 cm

3 **a** **i** 0.0766 **ii** 0.9998 **iii** 0.153
b 57.3

4 **a** 10.56% **b** 78.51%

5 Mean = 1.55 kg; sd = 0.194 kg

6 **a** 36.9% **b** $c = 69$

7 **a** 0.0228 **b** 0.0005 **c** 0.0206

8 **a** 3.04 **b** 350.27

9 1004 mL

10 **a** Small 0.1587; Med 0.7745; Large 0.0668
b $348.91

11 **a** **i** 0.1169 **ii** 17.7
b 0.0284

12 **a** A: 0.0228; B: 0.1587 **b** $c = \frac{34}{3}$

Exercise 16E

1 0.9632

2 0.2442

3 **a** 0.0478 **b** 0.2525

4 **a** 0.7834 **b** 0.0108

5 0.2819

6 **a** 0.0416 **b** 0.0038

Chapter 16 review

Technology-free questions

1 **a** $1 - p$ **b** $1 - p$ **c** $2p - 1$

2 **a** $a = -1$ **b** $b = 1$ **c** 0.5

3 $(x, y) \to \left(\frac{x-8}{3}, 3y\right)$

4 **a** $\frac{q}{p}$ **b** $1 - q$ **c** $\frac{1-p}{1-q}$

5 **a** $\Pr(Z < \frac{1}{2})$ **b** $\Pr(Z < -\frac{1}{2})$ **c** $\Pr(Z > \frac{1}{2})$
d $\Pr(-\frac{1}{2} < Z < \frac{1}{2})$ **e** $\Pr(-\frac{1}{2} < Z < 1)$

6 **a** 0.84 **b** 0.5 **c** 0.16 **d** 0.68

7 **a** 0.16 **b** 0.34 **c** 0.32 **d** 0.02

8 **a** 0.69 **b** 0.19 **c** 0.15 **d** 0.68

9 Best C, worst B

10 **a** 0.5 **b** $b = -1.5$

Multiple-choice questions

1 A **2** B **3** B **4** C **5** C **6** D
7 A **8** A **9** D **10** C

Extended-response questions

1

Category	Range
High	≥ 63
Moderate	[56, 62]
Average	[45, 55]
Little	[38, 44]
Low	≤ 37

2 0.92%

3 **a** **i** 0.1587 **ii** 0.9747 **iii** 0.0164
b $c = 53\,592$
c 3.7×10^{-11}

4 **a** 0.1056 **b** 0.3797 **c** 0.032 **d** 0.092
e 0.423

5 **a** $\mu = 60.1$, $\sigma = 0.2$ **b** 10%

6 **a** 0.9044 **b** 5.88 **c** 9.044
d 0.2651 **e** $17.61, 5.40281

7 **a** $\mu = 0$, $\sigma = 2.658$ **b** 0.882

8 **a** 0.1056 **b** 0.0803 **c** 0.5944

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Chapter 17

Exercise 17A

1 No; sample will be biased towards the type of movie being shown.

2 a No; biased towards shoppers.
b Randomly select a sample from telephone lists or an electoral roll.

3 No; only interested people will call, and they may call more than once.

4 a No; biased towards older, friendly or sick guinea pigs which may be easier to catch.
b Number guinea pigs and then generate random numbers to select a sample.

5 No; a student from a large school has less chance of being selected than a student from a small school.

7 a Unemployed will be under represented.
b Unemployed or employed may be under represented, depending on time of day.
c Unemployed will be over represented.
Use random sampling based on the whole population (e.g. electoral roll).

8 a Divide platform into a grid of 1 m^2 squares. Select squares using a random number generator to give two digits, one a vertical reference and one a horizontal reference.
b Yes, if crabs are fairly evenly distributed; otherwise, five squares may not be enough.

9 No; a parent's chance of selection depends on how many children they have at the school.

10 Not a random sample; only interested people will call, and they may call more than once.

11 People who go out in the evenings will not be included in the sample.

12 a All students at this school
b $p = 0.35$ **c** $\hat{p} = 0.42$

13 a 0.22 **b** $\hat{p}$

Exercise 17B

1 a 0.5 **b** $0, \frac{1}{3}, \frac{2}{3}, 1$

c

$\hat{p}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

d $\frac{1}{2}$

2 a $\frac{3}{5}$ **b** $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$

c

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$
$\Pr(\hat{P} = \hat{p})$	0.0036	0.0542	0.2384
$\hat{p}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$\Pr(\hat{P} = \hat{p})$	0.3973	0.2554	0.0511

d 0.3065 **e** 0.6924

3 a 0.5 **b** $0, \frac{1}{3}, \frac{2}{3}, 1$

c

$\hat{p}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$\Pr(\hat{P} = \hat{p})$	0.1	0.4	0.4	0.1

d 0.9

4 a 0.4 **b** $0, \frac{1}{3}, \frac{2}{3}, 1$

c

$\hat{p}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

d $\frac{1}{3}$

e $\frac{3}{5}$

5 a 0.5 **b** $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

c

$\hat{p}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

d $\frac{5}{16}$

6 a $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$

b

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{1}{32}$

c $\frac{3}{16}$ **d** $\frac{25}{26}$

7 a $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

b

$\hat{p}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

c $\frac{113}{625}$

8 $\mu = 0.5,\ \sigma = 0.25$

9 $\mu = 0.5,\ \sigma = 0.224$

10 $\mu = 0.2,\ \sigma = 0.2$

11 a 0.0845 **b** $\mu = 0.3,\ \sigma = 0.084$

12 a 0.1311 **b** 0.0655

13 a 0.3698 **b** 0.4606
c $\mu = 0.25,\ \sigma = 0.1083$

14 a 0.1844 **b** 0.7600 **c** 0.9683

Exercise 17C

1 0.2858 **2** 0.8568 **3** 0.1587
4 0.0092 **5** 0.0614

6 a 1 **b** 0.5000 **c** 0.0412

7 a 0.9545 **b** 164

8 a 0.9650 **b** 0.9650

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9 **a** 0.575 **b** 0.0139
10 **a** 0.848 **b** 0.0031 **c** Yes
11 121

