Answers

Chapter 1

Exercise 1A

1 **a** 3 **b** 9 **c** 1 **d** -8
e 5 **f** 2 **g** $\frac{5}{3}$ **h** $-\frac{7}{2}$
i $\frac{7}{3}$ **j** $\frac{20}{3}$ **k** $-\frac{10}{3}$ **l** $\frac{14}{5}$

2 **a** $a+b$ **b** $a-b$ **c** $\frac{b}{a}$ **d** ab
e $\frac{bc}{a}$

3 **a** $y=5$ **b** $t=5$ **c** $y=-\frac{3}{2}$ **d** $x=2$
e $a=\frac{11}{2}$ **f** $a=\frac{8}{3}$ **g** $y=136$ **h** $t=1$
i $x=12$ **j** $y=-\frac{9}{5}$ **k** $x=-7$ **l** $y=2$

4 **a** $\frac{4}{3}$ **b** -5 **c** 2

5 **a** -1 **b** 18 **c** $\frac{6}{5}$ **d** 23
e 0 **f** 10 **g** 12 **h** 8
i $-\frac{14}{5}$ **j** $\frac{12}{5}$ **k** $\frac{7}{2}$

6 **a** $-\frac{b}{a}$ **b** $\frac{e-d}{c}$ **c** $\frac{c}{a}-b$ **d** $\frac{b}{c-a}$
e $\frac{ab}{b+a}$ **f** $a+b$ **g** $\frac{b-d}{a-c}$ **h** $\frac{bd-c}{a}$

7 **a** $x=\frac{a^2+b^2+2ab}{ac+bc}=\frac{a+b}{c}$
b $x=\frac{3ab}{2a+b}$

8 **a** -18 **b** -78.2 **c** 16.75 **d** 28
e 34 **f** $\frac{3}{26}$

9 **a** $x=\frac{ab}{a-b-c}$ **b** $x=-\frac{2}{b+1}$

Exercise 1B

1 **a** $x+2=6,\ 4$ **b** $3x=10,\ \frac{10}{3}$
c $3x+6=22,\ \frac{16}{3}$ **d** $3x-5=15,\ \frac{20}{3}$
e $6(x+3)=56,\ \frac{19}{3}$ **f** $\frac{x+5}{4}=23,\ 87$

2 $A=\$8,\ B=\$24,\ C=\$16$
3 14 and 28
4 8 kg
5 1.3775 m^2
6 49, 50, 51
7 17, 19, 21, 23
8 4200 L
9 21
10 3 km
11 9 and 12 dozen
12 7.5 km/h
13 3.6 km
14 30, 6

Exercise 1C

1 **a** $x=-1,\ y=-1$ **b** $x=5,\ y=21$
c $x=-1,\ y=5$ **d** $x=5,\ y=19$
e $x=-4,\ y=-13$ **f** $x=-\frac{8}{5},\ y=-\frac{2}{5}$

2 **a** $x=8,\ y=-2$ **b** $x=-1,\ y=4$
c $x=7,\ y=\frac{1}{2}$

3 **a** $x=2,\ y=-1$ **b** $x=2.5,\ y=-1$
c $m=2,\ n=3$ **d** $x=2,\ y=-1$
e $s=2,\ t=5$ **f** $x=10,\ y=13$

4 **a** No solutions **b** Infinitely many solutions
c One solution **d** One solution

5 **a** $x=\frac{4}{3},\ y=\frac{7}{2}$ **b** $p=1,\ q=-1$
c $x=-1,\ y=\frac{5}{2}$

Exercise 1D

1 25, 113
2 22.5, 13.5
3 **a** \$70 **b** \$12 **c** \$3
4 **a** \$168 **b** \$45 **c** \$15

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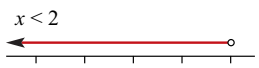
5 17 and 28 **6** 44 and 12
7 5 pizzas, 25 hamburgers
8 Started with 60 and 50; finished with 30 each
9 134 adults, 16 children **10** $\frac{7}{10}$
11 26 **12** 420 adults, 540 children
13 \$17 000 **14** 120 shirts, 300 ties
15 360 Outbacks, 300 Bush Walkers
16 2800 in Mydney, 3200 in Selbourne
17 20 kg at \$10, 40 kg at \$11 and 40 kg at \$12

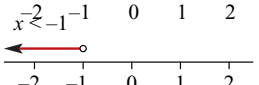
Exercise 1E

1 a $x < 1$ **b** $x > 13$ **c** $x \geq 3$ **d** $x \leq 12$
e $x \leq -6$ **f** $x > 3$ **g** $x < -2$ **h** $x \geq -8$
i $x \leq \frac{3}{2}$

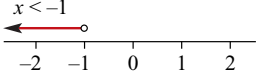
2 a $x < 2$



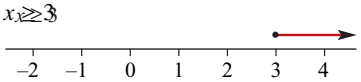
b $x < -1$



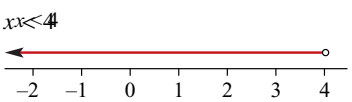
c $x < -1$



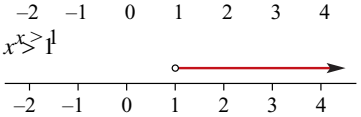
d $x \geq 3$



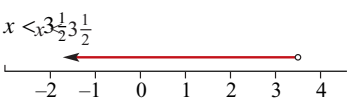
e $x < 4$



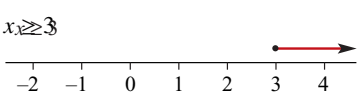
f $x > 1$



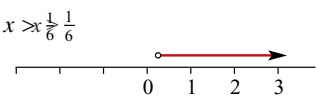
g $x < 3\frac{1}{2}$



h $x \geq 3$



i $x \geq \frac{1}{6}$



3 a $x > -\frac{1}{2}$ **b** $x < 2$ **c** $x > -5$
4 $3x < 20$, $x < \frac{20}{3}$, 6 pages
5 87
6 a $x \geq \frac{44}{21}$ **b** $x \leq \frac{52}{9}$
c $x \geq -\frac{52}{3(2a-5)}$ **d** $x \leq \frac{52}{9a}$

Exercise 1F

1 a 18 **b** 9 **c** 3 **d** −18
e 3 **f** 81 **g** 5 **h** 20

2 a $S = a + b + c$ **b** $P = xy$ **c** $C = 5p$
d $T = dp + cq$ **e** $T = 60a + b$
3 a 15 **b** 31.4 **c** 1000 **d** 12
e 314 **f** 720
4 a $V = \frac{c}{p}$ **b** $a = \frac{F}{m}$ **c** $P = \frac{I}{rt}$
d $r = \frac{w - H}{C}$ **e** $t = \frac{S - P}{Pr}$ **f** $r = \frac{R(V-2)}{V}$
5 a $T = 48$ **b** $b = 8$ **c** $h = 3.82$ **d** $b = 10$
6 a $(4a + 3w)$ m **b** $(h + 2b)$ m
c $3wh$ m^2 **d** $(4ah + 8ab + 6wb)$ m^2
7 a i $T = 2\pi(p + q) + 4h$ **ii** $88\pi + 112$
b $p = \frac{A}{\pi h} - q$
8 a $D = \frac{2}{3}$ **b** $b = 2$ **c** $n = \frac{60}{29}$ **d** $r = 4.8$
9 a $D = \frac{1}{2}bc(1 - k^2)$ **b** $k = \sqrt{1 - \frac{2D}{bc}}$
c $k = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$
10 a $P = 4b$ **b** $A = 2bc - c^2$ **c** $b = \frac{A + c^2}{2c}$
11 a $b = \frac{a^2 - a}{2}$ **b** $x = -\frac{ay}{b}$
c $r = \pm\sqrt{3q - p^2x^2}$ **d** $v = \pm\sqrt{u^2\left(1 - \frac{x^2}{y^2}\right)}$

Chapter 1 review

Technology-free questions

1 a 1 **b** $-\frac{3}{2}$ **c** $-\frac{2}{3}$ **d** −27
e 12 **f** $\frac{44}{13}$ **g** $\frac{1}{8}$ **h** 31
2 a $a - b$ **b** $\frac{cd - b}{a}$ **c** $\frac{d}{a} + c$
d $\frac{cb - a}{c - 1}$ **e** $\frac{2b}{c - a}$ **f** $\frac{1 - cd}{ad}$
3 a $x < \frac{2}{3}$ **b** $x \leq -148\frac{1}{2}$ **c** $x < \frac{22}{29}$ **d** $x \geq -\frac{7}{17}$
4 $x = 2(z + 3t)$, -10
5 a $d = e^2 + 2f$ **b** $f = \frac{d - e^2}{2}$ **c** $f = \frac{1}{2}$
6 400π cm^2
7 a 196π **b** $\frac{975\pi}{2}$
8 a $r = \frac{A}{\pi s}$ **b** $w = \frac{T - P}{Pr}$
c $r = \frac{n - p}{v^2}$ **d** $x = \frac{ac - b^2}{b}$
9 a $s = 75$ **b** $t = 8$
10 $5\sqrt{2}$ cm
11 12 m and 17 m
12 $m = 2$ and $n = 15$

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13 Mr Apollo earns \$100 000, Mr Adonis earns \$107 200 and Ms Aphrodite earns \$96 000

14 **a** $a = \frac{28}{11}, b = -\frac{9}{11}$ **b** $a = -\frac{11}{5}, b = -\frac{33}{5}$

15 5 hours travelling on highways

Multiple-choice questions

1 D **2** D **3** C **4** A **5** C **6** C
7 B **8** B **9** A **10** B **11** E **12** B

Extended-response questions

1 **a** $C = -\frac{10}{9}$ **b** $F = 86$ **c** $x = -40$
d $x = -62.5$ **e** $x = -\frac{160}{13}$ **f** $k = 5$

2 **a** $r = \frac{2uv}{u+v}$ **b** $m = \frac{v}{u}$

3 **a** $T = 6w + 6\ell$
b **i** $T = 8w$ **ii** $\ell = \frac{25}{6}, w = 12\frac{1}{2}$
c **i** $y = \frac{L - 6x}{8}$ **ii** $y = 22$
d $x = 10, y = 5$

4 **a** Distance Tom travelled = ut km
Distance Julie travelled = vt km
b **i** $t = \frac{d}{u+v}$ h
ii Distance from $A = \frac{ud}{u+v}$ km
c $t = 1.25$ h, distance from $A = 37.5$ km

5 **a** Average speed = $\frac{2uv}{u+v}$
b **i** $\frac{uT}{v}$ **ii** $\frac{vT + uT}{v}$

6 **a** $\frac{3}{a} + \frac{3}{b}$ **c** **i** $c = \frac{2ab}{a+b}$ **ii** $\frac{40}{3}$

7 **a** $\frac{x}{8}, \frac{y}{10}$ **b** $\frac{80(x+y)}{10x+8y} = \frac{560}{5x+4y}$
c $x = \frac{320}{9}, y = \frac{310}{9}$

8 The three lines intersect at the point (4, 3)

Chapter 2

Exercise 2A

1 **a** (5, 8) **b** $(\frac{1}{2}, \frac{1}{2})$ **c** (1.6, 0.7)
d (−0.7, 0.85)

2 $M_{AB}(3, 3)$, $M_{BC}(8, 3\frac{1}{2})$, $M_{AC}(6, 1\frac{1}{2})$

3 $C(6, 8.8)$

4 **a** (4, 4) **b** (2, −0.2) **c** (−2, 5) **d** (−4, −3)

5 $\left(\frac{1+a}{2}, \frac{4+b}{2}\right)$, $a = 9$, $b = -6$

6 **a** $5\sqrt{2} \approx 7.07$ **b** $\sqrt{17} \approx 4.12$
c $\sqrt{34} \approx 5.83$ **d** 13

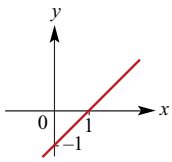
7 $\sqrt{97} + \sqrt{85} + \sqrt{104} \approx 29.27$

8 $PM = \sqrt{145} \approx 12.04$ **9** DN

Exercise 2B

1 **a** 4 **b** 2 **c** $\frac{1}{4}$ **d** −4 **e** 1 **f** −1
g $\frac{5}{4}$ **h** −2 **i** $-\frac{5}{4}$ **j** $\frac{4}{3}$ **k** 0

2 Any line parallel to the one shown
$y = x - 1$



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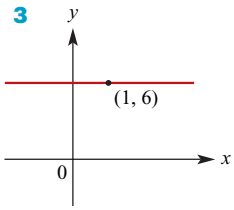

4 **a** $-\frac{1}{4}$ **b** $-\frac{5}{2}$ **c** −2 **d** −8 **e** 0 **f** −1
g 7 **h** 11 **i** −13 **j** 11 **k** 111 **l** 61

5 **a** −2 **b** $\frac{2}{5}$

6 **a** 54 **b** $\frac{5}{6}$

7 **a** 45° **b** 45° **c** 26.57° **d** 135°

8 **a** 45° **b** 26.57° **c** 161.57° **d** 49.4°
e 161.57° **f** 135°

9 **a** 1 **b** −1 **c** $\sqrt{3}$ **d** $-\sqrt{3}$

Exercise 2C

1 **a** $m = 3, c = 6$ **b** $m = -6, c = 7$
c $m = 3, c = -6$ **d** $m = -1, c = -4$

2 **a** $y = 3x + 5$ **b** $y = -4x + 6$
c $y = 3x - 4$

3 **a** $m = 3, c = -6$ **b** $m = 2, c = -4$
c $m = \frac{1}{2}, c = -2$ **d** $m = \frac{1}{3}, c = -\frac{5}{3}$

4 **a** $m = 2, c = -9$ **b** $m = -\frac{3}{4}, c = \frac{5}{2}$
c $m = -\frac{1}{3}, c = -2$ **d** $m = \frac{5}{2}, c = -2$

5 **a** $y = 3x - 11$ **b** $y = -2x + 9$

6 **a** $y = -\frac{1}{3}x + \frac{11}{3}$ **b** $y = -\frac{7}{5}x + 4$
c $y = -2x + 4$ **d** $y = \frac{11}{3}x - \frac{61}{3}$

7 **a** 2 **b** $y = 2x + 6$

8 **a** $y = 2x + 4$ **b** $y = -2x + 8$

9 **a** $y = 2x + 6$ **b** $y = -2x + 4$
c $y = -5x + 15$

10 **a** $y = -\frac{2}{3}x + 4$ **b** $y = -2x - 6$
c $y = -x + 4$ **d** $y = -\frac{3}{2}x + 3$

11 **a** $y = \frac{2}{3}x + 4$ **b** $y = \frac{2}{3}x - \frac{2}{3}$
c $y = \frac{1}{2}x + 1\frac{1}{2}$ **d** $y = -\frac{1}{2}x + 2$
e $y = x + 3.5$ **f** $y = -0.5x + 0.25$

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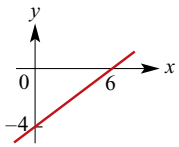
12 a $y = 4x + 4$ **b** $y = -\frac{2}{3}x$ **c** $y = -x - 2$
d $y = \frac{1}{2}x - 1$ **e** $y = 3\frac{1}{2}$ **f** $x = -2$

13 Yes **14** Only c

15 a $x = 4$ **b** $y = 11$ **c** $x = 11$ **d** $y = -1$

Exercise 2D

1 a (0, 4), (4, 0) **b** (0, −4), (4, 0)
c (0, −6), (−6, 0) **d** (0, 8), (−8, 0)

2 a


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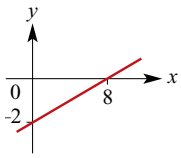

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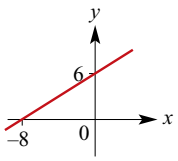

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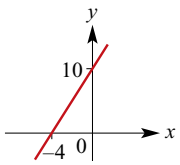

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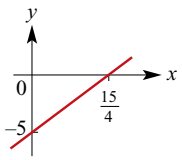

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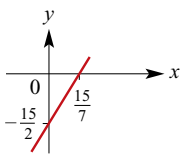

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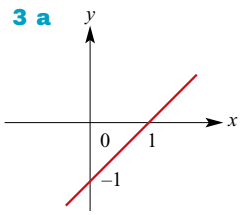

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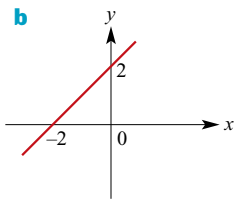

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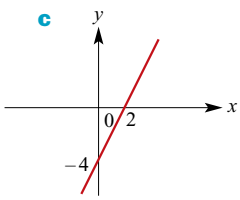

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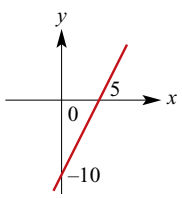

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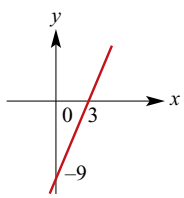

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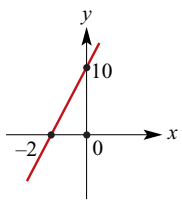

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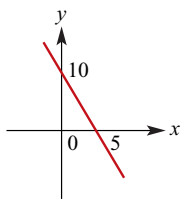

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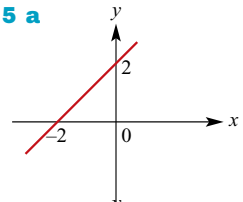

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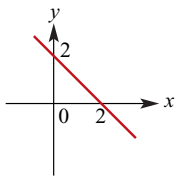

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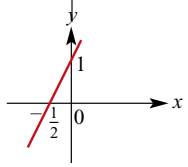

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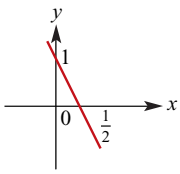

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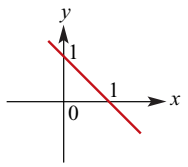

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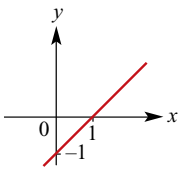

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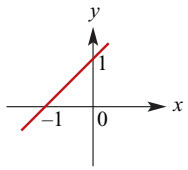

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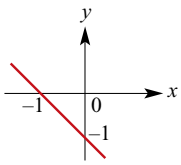

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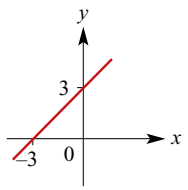

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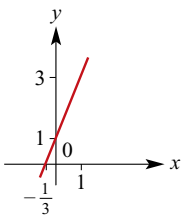

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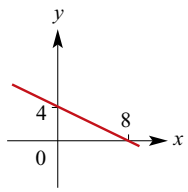

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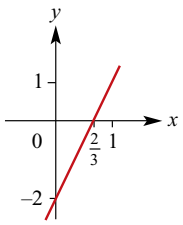

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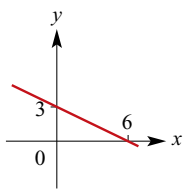

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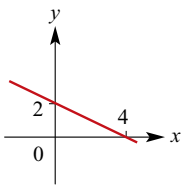

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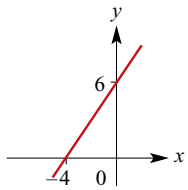

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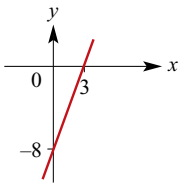

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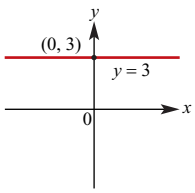
8 a



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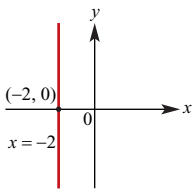

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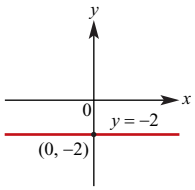

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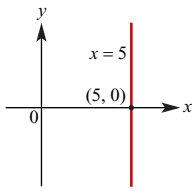
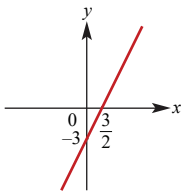


9 a 45° **b** 135° **c** 45° **d** 135°
e 63.43° **f** 116.57°

10 a 71.57° **b** 135° **c** 45° **d** 161.57°

11 $a = -4, b = \frac{4}{3}, d = -1, e = \frac{14}{3}$

12 a



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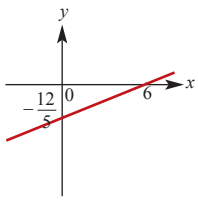

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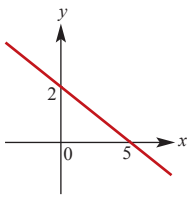

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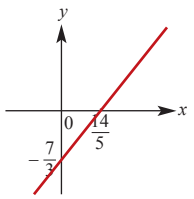


Exercise 2E

1 a $y = 2x - 10$ **b** $y = -\frac{1}{2}x$

c $y = -2x + 6$ **d** $y = \frac{1}{2}x - 4$

e $y = \frac{2}{3}x - \frac{14}{3}$ **f** $y = -\frac{3}{2}x + 4$

g $y = -\frac{1}{3}x - \frac{2}{3}$ **h** $y = 3x - 14$

2 Parallel lines: a, b, c; non-parallel lines: d

3 a $y = 4$ **b** $x = 2$ **c** $y = 4$ **d** $x = 3$

4 $y = 2x + 2$

5 $M(-1, 6)$, $y = 2x + 8$

6 $m_{BC} = -\frac{3}{5}$, $m_{AB} = \frac{5}{3}$

$\therefore m_{BC} \times m_{AB} = -\frac{3}{5} \times \frac{5}{3} = -1$

$\therefore \triangle ABC$ is a right-angled triangle

7 $m_{AB} = -2, m_{BC} = \frac{1}{2}$

8 $m_{RS} = -\frac{1}{2}, m_{ST} = 2 \therefore RS \perp ST$

$m_{UT} = -\frac{1}{2}, m_{ST} = 2 \therefore UT \perp ST$

(Also need to show $SR = UT$.)

$\therefore RSTU$ is a rectangle

9 $\ell = -\frac{16}{3}, m = \frac{80}{3}$

10 a $y = -\frac{1}{2}x + \frac{11}{2}$ **b** $B(1, 5)$ **c** $C(2, 7)$

Exercise 2F

1 $m = 5$

2 $c = 5$

3 a $y = -\frac{1}{m}x + 3$ **b** $m = \frac{1}{7}$

4 $m = 2$

5 a $x = \frac{3}{m}$ **b** $m = \frac{9}{5}$ **c** $m \geq 3$

d $y = -\frac{1}{m}x - 3$

6 a $x = -\frac{c}{2}$ **b** $c = -4$ **c** $c \geq -2$

d $y = -\frac{1}{2}x + c$

7 a $x = 4a$ **b** $m = \frac{12}{a}$ **c i** $a = 6$ **ii** $a = -6$

8 a $\frac{12}{b}$ **b** $-\frac{3}{b}$ **c i** $b = -3$ **ii** $b = \frac{3}{2}$

d $y = \frac{b}{3}x - \frac{4b}{3}$

9 a $\frac{c}{b}$

b i $b = c - 7$ **ii** $c \geq 14$

c i $c = 12, b = 2$ **ii** $c = 2\sqrt{2b}$

iii $c = \frac{10b}{\sqrt{b^2 + 1}}$

$c = 9.805\ldots, 9.950\ldots, 9.999\ldots$

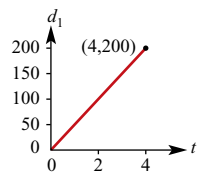
As b gets larger, c gets closer to 10

Exercise 2G

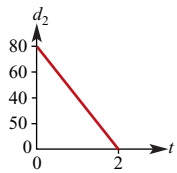
1 $w = 20n + 350$ for $n = 0, 1, 2, \ldots$

2 a $d_1 = 50t$ **b** $d_2 = 80 - 40t$

c Gradient = 50



Gradient = −40



3 a $V = 5t$

b $V = 10 + 5t$

4 a $v = 500 - 2.5t$

b Domain $0 \leq t \leq 200$; Range $0 \leq v \leq 500$

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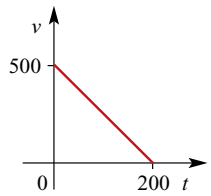
c

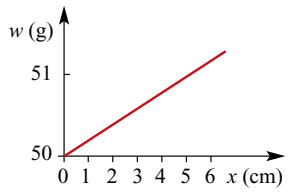


5 $C = 1.5n + 2.6$

6 **a** $C = 0.24x + 85$ **b** \$145

7 $d = 200 - 5t$

8 **a**



b $w = 0.2x + 50$ **c** $x = 12.5$ cm

9 **a** $C = 0.06n - 1$ **b** \$59

10 **a** $C = 5n + 175$ **b** Yes **c** \$175

Exercise 2H

1 Both lines have gradient -1, but their y-axis intercepts are 6 and $\frac{13}{2}$

2 $(t, 6 - t)$, where t is a real number

3 **a** $m = 4$ **b** $m \neq 4$ **c** $m = \frac{9}{5}$

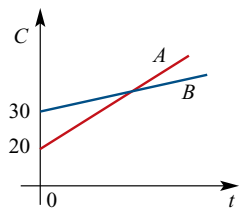
4 $k = 2, m = 5$

5 $k = 24, m = 0$

6 $m = -3$

7 **a** $m = -5$ **b** $m = 3$

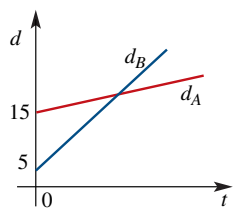
8 **a**



b $t = 5$

9 $b = 0.28$ and $a = 0.3$, $\frac{25}{7}$ m/s

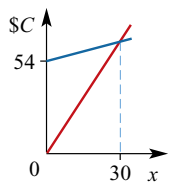
10 **a**



b 2 p.m.

11 **a** $C_T = 2.8x$, $C_B = 54 + x$

b

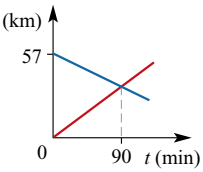


c More than 30

12 **a** $d_A = \frac{1}{3}t$

$d_M = 57 - \frac{3}{10}t$

b



c 10:30 a.m.

d Anne 30 km, Maureen 27 km

Chapter 2 review

Technology-free questions

1 **a** Midpoint = $(3, 2)$; Length = 4

b Midpoint = $\left(-\frac{1}{2}, -\frac{9}{2}\right)$; Length = $\sqrt{74}$

c Midpoint = $\left(5, \frac{5}{2}\right)$; Length = 5

2 **a** $\frac{9}{4}$ **b** $-\frac{10}{11}$ **c** Undefined

d -1 **e** $\frac{b}{a}$ **f** $-\frac{b}{a}$

3 **a** $y = 4x$ **b** $y = 4x + 5$

c $y = 4x + 2$ **d** $y = 4x - 5$

4 **a** $a = -2$ **b** $b = \frac{20}{3}$

5 $4y + 3x = -7$

6 $3y + 2x = -5$

7 **a** $y = 11$ **b** $y = 6x - 10$ **c** $3y + 2x = -3$

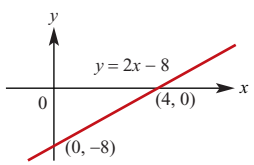
8 $\sqrt{3}y - x = 3\sqrt{3} - 2$

9 $y + x = 1$

10 $y = \frac{1}{3}x + \frac{20}{3}$

11 $a = 1, b = -\frac{1}{2}, d = 5, e = 3$

12 **a**



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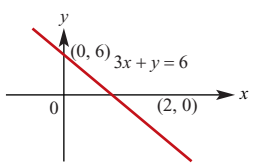

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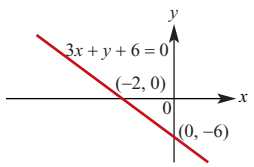

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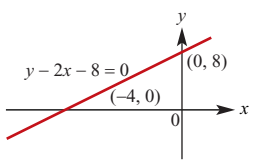

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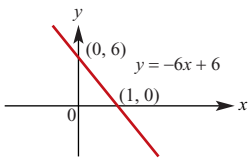

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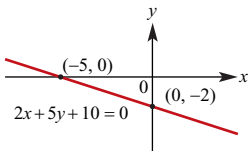


13 a $y = 2x + 2$

b i $-\frac{2}{a}$ **ii** $-2 < a < 0$

c $\left(\frac{1}{a-1}, \frac{1}{a-1} + 3\right)$

Multiple-choice questions

1 D **2** E **3** A **4** E **5** C **6** D
7 B **8** E **9** C **10** E **11** C

Extended-response questions

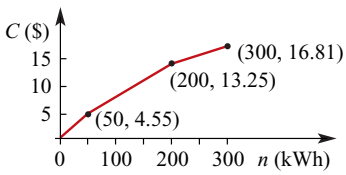
1 a $C = 550 + 190n$ **b** 12 days
c Fewer than 5 days

2 a Cost of the plug
b Cost per metre of cable **c** 1.8 **d** $11\frac{1}{9}$ m

3 a The maximum profit (when $x = 0$)
b 43 seats
c Profit reduces by \$24 for each empty seat

4 a i $C = 0.091n$ **ii** $C = 1.65 + 0.058n$
iii $C = 6.13 + 0.0356n$

b



i For 30 kWh, $C = 2.73$
ii For 90 kWh, $C = 6.87$
iii For 300 kWh, $C = 16.81$

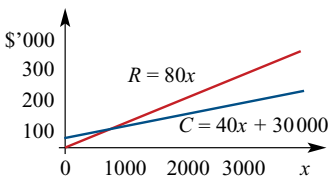
c 389.61 kWh

5 a $y = -\frac{7}{3}x + 14\frac{2}{3}$ **b** $20\frac{1}{3}$ km south

6 a No **b** $1\frac{41}{71}$ km to the east of H

7 a $C = 40x + 30\,000$ **b** \$45 **c** 5000
d $R = 80x$

e



f 751 **g** $P = 40x - 30\,000$

8 a Cost with method 1 is \$226.75; cost with method 2 is \$227; so method 1 is cheaper

b

	0	1000	2000	3000
Method 1	100	181.25	262.50	343.75
Method 2	110	185	260	335

c $C_1 = 0.08125x + 100$, $C_2 = 0.075x + 110$
d $x = 1600$

Chapter 3

Exercise 3A

1 a $2x - 8$ **b** $-2x + 8$ **c** $6x - 12$
d $-12 + 6x$ **e** $x^2 - x$ **f** $2x^2 - 10x$

2 a $6x + 1$ **b** $3x - 6$ **c** $x + 1$ **d** $5x - 3$

3 a $14x - 32$ **b** $2x^2 - 11x$
c $32 - 16x$ **d** $6x - 11$

4 a $2x^2 - 11x$ **b** $3x^2 - 15x$
c $-20x - 6x^2$ **d** $6x - 9x^2 + 6x^3$
e $2x^2 - x$ **f** $6x - 6$

5 a $6x^2 - 2x - 28$ **b** $x^2 - 22x + 120$
c $36x^2 - 4$ **d** $8x^2 - 22x + 15$
e $x^2 - (\sqrt{3} + 2)x + 2\sqrt{3}$
f $2x^2 + \sqrt{5}x - 5$ **g** $3x^2 + \sqrt{7}x - 14$
h $5x^2 + (10\sqrt{2} - 3)x - 6\sqrt{2}$
i $5x^2 - (3\sqrt{5} + 32\sqrt{10})x + 96\sqrt{2}$

6 a $6x^3 - 5x^2 - 14x + 12$ **b** $x^3 - 1$
c $24 - 20x - 8x^2 + 6x^3$ **d** $3x^2 + 4x + 3$
e $-10x^2 + 5x - 2$

7 a $x^2 - 8x + 16$ **b** $4x^2 - 12x + 9$
c $36 - 24x + 4x^2$ **d** $x^2 - x + \frac{1}{4}$
e $x^2 - 2\sqrt{5}x + 5$ **f** $x^2 - 4\sqrt{3}x + 12$

8 a $x^2 - 9$ **b** $4x^2 - 16$ **c** $81x^2 - 121$
d $4x^2 - 9$ **e** $4x^2 - 25$ **f** $x^2 - 5$
g $4x^2 - 27$ **h** $3x^2 - 7$

9 a $x^2 + y^2 - z^2 - 2xy$ **b** $4a^2 - 4ab + b^2 - c^2$
c $9w^2 + 8uz - 16z^2 - u^2$
d $4a^2 - 5b^2 + 4ac + c^2$

10 a i $x^2 + 2x + 1$ **ii** $(x + 1)^2$
b i $(x - 1)^2 + 2(x - 1) + 1$ **ii** x^2

11 a $4x^2 - 3$ **b** $\sqrt{21}x^2 - (5\sqrt{7} + 2\sqrt{3})x + 10$
c $10x^3 - x^2 - 28x + 15$
d $a^2 - 3ab + 2b^2 + bc - c^2$
e $a^2 - 4ab + 2ac + 4b^2 - 4bc + c^2$
f $a^3 + a^2b + a^2c - ab^2 - b^3 - b^2c$

Exercise 3B

1 a $2(x + 2)$ **b** $4(a - 2)$ **c** $3(2 - x)$
d $2(x - 5)$ **e** $6(3x + 2)$ **f** $8(3 - 2x)$

2 a $2x(2x - y)$ **b** $8x(a + 4y)$ **c** $6b(a - 2)$
d $2xy(3 + 7x)$ **e** $x(x + 2)$ **f** $5x(x - 3)$
g $-4x(x + 4)$ **h** $7x(1 + 7x)$ **i** $x(2 - x)$

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3 **a** $6x^2y^2(x+2)$ **b** $xy(7x-6y)$
c $2xy^2(4x+3)$

4 **a** $(x^2+1)(x+5)$ **b** $(x+3)(y+2)$
c $(x-1)(x+1)(y-1)(y+1)$
d $(a+b)(x+y)$ **e** $(a^2+1)(a-3)$
f $(2a-5)(b-6)$ **g** $(2x+5)(x-1)$
h $(x+2)^2(x-2)$ **i** $(x-a)(x+a)(x-b)$

5 **a** $(x-6)(x+6)$ **b** $(x-9)(x+9)$
c $(x-a)(x+a)$ **d** $(2x-9)(2x+9)$
e $(3x-4)(3x+4)$ **f** $(5x-y)(5x+y)$
g $3(x-4)(x+4)$ **h** $2(x-7)(x+7)$
i $3a(x-3)(x+3)$ **j** $(a-\sqrt{7})(a+\sqrt{7})$
k $(\sqrt{2}a-\sqrt{5})(\sqrt{2}a+\sqrt{5})$
l $(x-2\sqrt{3})(x+2\sqrt{3})$

6 **a** $(x-6)(x+2)$ **b** $(7+x)(3-x)$
c $3(x-1)(x+3)$ **d** $-5(2x+1)$
e $-24x$ **f** $-5(x+7)(x+1)$

7 **a** $(x-9)(x+2)$ **b** $(y-16)(y-3)$
c $(a-2)(a-12)$ **d** $(a+9)^2$
e $(x-8)(x+3)$ **f** $(x-12)(x+10)$

8 **a** $(3x-1)(x-2)$ **b** $(2x+1)(3x+2)$
c $(5x+3)(x+4)$ **d** $(2x+1)(x+4)$
e $(3x-2)(2x-5)$ **f** $(3x+1)(2x-3)$
g $(3x-2)(4x-3)$ **h** $(x-2)(5x+6)$
i $x(5x-6)(x-2)$

9 **a** $3(y-6)(y+2)$ **b** $2(x-7)(x-2)$
c $4(x-3)(x-6)$ **d** $3(x+2)(x+3)$
e $a(x+3)(x+4)$ **f** $3x(4-x)^2$

10 **a** $x(x+2)$ **b** $(2x-3)(x+2)$
c $2(2x+5)(x+2)$

11 **a** $(3-2x)(2x-1)$ **b** $(3x-4)(3x-2)$
c $(2x+5)(3x-4)$ **d** $(x+7)(2x-3)$
e $(x+7)(2x+3)$ **f** $(a+2)(3a-2)$

Exercise 3C

1 **a** 2 or 3 **b** 0 or 2 **c** 4 or 3 **d** 4 or 3
e 3 or −4 **f** 0 or 1 **g** $\frac{5}{2}$ or 6 **h** −4 or 4

2 **a** −0.65, 4.65 **b** −0.58, 2.58 **c** −2.58, 0.58

3 **a** 9, −8 **b** 4, 2 **c** 11, −3 **d** 4, −16
e 2, −7 **f** −3, 8

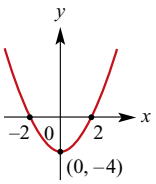
4 **a** $-\frac{3}{2}, -1$ **b** $\frac{1}{2}, \frac{3}{2}$ **c** $-\frac{2}{3}, -\frac{3}{2}$ **d** $-\frac{3}{2}, 2$
e $\frac{5}{6}, 3$ **f** $-\frac{3}{2}, 3$ **g** $\frac{1}{2}, \frac{3}{5}$ **h** $-\frac{3}{4}, \frac{2}{3}$
i $\frac{1}{2}$ **j** −5, 1 **k** 0, 3 **l** $\frac{1}{5}, 2$

5 **a** $-3, -\frac{1}{2}$ **b** $\frac{6}{5}, \frac{5}{3}$ **c** $-\frac{1}{2}, \frac{4}{3}$ **d** $\frac{1}{2}, 1$
e $-\frac{1}{2}, \frac{7}{2}$ **f** $\pm\frac{\sqrt{30}}{2}$

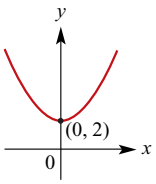
6 3 **7** 4 or 9 **8** $2, 2\frac{3}{8}$
9 13 **10** 50 **11** 6 cm, 2 cm
12 5 cm **13** \$90, \$60 **14** 42

Exercise 3D

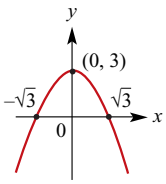
1 **a** **i** (0, −4)
ii $x = 0$
iii (−2, 0), (2, 0)



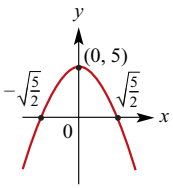
b **i** (0, 2)
ii $x = 0$
iii None



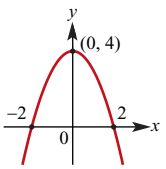
c **i** (0, 3)
ii $x = 0$
iii $(-\sqrt{3}, 0), (\sqrt{3}, 0)$



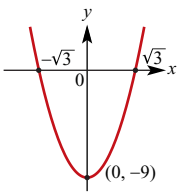
d **i** (0, 5)
ii $x = 0$
iii $\left(-\sqrt{\frac{5}{2}}, 0\right), \left(\sqrt{\frac{5}{2}}, 0\right)$



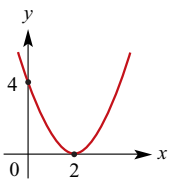
e **i** (0, 4)
ii $x = 0$
iii (−2, 0), (2, 0)



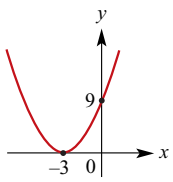
f **i** (0, −9)
ii $x = 0$
iii $(-\sqrt{3}, 0), (\sqrt{3}, 0)$



2 **a** **i** (2, 0)
ii $x = 2$
iii (2, 0)



b **i** (−3, 0)
ii $x = -3$
iii (−3, 0)



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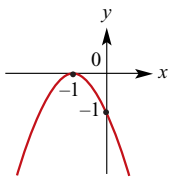
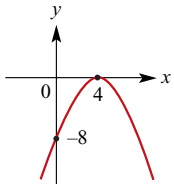
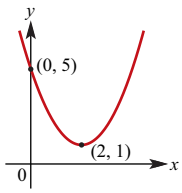
c **i** $(-1, 0)$
ii $x = -1$
iii $(-1, 0)$



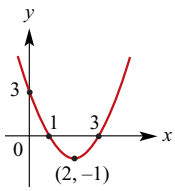
d **i** $(4, 0)$
ii $x = 4$
iii $(4, 0)$



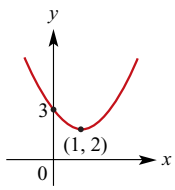
3 a **i** $(2, 1)$
ii $x = 2$
iii None



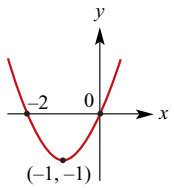
b **i** $(2, -1)$
ii $x = 2$
iii $(1, 0), (3, 0)$



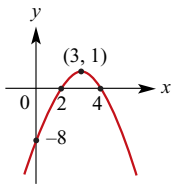
c **i** $(1, 2)$
ii $x = 1$
iii None



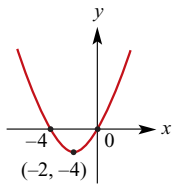
d **i** $(-1, -1)$
ii $x = -1$
iii $(-2, 0), (0, 0)$



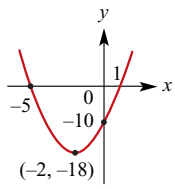
e **i** $(3, 1)$
ii $x = 3$
iii $(2, 0), (4, 0)$



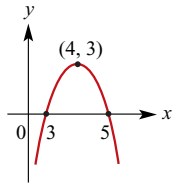
f **i** $(-2, -4)$
ii $x = -2$
iii $(-4, 0), (0, 0)$



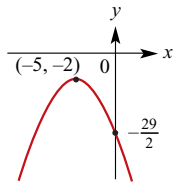
g **i** $(-2, -18)$
ii $x = -2$
iii $(-5, 0), (1, 0)$



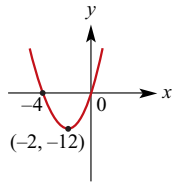
h **i** $(4, 3)$
ii $x = 4$
iii $(3, 0), (5, 0)$



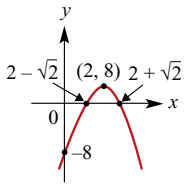
i **i** $(-5, -2)$
ii $x = -5$
iii None



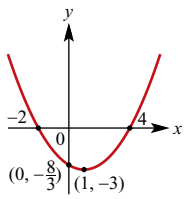
j **i** $(-2, -12)$
ii $x = -2$
iii $(0, 0), (-4, 0)$



k **i** $(2, 8)$
ii $x = 2$
iii $(2 - \sqrt{2}, 0)$, $(2 + \sqrt{2}, 0)$



l **i** $(1, -3)$
ii $x = 1$
iii $(-2, 0), (4, 0)$



Exercise 3E

1 a $x^2 - 2x + 1$ **b** $x^2 + 4x + 4$
c $x^2 - 6x + 9$ **d** $x^2 - 6x + 9$
e $x^2 + 4x + 4$ **f** $x^2 - 10x + 25$
g $x^2 - x + \frac{1}{4}$ **h** $x^2 - 3x + \frac{9}{4}$

2 a $(x - 2)^2$ **b** $(x - 6)^2$ **c** $-(x - 2)^2$
d $2(x - 2)^2$ **e** $-2(x - 3)^2$ **f** $\left(x - \frac{1}{2}\right)^2$
g $\left(x - \frac{3}{2}\right)^2$ **h** $\left(x + \frac{5}{2}\right)^2$

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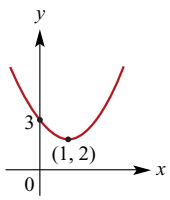
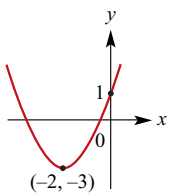
3 a $1 \pm \sqrt{2}$ **b** $2 \pm \sqrt{6}$ **c** $3 \pm \sqrt{7}$
d $\frac{5 \pm \sqrt{17}}{2}$ **e** $\frac{2 \pm \sqrt{2}}{2}$ **f** $-\frac{1}{3}, 2$
g $-1 \pm \sqrt{1-k}$ **h** $\frac{-1 \pm \sqrt{1-k^2}}{k}$
i $\frac{3k \pm \sqrt{9k^2-4}}{2}$

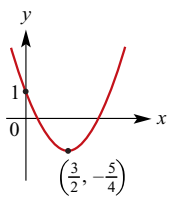
4 a $y = (x-1)^2 + 2$
t. pt $(1, 2)$



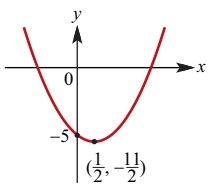
b $y = (x+2)^2 - 3$
t. pt $(-2, -3)$



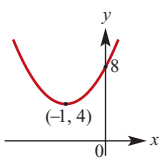
c $y = \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$
t. pt $\left(\frac{3}{2}, -\frac{5}{4}\right)$



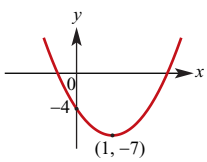
5 a $y = 2\left(x - \frac{1}{2}\right)^2 - \frac{11}{2}$
t. pt $\left(\frac{1}{2}, -\frac{11}{2}\right)$



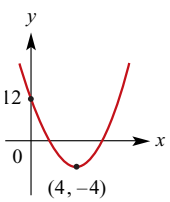
b $y = 4(x+1)^2 + 4$
t. pt $(-1, 4)$



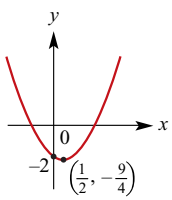
c $y = 3(x-1)^2 - 7$
t. pt $(1, -7)$



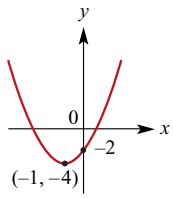
6 a $y = (x-4)^2 - 4$
t. pt $(4, -4)$



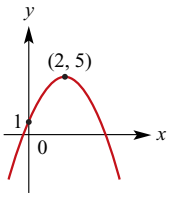
b $y = \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$
t. pt $\left(\frac{1}{2}, -\frac{9}{4}\right)$



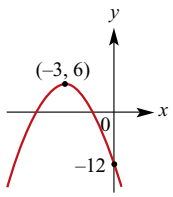
c $y = 2(x+1)^2 - 4$
t. pt $(-1, -4)$



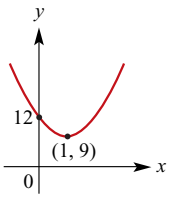
d $y = -(x-2)^2 + 5$
t. pt $(2, 5)$



e $y = -2(x+3)^2 + 6$
t. pt $(-3, 6)$



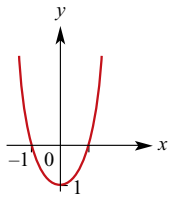
f $y = 3(x-1)^2 + 9$
t. pt $(1, 9)$



7 a $2(x+2)^2 - 5$ **b** $3(x+2)^2 - 14$
c $3\left(x - \frac{5k}{6}\right)^2 - \frac{25k^2 + 84}{12}$

Exercise 3F

1 a 7 **b** 7 **c** 1
2 a −2 **b** 8 **c** 4
3 a



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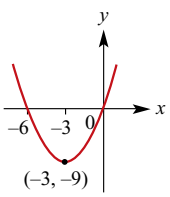

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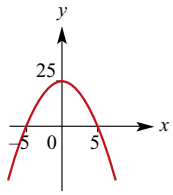

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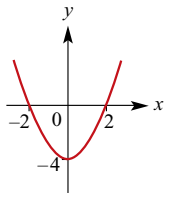

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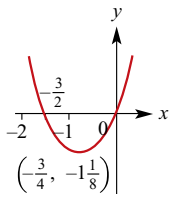
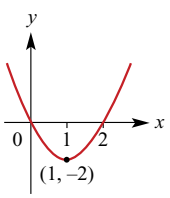


f



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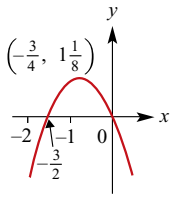

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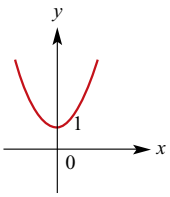

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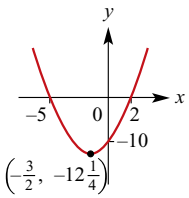
4 a



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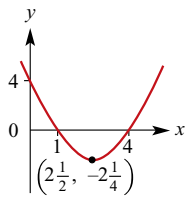

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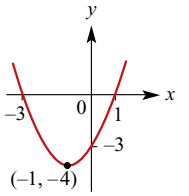

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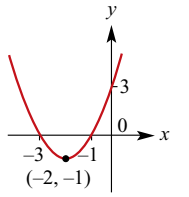

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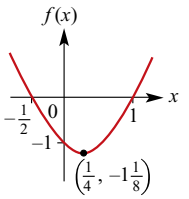

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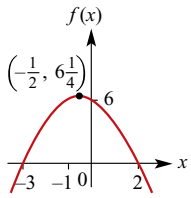

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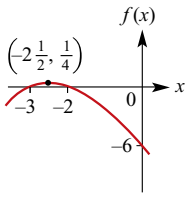

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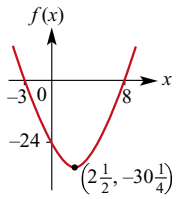

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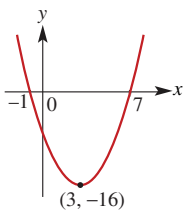

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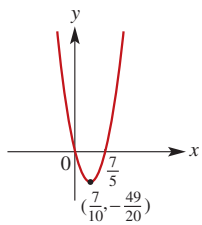

