Chapter 5

Linear relations and modelling

Chapter questions

- ▶ How do we use a formula?
- ▶ How do we create a table of values with and without a CAS calculator?
- ▶ How do we solve linear equations?
- ▶ How do we develop a formula?
- ▶ How do we determine the slope of a straight-line graph?
- ▶ How do we find the equation of a straight-line graph?
- ▶ How do we sketch a straight-line graph from its equation?
- ▶ How do we use straight-line graphs to model practical situations?
- ▶ How do we find the intersection of two linear graphs?
- ▶ What are simultaneous equations and how do we solve them?
- ▶ How can we use simultaneous equations to solve practical problems?
- ▶ What are piecewise linear graphs?
- ▶ What are step graphs?

Linear relations and equations connect two or more variables, and they produce a straight line when graphed. Many everyday situations can be described and investigated using a linear graph and its equation. Examples occur in technology, science and business, such as the depreciating value of a newly purchased car or the short-term growth of a newly planted tree. In this chapter, the properties of linear graphs and their equations will be applied to modelling linear growth and decay in the real world.

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5A Substitution of values into a formula and constructing a table of values

Learning intentions

- To be able to solve practical problems by substituting values into formulas.
- To be able to construct a table of values.

Substituting values into a formula

A **formula** is a mathematical relationship connecting two or more variables.

For example:

- $C = 45t + 150$ is a formula for relating the cost, C dollars, of hiring a plumber for t hours. C and t are the variables.
- $P = 4L$ is a formula for finding the perimeter of a square, where P is the perimeter and L is the side length of the square. P and L are the variables.

By substituting all known variables into a formula, we are able to find the value of an unknown variable.

Example 1 Using a formula

The cost of hiring a windsurfer is given by the rule:

$$C = 10 + 40t$$

where C is the cost, in dollars, and t is the time, in hours. How much will it cost to hire a windsurfer for 2 hours?

Explanation	Solution
1 Write the formula.	$C = 10 + 40t$
2 To determine the cost of hiring a windsurfer for 2 hours, substitute $t = 2$ into the formula.	$C = 10 + 40 \times 2$
3 Evaluate.	$C = 90$
4 Write your answer.	It will cost \$90 for a 2-hour hire.

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Now try this 1 Using a formula (Example 1)

The cost of hiring a bobcat is:

$$C = 330 + 80t$$

where C is the cost, in dollars, and t is the time, in hours. How much will it cost to hire a bobcat for 10 hours?

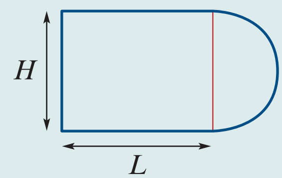
Hint 1 Substitute the value for t into the formula.

Hint 2 Remember that the answer needs to be given in \$.

Example 2 Using a formula

The perimeter of this shape can be given by the formula:

$$P = 2L + H\left(1 + \frac{\pi}{2}\right)$$



In this formula, L is the length of the rectangle and H is the height. Find the perimeter, to one decimal place, if $L = 16.1$ cm and $H = 3.2$ cm.

Note: π is the ratio of the circumference of any circle to its diameter. It is an *irrational* number. An approximate value of π is 3.14159, to 5 decimal places. Calculators have a special key for π.

Explanation	Solution
1 Write the formula.	$P = 2L + H\left(1 + \frac{\pi}{2}\right)$
2 Substitute values for L and H into the formula.	$P = 2 \times 16.1 + 3.2\left(1 + \frac{\pi}{2}\right)$
3 Evaluate.	$P = 40.4$ (to one decimal place)
4 Give your answer with correct units.	The perimeter is 40.4 cm.

Now try this 2 Using a formula (Example 2)

The perimeter of a rectangle can be given by the formula:

$$\mathrm{P} = 2L + 2W$$

In this formula, L is the length of the rectangle and W is the width. Find the perimeter if $L = 26.5$ cm and $W = 14.8$ cm.

Hint 1 Substitute the values for L and W into the formula.

Hint 2 Remember to write the answer with correct units.

