Chapter 1

Review of percentages and ratios

Chapter questions

► What is a percentage?

► How do we convert percentages to fractions or decimals?

► How do we convert fractions or decimals to percentages?

► How do we calculate percentage increase, percentage decrease and percentage change?

► What is GST and how do we calculate it?

► How do we express ratios in their simplest form?

► How do we divide quantities in given ratios?

► What is the unitary method?

► How do we solve practical problems involving ratios, percentages and the unitary method?

This chapter revises percentages and ratios. An understanding of these topics will be useful for our study of data (Chapter 2) and finance (Chapter 3).

Cambridge University Press

1A Percentages

Learning intentions

► To be able to express fractions and decimals as percentages.
► To be able to convert percentages to fractions or decimals.
► To be able to find a percentage of a quantity.
► To be able to compare two quantities.

Per cent is an abbreviation of the Latin words *per centum*, meaning 'by the hundred'. A **percentage** is a rate or a proportion expressed as a part of one hundred. The symbol used to indicate percentage is %. Percentages can be expressed as common fractions or as decimals.

For example: 17% (17 per cent) means 17 parts out of every 100.

$$17\% = \frac{17}{100} = 0.17$$

Conversions

1 To convert a fraction or a decimal to a percentage, multiply by 100.
2 To convert a percentage to a decimal or a fraction, divide by 100.

Example 1   Converting a fraction to a percentage

Express $\frac{36}{90}$ as a percentage.

Explanation

Method 1 (by hand)

1 Multiply the fraction $\frac{36}{90}$ by 100.
2 Evaluate and write your answer.

Method 2 (using CAS)

1 Enter $36 \div 90$ on the calculator.
2 Press % sign and EXE (Casio) or ENTER (Ti-Nspire).
3 Write your answer.

Solution

$$\frac{36}{90} \times 100$$

$$= 40\%$$

36/90%
40

Expressed as a percentage, $\frac{36}{90}$ is 40%.

Now try this 1 Converting a fraction to a percentage (Example 1)

Express $\dfrac{14}{70}$ as a percentage.

Hint 1 Multiply the fraction by 100.

Example 2 Converting a decimal to a percentage

Express 0.75 as a percentage.

Explanation	Solution
1 Multiply 0.75 by 100.	0.75×100
2 Evaluate and write your answer.	$= 75\%$

Now try this 2 Converting a decimal to a percentage (Example 2)

Express 0.25 as a percentage.

Hint 1 Multiply the decimal by 100.

Example 3 Converting a percentage to a fraction

Express 62% as a common fraction.

Explanation	Solution
1 As 62% means 62 out of 100, this can be written as a fraction $\dfrac{62}{100}$.	$62\% = \dfrac{62}{100}$
2 Simplify the fraction by dividing both the numerator and the denominator by 2.	$= \dfrac{62 \div 2}{100 \div 2}$ $= \dfrac{31}{50}$

Now try this 3 Converting a percentage to a fraction (Example 3)

Express 78% as a fraction.

Hint 1 Remember that % means "out of 100".
Hint 2 Simplify the fraction.

Example 4   Converting a percentage to a decimal

Express 72% as a decimal.

Explanation	Solution
1 Write 72% as a fraction out of 100 and express this as a decimal.	$\dfrac{72}{100} = 0.72$

Now try this 4   Converting a percentage to a decimal (Example 4)

Express 45% as a decimal.

Hint 1 Start by writing it as a fraction out of 100.

Finding a percentage of a quantity

To find a percentage *of* a number or a quantity, remember that, in mathematics, *of* means *multiply*.

Example 5   Finding a percentage of a quantity

Find 15% of $140.

Explanation	Solution
Method 1	15% of 140
1 Write out the problem and rewrite 15% as a fraction out of 100.	$= \dfrac{15}{100}$ of 140
2 Change *of* to *multiply*.	$= \dfrac{15}{100} \times 140$
3 Perform the calculation and write your answer.	$= 21$
Note: The above calculation can be performed on the ClassPad calculator.	15% of $140 is $21
Method 2 (using CAS)	
1 Enter 15%140 on the calculator.	15%140
2 Press EXE (Casio) or ENTER (Ti-Nspire).	21
3 Write your answer and include the $ sign.	$21

Now try this 5   Finding a percentage of a quantity (Example 5)

Find 30% of $90.

Hint 1 Write 30% as a fraction out of 100.
Hint 2 Remember *of* means *multiply*.

Comparing two quantities

One quantity or number may be expressed as a percentage of another quantity or number (both quantities must always be in the same units). Divide the quantity by what you are comparing it with and then multiply by 100 to convert it to a percentage.

Example 6   **Expressing a quantity as a percentage of another quantity**

There are 18 girls in a class of 25 students. What percentage of the class are girls?

Explanation	Solution
1 Work out the fraction of girls in the class.	$\text{Girls} = \dfrac{18}{25}$
2 Convert the fraction to a percentage by multiplying by 100.	$\dfrac{18}{25} \times 100$
3 Evaluate and write your answer.	$= 72$ 72% of the class are girls.

Now try this 6   **Expressing a quantity as a percentage of another quantity (Example 6)**

There are 10 faulty batteries in a box of 25 batteries. What percentage of batteries are faulty?

Hint 1   Work out the fraction of faulty batteries in the box.

Hint 2   Convert to a percentage by multiplying by 100.

Example 7   **Expressing a quantity as a percentage of another quantity with different units**

Express 76 mm as a percentage of 40 cm.

Explanation	Solution
1 First, convert 40 centimetres to millimetres by multiplying by 10, as there are 10 millimetres in 1 centimetre.	$40\,\text{cm} = 40 \times 10$ $= 400\,\text{mm}$
2 Write 76 millimetres as a fraction of 400 millimetres.	$\dfrac{76}{400}$
3 Multiply by 100 to convert to a percentage.	$\dfrac{76}{400} \times 100$
4 Evaluate and write your answer.	$= 19\%$

> **Now try this 7**   **Expressing a quantity as a percentage of another quantity with different units (Example 7)**
>
> Express 18 mm as a percentage of 60 cm.
>
> **Hint 1** Convert the 60 cm to mm by multiplying by 10.
>
> **Hint 2** Write 18 mm as a fraction of 600 mm and multiply by 100.