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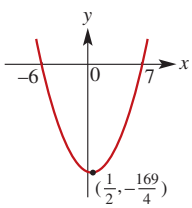
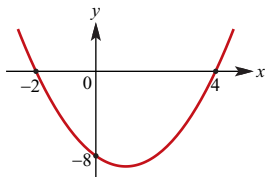


Exercise 3G

1 a $-2, 4$ **b**



c $-2 \le x \le 4$ **d** $x < -2$ or $x > 4$

2 a $x \le -2$ or $x \ge 3$ **b** $-4 < x < -3$

c $-4 \le x \le \frac{1}{2}$ **d** $x < 2$ or $x > 6$

e $2 < x < 3$ **f** $\frac{3}{2} \le x \le \frac{7}{2}$

g $-\frac{7}{2} < x < 2$ **h** $-2 \le x \le \frac{5}{2}$

i $x < -5$ or $x > \frac{5}{2}$

3 a $x < -5$ or $x > 5$ **b** $-\frac{2}{3} \le y \le \frac{2}{3}$

c $y < -4$ or $y > 4$ **d** $-\frac{6}{5} \le x \le \frac{6}{5}$

e $y \le -\frac{1}{4}$ or $y \ge \frac{1}{4}$ **f** $y < -\frac{5}{6}$ or $y > \frac{5}{6}$

4 a $x \ge 2$ or $x \le -4$ **b** $-3 < x < 8$

c $-2 \le x \le 6$ **d** $x > 3$ or $x < -\frac{3}{2}$

e $-\frac{3}{2} < x < -\frac{2}{3}$ **f** $-3 \le x \le -2$

g $x > \frac{2}{3}$ or $x < -\frac{3}{4}$ **h** $\frac{1}{2} \le x \le \frac{3}{5}$

i $-4 \le x \le 5$

j $\frac{1}{2}(5 - \sqrt{41}) \le p \le \frac{1}{2}(5 + \sqrt{41})$

k $y < -1$ or $y > 3$ **l** $x \le -2$ or $x \ge -1$

5 a $x \le \frac{-3 - \sqrt{29}}{2}$ or $x \ge \frac{-3 + \sqrt{29}}{2}$

b $\frac{5 - \sqrt{17}}{2} < x < \frac{5 + \sqrt{17}}{2}$

c $\frac{3 - \sqrt{17}}{4} \le x \le \frac{3 + \sqrt{17}}{4}$

d $\frac{-3 - \sqrt{41}}{2} < x < \frac{-3 + \sqrt{41}}{2}$

e $\frac{-7 - \sqrt{41}}{4} < x < \frac{-7 + \sqrt{41}}{4}$

f $x \le \frac{4 - \sqrt{6}}{2}$ or $x \ge \frac{4 + \sqrt{6}}{2}$

6 The square of any number is greater than or equal to zero.

7 The negative of the square of any number is less than or equal to zero.

8 $x^2 + 2x + 7 = (x + 1)^2 + 6$. For all x, we have $(x + 1)^2 \ge 0$ and so $(x + 1)^2 + 6 \ge 6$

9 $-x^2 - 2x - 7 = -(x + 1)^2 - 6$. For all x, we have $-(x + 1)^2 \le 0$ and so $-(x + 1)^2 - 6 \le -6$

10 a $x \le -\frac{13}{3}$ or $x \ge 0$ **b** $x < -3$ or $x > \frac{1}{3}$

c $x \le -3$ or $x \ge -1$

Exercise 3H

1 a **i** 40 **ii** $2\sqrt{10}$

b **i** 28 **ii** $2\sqrt{7}$

c **i** 172 **ii** $2\sqrt{43}$

d **i** 96 **ii** $4\sqrt{6}$

e **i** 189 **ii** $3\sqrt{21}$

2 a $1 + \sqrt{5}$ **b** $\frac{3 - \sqrt{5}}{2}$ **c** $\frac{1 + \sqrt{5}}{2}$ **d** $1 + 2\sqrt{2}$

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3 a $-3 \pm \sqrt{13}$ **b** $\frac{7 \pm \sqrt{61}}{2}$ **c** $\frac{1}{2}, 2$

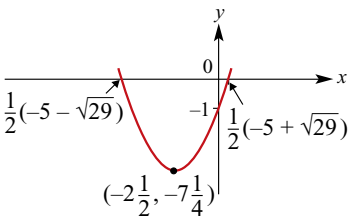
d $-1 \pm \frac{3}{2}\sqrt{2}$ **e** $-2 \pm \frac{3}{2}\sqrt{2}$ **f** $1 \pm \frac{\sqrt{30}}{5}$

g $1 \pm \frac{\sqrt{2}}{2}$ **h** $1, -\frac{3}{2}$ **i** $\frac{-3 \pm \sqrt{6}}{5}$

j $\frac{-13 \pm \sqrt{145}}{12}$ **k** $\frac{2 \pm \sqrt{4 - 2k^2}}{2k}$

l $\frac{2k \pm \sqrt{6k^2 - 2k}}{2(1-k)}$

4 a



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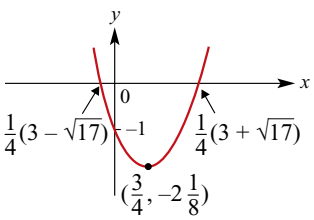

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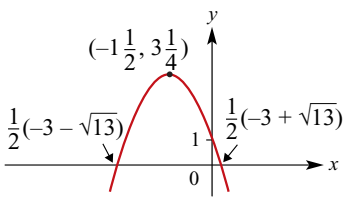

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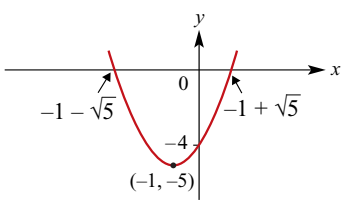

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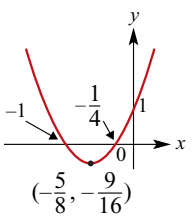

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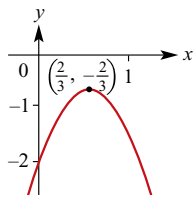

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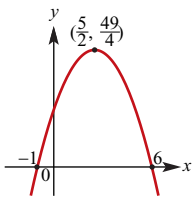

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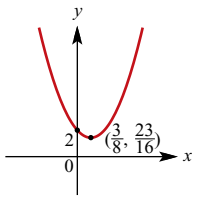

5 a $x = \frac{k \pm \sqrt{k(k+4)}}{2}$ **b** $x = \frac{1 \pm \sqrt{1 - 4k^2}}{2k}$

c $x = \frac{1 \pm \sqrt{1 - 4k^4}}{2k^2}$

Exercise 3I

1 a 20 **b** −12 **c** 25 **d** 41 **e** 41

2 a Crosses the x-axis **b** Does not cross
c Just touches the x-axis
d Crosses the x-axis **e** Does not cross
f Does not cross

3 a Two real solutions **b** No real solutions
c Two real solutions **d** Two real solutions
e Two real solutions **f** No real solutions

4 a $\Delta = 0$, one rational solution
b $\Delta = 1$, two rational solutions
c $\Delta = 17$, two irrational solutions
d $\Delta = 0$, one rational solution
e $\Delta = 57$, two irrational solutions
f $\Delta = 1$, two rational solutions

5 a **i** $-\sqrt{5} < m < \sqrt{5}$ **ii** $m = \pm\sqrt{5}$
iii $m > \sqrt{5}$ or $m < -\sqrt{5}$

b **i** $0 < m < \frac{4}{3}$ **ii** $m = \frac{4}{3}$
iii $m > \frac{4}{3}$ or $m < 0$

c **i** $-\frac{4}{5} < m < 0$ **ii** $m = 0$ or $m = -\frac{4}{5}$
iii $m < -\frac{4}{5}$ or $m > 0$

d **i** $-2 < m < 1$ **ii** $m = -2$ or $m = 1$
iii $m > 1$ or $m < -2$

6 $\Delta = (2m - n)^2$, a perfect square

7 $p > \frac{4}{3}$ **8** $p = -\frac{1}{2}$

9 a $p = \pm 3$ **b** $p > 1$ **c** $p > \frac{2}{3}$ **d** $p > 1$

10 $-2 < p < 8$

11 $\Delta = -4q^2 < 0$ for all $q \neq 0$

12 a $\Delta = 16m^2 - 96m + 176 = 16(m-3)^2 + 32$
b $\Delta \geq 32$; therefore two solutions

13 a $\Delta = 16$
b $\Delta > 0$; therefore two solutions

14 $\Delta = (m+4)^2$, a perfect square; therefore rational solutions

15 $\Delta = (m - 2n)^2$, a perfect square; therefore rational solutions

16 The graph will cross the x-axis twice

17 The graph will cross the x-axis twice

Exercise 3J

1 a $(1 - \sqrt{5}, -1 - \sqrt{5}), (1 + \sqrt{5}, -1 + \sqrt{5})$

b $(-3, 9), (2, 4)$ **c** $(-3, 9), \left(\frac{7}{4}, \frac{49}{16}\right)$

d $(1, 3), (2, 5)$

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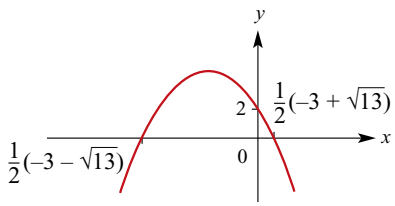
2 a $(2, 0), (-5, 7)$ **b** $(1, -3), (4, 9)$
c $(1, -3), (-3, 1)$ **d** $(-1, 1), (-3, -3)$
e $\left(\frac{1+\sqrt{33}}{2}, -3-\sqrt{33}\right), \left(\frac{1-\sqrt{33}}{2}, -3+\sqrt{33}\right)$
f $\left(\frac{5+\sqrt{33}}{2}, 23+3\sqrt{33}\right), \left(\frac{5-\sqrt{33}}{2}, 23-3\sqrt{33}\right)$

3 a Touch at $(2, 0)$ **b** Touch at $(3, 9)$
c Touch at $(-2, -4)$ **d** Touch at $(-4, -8)$

4 a $x = 8, y = 16$ or $x = -1, y = 7$
b $x = -\frac{16}{3}, y = 37\frac{1}{3}$ or $x = 2, y = 30$
c $x = \frac{4}{5}, y = 10\frac{2}{5}$ or $x = -3, y = 18$
d $x = 10\frac{2}{3}, y = 0$ or $x = 1, y = 29$
e $x = 0, y = -12$ or $x = \frac{3}{2}, y = -7\frac{1}{2}$
f $x = 1.14, y = 14.19$ or $x = -1.68, y = 31.09$

5 a -13
b i



ii $m = -6 \pm \sqrt{32} = -6 \pm 4\sqrt{2}$

6 $a = 3$ or $a = -1$

7 $b = 1$

8 $y = (2+2\sqrt{3})x - 4 - 2\sqrt{3}$ and $y = (2-2\sqrt{3})x - 4 + 2\sqrt{3}$

9 a $x = \frac{1}{2}(-k - \sqrt{k^2+16})$, $y = \frac{1}{2}k(-k - \sqrt{k^2+16})$ or $x = \frac{1}{2}(-k + \sqrt{k^2+16})$, $y = \frac{1}{2}k(-k + \sqrt{k^2+16})$
b $x = \frac{1}{2}(k+1 - \sqrt{k^2+2k+9})$, $y = \frac{1}{2}k(k+1 - \sqrt{k^2+2k+9})$ or $x = \frac{1}{2}(k+1 + \sqrt{k^2+2k+9})$, $y = \frac{1}{2}k(k+1 + \sqrt{k^2+2k+9})$
c $x = \frac{1}{2}(-k + \sqrt{k^2-4})$, $y = \frac{1}{2}(-k^2 + k\sqrt{k^2-4} + 10)$ or $x = \frac{1}{2}(-k - \sqrt{k^2-4})$, $y = \frac{1}{2}(-k^2 - k\sqrt{k^2-4} + 10)$

10 a $c = -\frac{1}{4}$ **b** $c > -\frac{1}{4}$

Exercise 3K

1 $a = -4, c = 6$

2 a $\Delta = b^2 - 16a$ **b** $a = \frac{b^2}{16}$ **c** $a = \frac{1}{4}, b = 2$

3 a $y = 2(x+2)(x-6)$ **b** $y = -2(x+2)^2 + 4$
c $y = -x^2 + 2x - 3$

4 $a = 2$ **5** $a = \frac{4}{7}, b = -\frac{24}{7}$

6 $a = -2, b = 1, c = 6$

7 a $y = -\frac{5}{16}x^2 + 5$ **b** $y = x^2$
c $y = \frac{1}{11}x^2 + \frac{7}{11}x$ **d** $y = x^2 - 4x + 3$
e $y = -\frac{5}{4}x^2 - \frac{5}{2}x + \frac{15}{4}$ **f** $y = x^2 - 4x + 6$

8 $y = \frac{5}{16}(x+1)^2 + 3$ **9** $y = -\frac{1}{2}(x^2 - 3x - 18)$

10 $y = (x+1)^2 + 3$ **11** $y = \frac{1}{180}x^2 - x + 75$

12 $y = 2x^2 - 4x$ **13** $y = x^2 - 2x - 1$

14 a C **b** B **c** D **d** A

15 a $y = a\left(x + \frac{1}{a}\right)^2 + a - \frac{1}{a}$ **b** $\left(-\frac{1}{a}, a - \frac{1}{a}\right)$
c $a = \pm 1$ **d** $-1 < a < 1$

16 $y = -2x^2 + 8x - 6$

17 a B **b** D

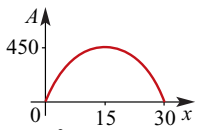
18 a $y = -2x^2 - x + 5$ **b** $y = 2x^2 - x - 5$

19 a $y = 2x^2 + \frac{5}{2}x - \frac{11}{2}$ **b** $y = 2x^2 - 3x + 1$

20 $r = -\frac{1}{8}t^2 + 2\frac{1}{2}t - 6\frac{3}{8}$

Exercise 3L

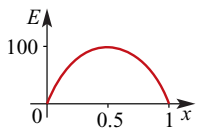
1 a $A = 60x - 2x^2$ **b**



c Maximum area = 450 m^2

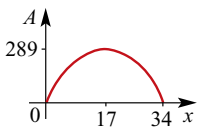
2 $A = x(10 - x)$; Maximum area = 25 m^2

3 a



b 0 and 1 **c** 0.5
d 0.23 and 0.77

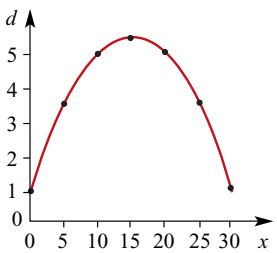
4 a $A = 34x - x^2$ **b**



c 289 cm^2

5 a

x	0	5	10	15	20	25	30
d	1	3.5	5	5.5	5	3.5	1



b i 5.5 m
ii $15 - 5\sqrt{7}$ m or $15 + 5\sqrt{7}$ m from the bat
iii 1 m above the ground

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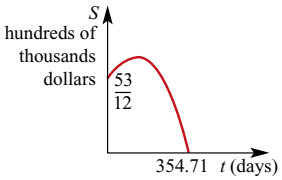
6 $a = -\frac{16}{15}, b = \frac{8}{5}, c = 0$

7 a $a = -\frac{7}{21\,600}, b = \frac{41}{400}, c = \frac{53}{12}$

b



c i \$1 236 666 **ii** \$59 259

Chapter 3 review

Technology-free questions

1 a $\left(x + \frac{9}{2}\right)^2$ **b** $(x + 9)^2$ **c** $\left(x - \frac{2}{5}\right)^2$

d $(x + b)^2$ **e** $(3x - 1)^2$ **f** $(5x + 2)^2$

2 a $-3x + 6$ **b** $-ax + a^2$

c $49a^2 - b^2$ **d** $x^2 - x - 12$

e $2x^2 - 5x - 12$ **f** $x^2 - y^2$

g $a^3 - b^3$ **h** $6x^2 + 8xy + 2y^2$

i $3a^2 - 5a - 2$ **j** $4xy$

k $2u + 2v - uv$ **l** $-3x^2 + 15x - 12$

3 a $4(x - 2)$ **b** $x(3x + 8)$

c $3x(8a - 1)$ **d** $(2 - x)(2 + x)$

e $a(u + 2v + 3w)$ **f** $a^2(2b - 3a)(2b + 3a)$

g $(1 - 6ax)(1 + 6ax)$ **h** $(x + 4)(x - 3)$

i $(x + 2)(x - 1)$ **j** $(2x - 1)(x + 2)$

k $(3x + 2)(2x + 1)$ **l** $(3x + 1)(x - 3)$

m $(3x - 2)(x + 1)$ **n** $(3a - 2)(2a + 1)$

o $(3x - 2)(2x - 1)$

4 a $x = 5$ or $x = -3$ **b** $x = 9$ or $x = 0$

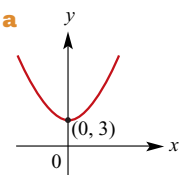
c $x = 2$ or $x = 3$ **d** $x = -1$ or $x = 25$

e $x = -3$ or $x = -2$ **f** $x = 6$

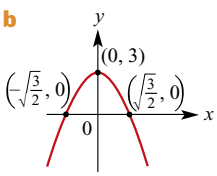
g $x = -\frac{1}{2}$ or $x = 3$ **h** $x = -\frac{5}{6}$ or $x = \frac{3}{2}$

i $x = -\frac{12}{5}$ or $x = 1$

5 a



b



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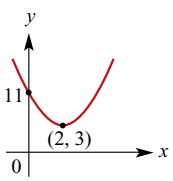

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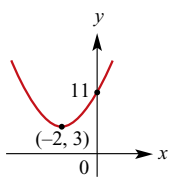

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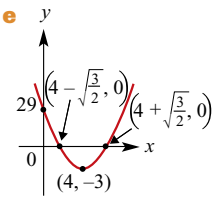

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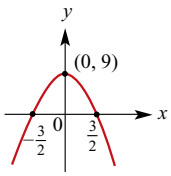
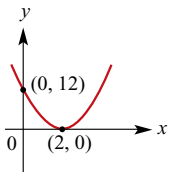


g



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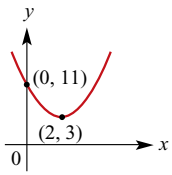
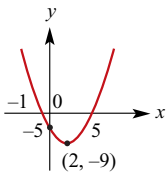
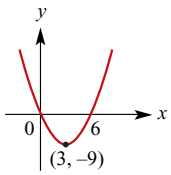


6 a



b



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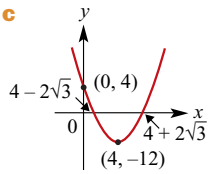
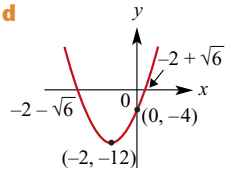


d



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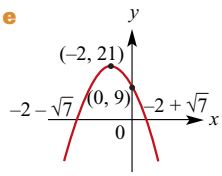
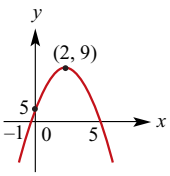
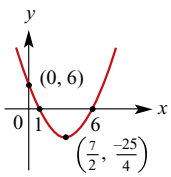


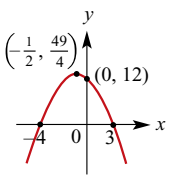
f



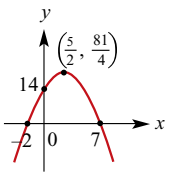
7 a $x = \frac{7}{2}$



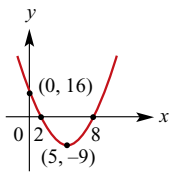
b $x = -\frac{1}{2}$



c $x = \frac{5}{2}$



d $x = 5$



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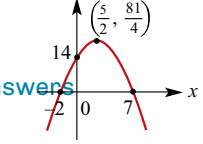
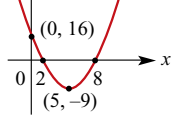
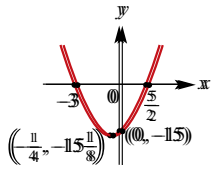
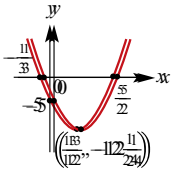




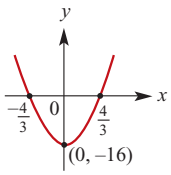
e $x = -\frac{11}{4}$



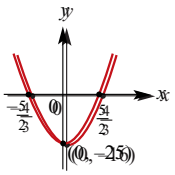
f $x = \frac{113}{112}$



g $x = 0$



h $x = 0$



8 $p = 1$ or $p = -\frac{3}{10}$

9 **a** $x \leqslant 0$ or $x \geqslant 1$
b $-2 - \sqrt{34} \leqslant x \leqslant -2 + \sqrt{34}$
c $-2 \leqslant x \leqslant \frac{1}{3}$ **d** $\frac{3}{2} \leqslant x \leqslant 5$

10 **a** $-3 + \sqrt{6}, -3 - \sqrt{6}$ **b** $\frac{-9 + \sqrt{33}}{2}, \frac{-9 - \sqrt{33}}{2}$ **c** $2 \pm \sqrt{2}$
d $\frac{-7 \pm 2\sqrt{33}}{4}$ **e** $2 - \sqrt{2}, 2 + \sqrt{2}$ **f** $\frac{-9 \pm \sqrt{93}}{6}$

11 $y = -\frac{3}{4}x(x - 5)$

12 $y = 3(x - 5)^2 + 2$

13 $m < -21 - 4\sqrt{29}$ or $m > -21 + 4\sqrt{29}$

14 $\frac{225}{4}$

15 $y = 5(x - 1)^2 + 5$

16 **a** $(3, 9), (-1, 1)$
b $\left(\frac{4 - \sqrt{38}}{2}, 27 - 4\sqrt{38}\right), \left(\frac{4 + \sqrt{38}}{2}, 27 + 4\sqrt{38}\right)$
c $\left(\frac{-7 - \sqrt{73}}{6}, 2\right), \left(\frac{-7 + \sqrt{73}}{6}, 2\right)$
d $\left(\frac{1}{2}, \frac{1}{2}\right), (-2, 8)$

17 **a** $y = 2(x + 4)(x - 1)$ **b** $y = -2(x + 1)^2 + 3$
c $y = 2x^2 - 2x - 3$

18 2.16 m

19 **a** $m = \pm\sqrt{8} = \pm 2\sqrt{2}$ **b** $m \leqslant -\sqrt{5}$ or $m \geqslant \sqrt{5}$

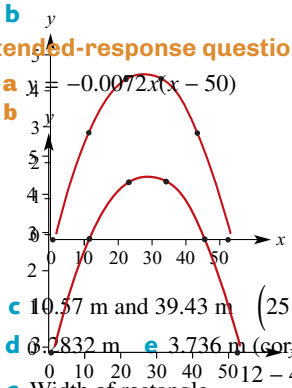
20 **a** $x = 0$ and $x = -b$ **b** $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$
c **i** $(0, 0), (1 - b, 1 - b)$ **ii** $b = 1$ **iii** $b \neq 1$

Multiple-choice questions

1 A	**2** C	**3** C	**4** E	**5** B
6 C	**7** E	**8** E	**9** D	**10** A
11 B	**12** E	**13** D		

Extended-response questions

1 **a** $y = -0.0072x(x - 50)$
b



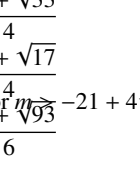
c 10.57 m and 39.43 m $\left(25 \pm \frac{25\sqrt{3}}{3}\text{ m}\right)$
d 3.2832 m **e** 3.736 m (correct to 3 d.p.)

2 **a** Width of rectangle $= \frac{12 - 4x}{3}$ m
Length of rectangle $= \frac{12 - 4x}{3}$ m
b $A = -\frac{16}{9}x^2 + 8x$
c Length for square $= \frac{96}{17}$ m and length for rectangle $= \frac{108}{17}$ m (≈ 5.65 m $\times$ 6.35 m)

3 **a** $V = 0.72x^2$ **b** 22 hours **c** 55.18 m

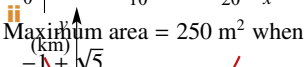
4 **a** $V = 10\,800x + 120x^2$
b $V = 46.6x + 3000x^2$ **c** $\frac{108}{17} \approx 6.35$ cm

5 **a** $t = 50 - \frac{5x}{2} - 1.2x$ **b** $A = 50x - \frac{5}{2}x^2$
c

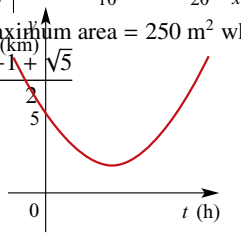


b **i** $16 - x$ **ii** $\sqrt{x^2 - 32x + 265}$
c 7.5 **d** 10.840 **e** 12.615

5 **a** **i** $y = \sqrt{64t^2 + 100(t - 0.5)^2}$
$= \sqrt{164t^2 - 100t + 25}$



ii



d Maximum area = 250 m^2 when $x = 10$ m

6 $x = \frac{-1 + \sqrt{5}}{2}$
iii $t = \frac{1}{2}$, 1:30 p.m.; $t = \frac{9}{82}$, 1:07 p.m.
iv $t \approx 0.305$, 1:18 p.m.; distance 3.123 km
b **i** $0, \frac{25}{41}$ **ii** $\frac{25 \pm 2\sqrt{269}}{82}$

6 **b** $2x + 2y = b$
c $8x^2 - 4bx + b^2 - 16a^2 = 0$
e **i** $x = 6 \pm \sqrt{14}, y = 6 \mp \sqrt{14}$
ii $x = y = \sqrt{2}a$
f $x = \frac{(5 \pm \sqrt{7})a}{4}, y = \frac{(5 \mp \sqrt{7})a}{4}$

7 **a** $b = -2, c = 4, h = 1$
b **i** $(x, -6 + 4x - x^2)$ **ii** $(x, x - 1)$
iii $(0, -1), (1, 0), (2, 1), (3, 2), (4, 3)$
iv $y = x - 1$

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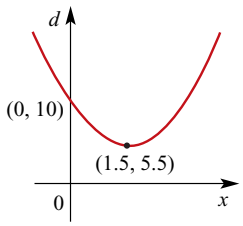
c **i** $d = 2x^2 - 6x + 10$

ii



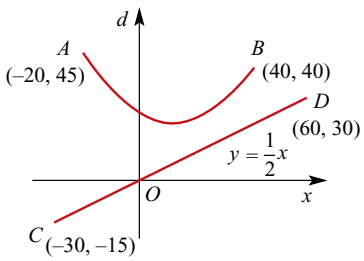
iii Min value of $d = 5.5$ when $x = 1.5$

8 a $45\sqrt{5}$

b **i** $y = \frac{1}{600}(7x^2 - 190x + 20\,400)$

ii $\left(\frac{190}{14}, \frac{5351}{168}\right)$

c



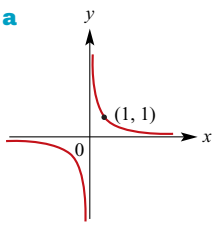
d **i** The distance (measured parallel to the y-axis) between path and pond

ii Minimum value $= \frac{473}{24}$ when $x = 35$

Chapter 4

Exercise 4A

1 a



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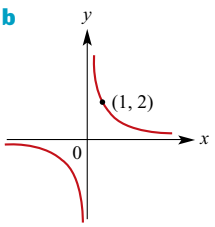

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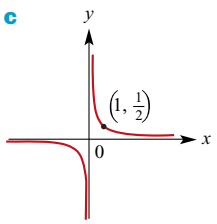

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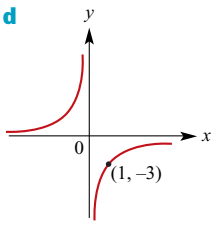

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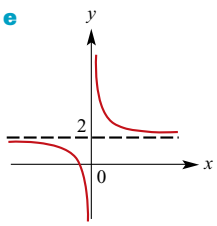

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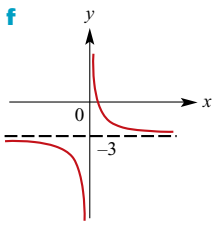

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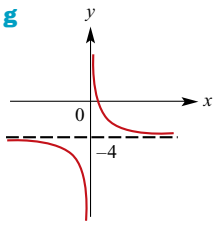

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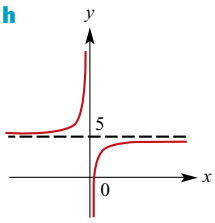

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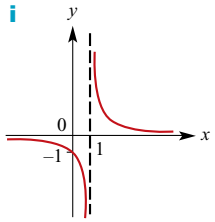

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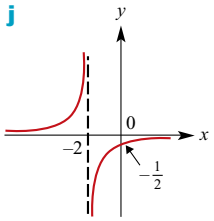

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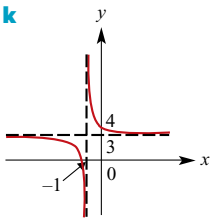

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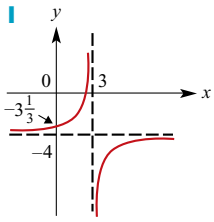


2 a $y = 0, x = 0$ **b** $y = 0, x = 0$

c $y = 0, x = 0$ **d** $y = 0, x = 0$

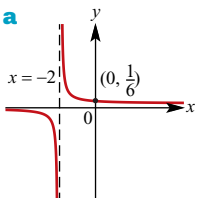
e $y = 2, x = 0$ **f** $y = -3, x = 0$

g $y = -4, x = 0$ **h** $y = 5, x = 0$

i $y = 0, x = 1$ **j** $y = 0, x = -2$

k $y = 3, x = -1$ **l** $y = -4, x = 3$

3 a



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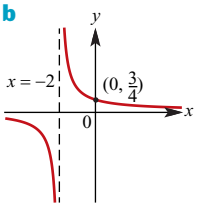

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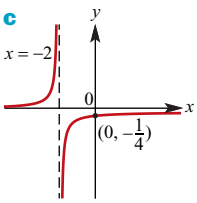

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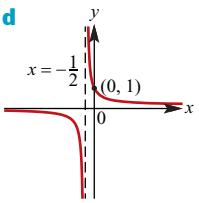
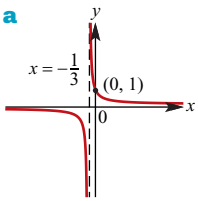


4 a



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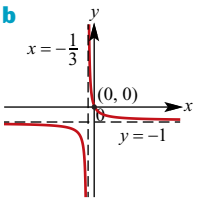

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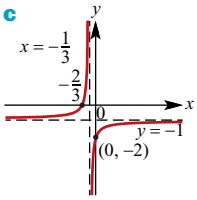

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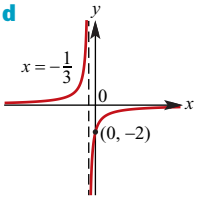

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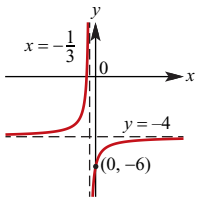
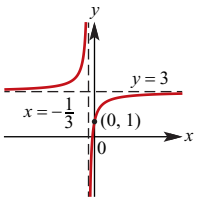
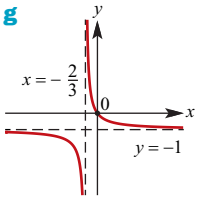
e x-axis intercept $-\frac{1}{2}$
y-axis intercept -6



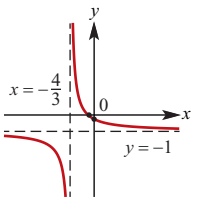
f x-axis intercept $-\frac{1}{9}$
y-axis intercept 1



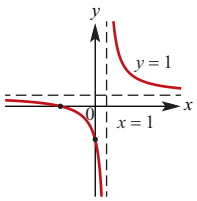
g



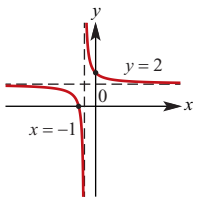
h x-axis intercept $-\frac{1}{3}$
y-axis intercept $-\frac{1}{4}$



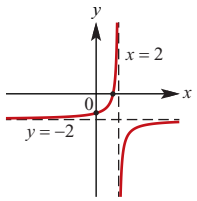
5 x-axis intercept -3
y-axis intercept -3



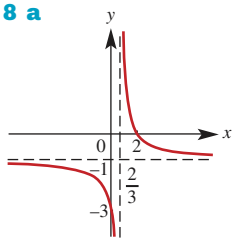
6 x-axis intercept $-\frac{3}{2}$
y-axis intercept 3



7 x-axis intercept $\frac{3}{2}$
y-axis intercept $-\frac{3}{2}$



8 a



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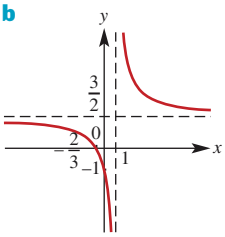

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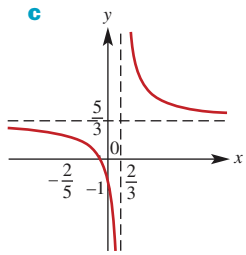
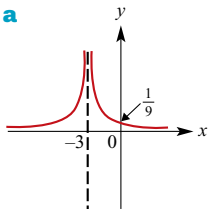


Exercise 4B

1 a



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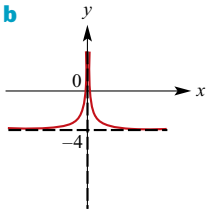

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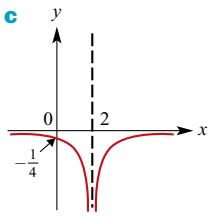

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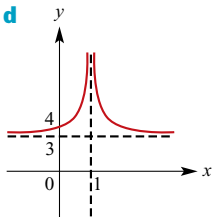

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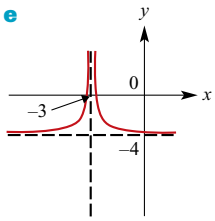

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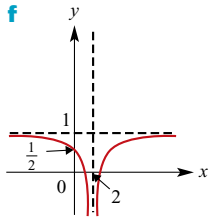

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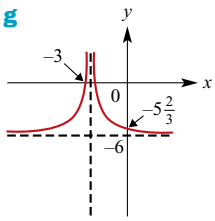

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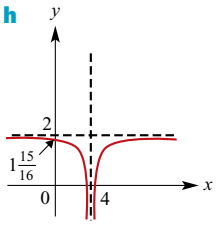
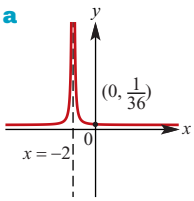


2 a $y = 0$, $x = -3$
b $y = -4$, $x = 0$
c $y = 0$, $x = 2$
d $y = 3$, $x = 1$
e $y = -4$, $x = -3$
f $y = 1$, $x = 2$
g $y = -6$, $x = -3$
h $y = 2$, $x = 4$

3 a



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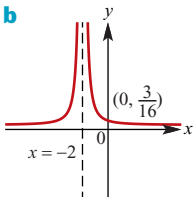

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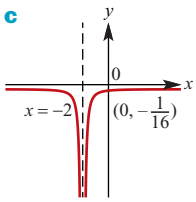

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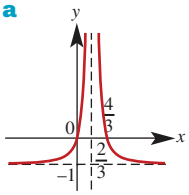
4 a



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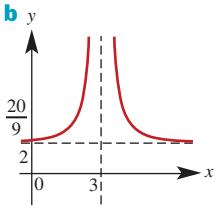

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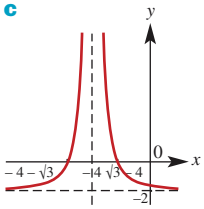
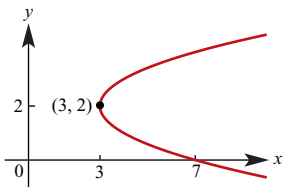


Exercise 4C

1 a



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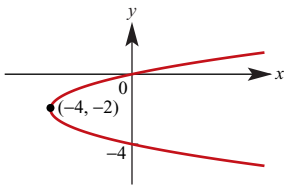

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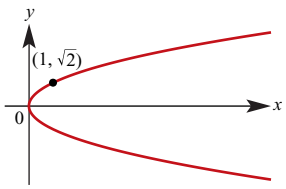

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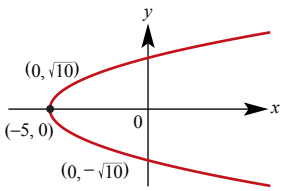

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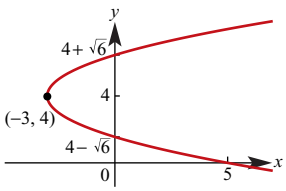

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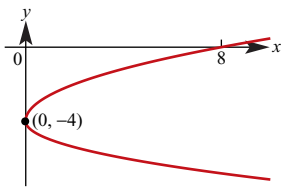

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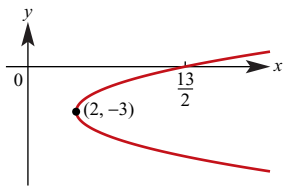

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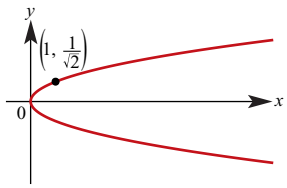
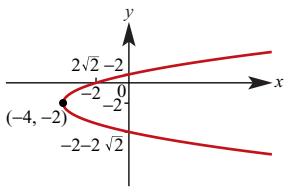


i



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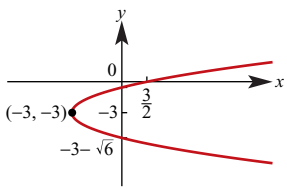

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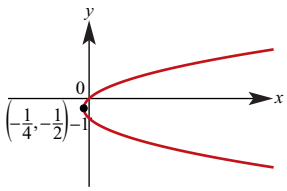

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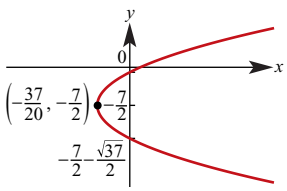

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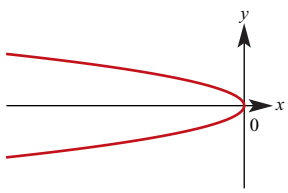

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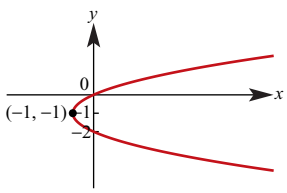

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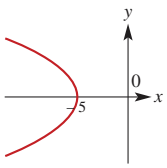
2 a



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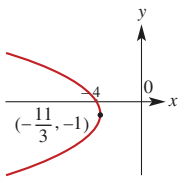

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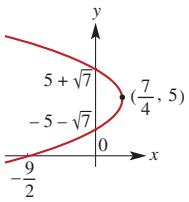
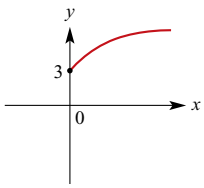
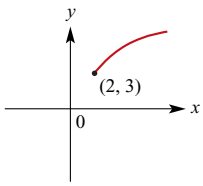


Exercise 4D

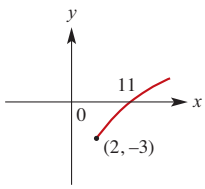
1 a $x \geq 0$ and $y \geq 3$



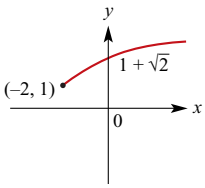
b $x \geq 2$ and $y \geq 3$



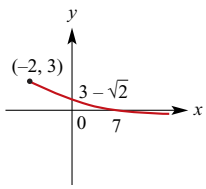
c $x \geq 2$ and $y \geq -3$



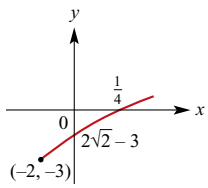
d $x \geq -2$ and $y \geq 1$



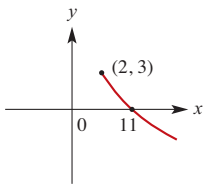
e $x \geq -2$ and $y \leq 3$



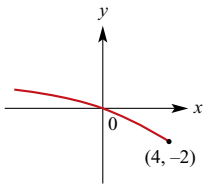
f $x \geq -2$ and $y \geq -3$



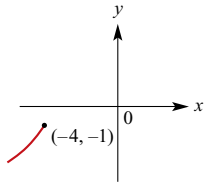
2 a $x \geq 2$ and $y \leq 3$



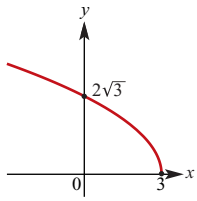
b $x \leq 4$ and $y \geq -2$



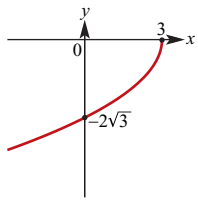
c $x \leq -4$ and $y \leq -1$



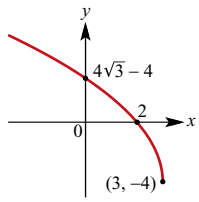
d $x \leq 3$ and $y \geq 0$



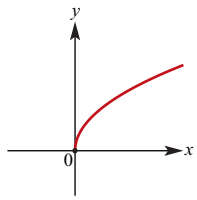
e $x \leq 3$ and $y \leq 0$



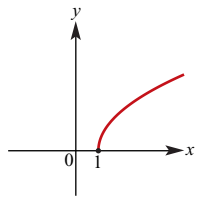
f $x \leq 3$ and $y \geq -4$



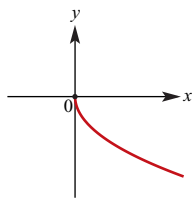
3 a $x \geq 0$ and $y \geq 0$



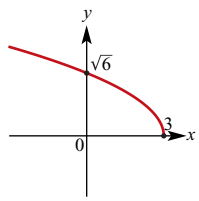
b $x \geq 1$ and $y \geq 0$



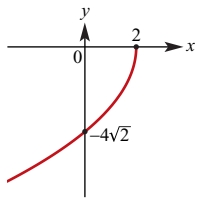
c $x \geq 0$ and $y \leq 0$



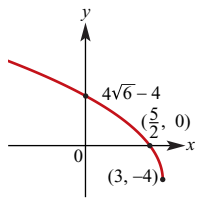
d $x \leq 3$ and $y \geq 0$



e $x \leq 2$ and $y \leq 0$



f $x \leq 3$ and $y \geq -4$



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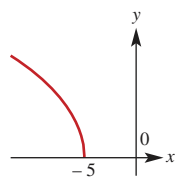

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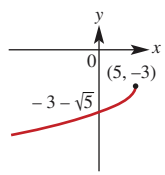

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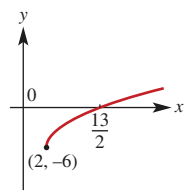

Exercise 4E

1 a $x^2 + y^2 = 9$ **b** $x^2 + y^2 = 16$

c $(x - 1)^2 + (y - 3)^2 = 25$

d $(x - 2)^2 + (y + 4)^2 = 9$

e $(x + 3)^2 + (y - 4)^2 = \frac{25}{4}$

f $(x + 5)^2 + (y + 6)^2 = (4.6)^2$

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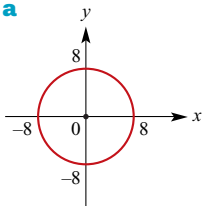
2 a $C(1, 3)$, $r = 2$ **b** $C(2, -4)$, $r = \sqrt{5}$
c $C(-3, 2)$, $r = 3$ **d** $C(-5, 4)$, $r = \sqrt{8}$

3 a



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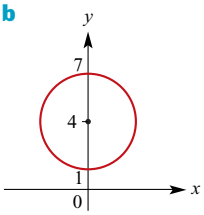

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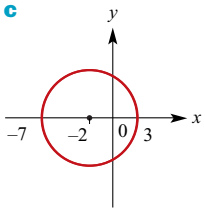

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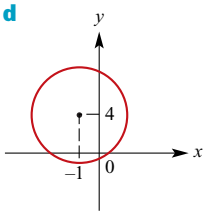

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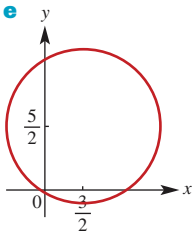

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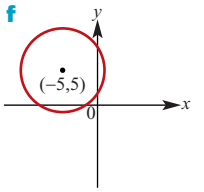
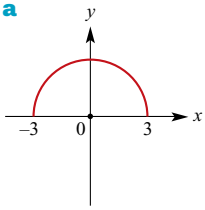


4 a $C(0, 3)$, $r = 5$ **b** $C(4, -6)$, $r = \sqrt{42}$
c $C(3, -2)$, $r = 2$ **d** $C(-2, 3)$, $r = 5$
e $C(4, -2)$, $r = \sqrt{19}$ **f** $C(\frac{1}{2}, -2)$, $r = \frac{3}{2}$

5 a



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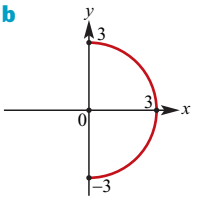

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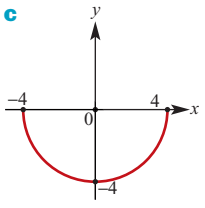

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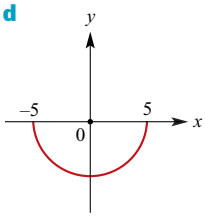

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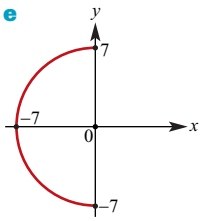

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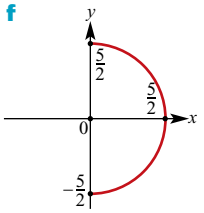

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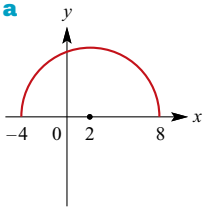

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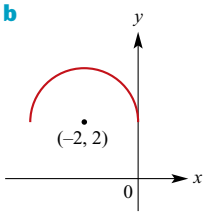

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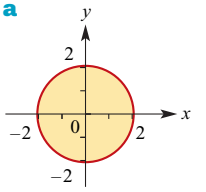

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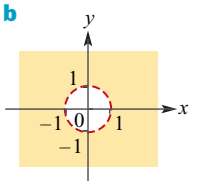

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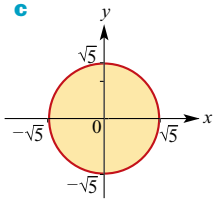

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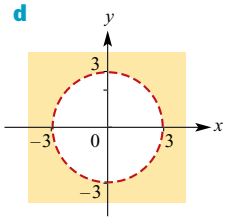

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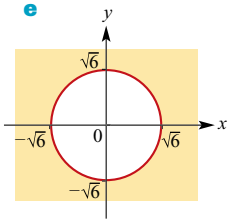

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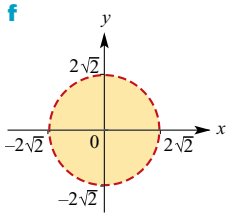

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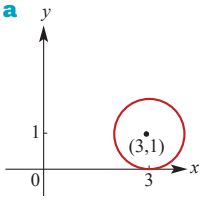

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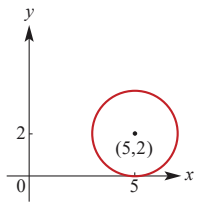

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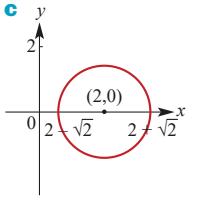

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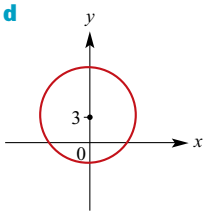

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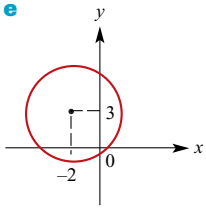

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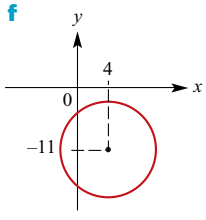

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Exercise 4F

1 $a = 5$

2 $a = -6, h = 3, k = 4$

3 $a = \frac{1}{2}, k = \frac{15}{2}$

4 $a = -16, h = 2, k = -4$

5 $a = 4\sqrt{2}$

6 $a = \frac{2\sqrt{3}}{3}, h = -2$

7 $a = 4, h = 5$

8 $h = 5, k = 2$

9 a $y = 3\sqrt{x-1} - 2$

b $y = \frac{1}{x-2} + 2$

c $y = -\frac{2}{x-1} - 2$

d $y = \sqrt{2-x} + 1$

e $y = \frac{1}{(x-2)^2} - 3$

f $(x-2)^2 + (y+2)^2 = 49$

10 a i $a = 8, b = 2$ **ii** $a = 8, b = -11$

b i $h = -\frac{241}{144}, k = -\frac{5}{6}$

ii $h = -\frac{241}{144}, k = -\frac{41}{6}$

11 a $(x-2)^2 + (y-1)^2 = 20$

b $(x+2)^2 + (y-3)^2 = 1$

c $(x+2)^2 + (y-3)^2 = 16$

d $(x-2)^2 + (y+3)^2 = 9$

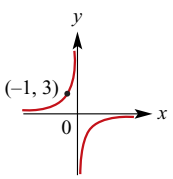
e $(x-4)^2 + (y-4)^2 = 20$

12 $(x-4)^2 + (y-5)^2 = 25$ and $(x+4)^2 + (y-5)^2 = 25$

Chapter 4 review

Technology-free questions

1 a



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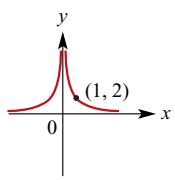