Constructing a table of values

We can use a formula to construct a **table of values**. This can be done by substitution (by hand) or using your TI-Nspire or ClassPad.

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Example 3 Constructing a table of values (by hand)

The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F, using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Explanation

Draw up a table of values for:

$F = \frac{9}{5}C + 32$, and then substitute values of $C = 0, 10, 20, 30, \ldots$ into the formula to find F.

Solution

If $C = 0$, $F = \frac{9}{5}(0) + 32 = 32$

If $C = 10$, $F = \frac{9}{5}(10) + 32 = 50$

and so on.

The table would then look as follows:

C	0	10	20	30	40	50	60	70	80	90	100
F	32	50	68	86	104	122	140	158	176	194	212

Now try this 3 Constructing a table of values (by hand) (Example 3)

The perimeter of a square of side length L is given by the formula:

$$P = 4L$$

Use this formula to construct a table of values for P, using values of L in intervals of 10 between $L = 0$ and $L = 100$.

How to construct a table of values using the TI-Nspire

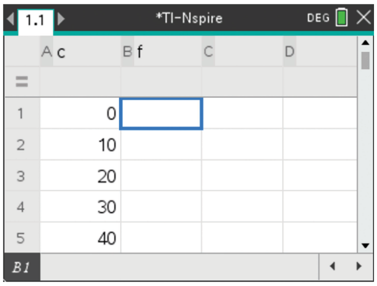
The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F, using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Steps

1. Start a new document: **Press** ctrl + N.
2. Select **Add Lists & Spreadsheet**. **Name** the lists c (for Celsius) and f (for Fahrenheit). Enter the data 0–100 in intervals of 10 into a list named c, as shown.



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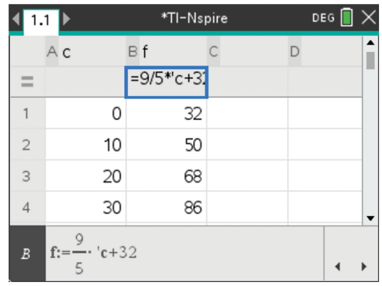
3 Place the cursor in the formula cell of column B (i.e. list f) and type in: **= 9 ÷ 5 × c + 32**.

Hint: If you typed in c you will need to select **Variable Reference** when prompted. This prompt occurs because c can also be a column name. Alternatively, pressing the var key and selecting **c** from the variable list will avoid this issue.

Press enter to display the values given.

Use the ▼ arrow to move down through the table.



How to construct a table of values using the ClassPad

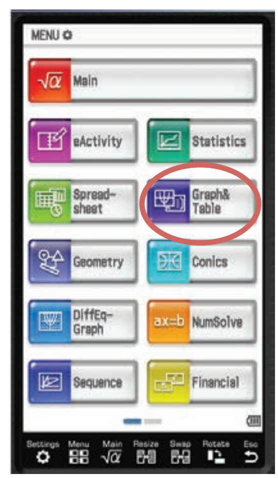
The formula for converting degrees Celsius to degrees Fahrenheit is given by:

$$F = \frac{9}{5}C + 32$$

Use this formula to construct a table of values for F, using values of C in intervals of 10 between $C = 0$ and $C = 100$.

Steps

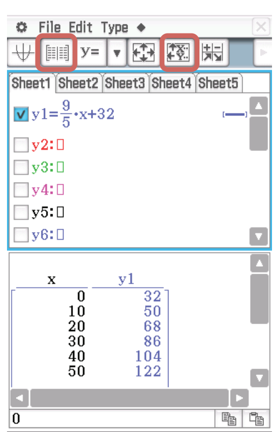
1 Enter the data into your calculator using the **Graph & Table** application. From the application menu screen, locate the built-in **Graph & Table** application, . Tap to open. Tapping Menu from the icon panel (just below the touch screen) will display the application menu if it is not already visible.