Section Summary
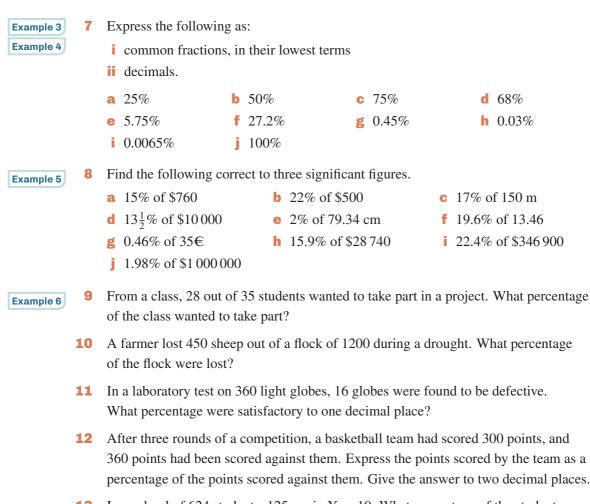
► To convert a fraction or a decimal to a percentage, multiply by 100.

► To convert a percentage to a decimal or a fraction, divide by 100.

► To find a percentage of a quantity, write the percentage as a fraction out of 100 and multiply by the quantity.

► To compare two quantities, make sure that they have the same units.

Skill-sheet

Exercise 1A

Building understanding

1 Express the following percentages as fractions out of 100.

   a 17%   **b** 94%   **c** 71%

2 Write the following percentages as fractions, simplifying where possible.

   a 7%   **b** 13%   **c** 50%   **d** 10%   **e** 20%

3 Rewrite the following fractions as percentages.

   a $\dfrac{11}{100}$ **b** $\dfrac{23}{100}$ **c** $\dfrac{79}{100}$

4 Convert the following fractions to percentages by multiplying by 100.

   a $\dfrac{1}{2}$ **b** $\dfrac{2}{5}$ **c** $\dfrac{1}{4}$

5 Convert the following decimals to percentages by multiplying by 100.

   a 0.78   **b** 0.37   **c** 0.561

Developing understanding

Example 1
Example 2

6 Express the following as percentages.

   a $\dfrac{1}{4}$ **b** $\dfrac{4}{5}$ **c** $\dfrac{3}{20}$ **d** $\dfrac{7}{10}$

   e 0.19   **f** 0.79   **g** 2.15   **h** 39.57

   i 0.073   **j** 1

Example 3
Example 4

7 Express the following as:

   i common fractions, in their lowest terms

   ii decimals.

  a 25%   **b** 50%   **c** 75%   **d** 68%

  e 5.75%   **f** 27.2%   **g** 0.45%   **h** 0.03%

  i 0.0065%   **j** 100%

Example 5

8 Find the following correct to three significant figures.

  a 15% of $760   **b** 22% of $500   **c** 17% of 150 m

  d $13\frac{1}{2}$% of $10 000   **e** 2% of 79.34 cm   **f** 19.6% of 13.46

  g 0.46% of 35€   **h** 15.9% of $28 740   **i** 22.4% of $346 900

  j 1.98% of $1 000 000

Example 6

9 From a class, 28 out of 35 students wanted to take part in a project. What percentage of the class wanted to take part?

10 A farmer lost 450 sheep out of a flock of 1200 during a drought. What percentage of the flock were lost?

11 In a laboratory test on 360 light globes, 16 globes were found to be defective. What percentage were satisfactory to one decimal place?

12 After three rounds of a competition, a basketball team had scored 300 points, and 360 points had been scored against them. Express the points scored by the team as a percentage of the points scored against them. Give the answer to two decimal places.

13 In a school of 624 students, 125 are in Year 10. What percentage of the students are in Year 10? Give your answer to the nearest whole number.

Example 7

14 Express 75 cm as a percentage of 2 m.

15 In a population of $3\frac{1}{4}$ million people, 2 115 000 are under the age of 16. Calculate the percentage, to two decimal places, of the population who are under the age of 16.

16 The cost of producing a chocolate bar that sells for $1.50 is 60c. Calculate the profit made on a bar of chocolate as a percentage of the production cost of a bar of chocolate.

Testing understanding

17 Kate and Tim were distributing letters. Tim had 5% more letters than Kate. Kate had 160 letters. How many did Tim have?

18 Martin and Simon were comparing their marble collections. Martin had 20% less marbles than Simon. Simon had 70 marbles. How many marbles did Martin have?

19 Ruby wanted to decorate the local hall for her party. She had 15 balloons. This was only 30% of the balloons that she required. How many balloons did she need in total?

1B Percentage increase and decrease

When increasing or decreasing a quantity by a given percentage, the percentage increase or decrease is always calculated as a percentage of the *original* quantity.

▶ **Example 8**   **Calculating the new price following a percentage increase**

Sally's daily wage of $175 is increased by 15%. Calculate her new daily wage.

Explanation	Solution
Method 1	
1 First find 15% of $175 by rewriting 15% as a fraction out of 100 and changing *of* to *multiply* (or use a calculator).	15% of 175 $$= \frac{15}{100} \times 175$$
2 Perform the calculation and write your answer.	$= 26.25$
3 As $175 is to be increased by 15%, add $26.25 to the original amount of $175.	$175 + 26.25$ $= 201.25$
4 Write your answer in a sentence.	Sally's new daily wage is $201.25.
Method 2	
1 An increase of 15% means that the new amount will be the original amount (in other words, 100%) plus an extra 15%. Find 115% of 175.	115% of 175 $$= \frac{115}{100} \times 175$$ $= 201.25$
2 Perform the calculation.	
3 Write your answer in a sentence.	Sally's new daily wage is $201.25.

Now try this 8   **Calculating the new price following a percentage increase (Example 8)**

Tom's weekly wage of $950 has increased by 10%. Calculate his new weekly wage.

Hint 1   Find 10% of $950.

Hint 2   Add this amount to $950.

Hint 3   **Alternatively**, an increase of 10% means the new weekly wage is $(100+10)\% = 110\%$ of the original weekly wage.

Example 9   **Calculating the new amount following a percentage decrease**

A primary school's fun run distance of 2.75 km is decreased by 20% for students in Years 2 to 4. Find the new distance.

Explanation	Solution
Method 1	
1 First find 20% of 2.75 by writing 20% as a fraction out of 100 and changing *of* to *multiply* (or use a calculator).	20% of 2.75 $= \dfrac{20}{100} \times 2.75$
2 Evaluate and write your answer.	$= 0.55$
3 As 2.75 km is to be decreased by 20%, subtract 0.55 km from the original 2.75 km.	$2.75 - 0.55$ $= 2.2$
4 Write your answer in a sentence.	The new distance is 2.2 km.
Method 2	
1 A decrease of 20% means that the new amount will be the original amount (100%) minus 20%. Find 80% of 2.75.	80% of 2.75 $= \dfrac{80}{100} \times 2.75$ $= 2.2$
2 Perform the calculation.	
3 Write your answer in a sentence.	The new distance is 2.2 km.