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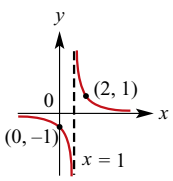

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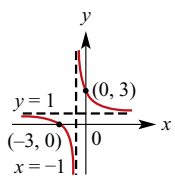

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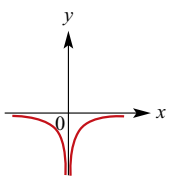

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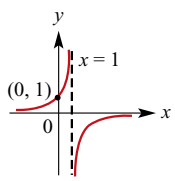

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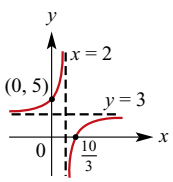

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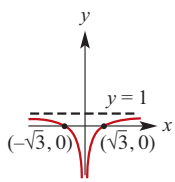

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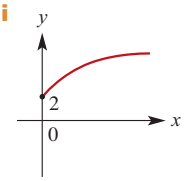

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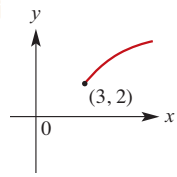

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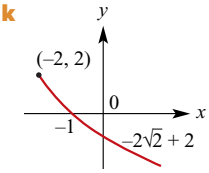

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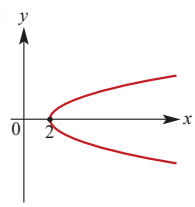

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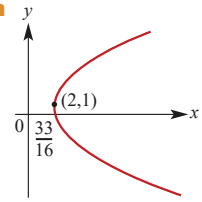

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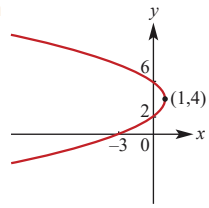


2 $a = -6, h = 2, k = 5$

3 $a = 81, h = -2, k = -5$

4 $a = 2\sqrt{2}, h = -\frac{5}{2}$

5 $a = 16, h = \frac{7}{16}$

6 a $(y+2)^2 = x + 6$; vertex $(-6, -2)$; axis intercepts $(-2, 0)$, $(0, -2 \pm \sqrt{6})$

b $(y+3)^2 = -2\left(x - \frac{5}{2}\right)$; vertex $\left(\frac{5}{2}, -3\right)$; axis intercepts $(-2, 0)$, $(0, -3 \pm \sqrt{5})$

c $(y+2)^2 = \frac{5}{2}\left(x + \frac{2}{5}\right)$; vertex $\left(-\frac{2}{5}, -2\right)$; axis intercepts $\left(\frac{6}{5}, 0\right)$, $(0, -1)$, $(0, -3)$

7 Touches at $(2, \frac{1}{2})$

8 a $A(-2, 0)$, $B(0, 3)$

9 a $(x-3)^2 + (y+2)^2 = 25$

b $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{25}{2}$

c $\left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{17}{8}$

d $(x+2)^2 + (y-3)^2 = 13$

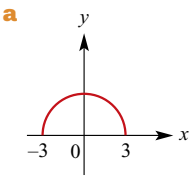
e $(x-3)^2 + (y-3)^2 = 18$

f $(x-2)^2 + (y+3)^2 = 13$

10 $C(-2, 3)$, $r = 6$

11 y-axis: $4\sqrt{6}$; x-axis: $2\sqrt{21}$

12 a



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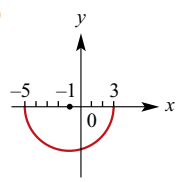

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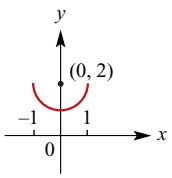

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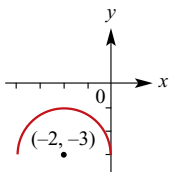


Multiple-choice questions

1 E **2** B **3** A **4** D **5** A **6** E
7 A **8** C **9** E **10** A **11** D

Extended-response questions

1 a (1, 1), (6, 6)

b $\left(\frac{-2a+7+\sqrt{25-12a}}{2}, \frac{7+\sqrt{25-12a}}{2}\right)$, $\left(\frac{-2a+7-\sqrt{25-12a}}{2}, \frac{7-\sqrt{25-12a}}{2}\right)$

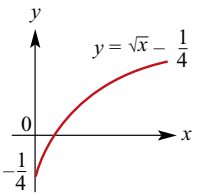
c i $a = \frac{25}{12}$ **ii** $a > \frac{25}{12}$

2 b $(-3+2\sqrt{2}, -2+2\sqrt{2})$, $(-3-2\sqrt{2}, -2-2\sqrt{2})$

c 8

d $(-3, -2)$

3 b ii $\left(\frac{1}{4}, \frac{1}{4}\right)$



c i $-\frac{1}{4} < k < 0$ **ii** $k = 0$ or $k < -\frac{1}{4}$

iii $k > 0$

4 a $0 < k < \frac{1}{4}$ **b** $k = \frac{1}{4}$ or $k \leq 0$

5 b i $a = \frac{1}{2}$, $(0, 1)$ **ii** $a = \frac{3}{25}$, $\left(-\frac{7}{75}, \frac{8}{25}\right)$

c i $\frac{1}{2}x + \frac{1}{2}y = 1$ **ii** $\frac{1}{5}x - \frac{2}{5}y = 1$

6 b i $a = \frac{1}{4}$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ **ii** $a = \frac{1}{16}$, $\left(\frac{1}{4}, \frac{1}{4}\right)$

c i $\frac{1}{4}x + y = 1$ **ii** $-\frac{1}{4}x - y = 1$

7 a $(x-10)^2 + y^2 = 25$ **c** $m = \pm\frac{\sqrt{3}}{3}$

d $P\left(\frac{15}{2}, \pm\frac{5\sqrt{3}}{2}\right)$ **e** $5\sqrt{3}$

8 a $x^2 + y^2 = 16$

b ii $m = \pm\frac{\sqrt{3}}{3}$; $y = \frac{\sqrt{3}}{3}x - \frac{8\sqrt{3}}{3}$

$y = -\frac{\sqrt{3}}{3}x + \frac{8\sqrt{3}}{3}$

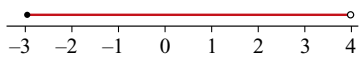
Chapter 5

Exercise 5A

1 a {7, 11} **b** {7, 11}
c {1, 2, 3, 5, 7, 11, 15, 25, 30}
d {1, 2, 3, 5, 7, 11, 15, 25, 30, 32}
e {1, 2, 3, 5, 7, 11, 15, 25, 30, 32}
f {1, 7, 11, 25, 30}

2 a {1, 2, 3, 5, 15} **b** {25, 30, 32}
c {2, 3, 5, 15} **d** {25, 30}

3 a



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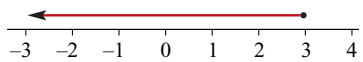

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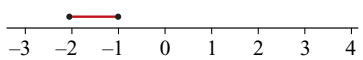

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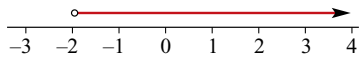

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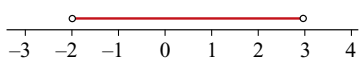

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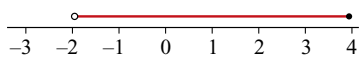


4 a $(-2, 1]$ **b** $[-3, 3]$ **c** $[-3, 2)$ **d** $(-1, 2)$

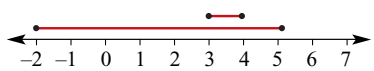
5 a $[-1, 2]$ **b** $(-4, 2]$ **c** $(0, \sqrt{2})$

d $\left(-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right]$ **e** $(-1, \infty)$ **f** $(-\infty, -2]$

g $(-\infty, \infty)$ **h** $[0, \infty)$ **i** $(-\infty, 0]$

6 a {7} **b** B, i.e. {7, 11, 25, 30, 32}
c $(2, \infty)$ **d** {30, 32}

7 a



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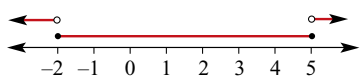

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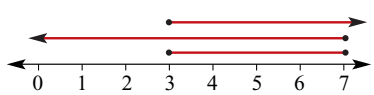

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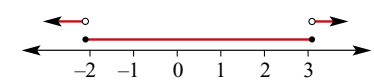
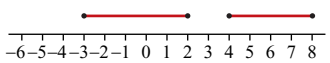


8 a $(-\infty, -2) \cup (-2, \infty)$ **b** $(-\infty, 3) \cup (3, \infty)$
c $(-\infty, 4) \cup (4, \infty)$

9 a



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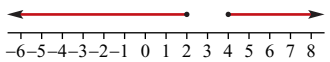

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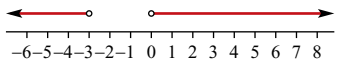

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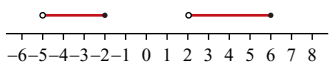

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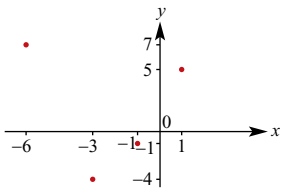
e

f

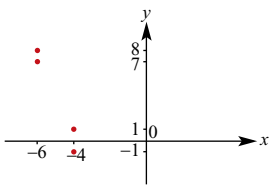
10 a $(-6, -3)$ **b** $\varnothing$ **c** $[-6, 0]$
d $[-1, 2]$ **e** $\{1\}$ **f** $(-10, -1]$

Exercise 5B

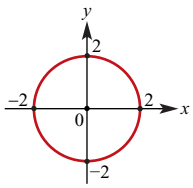
1 a Domain = $\{-3, -1, -6, 1\}$;
Range = $\{-4, -1, 7, 5\}$



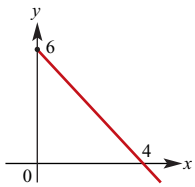
b Domain = $\{-4, -6\}$; Range = $\{-1, 1, 7, 8\}$



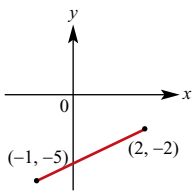
c Domain = $[-2, 2]$
Range = $[-2, 2]$



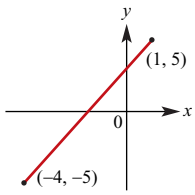
d Domain = $[0, \infty)$
Range = $(-\infty, 6]$



e Domain = $[-1, 2]$
Range = $[-5, -2]$

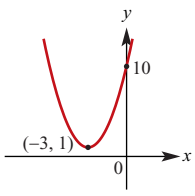


f Domain = $[-4, 1]$
Range = $[-5, 5]$

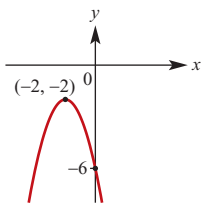


2 a Domain = $[-2, 2]$; Range = $[-1, 2]$
b Domain = $[-2, 2]$; Range = $[-2, 2]$
c Domain = $\mathbb{R}$; Range = $[-1, \infty)$
d Domain = $\mathbb{R}$; Range = $(-\infty, 4]$

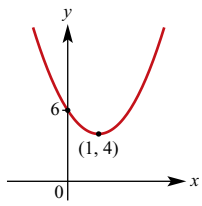
3 a $y = (x + 3)^2 + 1$
Range = $[1, \infty)$



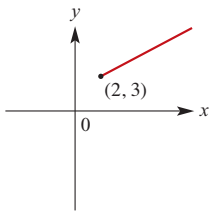
b $y = -(x + 2)^2 - 2$
Range = $(-\infty, -2]$



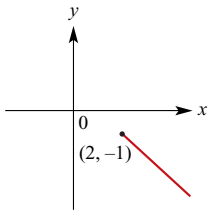
c $y = 2(x - 1)^2 + 4$
Range = $[4, \infty)$



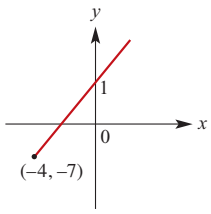
4 a Range = $[3, \infty)$



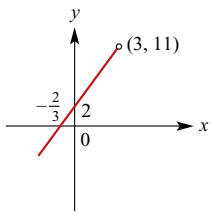
b Range = $(-\infty, -1]$



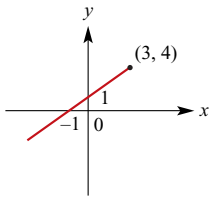
c Range = $[-7, \infty)$



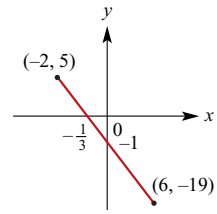
d Range = $(-\infty, 11)$



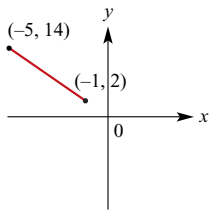
e Range = $(-\infty, 4]$



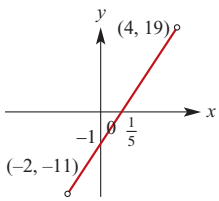
f Range = $[-19, 5]$



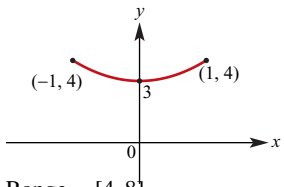
g Range = $[2, 14]$



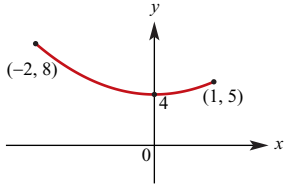
h Range = $(-11, 19)$



5 a Range = $[3, 4]$



b Range = $[4, 8]$



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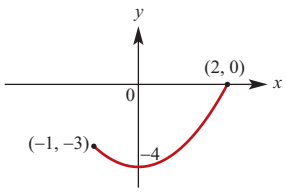
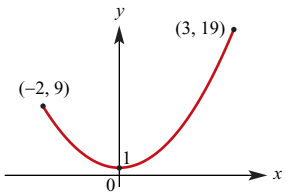
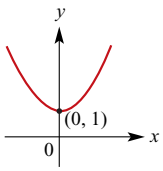
c Range = $[-4, 0]$



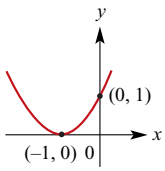
d Range = $[1, 19]$



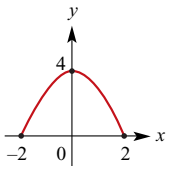
6 a Range = $[1, \infty)$



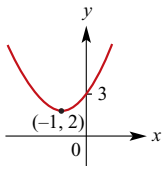
b Range = $[0, \infty)$



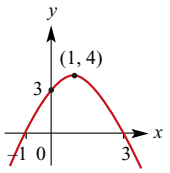
c Range = $[0, 4]$



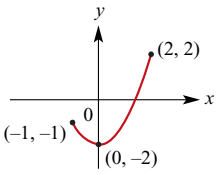
d Range = $[2, \infty)$



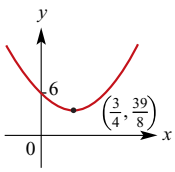
e Range = $(-\infty, 4]$



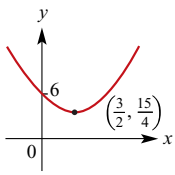
f Range = $[-2, 2]$



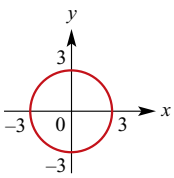
g Range = $\left[\frac{39}{8}, \infty\right)$



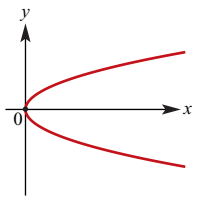
h Range = $\left[\frac{15}{4}, \infty\right)$



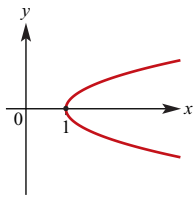
7 a Domain = $[-3, 3]$
Range = $[-3, 3]$



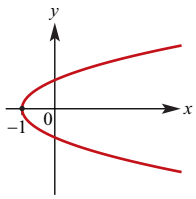
b Domain = $[0, \infty)$
Range = $\mathbb{R}$



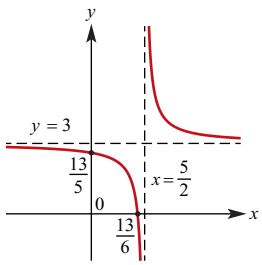
c Domain = $[1, \infty)$
Range = $\mathbb{R}$



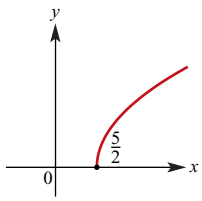
d Domain = $[-1, \infty)$
Range = $\mathbb{R}$



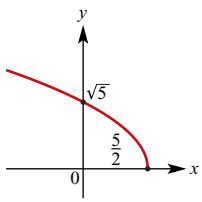
8 a Domain = $\mathbb{R} \setminus \left\{\frac{5}{2}\right\}$
Range = $\mathbb{R} \setminus \{3\}$



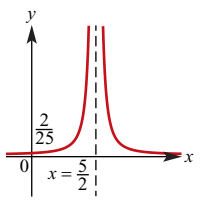
b Domain = $\left[\frac{5}{2}, \infty\right)$
Range = $[0, \infty)$



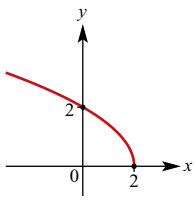
c Domain = $\left(-\infty, \frac{5}{2}\right]$
Range = $[0, \infty)$



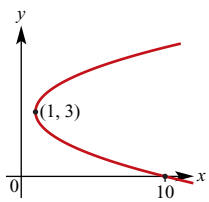
d Domain = $\mathbb{R} \setminus \left\{\frac{5}{2}\right\}$
Range = $(0, \infty)$



e Domain = $(-\infty, 2]$
Range = $[0, \infty)$



f Domain = $[1, \infty)$
Range = $\mathbb{R}$



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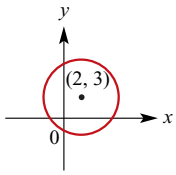
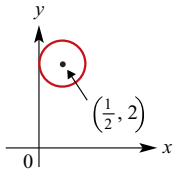
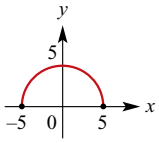
9 a Domain = $[-2, 6]$
Range = $[-1, 7]$



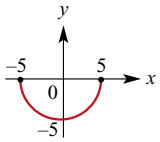
b Domain = $[0, 1]$
Range = $[1\frac{1}{2}, 2\frac{1}{2}]$



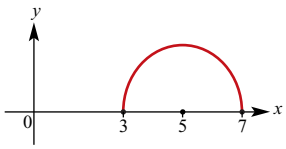
c Domain = $[-5, 5]$
Range = $[0, 5]$



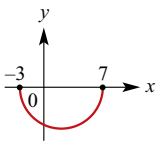
d Domain = $[-5, 5]$
Range = $[-5, 0]$



e Domain = $[3, 7]$; Range = $[0, 2]$



f Domain = $[-3, 7]$; Range = $[-5, 0]$



Exercise 5C

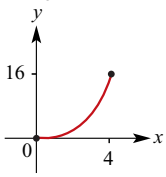
1 a Not a function; Domain = $\{0, 1, 2, 3\}$; Range = $\{1, 2, 3, 4\}$

b A function; Domain = $\{-2, -1, 0, 1, 2\}$; Range = $\{-5, -2, -1, 2, 4\}$

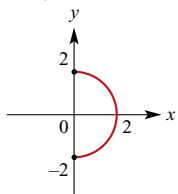
c Not a function; Domain = $\{-1, 0, 3, 5\}$; Range = $\{1, 2, 4, 6\}$

d A function; Domain = $\{1, 2, 4, 5, 6\}$; Range = $\{3\}$

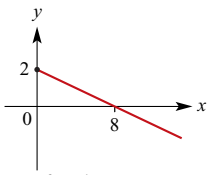
2 a A function
Domain = $[0, 4]$
Range = $[0, 16]$



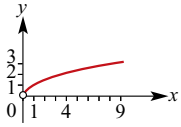
b Not a function
Domain = $[0, 2]$
Range = $[-2, 2]$



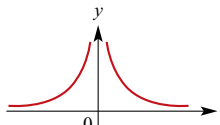
c A function
Domain = $[0, \infty)$
Range = $(-\infty, 2]$



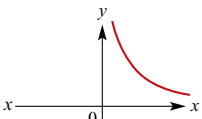
d A function
Domain = $[0, \infty)$
Range = $[0, \infty)$



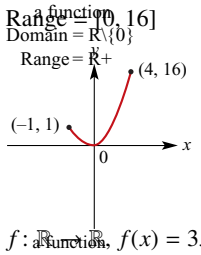
e A function
Domain = $\mathbb{R} \setminus \{0\}$
Range = $\mathbb{R}^+$



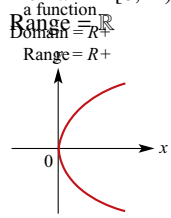
f A function
Domain = $\mathbb{R}^+$
Range = $\mathbb{R}^+$



g A function
Domain = $[-1, 4]$
Range = $[0, 16]$



h Not a function
Domain = $[0, \infty)$
Range = $\mathbb{R}$



3 a $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 3x + 2$

b $f: \mathbb{R} \to \mathbb{R}$, $f(x) = -\frac{3}{2}x + 6$

c $f: [0, \infty) \to \mathbb{R}$, $f(x) = 2x + 3$

d $f: [-1, 2] \to \mathbb{R}$, $f(x) = 5x + 6$

e $f: [-5, 5] \to \mathbb{R}$, $f(x) = -x^2 + 25$

f $f: [0, 1] \to \mathbb{R}$, $f(x) = 5x - 7$

4 a A function; Domain = $\mathbb{R}$; Range = $\{-2\}$

b Not a function; Domain = $\{3\}$; Range = $\mathbb{Z}$

c A function; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

d A function; Domain = $\mathbb{R}$; Range = $[5, \infty)$

e Not a function; Domain = $[-3, 3]$; Range = $[-3, 3]$

5 a **i** -3 **ii** 5 **iii** -5 **iv** 9
v $2x - 5$ **vi** $\frac{2}{a} - 3$

b **i** 4 **ii** -4 **iii** $\frac{4}{3}$ **iv** 2

c **i** 4 **ii** 36 **iii** 36 **iv** $(a-2)^2$

d **i** 0 **ii** $\frac{a}{1+a}$ **iii** $\frac{-a}{1-a} = \frac{a}{a-1}$ **iv** $1 - a$

6 a $5, 2t + 1$ **b** $x = \frac{5}{2}$ **c** $x = -\frac{1}{2}$
d $t = -1$ **e** $x \geq -1$ **f** $x \geq 1$

7 a 1 **b** $\frac{1}{6}$ **c** ± 3 **d** $-1, 4$ **e** $-1, 3$ **f** $-2, 3$

8 a $g(-1) = -1$, $g(2) = 8$, $g(-2) = 0$

b $h(-1) = 3$, $h(2) = 18$, $h(-2) = -14$

c **i** $g(-3x) = 9x^2 - 6x$
ii $g(x - 5) = x^2 - 8x + 15$
iii $h(-2x) = -16x^3 - 4x^2 + 6$
iv $g(x + 2) = x^2 + 6x + 8$
v $h(x^2) = 2x^6 - x^4 + 6$

9 a $f(2) = 5$, $f(-4) = 29$

b Range = $[-3, \infty)$

10 a $f(2) = 7$ **b** $x = 2$ **c** $x = -1$

11 a 2 **b** ± 1 **c** $x = \pm\sqrt{3}$

12 a $x = -1$ **b** $x > -1$ **c** $x = -\frac{6}{7}$

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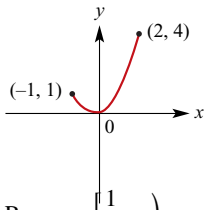
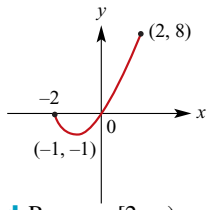
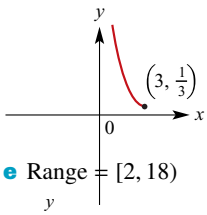
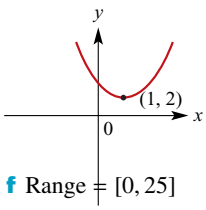
13 a Range = $[0, 4]$ **b** Range = $[-1, 8]$



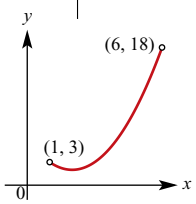


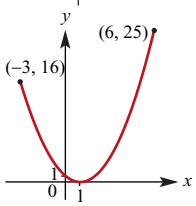
c Range = $\left[\frac{1}{3}, \infty\right)$ **d** Range = $[2, \infty)$





e Range = $[2, 18)$ **f** Range = $[0, 25]$





Exercise 5D

1 One-to-one functions: b, d, e, g

2 **i** Functions: a, c, d, f, g
ii One-to-one functions: c, g

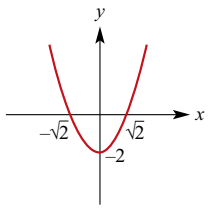
3 a Domain = $\mathbb{R}$; Range = $\mathbb{R}$
b Domain = $\mathbb{R}^+ \cup \{0\}$; Range = $\mathbb{R}^+ \cup \{0\}$
c Domain = $\mathbb{R}$; Range = $[1, \infty)$
d Domain = $[-3, 3]$; Range = $[-3, 0]$
e Domain = $\mathbb{R}^+$; Range = $\mathbb{R}^+$
f Domain = $\mathbb{R}$; Range = $(-\infty, 3]$
g Domain = $[2, \infty)$; Range = $\mathbb{R}^+ \cup \{0\}$
h Domain = $[\frac{1}{2}, \infty)$; Range = $[0, \infty)$
i Domain = $(-\infty, \frac{3}{2}]$; Range = $[0, \infty)$
j Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $\mathbb{R} \setminus \{0\}$
k Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $(-3, \infty)$
l Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $\mathbb{R} \setminus \{2\}$

4 a Domain = $[4, \infty)$; Range = $[0, \infty)$
b Domain = $(-\infty, 4]$; Range = $[0, \infty)$
c Domain = $[2, \infty)$; Range = $[3, \infty)$
d Domain = $\mathbb{R} \setminus \{4\}$; Range = $\mathbb{R} \setminus \{0\}$
e Domain = $\mathbb{R} \setminus \{4\}$; Range = $\mathbb{R} \setminus \{3\}$
f Domain = $\mathbb{R} \setminus \{-2\}$; Range = $\mathbb{R} \setminus \{-3\}$

5 a Domain = $\mathbb{R}$; Range = $\mathbb{R}$
b Domain = $\mathbb{R}$; Range = $[2, \infty)$
c Domain = $[-4, 4]$; Range = $[-4, 0]$
d Domain = $\mathbb{R} \setminus \{-2\}$; Range = $\mathbb{R} \setminus \{0\}$

6 $y = \sqrt{2 - x}$, Domain $(-\infty, 2]$, Range $[0, \infty)$
$y = -\sqrt{2 - x}$, Domain $(-\infty, 2]$, Range $(-\infty, 0]$

7 a

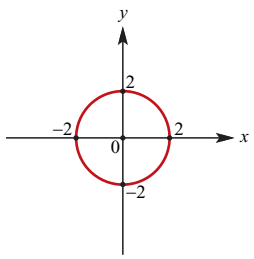


b $f_1 : [0, \infty) \to \mathbb{R}$, $f_1(x) = x^2 - 2$
$f_2 : (-\infty, 0] \to \mathbb{R}$, $f_2(x) = x^2 - 2$

8 b $f_1 : [1, \infty) \to \mathbb{R}$, $f_1(x) = x^2 - 2x + 4$
$f_2 : (-\infty, 1] \to \mathbb{R}$, $f_2(x) = x^2 - 2x + 4$

9 b $f_1 : (2, \infty) \to \mathbb{R}$, $f_1(x) = \dfrac{1}{(x-2)^2}$
$f_2 : (-\infty, 2) \to \mathbb{R}$, $f_2(x) = \dfrac{1}{(x-2)^2}$

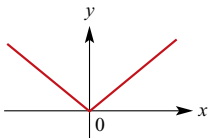
10 a Domain = $[-2, 2]$

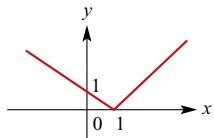


b $f_1 : [0, 2] \to \mathbb{R}$, $f_1(x) = \sqrt{4 - x^2}$
$f_2 : [0, 2] \to \mathbb{R}$, $f_2(x) = -\sqrt{4 - x^2}$
c $f_1 : [-2, 0] \to \mathbb{R}$, $f_1(x) = \sqrt{4 - x^2}$
$f_2 : [-2, 0] \to \mathbb{R}$, $f_2(x) = -\sqrt{4 - x^2}$

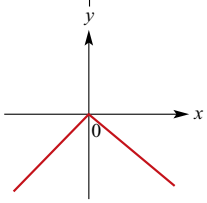
Exercise 5E

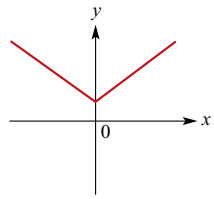
1 a Range = $[0, \infty)$ **b** Range = $[0, \infty)$



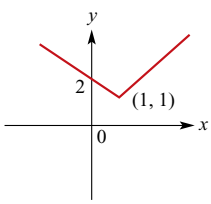


c Range = $(-\infty, 0]$ **d** Range = $[1, \infty)$





e Range = $[1, \infty)$



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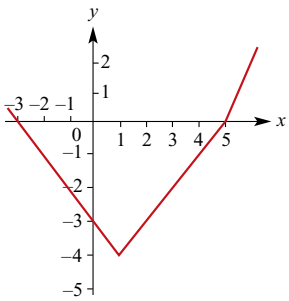
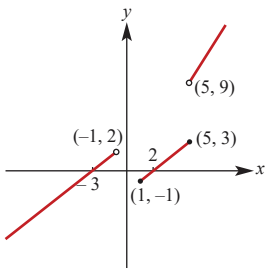
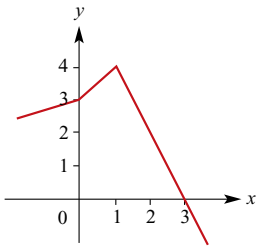
2 a



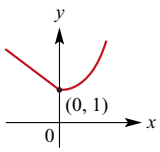
b



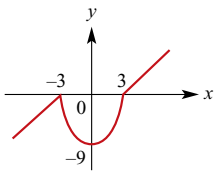
3 Range = $(-\infty, 4]$



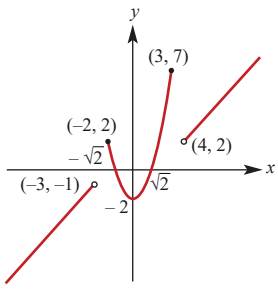
4 Range = $[1, \infty)$



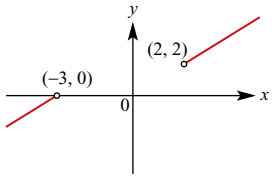
5 a Range = $\mathbb{R}$



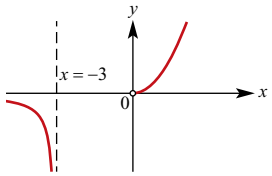
b Range = $\mathbb{R}$



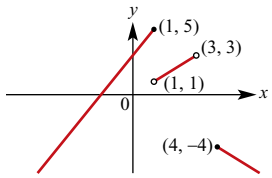
6 a Range = $(-\infty, 0) \cup (2, \infty)$



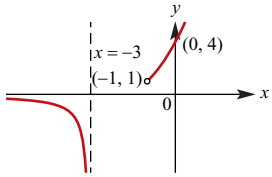
b Range = $\mathbb{R} \setminus \{0\}$



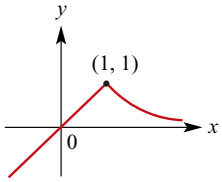
c Range = $(-\infty, 5]$



d Range = $\mathbb{R} \setminus [0, 1]$



7 Range = $(-\infty, 1]$



8 $f(x) = \begin{cases} x + 3, & -3 \leq x \leq -1 \\ -x + 1, & -1 < x \leq 2 \\ -\frac{1}{2}x, & 2 < x \leq 4 \end{cases}$

Exercise 5F

1 b i 25.06 **ii** 25.032 **iii** 25.2 **iv** 26

2 a $a = -3, b = \frac{1}{2}$ **b** 6

3 $f(x) = 7 - 5x$

4 a $f(0) = -\frac{9}{2}, f(1) = -3$ **b** 3

5 $f(x) = -7(x - 2)(x - 4)$

6 $f(x) = (x - 3)^2 + 7$, Range $= [7, \infty)$

7 $a = \frac{1}{10}, b = -\frac{9}{10}, c = 2$

8 $f(x) = -2(x - 1)(x + 5)$

$g(x) = -50(x - 1)\left(x + \frac{1}{5}\right)$

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9 a $k < -\frac{37}{12}$ **b** $k = -\frac{25}{12}$

Exercise 5G

1 a {(3, 1), (6, −2), (5, 4), (1, 7)}
Domain = {3, 6, 5, 1}; Range = {1, −2, 4, 7}

b {(3, 2), (6, −1), (−5, 4), (7, 1), (−4, 6)}
Domain = {3, 6, −5, 7, −4}
Range = {−1, 1, 2, 4, 6}

c {(3, 3), (−4, −2), (−1, −1), (1, −8)}
Domain = {3, 1, −1, −4}
Range = {3, −2, −1, −8}

d {(3, 1), (−7, −10), (−6, −7), (8, 2), (4, 11)}
Domain = {3, −7, −6, 8, 4}
Range = {1, −10, −7, 2, 11}

2 a $f^{-1}(x) = \frac{6-x}{2}$; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

b $f^{-1}(x) = 3 - x$
Domain = [−2, 2]; Range = [1, 5]

c $f^{-1}(x) = x - 4$
Domain = $(4, \infty)$; Range = $\mathbb{R}^+$

d $f^{-1}(x) = x - 4$
Domain = $(-\infty, 8]$; Range = $(-\infty, 4]$

e $f^{-1}(x) = 8 - \frac{x}{2}$
Domain = [2, 18]; Range = [−1, 7]

3 a $f^{-1}(x) = \sqrt{x}$
Domain = $\mathbb{R}^+ \cup \{0\}$; Range = $\mathbb{R}^+ \cup \{0\}$

b $f^{-1}(x) = 2 + \sqrt{x-3}$
Domain = $[3, \infty)$; Range = $[2, \infty)$

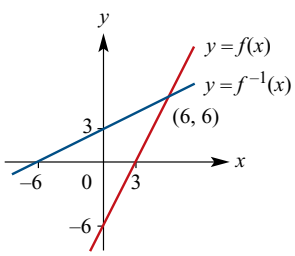
c $f^{-1}(x) = 4 - \sqrt{x-6}$
Domain = $[6, \infty)$; Range = $(-\infty, 4]$

d $f^{-1}(x) = 1 - x^2$
Domain = [0, 1]; Range = [0, 1]

4 a $f^{-1}(x) = \sqrt{16 - x^2}$
Domain = [0, 4]; Range = [0, 4]

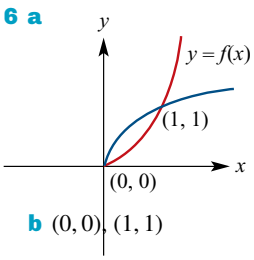
b $f^{-1}(x) = -4 + \sqrt{x-6}$
Domain = $[22, \infty)$; Range = $[0, \infty)$

5 a



b (6, 6)

6 a



b (0, 0), (1, 1)

7 $a = -\frac{1}{2}, b = \frac{5}{2}$

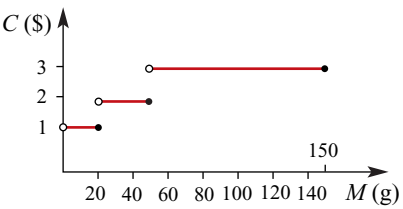
8 a $f^{-1}(x) = a - x^2$ **b** $a = 1$ or $a = 2$

Exercise 5H

1 $C = 0.30n + 80$, where n is the number of menus that are printed

2 a $C(m) = \begin{cases} 1.20 & \text{for } 0 < m \leq 20 \\ 2.00 & \text{for } 20 < m \leq 50 \\ 3.00 & \text{for } 50 < m \leq 150 \end{cases}$

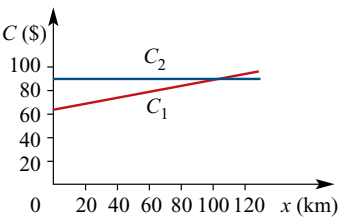
b



Domain = (0, 150]
Range = {1.20, 2.00, 3.00}

3 a $C_1 = 64 + 0.25x$, $C_2 = 89$

b



c $x > 100$ km

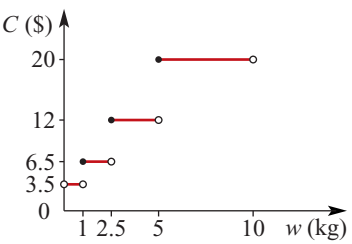
4 a Length = $(50 - x)$ cm

b $A(x) = x(50 - x)$

c $0 \leq x \leq 50$

d Maximum area = 625 cm^2 when $x = 25$

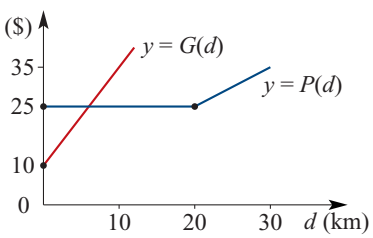
5 a



b i \$6.50 **ii** \$12 **iii** \$20

c Package them together

6 c



d i \$27.50 **ii** \$25

e Purple Taxi

f Greater than 6 km

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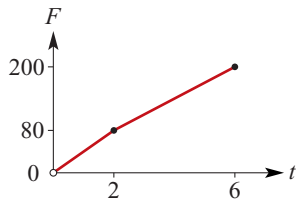
7 a $F(t) = \begin{cases} 40t & \text{for } 0 < t \leq 2 \\ 30t + 20 & \text{for } 2 < t \leq 6 \end{cases}$

b



c i \$60 **ii** \$95 **iii** \$125

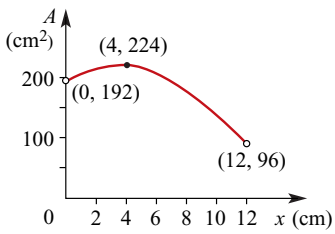
d \$35 per hour

8 a i $A = (8 + x)y - x^2$

ii $P = 2x + 2y + 16$

b i $A = 192 + 16x - 2x^2$ **ii** $0 < x < 12$

iii



iv 224 cm²

Chapter 5 review

Technology-free questions

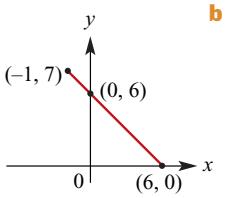
1 a $[-2, 4)$ **b** $[-2, 4]$ **c** $[1, 8)$ **d** $(-1, 6]$

e $(-4, -2] \cup (1, 5]$ **f** $(-4, -2] \cup (2, \infty)$

g $(-\infty, -3] \cup (1, \infty)$

2 a -16 **b** 26 **c** $-\frac{2}{3}$

3 a



b Range = $[0, 7]$

4 a Range = $\mathbb{R}$ **b** Range = $[-5, 4]$

c Range = $[0, 4]$ **d** Range = $(-\infty, 9]$

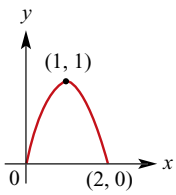
e Range = $[2, \infty)$ **f** Range = $\{-6, 2, 4\}$

g Range = $[0, \infty)$ **h** Range = $\mathbb{R} \setminus \{2\}$

i Range = $\mathbb{R}$ **j** Range = $[-1, 3]$

5 a $a = -15, b = \frac{33}{2}$ **b** Domain = $\mathbb{R} \setminus \{0\}$

6 a



b Range = $[0, 1]$

7 $a = 3, b = -5$

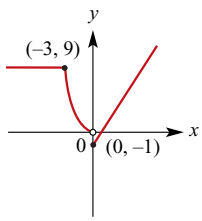
8 $a = -\frac{1}{2}, b = 2, c = 0$

9 a $\mathbb{R} \setminus \{2\}$ **b** $[2, \infty)$ **c** $[-5, 5]$

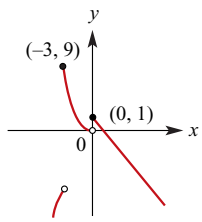
d $\mathbb{R} \setminus \{\frac{1}{2}\}$ **e** $[-10, 10]$ **f** $(-\infty, 4]$

10 One-to-one functions: b, c, d, e, f, g, j

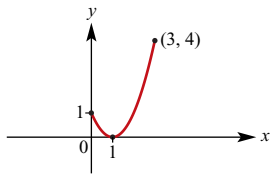
11 a



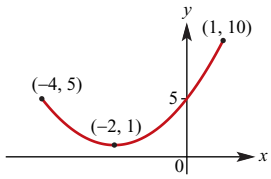
b



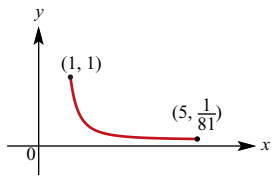
12 a Range = $[0, 4]$



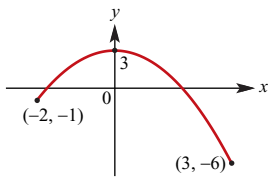
b Range = $[1, 10]$



c Range = $[\frac{1}{81}, 1]$



d Range = $[-6, 3]$



13 a Domain = $[1, \infty)$; Range = $[0, \infty)$

b Domain = $(-\infty, 1]$; Range = $[0, \infty)$

c Domain = $[0, \infty)$; Range = $(-\infty, 1]$

14 a Domain = $\mathbb{R} \setminus \{1\}$; Range = $\mathbb{R} \setminus \{0\}$

b Domain = $\mathbb{R} \setminus \{-1\}$; Range = $\mathbb{R} \setminus \{0\}$

c Domain = $\mathbb{R} \setminus \{1\}$; Range = $\mathbb{R} \setminus \{3\}$

15 a Domain = $[-1, 1]$; Range = $[0, 1]$

b Domain = $[-3, 3]$; Range = $[0, 3]$

c Domain = $[-1, 1]$; Range = $[3, 4]$

16 a $f^{-1}(x) = \frac{x + 2}{3}$; Domain = $[-5, 13]$

b $f^{-1}(x) = (x - 2)^2 - 2$; Domain = $[2, \infty)$

c $f^{-1}(x) = \sqrt{\frac{x}{3}} - 1$; Domain = $[0, \infty)$

d $f^{-1}(x) = -\sqrt{x} + 1$; Domain = $(0, \infty)$

17 a $2p + 5$ **b** $2(p + h) + 5$

c $2h$ **d** 2

18 -2

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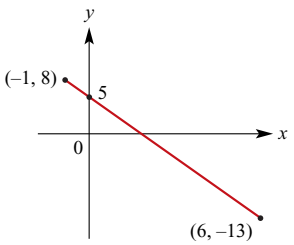
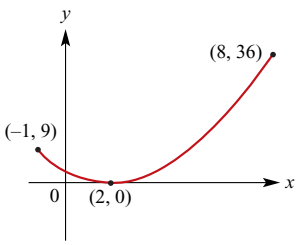
19 a $\left(-\infty, -\frac{15}{8}\right]$ **b** $[3\frac{7}{8}, \infty)$ **c** $(-\infty, 20]$
d $(-\infty, 3]$

20 a



b Range = $[-13, 8]$

21 a



b Range = $[0, 36]$

22 a $\{2, 4, 6, 8\}$ **b** $\{4, 3, 2, 1\}$
c $\{-3, 0, 5, 12\}$ **d** $\{1, \sqrt{2}, \sqrt{3}, 2\}$

Multiple-choice questions

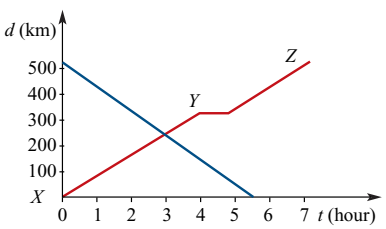
1 B **2** E **3** D **4** E **5** C
6 E **7** B **8** D **9** C **10** D

Extended-response questions

1 a 40 mg/L **b** 45 mg/L **c** 36 mg/L
d At $t = 9$ hours; $C = 9$ mg/L
e 54 mg/L (quite a lot)

2 a 3 m
b i 8 m **ii** 4.8 m **iii** 4 m
c $5 - \sqrt{10} \approx 1.84$ m, 8.5 m

3 a



Coach starting from X:

$$d = \begin{cases} 80t & \text{for } 0 \le t \le 4 \\ 320 & \text{for } 4 < t \le 4\frac{3}{4} \\ 80t - 60 & \text{for } 4\frac{3}{4} < t \le 7\frac{1}{4} \end{cases}$$

Range = $[0, 520]$

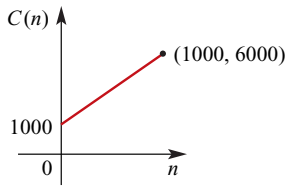
Coach starting from Z:

$d = 520 - \dfrac{1040t}{11}$ for $0 \le t \le 5\frac{1}{2}$

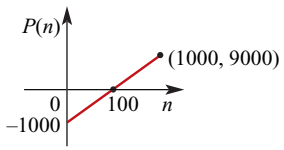
Range = $[0, 520]$

b The coaches pass $238\frac{1}{3}$ km from X

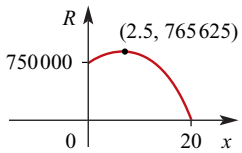
4 a i $C(n) = 1000 + 5n$, $n > 0$
ii



b i $P(n) = 15n - (1000 + 5n)$
$= 10n - 1000$
ii



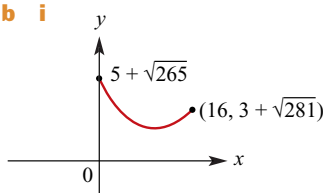
5 a $R = (50000 - 2500x)(15 + x)$
$= 2500(x + 15)(20 - x)$
b



c Price for max revenue = \$17.50

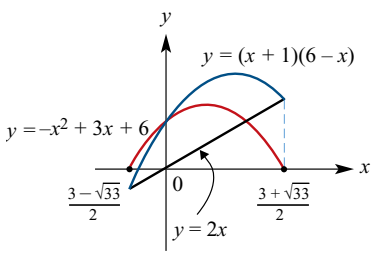
6 a $A(x) = \dfrac{x}{4}(2a - (6 - \sqrt{3})x)$ **b** $0 < x < \dfrac{a}{3}$

7 a i $d(x) = \sqrt{x^2 + 25} + \sqrt{(16 - x)^2 + 9}$
ii $0 \le x \le 16$
b i



ii 1.54
iii 3.40 or 15.04
c i Minimum is $8\sqrt{5}$, occurs when $x = 10$
ii Range = $[8\sqrt{5}, 5 + \sqrt{265}]$

8 a $A\left(\dfrac{3 + \sqrt{33}}{2}, 3 + \sqrt{33}\right)$, $B\left(\dfrac{3 - \sqrt{33}}{2}, 3 - \sqrt{33}\right)$
b i $d(x) = -x^2 + 3x + 6$
ii



c i Maximum value of $d(x)$ is 8.25
ii Range = $[0, 8.25]$
d $A(2.45, 12.25)$, $B(-2.45, -12.25)$
$d(x) = -x^2 + 6$
Maximum value of $d(x)$ is 6; Range = $[0, 6]$

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Chapter 6

Exercise 6A

1 **a** -3 **b** -1 **c** -7 **d** -15

2 **a** 0 **b** 0

3 **a** 6 **b** 9 **c** 26 **d** 11
e $a^3 + 4a^2 - 2a + 6$ **f** $8a^3 + 16a^2 - 4a + 6$

4 **a** $a = 4$ **b** $a = 4$ **c** $c = 6$
d $a = -33, b = -15$ **e** $a = -9, b = 23$

5 **a** $x^3 - 2x^2 - 2x + 2$ **b** $x^3 - x^2 + 2x$
c $x^3 - 2x^2 + 4x - 2$ **d** $3x^3 - 6x^2 + 3x$
e $-3x^4 + 8x^3 - 7x^2 + 2x$
f $-3x^3 - x^2 + 2x$ **g** $x^3 - x^2 - x + 2$
h $x^5 - x^4 - x^3 + x^2$

6 **a** $x^3 - 4x^2 + 7x - 6$ **b** $x^3 - 6x^2 + 11x - 12$
c $2x^3 - 5x^2 - x + 4$
d $x^3 + (b-2)x^2 + (c-2b)x - 2c$
e $2x^3 - 7x^2 - 10x - 3$