Exercise 17D

1 **a** 0.08 **b** 90%: (0.0354, 0.1246), 95%: (0.0268, 0.1332), 99%: (0.0101, 0.1499) Interval width increases as confidence level increases
2 **a** 0.192 **b** 90%: (0.1510, 0.2330), 95%: (0.1432, 0.2408), 99%: (0.1278, 0.2562) Interval width increases as confidence level increases
3 **a** 0.2 **b** (0.1069, 0.2931)
4 (0.2888, 0.3712)
5 **a** (0.4761, 0.5739) **b** (0.5095, 0.5405)
c The second interval is narrower because the sample size is larger
6 **a** (0.7895, 0.9065) **b** (0.8295, 0.8665)
c The point estimate for both samples is the same, but the second interval is narrower because the sample size is larger, and does not include 0.9. This would cause us to doubt the manufacturer's claim.
7 90%: (0.5194, 0.6801), 95%: (0.5040, 0.6960), 99%: (0.4738, 0.7262); Interval width increases as confidence level increases
8 90%: (0.5111, 0.5629), 95%: (0.5061, 0.5679), 99%: (0.4964, 0.5776); Interval width increases as confidence level increases
9 1537
10 174
11 **a** 1549 **b** 3484
c Reducing margin of error by 1% requires the sample size to be more than doubled
12 **a** 2017 **b** 2401
c **i** $M = 1.8\%$ **ii** $M = 2.2\%$
d 2401, as this ensures that M is 2% or less, whoever is correct

Chapter 17 review

Technology-free questions

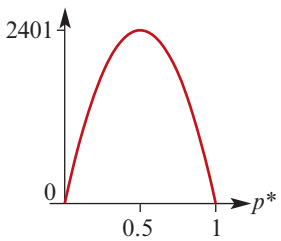
1 **a** All employees of the company **b** $p = 0.35$
c $\hat{p} = 0.40$
2 **a** No; only people already interested in yoga
b Use electoral roll
3 **a** $\frac{3}{5}$ **b** $\{\frac{1}{3}, \frac{2}{3}, 1\}$ **c** $\frac{3}{10}$
4 700
5 **a** $\frac{k}{100}$ **b** $\frac{k}{100} \pm \frac{1.96\sqrt{k(100-k)}}{1000}$
6 **a** $\hat{p} = 0.9$ **b** $M = \frac{0.588}{\sqrt{n}}$
c Margin of error would decrease by a factor of $\sqrt{2}$
7 **a** $\frac{1}{2}$ **b** 625
8 **a** 38 **b** $(0.95)^{40}$
9 **a** 45 **b** $5.9(0.9)^{49}$
10 **a** 0.60 **b** 0.10 **c** Increase sample size

Multiple-choice questions

1 B **2** C **3** D **4** E **5** C
6 E **7** B **8** E **9** C **10** E
11 A **12** B **13** C **14** D

Extended-response questions

1 **a**



b 0.5 **c** 2401
2 **a** **i** 0.1593
ii 0.5037
b 525
3 **a** (0.4730, 0.6670)
b **i** 0.7738 **ii** 0.0000003
iii 0.2262 **iv** 4.75
c (0.5795, 0.6645)
4 **a** $p = \frac{500}{N}$ **b** $\hat{p} = 0.15$
c $N = 3333.33 \approx 3333$ **e** (2703, 4348)

Chapter 18

Technology-free questions

1 **a** $\frac{17}{750}$ **b** $\frac{7}{17}$
2 **a** $\frac{1}{625}$ **b** $\frac{624}{625}$ **c** $\left(\frac{4}{5}\right)^{24}$

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3 a $\frac{\pi}{2}$ **b** 2 **c** $\frac{2-\sqrt{2}}{2}$ **d** $3-2\sqrt{2}$

4 a $\frac{1}{5}$ **b** $\frac{4}{9}$ **c** 1.7 **d** 2.01

5 a $\frac{1}{36}$ **b** $\frac{20}{27}$ **c** 3 **d** 3 **e** $\frac{14}{27}$ **f** $\frac{13}{20}$

6 a $\frac{3}{28}$ **b** $\frac{3}{14}$ **c** $\frac{5}{7}$ **d** $\frac{5}{14}$ **e** $\frac{5}{28}$

7 a $\frac{1}{7}$ **b** $\frac{1}{3}$

8 a $\frac{8-5p}{20}$ **b** $\frac{3}{8-5p}$

9 a $a = 0.34,\ b = 0.06$ **b** 1.6644

10 a 0.75 **b** 0.28

11 b $\frac{6^{\frac{4}{3}}}{4}$

12 a 2 **b** $\frac{23 \times 7^{15}}{8^{16}}$

13 0.5

14 a 0.8 **b** 0.6

15 $\frac{1-q}{2}$

16 $a = \frac{1}{2}$

17 a $(1-p)^3$ **b** $p = \frac{1}{3}$

18 b $1 - \frac{\sqrt{3}}{2}$ **c** $\sqrt{\frac{\pi}{6}}$

19 a $\frac{3}{10}$ **b** $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$ **c** $\frac{7}{24}$