> **Now try this 9** **Calculating the new amount following a percentage decrease (Example 9)**
>
> The local council's fun run of 12 km is decreased by 8%. Find the new distance.
>
> Hint 1 Find 8% of 12 km.
>
> Hint 2 Subtract this amount from 12 km.
>
> Hint 3 **Alternatively**, a decrease of 8% means the new distance is $(100 - 8)\% = 92\%$ of the original distance.

> **Example 10** **Calculating a new price with a percentage discount**
>
> If a shop offers a discount of 15% on items in a sale, what would be the sale price of a pair of jeans originally priced at $95?
>
> | Explanation | Solution |
> |---|---|
> | **Method 1** | |
> | **1** Find 15% of 95. | 15% of $95 = \dfrac{15}{100} \times 95$
 $= 14.25$ |
> | **2** As jeans are discounted by 15%, this is a decrease, so we need to subtract the discounted price of $14.25 from the original price of $95. | $95 - 14.25$
 $= 80.75$ |
> | **3** Write your answer in a sentence. | The sale price would be $80.75. |
> | **Method 2** | |
> | **1** A discount of 15% means that the new amount is 85% of 95. | 85% of 95
 $= \dfrac{85}{100} \times 95$ |
> | **2** Perform the calculation. | $= 80.75$ |
> | **3** Write your answer in a sentence. | The sale price would be $80.75. |

> **Now try this 10** **Calculating a new price with a percentage discount (Example 10)**
>
> A shoe shop offers a discount of 20%. What would be the sale price of a pair of shoes originally priced at $150?
>
> Hint 1 Find 20% of $150.
>
> Hint 2 Subtract this amount from $150 to obtain the new price.
>
> Hint 3 **Alternatively**, 20% discount means the new price of the shoes is $(100 - 20)\% = 80\%$ of the original price.

© Peter Jones et al 2023
Cambridge University Press

Finding a percentage change

If we are given the original price and the new price of an item, we can find the percentage change. To find the percentage change, we compare the change (increase or decrease) with the original number.

> **Percentage change**
>
> $$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100$$
>
> Thus:
>
> $$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$
>
> $$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Example 11   Calculating a percentage increase

A university increased its total size at the beginning of an academic year by 3000 students. If the previous number of students was 35 000, by what percentage, to two decimal places, did the student population increase?

Explanation

1 To find the percentage increase, use the formula:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Substitute increase as 3000 and original as 35 000.

2 Evaluate.

3 Write your answer to two decimal places.

Solution

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

$$= \frac{3000}{35\,000} \times 100$$

$$= 8.5714\ldots$$

Student population increased by 8.57%.

Now try this 11   Calculating a percentage increase (Example 11)

The number of students studying VCE at Hambrook High School increased by 70 students in one year. If the previous number of students studying VCE was 365, by what percentage, to two decimal places, did the VCE student population increase?

Hint 1 Use the formula for percentage increase and substitute in relevant values.

Hint 2 $\text{Percentage increase} = \dfrac{\text{increase}}{\text{original}} \times 100$

Example 12   Calculating the percentage discount

Calculate the percentage discount obtained when a calculator with a normal price of $38 is sold for $32, to the nearest whole per cent.

Explanation	**Solution**
1 Find the amount of discount given by subtracting the new price, $32, from the original price, $38.	Discount $= \$38 - \$32$ $= \$6$
2 To find the percentage discount, use the formula: $\text{Percentage discount} = \dfrac{\text{discount}}{\text{original}} \times 100$ Substitute discount as 6 and original as 38, and evaluate.	$\text{Percentage discount} = \dfrac{\text{discount}}{\text{original}} \times 100$ $= \dfrac{6}{38} \times 100$ $= 15.7895\ldots$
3 Write your answer to the nearest whole per cent.	The percentage discount is 16%.

Now try this 12   Calculating the percentage discount (Example 12)

Calculate the percentage discount obtained when an iPad with a normal price of $599 is sold for $529. Give your answer to the nearest whole per cent.

Hint 1   Find the amount of discount in $.

Hint 2   Use the formula for % discount. Percentage discount $= \dfrac{\text{discount}}{\text{original}} \times 100$

Section Summary

► When increasing or decreasing a quantity by a certain percentage, calculate the percentage of the original quantity.

► The formula for % change is: Percentage change $= \dfrac{\text{change}}{\text{original}} \times 100$.

Exercise 1B

Building understanding

1 A $190 watch is for sale with a 20% discount.

  a By finding 20% of $190, work out how much the discount is.

  b What will be the new sale price of the watch?

Example 8

2 Sally's hourly work rate of $30 is increased by 10%.

  a By how much will her hourly rate be increased?

  b What is her new hourly rate?

Example 9

3 The price of a $25 book is decreased by 5%.

  a By how much is it decreased?

  b What is the new price of the book?

Developing understanding

4 A jewellery store has a promotion of 20% discount on all watches.

  a How much discount will you get on a watch marked $185?

  b What is the sale price of the watch?

Example 10

5 A store gave different savings discounts on a range of items in a sale.
Copy and complete the following table. (Round to the nearest cent.)

	Normal price	% Discount	Saving	Sale price
a	$89.99	5		
b	$189.00	10		
c	$499.00	15		
d	$249.00	20		
e	$79.95	22.5		
f	$22.95	25		
g	$600.00	27.5		
h	$63.50	30		
i	$1000.00	33		

6   In a particular shop the employees are given a $12\frac{1}{2}\%$ discount on any items they purchase. Calculate the actual price an employee would pay for each of the following:

   a $1200 laptop   **b** $1400 smart television

   c $246 mini drone   **d** $750 digital camera

7   A clothing store offers 6% discount for cash sales. A customer who paid cash purchased the following items:

   ■ A pair of jeans at $95.95   ■ A leather belt at $29.95   ■ Two jumpers at $45 each.

   Calculate, to the nearest cent:

   a the total savings   **b** the actual amount paid for the goods.

8   The number of cars of a particular model produced by a company is increased from 14 000 by 6% over a 12-month period. In 12 months' time, what is the new production figure?

9   If a new car is sold for $23 960, and three years later it is valued at $18 700, calculate the percentage decrease (called depreciation) to two decimal places.