7 **a** $x^3 + (b+1)x^2 + (c+b)x + c$
b $b = -8, c = 12$ **c** $(x+1)(x-6)(x-2)$

8 $b = -3, c = -11$

Exercise 6B

1 **a** $x^2 + 2x + \dfrac{3}{x-1}$ **b** $2x^2 - x - 3 + \dfrac{6}{x+1}$
c $3x^2 - 10x + 22 - \dfrac{43}{x+2}$
d $2x^2 + 3x + 10 + \dfrac{28}{x-3}$

2 **a** $x^2 - x + 4 - \dfrac{8}{x+1}$ **b** $2x^2 - 8x + 49 - \dfrac{181}{x+4}$
c $x^2 + x - 3 + \dfrac{11}{x+3}$ **d** $x^2 - x + 4 + \dfrac{8}{x-2}$

3 **a** $x^2 - 2x + 5$ **b** $2x^2 - 2x - 6$
c $x^2 - 2x - 6$ **d** $3x^2 - x - 6$

4 **a** Quotient $x^2 - 3$; Remainder 7
b Quotient $x^2 + 2x + 15$; Remainder 71
c Quotient $2x^2 - 3x$; Remainder -7
d Quotient $5x^2 + 20x + 77$; Remainder 315

5 **a** $\dfrac{1}{2}x^2 + \dfrac{7}{4}x - \dfrac{3}{8} + \dfrac{103}{8(2x+5)}$
b $x^2 + 2x - 3 - \dfrac{2}{2x+1}$

6 **a** $x^2 + 2x - 15$
b $\dfrac{1}{3}x^2 - \dfrac{8}{9}x - \dfrac{8}{27} + \dfrac{19}{27(3x-1)}$

7 **a** $x^2 + 3x + 8 + \dfrac{9}{x-1}$
b $x^2 - \dfrac{x}{2} + \dfrac{9}{4} + \dfrac{21}{4(2x-1)}$

8 **a** Quotient $2x - 6$; Remainder 0
b Quotient $x - 6$; Remainder -2
c Quotient $2x - 6$; Remainder 42
d Quotient $x^2 - 4x + 2$; Remainder $-x + 7$

9 **a** Quotient $x^2 - 3x + 7$; Remainder $-10x + 9$
b Quotient $x^2 + x - \dfrac{3}{2}$; Remainder $\dfrac{15}{2}x + 16$

Exercise 6C

1 **a** -2 **b** -29 **c** 15 **d** 4 **e** 7
f -12 **g** 0 **h** -5 **i** -8

2 **a** $a = -3$ **b** $a = 2$ **c** $a = 4$ **d** $a = -10$

3 **a** $P(1) = 0$ **b** $P(1) = 0$
c $P(-2) = 0$ **d** $P(\frac{3}{2}) = 0$

4 **a** 6 **b** 28 **c** $-\dfrac{1}{3}$

5 **a** $(x-1)(x+1)(2x+1)$ **b** $(x+1)^3$
c $(x-1)(6x^2 - 7x + 6)$
d $(x-1)(x+5)(x-4)$ **e** $(x+1)^2(2x-1)$
f $(x+1)(x-1)^2$ **g** $(x-2)(4x^2 + 8x + 19)$
h $(x+2)(2x+1)(2x-3)$

6 1

7 **a** $(2x-3)(x^2 - 2x + 5)$
b $(2x+1)(x^2 - 2x + 5)$
c $(2x+1)(x-1-\sqrt{6})(x-1+\sqrt{6})$
d $(2x+3)(x-1-\sqrt{2})(x-1+\sqrt{2})$

8 **a** $(x-1)(x^2 + x + 1)$
b $(x+4)(x^2 - 4x + 16)$
c $(3x-1)(9x^2 + 3x + 1)$
d $(4x-5)(16x^2 + 20x + 25)$
e $(1-5x)(1+5x+25x^2)$
f $(3x+2)(9x^2 - 6x + 4)$
g $(4m-3n)(16m^2 + 12mn + 9n^2)$
h $(3b+2a)(9b^2 - 6ab + 4a^2)$

9 **a** $(x+2)(x^2 - x + 1)$
b $(3x+2)(x-1)(x-2)$
c $(x-3)(x+1)(x-2)$
d $(3x+1)(x+3)(2x-1)$

10 $a = 3, b = -3, P(x) = (x-1)(x+3)(x+1)$

11 **b** **i** n odd **ii** n even

12 **a** $a = 1, b = 1$ **b** **i** $P(x) = x^3 - 2x^2 + 3$

Exercise 6D

1 **a** 1, -2, 4 **b** 4, 6 **c** $\dfrac{1}{2}$, 3, $-\dfrac{2}{3}$ **d** 0, -3, $\dfrac{5}{2}$

2 **a** -2, 0, 4 **b** $0, -1 \pm 2\sqrt{3}$ **c** -5, 0, 8
d $0, -1 \pm \sqrt{17}$

3 **a** 1 **b** -1 **c** $5, \pm\sqrt{10}$ **d** $\pm 4, a$

4 **a** 2, 3, -5 **b** $-1, -\dfrac{2}{3}, 3$ **c** $1, -\sqrt{2}, \sqrt{2}$
d $-\dfrac{2}{5}, -4, 2$ **e** $-\dfrac{1}{2}, \dfrac{1}{3}, 1$ **f** $-2, -\dfrac{3}{2}, 5$

5 **a** -6, 2, 3 **b** $-2, -\dfrac{2}{3}, \dfrac{1}{2}$ **c** 3
d -1 **e** -1, 3 **f** $3, -2 \pm \sqrt{3}$

6 **a** $0, \pm 2\sqrt{2}$ **b** $1 + 2\sqrt[3]{2}$ **c** -2
d -5 **e** $\dfrac{1}{10}$

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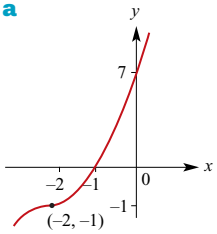
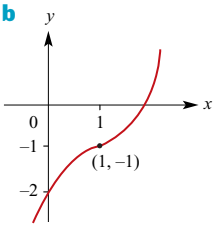
7 a $2(x-9)(x-13)(x+11)$
b $(x+11)(x+3)(2x-1)$
c $(x+11)(2x-9)(x-11)$
d $(2x-1)(x+11)(x+15)$

Exercise 6E

1 a



b



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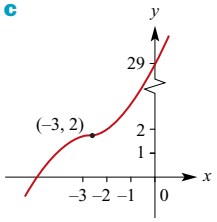

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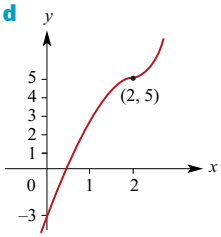

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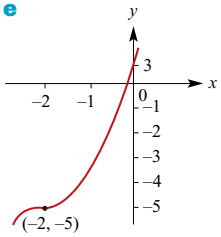
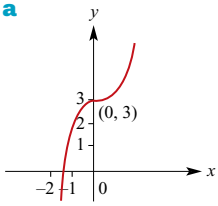


2 a



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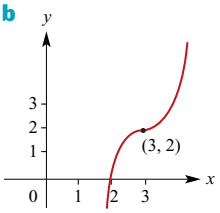

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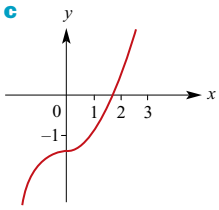

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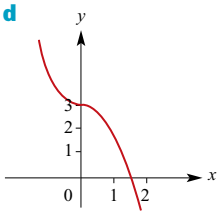
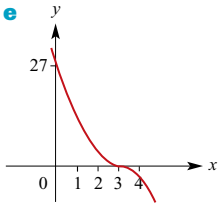


e



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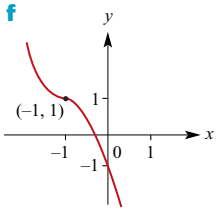
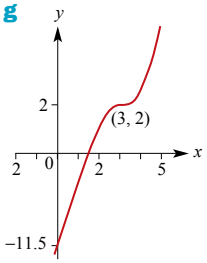


g



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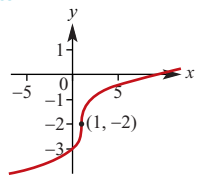

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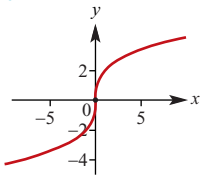

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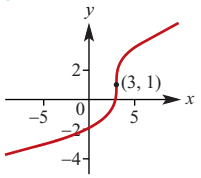

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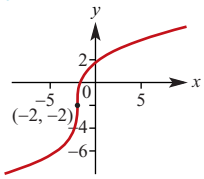

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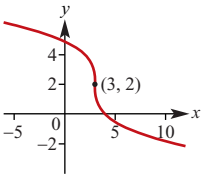

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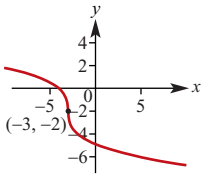


4 a $f^{-1}(x) = \left(\dfrac{x-3}{2}\right)^{\frac{1}{3}}$

b $f^{-1}(x) = \dfrac{x^3}{27}$

c $f^{-1}(x) = \left(\dfrac{x-1}{2}\right)^{\frac{1}{3}} - 1$

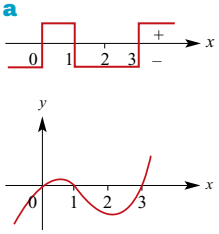
d $f^{-1}(x) = \dfrac{(x+2)^3}{8} - 3$

e $f^{-1}(x) = 1 - \dfrac{(x-4)^3}{8}$

f $f^{-1}(x) = -\dfrac{(x+1)^3}{8} - 2$

Exercise 6F

1 a



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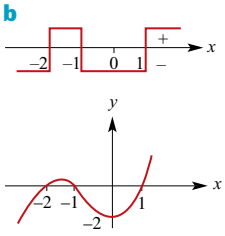

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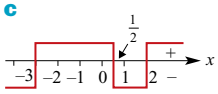

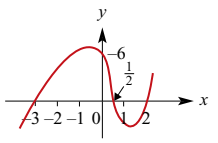

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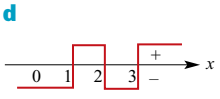

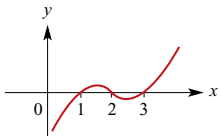

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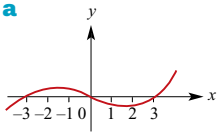

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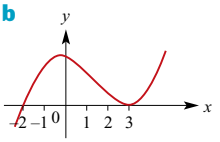

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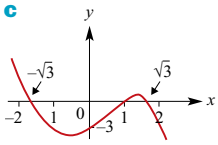

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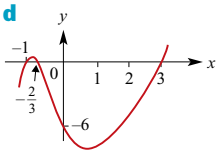

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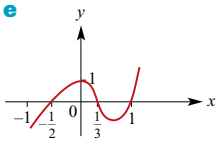

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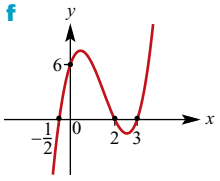

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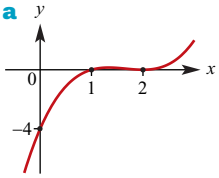

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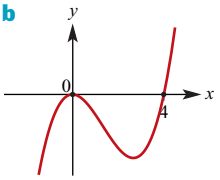

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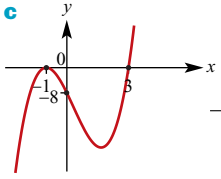

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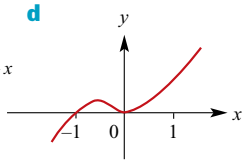

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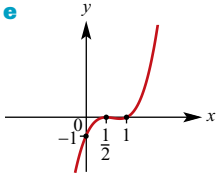

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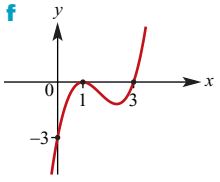

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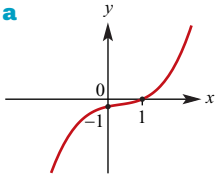

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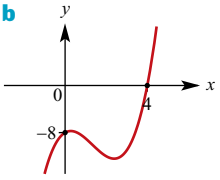

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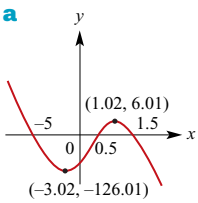

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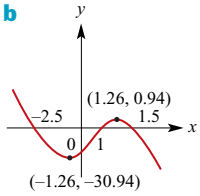

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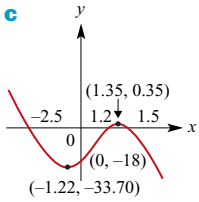

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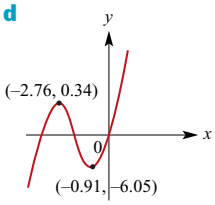

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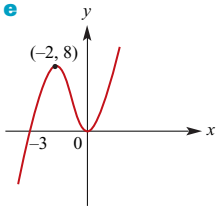

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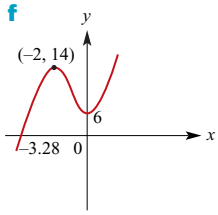

6 $f(x) = (x + 1)^2(x - 3)$, so graph just touches the x-axis at $x = -1$ and cuts it at $x = 3$

Exercise 6G

1 a $(-\infty, -2] \cup [1, 3]$ **b** $[-2, -1] \cup [4, \infty)$
c $(-\infty, 1)$ **d** $(-2, 0) \cup (3, \infty)$
e $(-\infty, -1]$ **f** $[1, \infty)$
g $(4, \infty)$ **h** $(-\infty, -3]$

2 a $(-2, 0) \cup (2, \infty)$ **b** $(-\infty, 0) \cup (0, 5)$
c $(-\infty, 0] \cup \{2\}$ **d** $(-3, 0) \cup (3, \infty)$
e $[6, \infty)$ **f** $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, 3)$

Exercise 6H

1 a $a = 11$ **b** $a = 2$ **c** $a = \frac{4}{3}, b = \frac{44}{3}$

2 a $y = -\frac{1}{8}(x + 2)^3$ **b** $y - 2 = -\frac{1}{4}(x - 3)^3$

3 $y = 2x(x - 2)^2$

4 $y = -2x(x + 4)^2$

5 $y = -2(x - 1)(x - 3)(x + 1)$

6 $a = 36$

7 a $y = (x - 3)^3 + 2$ **b** $y = \frac{23}{18}x^3 + \frac{67}{18}x^2$
c $y = 5x^3$

8 a $y = -\frac{1}{3}x^3 + \frac{4}{3}x$ **b** $y = \frac{1}{4}x(x^2 + 2)$

9 a $y = -4x^3 - 50x^2 + 96x + 270$
b $y = 4x^3 - 60x^2 + 80x + 26$
c $y = x^3 - 2x^2 + 6x - 4$
d $y = 2x^3 - 3x$
e $y = 2x^3 - 3x^2 - 2x + 1$
f $y = x^3 - 3x^2 - 2x + 1$
g $y = -x^3 - 3x^2 - 2x + 1$

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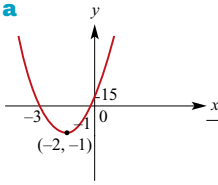
Exercise 6I

1 a



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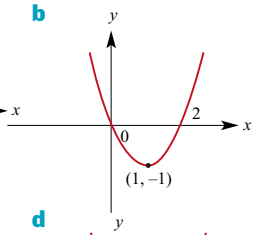

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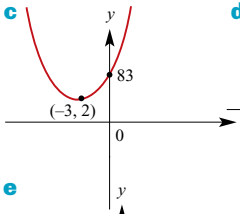

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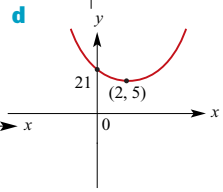

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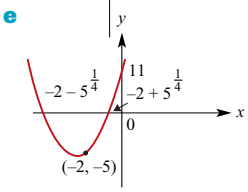

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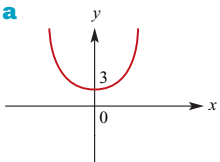

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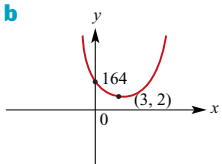

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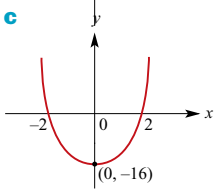

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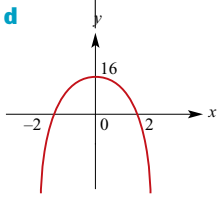

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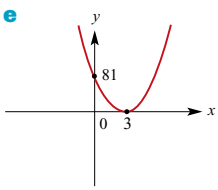

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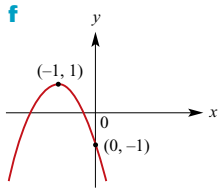


3 a $x = 0$ or $x = 3$

b $x = 2$ or $x = -1$ or $x = 5$ or $x = -3$

c $x = 0$ or $x = -2$ **d** $x = 0$ or $x = 6$

e $x = 0$ or $x = 3$ or $x = -3$

f $x = 3$ or $x = -3$

g $x = 0$ or $x = 4$ or $x = -4$

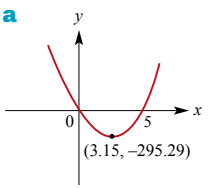
h $x = 0$ or $x = 4$ or $x = 3$

i $x = 0$ or $x = 4$ or $x = 5$

j $x = 2$ or $x = -2$ or $x = 3$ or $x = -3$

k $x = 4$ **l** $x = -4$ or $x = 2$

4 a



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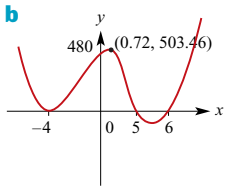

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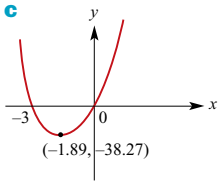

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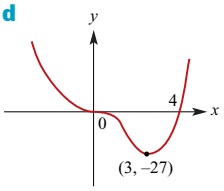

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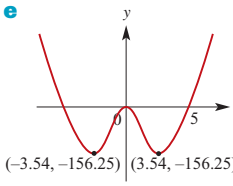

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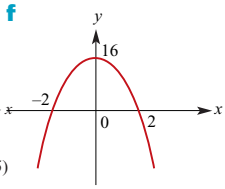

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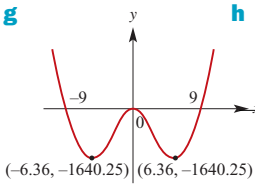

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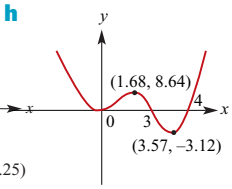

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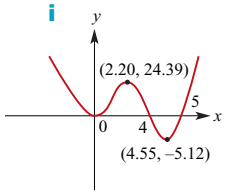

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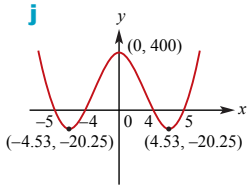

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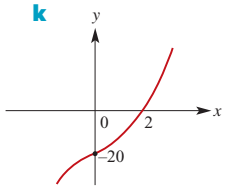

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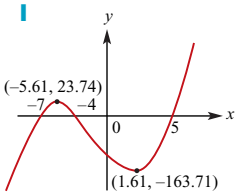
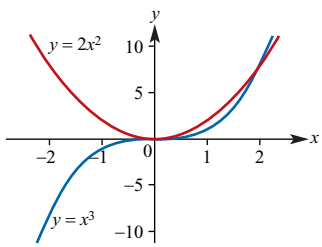


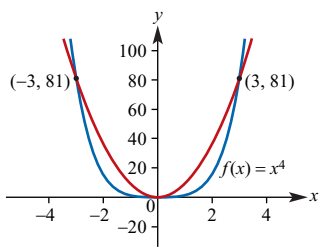
5 a Even **b** Odd **c** Even **d** Odd

6 a



b $x = 0$, $x = 2$ **c** $\{x : x \leq 2\}$

7 a



b $x = 0$, $x = 3$, $x = -3$

c $\{x : -3 \leq x \leq 3\}$

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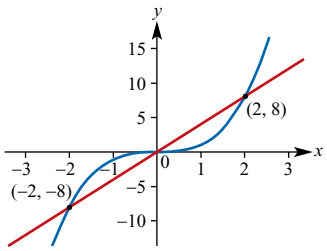
8 a



b $x = 0$, $x = 2$, $x = -2$

c $\{x : x \leq -2\} \cup \{x : 0 \leq x \leq 2\}$

9 a 0 **b** 2 **c** 1 **d** 4 **e** 3 **f** 1

Exercise 6J

1 a Length of each edge $= 20 - 2x$

b $V(x) = 4x(10 - x)^2$

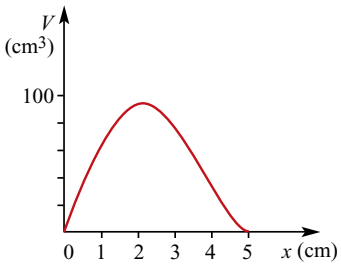
c $V(5) = 500$; Volume of box $= 500$ cm^3

d $x = 5$ or $x = \frac{5}{2}(3 - \sqrt{5})$

2 a $\ell = 12 - 2x$, $w = 10 - 2x$

b $V = 4x(6 - x)(5 - x)$

c



d $V = 80$ **e** $x = 3.56$ or $x = 0.51$

f $V_{\max} = 96.8$ cm^3 when $x = 1.81$

3 a Surface area $= x^2 + 4xh$

b $h = \frac{75 - x^2}{4x}$ **c** $V = \frac{1}{4}(75x - x^3)$

d i $\frac{71}{2}$ **ii** $\frac{125}{2}$ **iii** 22

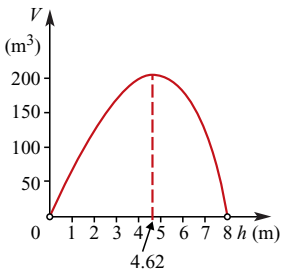
e $x = -2 + 3\sqrt{7}$

4 a $h = 60 - 20x$ **b** $V = 600x^2(3 - x)$

c 0 **d** $x = 1$ or $x = 1 + \sqrt{3}$

5 a $x = \sqrt{64 - h^2}$ **b** $V = \frac{\pi h}{3}(64 - h^2)$

c



d Domain $= \{h : 0 < h < 8\}$ **e** 64π

f $h = 2.48$ or $h = 6.47$

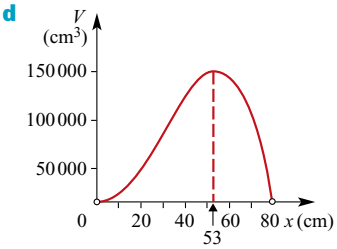
g $V_{\max} \approx 206.37$ m^3 when $h = 4.62$

6 a $h = 160 - 2x$

b $V = x^2(160 - 2x)$

c Domain $= (0, 80)$

d



e $x = 20.498$ or $x = 75.63$

f $V_{\max} \approx 151\,703.7$ cm^3 when $x \approx 53$

Exercise 6K

1 a 1.32 **b** 1.164 **c** 1.124 or 1.451

d 2.151 **e** −1.75

2

```
define f(x):
  return −x^3 + 3x + 6

a ← 2
b ← 3
m ← 2.5
while b − a > 2 × 0.001
  if f(a) × f(m) < 0 then
    b ← m
  else
    a ← m
  end if
  m ← (a + b)/2
  print a, m, b, f(a), f(m), f(b)
end while
print m
```

3 a $f(1) = 2$, $f(2) = -8$; $[1, 2]$

b $x = 1.29$

4 a

	a	m	b
Pass 1	−3.5	−3.25	−3
Pass 2	−3.25	−3.125	−3
Pass 3	−3.25	−3.1875	−3.125

b

	a	m	b
Pass 1	−3	−2.75	−2.5
Pass 2	−3	−2.875	−2.75
Pass 3	−2.875	−2.8125	−2.75

c

	a	m	b
Pass 1	−1	−0.75	−0.5
Pass 2	−0.75	−0.625	−0.5
Pass 3	−0.75	−0.6875	−0.625

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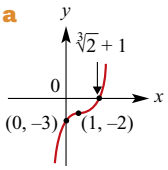
Chapter 6 review

Technology-free questions

1 a



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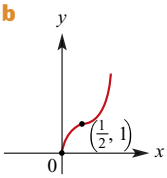

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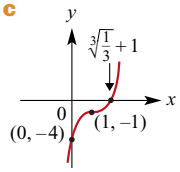

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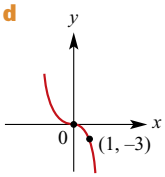

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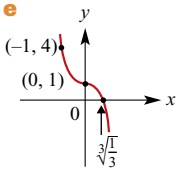

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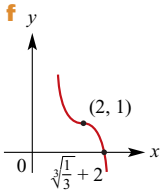

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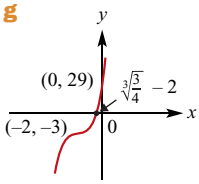

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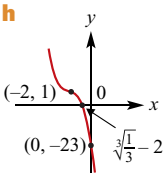

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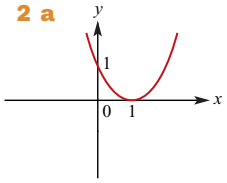

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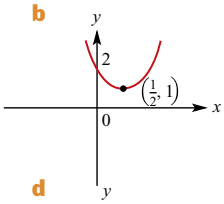

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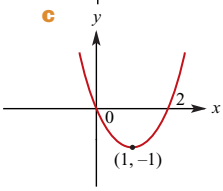

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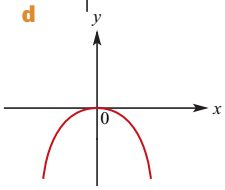

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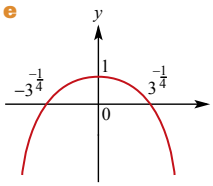

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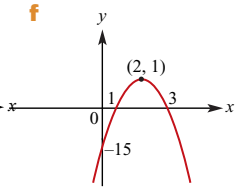

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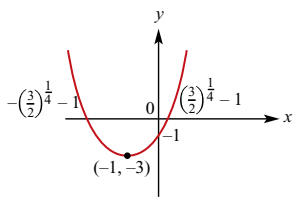

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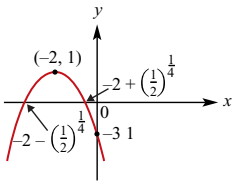


3 a $x = 2,\ x = -\dfrac{1}{2},\ x = -3$

b $x = 2,\ x = \dfrac{\sqrt{17}+1}{4},\ x = \dfrac{1-\sqrt{17}}{4}$

c $x = -1,\ x = 2,\ x = 6$

4 a $P\left(\dfrac{3}{2}\right) = 0$ and $P(-2) = 0$; $(3x+1)$

b $x = -2,\ \dfrac{1}{2},\ 3$

c $x = -1,\ -\sqrt{11},\ +\sqrt{11}$

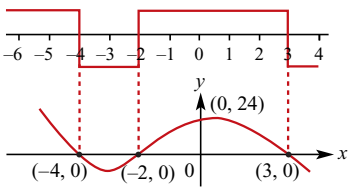
d i $P\left(\dfrac{1}{3}\right) = 0$ **ii** $(3x-1)(x+3)(x-2)$

5 a $f(1) = 0$

b $(x-1)(x^2 + (1-k)x + k + 1)$

6 $a = 3,\ b = -24$

7 a



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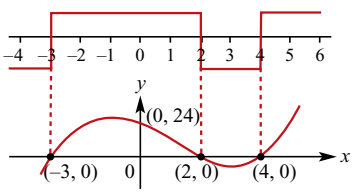

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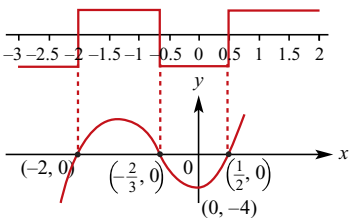

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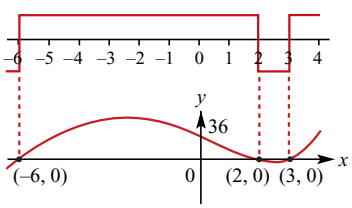

8 a -41 **b** 12 **c** $\dfrac{43}{9}$

9 $y = -\dfrac{2}{5}(x+2)(x-1)(x-5)$

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10 $y = \frac{2}{81}x(x+4)^2$

11 a $a = 3, b = 8$

b $(x+3)(2x-1)(x-1)$

12 a $(\infty, -4] \cup \{3\}$ **b** $(-\infty, -4] \cup [-3, 2]$

c $(-\infty, -1) \cup (2, 3)$

13 a Dilation of factor 2 from the x-axis, then translation of 1 unit in the positive direction of the x-axis and 3 units in the positive direction of the y-axis

b Reflection in the x-axis, then translation of 1 unit in the negative direction of the x-axis and 2 units in the positive direction of the y-axis

c Dilation of factor $\frac{1}{2}$ from the y-axis, then translation of $\frac{1}{2}$ unit in the negative direction of the x-axis and 2 units in the negative direction of the y-axis

Multiple-choice questions

1 B **2** D **3** A **4** D **5** A
6 C **7** B **8** B **9** D **10** B

Extended-response questions

1 a $V = \pi r^2(6-r)$ **b** $0 \leq r \leq 6$

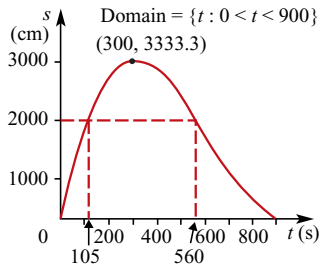
c $V(3) = 27\pi$ **d** $r = 3$ or $r = \frac{3}{2}(1+\sqrt{5})$

e Maximum ≈ 100.53 (correct to 2 d.p.)

2 a $v = \frac{1}{32\,400}(t-900)^2$

b $s = \frac{t}{32\,400}(t-900)^2$

c



d No, it is not feasible as the maximum range of the taxi is less than 3.5 km (≈ 3.33 km)

e Maximum speed $\approx \frac{2000}{105} = 19$ m/s

Minimum speed $\approx \frac{2000}{560} = 3.6$ m/s

3 a $a = -\frac{43}{15\,000}$, $b = 0.095$, $c = -\frac{119}{150}$, $d = 15.8$

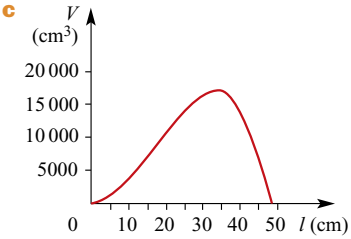
b i (5.59, 13.83) **ii** (0, 15.8)

4 a $R = a(x-5)^3 + 10$ **b** $a = \frac{2}{25}$

c $R = \frac{12}{343}(x-7)^3 + 12$

5 a 4730 cm^2 **b** $V = \ell^2(\sqrt{2365} - \ell)$

c



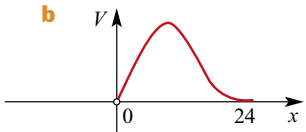
d i $\ell = 23.69$ or $\ell = 39.79$

ii $\ell = 18.1$ or $\ell = 43.3$

e $V_{max} \approx 17\,039$ cm^3 when $\ell \approx 32.42$ cm

6 a $V = (96-4x)(48-2x)x = 8x(24-x)^2$

b



i $0 < x < 24$

ii $V_{max} = 16\,384$ cm^3 when $x = 8.00$

c 15 680 cm^3 **d** 14 440 cm^3 **e** 9720 cm^3

Chapter 7

Exercise 7A

1 a $(-1, 1)$ **b** $(-5, 8)$ **c** $(-6, 2)$

d $(-7, 9)$ **e** $(-5, 3)$

2 a A translation of 5 units in the negative direction of the x-axis and 3 units in the positive direction of the y-axis

b A translation of 6 units in the positive direction of the x-axis and 15 units in the negative direction of the y-axis

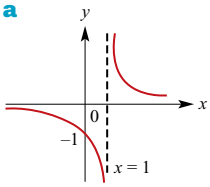
c A translation of 12 units in the negative direction of the x-axis and 17 units in the positive direction of the y-axis

3 a $g(x) = \frac{1}{x-2} - 1$ **b** $g(x) = \frac{1}{(x-4)^2} + 3$

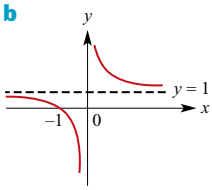
c $g(x) = (x+2)^2 - 3$ **d** $g(x) = (x-4)^3 - 2$

e $g(x) = \sqrt{x-2} - 1$

4 a



b



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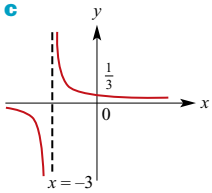

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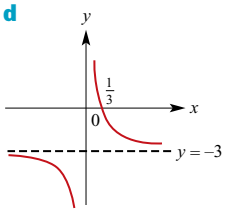

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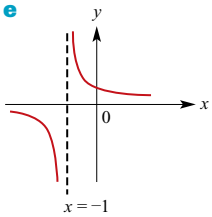

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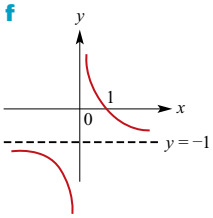

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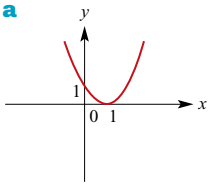

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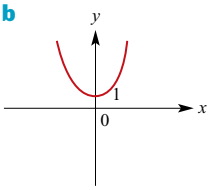

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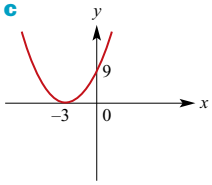

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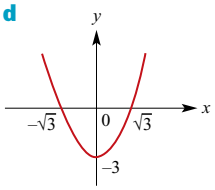

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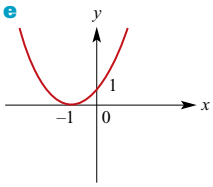

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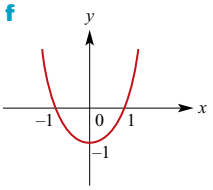

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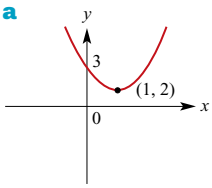

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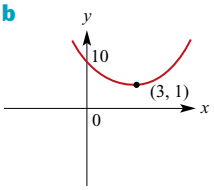

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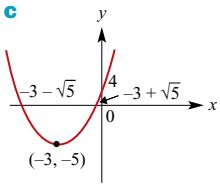

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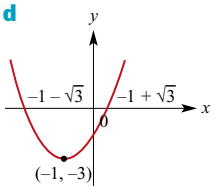

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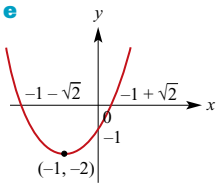

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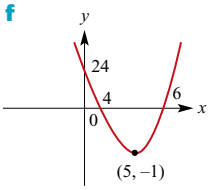


Exercise 7B

1 a $(-2, 3)$ **b** $(2, -3)$ **c** $(-2, -12)$ **d** $(-8, -3)$

2 a i $y = 4x^2$ **ii** $y = \frac{x^2}{25}$ **iii** $y = \frac{2x^2}{3}$

iv $y = 4x^2$ **v** $y = -x^2$ **vi** $y = x^2$

b i $y = \frac{1}{4x^2}$ **ii** $y = \frac{25}{x^2}$ **iii** $y = \frac{2}{3x^2}$

iv $y = \frac{4}{x^2}$ **v** $y = -\frac{1}{x^2}$ **vi** $y = \frac{1}{x^2}$

c i $y = \frac{1}{2x}$ **ii** $y = \frac{5}{x}$ **iii** $y = \frac{2}{3x}$

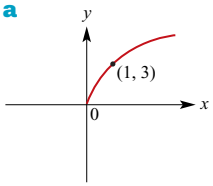
iv $y = \frac{4}{x}$ **v** $y = -\frac{1}{x}$ **vi** $y = -\frac{1}{x}$

d i $y = \sqrt{2x}$ **ii** $y = \sqrt{\frac{x}{5}}$ **iii** $y = \frac{2\sqrt{x}}{3}$

iv $y = 4\sqrt{x}$ **v** $y = -\sqrt{x}$

vi $y = \sqrt{-x}$, $x \leq 0$

3 a



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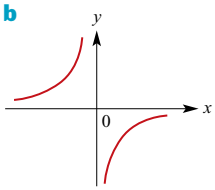

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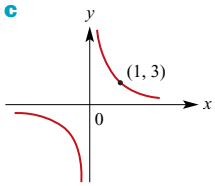

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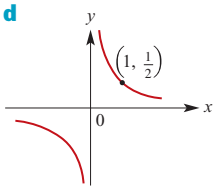

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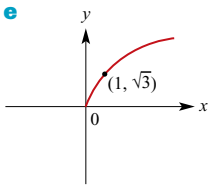

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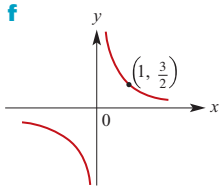

Exercise 7C

1 a $(5, -2)$ **b** $(6, 1)$ **c** $(3, -5)$

2 a $y = 3\sqrt{x-2}$ **b** $y = -\sqrt{x+3}$

c $y = -3\sqrt{x}$ **d** $y = -\sqrt{\frac{x}{2}}$

e $y = 2\sqrt{x-2} - 3$

3 a $y = \frac{3}{x-2}$ **b** $y = -\frac{1}{x+3}$ **c** $y = -\frac{3}{x}$

d $y = -\frac{2}{x}$ **e** $y = \frac{2}{x-2} - 3$

4 a $y = 3(x-2)^{\frac{1}{3}}$ **b** $y = -(x+3)^{\frac{1}{3}}$

c $y = -3x^{\frac{1}{3}}$ **d** $y = -\left(\frac{x}{2}\right)^{\frac{1}{3}}$

e $y = 2(x-2)^{\frac{1}{3}} - 3$

5 a $y = \frac{1}{9}(x-6)^2$ **b** $y = \frac{1}{9}(x-2)^2$

6 a $y = -\frac{1}{x+2}$ **b** $y = \frac{1}{2-x}$

7 a $y = 2x^2$ **b** $y = -2(x-2)^2$

c $y = \frac{2}{9}(x+6)^2 + 3$

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8 a $y = \dfrac{6}{x-6} + 5$ **b** $y = \dfrac{6}{x-10} + 5$

9 a $a = 0$ and $k = 1$, or $a = \dfrac{4}{3}$ and $k = 9$

b $a = \sqrt{2}$ and $k = 3 + 2\sqrt{2}$, or $a = -\sqrt{2}$ and $k = 3 - 2\sqrt{2}$

Exercise 7D

1 a i A dilation of factor 2 from the x-axis, then a translation of 1 unit in the positive direction of the x-axis and 3 units in the positive direction of the y-axis

ii A reflection in the x-axis, then a translation of 1 unit in the negative direction of the x-axis and 2 units in the positive direction of the y-axis

iii A dilation of factor $\frac{1}{2}$ from the y-axis, then a translation of $\frac{1}{2}$ unit in the negative direction of the x-axis and 2 units in the negative direction of the y-axis

b i A dilation of factor 2 from the x-axis, then a translation of 3 units in the negative direction of the x-axis

ii A translation of 3 units in the negative direction of the x-axis and 2 units in the positive direction of the y-axis

iii A translation of 3 units in the positive direction of the x-axis and 2 units in the negative direction of the y-axis

c i A translation of 3 units in the negative direction of the x-axis and 2 units in the positive direction of the y-axis

ii A dilation of factor $\frac{1}{3}$ from the y-axis, then a dilation of factor 2 from the x-axis

iii A reflection in the x-axis, then a translation of 2 units in the positive direction of the y-axis

2 a $(x, y) \to \left(x - 3, \dfrac{y+7}{5}\right)$

b $(x, y) \to (3x + 2, y - 5)$

c $(x, y) \to \left(3x + 1, -\dfrac{y-7}{3}\right)$

d $(x, y) \to \left(-(x - 4), \dfrac{y}{2}\right)$

e $(x, y) \to \left(-(x - 4), \dfrac{15 - y}{2}\right)$

3 a $(x, y) \to \left(\dfrac{2x}{3}, y - 5\right)$

b $(x, y) \to (x + 3, 3y + 7)$

c $(x, y) \to \left(\dfrac{x+4}{3}, y - 9\right)$

d $(x, y) \to \left(9 - x, \dfrac{5y}{2}\right)$

e $(x, y) \to \left(2 - x, \dfrac{27 - 5y}{2}\right)$

4 a $a = 2$ and $b = 3$

b A dilation of factor 2 from the x-axis, then a translation of 2 units in the positive direction of the x-axis and 3 units in the positive direction of the y-axis

Exercise 7E

1 $y = -\dfrac{2x^2}{9} - \dfrac{2x}{3} - 4$ **2** $y = -\dfrac{x^3}{32} - x$

3 $y = -\dfrac{3x}{4} + \dfrac{9}{2}$ **4** $y = \dfrac{x}{4} - 4$

5 a $y = \dfrac{x}{4} + \dfrac{9}{2}$ **b** $y = \dfrac{x}{4} + \dfrac{21}{4}$ **c** $y = 3x + 26$

6 A dilation of factor 3 from the x-axis, then a translation of 1 unit in the positive direction of the x-axis and 22 units in the negative direction of the y-axis

7 A reflection in the x-axis, then a dilation of factor 2 from the x-axis, then a translation of 5 units in the negative direction of the x-axis and 31 units in the negative direction of the y-axis

8 a $y = \dfrac{6}{x-8} + 2$

b A dilation of factor $\frac{2}{3}$ from the x-axis, then a translation of 6 units in the positive direction of the y-axis

9 a $y = -5(2x + 3)^2$

b A reflection in the x-axis, then a dilation of factor 5 from the x-axis, then a dilation of factor $\frac{1}{2}$ from the y-axis, then a translation of $\frac{3}{2}$ units in the negative direction of the x-axis

10 a $y = -\dfrac{2}{(x-3)^2} + 4$

b A reflection in the x-axis, then a dilation of factor 2 from the x-axis, then a translation of 3 units in the positive direction of the x-axis and 4 units in the positive direction of the y-axis

11 a $a = -1,\ h = -1,\ b = 1,\ k = 1$

b $y = (x + 1)^2 + 1$

12 a $a = -1,\ h = -1,\ b = 14,\ k = -45$

b $y = 14x^2 - 69$

13 a $a = 11,\ h = -15,\ b = 0,\ k = 5$

b $y = 5$

14 a $a = \dfrac{1}{2}$ **b** $\left(-\dfrac{1}{3}, \dfrac{1}{9}\right)$

15 a $a = \dfrac{1 + \sqrt{2}}{2}$ **b** $\left(\dfrac{2 + \sqrt{2}}{2}, \dfrac{2 + \sqrt{2}}{2}\right)$

16 a $a = \dfrac{1}{4}$ **b** $(-1, 2)$

17 a $(0, 0), (6, 0)$ **b** $(-1, 0), (2, 0)$

c $(3, 0), (8, 0)$

18 a $y = 3x - 12$ **b** $y = \dfrac{1}{9}(x - 2)^3$ **c** $(5, 3)$

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19 a $f^{-1}: \mathbb{R} \setminus \{0\} \to \mathbb{R}, f^{-1}(x) = \frac{1}{5x} + \frac{2}{5}$

b $(x, y) \to \left(x - \frac{2}{5}, y + \frac{2}{5}\right)$

21 a $T^{-1}(x, y) = \left(\frac{x-3}{2}, -\frac{y}{4}\right)$

b $T^{-1}(x, y) = \left(3 - x, -\frac{y}{4}\right)$

c $T^{-1}(x, y) = \left(2x - 6, \frac{5-y}{2}\right)$

Chapter 7 review

Technology-free questions

1 a $(-1, 12)$ **b** $(-3, 3)$ **c** $(-1, -3)$
d $(1, 3)$ **e** $(3, -1)$

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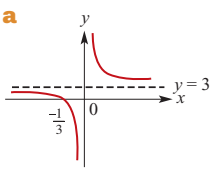

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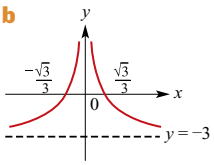

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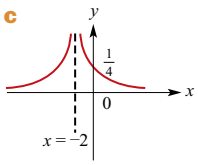

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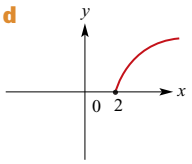

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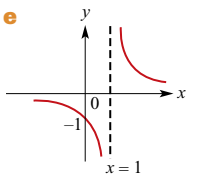

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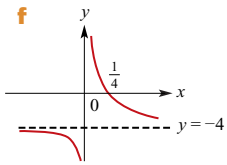

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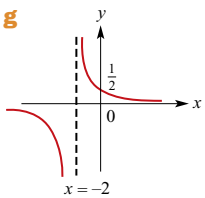

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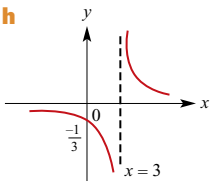

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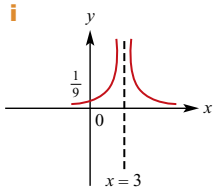

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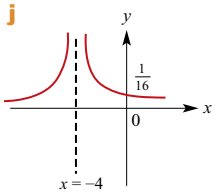

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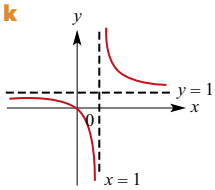

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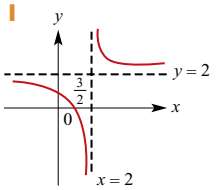
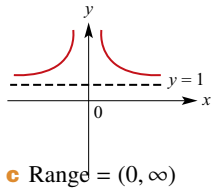
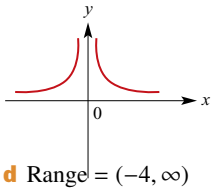
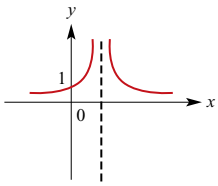


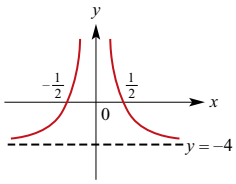
3 a Range $= (1, \infty)$ **b** Range $= (0, \infty)$





c Range $= (0, \infty)$ **d** Range $= (-4, \infty)$





4 a $(x, y) \to (x + 2, 2y + 3)$

b $x = x' - 2,\ y = \frac{y' - 3}{2}$

5 a i $(x, y) \to (x - 1, 3y + 2)$

ii $(x, y) \to (x - 2, -2y + 3)$

iii $(x, y) \to \left(\frac{x-1}{3}, y - 1\right)$

b i $(x, y) \to (x - 2, 4y)$

ii $(x, y) \to (x - 6, y - 12)$

iii $(x, y) \to (x + 3, 4y - 5)$

c i $(x, y) \to (x + 4, y + 2)$

ii $(x, y) \to \left(\frac{x}{2}, 2y\right)$

iii $(x, y) \to (x, -2y + 3)$

6 a $(x, y) \to (3x - 2, -y + 3)$

b $x = \frac{x' + 2}{3},\ y = -y' + 3$

7 a $y = \frac{1}{2x - 4}$ **b** $y = \frac{1}{2x - 3} + 3$

c $y = \frac{1}{2x - 11} + 5$

Multiple-choice questions

1 C **2** D **3** A **4** B **5** E **6** B
7 D **8** D **9** E **10** A **11** B **12** B

Extended-response questions

1 a $k = \frac{1}{4}$ **b** $h = -\frac{1}{4}$

2 a $h = -1 \pm 2\sqrt{2}$ **b** $a = \pm 2\sqrt{2}$

c $a = -8, b = 16$

3 a $k = 10$

b i $h > 2 + \sqrt{10}$ **ii** $h < 2 - \sqrt{10}$

iii $2 - \sqrt{10} < h < 2 + \sqrt{10}$

4 a $y = \frac{x^2}{4}$ **b** $y = \frac{4}{3}(x - 1), C(\frac{1}{4}, -1)$

c The line segment with endpoints $A(4, 4)$ and $C'(-1, \frac{1}{4})$ is a chord of the parabola $y = \frac{x^2}{4}$ passing through $B'(0, 1)$

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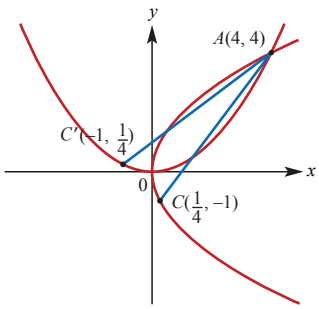
d