20 64

21 a i $\Pr(\text{Black}) = \frac{4n-3}{n(n+1)}$

ii $\Pr(\text{White}) = \frac{n^2-3n+3}{n(n+1)}$

b $\frac{(n-3)^2}{n^2-3n+3}$

Multiple-choice questions

1 E **2** B **3** D **4** B **5** C **6** A
7 B **8** E **9** D **10** E **11** C **12** B
13 D **14** C **15** B **16** E **17** E **18** A
19 A **20** D **21** D **22** C **23** E **24** B
25 C **26** D **27** C **28** E **29** A **30** B
31 E **32** C **33** A **34** B **35** B **36** B
37 B **38** D **39** B **40** C **41** B **42** C

Extended-response questions

1 a 0.0968 **b** 2.96kg **c** 3.5kg **d** 0.1464
e 2.61 kg **f** 0.227 **g** 0.560 **h** 0.79
i 9

2 a 99.2°C **b** 0.928 **c** 100°C or 105°C
d $\mu = 96.2,\ \sigma = 7.9$ **e** 0.608 **f** 22
g 0.08 **h** 75

3 a i 0.7437 **ii** 0.2525

b

f	0	3000	10 000
$\Pr(F = f)$	0.4387	0.3125	0.2488

c $E(F) = \$3425.50$, $sd(F) = \$3994.80$
d (0.084, 0.162) **e** 5.0 **f** 0.0187

4 a $\frac{3}{64}$ **b** 9.6 sec **c** 25 **d** 0.1625
e 0.5559 **f** 12.0
g i 0.8863 **ii** 8
iii $E(\hat{P}) = 0.033$, $sd(\hat{P}) = 0.022$
h 60%

5 b $\frac{9}{16}$

6 \$0.76

7 a $P = \begin{cases} 0.76x - 0.5s, & x \leq s \\ 0.5s - 0.25x, & x > s \end{cases}$

b \$5.95

c $E(P) = \sum_{x=24}^{s} (0.75x - 0.5s)p(x) + \sum_{x=s+1}^{30} (0.5s - 0.25x)p(x)$

d 27

8 a i $\frac{1}{6}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{6}$

b i $\frac{4}{25}$ **ii** $\frac{41}{100}$

c $\frac{121}{600}$

9 a 0.0436 **b** 26.67% **c** 183 **d** 59 271

10 a i 0.1587 **ii** 511.63
b 0.1809

11 a i $\frac{1}{2500}$ **ii** $\frac{16}{3}$ **iii** 0.8281 **iv** 0.7677
b 0.9971

12 a 0.1056 **b** 1027.92 g

13 a i 0.0105 **ii** 0.0455
b 0.4396
c $\frac{1149}{1909}$

14 a i $\mu = 4.25$ **ii** $\sigma = 0.9421$ **iii** 0.94 **iv** 0.9
b i Binomial **ii** 18 **iii** 1.342 **iv** 0.3917

15 a (0.0814, 0.1186) **b** (0.0792, 0.1208)
c Larger sample of females
d 900 of each sex **e** 0.078 or 0.922

Algorithms and pseudocode

See online solutions

Chapter 19

Technology-free questions

1 $f(g(x)) = (3x+1)^2 + 6 = 9x^2 + 6x + 7$

2 $k = -1 - \sqrt{13}$ or $k = -1 + \sqrt{13}$

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3 $y = -\dfrac{6}{x}$

Reflection in x-axis, dilation of factor 2 from the y-axis, dilation of factor 3 from the x-axis; Alternatively: reflection in the x-axis, dilation of factor 6 from the x-axis

4 a $f'(x) = 21x^6(5x^2 - 3)^6(5x^2 - 1)$

b $f'(0) = 2$

5 a $x(1 + 2\log_e(2x))$ **b** $f'\left(\dfrac{\pi}{2}\right) = \dfrac{-2}{(\pi + 1)^2}$

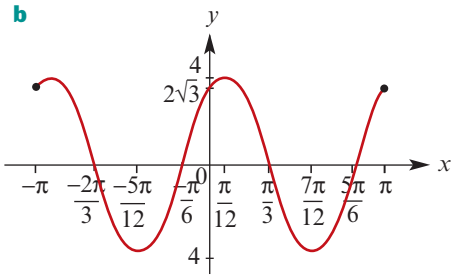
6 a $f'(x) = 2\cos(2x)e^{\sin(2x)}$

b $f'\left(\dfrac{\pi}{3}\right) = 8\pi - 3\sqrt{3}$

7 $x = \dfrac{(4n + 1)\pi}{8}, n \in \mathbb{Z}$

8 a Amplitude = 4; Period = π

b



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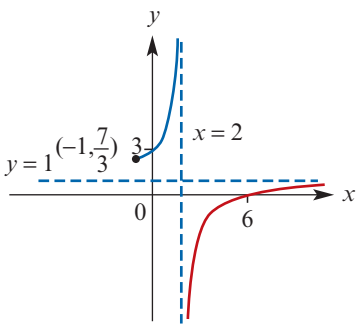