10   A leading tyre manufacturer claims that a new tyre will average 12% more life than a previous tyre. The owner of a taxi fleet finds that the previous tyre averaged 24 000 km before replacement. How many kilometres should the new tyre average?

Example 12 **11**   Calculate the percentage discount for each of the following, to the nearest whole number.

	Normal price	Selling price	% Discount
a	$60.00	$52.00	
b	$250.00	$185.00	
c	$5000.00	$4700.00	
d	$3.80	$2.90	
e	$29.75	$24.50	
f	$12.95	$10.00	

12   A second-hand car advertised for sale at $13 990 was sold for $13 000. Calculate, to two decimal places, the percentage discount obtained by the purchaser.

13   A sport shop advertised the following items in their end-of-year sale. Calculate the percentage discount for each of the items, to the nearest whole number.

		Normal price	Selling price	% Discount
a	Shoes	$79.99	$65.00	
b	12-pack of golf balls	$29.99	$19.99	
c	Exercise bike	$1099.00	$599.00	
d	Basketball	$49.99	$39.99	
e	Sports socks	$14.95	$10.00	
f	Hockey stick	$299.00	$250.00	

Example 11

14   Find the percentage increase that has been applied in each of the following:
   a   a book that is increased from $20 to $25
   b   an airfare that is increased from $300 to $420
   c   accommodation costs that are increased from $540 to $580.50.

Testing understanding

15   Which results in the larger sum of money, increasing $50 by 10% or decreasing $60 by 8%?

16   William is 1.86 m tall and Jonathon is 1.92 m tall. What percentage taller is Jonathon than William? Give your answer to two decimal places.

1C Goods and Services Tax (GST)

Learning intentions

► To be able to calculate GST.

► To be able to find the cost of items that include GST.

► To be able to find the cost of items prior to GST being added.

The goods and services tax (GST) is a tax of 10% that is added to the price of most goods and services in Australia. GST is an example of percentage increase.

To find the GST payable, you find 10% of the original amount.

$$10\% = \frac{10}{100} = \frac{1}{10}$$

so finding 10% of the original amount is the same as dividing the original amount by 10.

Thus, Amount of GST $= \dfrac{\text{cost without GST}}{10}$

Example 13   Calculating the amount of GST when the cost without GST is known

If the cost of electricity in one quarter is $288.50, how much GST will be added?

Explanation

1 Use the rule for finding the amount of GST when cost without GST is known.

2 Substitute in $288.50 for cost without GST. Or, use a calculator to find 10% of $288.50.

3 Write your answer.

Solution

Amount of GST $= \dfrac{\text{cost without GST}}{10}$

Amount of GST $= \dfrac{288.50}{10} = 28.85.$

GST is $28.85.

Now try this 13   Calculating the amount of GST when the cost without GST is known (Example 13)

The cost of installing a sliding door is $850. How much GST will be added to this cost?

Hint 1 Use the rule, amount of GST $= \dfrac{\text{cost without GST}}{10}$ and substitute the known value.

Example 14 Calculating the new price with GST

The bill for the services that a plumber provides is $650. GST needs to be added. What is the amount of the final bill with GST included?

Explanation	**Solution**
Method 1	
1 GST is 10%. Find 10% of $650.	10% of $650 = \dfrac{10}{100} \times 650 = 65$
2 As $650 is to be increased by 10%, add $65 to the original amount of $650.	$650 + 65 = 715$
3 Write your answer.	Amount of final bill with GST is $715.
Method 2	
1 An increase of 10% means that the new amount will be the original amount (in other words, 100%) plus an extra 10%. Find 110% of 650.	110% of 650 $= \dfrac{110}{100} \times 650 = 715$
2 Write your answer.	Amount of final bill with GST is $715.

Now try this 14 Calculating the new price with GST (Example 14)

The price of a washing machine is $1400. GST needs to be added. What is the price of the washing machine with GST added?

Hint 1 Find 10% of $1400.

Hint 2 Add this amount to $1400.

Hint 3 **Alternatively**, an increase of 10% means that the new price is
$(100+10)\% = 110\%$ of the initial price.

Another way to evaluate cost prices with GST is to multiply the original amount by 1.1 as this is the same as multiplying by $\dfrac{110}{100}$ (finding 110%). Thus:

$$\textbf{cost with GST} = \text{cost } \textbf{without } \text{GST} \times 1.1$$

We can then rearrange this equation to find the cost without GST:

$$\textbf{cost without GST} = \frac{\text{cost } \textbf{with } \text{GST}}{1.1}$$

This can be summarised as follows:

To find the cost with GST, multiply the original cost (cost without GST) by 1.1.

To find the original cost (cost without GST), divide the cost with GST by 1.1.

Example 15   Calculating the original cost (cost without GST)

An electric guitar costs $2299 including GST. What is the price without GST?

Explanation	Solution
1 To find the original cost, divide the cost with GST by 1.1.	$2299 \div 1.1 = 2090$
2 Write your answer.	The price without GST is $2090.

Now try this 15   Calculating the original cost (cost without GST) (Example 15)

A drum kit sells for $737 including GST. What is the price without GST?

Hint 1 Divide the original price by 1.1.

We have seen that we can calculate the amount of GST from the original cost (cost without GST).

$$\textbf{Amount of GST} = 10\% \text{ of the original cost (cost without GST)}$$
$$= \frac{\text{cost without GST}}{10}$$

We can also find the amount of GST when we know the final cost (cost with GST).

Remember that: Original cost + 10% of the original cost = cost with GST

so,   110% of the original cost (cost without GST) = cost with GST.

This can be written as: $\dfrac{110}{100} \times \text{cost without GST} = \text{cost with GST}$

or, $\dfrac{11}{10} \times \text{cost without GST} = \text{cost with GST}.$

Dividing both sides by 11 gives:

$$\frac{\text{cost without GST}}{10} = \frac{\text{cost with GST}}{11}$$

But remember: $\dfrac{\text{cost \textbf{without} GST}}{10} = \text{Amount of GST}$

So, $\textbf{Amount of GST} = \dfrac{\text{cost \textbf{with} GST}}{11}$

We can now find the amount of GST added when we know either the original amount (cost without GST) or when we know the cost with GST.

$$\text{Amount of GST} = \frac{\text{cost \textbf{without} GST}}{10} \quad \text{and} \quad \text{Amount of GST} = \frac{\text{cost \textbf{with} GST}}{11}$$

Example 16   Calculating the amount of GST when the cost with GST is known

A desktop computer sells for $935, including GST. What is the amount of GST that has been added?