5 a Dilation of factor $\frac{1}{225}$ from the x-axis

b $x' = x, y' = -y$

c $x' = x + 45, y' = y + 9$

d $x' = x + 45, y' = -\frac{1}{225}y + 9$

6 a $x = 5$ or $x = -2$ or $x = 7$

b $x = 1$ or $x = -6$ or $x = 3$

c $k = -60$,
$x = 0$ or $x = 2 + \sqrt{21}$ or $x = 2 - \sqrt{21}$

d $h = -3$ or $h = -5$ or $h = 4$

e $-5 < h < -3$

7 a $(4, 6)$

b $(x, y) \to (6 - x, y)$

c $(x, y) \to (6 - x, y)$

d i ■ Translation of m units in the negative direction of the x-axis
■ Reflection in the y-axis
■ Translation of m units in the positive direction of the x-axis

ii $(x, y) \to (2m - x, y)$

e i ■ Translation of n units in the negative direction of the y-axis
■ Reflection in the x-axis
■ Translation of n units in the positive direction of the y-axis

ii $(x, y) \to (x, 2n - y)$

f i $y = -x + 3$ **ii** $y = -x + 6$
iii $y = (6 - x)^2$ **iv** $y = (3 - x)^2$

8 a $A'(-1, 3)$ **b i** $\frac{1}{3}$ **ii** -3

c i $\frac{q}{p}$ **ii** $A'(-q, p)$ **d** $(x, y) \to (-y, x)$

e i $y = -x$ **ii** $x = -y^2$

iii $x^2 + y^2 = 1$ **iv** $y = -\frac{1}{x}$

Chapter 8

Technology-free questions

1 $a = 2$, $a = \frac{1}{2}$ or $a = \frac{2}{3}$

2 a $(x + 4)^2 - 7$ **b** $x = -4$

3 $[1, 9]$

4 $\left(\frac{y + 3}{2}\right)^2 = x$

5 ■ Dilation of factor 3 from the x-axis
■ Translation of 4 units in the negative direction of the y-axis
■ Translation of 2 units in the positive direction of the x-axis

6 a $y = \frac{-11}{x + 1} + 4$ **b** $(0, -7)$, $\left(\frac{7}{4}, 0\right)$

c $x = -1$, $y = 4$

7 a $M\left(\frac{1}{2}, -\frac{3}{2}\right)$, $N\left(\frac{3}{2}, 4\right)$ **b** $m_{BC} = m_{MN} = \frac{11}{2}$

8 a -6 **b** 69 **c** -15

9 a $12a^2 - 4$ **b** $3a^2 - 6a - 1$ **c** $12a$

10 a No **b** $x = -\frac{3}{7}$ **c** $x \leq -\frac{3}{7}$ **d** $k = -\frac{3}{16}$

11 $x = 2$ and $y = 3$, or $x = 3$ and $y = 2$

12 $AB = BC = CD = DA = 5\sqrt{2}$,
$m_{BC} = m_{AD} = 1$ and $m_{AB} = m_{CD} = -7$

13 a $y = (x + 2)^2 - 13$ **b** $y = \left(x - \frac{3}{2}\right)^2 - \frac{53}{4}$

c $y = 2\left(x - \frac{3}{4}\right)^2 + \frac{79}{8}$

14 a $\left(\frac{1 - \sqrt{41}}{2}, 3 - 2\sqrt{41}\right)$, $\left(\frac{1 + \sqrt{41}}{2}, 3 + 2\sqrt{41}\right)$

b $(2, 6)$ **c** $(-4, 14)$

15 a $x < \frac{-3 - \sqrt{29}}{2}$ or $x > \frac{-3 + \sqrt{29}}{2}$

b $x \leq \frac{5 - \sqrt{65}}{4}$ or $x \geq \frac{5 + \sqrt{65}}{4}$ **c** $x \geq -4$

d $\frac{1}{2} \leq x \leq 3$ or $x \leq -4$ **e** $x \leq 4$

16 a $\mathbb{R} \setminus \{\frac{5}{2}\}$ **b** $(-\infty, 5]$ **c** $\mathbb{R}$

d $\mathbb{R} \setminus \{2\}$ **e** $\mathbb{R}$ **f** $\mathbb{R} \setminus \{\frac{2}{3}\}$

17 $p = -38$, $(x - 3)(x + 4)(3x - 2)$

18 $a = -5$, $R = -35$

19 a $f^{-1}: [1, 4] \to \mathbb{R}$, $f^{-1}(x) = \sqrt{x}$

b $f^{-1}: [0, 3] \to \mathbb{R}$, $f^{-1}(x) = 2 - x$

c $f^{-1}: (-4, \infty) \to \mathbb{R}$, $f^{-1}(x) = -\sqrt{x + 4}$

d $f^{-1}: [3, \infty) \to \mathbb{R}$, $f^{-1}(x) = 2 - (x - 3)^2$

e $f^{-1}: \mathbb{R} \to \mathbb{R}$, $f^{-1}(x) = (x - 8)^{\frac{1}{3}} + 2$

20 a $3b + 2f = 18.20$ **b** $\$2.80$

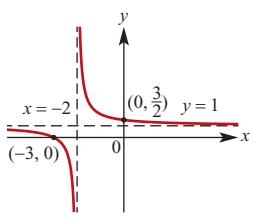
21 a $k = 1$ **b** $k = -16$

22 a $\frac{2}{5}$ **b** $2y + 5x - 17 = 0$

23 a $a\left(x + \frac{1}{a}\right)^2 + \frac{a^2 - 1}{a}$ **b** $\left(-\frac{1}{a}, \frac{a^2 - 1}{a}\right)$

c $a = \pm 1$ **d** $a \in (-1, 1)$

24 a



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b $A\left(0, \frac{3}{2}\right)$, $B(-3, 0)$ **c** $y = \frac{1}{2}x + \frac{3}{2}$

d $\left(-\frac{3}{2}, \frac{3}{4}\right)$ **e** $y = -2x - \frac{9}{4}$

25 $y = \frac{3}{2}x^2 + \frac{5}{2}x + 2$

26 a $a = 3,\ b = 17$ **b** $x = -1, 2$ or $\frac{7}{2}$

c $x = -\frac{1}{3}, \frac{2}{3}$ or $\frac{7}{6}$

27 a $k = -2$ **b** $k \neq -2$ **c** $k < -1$ and $k \neq -2$

28 $b = 4,\ c = 1$

29 a i $x + 6$ cm **ii** $x(x + 6)$ cm^2

iii Two sides $5x$ cm^2; two sides $5(x + 6)$ cm^2

iv $x^2 + 26x + 60$ cm^2

b $x = 4$

30 $a = 2,\ h = 3,\ k = 4$

Multiple-choice questions

1 B	**2** D	**3** C	**4** D	**5** D
6 C	**7** C	**8** A	**9** E	**10** B
11 A	**12** E	**13** B	**14** D	**15** D
16 E	**17** B	**18** D	**19** D	**20** B
21 D	**22** D	**23** B	**24** D	**25** D
26 B	**27** C	**28** A	**29** A	**30** B
31 C	**32** D	**33** E	**34** E	**35** C
36 C	**37** C	**38** A		

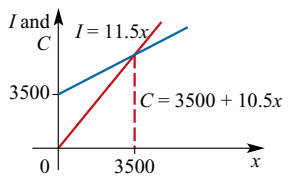
Extended-response questions

1 a $x \leq a$ **b** $\left(\frac{\sqrt{4a+1}-1}{2}, \frac{\sqrt{4a+1}-1}{2}\right)$

c $a = 2$ **d** $a = 6$ **e** $a = c^2 + c$

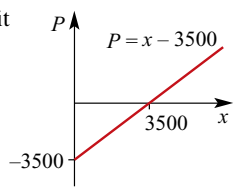
2 a $C = 3500 + 10.5x$ **b** $I = 11.5x$

c



d 3500

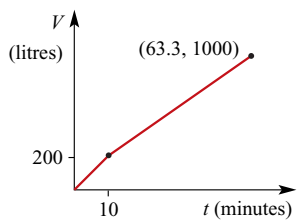
e Profit



f 5500

3 a 200 L **b** $V = \begin{cases} 20t & 0 \leq t \leq 10 \\ 15t + 50 & 10 < t \leq \frac{190}{3} \end{cases}$

c



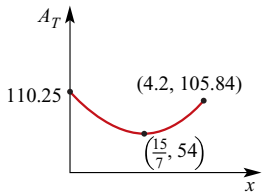
4 a $A_R = 6x^2$

b $A_S = (10.5 - 2.5x)^2$

c $0 \leq x \leq 4.2$

d $A_T = 12.25x^2 - 52.5x + 110.25$

e



f 110.25 cm^2 (when area of rectangle = 0)

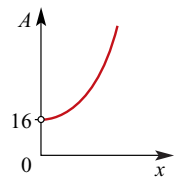
g Rectangle 9×6 and square 3×3,
or rectangle $\frac{27}{7} \times \frac{18}{7}$ and square $\frac{51}{7} \times \frac{51}{7}$

5 a 20 m **b** 20 m **c** 22.5 m

6 a $A = 10x^2 + 28x + 16$

b i 54 cm^2 **ii** 112 cm^2

c 3 cm **d**



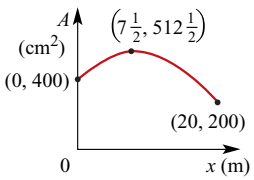
e $V = 2x^3 + 8x^2 + 8x$ **f** $x = 3$ **g** $x = 6.66$

7 a i $A = (10 + x)y - x^2$ **ii** $P = 2(y + x + 10)$

b i $A = 400 + 30x - 2x^2$

ii $512\frac{1}{2}$ m^2 **iii** $0 \leq x \leq 20$

iv

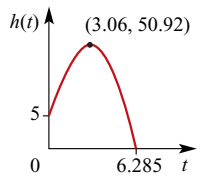


8 a $A = 6x^2 + 7xy + 2y^2$

c i $x = 0.5$ m **ii** $y = 0.25$ m

9 a 50.9 m **b** $t = 6.12$ seconds

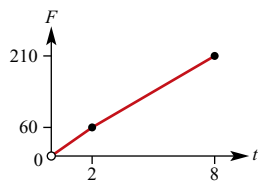
c



d 6.285 seconds

10 a $F(t) = \begin{cases} 30t & \text{for } 0 < t \leq 2 \\ 25t + 10 & \text{for } 2 < t \leq 8 \end{cases}$

b



c i \$45 **ii** \$60 **iii** \$122.50

d $G(t) = -\frac{5}{8}t^2 + \frac{125}{4}t,\ 0 < t \leq 8$

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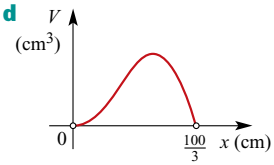
11 a $h = 100 - 3x$ **b** $V = 2x^2(100 - 3x)$

c $0 < x < \frac{100}{3}$ **d**



e i $x = 18.142$ or $x = 25.852$

ii $x = 12.715$ or $x = 29.504$

f $V_{max} = 32\,921.811$ cm³ when $x = 22.222$

g i $S = 600x - 14x^2$

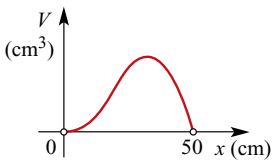
ii $S_{max} = \frac{45000}{7}$ cm² when $x = \frac{150}{7}$

h $x = 3.068$ or $x = 32.599$

12 a i $y = 250 - 5x$

ii $V = x^2(250 - 5x) = 5x^2(50 - x)$

b



c $(0, 50)$

d $x = 11.378$ or $x = 47.813$

e $V_{max} = 92\,592.59$ cm³ when $x = 33.33$ and $y = 83.33$

Investigations

See solutions supplement

Chapter 9

Exercise 9A

1 $\{H, T\}$

2 $\{1, 2, 3, 4, 5, 6\}$

3 a $\{0, 1, 2, 3, \ldots\}$ **b** $\{0, 1, 2, 3, \ldots, 41\}$

c $\{1, 2, 3, \ldots\}$

4 a $\{2, 4, 6\}$ **b** $\{FFF\}$ **c** $\varnothing$

5 a $\frac{1}{2}$ **b** $\frac{3}{10}$ **c** $\frac{3}{20}$

6 a $\frac{4}{15}$ **b** $\frac{2}{3}$ **c** $\frac{4}{15}$

7 a $\frac{1}{365}$ **b** $\frac{30}{365}$ **c** $\frac{6}{73}$ **d** $\frac{18}{73}$

8 a $\frac{1}{9}$ **b** $\frac{1}{3}$ **c** $\frac{5}{9}$ **d** $\frac{4}{9}$

9 $\frac{1}{3}$

10 0.4

11 a $\frac{1}{3}$ **b** $\frac{1}{8}$ **c** $\frac{1}{4}$

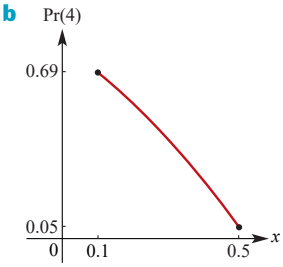
12 a $\frac{1}{7}$ **b** $\frac{5}{7}$

13 $\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{2}{5}$

14 a $\frac{1}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13}, \frac{4}{13}$ **b** $\frac{9}{13}$

15 a $\Pr(4) = 0.8 - x - x^2$

b



c 0.69

Exercise 9B

1 a $\frac{17}{50}$ **b** $\frac{1}{10}$ **c** $\frac{4}{15}$ **d** $\frac{1}{200}$

2 a No **b** Answers will vary

c Answers will vary **d** Yes

e As the number of trials approaches infinity, the relative frequency approaches the value of the probability

3 $\Pr(\text{a 6 from first die}) \approx \frac{78}{500} = 0.156$

$\Pr(\text{a 6 from second die}) \approx \frac{102}{700} \approx 0.146$

Choose first die, as higher probability of a 6

4 a $\frac{17}{20}$ **b** $\frac{4}{5}$ **c** $\frac{9}{10}$ **d** 51

5 0.445

6 a $\frac{\pi}{4}$ **b** $\frac{\pi}{4} \approx 0.7855$

7 $\frac{3}{4}$

8 a $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$

9 a $\frac{4\pi}{25}$ **b** $1 - \frac{4\pi}{25} \approx 0.4973$

10 a i x^2 **ii** $\frac{1}{4}\pi x^2$ **iii** $\frac{1}{16}\pi x^2$

b i $\frac{\pi}{16}$ **ii** $\frac{3\pi}{16}$ **iii** $1 - \frac{\pi}{4}$

Exercise 9C

1 a $\frac{1}{4}$ **b** $\frac{1}{4}$

2 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

3 a $\frac{1}{2}$ **b** $\frac{1}{18}$ **c** $\frac{5}{18}$

4 a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{7}{12}$

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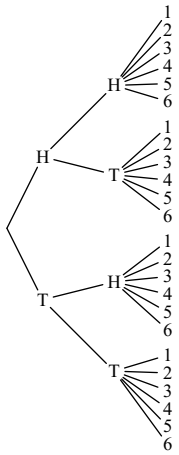
5 **a** $\frac{3}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{8}$ **d** $\frac{1}{8}$

6 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$

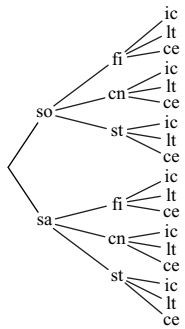
7 $\frac{1}{4}$

8 **a**



b **i** $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{2}$

9 **a**



b **i** $\frac{1}{18}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{6}$ **iv** $\frac{2}{3}$

c **i** $\frac{1}{36}$ **ii** $\frac{1}{2}$ **iii** $\frac{5}{12}$ **iv** $\frac{1}{12}$

10 **a**

		2nd ball				
		1	2	3	4	5
1st ball	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

b **i** $\frac{4}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{25}$

Exercise 9D

1 **a** {1, 2, 3, 4, 6} **b** {2, 4}
c {5, 6, 7, 8, 9, 10} **d** {1, 3}
e {1, 3, 5, 6, 7, 8, 9, 10}
f {5, 7, 8, 9, 10}

2 **a** {1, 2, 3, 5, 6, 7, 9, 10, 11}
b {1, 3, 5, 7, 9, 11} **c** {2, 4, 6, 8, 10, 12}
d {1, 3, 5, 7, 9, 11} **e** {1, 3, 5, 7, 9, 11}

3 **a** {E, H, M, S} **b** {C, H, I, M}
c {A, C, E, I, S, T} **d** {H, M}
e {C, E, H, I, M, S} **f** {H, M}

4 **a** 20 **b** 45

5 **a** $\frac{2}{3}$ **b** 0 **c** $\frac{1}{2}$ **d** $\frac{5}{6}$

6 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}, \frac{2}{3}$

7 **a** $\frac{7}{18}$ **b** $\frac{4}{18} = \frac{2}{9}$ **c** $\frac{2}{18} = \frac{1}{9}$ **d** $\frac{1}{2}$

8 **a** $\frac{3}{10}$ **b** $\frac{1}{5}$ **c** $\frac{1}{10}$ **d** $\frac{2}{5}$

9 $\Pr(A \cup B) = 0.7$

10 $\Pr(A \cup B) = 0.47$

11 **a** $\Pr(A \cap B) = 0.28$ **b** $\Pr(A \cup B) = 0.45$

12 **a** $\Pr(A \cap B) = 0.45$ **b** $\Pr(A \cup B) = 0.58$

13 **a** $\Pr(A \cap B) = 0$ **b** $\Pr(A \cup B) = 0.7$

14 **a** $\Pr(A \cap B) = 0$ **b** $\Pr(A \cup B) = 0.23$

15 $\Pr(A \cap B) = 0.2$

16 $\Pr(A \cap B) = 0.05$

17 $\Pr(A \cup B') = 0.7$

18 0.32

19 **a** 0.43 **b** 0.29

Exercise 9E

1 **a** 0.2 **b** 0.5 **c** 0.3 **d** 0.7

2 **a** 0.75 **b** 0.4 **c** 0.87 **d** 0.48

3 **a** 0.63 **b** 0.23 **c** 0.22 **d** 0.77

4 **a** 0.45 **b** 0.40 **c** 0.25 **d** 0.70

5 **a** 0.6 **b** 0.1 **c** 0.9 **d** 0.9

6 **a** 95% **b** 5%

7 **a** $\frac{8}{15}$ **b** $\frac{7}{10}$ **c** $\frac{2}{15}$ **d** $\frac{1}{3}$

8 **a** 0.8 **b** 0.57 **c** 0.28 **d** 0.08

9 **a** 0 **b** 1 **c** $\frac{1}{5}$ **d** $\frac{1}{3}$

10 **a** 0.88 **b** 0.58 **c** 0.30 **d** 0.12

Exercise 9F

1 $\frac{1}{4}$ **2** $\frac{1}{3}$ **3** $\frac{7}{19}$ **4** $\frac{1}{6}$

5 **a** $\frac{4}{17}$ **b** $\frac{4}{7}$

6 $\frac{7}{12}$

7 **a** $\frac{65}{284}$ **b** $\frac{137}{568}$ **c** $\frac{21}{65}$ **d** $\frac{61}{246}$

8 **a** 0.06 **b** 0.2

9 **a** $\frac{4}{7}$ **b** 0.3 **c** $\frac{15}{22}$

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10 **a** 0.2 **b** 0.5 **c** 0.4

11 **a** 0.2 **b** $\frac{10}{27}$ **c** $\frac{1}{3}$

12 **a** 0.3 **b** 0.75

13 16%

14 $\frac{1}{5}$

15 $0.230\,808 \approx 0.231$

16 **a** $\frac{15}{28}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{2}{5}$
e $\frac{3}{7}$ **f** $\frac{8}{13}$ **g** $\frac{5}{28}$ **h** $\frac{3}{14}$

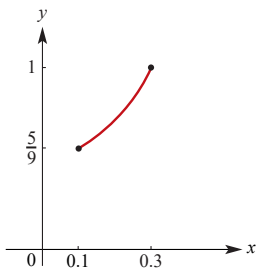
17 0.4, 68%

18 **a** **i** 0.444 **ii** 0.4 **iii** 0.35 **iv** 0.178 **v** 0.194
b 0.372
c **i** 0.478 **ii** 0.425

19 **a** $\frac{1}{6}$ **b** $\frac{53}{90}$ **c** $\frac{15}{53}$

20 **a** $B \subseteq A$ **b** $A \cap B = \varnothing$ **c** $A \subseteq B$

21 **a** $y = \dfrac{0.25}{0.55 - x}$



b Maximum 1; minimum $\frac{5}{9}$

22 **a** $\dfrac{x^2 + (x+4)^2}{(2x+4)^2}$ **b** $x = 10$

23 **a** $\dfrac{\pi x^2}{2500}$ **b** $2\left(\dfrac{\pi x^2}{2500}\right)\left(1 - \dfrac{\pi x^2}{2500}\right)$
c $x = \dfrac{25\sqrt{2}}{\sqrt{\pi}} \approx 19.95$; maximum 0.5

Exercise 9G

1 Pr(male and support guns) = 0.35;
Pr(male) × Pr(support guns) = 0.39 ≠ 0.35;
therefore not independent

2 Pr(male and prefer sport) = 0.45;
Pr(male) × Pr(prefer sport) = 0.45;
therefore independent

3 Pr(speeding and serious) ≈ 0.112;
Pr(speeding) × Pr(serious) ≈ 0.095 ≠ 0.112;
therefore not independent

4 **a** Yes **b** Yes **c** No

5 $\Pr(A) \times \Pr(B) = \frac{3}{6} \times \frac{2}{6} = \frac{1}{6} = \Pr(A \cap B)$

6 No

7 **a** 0.6 **b** 0.42 **c** 0.88

8 0.6

9 **a** 0.35 **b** 0.035 **c** 0.1225 **d** 0.025

10 **a** $\frac{4}{15}$ **b** $\frac{1}{15}$ **c** $\frac{133}{165}$ **d** $\frac{6}{11}$ **e** $\frac{4}{15}$
No, as $\Pr(L \mid F) \neq \Pr(L)$

11 $\Pr(A) \times \Pr(B) = \frac{20}{36} \times \frac{9}{36} = \frac{5}{36} = \Pr(A \cap B)$

12 **a** 0.35 **b** 0.875

13 **a** $\frac{18}{65}$ **b** $\frac{12}{65}$ **c** $\frac{23}{65}$ **d** $\frac{21}{65}$ **e** $\frac{4}{65}$
f $\frac{8}{65}$ **g** $\frac{2}{15}$ **h** $\frac{8}{21}$; No

14 **a** **i** 0.75 **ii** 0.32 **iii** 0.59 **b** No **c** No

15 Minimum 0.2775 when $\Pr(A) = \Pr(B) = 0.15$

Exercise 9H

1 Approx. 0.125
2 Approx. 0.5
3 Approx. 0.033
4 Approx. 29.29
5 Approx. 0.53
6 Approx. 2.5
7 Approx. 0.75

Exercise 9I

1 Change `if-then` block:

```
if outcome = 5 or outcome = 6 then
  count ← count + 1
end if
```

2 Change `while` loop:

```
while outcome ≠ 5 and outcome ≠ 6
  outcome ← randint(1, 6)
  count ← count + 1
end while
```

3 **a** 100 000 families
b 0 or 1
c The first child in the family is a girl
d The number of families with three girls
e **i** 1 **ii** 1 **iii** 2
f **i** *child1* + *child2* + *child3* = 0
ii *child1* + *child2* + *child3* = 2
iii *child1* + *child2* + *child3* ≥ 1

4 **a**
- `for` loop: simulates 1000 shoppers
- `while` loop: simulates one shopper making purchases until they get all three toys
- *count*: running tally of the number of purchases for the current shopper
- *sum*: running total of the number of purchases required by all shoppers
- *toy*: the toy from the current purchase (value 1, 2 or 3)
- *t1*, *t2*, *t3*: given value 1 when the current shopper gets the corresponding toy
- `if-then` block: used to update the value of *t1*, *t2* or *t3* based on the current toy

b Use variables *t1*, *t2*, . . . , *t10* and continue the `while` loop until all are non-zero

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5 a $\pi \approx 4 \times \dfrac{count}{N}$

b Change `for` loop:

```
for i from 1 to N
  x ← random( ) – 0.5
  y ← random( ) – 0.5
  if 0.25² ≤ x² + y² ≤ 0.5² then
    count ← count + 1
  end if
end for
```

6 a

```
total ← 0
for i from 1 to 10
  x ← 20 × random( ) – 10
  y ← 20 × random( ) – 10
  if –1 ≤ x ≤ 1 and –1 ≤ y ≤ 1 then
    score ← 10
  else if –6 ≤ x ≤ 6 and –6 ≤ y ≤ 6
  then
    score ← 5
  else
    score ← 1
  end if
  total ← total + score
end for
print total
```

b

```
sum ← 0
for j from 1 to 1000
  total ← 0
  [insert for loop from part a]
  sum ← sum + total
end for
print sum / 1000
```

c

```
successes ← 0
for j from 1 to 100 000
  hit ← 0
  for i from 1 to 50
    x ← 20 × random( ) – 10
    y ← 20 × random( ) – 10
    if –1 ≤ x ≤ 1 and –1 ≤ y ≤ 1 then
      hit ← 1
    end if
  end for
  if hit = 1 then
    successes ← successes + 1
  end if
end for
print successes / 100 000
```

7

```
sum ← 0
for i from 1 to 1000
  bar ← 0
  count ← 0
  while bar ≠ 5
    bar ← randint(1, 5)
    count ← count + 1
  end while
  sum ← sum + count
end for
print sum / 1000
```

8 a

```
total ← 0
count ← 0
for i from 1 to 6
  for j from 1 to 6
    for k from 1 to 6
      total ← total + 1
      if i + j + k = 15 then
        count ← count + 1
      end if
    end for
  end for
end for
print count / total
```

Chapter 9 review

Technology-free questions

1 a $\frac{1}{6}$ **b** $\frac{5}{6}$

2 a $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

3 a $\frac{9}{25}$ **b** $\frac{87}{245}$

4 $\frac{4}{15}$

5 a {156, 165, 516, 561, 615, 651} **b** $\frac{2}{3}$ **c** $\frac{1}{3}$

6 a $\frac{5}{12}$ **b** $\frac{1}{4}$

7 a 0.036 **b** 0.027 **c** 0.189 **d** 0.729

8 a $\frac{1}{27}$ **b** $\frac{4}{27}$ **c** $\frac{4}{9}$ **d** $\frac{20}{27}$

9 No

10 a 0.5 **b** 0 **c** 1

11 a $\frac{7}{18}$ **b** $\frac{1}{2}$

12 a 0.2 **b** 0.4

13 a 0.7 **b** 0.3 **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

14 a $B \subseteq A$ **b** $A \cap B = \varnothing$

c A and B are independent

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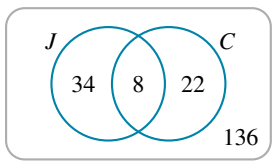
Multiple-choice questions

1 B **2** C **3** A **4** C **5** A
6 D **7** E **8** A **9** B **10** E
11 C **12** A **13** B **14** D

Extended-response questions

1 a



b 56
c i 0.28 **ii** 0.68 **iii** 0.14 **iv** No

2 a 18 days **b** 0.072 **c** 0.15
d 0.148 **e** 0.34
f i 12 days **ii** 0.18

3 a A: $\frac{3}{28}$; B: $\frac{3}{4}$ **b** A: $\frac{9}{64}$; B: $\frac{49}{64}$
c $\frac{1}{8}$ **d** $\frac{9}{58}$

4 a $\frac{4}{5}$ **b** 0.69 **c** 0.208 **d** $\frac{21}{26}$

5 a i 0.1963 **ii** 0.1610 **iii** 0.6427
b i 0.0259 **ii** 0.0632
c i 0.2782 **ii** 0.4535

6 a i $\frac{1}{2}$ **ii** $\frac{5}{22}$ **iii** $\frac{1}{2}$ **b i** $\frac{30}{n(n-1)}$ **ii** 9

7 a 0.65% **b** 0.69
c More than 900 components

8 b $\frac{1}{5}(\alpha^2 + 5\beta + 3\alpha\beta - 4\beta^2)$
c $\alpha = \frac{13}{15}, \beta = \frac{8}{15}$

Chapter 10

Exercise 10A

1 a 11 **b** 12 **c** 37 **d** 29
2 a 60 **b** 500 **c** 350 **d** 512
3 a 128 **b** 160
4 20 **5** 63 **6** 26
7 240 **8** 260 000 **9** 17 576 000
10 30 **11** $m \geq 14$

Exercise 10B

1 a 6 **b** 120 **c** 5040 **d** 2 **e** 1 **f** 1
2 a 20 **b** 72 **c** 6 **d** 56 **e** 120 **f** 720
3 120 **4** 5040 **5** 24
6 720 **7** 720 **8** 336
9 a 5040 **b** 210
10 a 120 **b** 120
11 a 840 **b** 2401
12 a 480 **b** 1512
13 a 60 **b** 24 **c** 252
14 a 150 **b** 360 **c** 1560
15 a 720 **b** 48
16 a $n \geq 12$ **b** $n \geq 14$

Exercise 10C

1 a 3 **b** 3 **c** 6 **d** 4
2 a 10 **b** 10 **c** 35 **d** 35
3 a 190 **b** 100 **c** 4950 **d** 31 125
4 a 20 **b** 7 **c** 28 **d** 1225
5 1716 **6** 2300
7 133 784 560 **8** 8 145 060 **9** 18
10 a 5 852 925 **b** 1 744 200
11 100 386
12 a 792 **b** 336
13 a 150 **b** 75 **c** 6 **d** 462 **e** 181
14 a 8 436 285 **b** 3003 **c** 66 **d** 2 378 376
15 186 **16** 32 **17** 256
18 31 **19** 57
20 a 10 **b** 21
21 $n \geq 8$
22 a ${}^{n+m}C_3 = \frac{1}{6}(n+m)(n+m-1)(n+m-2)$
b ${}^nC_1 \times {}^mC_2 + {}^nC_2 \times {}^mC_1 = \frac{1}{2}nm(n+m-2)$
c $n \leq 9$

Exercise 10D

1 a 0.5 **b** 0.5
2 0.375
3 a 0.2 **b** 0.6 **c** 0.3
4 0.2 **5** $\frac{329}{858}$
6 a $\frac{2^7}{2^8-1} \approx 0.502$ **b** $\frac{56}{255}$ **c** $\frac{73}{85}$
7 a $\frac{5}{204}$ **b** $\frac{35}{136}$
8 a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{17}{21}$ **d** $\frac{34}{35}$
9 a $\frac{25}{49}$ **b** $\frac{24}{49}$ **c** $\frac{3}{7}$ **d** $\frac{1}{5}$
10 a $\frac{5}{42}$ **b** $\frac{20}{21}$ **c** $\frac{15}{37}$
11 a $\frac{g(8-g)(7-g)}{112}$ **b** $g = 2$ or $g = 3$
12 a $\frac{90(n-6)}{n(n-1)(n-2)}$ **b** $n = 8$ or $n = 9$

Chapter 10 review

Technology-free questions

1 a 499 500 **b** 1 000 000 **c** 1 000 000
2 $n = 9$ **3** 120 **4** $8n$

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5 a $\frac{1}{6}(a+b)(a+b-1)(a+b-2)$

b $\frac{1}{2}ab(a-1)$

6 a 325 **b** $\frac{23}{65}$

7 a 10 **b** $n = 11$

8 1200

9 a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{3}{28}$

Multiple-choice questions

1 E **2** D **3** A **4** D **5** C
6 B **7** C **8** A **9** E **10** E

Extended-response questions

1 a 96 **b** 120 **c** 160 **d** $m = 14, 16$ or 18

2 a 720 **b** 48 **c** 336 **d** $\frac{7}{15}$ **e** $n = 12$

3 a 1365 **b** 210 **c** 1155 **d** $\frac{11}{13}$ **e** $\frac{40}{77}$

4 a i 210 **ii** 100 **iii** $\frac{10}{21}$ **iv** $\frac{1}{21}$

b i $\frac{1}{24}(n+m)(n+m-1)(n+m-2)(n+m-3)$

ii $\frac{1}{4}nm(n-1)(m-1)$ **iii** $n = 4, 6, 8$ or 10

5 Div. 1: 1.228×10^{-7} Div. 2: 1.473×10^{-6}
Div. 3: 2.726×10^{-5} Div. 4: 1.365×10^{-3}
Div. 5: 3.362×10^{-3}

6 a 1.227×10^{-3} **b** 3.271×10^{-3}

Chapter 11

Exercise 11A

1 a No **b** No **c** Yes **d** No **e** No

2 a $\Pr(X = 2)$ **b** $\Pr(X > 2)$ **c** $\Pr(X \geq 2)$
d $\Pr(X < 2)$ **e** $\Pr(X \geq 2)$ **f** $\Pr(X > 2)$
g $\Pr(X \leq 2)$ **h** $\Pr(X \geq 2)$ **i** $\Pr(X \leq 2)$
j $\Pr(2 < X < 5)$

3 a {2} **b** {3, 4, 5} **c** {2, 3, 4, 5}
d {0, 1} **e** {0, 1, 2} **f** {2, 3, 4, 5}
g {3, 4, 5} **h** {2, 3, 4} **i** {3, 4}

4 a 0.2 **b** 0.5 **c** 0.3 **d** 0.35 **e** 0.9

5 a $\frac{1}{15}$ **b** $\frac{3}{5}$

6 a 0.09 **b** 0.69

7 a 0.49 **b** 0.51 **c** 0.74

8 a 0.6 **b** 0.47 **c** $\frac{2}{3}$

9 a $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ **b** $\frac{3}{8}$

c

x	0	1	2	3
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

d $\frac{7}{8}$ **e** $\frac{4}{7}$

10 a {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} **b** $\frac{1}{6}$

c

y	2	3	4	5	6	7	8	9	10	11	12
$p(y)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

11 a {1, 2, 3, 4, 5, 6} **b** $\frac{7}{36}$

c

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

12 a

y	−3	−2	1	3
$p(y)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

b $\frac{7}{8}$

Exercise 11B

1 0.378 **2** $\frac{28}{57} \approx 0.491$

3 $\frac{12}{13} \approx 0.923$ **4** $\frac{60}{253} \approx 0.237$ **5** 0.930

6 0.109 **7** 6 **8** 27

Exercise 11C

1 a 0.185 **b** 0.060

2 a 0.194 **b** 0.930

3 a 0.137 **b** 0.446 **c** 0.554

4 a 0.008 **b** 0.268 **c** 0.468

5 a 0.056 **b** 0.391

6 0.018

7 a $\Pr(X = x) = \binom{5}{x}(0.1)^x(0.9)^{5-x}$
for $x = 0, 1, 2, 3, 4, 5$

x	0	1	2	3	4	5
$p(x)$	0.590	0.328	0.073	0.008	0.000	0.000

b Most probable number is 0

8 0.749 **9** 0.021 **10** 0.540 **11** $\frac{175}{256}$

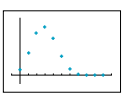
12 a 0.988 **b** 0.9999 **c** 8.1×10^{-11}

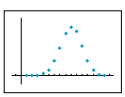
13 a 0.151 **b** 0.302

14 5.8%

15 a i 0.474 **ii** 0.224 **iii** 0.078
b Answers will vary – about 5 or more

16 0.014

17 

18 

19 a 5 **b** 8

20 a 13 **b** 22

21 a 16 **b** 29

22 a 45 **b** 59

23 a 0.3087 **b** $\frac{0.3087}{1-(0.3)^5} \approx 0.3095$

24 Maximum 0.346 when $p = 0.6$

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Chapter 11 review

Technology-free questions

1 a 0.92 **b** 0.63 **c** 0.8

2

x	1	2	3	4
$p(x)$	0.25	0.28	0.30	0.17

3

x	2	3	4
$p(x)$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

4 a

1st choice \ 2nd choice	1	2	3	6	7	9
1	2	3	4	7	8	10
2	3	4	5	8	9	11
3	4	5	6	9	10	12
6	7	8	9	12	13	15
7	8	9	10	13	14	16
9	10	11	12	15	16	18

b {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18}

c

x	2	3	4	5	6	7	8	9
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{4}{36}$

x	10	11	12	13	14	15	16	18
$p(x)$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

5 a 0.3 **b** 6

6 a $\frac{9}{64}$ **b** $\frac{37}{64}$

7 a $\frac{16}{81}$ **b** $\frac{32}{81}$ **c** $\frac{11}{27}$

8 a $\binom{7}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^4$

b $\left(\frac{3}{4}\right)^7 + 7\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^6 + 21\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^5$

9 a $\left(\frac{p}{100}\right)^{15}$ **b** $15\left(\frac{p}{100}\right)^{14}\left(1 - \frac{p}{100}\right)$

c $\left(\frac{p}{100}\right)^{15} + 15\left(\frac{p}{100}\right)^{14}\left(1 - \frac{p}{100}\right) + 105\left(\frac{p}{100}\right)^{13}\left(1 - \frac{p}{100}\right)^2$

10 a $\frac{117}{125}$ **b** $m = 5$

Multiple-choice questions

1 B **2** A **3** C **4** A **5** E
6 C **7** A **8** D **9** B **10** E

Extended-response questions

1 a i 0.24 **ii** 0.06 **iii** 0.16 **iv** 0.54

b i

x	1	2	3	4
$p(x)$	0.54	0.16	0.06	0.24

ii 0.46

2 a i 0.1 **ii** 0.6 **iii** $\frac{2}{3}$ **b i** 0.0010 **ii** 0.2001

3 a $\frac{3}{5}$ **b i** $\frac{7}{40}$ **ii** $\frac{3}{10}$ **c i** $\frac{11}{20}$ **ii** $\frac{3}{8}$

4 a $\Pr(X = x) = \binom{20}{x}(0.2)^x(0.8)^{20-x}$ for $x = 0, 1, \ldots, 20$

b 4 **c** 0.003 **d** 5.320×10^{-6}

5 a i 0.9898 **ii** 0.9224

b 0.8

c $0.0860 < p < 0.3619$

6 a 0.401 **b** $n \geq 45$

7 a $1 - q^2$ **b** $1 - 4q^3 + 3q^4$ **c** $\frac{1}{3} < q < 1$

8 a 0.5357 **b** 0.3225 or 0.3195

Chapter 12

Technology-free questions

1 a $\frac{1}{9}$ **b** $\frac{8}{9}$

2 a {348, 384, 438, 483, 834, 843}, $n(\varepsilon) = 6$

b $\frac{2}{3}$ **c** $\frac{2}{3}$

3 a $\frac{1}{4}$ **b** $\frac{81}{256}$

4 a

x	1	2	3	4	5	6
$p(x)$	$\frac{2}{17}$	$\frac{2}{17}$	$\frac{2}{17}$	$\frac{8}{17}$	$\frac{2}{17}$	$\frac{1}{17}$

b $\frac{81}{289}$

5 $\frac{1}{4}$

6 a 0.48 **b** 0.56

7 a 0.4 **b** 0.2 **c** 0.7

8 a i 0.05 **ii** $\frac{0.05}{p}$

b $0.05 \leq p \leq 0.2$

9 a 720 **b** $\frac{1}{30}$

10 a $\frac{7}{22}$ **b** $\frac{37}{44}$ **c** $\frac{2}{37}$

11 a $k = 0.2$

b i 0.7 **ii** 0.7 **iii** $\frac{2}{3}$

c 0.26

12 a 0.038 **b** $\frac{10}{19}$

13 a $\left(\frac{x}{100}\right)^{10}$ **b** $10\left(\frac{x}{100}\right)^9\left(1 - \frac{x}{100}\right)$

c $\left(\frac{x}{100}\right)^{10} + 10\left(\frac{x}{100}\right)^9\left(1 - \frac{x}{100}\right) + 45\left(\frac{x}{100}\right)^8\left(1 - \frac{x}{100}\right)^2$

14 a $p = \frac{3}{11}$ **b** $1 - \left(\frac{8}{11}\right)^{10}$

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Multiple-choice questions

1 E	**2** C	**3** E	**4** B	**5** E
6 E	**7** C	**8** C	**9** B	**10** D
11 D	**12** D	**13** E	**14** A	**15** E
16 E	**17** B	**18** C	**19** C	**20** A
21 E	**22** E	**23** C	**24** D	**25** D

Extended-response questions

1 a i $\frac{15}{28}$ **ii** $\frac{37}{56}$ **iii** $\frac{43}{49}$

b i $\frac{9}{14}$ **ii** $\frac{135}{392}$

2 a $\frac{1}{9}$ **b** $\frac{13}{36}$ **c** $\frac{1}{2}$ **d** $\frac{13}{36}$

3 a i 0.0027 **ii** 0.12 **iii** 0.17 **iv** 0.72

b Maximum 0.302 when $p = 0.8$

4 a $\frac{59}{120}$ **b** $\frac{45}{59}$

c i 0.9844 **ii** 0.2627

5 a $\frac{167}{360}$

b i $\frac{108}{193}$ **ii** $\frac{45}{193}$

6 a i $\frac{1}{9}$ **ii** $\frac{5}{18}$

b i $\frac{1}{81}$ **ii** $\frac{13}{324}$

7 a i $m = 30$, $q = 35$, $s = 25$

ii $m + q = 65$

b $\frac{3}{10}$ **c** $\frac{7}{12}$

8 a 60 **b** 8 **c** 0.1

9 a $\frac{1}{60}$ **b** $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{6}{13}$

10 a i 10 000 cm^2 **ii** 400 cm^2 **iii** 6400 cm^2

b i 0.04 **ii** 0.12 **iii** 0.64

c i 0.0016 **ii** 0.000 64

11 b $\frac{1}{4}(\alpha^2 + 4\beta + 2\alpha\beta - 3\beta^2)$

c $\alpha = \frac{1}{2}$, $\beta = \frac{1}{3}$

12 a i 0.328 **ii** 0.205 **iii** 0.672

b i 11 **ii** 18

Investigations

See solutions supplement

Chapter 13

Exercise 13A

1 a x^5 **b** $8x^7$ **c** x^2 **d** $2x^3$

e a^6 **f** 2^6 **g** x^2y^2 **h** x^4y^6

i $\frac{x^3}{y^3}$ **j** $\frac{x^6}{y^4}$

2 a 3^{17} **b** x^7y^5 **c** 3^{4x+3} **d** $30a^5b^6$

3 a x^2y **b** b^{4x+1} **c** $4a^5b$

4 a $\frac{1}{49}$ **b** 64 **c** $\frac{8}{125}$

5 a b^{10} **b** 729 **c** b^4

6 a $\frac{27a^8b}{16}$ **b** $\frac{125b^6}{c^9}$

7 a 64 **b** $-27a^3$ **c** $-96a^3$

8 a 2^{-2n} **b** 2^4 **c** $\frac{5^{2n}}{2^{2n}}$

9 a x^9 **b** 2^{16} **c** 3^{17} **d** q^8p^9

e $a^{11}b^3$ **f** 2^8x^{18} **g** $m^{11}n^{12}p^{-2}$

h $2a^5b^{-2}$

10 a x^2y^3 **b** $8a^8b^3$ **c** x^5y^2 **d** $\frac{9}{2}x^2y^3$

11 a $\frac{1}{n^4p^5}$ **b** $\frac{2x^8z}{y^4}$ **c** $\frac{b^5}{a^5}$ **d** $\frac{a^3b}{c}$

e $a^{n+2}b^{n+1}c^{n-1}$

12 a 3^{17n} **b** 2^{3-n} **c** $\frac{3^{4n-11}}{2^2}$ **d** $2^{n+1}3^{3n-1}$

e 5^{3n-2} **f** $2^{3x-3} \times 3^{-4}$

g $3^{6-n} \times 2^{-5n}$ **h** $3^3 = 27$ **i** 6

13 a $2^{12} = 4096$ **b** $5^5 = 3125$

c $3^3 = 27$

Exercise 13B

1 a 25 **b** 27 **c** $\frac{1}{9}$ **d** 16

e $\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $\frac{1}{25}$ **h** 16

i $\frac{1}{10\,000}$ **j** 1000 **k** 27 **l** $\frac{3}{5}$

m -2 **n** $\frac{1}{625}$ **o** 16 **p** 343

2 a $a^{\frac{1}{6}}b^{-\frac{7}{6}}$ **b** $a^{-6}b^{\frac{9}{2}}$ **c** $3^{-\frac{7}{3}} \times 5^{-\frac{7}{6}}$

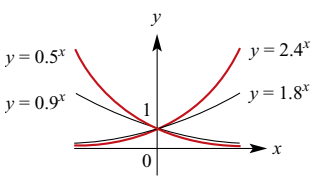
d $\frac{1}{4}$ **e** x^6y^{-8} **f** $a^{\frac{14}{15}}$

3 a $(2x-1)^{\frac{3}{2}}$ **b** $(x-1)^{\frac{5}{2}}$ **c** $(x^2+1)^{\frac{3}{2}}$

d $(x-1)^{\frac{7}{2}}$ **e** $x(x-1)^{-\frac{1}{2}}$ **f** $(5x^2+1)^{\frac{4}{3}}$

Exercise 13C

1



- All pass through (0, 1)
- Horizontal asymptote $y = 0$
- Increasing for base > 1
- Decreasing for base < 1

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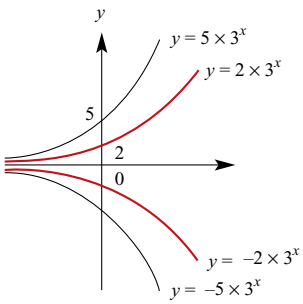
2

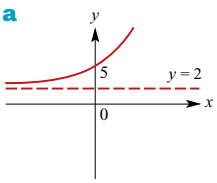


- $y = a \times b^x$ has y-axis intercept at $(0, a)$
- Horizontal asymptote $y = 0$
- Graphs c and d are reflections in the x-axis of graphs a and b

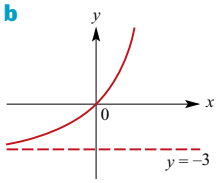
3 a $y = 8.574$ **b** $x = 3.807$

4 $x = 0.778$

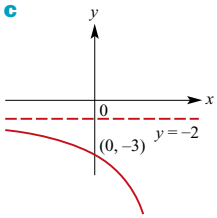
5 a



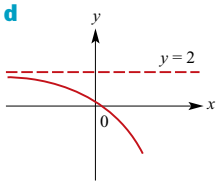
b



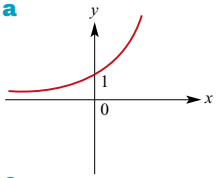
c



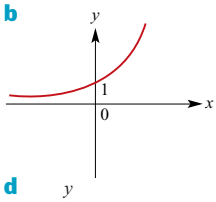
d



6 a



b



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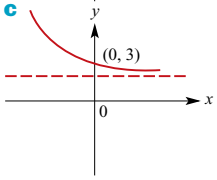

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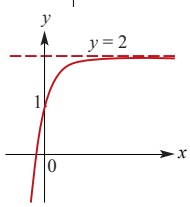


Exercise 13D

1 a 3 **b** 3 **c** $\frac{1}{2}$ **d** $\frac{3}{4}$ **e** $\frac{1}{3}$
f 4 **g** 2 **h** 3 **i** 3

2 a 1 **b** 2 **c** $-\frac{3}{2}$ **d** $\frac{4}{3}$ **e** -1
f 8 **g** 3 **h** -4 **i** 8 **j** 4
k $3\frac{1}{2}$ **l** 6 **m** $7\frac{1}{2}$

3 a $\frac{4}{5}$ **b** $\frac{3}{2}$ **c** $5\frac{1}{2}$

4 a 0 **b** 0, -2 **c** 1, 2 **d** 0, 1

5 a 2.32 **b** 1.29 **c** 1.26 **d** 1.75

6 a $x > 2$ **b** $x > \frac{1}{3}$ **c** $x \leq \frac{1}{2}$ **d** $x < 3$
e $x < \frac{3}{4}$ **f** $x > 1$ **g** $x \leq 3$

Exercise 13E

1 a 7 **b** 4 **c** 3 **d** -1

2 a $\log_2(10a)$ **b** 1 **c** $\log_2\left(\frac{9}{4}\right)$ **d** 1
e $3 \log_2 a$ **f** 9 **g** $-\log_5 6$ **h** -2

3 a 3 **b** 4 **c** -7 **d** -3
e 4 **f** -3 **g** 4 **h** -6
i -9 **j** -1 **k** 4 **l** -2

4 a 2 **b** 7 **c** 9 **d** 1
e $\frac{5}{2}$ **f** $\log_x(a^5)$ **g** 3 **h** 1

5 a 2 **b** 27 **c** $\frac{1}{125}$ **d** 8
e 30 **f** $\frac{2}{3}$ **g** 8 **h** 64
i 4 **j** 10

6 a 5 **b** 32.5 **c** 22 **d** 20
e $\frac{3 \pm \sqrt{17}}{2}$ **f** 3 or 0

7 $2 + 3a - \frac{5c}{2}$ **9** 10

10 a 4 **b** $\frac{6}{5}$ **c** 3 **d** 10
e 9 **f** 2

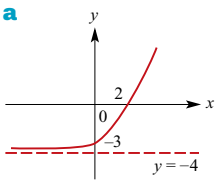
Exercise 13F

1 a 2.81 **b** -1.32 **c** 2.40 **d** 0.79
e -2.58 **f** -0.58

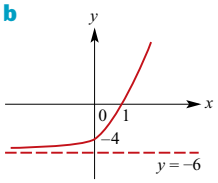
2 a 1.90 **b** 3.10 **c** -0.68

3 a $x > 3$ **b** $x < 1.46$ **c** $x < -1.15$
d $x \leq 2.77$ **e** $x \geq 1.31$