10 a $f^{-1}(x) = \log_e\left(\dfrac{x + 3}{5}\right) + 1$

b dom $f^{-1} = (-3, \infty)$

11 $x = -\dfrac{2\pi}{15}$ or $x = \dfrac{2\pi}{15}$

13 $\dfrac{1}{4}(e^4 - 1)$

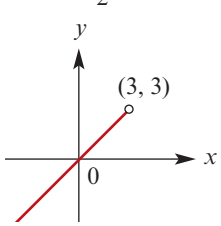
14 a $c = 6$

b $4a + b - 3 = 0,\ 3a + b = 0$

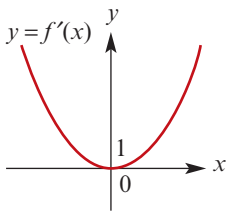
c $a = 3,\ b = -9$

15 a $g^{-1}(x) = \dfrac{1}{2}\log_e(3 - x)$, dom $g^{-1} = (-\infty, 3)$

b



16 a



b $f'(x) = \begin{cases} -8x^3 & \text{if } x \leq 0 \\ 8x^3 & \text{otherwise} \end{cases}$

17 $\dfrac{-1}{3}\log_e(1 - 3x) + c$ **18** $f^{-1}(x) = \dfrac{x}{2(x - 3)}$

19 $x = \dfrac{-\pi}{6}$ or $x = \dfrac{\pi}{3}$

20 a 0.5 **b** 0.68 **c** 0.32

21 a $\dfrac{1}{6}$ **b** $\sqrt{31}$

22 $\dfrac{-2}{3}, \dfrac{2}{3}$

23 a $A = 32a - 8a^3$

b Max value $A = \dfrac{128\sqrt{3}}{9}$ when $a = \dfrac{2\sqrt{3}}{3}$

24 $b = 3$

25 a 0.36 **b** 0.5625

26 a \$0.65 **b** 0.425

27 0.37

28 a $h = \dfrac{4000}{x^2}$ **c** $2000(2 + \sqrt{2})$

29 a $E(X) = 1$

b i {0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0}

ii $\dfrac{19 \times 9^9}{10^{10}}$

30 $p = 0.3$

31 $\dfrac{q + 1}{2}$

Multiple-choice questions

1 B	**2** A	**3** B	**4** C	**5** D
6 A	**7** D	**8** C	**9** D	**10** A
11 A	**12** B	**13** D	**14** B	**15** C
16 A	**17** A	**18** E	**19** E	**20** E
21 C	**22** E	**23** D	**24** C	

Extended-response questions

1 a i $\left(\dfrac{1}{2}, 8\right)$ **ii** Minimum

b ii $A = \dfrac{x}{12}(60 - 5x)$ **iii** Max area 15 cm^2

2 a $p = 1,\ q = 3,\ k = 2$

b i $m = -2$ **ii** $y = -2x^3 + 10x^2 - 14x + 6$

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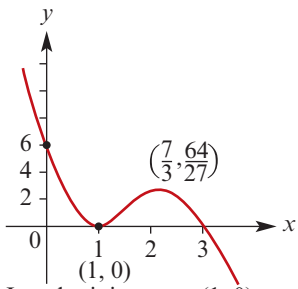
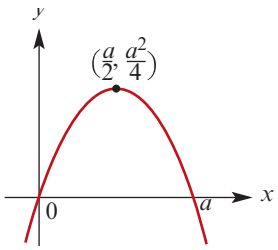
iii



Local minimum at (1, 0)

Local maximum at $(\frac{7}{3}, \frac{64}{27})$

3 a



b $\frac{a^3}{6}$ square units

c **i** $y = \frac{2a^2}{9}, y = \frac{2a^2}{9}$ **ii** $\frac{a^3}{162}$ square units

4 a $y = -\frac{1}{2}x + \frac{3}{2}$

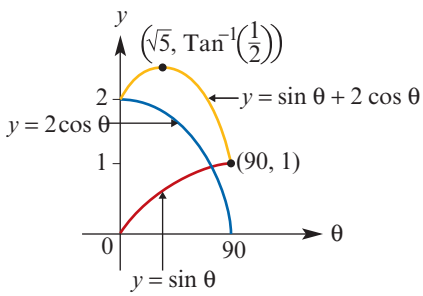
b **i** $\frac{dy}{d\theta} = \cos\theta - 2\sin\theta$

ii $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$

iii (26.57, 2.2361); exact: $(\tan^{-1}(\frac{1}{2}), \sqrt{5})$

iv $r = \sqrt{5}$, $\alpha = 63.435°$

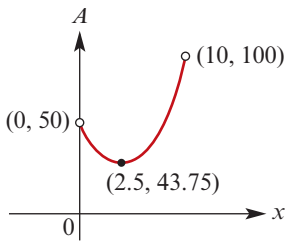
v



c **i** $Q(2\sin\theta, 2\cos\theta)$ **iii** $\theta = 74.4346°$

5 a **i** $A = x^2 - 5x + 50$ **ii** (0, 10)

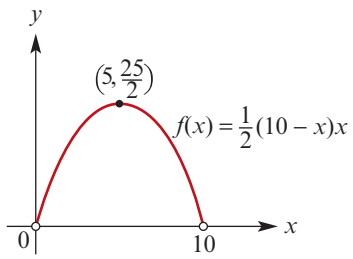
iii



iv Minimum area = 43.75 cm^2

b **i** $f(x) = \frac{1}{2}(10 - x)x$

ii



c $AYX : OXYZ : ABY : CBYZ = 1 : 2 : 2 : 3$

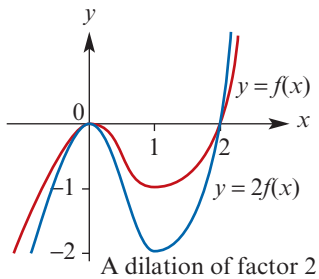
6 a **i** $f'(t) = -100e^{-\frac{t}{10}}(t^2 - 30t + 144)$

ii $f''(t) = 10e^{-\frac{t}{10}}(t^2 - 50t + 444)$

b **i** $t \in (6, 24)$ **ii** $t \in (11.546, 35)$; exact: $t \in (25 - \sqrt{181}, 35]$