Explanation	Solution
1 Use the rule for finding the amount of GST when cost with GST is known.	Amount of GST $= \dfrac{\text{cost with GST}}{11}$
2 Substitute in \$935 for cost with GST.	Amount of GST $= \dfrac{935}{11} = 85$
3 Write your answer.	GST is \$85

Now try this 16   **Calculating the amount of GST when the cost with GST is known (Example 16)**

An iPad sells for \$1155, including GST. What is the amount of GST that has been added?

Hint 1 Use the rule: Amount of GST $= \dfrac{\text{cost with GST}}{11}$ and substitute the known value.

Section Summary

► GST stands for goods and services tax and is a tax of 10%.

► To find the amount of GST when the cost without GST is known, divide by 10.

► To find the amount of GST when the cost with GST is known, divide by 11.

► To calculate the cost with GST, multiply the original cost by 1.1.

► To calculate the original cost, divide the cost with GST by 1.1.

Skill-sheet

Exercise 1C

Building understanding

1 Find 10% of the following:

  a \$900   **b** \$760   **c** \$599   **d** \$65   **e** \$2572   **f** \$48 755

Example 13

2 Find the GST payable on each of the following (give your answer to the nearest cent).

  a a gas bill of \$121.30   **b** an electrician's bill of \$367.50

  c a TV costing \$1085.50   **d** gardening services of \$395

Developing understanding

Example 14

3 The following prices are without GST. Find the price after GST has been added.

  a a dress worth \$139   **b** a bedroom suite worth \$2678

  c a home movie theatre worth \$9850   **d** painting services of \$1395

Example 15

4 If a computer is advertised for \$2399 including GST, how much would the computer have cost without GST to the nearest cent?

Example 16   **5**   What is the amount of the GST that has been added if the price of a car is advertised as $39 990 including GST to the nearest cent?

6   A gas bill is $109.78 after GST is added.

    a   What was the price before GST was added?

    b   How much GST must be paid?

Testing understanding

7   **a**   How much was the GST on a car that sold for $57 300 (to the nearest cent)?

    b   What was the pre-GST price of the car (to the nearest cent)?

8   Chris buys a camera for $1599, including GST. How much of this price is GST (to the nearest cent)?

9   John sees a lawn mower advertised for $765, including GST. He then sees another lawn mower for $695 which has not as yet had GST added. Which lawn mower should he buy and how much will he save?

1D   Ratio and proportion

Ratios are used to numerically compare the values of two or more quantities.

A **ratio** can be written as $a : b$ (read as 'a to b'). It can also be written as a fraction $\dfrac{a}{b}$.

The order of the numbers in a ratio is important. $a : b$ is *not* the same as $b : a$.

Example 17   **Expressing quantities as a ratio**

In a Year 10 class of 26 students, there are 14 girls and 12 boys. Express the number of girls to boys as a ratio.

Solution

As there are 14 girls and 12 boys, the ratio of girls to boys is 14 : 12.

Note: This could also be written as a fraction $\dfrac{14}{12}$.

Example 18   **Expressing more than two quantities as a ratio**

A survey of the same group of 26 students showed that 10 students walked to school, 11 came by public transport and 5 were driven by their parents. Express as a ratio the number of students who walked to school, to the number of students who came by public transport, to the number of students who were driven to school.

Solution

The order of the numbers in a ratio is important.

10 students walked, 11 used public transport and 5 were driven, so the ratio is $10 : 11 : 5$.

> **Now try this 18** — **Expressing more than two quantities as a ratio (Example 18)**
>
> It was found that, out of a group of 20 students, English was the favourite subject of 8 students, Mathematics was the favourite subject of 11 students and Computer Science was the favourite subject of 1 student. Express as a ratio the number of students whose favourite subject was English, to the number of students whose favourite subject was Mathematics, to the number of students whose favourite subject was Computer Science.
>
> **Hint 1** Express the numbers as a ratio, ensuring that the order is correct.

Expressing ratios in their simplest form

Ratios can be simplified by dividing through by a common factor or by multiplying each term as required.

Example 19 **Simplifying ratios**

Simplify the following ratios.

a $15:20$ **b** $0.4:1.7$ **c** $\frac{3}{4}:\frac{5}{3}$

Explanation

a 1 Divide both 15 and 20 by 5.

2 Evaluate and write your answer.

b 1 Multiply both 0.4 and 1.7 by 10 to give whole numbers.

2 Evaluate and write your answer.

c Method 1

1 Multiply both fractions by 4.

2 Multiply both sides of the ratio by 3.

3 Write your answer.

c Method 2

1 Multiply both $\frac{3}{4}$ and $\frac{5}{3}$ by the lowest common multiple (LCM) of 3 and 4, which is 12, to eliminate fractions.

2 Evaluate and write your answer.

Solution

$$15:20$$
$$=\frac{15}{5}:\frac{20}{5}$$
$$=3:4$$

$$0.4:1.7$$
$$=0.4\times10:1.7\times10$$
$$=4:17$$

$$\frac{3}{4}\times4:\frac{5}{3}\times4$$
$$=3:\frac{20}{3}$$
$$=3\times3:\frac{20}{3}\times3$$
$$=9:20$$

$$\frac{3}{4}:\frac{5}{3}$$
$$=\frac{3}{4}\times12:\frac{5}{3}\times12$$
$$=9:20$$

In each of the above examples, the ratios are equivalent and the information is unchanged. For example, the ratio:

   12 : 8 is equivalent to the ratio 24 : 16 (multiply both 12 and 8 by 2)

and

   12 : 8 is also equivalent to the ratio 3 : 2 (divide both 12 and 8 by 4).

Now try this 19   Simplifying ratios (Example 19)

Express 14 : 49 as a ratio in simplest form.

Hint 1 Find the largest common factor of 14 and 49, and divide both 14 and 49 by this factor.

Example 20   Simplifying ratios with different units

Express 15 cm to 3 m as a ratio in its simplest form.

Explanation

1 Write down the ratio.

2 Convert 3 m to cm by multiplying 3 m by 100, so that both parts of the ratio will be in the same units.

3 Simplify the ratio by dividing both 15 and 300 by 15.

4 Write your answer.

Solution

$15 \text{ cm} : 3 \text{ m}$

$15 \text{ cm} : 3 \times 100 \text{ cm}$

$= 15 \text{ cm} : 300 \text{ cm}$

$= 15 : 300$

$= \dfrac{15}{15} : \dfrac{300}{15}$

$= 1 : 20$

Now try this 20   Simplifying ratios with different units (Example 20)

Express 52 mm to 8 cm as a ratio in simplest form.