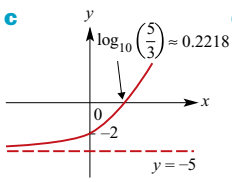
4 a



b



c



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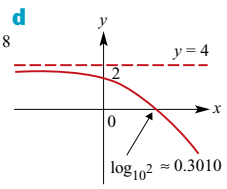
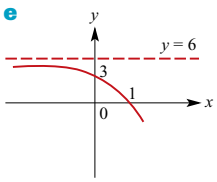
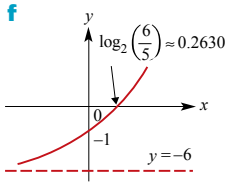


e



f



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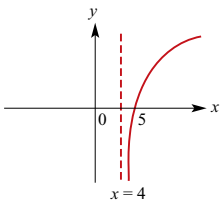
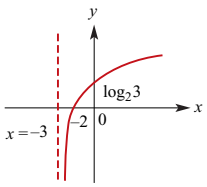
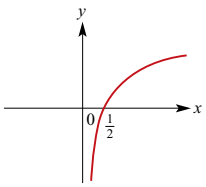
Exercise 13G

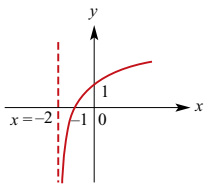
1 a Domain = $(4, \infty)$ **b** Domain = $(-3, \infty)$



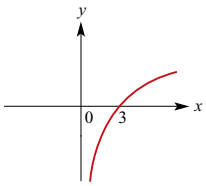


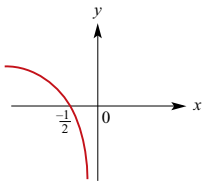
c Domain = $(0, \infty)$ **d** Domain = $(-2, \infty)$



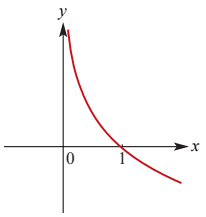


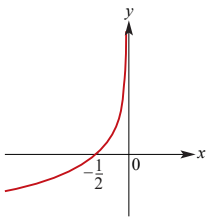
e Domain = $(0, \infty)$ **f** Domain = $(-\infty, 0)$



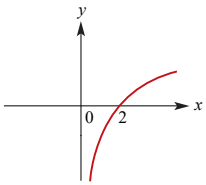


g Domain = $(0, \infty)$ **h** Domain = $(-\infty, 0)$





i Domain = $(0, \infty)$



2 a $y = 2\log_{10} x$ **b** $y = 10^{\frac{1}{3}x}$

c $y = \frac{1}{3}\log_{10} x$ **d** $y = \frac{1}{3}10^{\frac{1}{2}x}$

3 a $f^{-1}(x) = \log_3(x - 2)$

b $f^{-1}(x) = 2^x + 3$

c $f^{-1}(x) = \log_3\left(\frac{x-2}{4}\right)$

d $f^{-1}(x) = \log_5(x + 2)$

e $f^{-1}(x) = \frac{1}{3} \times 2^x$

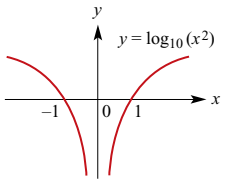
f $f^{-1}(x) = 3 \times 2^x$

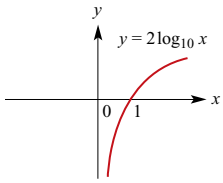
g $f^{-1}(x) = 2^x - 3$

h $f^{-1}(x) = \log_3\left(\frac{x+2}{5}\right)$

4 a 0.64 **b** 0.40

5





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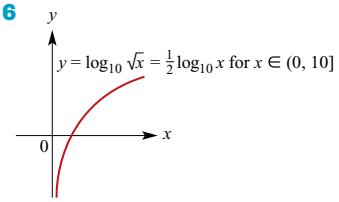

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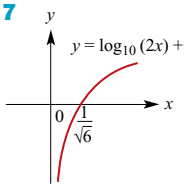
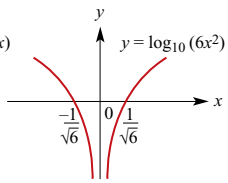




Exercise 13H

1 a $N = 1000 \times 2^{\frac{t}{15}}$ **b** 50 minutes

2 79 726 years

3 7575 years

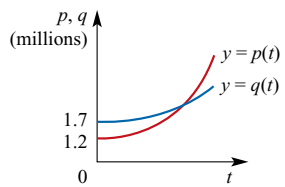
4 a 535 millibars **b** 7331 metres

5 22 hours later, i.e, 10:00 am Monday

6 6.4°C

7 $t > 18.668\ldots$

8 a



b i $t = 12.56\ldots$ (mid 1962)

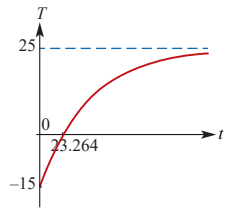
ii $t = 37.56\ldots$ (mid 1987)

9 a $k = \log_{10}\left(\frac{5}{4}\right)$ **b** 7.213 hours

10 a $T \to 25$; room temperature is 25°C

b 23.264 minutes

c



d The ice-cream approaches but never reaches room temperature

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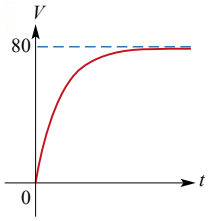
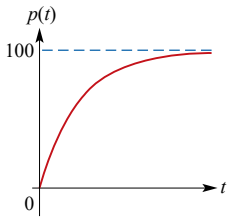
11 a 80 m/s **b** 1.577 seconds

c



12 a $p(t) \to 100$ **b** 25.237 days

c



13 a $y = 3 \times 5^x$ **b** $y = 4 \times (\frac{1}{2})^x$ **c** $y = 5 \times (\frac{3}{2})^x$

14 a $N = 1000 \times 10^{\frac{t}{5}}$ **b** 210 minutes

c 15 hours **d** 251 189 bacteria

15 $a = 6 \times \left(\frac{10}{3}\right)^{-\frac{2}{3}}$ and $k = \frac{1}{3}\log_{10}\left(\frac{10}{3}\right)$

16 $y = 1.5 \times 0.575^x$

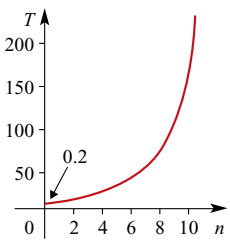
17 $p = 2.5 \times 1.35^t$

18 a

Cuts, n	Sheets	Thickness, T (mm)
0	1	0.2
1	2	0.4
2	4	0.8
3	8	1.6
4	16	3.2
5	32	6.4
6	64	12.8
7	128	25.6
8	256	51.2
9	512	102.4
10	1024	204.8

b $T = 0.2 \times 2^n$

c



d 214 748.4 m

19 $d_0 = 41.92$, $m = 0.094$

Chapter 13 review

Technology-free questions

1 a a^4 **b** $\frac{1}{b^2}$ **c** $\frac{1}{m^2n^2}$ **d** $\frac{1}{ab^6}$

e $\frac{3a^6}{2}$ **f** $\frac{5}{3a^2}$ **g** a^3 **h** $\frac{n^8}{m^4}$

i $\frac{1}{p^2q^4}$ **j** $\frac{8}{5a^{11}}$ **k** $2a$ **l** $a^2 + a^6$

2 a $\log_2 7$ **b** $\frac{1}{2}\log_2 7$ **c** $\log_{10} 2$

d $\log_{10}\left(\frac{18}{5}\right)$ **e** $1 + \log_{10} 11$

f $1 + \log_{10} 101$ **g** $\frac{1}{5}\log_2 100$

h $-\log_2 10$

3 a 2 **b** 3 **c** 3 **d** -1

4 a $\log_a(xy)$ **b** $\log_2(x(x+3))$

c $\log_b\left(\frac{2x}{3y}\right)$ **d** $\log_a 8$

e -2 **f** $6\log_3 x$

5 a $\frac{5}{2}$ **b** 1 **c** -2 **d** -4 **e** $\frac{8}{3}$ **f** -1

6 a $\frac{3}{2}$ **b** $\frac{5}{3}$ **c** $\frac{4}{3}$ **d** $\frac{1}{2}$

7 a 9 **b** 8 **c** 2 **d** 26

8 a 6 **b** 7 **c** 2 **d** 0

e 3 **f** -2 **g** -3 **h** 4

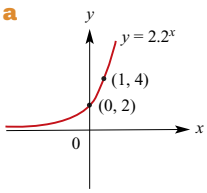
9 a $\log_{10} 6$ **b** $\log_{10} 6$ **c** $\log_{10}\left(\frac{a^2}{b}\right)$

d $\log_{10}\left(\frac{a^2}{25000}\right)$ **e** $\log_{10} y$ **f** $\log_{10}\left(\frac{a^2b^3}{c}\right)$

10 a $x = 3$ **b** $x = 3$ or $x = 0$

c $x = 1$ **d** $x = 2$ or $x = 3$

11 a



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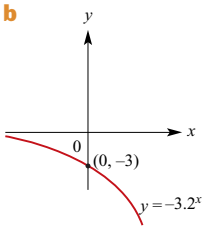

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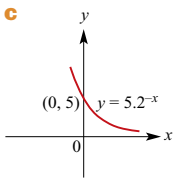

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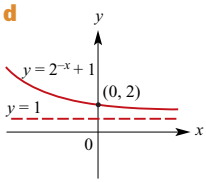

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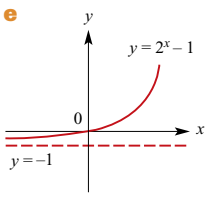

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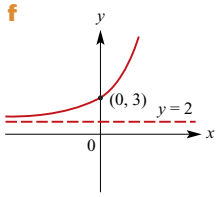
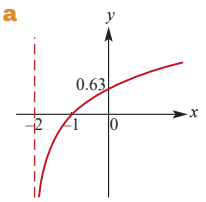
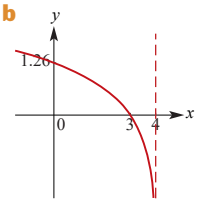


12 a



b



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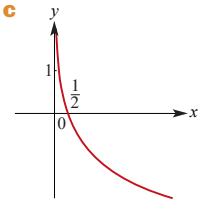
c



13 a $x = 6$ **b** $x = \sqrt{17}$ **c** $x = \dfrac{1 + 11\sqrt{6}}{5}$

14 a $x = 1$ or $x = 2$ **b** $x = -1$

15 $x = 1$ **17** 3

18 a $k = \dfrac{1}{7}$ **b** $q = \dfrac{3}{2}$

19 a $a = \dfrac{1}{2}$ **b** $y = -4$ or $y = 20$

Multiple-choice questions

1 C **2** A **3** C **4** C **5** A
6 B **7** A **8** A **9** A **10** A

Extended-response questions

1 a $(\frac{1}{2})^{3n}$ **b** $(\frac{1}{2})^{5n-2}$ **c** $n = 3$

2 a $729(\frac{1}{4})^n$ **b** $128(\frac{1}{2})^n$ **c** 4 times

3 a Batch 1: $15(0.95)^n$; Batch 2: $20(0.94)^n$
b 32 years

4 a X \$1.82 Y \$1.51 Z \$2.62
b X \$4.37 Y \$4.27 Z \$3.47
c Intersect at $t = 21.784\ldots$ and $t = 2.090\ldots$; therefore Mar 2019 until Oct 2020
d Mar 2020 until Oct 2020; approx. 8 months

5 a 13.81 years **b** 7.38 years

6 a Temperature $= 87.065 \times 0.94^t$
b i 87.1°C **ii** 18.56°C
c Temperature $= 85.724 \times 0.94^t$
d i 85.72°C **ii** 40.82°C
e 28.19 minutes

7 a $a = 0.2$ and $b = 5$
b i $z = x \log_{10} b$ **ii** $a = 0.2$ and $k = \log_{10} 5$

8 a $y = 2 \times 1.585^x$ **b** $y = 2 \times 10^{0.2x}$
c $x = 5 \log_{10}\left(\dfrac{y}{2}\right)$

Chapter 14

Exercise 14A

1 a $\frac{\pi}{3}$ **b** $\frac{4\pi}{5}$ **c** $\frac{4\pi}{3}$ **d** $\frac{11\pi}{6}$ **e** $\frac{7\pi}{3}$ **f** $\frac{8\pi}{3}$

2 a 120° **b** 150° **c** 210° **d** 162°
e 100° **f** 324° **g** 220° **h** 324°

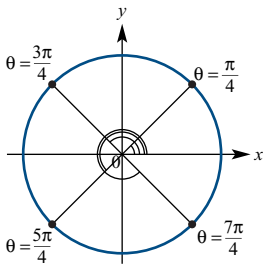
3 a 34.38° **b** 108.29° **c** 166.16° **d** 246.94°
e 213.14° **f** 296.79° **g** 271.01° **h** 343.77°

4 a 0.66 **b** 1.27 **c** 1.87 **d** 2.81
e 1.47 **f** 3.98 **g** 2.38 **h** 5.74

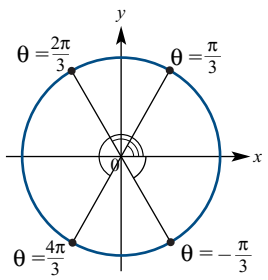
5 a −60° **b** −720° **c** −540° **d** −180°
e 300° **f** −330° **g** 690° **h** −690°

6 a -2π **b** -3π **c** $-\frac{4\pi}{3}$ **d** -4π
e $-\frac{11\pi}{6}$ **f** $-\frac{7\pi}{6}$

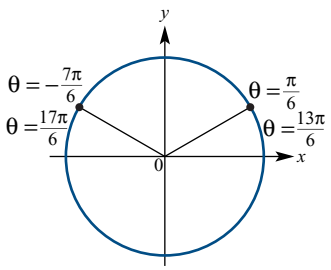
7 a



b



c



Exercise 14B

1 a 0, 1 **b** −1, 0 **c** 1, 0 **d** 1, 0
e 0, −1 **f** 1, 0 **g** −1, 0 **h** 0, 1

2 a 0.95 **b** 0.75 **c** −0.82 **d** 0.96
e −0.5 **f** −0.03 **g** −0.86 **h** 0.61

3 a 0, −1 **b** −1, 0 **c** −1, 0 **d** −1, 0
e −1, 0 **f** 0, −1 **g** 0, −1 **h** 0, −1

Exercise 14C

1 a 0 **b** 0 **c** Undefined **d** 0
e Undefined **f** Undefined

2 a −34.23 **b** −2.57 **c** −0.97 **d** −1.38
e 0.95 **f** 0.75 **g** 1.66

3 a 0 **b** 0 **c** 0 **d** 0 **e** 0 **f** 0

Exercise 14D

1 a −0.42 **b** −0.7 **c** −0.42 **d** −0.38
e 0.42 **f** −0.38 **g** −0.7 **h** 0.7

2 a −0.7 **b** −0.6 **c** −0.4 **d** −0.6
e −0.7 **f** −0.7 **g** 0.4 **h** 0.6

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3 **a** $\frac{5\pi}{6}$ **b** $\frac{7\pi}{6}$ **c** $\frac{11\pi}{6}$

4 **a** $-\frac{1}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{2}$ **d** $-\frac{\sqrt{3}}{2}$
e $-\sqrt{3}$ **f** $-\sqrt{3}$

5 **a** $-\frac{\sqrt{3}}{2}$ **b** $\frac{1}{2}$ **c** $-\sqrt{3}$ **d** $-\frac{\sqrt{3}}{2}$ **e** $-\frac{1}{2}$

6 **a** $a = 0.7660$, $b = 0.6428$
b $c = -0.7660$, $d = 0.6428$
c **i** $\cos 140° = -0.7660$, $\sin 140° = 0.6428$
ii $\cos 140° = -\cos 40°$

7 **a** 120° **b** 240° **c** −60° **d** 120°
e 240° **f** 300°

Exercise 14E

1 **a** $\sin = \frac{\sqrt{3}}{2}$, $\cos = -\frac{1}{2}$, $\tan = -\sqrt{3}$

b $\sin = \frac{1}{\sqrt{2}}$, $\cos = -\frac{1}{\sqrt{2}}$, $\tan = -1$

c $\sin = -\frac{1}{2}$, $\cos = -\frac{\sqrt{3}}{2}$, $\tan = \frac{1}{\sqrt{3}}$

d $\sin = -\frac{\sqrt{3}}{2}$, $\cos = -\frac{1}{2}$, $\tan = \sqrt{3}$

e $\sin = -\frac{1}{\sqrt{2}}$, $\cos = \frac{1}{\sqrt{2}}$, $\tan = -1$

f $\sin = \frac{1}{2}$, $\cos = \frac{\sqrt{3}}{2}$, $\tan = \frac{1}{\sqrt{3}}$

g $\sin = \frac{\sqrt{3}}{2}$, $\cos = \frac{1}{2}$, $\tan = \sqrt{3}$

h $\sin = -\frac{1}{\sqrt{2}}$, $\cos = -\frac{1}{\sqrt{2}}$, $\tan = 1$

i $\sin = \frac{\sqrt{3}}{2}$, $\cos = \frac{1}{2}$, $\tan = \sqrt{3}$

j $\sin = -\frac{\sqrt{3}}{2}$, $\cos = \frac{1}{2}$, $\tan = -\sqrt{3}$

2 **a** $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $-\frac{1}{\sqrt{3}}$
d $-\frac{1}{2}$ **e** $-\frac{1}{\sqrt{2}}$ **f** $\sqrt{3}$
g $-\frac{\sqrt{3}}{2}$ **h** $\frac{1}{\sqrt{2}}$ **i** $-\frac{1}{\sqrt{3}}$

3 **a** $-\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $\frac{1}{\sqrt{3}}$
d Undefined **e** 0 **f** $-\frac{1}{\sqrt{2}}$
g $\frac{1}{\sqrt{2}}$ **h** −1

4 **a** $\sin(0.1) = 0.099833\ldots$
b $\sin(0.2) = 0.198669\ldots$
c $\sin(-0.1) = -0.099833\ldots$
d $\sin(-0.2) = -0.198669\ldots$

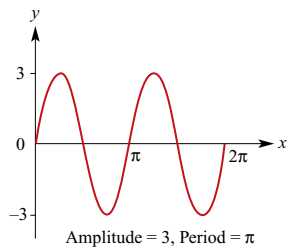
Exercise 14F

1 **a** **i** 2π **ii** 2 **b** **i** π **ii** 3 **c** **i** $\frac{2\pi}{3}$ **ii** $\frac{1}{2}$
d **i** 4π **ii** 3 **e** **i** $\frac{2\pi}{3}$ **ii** 4 **f** **i** $\frac{\pi}{2}$ **ii** $\frac{1}{2}$
g **i** 4π **ii** 2 **h** **i** 2 **ii** 2 **i** **i** 4 **ii** 3

2 **a** Dilation of factor 3 from the x-axis;
Amplitude = 3; Period = 2π
b Dilation of factor $\frac{1}{5}$ from the y-axis;
Amplitude = 1; Period $\frac{2\pi}{5}$
c Dilation of factor 3 from the y-axis;
Amplitude = 1; Period = 6π
d Dilation of factor 2 from the x-axis and dilation of factor $\frac{1}{5}$ from the y-axis;
Amplitude = 2; Period = $\frac{2\pi}{5}$

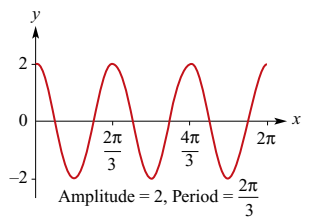
3 **a** Dilation of factor $\frac{1}{5}$ from the y-axis and reflection in the x-axis;
Amplitude = 1; Period = $\frac{2\pi}{5}$
b Reflection in the y-axis;
Amplitude = 1; Period = 2π
c Dilation of factor 3 from the y-axis and dilation of factor 2 from the x-axis;
Amplitude = 2; Period = 6π
d Dilation of factor 2 from the y-axis, dilation of factor 4 from the x-axis and reflection in the x-axis; Amplitude = 4; Period = 4π
e Dilation of factor 3 from the y-axis, dilation of factor 2 from the x-axis and reflection in the y-axis; Amplitude = 2; Period = 6π

4 **a**



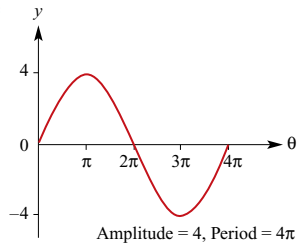
Amplitude = 3, Period = π

b



Amplitude = 2, Period = $\frac{2\pi}{3}$

c



Amplitude = 4, Period = 4π

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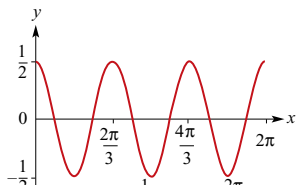
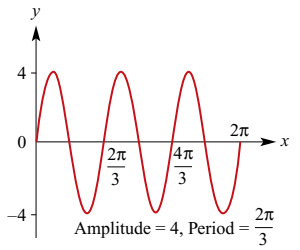
d



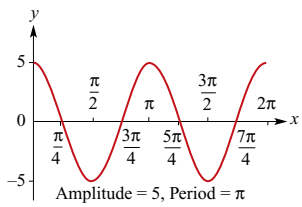
Amplitude = $\frac{1}{2}$, Period = $\frac{2\pi}{3}$

e



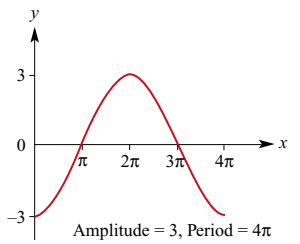
Amplitude = 4, Period = $\frac{2\pi}{3}$

f



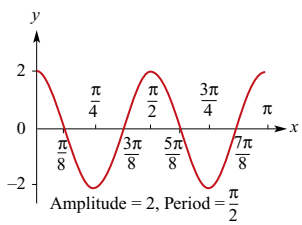
Amplitude = 5, Period = π

g



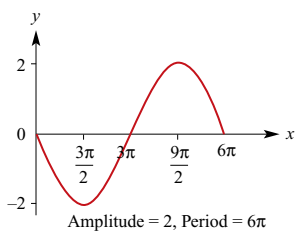
Amplitude = 3, Period = 4π

h



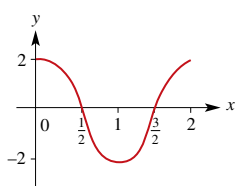
Amplitude = 2, Period = $\frac{\pi}{2}$

i

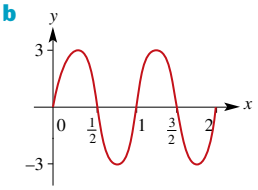


Amplitude = 2, Period = 6π

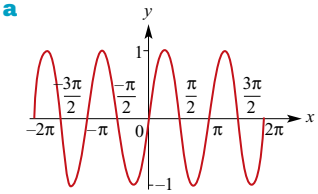
5 a



b



6 a



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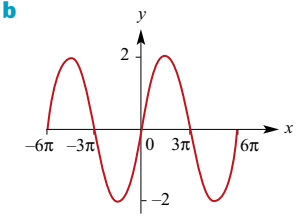
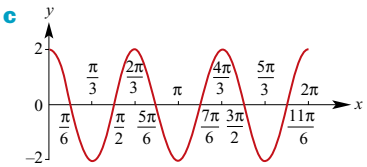


c



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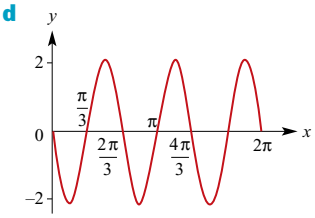

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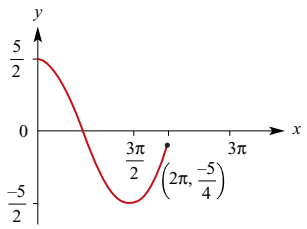
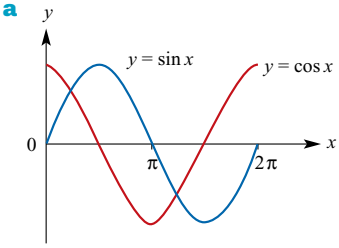


8 a



b $\frac{\pi}{4}, \frac{5\pi}{4}$

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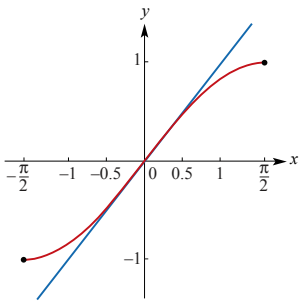
9



Exercise 14G

1 a $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

b $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

c $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

2 a 0.93, 2.21 **b** 4.30, 1.98 **c** 3.50, 5.93

d 0.41, 2.73 **e** 2.35, 3.94 **f** 1.77, 4.51

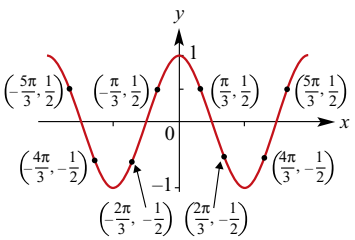
3 a 150, 210 **b** 30, 150 **c** 120, 240

d 120, 240 **e** 60, 120 **f** 45, 135

4 a $\frac{\pi}{6}, \frac{11\pi}{6}$ **b** $\frac{5\pi}{4}, \frac{7\pi}{4}$ **c** $\frac{\pi}{4}, \frac{7\pi}{4}$

5 a $\frac{3\pi}{4}, -\frac{3\pi}{4}$ **b** $\frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\frac{2\pi}{3}, -\frac{2\pi}{3}$

6



7 a $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

b $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

c $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

d $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$

e $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$

f $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

8 a 2.034, 2.678, 5.176, 5.820

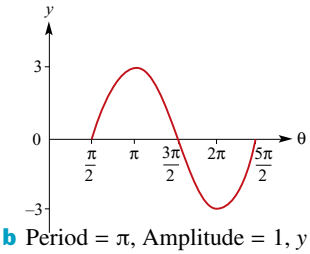
b 1.893, 2.820, 5.034, 5.961

c 0.580, 2.562, 3.721, 5.704

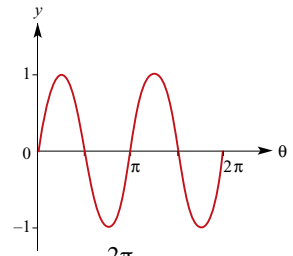
d 0.309, 1.785, 2.403, 3.880, 4.498, 5.974

Exercise 14H

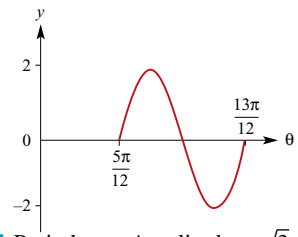
1 a Period = 2π, Amplitude = 3, $y = \pm 3$



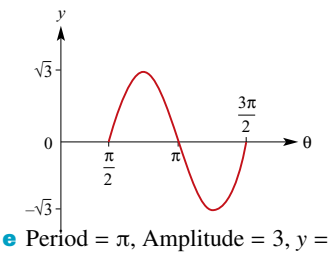
b Period = π, Amplitude = 1, $y = \pm 1$



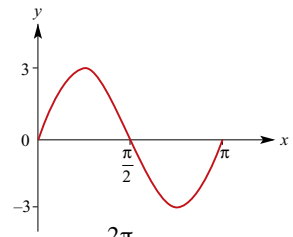
c Period = $\frac{2\pi}{3}$, Amplitude = 2, $y = \pm 2$



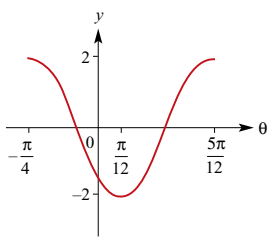
d Period = π, Amplitude = $\sqrt{3}$, $y = \pm\sqrt{3}$



e Period = π, Amplitude = 3, $y = \pm 3$



f Period = $\frac{2\pi}{3}$, Amplitude = 2, $y = \pm 2$



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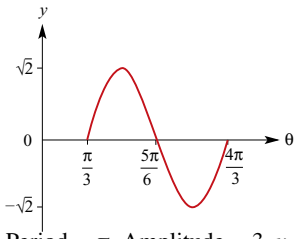
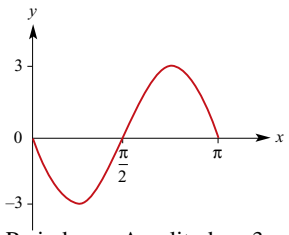
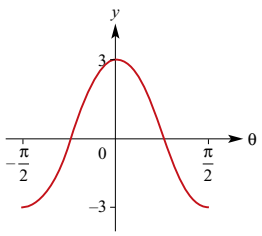
g Period = π, Amplitude = $\sqrt{2}$, $y = \pm\sqrt{2}$



h Period = π, Amplitude = 3, $y = \pm 3$

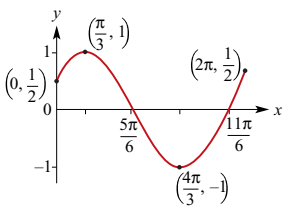


i Period = π, Amplitude = 3, $y = \pm 3$



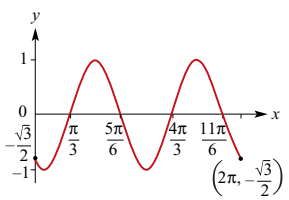
2 a $f(0) = \frac{1}{2}$, $f(2\pi) = \frac{1}{2}$

b



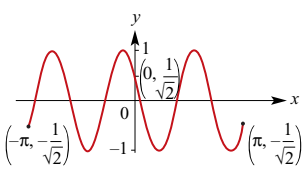
3 a $f(0) = -\frac{\sqrt{3}}{2}$, $f(2\pi) = -\frac{\sqrt{3}}{2}$

b



4 a $f(-\pi) = -\frac{1}{\sqrt{2}}$, $f(\pi) = -\frac{1}{\sqrt{2}}$

b



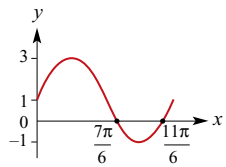
5 a $y = 3\sin\left(\frac{x}{2}\right)$ **b** $y = 3\sin(2x)$

c $y = 2\sin\left(\frac{x}{3}\right)$ **d** $y = \sin 2\left(x - \frac{\pi}{3}\right)$

e $y = \sin\frac{1}{2}\left(x + \frac{\pi}{3}\right)$

Exercise 14I

1 a



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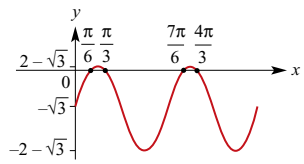

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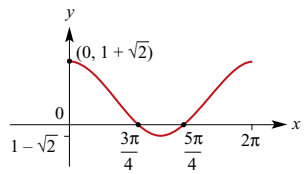

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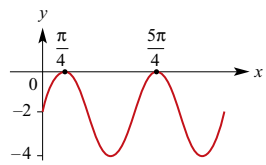
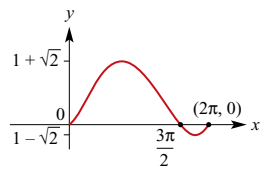
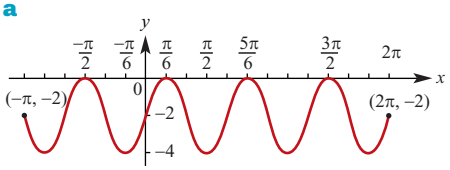


e



2 a



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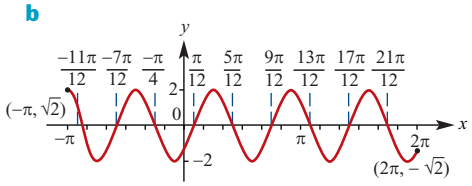

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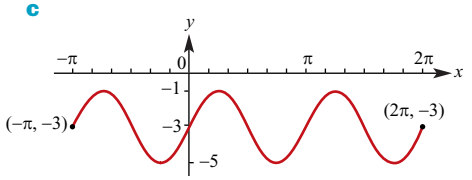

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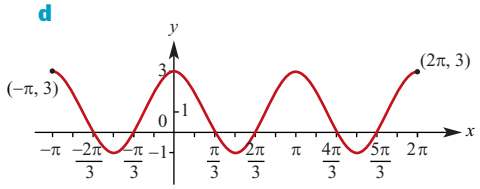
d



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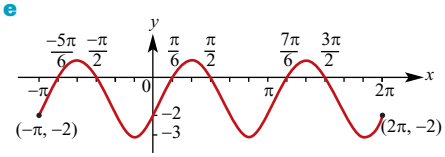

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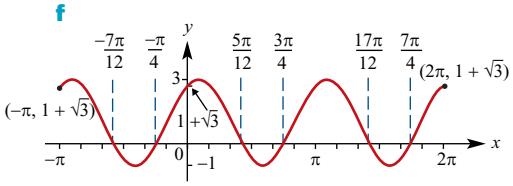
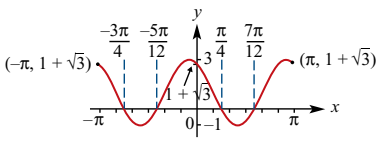


3 a



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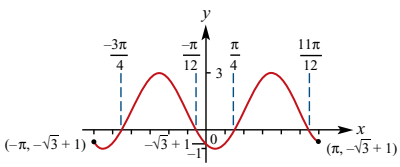

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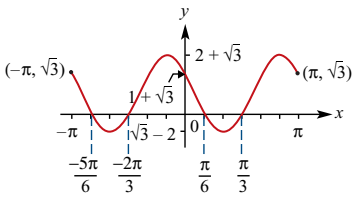


Exercise 14J

1 a 0.6 **b** 0.6 **c** −0.7 **d** 0.3
e −0.3 **f** $\frac{10}{7}$ **g** −0.3 **h** 0.6
i −0.6 **j** −0.3

2 a $\frac{\pi}{3}$ **b** $\frac{\pi}{3}$ **c** $\frac{5\pi}{12}$ **d** $\frac{\pi}{14}$

3 $\sin x = -\frac{4}{5}$, $\tan x = -\frac{4}{3}$

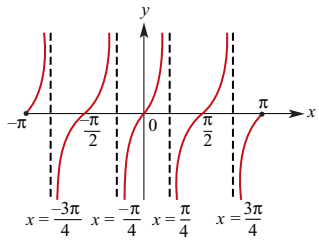
4 $\cos x = -\frac{12}{13}$, $\tan x = -\frac{5}{12}$

5 $\sin x = -\frac{2\sqrt{6}}{5}$, $\tan x = -2\sqrt{6}$

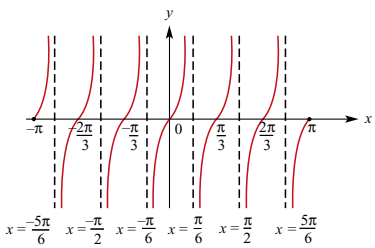
Exercise 14K

1 a $\frac{\pi}{4}$ **b** $\frac{3\pi}{2}$ **c** $\frac{\pi}{2}$

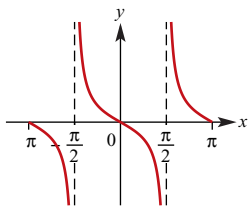
2 a



b



c



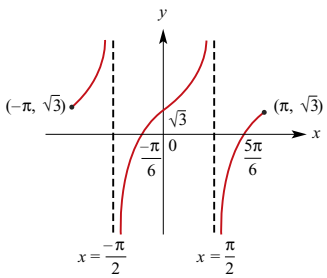
3 a $-\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$

b $-\frac{17\pi}{18}, -\frac{11\pi}{18}, -\frac{5\pi}{18}, \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}$

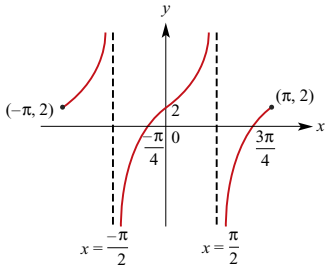
c $-\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}$

d $-\frac{13\pi}{18}, -\frac{7\pi}{18}, -\frac{\pi}{18}, \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}$

4 a



b



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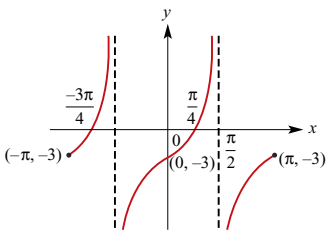
c



Exercise 14L

1 a 0.74 **b** 0.51 **c** 0.82, −0.82 **d** 0, 0.88

2 $y = a\sin(b\theta + c) + d$

a $a = 1.993$, $b = 2.998$, $c = 0.003$, $d = 0.993$

b $a = 3.136$, $b = 3.051$, $c = 0.044$, $d = -0.140$

c $a = 4.971$, $b = 3.010$, $c = 3.136$, $d = 4.971$

Exercise 14M

1 a $x = \dfrac{(12n+1)\pi}{6}$ or $x = \dfrac{(12n+5)\pi}{6}$, $n \in \mathbb{Z}$

b $x = \dfrac{(12n \pm 1)\pi}{18}$, $n \in \mathbb{Z}$

c $x = \dfrac{(3n+2)\pi}{3}$, $n \in \mathbb{Z}$

2 a $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$ **b** $\dfrac{\pi}{18}, \dfrac{11\pi}{18}$ **c** $\dfrac{2\pi}{3}, \dfrac{5\pi}{3}$

3 $x = n\pi$ or $x = \dfrac{(4n-1)\pi}{4}$, $n \in \mathbb{Z}$;

$x = -\dfrac{5\pi}{4}, -\pi, -\dfrac{\pi}{4}, 0, \dfrac{3\pi}{4}, \pi$ or $\dfrac{7\pi}{4}$

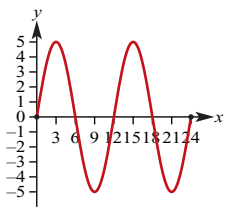
4 $x = \dfrac{n\pi}{3}$, $n \in \mathbb{Z}$; $x = -\pi, -\dfrac{2\pi}{3}, -\dfrac{\pi}{3}$ or 0

5 $x = \dfrac{6n-1}{12}$ or $x = \dfrac{3n+2}{6}$, $n \in \mathbb{Z}$;

$x = -\dfrac{2}{3}, -\dfrac{7}{12}, -\dfrac{1}{6}, -\dfrac{1}{12}, \dfrac{1}{3}, \dfrac{5}{12}, \dfrac{5}{6}$ or $\dfrac{11}{12}$

Exercise 14N

1 a



b $t = 3$ and $t = 15$

c 5 m above mean sea level

d $\dfrac{5\sqrt{3}}{2}$ m above mean sea level

e $\dfrac{5\sqrt{3}}{2}$ m above mean sea level

f $t \in [1, 5] \cup [13, 17]$

2 a 5 metres **b** 1 metre

c $t = 0.524$, 2.618 or 4.712 seconds

d $t = 0$, 1.047 or 2.094 seconds

e Particle oscillates between $x = 1$ and $x = 5$

3 a 7 m **b** 3 m

c $t = \dfrac{1}{4}, \dfrac{5}{4}, \dfrac{9}{4}, \dfrac{13}{4}$ or $\dfrac{17}{4}$

d $t = \dfrac{1}{12}, \dfrac{5}{12}, \dfrac{13}{12}, \dfrac{17}{12}, \dfrac{25}{12}$ or $\dfrac{29}{12}$

e Particle oscillates between $x = 3$ and $x = 7$

4 a **i** 10 **ii** $10 + 5\sqrt{3}$ **iii** $10 + 5\sqrt{3}$

iv $10 - 5\sqrt{3}$ **v** $10 - 5\sqrt{3}$

b 6 seconds **c** 20 metres

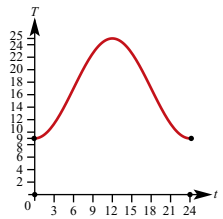
d $\dfrac{1}{2}, \dfrac{5}{2}, \dfrac{13}{2}, \dfrac{17}{2}$ s **e** $\dfrac{7}{2}, \dfrac{11}{2}, \dfrac{19}{2}, \dfrac{23}{2}$ s

5 a 9°C

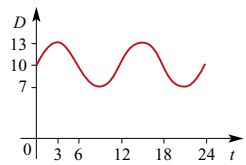
b Maximum = 25°C; Minimum = 9°C

c Between 7:28 and 16:32

d



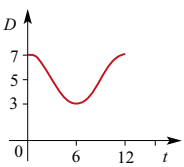
6 a



b $\{t : D(t) \geq 8.5\} = [0, 7] \cup [11, 19] \cup [23, 24]$

c 12.9 m

7 a $p = 5$, $q = 2$ **b**



c A ship can enter 2 hours after low tide

Chapter 14 review

Technology-free questions

1 a $\dfrac{11\pi}{6}$ **b** $\dfrac{9\pi}{2}$ **c** 6π **d** $\dfrac{23\pi}{4}$ **e** $\dfrac{3\pi}{4}$

f $\dfrac{9\pi}{4}$ **g** $\dfrac{13\pi}{6}$ **h** $\dfrac{7\pi}{3}$ **i** $\dfrac{4\pi}{9}$

2 a 150° **b** 315° **c** 495° **d** 45°

e 1350° **f** −135° **g** −45° **h** −495°

i −1035°

3 a $\dfrac{1}{\sqrt{2}}$ **b** $\dfrac{1}{\sqrt{2}}$ **c** $-\dfrac{1}{2}$ **d** $-\dfrac{\sqrt{3}}{2}$

e $\dfrac{\sqrt{3}}{2}$ **f** $-\dfrac{1}{2}$ **g** $\dfrac{1}{2}$ **h** $-\dfrac{1}{\sqrt{2}}$

4 a $2, 4\pi$ **b** $3, \dfrac{\pi}{2}$ **c** $\dfrac{1}{2}, \dfrac{2\pi}{3}$ **d** $3, \pi$

e $4, 6\pi$ **f** $\dfrac{2}{3}, 3\pi$

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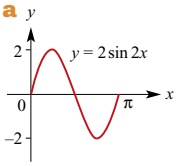
5 a



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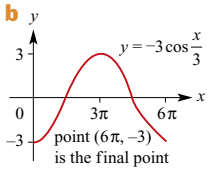

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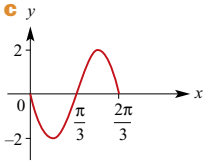

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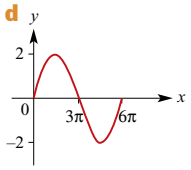

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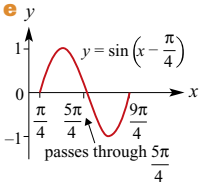

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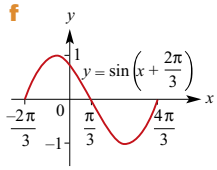
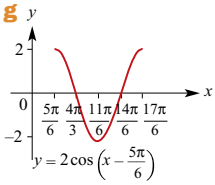
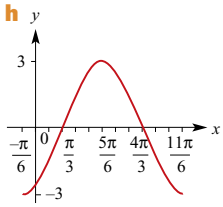


g



h



6 a $-\frac{2\pi}{3}, -\frac{\pi}{3}$ **b** $-\frac{\pi}{3}, -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$

c $\frac{\pi}{6}, \frac{3\pi}{2}$ **d** $\frac{7\pi}{6}$ **e** $\frac{\pi}{2}, \frac{7\pi}{6}$

Multiple-choice questions

1 C **2** D **3** E **4** C **5** E
6 D **7** E **8** E **9** C **10** B

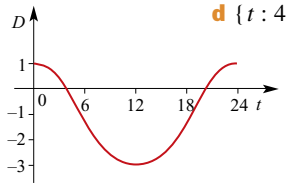
Extended-response questions

1 a i 1.83×10^{-3} hours **ii** 11.79 hours

b 26 April ($t = 3.86$), 14 August ($t = 7.48$)

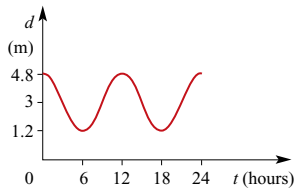
2 a 19.5°C **b** $D = -1 + 2\cos\left(\frac{\pi t}{12}\right)$

c



d $\{t : 4 < t < 20\}$

3 a

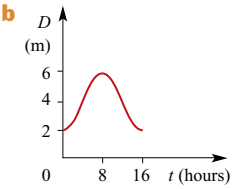


b 3 a.m., 3 p.m., 3 a.m.

c 9 a.m., 9 p.m. **d** 10:03 a.m.

e i 6:12 p.m. **ii** 5 trips

4 b



c $t = 16$ (8 p.m.)

d $t = 4$ and $t = 12$ (8 a.m. and 4 p.m.)

e i 1.5 m **ii** 2.086 m

f 9 hours 17 minutes

Chapter 15

Technology-free questions

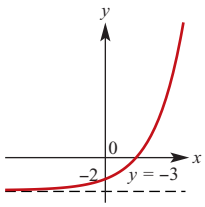
1 a $-24a^{10}$ **b** $\frac{a^3}{2b^2}$ **c** $\frac{3}{4x^5}$ **d** 8

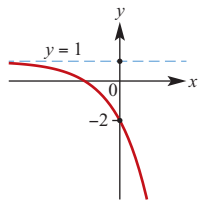
e $\frac{y^{\frac{2}{3}}}{x^{\frac{1}{6}}}$ **f** $\frac{1}{(2x-1)^{\frac{1}{2}}}$

2 a $\frac{25}{9}$ **b** 16 **c** 81 **d** $-\frac{1}{3}$

3 a $2^{6n} \times 3^{3n}$ **b** 12 **c** $\log_{10} 36$ **d** −3

4 a Range = $(-3, \infty)$ **b** Range = $(-\infty, 1)$





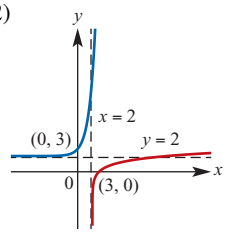
5 a $x = 3$ **b** $x = 0$ or $x = 2$ **c** $x > 4$

d $x = 7$ **e** $x = 1$

6 a $x = \log_2 5$ **b** $x = \frac{1}{3}(\log_5(10) - 1)$

c $x > \frac{\log_{10} 0.2}{\log_{10} 0.6}$

7 $f^{-1}(x) = \log_3(x - 2)$



8 a $\frac{\pi}{3}$ **b** $\frac{3\pi}{2}$ **c** $\frac{7\pi}{9}$

9 a −1 **b** 0 **c** 0 **d** Undefined

10 a −0.3 **b** −0.5 **c** 1.6 **d** −0.6

e 0.1 **f** $\frac{4}{5}$

11 a $\frac{\sqrt{3}}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** −1 **d** $\frac{1}{2}$

e $\frac{1}{\sqrt{2}}$ **f** $-\sqrt{3}$

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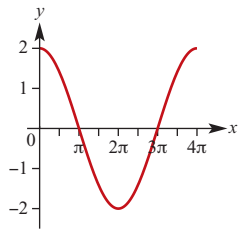
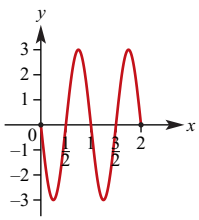
12 a Period = 4π; Amplitude = 2

b



c Dilation of factor 2 from the x-axis and dilation of factor 2 from the y-axis

13



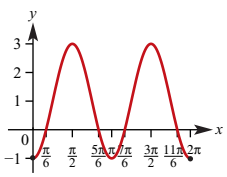
14 a $-\frac{7\pi}{6}, -\frac{5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

b $-\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

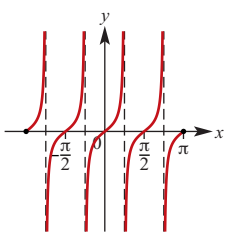
c $-\frac{17\pi}{12}, -\frac{13\pi}{12}, -\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

d $-\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

15



16



17 a $x = \frac{\pi}{6} + 2n\pi$ or $x = \frac{5\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$

b $x = \pm\frac{\pi}{6} + 2n\pi$, $n \in \mathbb{Z}$

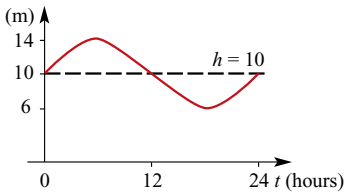
c $x = -\frac{\pi}{8} + \frac{n\pi}{2}$, $n \in \mathbb{Z}$

Multiple-choice questions

1 B	**2** B	**3** B	**4** E	**5** D
6 A	**7** D	**8** C	**9** B	**10** A
11 A	**12** D	**13** A	**14** D	**15** D
16 D	**17** A	**18** E	**19** D	**20** D
21 E	**22** A	**23** E	**24** B	**25** D
26 B				

Extended-response questions

1 a



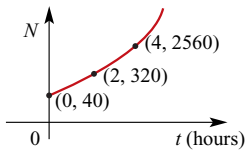
b $t = 3.2393$ and $t = 8.7606$

c $t \in [0.9652, 11.0348]$

2 a 40 bacteria

b **i** 320 **ii** 2560 **iii** 10 485 760

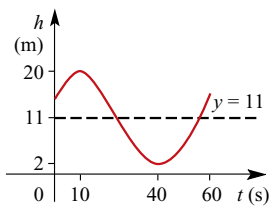
c



d 40 minutes ($= \frac{2}{3}$ hours)

3 a 60 seconds

b

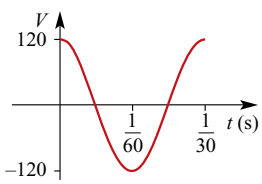


c [2, 20]

d First at height 2 metres after 40 seconds; then every 60 seconds after this first time

e At $t = 0$, $t = 20$ and $t = 60$, for $t \in [0, 60]$

4 a



b $t = \frac{1}{180}$ s

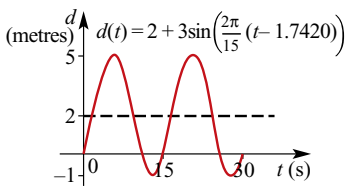
c $t = \frac{k}{30}$ s, for $k = 0, 1, 2, \ldots$

5 a **i** Period = 15 seconds

ii Amplitude = 3 **iii** $c = \frac{2\pi}{15}$

b $h = 1.74202$

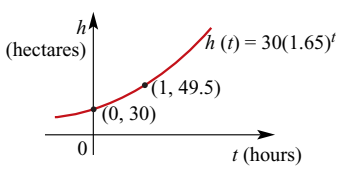
c



6 a **i** 30 **ii** 49.5 **iii** 81.675

b $k = 1.65$ **c** 6.792 hours

d



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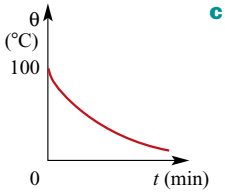
7 a

t	0	1	2	3	4	5
θ	100	60	40	30	25	22.5

b



c 1 minute

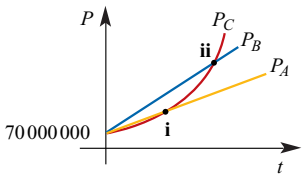
d 27.071

8 a $P_A = 70\,000\,000 + 3\,000\,000t$

$P_B = 70\,000\,000 + 5\,000\,000t$

$P_C = 70\,000\,000 \times 1.3^{\frac{t}{10}}$

b



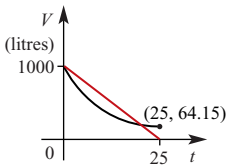
c i 35 years **ii** 67 years

9 a 2.378 km^2 **b** 17.288 hours

c $13.288 \leq t \leq 19.628$

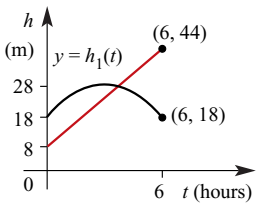
10 a $V_1(0) = V_2(0) = 1000$

b



c 64.15 litres **d** $t = 0$ and $t = 23.00$

11 a



b 3:19 a.m. to nearest minute ($t = 3.31$)

c i 9 a.m. **ii** $8 + 6t$ metres

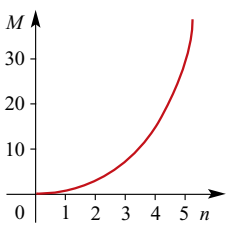
12 a

n	1	2	3	4
M	1	3	7	15

b $M = 2^n - 1$

n	5	6	7
M	31	63	127

c

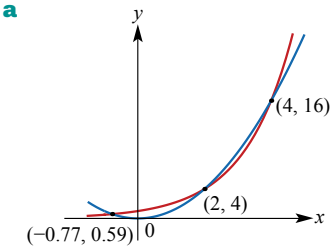


d

Three discs	1	2	3
Times moved	4	2	1

Four discs	1	2	3	4
Times moved	8	4	2	1

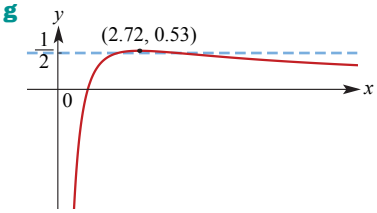
13 a



b i (2, 4), (4, 16) **ii** (−0.77, 0.59)

c $-0.77 < x < 2$ or $x > 4$

g



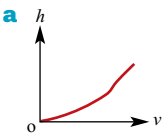
Investigations

See solutions supplement

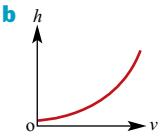
Chapter 16

Exercise 16A

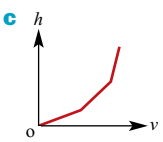
1 a



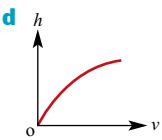
b



c



d

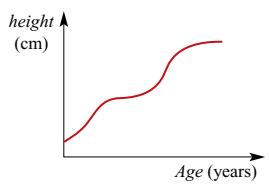


2 For the first 2 minutes, the particle travels a distance of 4 m with its speed increasing. For the next 4 minutes, it travels 4 m at constant speed. Then it turns back and returns to its starting point O, travelling at a constant speed and taking 8 minutes to reach O.