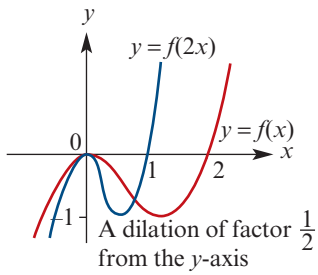
iii $t \in (11.546, 24)$

7 a **i**



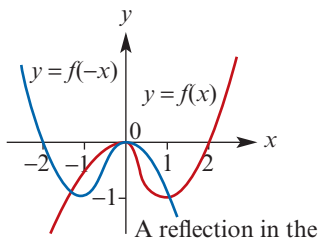
A dilation of factor 2 from the x-axis

ii



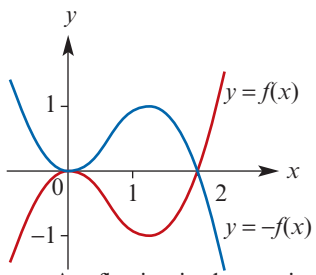
A dilation of factor $\frac{1}{2}$ from the y-axis

iii



A reflection in the y-axis

iv



A reflection in the x-axis

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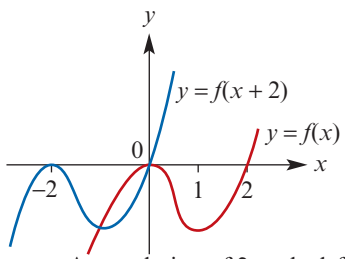
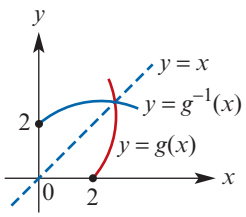
v



A translation of 2 to the left

b f is not one-to-one

c



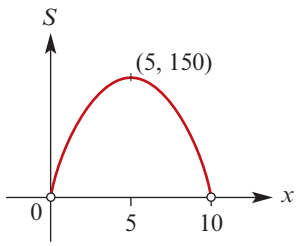
d i Gradient = 15 **ii** Gradient = $\frac{1}{15}$

8 a i 0.995 **ii** $x = \pm 0.2$

b i $h(x) = \frac{1}{2}(x - \pi)^2 - 1$ **ii** −0.989 98

9 a $S = (60x - 6x^2)$ **b** $0 < x < 10$

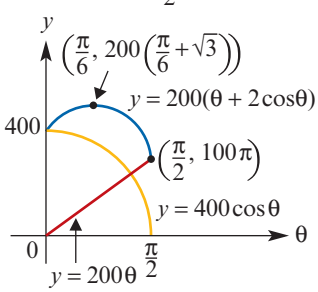
c $x = 5$ **d**



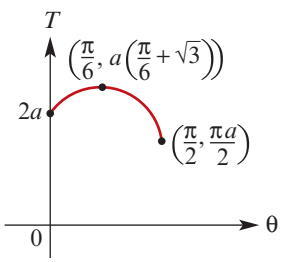
10 a i $OP = \frac{1}{\sin\theta}$ **ii** $BQ = \frac{1 - \cos\theta}{\sin\theta}$

d Min value $S = \frac{\sqrt{3}}{2}$ when $AP = \frac{2\sqrt{3} - 3}{3}$

11 b



c



Minimum value for T is $\frac{\pi a}{2}$

12 a iii $x = 1$ or $x = k - 2$

b i $b = 3 - 2a,\ c = a - 2$

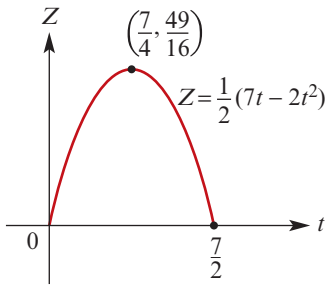
ii $h = a - 2$

iii $a = 0,\ b = 3,\ c = -2$

iv $a = -1,\ b = 5,\ c = -3$

13 a $Z = \frac{1}{2}(7t - 2t^2)$

b



c Max value $Z = \frac{49}{16}$ when $t = \frac{7}{4}$

14 a i $\frac{3}{8}$ **ii** $\frac{4}{15}$

b i $\frac{27}{125}$ **ii** $\frac{8}{125}$ **iii** $\frac{38}{125}$

15 a $k = \frac{b}{a^2}$

b i $y = \frac{b}{2a}x + \frac{b}{2}$ **ii** $\left(\frac{-a}{2}, \frac{b}{4}\right)$

d $S_1 : S_2 = 27 : 37$

16 a i 0.9332 **ii** 0.0668 **iii** 0.1151 **iv** 0.1151

b i 33.3% **ii** 866.4 **iii** 199.4

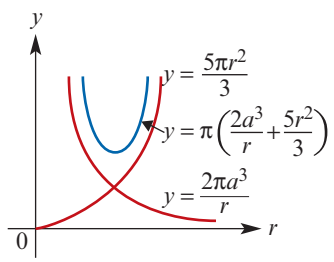
17 $90 - 8\sqrt{3}$ metres from A towards E