Hint 1 Convert 8 cm to mm so that both parts of the ratio are in the same units.
Hint 2 Simplify the ratio.

Example 21   Finding missing values in a ratio

Find the missing value for the equivalent ratios $3 : 7 = \boxed{} : 28$.

Explanation

1 Let the unknown value be x and write the ratios as fractions.

2 Solve for x.

Solution

$3 : 7 = x : 28$

$\dfrac{3}{7} = \dfrac{x}{28}$

Method 1 (by hand)

1 Multiply both sides of the equation by 28.

$$\frac{3}{7} \times 28 = \frac{x}{28} \times 28$$
$$x = 12$$

2 Evaluate and write your answer.

$$3 : 7 = 12 : 28$$

Method 2 (using CAS)

Use the solve function.

$$\text{solve}\left(\frac{3}{7} = \frac{x}{28}, x\right)$$
$$x = 12$$

Now try this 21 Finding missing values in a ratio (Example 21)

Find the missing value for the equivalent ratios $4 : 11 =$ ☐ $: 88$.

Hint 1 Let the unknown value be x and write the ratios as fractions.

Hint 2 Solve for x.

Section Summary

▶ When ratios are written in terms of the smallest possible whole numbers, they are expressed in their simplest form. (The highest common factor of the two numbers is 1.)

▶ The order of the numbers in a ratio is important. $3 : 5$ is not the same as $5 : 3$.

▶ Both parts of a ratio must be expressed in the same unit of measurement.

Skill-sheet

Exercise 1D

Building understanding

1 A shopper buys 17 wholemeal rolls and 9 sourdough rolls. Find:

  a the ratio of the number of wholemeal rolls to the number of sourdough rolls

  b the ratio of the number of sourdough rolls to the number of wholemeal rolls

  c the ratio of the number of wholemeal rolls to the total number of rolls.

2 Simplify the following ratios.

  a $25 : 75$ **b** $0.2 : 0.5$ **c** $32 \text{ mm} : 5 \text{ cm}$

Developing understanding

Example 17

3 A survey of a group of 50 Year 11 students in a school showed that 35 of them have a part-time job and 15 do not. Express the number of students having a part-time job to those who do not as a ratio.

Example 18

4   The table opposite shows the average life expectancy of some animals.

Find the ratios between the life expectancies of the following animals.

a   Whale to horse
b   Elephant to kangaroo
c   Whale to tortoise
d   Chimpanzee to mouse
e   Horse to mouse to whale

Animal	Life expectancy
Chimpanzee	40 years
Elephant	70 years
Horse	40 years
Kangaroo	9 years
Tortoise	120 years
Mouse	4 years
Whale	80 years

Example 19

5   Express the following ratios in their simplest forms.

a $12 : 15$ b $10 : 45$ c $22 : 55 : 33$ d $1.3 : 3.9$

e $2.7 : 0.9$ f $\dfrac{5}{3} : \dfrac{1}{4}$ g $18 : 8$

Example 20

6   Express the following ratios in their simplest form after making sure that each quantity is expressed in the same units.

a   60 L to 25 L   b $2.50 to $50
c   75 cm to 2 m   d   5 kg to 600 g
e   15 mm to 50 cm to 3 m   f   1 km to 1 m to 1 cm
g   5.6 g to 91 g   h $30 to $6 to $1.20 to 60c

Example 21

7   For each of the following equivalent ratios, find the missing value.

a $1 : 4 = \boxed{} : 20$
b $15 : 8 = 135 : \boxed{}$
c $600 : 5 = \boxed{} : 1$
d $2 : 5 = 2000 : \boxed{}$
e $3 : 7 = \boxed{} : 56$

8   Which of the following statements are true and which are false? For those that are false, suggest a correct replacement statement, if possible.

a   The ratio $4 : 3$ is the same as $3 : 4$.
b   The ratio $3 : 4$ is equivalent to $20 : 15$.
c $9 : 45$ is equivalent to $1 : 5$.
d   The ratio 60 to 12 is equivalent to 15 to 3, which is the same as 4 to 1.
e   If the ratio of a father's age to his daughter's age is $7 : 1$, then the girl is 7 years old when her father is 56.
f   If my weekly allowance is $\dfrac{5}{8}$ of that of my friend, then the ratio of my monthly allowance to the allowance of my friend is $20 : 32$.

Testing understanding

9 The following recipe is for Anzac biscuits.

> **Anzac biscuits (makes 25)**
>
> 100 grams rolled oats   60 grams desiccated coconut
>
> 175 grams plain flour   125 grams soft brown sugar
>
> 125 grams butter   3 tablespoons boiling water
>
> 2 tablespoons golden syrup   1 teaspoon bicarb soda

a What is the unsimplified ratio of rolled oats : coconut : flour : brown sugar : butter?

b Simplify the ratio from part **a**.

c You want to adapt the recipe to make 75 biscuits. What quantity of each ingredient do you need?

1E Dividing quantities in given ratios

If a quantity is divided in the ratio 5 : 3, there are $5 + 3 = 8$ parts in total. After division, one allocation has 5 parts and the other 3. That is, if you divide \$8 between two people in the ratio 5 : 3, one person gets \$5 and the other \$3.

Example 22   Dividing quantities in given ratios

Calculate the number of students in each class if 60 students are divided into classes in the following ratios.

a 1 : 3   **b** 1 : 5   **c** 1 : 2 : 7

Explanation	Solution
a 1 Add up the total number of parts. (Remember that a 1 : 3 ratio means that there is 1 part for every 3 parts).	The total number of parts is $1 + 3 = 4$.
2 Divide the number of students (60) by the number of parts (4) to give the number of students in one group.	$60 \div 4 = 15$ One group of students will have $1 \times 15 = 15$ students.
3 Work out how many students are in the other group by multiplying the number of parts (3) by the number of students in one group (15).	The other group will have $3 \times 15 = 45$ students.
4 Check this gives a total of 60 students and write your answer.	$15 + 45 = 60$ The two groups will have 15 and 45 students.

b 1 Add up the total number of parts. (Remember that a 1 : 5 ratio means that there is 1 part for every 5 parts.)	The total number of parts is $1 + 5 = 6$.
2 Divide the number of students (60) by the number of parts (6) to give the number of students in one group.	$60 \div 6 = 10$ One group of students will have $1 \times 10 = 10$ students.
3 Work out how many students in the other group by multiplying the number of parts (5) by the number of students in one group (10).	The other group will have $5 \times 10 = 50$ students.
4 Check this gives a total of 60 students and write your answer.	$10 + 50 = 60$. The two groups will have 10 and 50 students.
c 1 To divide 60 students into classes in the ratio 1 : 2 : 7, first add up the total number of parts.	The total number of parts is $1 + 2 + 7 = 10$.
2 Divide the number of students (60) by the number of parts (10) to give the number of students in one group.	$60 \div 10 = 6$ One group of students will have $1 \times 6 = 6$ students.
3 Work out how many students in the other two groups by multiplying the number of parts (2) and (7) by the number of students in one group (6).	The other groups will have $2 \times 6 = 12$ students and $7 \times 6 = 42$ students.
4 Check that this gives 60 students and write your answer.	$6 + 12 + 42 = 60$ The three groups will have 6, 12 and 42 students.