Note: *For Questions 3–6, there may be more than one correct answer.*

3 C is the most likely

4



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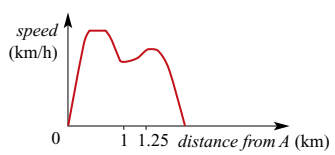

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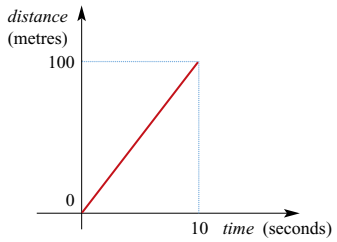
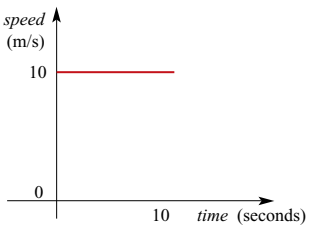
6 C and B are the most likely

7 a



b



8 D

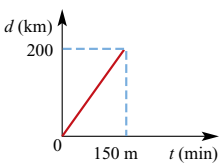
9 C

10 a $(-4, 0)$ **b** $[-7, -4) \cup (0, 3]$

11 a $(-3, 0)$ **b** $[-5, -3) \cup (0, 2]$

Exercise 16B

1 $\frac{4}{3}$ km/min = 80 km/h



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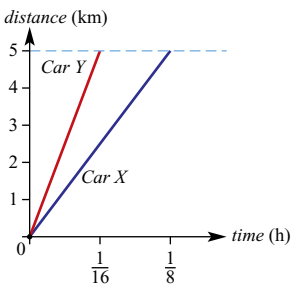

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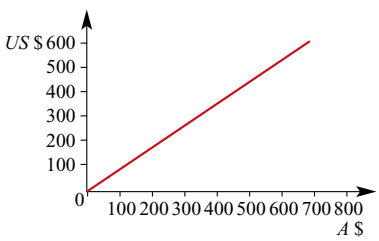


4 a 60 km/h **b** 3 m/s

c 400 m/min = 24 km/h = $6\frac{2}{3}$ m/s

d 35.29 km/h (correct to 2 d.p.)

e 20.44 m/s (correct to 2 d.p.)

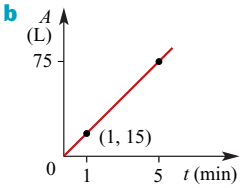
5 a 8 litres/min **b** 50 litres/min

c $\frac{135}{13}$ litres/min

6 a

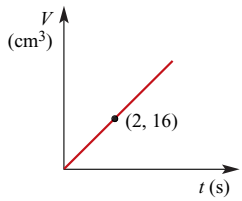
t	0	0.5	1	1.5	2	3	4	5
A	0	7.5	15	22.5	30	45	60	75

b



7 $\frac{\$200}{13}$ per hour = \$15.38 per hour

8



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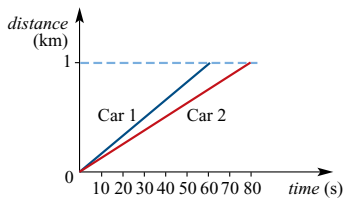


Exercise 16C

1 3.2 m/s

2 a 2 **b** 7 **c** $-\frac{1}{2}$ **d** $\frac{1-\sqrt{5}}{4}$

3 a $-\frac{25}{7}$ **b** $-\frac{18}{7}$ **c** 4 **d** $\frac{4b}{3a}$

4 a 4 m/s **b** 32 m/s

5 a \$2450.09 **b** \$150.03 per year

6 3.125 cm/min

7 C

Exercise 16D

1 7.19

2 a 0.015 **b** $\frac{1}{60} \approx 0.0167$

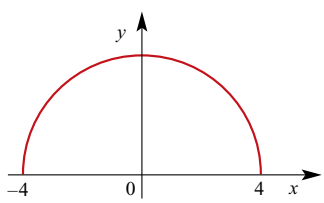
3 a i 9 **ii** 4.3246 **iii** 2.5893

b 2.30

4 a 25°C at 16:00 **b** ≈ 3°C/h **c** −2.5°C/h

5 −0.5952

6



a 0 **b** −0.6 **c** −1.1

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7 4

8 a 16 m^3/min **b** 10 m^3/min

9 a 18 million/min **b** 8.3 million/min

10 a 620 m^3/min flowing out
b 4440 m^3/min flowing out
c 284 000 m^3/min flowing out

11 a 7 **b** 9 **c** 2 **d** 35

12 a 28 **b** 12

13 a 10 **b** 4

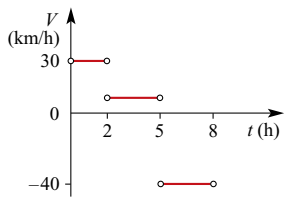
14 a i $\frac{2}{\pi} \approx 0.637$ **ii** $\frac{2\sqrt{2}}{\pi} \approx 0.9003$
iii 0.959 **iv** 0.998
b 1

Exercise 16E

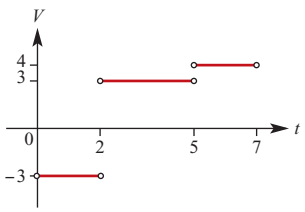
1 a 4 m/s **b** 1.12 m/s

2 a i 30 km/h **ii** $\frac{20}{3}$ km/h **iii** −40 km/h

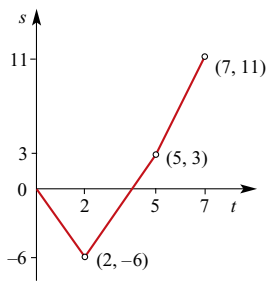
c



3 a



b



4 a $t = 2.5$ **b** $0 \leq t < 2.5$ **c** 6 m
d 5 s **e** 3 m/s

5 a $t = 6$ **b** 15 m/s **c** 17.5 m/s
d 20 m/s **e** −10 m/s **f** −20 m/s

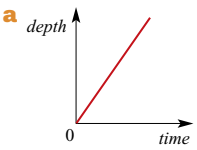
6 a 11 m/s **b** 15 m **c** 1 s
d 2.8 s **e** 15 m/s

7 a $t = 2$, $t = 3$ and $t = 8$
b $0 < t < 2.5$ and $t > 6$
c $t = 2.5$ and $t = 6$

Chapter 16 review

Technology-free questions

1 a



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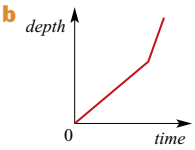
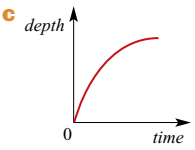


c



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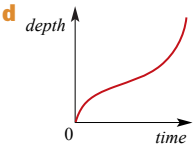
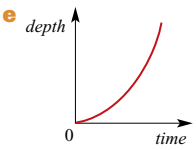
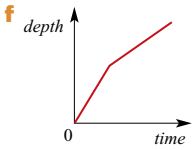


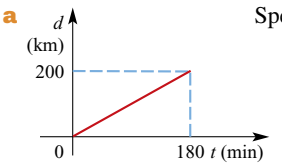
e



f

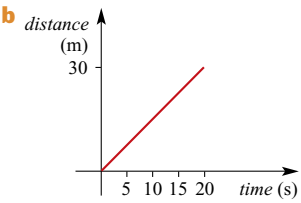


2 a

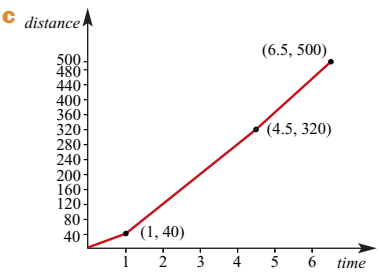


Speed $= \frac{200}{3}$ km/h
$= \frac{10}{9}$ km/min

b



c



3 36 cm^2/cm

4 a 1 **b** 13

5 a −2 m/s **b** −12.26 m/s **c** −14 m/s

Multiple-choice questions

1 C **2** B **3** B **4** E **5** D
6 E **7** D **8** C

Extended-response questions

1 a i 9.8 m/s **ii** 29.4 m/s
b i $4.9(8h - h^2)$
ii $4.9(8 - h)$
iii 38.22 m/s, 38.71 m/s, 38.995 m/s, 39.151 m/s, 39.1951 m/s

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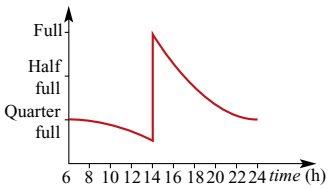
2



3 **a** $b + a \ (a \neq b)$ **b** 3 **c** 4.01

4 **a** $2\frac{2}{3}$, $1\frac{3}{5}$; Gradient $= -1\frac{1}{15}$
b 2.1053, 1.9048; Gradient $= -1.003$
c -1.000025 **d** -1.0000003

5 **a** **i** 2.5×10^8 **ii** 5×10^8
b 0.006 billion/year
c **i** 0.004 billion/year **ii** 0.015 billion/year
d 25 years after 2020

6 **a** **i** 1049.1 **ii** 1164.3 **iii** 1297.7 **iv** 1372.4
b At 2.8 the gradient is 1452.8

7 **a** $a^2 + ab + b^2$ **b** 7 **c** 12.06
d $3b^2$

8 **a** B **b** A **c** 25 m **d** 45 s
e 0.98 m/s, 1.724 m/s, 1.136 m/s

9 **a** m **b** cm **c** $-m$

Chapter 17

Exercise 17A

1 **a** $-2 - h$ **b** -2

2 **a** $5 + h$ **b** 5

3 $2x - 2$ **4** 32

5 2000 m/s **6** 7 per day

7 **a** $10x^2$ **b** 20 **c** 1
d $3x^2 + 1$ **e** $30x^2 + 1$ **f** 5

8 **a** $2x + 2$ **b** 13 **c** $3x^2 + 4x$

9 **a** $5 + 3h$ **b** 5.3 **c** 5

10 **a** $-\dfrac{1}{2+h}$ **b** -0.48 **c** $-\dfrac{1}{2}$

11 **a** $6 + h$ **b** 6.1 **c** 6

12 **a** $6x$ **b** 4 **c** 0
d $6x + 4$ **e** $6x^2$ **f** $8x - 5$
g $-2 + 2x$ **h** $2 - 3x^2$ **i** $2 - 6x$

13 $4x^3$

14 **a** **i** $f'(2) \approx 4 + h$ **ii** $f'(2) \approx 4$
b **i** $f'(2) \approx 12 + 6h + h^2$ **ii** $f'(2) \approx 12 + h^2$
c **i** $f'(2) \approx 14 + 6h + h^2$ **ii** $f'(2) \approx 14 + h^2$
In each case, the approximation given by **ii** is better than the approximation given by **i**

Exercise 17B

1 **a** $2x + 4$ **b** 2 **c** $3x^2 - 1$ **d** $x - 3$
e $15x^2 + 6x$ **f** $-3x^2 + 4x$

2 **a** $12x^{11}$ **b** $21x^6$ **c** 5 **d** 5
e 0 **f** $10x - 3$ **g** $50x^4 + 12x^3$
h $8x^3 - x^2 - \frac{1}{2}x$

3 **a** 6 **b** 20 **c** 5 **d** 10 **e** 0
f 7 **g** 31 **h** 7 **i** -34

4 **a** 60 **b** -16 **c** 57 **d** 168

5 **a** 7 **b** 2 **c** -16 **d** 11

6 **a** $2t$ **b** $-15t^2 + 1$ **c** $2x^3 - 2x$

7 **a** -1 **b** 0 **c** $12x^2 - 3$
d $x^2 - 1$ **e** $2x + 3$ **f** $18x^2 - 8$
g $8x - 4$ **h** -1 **i** $15x^2 + 3x$

8 **a** $2(x + 4)$ **b** $48t^2 + 16t - 7$ **c** $2x$

9 **a** **i** 3 **ii** $3a^2$ **b** $3x^2$

10 **a** $\dfrac{dy}{dx} = 3(x-1)^2 \geq 0$ for all x;
therefore gradient of graph ≥ 0 for all x
b $\dfrac{dy}{dx} = 1$ for $x \neq 0$
c $18x + 6$

11 **a** 1, Gradient = 2 **b** 1, Gradient = 1
c 3, Gradient = -4 **d** -5, Gradient = 4
e 28, Gradient = -36 **f** 9, Gradient = -24

12 **a** **i** $4x - 1$, 3, $(\frac{1}{2}, 0)$
ii $\dfrac{1}{2} + \dfrac{2}{3}x$, $\dfrac{7}{6}$, $\left(\dfrac{3}{4}, \dfrac{25}{16}\right)$
iii $3x^2 + 1$, 4, $(0, 0)$
iv $4x^3 - 31$, -27, $(2, -46)$
b Coordinates of the point where gradient is 1

13 **a** $6t - 4$ **b** $-2x + 3x^2$ **c** $-4z - 4z^3$
d $6y - 3y^2$ **e** $6x^2 - 8x$ **f** $19.6t - 2$

14 **a** $(4, 16)$ **b** $(2, 8), (-2, -8)$ **c** $(0, 0)$
d $\left(\dfrac{3}{2}, -\dfrac{5}{4}\right)$ **e** $(2, -12)$ **f** $\left(-\dfrac{1}{3}, \dfrac{4}{27}\right), (1, 0)$

Exercise 17C

1 **a** $-\dfrac{1}{(x-3)^2}$ **b** $-\dfrac{1}{(x+2)^2}$

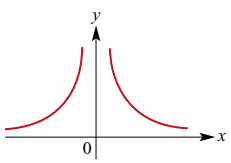
2 **a** $-\dfrac{2}{x^3}$ **b** $-\dfrac{4}{x^5}$

3 **a** $-6x^{-3} - 5x^{-2}$ **b** $-6x^{-3} + 10x$
c $-15x^{-4} - 8x^{-3}$ **d** $6x - \dfrac{20}{3}x^{-5}$
e $-12x^{-3} + 3$ **f** $3 - 2x^{-2}$

4 **a** $-2z^{-2} - 8z^{-3}, \ z \neq 0$ **b** $-9z^{-4} - 2z^{-3}, \ z \neq 0$
c $\dfrac{1}{2}, \ z \neq 0$ **d** $18z + 4 - 18z^{-4}, \ z \neq 0$
e $2z^{-3}, \ z \neq 0$ **f** $-\dfrac{3}{5}, \ z \neq 0$

5 **a** $f'(x) = 12x^3 + 18x^{-4} - x^{-2}$
b $f'(x) = 20x^3 - 8x^{-3} - x^{-2}$

6



a Gradient of $PQ = \dfrac{-2-h}{(1+h)^2}$ **b** -2

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7 a $11\frac{3}{4}$ **b** $\frac{1}{8}$ **c** -1 **d** 5

8 a $-\frac{1}{2}$ **b** $\frac{1}{2}$

9 $f'(x) = -\frac{1}{x^2} < 0$ for all $x \neq 0$

Exercise 17D

1 Graphs b and d

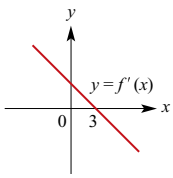
2 Graphs a, b and e

3 a $x = 1$ **b** $x = 1$ **c** $x > 1$ **d** $x < 1$
e $x = \frac{1}{2}$

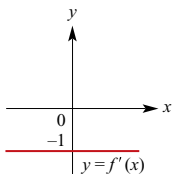
4 a i $(-1, 1.5)$ **ii** $(-\infty, -1) \cup (1.5, \infty)$
iii $\{-1, 1.5\}$
b i $(-\infty, -3) \cup (\frac{1}{2}, 4)$ **ii** $(-3, \frac{1}{2}) \cup (4, \infty)$
iii $\{-3, \frac{1}{2}, 4\}$

5 a B **b** C **c** D **d** A **e** F **f** E

6 a



b



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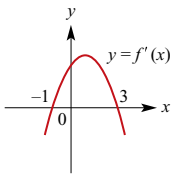

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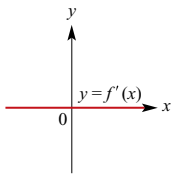
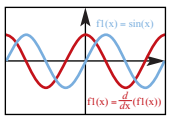


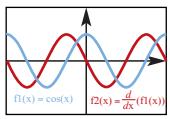
7 a $(3, 0)$ **b** $(4, 2)$

8 a $(\frac{1}{2}, -6\frac{1}{4})$ **b** $(0, -6)$

9 a



b



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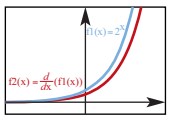


10 a i 66.80° **ii** 42.51°
b (0.5352, 0.2420)
c No

11 a $(0.6)t^2$ **b** 0.6 m/s, 5.4 m/s, 15 m/s

12 a $a = 2, b = -5$ **b** $\left(\frac{5}{4}, -\frac{25}{8}\right)$

13 a Height = 450 000 m; Speed = 6000 m/s
b $t = 25$ s

Exercise 17E

1 a $\frac{x^4}{8} + c$ **b** $x^3 - 2x + c$
c $\frac{5x^4}{4} - x^2 + c$ **d** $\frac{x^4}{5} - \frac{2x^3}{3} + c$
e $\frac{x^3}{3} - x^2 + x + c$ **f** $\frac{x^3}{3} + x + c$
g $\frac{z^4}{2} - \frac{2z^3}{3} + c$ **h** $\frac{4t^3}{3} - 6t^2 + 9t + c$
i $\frac{t^4}{4} - t^3 + \frac{3t^2}{2} - t + c$

2 $f(x) = x^4 + 2x^3 + 2x$

3 $y = 2x^3 + 12$

4 a $y = x^2 - x$ **b** $y = 3x - \frac{x^2}{2} + 1$
c $y = \frac{x^3}{3} + x^2 + 2$ **d** $y = 3x - \frac{x^3}{3} + 2$
e $y = \frac{2x^5}{5} + \frac{x^2}{2}$

5 a $V = \frac{t^3}{3} - \frac{t^2}{2} + \frac{9}{2}$ **b** $\frac{1727}{6} \approx 287.83$

6 $f(x) = x^3 - x + 2$

7 a B **b** $w = 2000t - 10t^2 + 100\,000$

8 $f(x) = 5x - \frac{x^2}{2} + 4$

9 $f(x) = \frac{x^4}{4} - x^3 - 2$

10 a $k = 8$ **b** $(0, 7)$

11 $8\frac{2}{3}$

12 a $k = -4$ **b** $y = x^2 - 4x + 9$

13 a $k = -32$ **b** $f(7) = 201$

14 $y = \frac{1}{3}(x^3 - 5)$

Exercise 17F

1 a 15 **b** 1 **c** $-3\frac{1}{2}$ **d** $-2\frac{1}{2}$
e 0 **f** 4 **g** 2 **h** $2\sqrt{3}$
i -2 **j** 12 **k** $\frac{11}{9}$ **l** $\frac{1}{4}$

2 a -1 **b** 0 **c** 1 **d** 1 **e** $\frac{3}{2}$ **f** 0

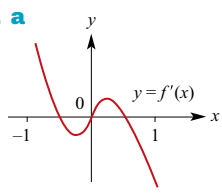
3 a 3, 4 **b** 7

4 a Discontinuity at 0, as $f(0) = 0$,
$\lim_{x \to 0^+} f(x) = 0$ but $\lim_{x \to 0^-} f(x) = 2$
b Discontinuity at 1, as $f(1) = 3$,
$\lim_{x \to 1^+} f(x) = 3$ but $\lim_{x \to 1^-} f(x) = -1$
c Discontinuity at 0, as $f(0) = 1$,
$\lim_{x \to 0^+} f(x) = 1$ but $\lim_{x \to 0^-} f(x) = 0$

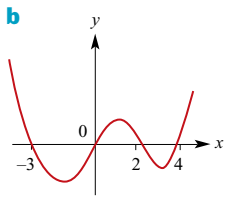
5 $x = 1$

Exercise 17G

1 a



b



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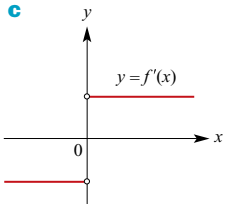
c



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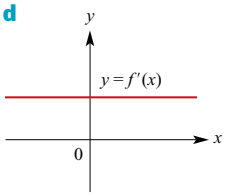

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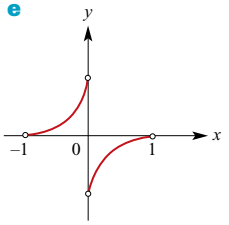
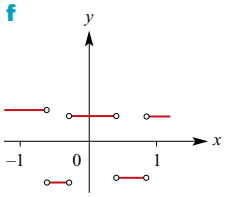
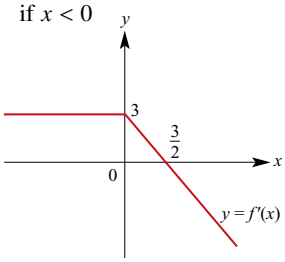


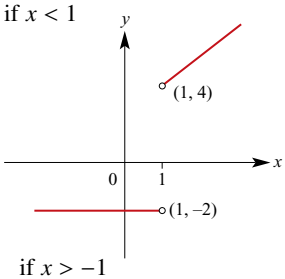
f



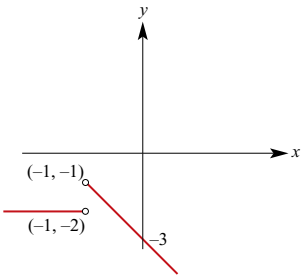
2 $f'(x) = \begin{cases} -2x + 3 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$



3 $f'(x) = \begin{cases} 2x + 2 & \text{if } x > 1 \\ -2 & \text{if } x < 1 \end{cases}$



4 $f'(x) = \begin{cases} -2x - 3 & \text{if } x > -1 \\ -2 & \text{if } x < -1 \end{cases}$



Chapter 17 review

Technology-free questions

1 **a** 3 **b** $-2x$ **c** $2x + 5$
d $3x^2 + 1$ **e** $2x + 2$ **f** $6x - 1$

2 **a** $6x - 2$ **b** 0 **c** $4 - 4x$
d $4(20x - 1)$ **e** $6x + 1$ **f** $-6x - 1$

3 **a** -1 **b** 0 **c** $\frac{4x + 7}{4}$ **d** $\frac{4x - 1}{3}$
e x

4 **a** 28; 46 **b** 8; −21 **c** 2; −8 **d** 2; −3

5 **a** $\left(\frac{3}{2}, -\frac{5}{4}\right)$ **b** $(2, -12)$ **c** $\left(-\frac{1}{3}, \frac{4}{27}\right)$, $(1, 0)$
d $(-1, 8)$, $(1, 6)$ **e** $(0, 1)$, $\left(\frac{3}{2}, -\frac{11}{16}\right)$
f $(3, 0)$, $(1, 4)$

6 **a** $x = \frac{1}{2}$ **b** $x = \frac{1}{2}$ **c** $x > \frac{1}{2}$ **d** $x < \frac{1}{2}$
e $x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}$ **f** $x = \frac{5}{8}$

7 **a** $-4x^{-5}$ **b** $-6x^{-4}$ **c** $\frac{2}{3x^3}$ **d** $\frac{4}{x^5}$
e $-\frac{15}{x^6}$ **f** $-\frac{2}{x^3} - \frac{1}{x^2} = -\frac{2 + x}{x^3}$
g $-\frac{2}{x^2}$ **h** $10x + \frac{2}{x^2}$

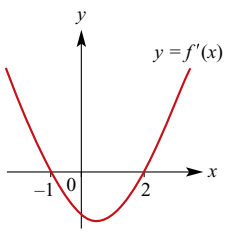
8 **a** $a = 2, b = -1$ **b** $\left(\frac{1}{4}, -\frac{1}{8}\right)$

9 **a** $\frac{x}{2} + c$ **b** $\frac{x^3}{6} + c$ **c** $\frac{x^3}{3} + \frac{3x^2}{2} + c$
d $\frac{4x^3}{3} + 6x^2 + 9x + c$ **e** $\frac{at^2}{2} + c$ **f** $\frac{t^4}{12} + c$
g $\frac{t^3}{3} - \frac{t^2}{2} - 2t + c$ **h** $-\frac{t^3}{3} + \frac{t^2}{2} + 2t + c$

10 $f(x) = x^2 + 5x - 25$

11 **a** $f(x) = x^3 - 4x^2 + 3x$ **b** 0, 1, 3

12



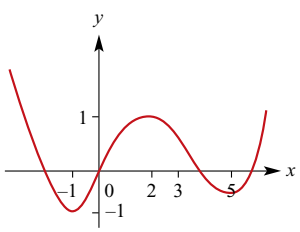
13 **a** $(-1, 4)$ **b** $(-\infty, -1) \cup (4, \infty)$ **c** $\{-1, 4\}$

Multiple-choice questions

1 D **2** B **3** E **4** B **5** C
6 C **7** A **8** E **9** A **10** D

Extended-response questions

1



2 $y = \frac{7}{36}x^3 + \frac{1}{36}x^2 - \frac{20}{9}x$

3 **a** **i** 71.57° **ii** 89.58°
b 2 km

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4 a 0.12, −0.15
b $x = 2$, $y = 2.16$; Height 2.16 km
5 a $t = \sqrt[3]{250}$, 11.9 cm/s **b** 3.97 cm/s
6 a At $x = 0$, gradient is −2; at $x = 2$, gradient is 2. Angles of inclination to the positive direction of the x-axis are supplementary.

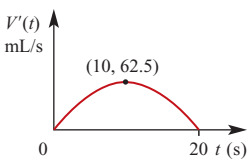
Chapter 18

Exercise 18A

1 a $y = 4x - 4$, $4y + x = 18$
b $y = 12x - 15$, $12y + x = 110$
c $y = -x + 4$, $y = x$
d $y = 6x + 2$, $6y + x = 49$
2 $y = 2x - 10$
3 $y = 2x - 1$, $y = 2x - \frac{8}{3}$;
Both have gradient 2; Distance apart $= \frac{\sqrt{5}}{3}$
4 $y = 3x + 2$, $y = 3x + 6$
5 a Both tangents have gradient 2 **b** $(0, -3)$
6 $(3, 12)$, $(1, 4)$
7 a $y = 10x - 16$ **b** $(-4, -56)$
8 a $y = 5x - 1$ **b** $(2, 4)$, $(4, -8)$

Exercise 18B

1 a 36; $\frac{36}{1} = 36$ **b** $48 - 12h$ **c** 48
2 a $1200t - 200t^2$ **b** \$1800 per month
c At $t = 0$ and $t = 6$
3 a $30 - 4P$
b 10, −10
c For $P < 7.5$ revenue increases as P increases
4 a 50 people per year **b** 0 people per year
c Decreasing by 50 people per year
5 a **i** 0 mL **ii** $833\frac{1}{3}$ mL
b $V'(t) = \frac{5}{8}(20t - t^2)$
c



6 a 0.6 km^2 **b** 0.7 km^2/h

Exercise 18C

1 a $(3, -6)$ **b** $(3, 2)$ **c** $(2, 2)$ **d** $(4, 48)$
e $(0, 0)$, $(2, -8)$ **f** $(0, -10)$, $(2, 6)$
2 $a = 2$, $b = -8$, $c = -1$
3 $a = -\frac{1}{2}$, $b = 1$, $c = 1\frac{1}{2}$
4 a $a = 2$, $b = -5$ **b** $\left(\frac{5}{4}, -\frac{25}{8}\right)$
5 $a = -8$
6 $a = 6$
7 a $(2.5, -12.25)$ **b** $\left(\frac{7}{48}, -\frac{625}{96}\right)$
c $(0, 27)$, $(3, 0)$ **d** $(-2, 48)$, $(4, -60)$
e $(-3, 4)$, $(-1, 0)$ **f** $(-1.5, 0.5)$
8 $a = -1$, $b = 2$
9 $a = -\frac{2}{9}$, $b = \frac{3}{2}$, $c = -3$, $d = 7\frac{1}{2}$

Exercise 18D

1 a

x		1		3	
$f'(x)$	−	0	+	0	−
shape of f	\	⎽	/	‾	\

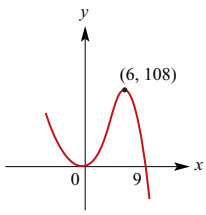
Local minimum at $x = 1$;
local maximum at $x = 3$

b

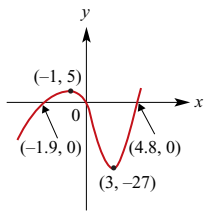
x		2		5	
$f'(x)$	+	0	−	0	−
shape of f	/	‾	\	⎽	\

Local maximum at $x = 2$;
stationary point of inflection at $x = 5$

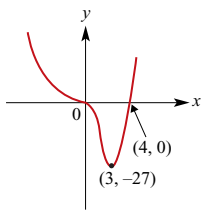
2 a min $(0, 0)$
max $(6, 108)$



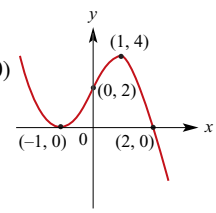
b min $(3, -27)$
max $(-1, 5)$



c inflection $(0, 0)$
min $(3, -27)$



3 a $(0, 0)$ max; $\left(\frac{8}{3}, -\frac{256}{27}\right)$ min
b $(0, 0)$ min; $(2, 4)$ max **c** $(0, 0)$ min
d $\left(\frac{10}{3}, -\frac{200\,000}{729}\right)$ min; $(0, 0)$ inflection
e $(3, -7)$ min; $\left(\frac{1}{3}, \frac{67}{27}\right)$ max
f $(6, -36)$ min; $\left(\frac{4}{3}, \frac{400}{27}\right)$ max
4 a max at $(1, 4)$
min at $(-1, 0)$
intercepts $(2, 0)$, $(-1, 0)$



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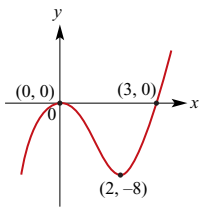
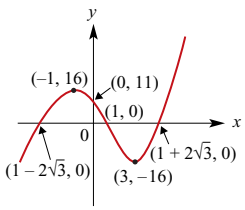
b min at $(2, -8)$
max at $(0, 0)$
intercepts $(3, 0)$, $(0, 0)$

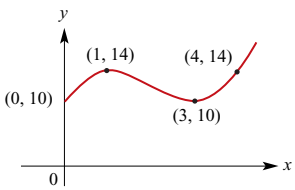


c min at $(3, -16)$
max at $(-1, 16)$
intercepts $(0, 11)$,
$(1 \pm 2\sqrt{3}, 0)$, $(1, 0)$



5 a Local maximum
b Stationary point of inflection

6 a $(-\infty, 1) \cup (3, \infty)$
b (1, 14) max; (3, 10) min
c

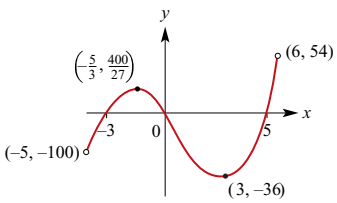


7 $\{x : -2 < x < 2\}$

8 a $x \in (-1, 1)$ **b** $x \in (-\infty, -1) \cup (1, \infty)$

9 a $x = -\frac{5}{3}$, $x = 3$

b max at $\left(-\frac{5}{3}, \frac{400}{27}\right)$, min at $(3, -36)$
intercepts $(5, 0)$, $(0, 0)$, $(-3, 0)$



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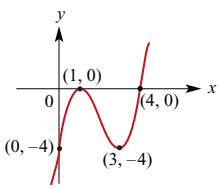

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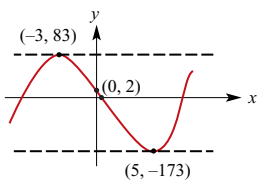
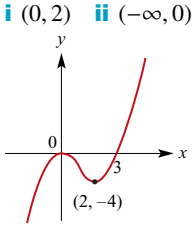
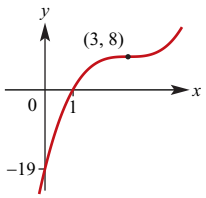


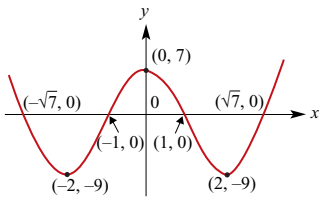
12 a i $(0, 2)$ **ii** $(-\infty, 0) \cup (2, \infty)$ **iii** $\{0, 2\}$
b



13 Stationary point of inflection at $(3, 8)$



14 min at $(-2, -9)$ and $(2, -9)$; max at $(0, 7)$
intercepts $(\pm\sqrt{7}, 0)$, $(\pm 1, 0)$, $(0, 7)$



Exercise 18E

1 2500 cm^2

2 25

3 2

4 a $V = (6 - 2x)^2 x$
b $V_{\max} = 16$ m^3 when $x = 1$

5 a i 0.9375 m **ii** 2.5 m **iii** 2.8125 m
b $x = \frac{40}{3}$, $y = \frac{80}{27}$
c i $x = 11.937$, $x = 1.396$ **ii** $x = 14.484$

6 b $V = \frac{75x - x^3}{2}$ **c** 125 cm^3 **d** 118 cm^3

7 256π

8 $x = \frac{5}{3}(9 - \sqrt{21})$

9 Absolute max = 2; Absolute min = −30

10 Absolute max = 6; Absolute min = −9

11 Absolute max = 32; Absolute min = −8

12 Absolute max = 1050; Absolute min = −8

13 b $\frac{dV}{dx} = 30x - 36x^2$ **c** $\frac{125}{36}$
d $\frac{432}{125}$ when $x = 0.8$ **e** $\frac{125}{36}$ when $x = \frac{5}{6}$

14 a $15 \leq y \leq 18$ **b** Max 75, min 36

15 a $\frac{125\,000}{27}$ **b** 3000 **c** $\frac{125\,000}{27}$

16 b $\frac{dA}{dx} = \frac{1}{8}(2x - 10)$ **c** $x = 5$ **d** $\frac{25}{8}$ m^2

Exercise 18F

1 a $x = 11$ **b** $x = -16$

2 a −12 cm/s **b** $t = 6$, $x = -25$
c −9 cm/s **d** 9 cm/s

3 a −3 cm/s **b** $2\sqrt{3}$ s

4 a $x = 5$ cm, $v = 0$ cm/s, $a = -12$ cm/s^2
b $t = 0$, $x = 5$, $a = -12$; $t = 1$, $x = 3$, $a = 12$

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5 **a** 2 m/s^2 **b** 50 m/s^2

6 **a** 3.5 s **b** 2 m/s^2 **c** 14.5 m
d $t = 2.5$ s; particle is 1.25 m to the left of O

7 **a** 0 s, 1 s, 2 s
b 2 m/s, −1 m/s, 2 m/s; −6 m/s^2, 0 m/s^2, 6 m/s^2
c 0 m/s

8 **a** 12 cm to the right of O
b 2 cm to the right of O
c Moving to the left at 7 cm/s
d $t = 3.5$ s; particle is 0.25 cm to the left of O
e −2 cm/s **f** 2.9 cm/s

9 **a** 3 cm to the left of O, moving to the right at 24 cm/s
b $v = 3t^2 - 22t + 24$
c At $\frac{4}{3}$ s and 6 s
d $11\frac{22}{27}$ cm to the right of O and 39 cm to the left of O
e $4\frac{2}{3}$ s
f $a = 6t - 22$
g When $t = \frac{11}{3}$ s and the particle is $13\frac{16}{27}$ cm left of O moving to the left at $16\frac{1}{3}$ cm/s

10 **a** 18 m/s^2, 54 m/s^2, 114 m/s^2 **b** 58 m/s^2

11 When $t = 2$ s, $v = 6$ cm/s, $a = -14$ cm/s^2
When $t = 3$ s, $v = -5$ cm/s, $a = -8$ cm/s^2
When $t = 8$ s, $v = 30$ cm/s, $a = 22$ cm/s^2

12 **a** $t = 4$ s and $t = -1$ s **b** $t = \frac{3}{2}$ s

Exercise 18G

1 **a** $f'(x) = (x-2)(3x-2(b+1))$
b $(2, 0)$, $\left(\frac{2(b+1)}{3}, -\frac{4(b-2)^3}{27}\right)$ **d** $b = 5$

2 **a** $(0, 0)$, $(9, -2187)$
b (a, b), $(9+a, -2187+b)$

3 **a** **i** $\left(-\infty, \frac{1}{2a}\right)$ **ii** $\left(\frac{1}{2a}, \infty\right)$
b $y = -x + \frac{1}{a}$ **c** $y = x - \frac{1}{a}$ **d** $\left(-\infty, \frac{1}{4a}\right]$

4 **a** $(a, 0)$, $\left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$
b Local minimum at $(a, 0)$
Local maximum at $\left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$
c **i** $y = (a-1)^2(x-1)$ **ii** $y = 0$
iii $y = -\frac{(a-1)^2}{4}(x-a)$

5 **a** **i** $2(a-2)$ **ii** $m = 2(a-2)$
b $P(a, (a-2)^2)$
c $y = 2(a-2)x - a^2 + 4$ **d** $\frac{a+2}{2}$

6 **a** $h = 2$ **b** $a = 3$ **c** $a = -16$, $b = -24$

7 **a** $(0, 0)$ **b** (a, b)

8 **a** $f'(x) = 2(x-1)(x-b)(2x-b-1)$
b $(1, 0)$, $(b, 0)$, $\left(\frac{b+1}{2}, \frac{(b-1)^4}{16}\right)$ **c** $b = 3$

Exercise 18H

1 **a** 1.32 **b** 1.164 **c** 1.124 or 1.451
d 2.151 **e** −1.75

2 1.44225 **3** 1.618

4 **a** **ii** 1.869565, 1.799452, 1.796328
b **ii** 1.967742, 1.966904, 1.966903
c **ii** 0.636364, 0.618382, 0.618034
d **ii** 3.245161, 3.225355, 3.225240

Chapter 18 review

Technology-free questions

1 **a** $\frac{dy}{dx} = 4 - 2x$ **b** 2 **c** $y = 2x + 1$

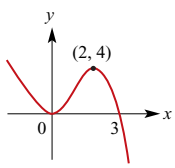
2 **a** $3x^2 - 8x$ **b** −4 **c** $y = -4x$
d $(0, 0)$

3 **a** $3x^2 - 12$; $x = \pm 2$
b Local minimum at $x = 2$
Local maximum at $x = -2$
c $x = 2$, $y = -14$; $x = -2$, $y = 18$

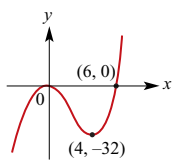
4 **a** Stationary point of inflection at $x = 0$
b Maximum at $x = 0$
c Min at $x = 3$, max at $x = 2$
d Min at $x = 2$, max at $x = -2$
e Max at $x = 2$, min at $x = -2$
f Max at $x = 3$, min at $x = 1$
g Max at $x = 4$, min at $x = -3$
h Max at $x = 3$, min at $x = -5$

5 **a** $\left(-\frac{2}{3}, -\frac{16}{9}\right)$ minimum, $\left(\frac{2}{3}, \frac{16}{9}\right)$ maximum
b $(-1, 0)$ maximum, $(2, -27)$ minimum
c $\left(\frac{2}{3}, \frac{100}{27}\right)$ maximum, $(3, -9)$ minimum

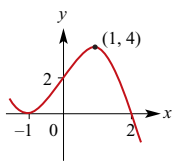
6 **a**



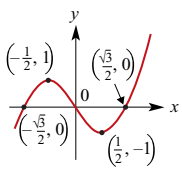
b



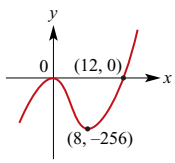
c



d



e



7 **a** C **b** A **c** B

8 **a** 20 m **b** 6 s **c** 40 m/s

9 72

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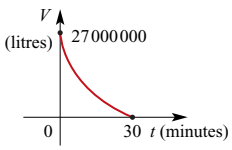
Multiple-choice questions

1 D **2** E **3** E **4** A **5** C **6** D
7 D **8** A **9** A **10** C **11** A **12** A

Extended-response questions

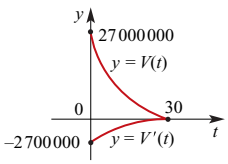
1 a -14 m/s **b** -8 m/s^2

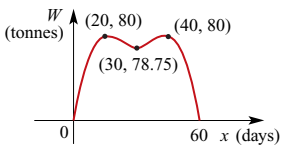
2 a


b i 17.4 minutes **ii** 2.9 minutes

c $\frac{dV}{dt} = -3000(30 - t)^2$

d 30 minutes **e** 28.37 minutes

f


3 a


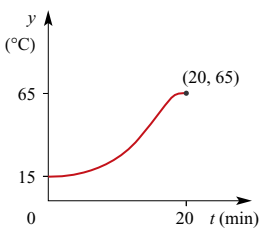
b From 5.71 days until 54.29 days

c When $x = 20$ and when $x = 40$, $\frac{dW}{dx} = 0$;
When $x = 60$, $\frac{dW}{dx} = -12$ tonnes per day

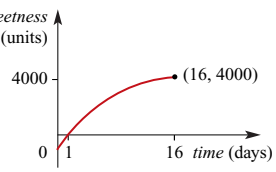
d When $x = 30$, $W = 78.75$

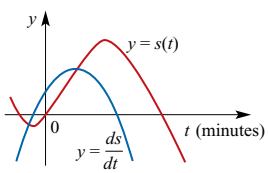
4 a 15°C

b 0°C/min, $\frac{45}{16}$ °C/min, $\frac{15}{4}$ °C/min, $\frac{45}{16}$ °C/min, 0°C/min

c


5 a 768 units/day **b** 432, 192, 48, 0

c $t = 16$ **d**


6 a


b 11:59 a.m., 12:03 p.m.

c $\frac{5}{27}$ km, 1 km

d $\frac{8}{27}$ km/min $= 17\frac{7}{9}$ km/h

e $\frac{1}{3}$ km/min $= 20$ km/h

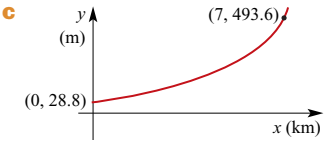
7 a $0 \leq t \leq 12$

b i 27 L/h **ii** 192 L/h

8 a -1 m/s **b** $k = 4$ **c** -4 m/s

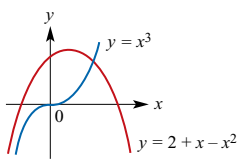
9 a 28.8 m

b 374.4

c


d Path gets too steep after 7 km

e i 0.0384 **ii** 0.0504 **iii** 0.1336

10 a


b For $x \leq 0$, the minimum vertical distance occurs when $x = -1$; Min distance = 1 unit