18 a i $y = -e^{-n}x + e^{-n}(n + 1)$ **ii** $x = n + 1$

b i $\frac{1}{e^n}\left(1 - \frac{1}{e}\right)$ **ii** $e : e - 2$

19 b i $h = \frac{3a^3 - 2r^3}{3r^2}$ **ii** $S = \pi\left(\frac{2a^3}{r} + \frac{5r^2}{3}\right)$

c i



ii Local minimum at $\left((\sqrt[3]{0.6})a,\ \pi a^2\left(\frac{2}{\sqrt[3]{0.6}} + \frac{5}{3}(\sqrt[3]{0.6})^2\right)\right)$

20 a 0.0023

b

Q	$s - 1$	-1
$Pr(Q = q)$	$\frac{3}{4}$	$\frac{1}{4}$

c $E(Q) = \frac{3}{4}s - 1,\ sd(Q) = \frac{\sqrt{3}}{4}s$

21 a 0.091 21 **b** 0.2611 **c** 0.275

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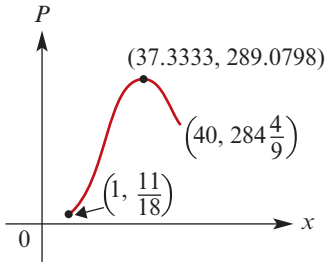
22 a $\frac{dP}{dx} = \frac{1}{90}(112x - 3x^2)$

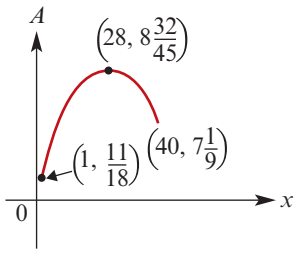
b i



ii Max value of P is 289.0798 tonnes

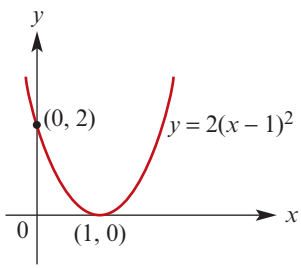
c $A = \frac{x}{90}(56 - x)$

i

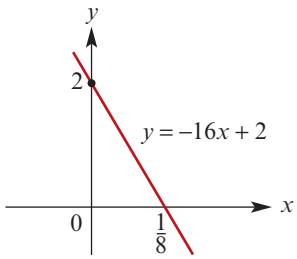


ii Max value of A is $8\frac{32}{45}$ tonnes per miner, occurs when $x = 28$

23 a i



ii



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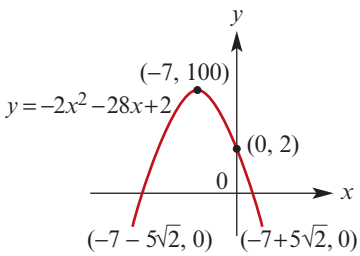


b $\left(\frac{2-3k}{k+2}, \frac{-(2-3k)^2 + 2(k+2)}{k+2}\right)$

i $\left(-2, \frac{2}{3}\right)$ **ii** $\left\{\frac{2}{3}\right\}$

iii $(-\infty, -2) \cup \left(0, \frac{14}{9}\right)$ **iv** $(-2, 0) \cup \left(\frac{14}{9}, \infty\right)$

c $k < -2$

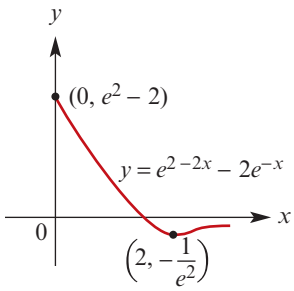
d i $k = 0$ or $k = \frac{14}{9}$ **ii** $0 < k < \frac{14}{9}$

24 a $x = 2 - \log_e 2$

b i $\frac{dy}{dx} = -2e^{2-2x} + 2e^{-x}$ **ii** $x = 2$

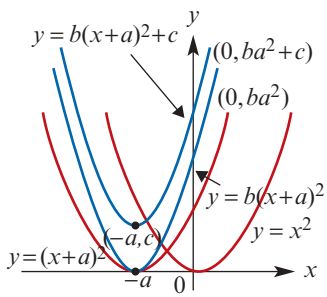
iii $\left(2, -\frac{1}{e^2}\right)$

iv



c $\left(-\frac{1}{e^2}, 0\right)$

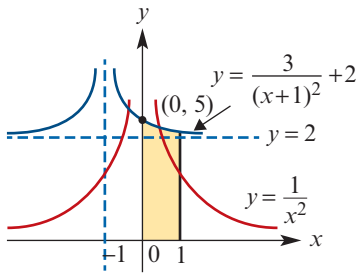
25 a



c Dilation of factor 3 from the x-axis, then translation 1 unit to the left and 2 units up

d $\frac{7}{2}$

e



26 a i $y = 50$ **ii** $y = x - 25$

b $a = -\frac{1}{15}$, $c = -\frac{25}{3}$

c i 1250 square units

ii $\frac{14\,375}{18}$ square units

iii $\frac{36\,875}{18}$ square units

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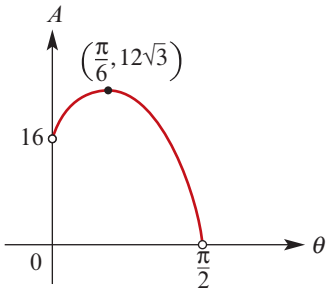
27 c $\theta = \frac{\pi}{6}$

d



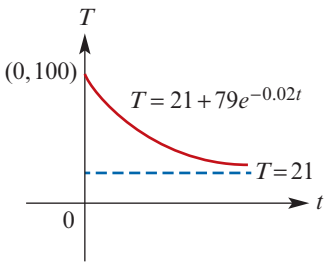
Maximum value of A is $12\sqrt{3}$

28 a $\mu = 5.0290,\ \sigma = 0.0909$ **b** \$409.28

29 a $k = \frac{1}{10}\log_e\left(\frac{79}{63}\right) \approx 0.02,\ A = 79$

b Approx 2:44 p.m.