Now try this 22   **Dividing quantities in given ratios (Example 22)**

Divide $36 in the ratio 1 : 2 : 3.

Hint 1   Add up the total number of parts.

Hint 2   Divide $36 by this total to give the amount of one part.

Hint 3   Multiply this amount by each quantity in the ratio.

Section Summary

► To divide a quantity in a given ratio **a** : **b**, divide the quantity by the sum of **a** and **b**. This gives the allocation of one part. Multiply this one part value by **a** and then by **b** to give the required allocations.

Exercise 1E

Building understanding

1 Divide 96 in each of the following ratios.

   a 1:1　　　　　**b** 2:1　　　　　**c** 5:3　　　　　**d** 19:5

Developing understanding

Example 22a, b

2 If a 40 m length of rope is cut in the following ratios, what will be the lengths of the individual pieces of rope?

   a 4 : 1　　　　**b** 1 : 7　　　　**c** 6 : 2　　　　**d** 4 : 4

Example 22c

3 If a sum of $500 were shared among a group of people in the following ratios, how much would each person receive?

   a 6 : 4　　**b** 1 : 4 : 5　　**c** 1 : 8 : 1　　**d** 8 : 9 : 8　　**e** 10 : 5 : 4 : 1

4 A basket contains bananas, mangos and pineapples in the ratio 10 : 1 : 4. If there are 20 pineapples in the basket, calculate:

   a the number of bananas

   b the number of mangos

   c the total number of pieces of fruit in the basket.

5 7.5 litres of a mixed cordial drink is required for a children's party. If the ratio of cordial to water is 1 : 4,

   a how many litres of cordial are required?

   b how many litres of water are required?

6 The scale on a map is 1 : 20 000 (in cm). If the measured distance on the map between two historical markers is 15 centimetres, what is the actual distance between the two markers in kilometres?

Testing understanding

7 Parents divide their lottery winnings of $3 600 000 in the ratio 4 : 3 : 3 amongst their daughter and two sons. How much does each receive?

1F Unitary method

Ratios can be used to calculate unit prices, i.e. the price of one item. This method is known as the *unitary method* and can be used to solve a range of ratio problems.

Example 23   Using the unitary method

If 24 golf balls cost $86.40, how much do 7 golf balls cost?

Explanation	Solution
1 Find the cost of one golf ball by dividing $86.40 (the total cost) by 24 (the number of golf balls).	$86.40 \div 24 = \$3.60$
2 Multiply the cost of one golf ball ($3.60) by 7.	$\$3.60 \times 7 = \$25.20$
3 Write your answer.	7 golf balls cost $25.20.

Now try this 23   Using the unitary method (Example 23)

If 8 pens cost $32, how much do 15 pens cost?

Hint 1 Divide $32 by 8 to find the cost of one pen.
Hint 2 Multiply this cost by 15.

Section Summary

The unitary method first identifies the total number of parts involved, then determines the allocation to one part by division and then finds the allocation to a greater number of parts by multiplication.

For example: If 15 items cost $45, then one item costs $45 \div 15 = $3, and 3 items cost $9.

Exercise 1F

Building understanding

1 a Twelve mangos cost $36.

    i How much does one mango cost?

    ii How much do 11 mangos cost?

    iii How many mangos could I buy for $18?

  b If 14 oranges cost $12, how many oranges can I buy for:

    i $6   **ii** $36   **iii** $48

Developing understanding

Example 23

2 Use the unitary method to answer the following questions.

   a If 12 cakes cost $14.40, how much do 13 cakes cost?

   b If a clock gains 20 seconds in 5 days, how much does the clock gain in three weeks?

   c If 17 textbooks cost $501.50, how much would 30 textbooks cost?

   d If an athlete can run 4.5 kilometres in 18 minutes, how far could she run in 40 minutes at the same pace?

3 If one tin of red paint is mixed with four tins of yellow paint, it produces five tins of orange paint. How many tins of the red and yellow paint would be needed to make 35 tins of the same shade of orange paint?

4 If a train travels 165 kilometres in 1 hour 50 minutes at a constant speed, calculate how far it could travel in:

   a 3 hours

   b $2\frac{1}{2}$ hours

   c 20 minutes

   d 70 minutes

   e 3 hours and 40 minutes

   f $\frac{3}{4}$ hour

5 Ice creams are sold in two different sizes. A 35 g cone costs $1.25 and a 73 g cone costs $2.00. Which is the better buy?

6 A shop sells 2 L containers of Brand A milk for $2.99, 1 L of Brand B milk for $1.95 and 600 mL of Brand C milk for $1.42. Calculate the best buy.

7 You need 6 eggs to bake 2 chocolate cakes. How many eggs will you need to bake 17 chocolate cakes?

8 A car uses 45 litres of petrol to travel 495 kilometres. Under the same driving conditions, calculate:

   a how far the car could travel on 50 litres of petrol

   b how much petrol the car would use to travel 187 kilometres.

Testing understanding

9 If 110% of an amount is $330, how much is 100%?

10 At a certain time, one Australian dollar bought US$0.72. Determine how many Australian dollars you would get for US$180.
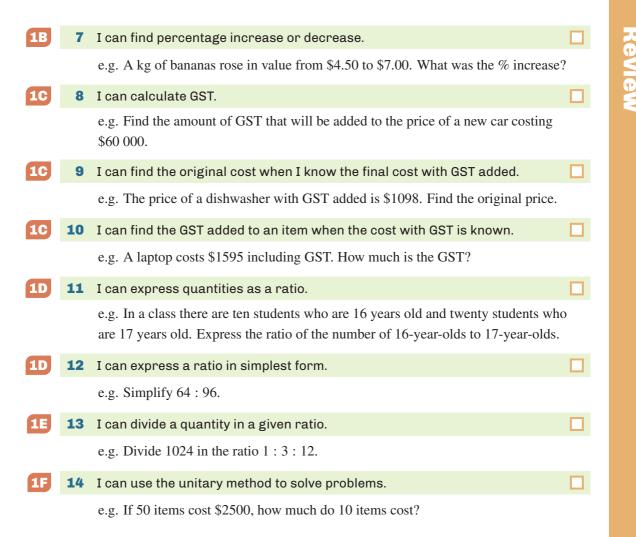
Review

Key ideas and chapter summary

Assignment

Percentages   To convert a fraction or a decimal to a percentage, multiply by 100.