11 8 mm for maximum; $\frac{4}{3}$ mm for minimum

12 a $y = 5 - x$ **b** $P = x(5 - x)$

c Maximum 6.25 when $x = 2.5$, $y = 2.5$

13 a $y = 10 - 2x$ **b** $A = x^2(10 - 2x)$

c Maximum $\frac{1000}{27}$ when $x = \frac{10}{3}$, $y = \frac{10}{3}$

14 $20\sqrt{10}$

15 a $y = 8 - x$ **b** $s = x^2 + (8 - x)^2$ **c** 32

16 $\frac{4}{3}, \frac{8}{3}$

17 Maximum area is 625 m^2 for 25 m $\times$ 25 m

18 $x = 12$

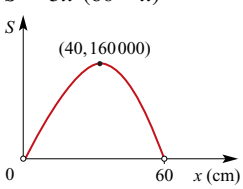
19 32

20 Maximum value of P is 2500

21 Maximum area is 2 km^2 for 2 km $\times$ 1 km

22 $p = \frac{3}{2}, q = \frac{8}{3}$

23 a $y = 60 - x$ **b** $S = 5x^2(60 - x)$

c $0 < x < 60$ **d**


e $x = 40$, $y = 20$ **f** 74 005

24 12°C

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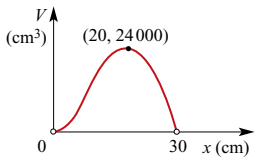
25 b $0 < x < 30$ **c**



d 20 cm, 40 cm, 30 cm

e $x = 14.82$ or $x = 24.4$

26 b Maximum when $x = 3$ and $y = 18$

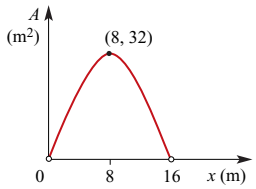
27 a Use 44 cm for circle and 56 cm for square

b Use all the wire for the circle

28 Length 7.2 metres, width 4.5 metres

29 a $A = xy$ **b** $A = \left(8 - \frac{x}{2}\right)x$

c $0 < x < 16$ **d**



e 32 m²

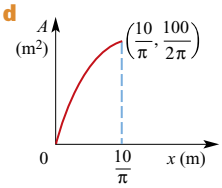
30 $h = 1188$, $a = 937$

31 $\frac{25}{6}$ m²

32 b $x = 1$ **c** $a = 0.1$ **d** $(2, 0.2)$

33 a $y = 10 - \pi x$ **b** $0 \leq x \leq \frac{10}{\pi}$

c $A = \frac{x}{2}(20 - \pi x)$ **d**



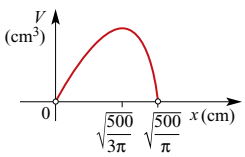
e Maximum when $x = \frac{10}{\pi}$

f A semicircle

34 a $h = \frac{500}{\pi x} - x$ **b** $V = 500x - \pi x^3$

c $\frac{dV}{dx} = 500 - 3\pi x^2$ **d** $x = 10\sqrt{\frac{5}{3\pi}} \approx 7.28$

e



f 2427.89 cm³

g $x = 2.05$, $h = 75.41$ or $x = 11.46$, $h = 2.42$

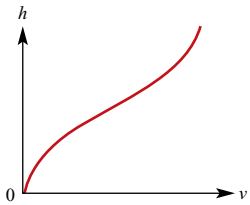
35 a $r = 4.3$ cm, $h = 8.6$ cm

b $r = 4.3$ cm, $h = 8.6$ cm

Chapter 19

Technology-free questions

1



2 a 1 cm/s **b** 41 cm/s

3 a i −4 **ii** −3 **b** $-2 - h$ **c** −2

4 $x - 1$

5 a $6x^2 - 1$ **b** $2x + 1$ **c** 1

6 a 13 **b** 10

7 a $x = 0$ or $x = \frac{1}{2}$ **b** $x = \frac{1}{4}$ **c** $x < \frac{1}{4}$

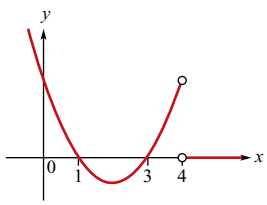
d $x > \frac{1}{4}$ **e** $x = \frac{11}{4}$

8 a $-6x^{-4} + x^{-2}$ **b** $\frac{2z - 9}{z^4}$

9 Tangent $y = -3x - 1$; Normal $y = \frac{1}{3}x - \frac{13}{3}$

10 Local max $\left(2, \frac{11}{3}\right)$, local min $\left(3, \frac{7}{2}\right)$

11



12 Local max $\left(-\frac{2}{\sqrt{3}}, \frac{32}{3\sqrt{3}}\right)$

Local min $\left(\frac{2}{\sqrt{3}}, -\frac{32}{3\sqrt{3}}\right)$

13 a $\frac{x^5}{10} + c$ **b** $3x^3 - 12x^2 + 16x + c$

c $\frac{3x^4}{4} - 2x^2 + c$

14 a $f(x) = x^2 - 1$

b $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + \frac{19}{3}$

c $f(x) = \frac{x^3}{3} - x^2 + x + \frac{7}{3}$

Multiple-choice questions

1 B	**2** A	**3** B	**4** A	**5** B
6 A	**7** D	**8** B	**9** C	**10** A
11 C	**12** B	**13** A	**14** C	**15** C
16 D	**17** E	**18** C	**19** D	**20** E
21 D	**22** A	**23** A	**24** C	**25** D
26 B	**27** B	**28** B	**29** C	**30** D
31 C	**32** A	**33** A	**34** C	**35** C
36 D	**37** D	**38** D	**39** B	

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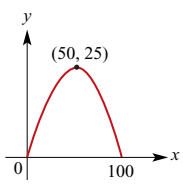
Extended-response questions

1 a 100 **b** $\frac{dy}{dx} = 1 - 0.02x$

c $x = 50, y = 25$ **d**



e i (25, 18.75) **ii** (75, 18.75)

2 a $\left(66\frac{2}{3}, 14\frac{22}{27}\right)$

b i 0.28 **ii** −0.32 **iii** −1

c A gradual rise to the turning point and a descent which becomes increasingly steep (in fact, alarmingly steep)

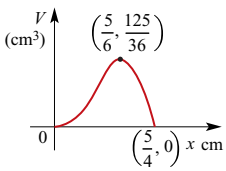
d Smooth out the end of the trip

3 a $h = 5 - 4x$ **c** $0 < x < \frac{5}{4}$

d $\frac{dV}{dx} = 30x - 36x^2$

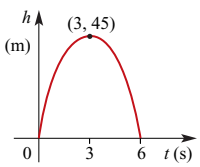
e $\left\{0, \frac{5}{6}\right\}$; Maximum volume $= 3\frac{17}{36}$ cm^3

f



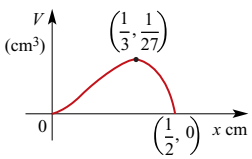
4 a $\frac{dh}{dt} = 30 - 10t$ **b** 45 m

c



5 a $A = 4x - 6x^2$ **b** $V = x^2 - 2x^3$

c



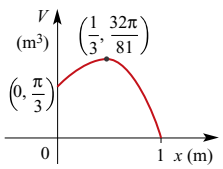
d $\frac{1}{3}$ cm $\times \frac{1}{3}$ cm $\times \frac{1}{3}$ cm; Volume $= \frac{1}{27}$ cm^3

6 a i $r = \sqrt{1 - x^2}$ **ii** $h = 1 + x$

c $0 < x < 1$

d i $\frac{dV}{dx} = \frac{\pi}{3}(1 - 2x - 3x^2)$ **ii** $\{\frac{1}{3}\}$ **iii** $\frac{32\pi}{81}$ m^3

e



7 a 1000 insects **b** 1366 insects

c i $t = 40$ **ii** $t = 51.70$

d 63.64

e i $\frac{1000 \times 2^{\frac{3}{4}}\left(2^{\frac{h}{20}} - 1\right)}{h}$

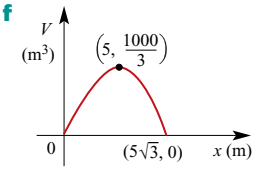
ii Consider h decreasing and approaching zero; instantaneous rate of change = 58.286 insects/day

8 a $h = \frac{150 - 2x^2}{3x}$ **b** $V = \frac{2}{3}(150x - 2x^3)$

c $\frac{dV}{dx} = 2(50 - 2x^2)$ **d** $0 < x < 5\sqrt{3}$

e $\frac{1000}{3}$ m^3 when $x = 5$

f



9 a 10

c i $h = 2.5x$

d $V = 40(420x - 135x^2)$

e i $x = \frac{14}{9}, y = \frac{140}{9}$ **ii** 13 066$\frac{2}{3}$ m^3

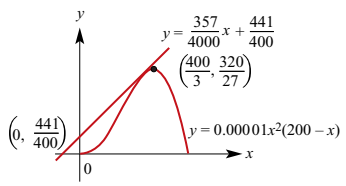
10 a $a = 200, k = 0.000\,01$

b i $\frac{400}{3}$ **ii** $\frac{320}{27}$ **c i** $\frac{8379}{800}$ **ii** $\frac{357}{4000}$

d i $y = \frac{357}{4000}x + \frac{441}{400}$ **ii** $\frac{441}{400}$

e 0.09975

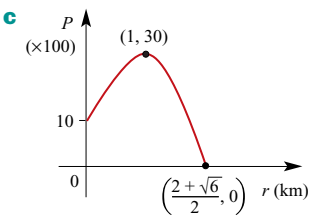
f



11 a 10 000 people/km^2

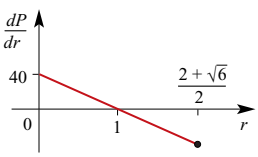
b $0 \leq r \leq \frac{2 + \sqrt{6}}{2}$

c



d i $\frac{dP}{dr} = 40 - 40r$ **ii** 20, 0, −40

iii



e At $r = 1$

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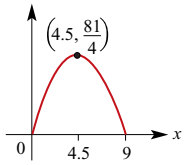
12 a $y = ax - x^2$ **b** $0 < x < a$ **c** $\frac{a^2}{4}, \frac{a}{2}$

d Negative coefficient of x^2 for quadratic function

e i

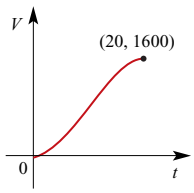


ii $0 < y \leq \frac{81}{4}$

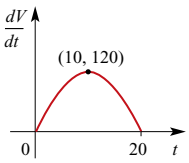
13 a i 0 mL **ii** 1600 mL

b $\frac{dV}{dt} = 0.6(40t - 2t^2)$

c



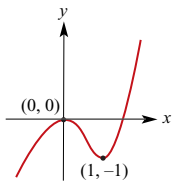
d



14 a $-1 = a + b$

b $0 = 3a + 2b$, $a = 2$, $b = -3$

c



15 a i $80 - 2x$ **ii** $h = \frac{\sqrt{3}}{2}x$

b $A = \frac{\sqrt{3}}{4}x(160 - 3x)$ **c** $x = \frac{80}{3}$

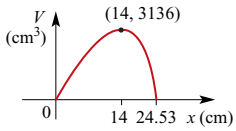
16 a $y = \frac{1400 - 2x^2 - 8x}{4x}$

b $V = -\frac{x^3}{2} - 2x^2 + 350x$

c $\frac{dV}{dx} = -\frac{3}{2}x^2 - 4x + 350$

d $x = 14$

e



f Maximum volume is 3136 cm³

g $x = 22.83$ and $y = 1.92$, or $x = 2.94$ and $y = 115.45$

Investigations

See solutions supplement

Chapter 20

Exercise 20A

1 a $30(x - 1)^{29}$

b $100(x^4 - 2x^9)(x^5 - x^{10})^{19}$

c $4(1 - 3x^2 - 5x^4)(x - x^3 - x^5)^3$

d $8(x + 1)^7$

e $-4(x + 1)(x^2 + 2x)^{-3}$

f $-6(x + x^{-2})(x^2 - 2x^{-1})^{-4}$

2 a $24x^2(2x^3 + 1)^3$ **b** 648

3 a $-\frac{1}{16}$ **b** $-\frac{3}{256}$

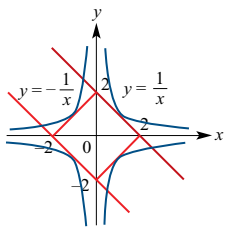
4 a $-\frac{2}{9}$ **b** $\left(-3, -\frac{1}{3}\right), \left(0, \frac{1}{3}\right)$

5 a $-\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $y = -x + 2$

d $y = x - 2$

e At P, $y = x + 2$; at Q, $y = -x - 2$; $(-2, 0)$

f



Exercise 20B

1 a $\frac{1}{3}x^{-\frac{2}{3}}$ **b** $\frac{3}{2}x^{\frac{1}{2}}$, $x > 0$

c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}$, $x > 0$ **d** $x^{-\frac{1}{2}} - 5x^{\frac{2}{3}}$, $x > 0$

e $-\frac{5}{6}x^{-\frac{11}{6}}$, $x > 0$ **f** $-\frac{1}{2}x^{-\frac{3}{2}}$, $x > 0$

2 a $x(1 + x^2)^{-\frac{1}{2}}$ **b** $\frac{1}{3}(1 + 2x)(x + x^2)^{-\frac{2}{3}}$

c $-x(1 + x^2)^{-\frac{3}{2}}$ **d** $\frac{1}{3}(1 + x)^{-\frac{2}{3}}$

3 a i $\frac{4}{3}$ **ii** $\frac{4}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$

4 a $\{x : 0 < x < 1\}$ **b** $\{x : x > (\frac{2}{3})^6\}$

5 a $-5x^{-\frac{1}{2}}(2 - 5\sqrt{x})$ **b** $3x^{-\frac{1}{2}}(3\sqrt{x} + 2)$

c $-4x^{-3} - \frac{3}{2}x^{-\frac{5}{2}}$ **d** $\frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{3}{2}}$

e $\frac{15}{2}x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

Exercise 20C

1 a $-\frac{3}{x} + c$ **b** $3x^2 - \frac{2}{3x^3} + c$

c $\frac{4}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$ **d** $\frac{9}{4}x^{\frac{4}{3}} - \frac{20}{9}x^{\frac{9}{4}} + c$

e $\frac{3}{2}z^2 - \frac{2}{z} + c$ **f** $\frac{12}{7}x^{\frac{7}{4}} - \frac{14}{3}x^{\frac{3}{2}} + c$

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2 a $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - \frac{22}{3}$

b $y = \frac{3}{2} - \frac{1}{2x^2}$ **c** $y = \frac{3}{2}x^2 - \frac{1}{x} + \frac{9}{2}$

3 $f(x) = x^3 + \frac{1}{x} - \frac{17}{2}$

4 $s = \frac{3t^2}{2} + \frac{8}{t} - 8$

5 $y = 5$

6 a $a = 2$ **b** $y = x^2 + 1$

7 $y = \frac{x^3}{3} + \frac{7}{3}$

Exercise 20D

1 a $6x$ **b** 0 **c** $108(3x + 1)^2$

d $-\frac{1}{4}x^{-\frac{3}{2}} + 18x$

e $306x^{16} + 396x^{10} + 90x^4$

f $10 + 12x^{-3} + \frac{9}{4}x^{-\frac{1}{2}}$

2 a $18x$ **b** 0 **c** 12 **d** $432(6x + 1)^2$

e $300(5x + 2)^2$ **f** $6x + 4 + 6x^{-3}$

3 -9.8 m/s^2

4 a i -16 **ii** 4 m/s **iii** $\frac{7}{4}$ m/s **iv** -32 m/s

b $t = 0$

c -8 m/s

Exercise 20E

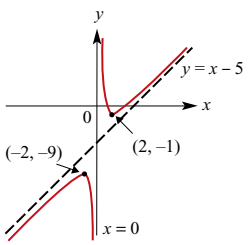
1 a $(\frac{1}{2}, 4), (-\frac{1}{2}, -4)$ **b** $y = \frac{15}{4}x + 1$

2 $\pm\frac{1}{2}$

3 $\frac{1}{2}$

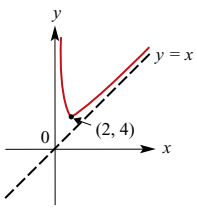
4 a $(4, 0), (1, 0)$ **b** $y = x - 5$, $x = 0$

c $(2, -1)$ min, $(-2, -9)$ max

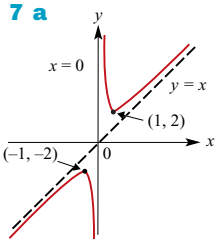


5 3

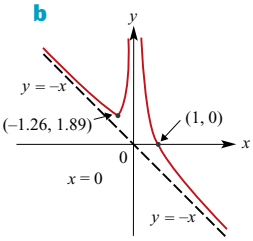
6 4



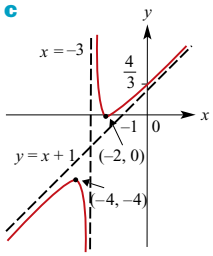
7 a



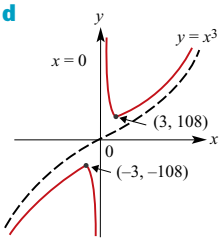
b



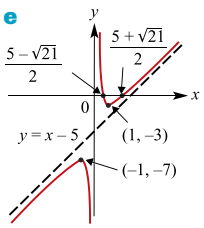
c



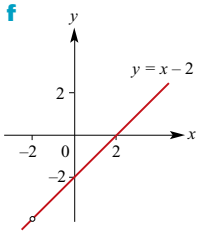
d



e



f



Chapter 20 review

Technology-free questions

1 a $\frac{1}{2x^{\frac{1}{2}}}$ **b** $\frac{1}{3x^{\frac{2}{3}}}$ **c** $\frac{2}{3x^{\frac{4}{3}}}$ **d** $\frac{4}{3}x^{\frac{1}{3}}$

e $-\frac{1}{3x^{\frac{4}{3}}}$ **f** $-\frac{1}{3x^{\frac{4}{3}}} + \frac{6}{5x^{\frac{2}{5}}}$

2 a $8x + 12$ **b** $24(3x + 4)^3$

c $\frac{1}{(3 - 2x)^{\frac{3}{2}}}$ **d** $-\frac{2}{(3 + 2x)^2}$

e $-\frac{4}{3(2x - 1)^{\frac{5}{3}}}$ **f** $-\frac{3x}{(2 + x^2)^{\frac{3}{2}}}$

g $\frac{1}{3}\left(4x + \frac{6}{x^3}\right)\left(2x^2 - \frac{3}{x^2}\right)^{-\frac{2}{3}}$

3 a $-\frac{1}{x^2} + c$ **b** $\frac{2x^{\frac{5}{2}}}{5} - \frac{4x^{\frac{3}{2}}}{3} + c$

c $\frac{3x^2}{2} + 2x + c$ **d** $-\frac{6x + 1}{2x^2} + c$

e $\frac{5x^2}{2} - \frac{4x^{\frac{3}{2}}}{3} + c$ **f** $\frac{20x^{\frac{7}{4}}}{7} - \frac{3x^{\frac{4}{3}}}{2} + c$

g $2x - \frac{2x^{\frac{3}{2}}}{3} + c$ **h** $-\frac{3x + 1}{x^2} + c$

4 $s = \frac{1}{2}t^2 + 3t + \frac{1}{t} + \frac{3}{2}$

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5 **a** $\frac{1}{6}$ **b** -2 **c** $-\frac{1}{16}$ **d** -2 **e** $\frac{1}{6}$ **f** 0

6 $\left(\frac{1}{2}, 2\right), \left(-\frac{1}{2}, -2\right)$

7 $\left(\frac{1}{16}, \frac{1}{4}\right)$

Multiple-choice questions

1 B **2** D **3** A **4** A **5** A
6 E **7** A **8** B **9** A **10** D

Extended-response questions

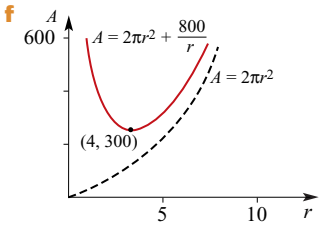
1 **a** $h = \frac{400}{\pi r^2}$

c $\frac{dA}{dr} = 4\pi r - \frac{800}{r^2}$

d $r = \left(\frac{200}{\pi}\right)^{\frac{1}{3}} \approx 3.99$

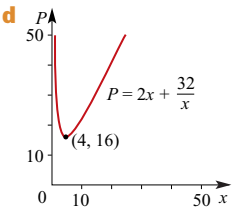
e $A = 301 \text{ cm}^2$

f



2 **a** $y = \frac{16}{x}$ **c** $x = 4, P = 16$

d



3 **a** $OA = \frac{120}{x}$ **b** $OX = \frac{120}{x} + 7$

c $OZ = x + 5$ **d** $y = 7x + \frac{600}{x} + 155$

e $x = \frac{10\sqrt{42}}{7} \approx 9.26 \text{ cm}$

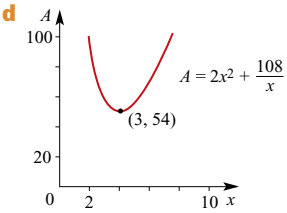
4 **a** $A(-2, 0), B(0, \sqrt{2})$ **b** $\frac{dy}{dx} = \frac{1}{2\sqrt{x+2}}$

c **i** $\frac{1}{2}$ **ii** $2y - x = 3$ **iii** $CD = \frac{3\sqrt{5}}{2}$

d $x > -\frac{7}{4}$

5 **a** $h = \frac{18}{x^2}$ **c** $x = 3, h = 2$

d



6 **a** $y = \frac{250}{x^2}$ **c** $\frac{dS}{dx} = 24x - \frac{3000}{x^2}$

d $S_{\min} = 900 \text{ cm}^2$

Chapter 21

Exercise 21A

1 62 square units **2** 80 square units
3 60.90625 square units **4** 68 square units
5 **a** 13.2 **b** 10.2 **c** 11.7
6 **a** 4.375 **b** 4.48
7 **a** 36.8 **b** 36.75
8 $\pi \approx 3.13$
9 **a** 4.371 **b** 1.128
10 109.5 m^2

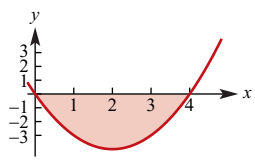
Exercise 21B

1 **a** $\frac{7}{3}$ **b** $16\frac{1}{4}$ **c** $\frac{9}{4}$ **d** 9
e $\frac{15}{4}$ **f** $49\frac{1}{2}$ **g** $15\frac{1}{3}$ **h** 30

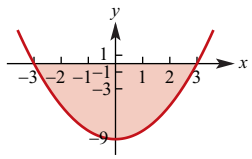
2 4 **3** 9 **4** $\frac{4}{3}$ **5** 4

Exercise 21C

1 $-\frac{32}{3}$



2 -36



3 36
4 $\frac{37}{12}$
5 **a** 8 **b** 16 **c** -4
6 **a** -12 **b** 36 **c** 20
7 **a** 24, 21, 45 **b** 4, -1, 3
8 4.5 square units
9 $166\frac{2}{3}$ square units
10 $\frac{37}{12}$ square units
11 **a** 1 **b** 1 **c** 14 **d** 31 **e** $2\frac{1}{4}$ **f** 0
12 **a** $\frac{4}{3}$ **b** $\frac{1}{6}$ **c** $121\frac{1}{2}$ **d** $\frac{1}{6}$
e $4\sqrt{3} \approx 6.93$ **f** 108

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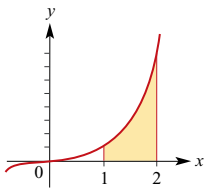
Chapter 21 review

Technology-free questions

1 **a** 3 **b** 6 **c** 114 **d** $\frac{196}{3}$ **e** 5

2 **a** $\frac{14}{3}$ **b** $48\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{15}{16}$ **e** $\frac{16}{15}$

3 $\frac{15}{4}$ square units



4 $4\frac{1}{2}$ square units **5** $21\frac{1}{12}$ square units

6 **a** $B(1, 3), C(3, 3)$ **b** 6 **c** $\frac{4}{3}$

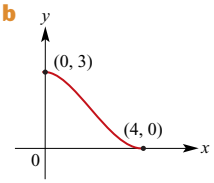
Multiple-choice questions

1 C **2** D **3** A **4** D **5** B
6 B **7** D **8** B **9** C **10** A

Extended-response questions

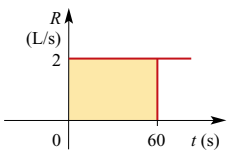
1 **a** $y = \frac{9}{32}\left(\frac{x^3}{3} - 2x^2\right) + 3$

b

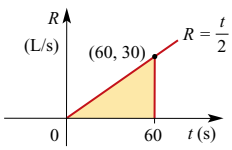


c Yes, for $x \in \left(\frac{4}{3}, \frac{8}{3}\right)$

2 **a** **i** 120 L **ii**



b **i** 900 L **ii**



iii $900a^2$ L

c **i** 7200 square units

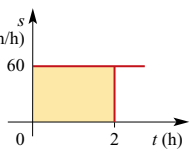
ii Volume of water which has flowed into the container after 1 minute

iii 66.94 s

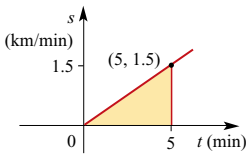
3 **a** 27 square units **b** $y = \frac{3}{25}(x - 4)^2$

c $\frac{189}{25}$ square units **d** $\frac{486}{25}$ square units

4 **a**



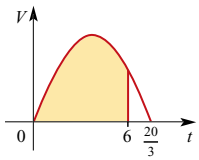
b **i**



ii 3.75 km

c **i** $20 - 6t$ m/s^2

ii



iii 144 metres

5 **a** **i** 4 m **ii** 16 m

b **i** 0.7 **ii** −0.8

c **i** $x = \frac{100}{3}$ **ii** $\frac{500}{27}$ m

d $\frac{3125}{6}$ m^2

e **i** $B(15 + 5\sqrt{33}, 12)$

ii $R = 60\sqrt{33} - 60$, $q = 20$, $p = 15 + 5\sqrt{33}$

6 **a** **i** 9 **ii** $y = 9x - 3$ **iii** $y = 3x^2 + 3x$

b **i** $12 + k$ **ii** $k = -7$ **iii** $y = 3x^2 - 7x + 12$

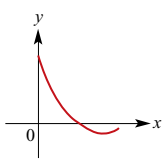
7 **a** 6 m^2 **b** **i** $y = x - \frac{1}{2}$ **ii** $\left(x^2 - \frac{1}{4}\right)$ m^2

c **i** $y = \frac{1}{2}x^2$, $P(-2, 2), S(2, 2)$ **ii** $\frac{16}{3}$ m^2

8 **a** $y = 7 \times 10^{-7}x^3 - 0.00116x^2 + 0.405x + 60$

b 100 m

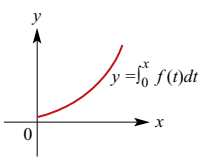
c **i**



ii (0, 60)

d 51 307 m^2

9 **a**



b $x = 2.988$

10 **a** 256.5624

b **i** Change first line inside `for` loop:

```
strip ← f(left) × h
```

ii Change first line inside `for` loop:

```
strip ← f(right) × h
```

c Change first five lines:

```
define f(x):
  return 2^x

a ← 0
b ← 3
n ← 100
```

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Chapter 22

Technology-free questions

1 a $\frac{25}{12}$ **b** $\frac{77}{60}$ **c** $\frac{101}{60}$

2 a $\frac{9}{2}x^{\frac{1}{2}}$ **b** $\frac{1}{5}x^{-\frac{4}{5}}$ **c** $\frac{10}{3}x^{-\frac{8}{3}}$ **d** $10x^{\frac{2}{3}}$
e $-\frac{1}{5}x^{-\frac{6}{5}}$ **f** $-\frac{2}{3}x^{-\frac{5}{3}} - 3x^{\frac{1}{2}}$

3 a $6(3x+5)$ **b** $-8(2x+7)^3$
c $\frac{2}{3}(5-2x)^{-\frac{4}{3}}$ **d** $-\frac{12}{(5+3x)^2}$
e $-\frac{2}{3}(x-1)^{-\frac{5}{3}}$ **f** $-9x(2+3x^2)^{-\frac{3}{2}}$
g $\frac{1}{3}\left(2x^3 - \frac{5}{x}\right)^{-\frac{2}{3}}\left(6x^2 + \frac{5}{x^2}\right)$

4 $x = \frac{t^2}{2} + 4t + \frac{3}{t} - \frac{3}{2}$

5 a $\frac{1}{27}$ **b** -3 **c** $-\frac{3}{8}$

6 $\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}, 4^{\frac{1}{3}}\right)$

7 $\left(\left(\frac{1}{6}\right)^{\frac{3}{2}}, \left(\frac{1}{6}\right)^{\frac{1}{2}}\right), \left(-\left(\frac{1}{6}\right)^{\frac{3}{2}}, -\left(\frac{1}{6}\right)^{\frac{1}{2}}\right)$

8 a $x^3 + x + c$ **b** $-t^3 - \frac{1}{2}t^2 + 2t + c$
c $\frac{2}{3}x^{\frac{3}{2}} + c$ **d** $\frac{4}{5}x^{\frac{5}{2}} + \frac{3}{4}x^{\frac{4}{3}} + c$

9 a $\frac{2}{3}$ **b** $\frac{5}{6}$

10 a $\frac{1}{6}$ square units **b** $\frac{1}{2}$ square units

Multiple-choice questions

1 E **2** C **3** E **4** D **5** A
6 A **7** E **8** C **9** E **10** C
11 B **12** C **13** C

Chapter 23

Technology-free questions

1 $x = 4$ **2** $t = \frac{2d-b}{a-2c}$ **3** $x \geq -\frac{3}{2}$

4 a -12 **b** 3 **c** 100

5 15

6 $x \leq \frac{37}{5}$

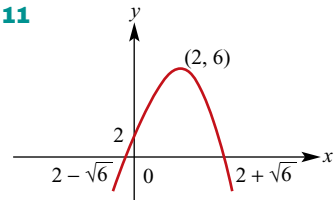
7 $a = 7.9$

8 a $\left(\frac{a+8}{2}, \frac{b+14}{2}\right)$ **b** $a = 2, b = 6$

9 a $4y - 3x = 30$ **b** $\frac{25}{2}$

10 a $(2, \frac{1}{2})$ **b** $\sqrt{445}$ **c** $11x + 18y = 31$
d $22y - 36x + 61 = 0$

11



12 $f(x) = \frac{9}{8}(x-2)^2 - 6$

13 $a = -2$

14 a $w = 1500 - 9x$ **b** $V = 20x^2(1500 - 9x)$
c $0 \leq x \leq \frac{500}{3}$ **d** 120 000 000 cm^3

15 a $\frac{16}{81}$ **b** $\frac{28}{153}$

16 $\frac{1}{3}$

17 a $\frac{1}{2}$ **b** $\frac{1}{3}$

18 0.42

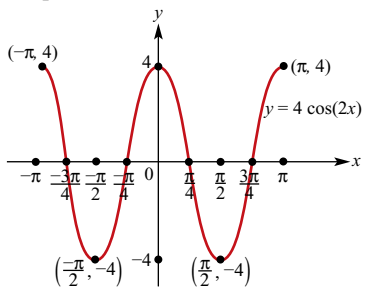
19 $-\frac{\pi}{9}, \frac{\pi}{9}$

20 a $c = 6$ **b** $0 = -8a - 2b + 6, 0 = 3a + b$
c $a = 3, b = -9$

21 $a = -48$

22 a Amplitude = 4; Period = π

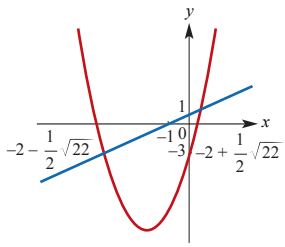
b



23 a $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{4}$

24 a $\left(-\frac{7}{4}, -\frac{3}{4}\right)$

b



25 a $x = 3$ **b** $x = -\frac{5}{2}$ or $x = 1$

26 $k = 4$, $y = -2x^2 + 4x + 3$

27 $a = \frac{1}{3}$ **28** $y = -\frac{2x^2}{9}$ **29** $b = -\frac{1}{3}$

30 Intersect at $(-3, -27)$; both curves have gradient 27 at this point

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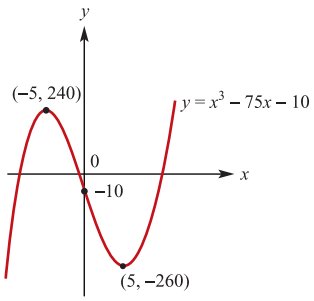
31 a $(5, -260), (-5, 240)$



b $-260 \leq p \leq 240$

32 a $\mathbb{R} \setminus \{3\}$ **b** $\mathbb{R} \setminus \{2\}$ **c** $(-\infty, 2]$
d $[4, \infty)$ **e** $(-\infty, 5)$

Multiple-choice questions

1 B	**2** A	**3** D	**4** D	**5** D
6 C	**7** C	**8** E	**9** A	**10** B
11 A	**12** B	**13** A	**14** A	**15** E
16 B	**17** C	**18** B	**19** A	**20** C
21 A	**22** B	**23** B	**24** D	**25** D
26 A	**27** C	**28** B	**29** A	**30** E
31 C				

Extended-response questions

1 a i $-b$ **ii** $x = \frac{b}{2}$

b i $S(b) = \frac{b}{2}(32 - b^2)$ **ii** $b = 2$

2 a $a = 4$ **b** $\left(\frac{1}{4}, 4\right)$ **c** $\frac{34}{15}$

3 a $v = 4t - 6$ **b** 6 cm/s **c** 0 cm/s
d 9 cm **e** 3 cm/s

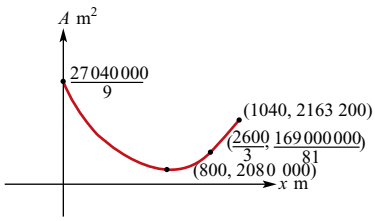
4 a i $(-\infty, 0) \cup \left(\frac{2a}{3}, \infty\right)$ **ii** $\left(0, \frac{2a}{3}\right)$

b $y = -a^2(x - a)$ **c** $y = \frac{x}{a^2} - \frac{1}{a}$ **d** $\frac{a^4}{12}$

5 a $y = 13 - 9x$ **b** $A = 156x - 60x^2$
c $x = \frac{13}{10}, y = \frac{13}{10}$

6 b 2 080 000 m² when $x = 800, y = 1200$

c



7 a $\{(0, 1), (0, 3), (0, 5), (0, 7), (0, 9), (0, 11),$
$(2, 1), (2, 3), (2, 5), (2, 7), (2, 9), (2, 11),$
$(4, 1), (4, 3), (4, 5), (4, 7), (4, 9), (4, 11),$
$(6, 1), (6, 3), (6, 5), (6, 7), (6, 9), (6, 11),$
$(8, 1), (8, 3), (8, 5), (8, 7), (8, 9), (8, 11),$
$(10, 1), (10, 3), (10, 5), (10, 7), (10, 9), (10, 11)\}$

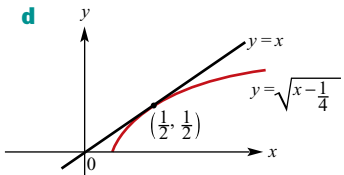
b i $\frac{1}{36}$ **ii** $\frac{5}{36}$ **iii** $\frac{5}{36}$ **c** $\frac{2}{13}$

8 a $[2c + 4, 2d + 4]$ **b** $g(x) = 3 - f\left(\frac{x - 4}{2}\right)$

d $g: [6, 8] \to \mathbb{R}, g(x) = 3 - 2^{\frac{x-4}{2}}$

9 a $x \geq 2a$ **b** $x = \frac{1 \pm \sqrt{1 - 8a}}{2}$ **c** $a = \frac{1}{8}$

d



10 a 0.343 **b** 0.399

11 a 0.4219 **b** 0.2156 **c** 0.6125

12 a $[-mb + 3, -ma + 3]$ **b** $f^{-1}(x) = -\frac{1}{m}x + \frac{3}{m}$

c $\left(\frac{a + b}{2}, \frac{-m(a + b) + 6}{2}\right)$

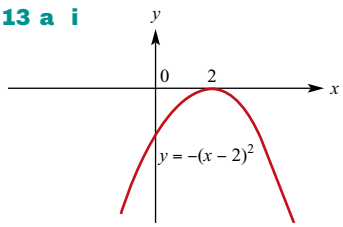
d $2ym - 2x = -m^2(a + b) + 6m - (a + b)$

e $y = -mx - 3m + 8$
$A'(a - 3, -ma + 8), B'(b - 3, -mb + 8)$

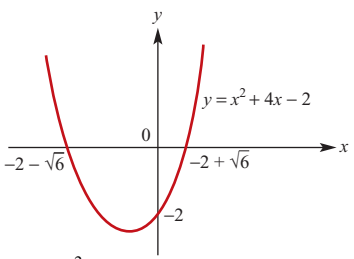
f $y = mx + 3$
$A'(-a, -ma + 3), B'(-b, -mb + 3)$

g $b = 12, m = \frac{7}{6}$

13 a i



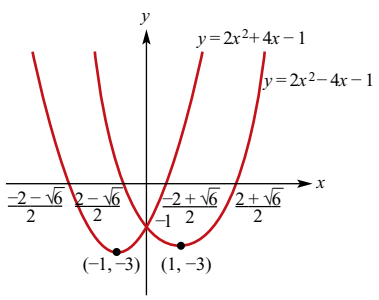
ii



b $\left(\frac{2}{1 - p}, \frac{p^2 - 5p}{p - 1}\right)$ **c** $p = 0$ or $p = 5$

d $0 < p < 5$ and $p \neq 1$

e $y = 2x^2 - 4x - 1$



14 a $k = \frac{\pi}{6}$ **b** $115\sqrt{2}$ cm **c** $115\sqrt{3}$ cm

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Appendix A

Exercise A1

1 a

	A
Step 1	3
Step 2	1

b

	A
Step 1	3
Step 2	24

c

	A
Step 1	3
Step 2	127

d

	A
Step 1	3
Step 2	14
Step 3	24

e

	A
Step 1	3
Step 2	18
Step 3	93

f

	A
Step 1	3
Step 2	18
Step 3	3

2 a

	A	B
Step 1	2	
Step 2	2	6
Step 3	10	6

b

	A	B
Step 1	2	
Step 2	2	6
Step 3	2	10

c

	A	B
Step 1	3	
Step 2	6	
Step 3	6	12

d

	A	B	C
Step 1	2		
Step 2	2	6	
Step 3	2	6	6
Step 4	6	6	6
Step 5	6	6	6

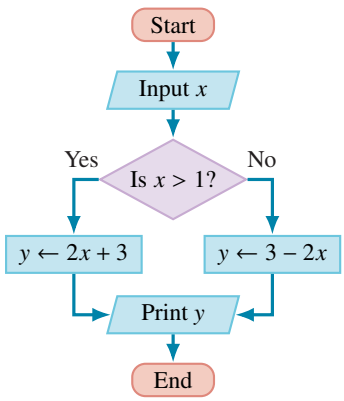
e

	A	B	C
Step 1	2		
Step 2	2	6	
Step 3	2	2	
Step 4	2	2	2
Step 5	2	2	2

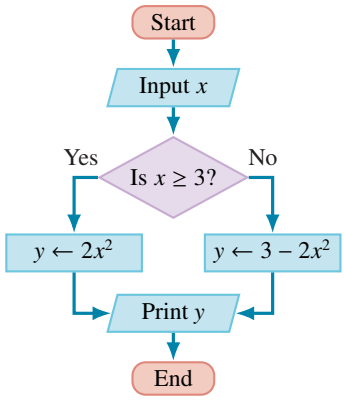
f

	A	B	C
Step 1	2		
Step 2	2	6	
Step 3	2	6	8
Step 4	2	14	8
Step 5	22	14	8

3 a



b

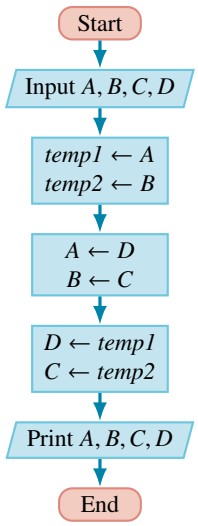


4 a Step 1 Input A, B, C, D
Step 2 $temp1 \leftarrow A$, $temp2 \leftarrow B$
Step 3 $A \leftarrow D$, $B \leftarrow C$
Step 4 $D \leftarrow temp1$, $C \leftarrow temp2$
Step 5 Print A, B, C, D

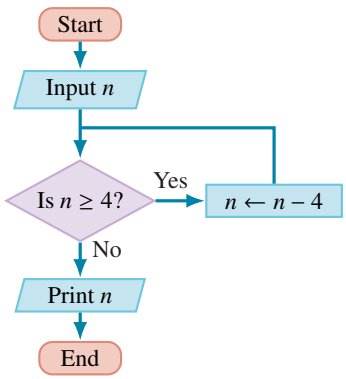
b

	A	B	C	D	$temp1$	$temp2$
Step 1	1	2	3	4		
Step 2	1	2	3	4	1	2
Step 3	4	3	3	4	1	2
Step 4	4	3	2	1	1	2

c



5 a



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b i

	n
Initial	15
Pass 1	11
Pass 2	7
Pass 3	3

ii

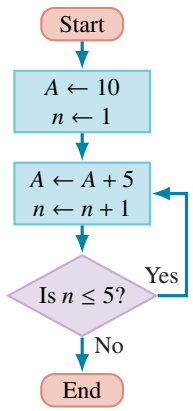
	n
Initial	12
Pass 1	8
Pass 2	4
Pass 3	0

iii

	n
Initial	3

Exercise A2

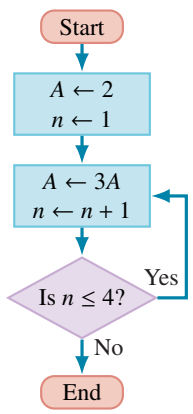
1 a i



ii

	n	A
Initial	1	10
Pass 1	2	15
Pass 2	3	20
Pass 3	4	25
Pass 4	5	30
Pass 5	6	35

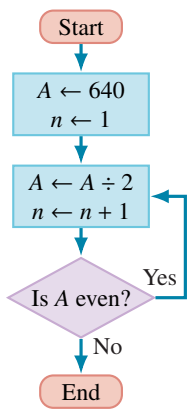
b i



ii

	n	A
Initial	1	2
Pass 1	2	6
Pass 2	3	18
Pass 3	4	54
Pass 4	5	162

c i



ii

	n	A
Initial	1	640
Pass 1	2	320
Pass 2	3	160
Pass 3	4	80
Pass 4	5	40
Pass 5	6	20
Pass 6	7	10
Pass 7	8	5

2

	sum	n	$f(n)$
Initial	0	1	195
Pass 1	195	2	190
Pass 2	385	3	185
Pass 3	570	4	180
Pass 4	750	5	175
Pass 5	925	6	170
Pass 6	1095	7	165

3 a 1.4167

b i 2.2381 **ii** 18.5742

iii 39.5348 **iv** 88.6341

4 a

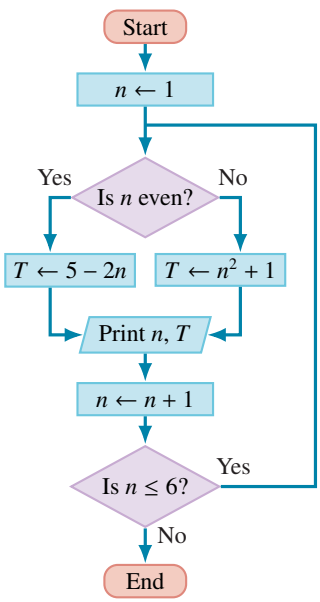
	product	n
Initial	1	1
Pass 1	1	2
Pass 2	2	3
Pass 3	6	4
Pass 4	24	5
Pass 5	120	6

b The product of the first five natural numbers

5 Step 1 *product* ← 1 and $n \leftarrow 1$
Step 2 *product* ← *product* × n^2
Step 3 $n \leftarrow n + 1$
Step 4 Repeat from Step 2 while $n \leq 10$
Step 5 Print *product*

6 a Step 1 $n \leftarrow 1$
Step 2 If n is even, then $T \leftarrow 5 - 2n$
Otherwise $T \leftarrow n^2 + 1$
Step 3 Print n, T
Step 4 $n \leftarrow n + 1$
Step 5 Repeat from Step 2 while $n \leq 6$

b



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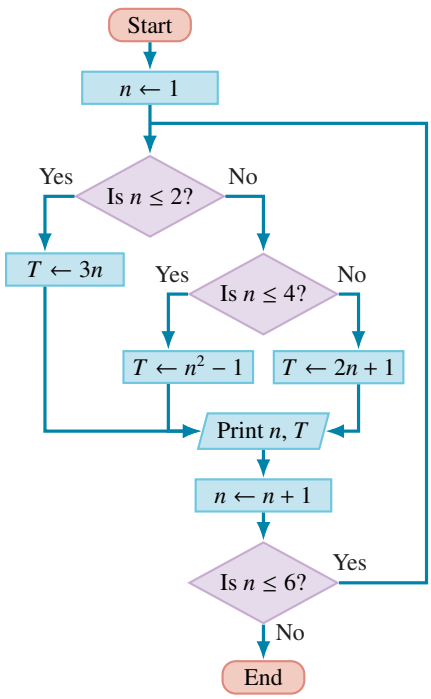
c

	n	T
Pass 1	1	2
Pass 2	2	1
Pass 3	3	10
Pass 4	4	−3
Pass 5	5	26
Pass 6	6	−7

7 a Step 1 $n \leftarrow 1$
Step 2a If $n \leq 2$, then $T \leftarrow 3n$
Step 2b Else if $n \leq 4$, then $T \leftarrow n^2 - 1$
Step 2c Else $T \leftarrow 2n + 1$
Step 3 Print n, T
Step 4 $n \leftarrow n + 1$
Step 5 Repeat from Step 2 while $n \leq 6$

b



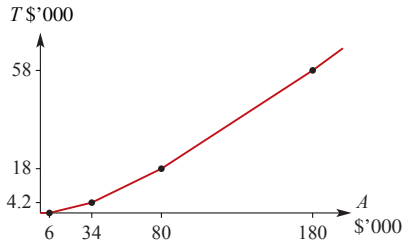
c

	n	T
Pass 1	1	3
Pass 2	2	6
Pass 3	3	8
Pass 4	4	15
Pass 5	5	11
Pass 6	6	13

8 a $f(A) =$

$$\begin{cases} 0 & \text{if } A \leq 6000 \\ 0.15A - 900 & \text{if } 6000 < A \leq 34\,000 \\ 0.3A - 6000 & \text{if } 34\,000 < A \leq 80\,000 \\ 0.4A - 14\,000 & \text{if } 80\,000 < A \leq 180\,000 \\ 0.45A - 23\,000 & \text{if } A > 180\,000 \end{cases}$$

b



c Input A
If $A \leq 6000$, then $T \leftarrow 0$
Else if $A \leq 34\,000$, then $T \leftarrow 0.15A - 900$
Else if $A \leq 80\,000$, then $T \leftarrow 0.3A - 6000$
Else if $A \leq 180\,000$, then $T \leftarrow 0.4A - 14\,000$
Else $T \leftarrow 0.45A - 23\,000$
Print T

Exercise A3

1

```
input a, b
if a ≤ b then
  print a
else
  print b
end if
```

2

```
input n
if n ≤ 4 then
  T ← n
else if n ≤ 10 then
  T ← n + 1
else
  T ← n + 2
end if
print T
```

3 a

	i	sum
Initial		0
Pass 1	1	1
Pass 2	2	3
Pass 3	3	6
Pass 4	4	10
Pass 5	5	15

b 15
c The sum of the first five natural numbers

4 a 4 **b** 8 **c** 24

5 a 1 **b** 3 **c** 21

6 a

```
input n
sum ← 0
for i from 1 to n
  sum ← sum + 1 ÷ (i × (i + 1))
end for
print sum
```

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b

	i	sum
Initial		0
Pass 1	1	0.5
Pass 2	2	0.666667
Pass 3	3	0.75
Pass 4	4	0.8

7 a

```
count ← 0
remainder ← 80
while remainder ≥ 13
    count ← count + 1
    remainder ← remainder – 13
end while
print count, remainder
```

b

	$count$	$remainder$
Initial	0	80
Pass 1	1	67
Pass 2	2	54
Pass 3	3	41
Pass 4	4	28
Pass 5	5	15
Pass 6	6	2

8 a $a = 7, b = 21$ **b** $a = 7, b = 20$
c $a = 7, b = 17$

9 a

```
A ← 2
while A^2 > 7 + 10^-4 or A^2 < 7 – 10^-4
    A ← 0.5 × (A + 7 / A)
    print A, A^2
end while
```

b

	A	A^2
Initial	2	4
Pass 1	2.75	7.5625
Pass 2	2.647727	7.010460
Pass 3	2.645752	7.000004

10

	n	a	b	c
Initial		1	2	
Pass 1	1	1	1	1
Pass 2	2	−1	1	−1
Pass 3	3	4	−1	4
Pass 4	4	−17	4	−17
Pass 5	5	89	−17	89

11 a

	A	A^3
Initial	1.3	2.197
Pass 1	1.241716	1.914550
Pass 2	1.269426	2.045606
Pass 3	1.255275	1.977957
Pass 4	1.262270	2.011206
Pass 5	1.258753	1.994444

b Finds an approximation to $\sqrt[3]{2}$

c

```
A ← 2.5
while A^3 > 20.01 or A^3 < 19.99
    A ← 0.5 × (A + 20 / A^2)
    print A, A^3
end while
```

12

```
for n from 1 to 20
    if n ≤ 5 then
        T ← n^2
    else if n ≤ 10 then
        T ← (n – 5)^2
    else if n ≤ 15 then
        T ← (n – 10)^2
    else
        T ← (n – 15)^2
    end if
    print T
end for
```

13 a

```
define gradient(x1, y1, x2, y2):
    if x1 ≠ x2 then
        gradient ← (y2 – y1) / (x2 – x1)
    else
        gradient ← 'undefined'
    end if
    return gradient
```

b

```
define distance(x1, y1, x2, y2):
    distance ← √((x2 – x1)^2 + (y2 – y1)^2)
    return distance
```

c

```
define intercept(m, c):
    if m ≠ 0 then
        intercept ← –c / m
    else
        intercept ← 'undefined'
    end if
    return intercept
```

d

```
define discriminant(a, b, c):
    discriminant ← b^2 – 4ac
    return discriminant
```

14 a

a	b	c
		0
1	1	1
1	2	2
2	1	3
2	2	4
3	1	5
3	2	6

b

a	b	c
		0
1	1	1
1	2	3
2	1	5
2	2	9
3	1	12
3	2	18

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