c



d Average rate of change = −1.6°C/minute

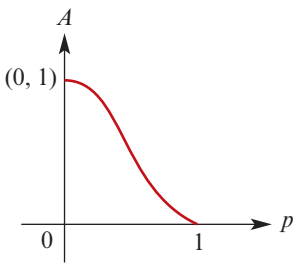
e i 2.0479°C/minute

ii −0.8826°C/minute

30 a $\frac{3}{16}$

b $b = 4$

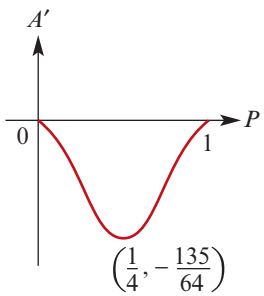
c



d i 0.076 **ii** 0.657

e i $A'(p) = -20p(1-p)^3$

ii



iii $p = \frac{1}{4}$

iv Most rapid rate of change of probability occurs when $p = \frac{1}{4}$

31 a 91.125 cm

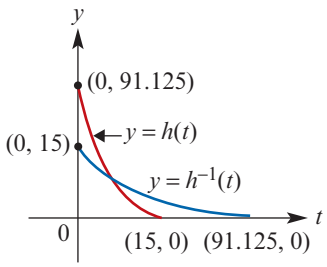
b [0, 15]

c $V = 0.64(4.5 - 0.3t)^3$

d h is a one-to-one function;

$h^{-1}(t) = 15 - \frac{10t^{\frac{1}{3}}}{3}$, dom h^{-1} = [0, 91.125]

e



32 a 0.065 36

b i 0.6595 **ii** 0.198 14

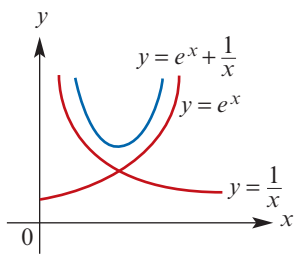
c i 23.3% **ii** $c = 0.1075$

33 a i 0.32 **ii** 0.18 **iii** 0.5

b 0.64

c i 0.043 95 **ii** 0.999 **iii** $\frac{7}{128}$

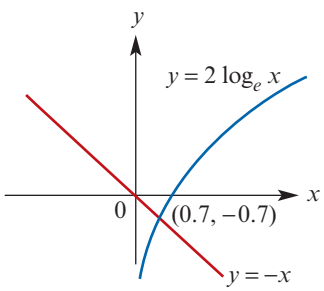
34 a b



c $y = \frac{1}{x} + e^x,\ \frac{dy}{dx} = -\frac{1}{x^2} + e^x$

d ii $2\log_e x < 0,\ x \in (0, 1)$

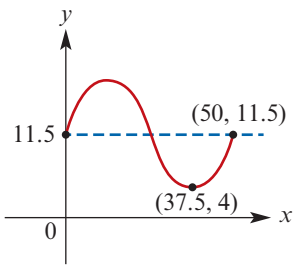
iii



iv (0.7, 3.4)

35 a i $m = 12.5,\ n = 15,\ d = 37.5,\ a = 7.5,\ b = 7.5$

ii



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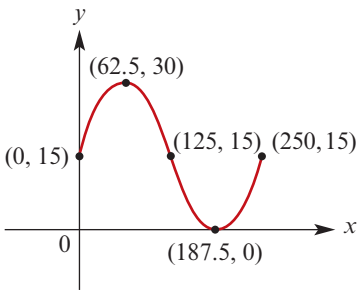
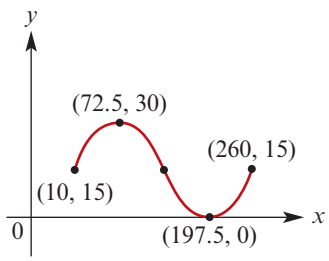
b (2.704, 10), (22.296, 10)

c



d **i** $h(x) = 15 + 15\sin\left(\frac{\pi(x-10)}{125}\right)$

ii



36 a $k = 4$

b **i** $\mathrm{E}(X) = \frac{13}{6}$ **ii** $\frac{10-\sqrt{2}}{4}$ **iii** $\frac{\sqrt{2}}{12}$

c 0.1857

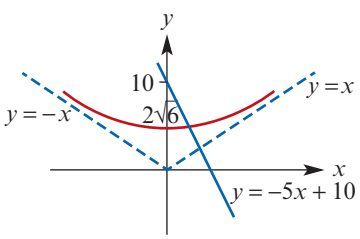
37 a $k = \frac{2}{a^2}$ **b** $\mathrm{E}(X) = \frac{a}{3}$, $\mathrm{Var}(X) = \frac{a^2}{18}$

c $\frac{6-4\sqrt{2}}{9}$ **d** $a = 1000(\sqrt{2}+2)$

38 a 7 **c** $y = \frac{1}{10}x - \log_e 10$ **d** **ii** 36.852

39 a $\frac{dy}{dx} = \frac{x}{\sqrt{x^2+24}}$ **b** $(0, 2\sqrt{6})$ **c** Even

d



e $y = -5x + 10$ **f** 14 units/second

h $12\log_e(\sqrt{7}-1) - 2\sqrt{7} + \frac{35}{2}$

40 a **i** 0 **ii** −0.6745 **iii** 0.6745
iv 1.3490 **v** 99.3% **vi** 0.7%

b **i** μ **ii** $\mu - 0.645\sigma$
iii $\mu + 0.645\sigma$ **iv** 1.3490σ
v 0.9930% **vi** 0.7%