To convert a percentage to a decimal or a fraction, divide by 100.

$$\text{Percentage change} = \frac{\text{change}}{\text{original quantity or price}} \times 100$$

GST   GST stands for goods and services tax and is a tax of 10%.

To find the amount of GST when the cost without GST is known, divide by 10.

To find the amount of GST when the cost with GST is known, divide by 11.

To calculate the cost with GST, multiply the original cost by 1.1.

To calculate the original cost, divide the cost with GST by 1.1.

Ratios   Order of figures in a ratio is important. 4 : 3 is not the same as 3 : 4.

Ratios can be simplified, e.g. 6 : 2 = 3 : 1.

Unitary method   The unitary method first identifies the total number of parts involved, then determines the allocation to one part by division and then finds the allocation to a greater number of parts by multiplication.

Skills checklist

Checklist

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

1A **1** I can convert fractions to percentages. ☐

e.g. What is $\frac{3}{4}$ as a percentage?

1A **2** I can convert decimals to percentages. ☐

e.g. What is 0.67 as a percentage?

1A **3** I can convert percentages to fractions. ☐

e.g. What is 40% as a fraction?

1A **4** I can convert percentages to decimals. ☐

e.g. What is 85% as a decimal?

1A **5** I can find a percentage of a quantity. ☐

e.g. What is 30% of $50?

1B **6** I can increase or decrease a value by a percentage. ☐

e.g. Decrease $150 by 20%.

Review

1B   **7**   I can find percentage increase or decrease.   ☐

e.g. A kg of bananas rose in value from \$4.50 to \$7.00. What was the % increase?

1C   **8**   I can calculate GST.   ☐

e.g. Find the amount of GST that will be added to the price of a new car costing \$60 000.

1C   **9**   I can find the original cost when I know the final cost with GST added.   ☐

e.g. The price of a dishwasher with GST added is \$1098. Find the original price.

1C   **10**   I can find the GST added to an item when the cost with GST is known.   ☐

e.g. A laptop costs \$1595 including GST. How much is the GST?

1D   **11**   I can express quantities as a ratio.   ☐

e.g. In a class there are ten students who are 16 years old and twenty students who are 17 years old. Express the ratio of the number of 16-year-olds to 17-year-olds.

1D   **12**   I can express a ratio in simplest form.   ☐

e.g. Simplify 64 : 96.

1E   **13**   I can divide a quantity in a given ratio.   ☐

e.g. Divide 1024 in the ratio 1 : 3 : 12.

1F   **14**   I can use the unitary method to solve problems.   ☐

e.g. If 50 items cost \$2500, how much do 10 items cost?

Review

Multiple-choice questions

1   56% as a fraction in its simplest form is:

   A 0.56   **B** $\frac{56}{100}$ **C** $\frac{0.56}{100}$ **D** $\frac{14}{25}$ **E** $\frac{28}{50}$

2   15% of $1600 is equal to:

   A $24   **B** $150   **C** $240   **D** $1840   **E** $24 000

3   An item with a cost price of $450 is marked up by 30%. Its selling price is:

   A $585   **B** $135   **C** $480   **D** $1350   **E** $463.50

4   A motorbike's original price is $19 990. The cost with GST added is:

   A $1817.27   **B** $1999   **C** $18 172.72   **D** $20 000   **E** $21 989

5   A box contains 5 green marbles, 7 blue marbles and 3 yellow marbles. The ratio of blue marbles to total marbles is:

   A 7 : 5 : 3   **B** 7 : 8   **C** 7 : 15   **D** 5 : 7 : 3   **E** 5 : 7 : 3 : 15

6   $750 is divided in the ratio 1 : 3 : 2. The smallest share is:

   A $250   **B** $125   **C** $375   **D** $750   **E** $150

7   In simplest ratio form, the ratio of 450 grams to 3 kilograms is:

   A 3 : 20   **B** 450 : 3   **C** 9 : 60   **D** 150 : 1   **E** 15 : 100

Short-answer questions

1   Express the following percentages as decimals.

   a 75%   **b** 40%   **c** 27.5%

2   Express the following percentages as fractions in their lowest terms.

   a 10%   **b** 20%   **c** 22%

3   Evaluate the following.

   a 30% of 80   **b** 15% of $70   **c** $12\frac{1}{2}$% of $106

4   A new smart TV was valued at $1038. During a sale it was discounted by 5%.

   a What was the amount of discount?

   b What was the sale price?

5   Tom's weekly wage of $750 is increased by 15%. What is his new weekly wage?

6   A 15-year-old girl working at a local bakery is paid $12.50 per hour. Her pay will increase to $15 per hour when she turns 16. What will be the percentage increase to her pay (to the nearest whole number)?

7   A leather jacket is reduced from $516 to $278. Calculate the percentage discount (to the nearest whole number).

8   Find the GST that needs to be added to the following items.

   a   a bookcase costing $279

   b   a lawn mower costing $445

9   The item costs below include GST. How much is the GST for each item?

   a   a hedge trimmer for $198

   b   a sewing machine for $2090

10   True or false?

   a   The ratio 3 : 2 is the same as 2 : 3.

   b   1 : 5 = 3 : 12

11   If a sum of $800 were to be shared among a group of people in the following ratios, how much would each person receive?

   a 4 : 6   **b** 1 : 4   **c** 2 : 3 : 5   **d** 2 : 2 : 4

12   A recipe for pizza dough requires 3 parts wholemeal flour for each 4 parts of plain flour. How many cups of wholemeal flour are needed if 24 cups of plain flour are used?

13   The scale on a map is 1 : 1000 (in cm). Find the actual distance (in metres) between two markers if the distance between the two markers on a map is:

   a 2.7 cm   **b** 140 mm

14   If 5 kilograms of mincemeat costs $50, how much does 2 kilograms cost?

15   A truck uses 12 litres of petrol to travel 86 km. How far will it travel on 42 litres?