41 a $k = n + 1$ **b** $\mathrm{E}(X) = \frac{n+1}{n+2}$

c $\frac{n+1}{(n+2)^2(n+3)}$ **d** Median $= \sqrt[n+1]{\frac{1}{2}}$

42 a **i** $\frac{1}{1-b}$ **ii** $1 + \sqrt{b-1}$

b **i** 1 **ii** $1 + e^{-7}$

c **i** $\frac{b(b-2)}{2(b-1)}$ **ii** $9 + \sqrt{65}$

d e

43 a **i** $\frac{-(b+1)}{b^2}$ **ii** $\left(\frac{2b^2}{b+1}\right)^{\frac{1}{3}}$

b **i** $\frac{(b^2+1)(b-1)}{2b^2}$ **ii** $b = 3$

44 a Local maximum $\left(\frac{m}{n}, \left(\frac{m}{n}\right)^m e^{n-m}\right)$

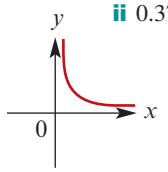
b $\left(\frac{m-1}{n}, \left(\frac{m-1}{n}\right)^m e^{n-m+1}\right)$

c **i** $\frac{4}{e^2}$ **ii** $1 - \frac{5}{e^2}$

45 a **i** q **ii** $\frac{1}{q}$ **iii** $\frac{1}{q^2}$

b $\frac{2}{3}$

c **i**



ii 0.37 **iii** 69.31

46 a $\frac{527}{1000}$ **b** (0.4961, 0.5580)

c 67% **d** 99.9955%

Appendix B

Exercise B1

1 63 **2** 26 **3** 336

4 a 5040 **b** 210

5 a 120 **b** 120

6 18

7 a 5 852 925 **b** 1 744 200

8 100 386

9 a 792 **b** 336

10 a 200 **b** 75 **c** 6 **d** 462
e 81

Exercise B2

1 a $\sum_{i=1}^{4} i^3 = 1 + 8 + 27 + 64 = 100$

b $\sum_{k=1}^{5} k^3 = 1 + 8 + 27 + 64 + 125 = 225$

c $\sum_{i=1}^{5} (-1)^i i = -1 + 2 - 3 + 4 - 5 = -3$

d $\frac{1}{5}\sum_{i=1}^{5} i = \frac{1}{5}(1 + 2 + 3 + 4 + 5) = 3$

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e $\sum_{i=1}^{6} i = 1 + 2 + 3 + 4 + 5 + 6 = 21$

f $\sum_{k=1}^{4} (k-1)^2 = 0 + 1 + 4 + 9 = 14$

g $\frac{1}{3}\sum_{i=1}^{4} (i-2)^2 = \frac{1}{3}(1 + 0 + 1 + 4) = 2$

h $\sum_{i=1}^{6} i^2 = 1 + 4 + 9 + 16 + 25 + 36 = 91$

2 a $\sum_{i=1}^{n} i$ **b** $\sum_{i=1}^{11} x_i$ **c** $\frac{1}{10}\sum_{i=1}^{10} x_i$

d $\sum_{i=1}^{n+1} i^4$ **e** $\sum_{i=1}^{5} \frac{1}{i}$

3 a $x + x^2 + x^3 + \cdots + x^n$

b $32 + 16x + 8x^2 + 4x^3 + 2x^4 + x^5$

c $3^6 + (2x) \cdot 3^5 + (2x)^2 \cdot 3^4 + (2x)^3 \cdot 3^3 + (2x)^4 \cdot 3^2 + (2x)^5 \cdot 3 + (2x)^6$

d $1 + (x - x_1) + (x - x_2)^2 + (x - x_3)^3 + (x - x_4)^4$

4 a $\sum_{i=0}^{5} x^{5-i} \cdot 3^i$ **b** $\sum_{i=0}^{5} x^{5-i} \cdot (-3)^i$

c $\sum_{i=0}^{2} x^{2-i} \cdot 2^{2-i}$ **d** $\sum_{i=0}^{3} (2x)^{3-i} \cdot 3^i$

Exercise B3

1 a $x^6 + 36x^5 + 540x^4 + 4320x^3 + 19\,440x^2 + 46\,656x + 46\,656$

b $32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1$

c $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$

d $64x^6 + 576x^5 + 2160x^4 + 4320x^3 + 4860x^2 + 2916x + 729$

e $64x^6 - 1152x^5 + 8640x^4 - 34\,560x^3 + 77\,760x^2 - 93\,312x + 46\,656$

f $16x^4 - 96x^3 + 216x^2 - 216x + 81$

g $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

h $x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 + 210x^4 + 120x^3 + 45x^2 + 10x + 1$

2 a $-960x^3$ **b** $960x^3$

c $-960x^3$ **d** $192\,456x^5$

e $1732\,104x^5$ **f** $-25\,344b^7x^5$

3 $-\frac{16}{243}x^7$ **4** $-336\,798x^6$

5 $(-x + 1)^{11} = -x^{11} + 11x^{10} - 55x^9 + 165x^8 - 330x^7 + 462x^6 - 462x^5 + 330x^4 - 165x^3 + 55x^2 - 11x + 1$

6 a 40 **b** -160 **c** -80

d 181 440 **e** 432 **f** 1080

7 83 026 944 **8** -768

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