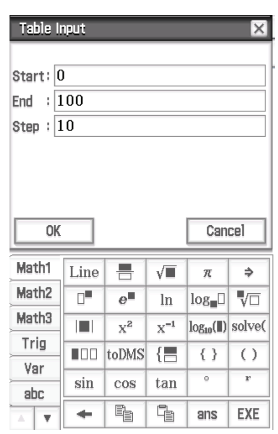


2 **a** Adjacent to **y1=** type in the formula $\frac{9}{5}x + 32$. Then press EXE.

b Tap the **Table Input** ( ) icon to set the table entries as shown and tap OK.

c Tap the icon to display the required table of values. Scrolling down will show more values in the table.





Note: $y1$ is used to represent the variable F and x is used to represent the variable C.

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Section Summary

- A **formula** is a mathematical relationship connecting two or more variables.
- A **table of values** can be found by substitution (by hand) or using a TI-Nspire or ClassPad.

Exercise 5A

Building understanding

Example 1

1 The cost of hiring a removalist has a fixed fee of \$300 and is given by the rule:

$$C = 300 + 120t$$

where C is the total cost in dollars and t is the number of hours for which the removalist is hired.

a Substitute $t = 1$ into the rule to find the cost of hiring the removalist for one hour.

b Substitute $t = 2$ into the rule to find the cost of hiring the removalist for two hours.

Example 2

2 The perimeter of a rectangle is given by the formula:

$$P = 2L + 2W$$

In this formula, L is the length of the rectangle and W is the width.

a Substitute $L = 4$ and $W = 2$ to find the perimeter of a rectangle with length 4 cm and width 2 cm.

b Substitute $L = 5.8$ and $W = 3.5$ to find the perimeter of a rectangle with length 5.8 cm and width 3.5 cm.

c What values for L and W would you substitute in to find the perimeter of a rectangle with length 7.9 cm and width 2.7 cm?

Example 3

3 A football club wishes to purchase a number of pies at a cost of \$3 each. How much does it cost for:

a 43 pies? **b** 44 pies?

c If C is the cost (\$) and x is the number of pies, complete the table by putting your answers to part **a** and **b** into the table. Then find the unknown values, showing the amount of money needed to purchase from 40 to 45 pies.

x	40	41	42	43	44	45
C(\$)						

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Developing understanding

4 The cost of hiring a dance hall is given by the rule:

$$C = 1200 + 50t$$

where C is the total cost, in dollars, and t is the number of hours for which the hall is hired.
Find the cost of hiring the hall for:

a 4 hours **b** 6 hours **c** 4.5 hours

5 The distance, d km, travelled by a car in t hours at an average speed of v km/h is given by the formula: $d = v \times t$.
Find the distance travelled by a car travelling at a speed of 95 km/h for 4 hours.

6 Taxi fares are calculated using the formula:

$$F = 4 + 1.3K$$

where K is the distance travelled, in kilometres, and F is the cost of the fare in dollars.
Find the costs of the following trips.

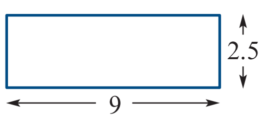
a 5 km **b** 8 km **c** 20 km

7 The circumference, C, of a circle with a radius, r, is given by $C = 2\pi r$.
Find, to two decimal places, the circumferences of the circles with the following radii.

a An earring with $r = 3$ mm **b** A circular garden bed with $r = 7.2$ m

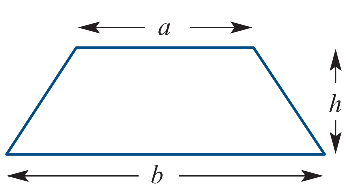
8 If the area of a rectangle is given by $A = L \times W$, find the value of A for the following rectangles.

a $L = 3$ and $W = 4$ **b** $L = 15$ and $W = 8$ **c**



9 The area of a trapezium, as shown, is $A = \frac{1}{2}h(a + b)$.
Find A if:

a $h = 1, a = 3, b = 5$
b $h = 5, a = 2.5, b = 3.2$



10 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is:

$$C = \frac{5}{9}(F - 32)$$

Use this formula to convert the following temperatures to degrees Celsius. Give your answers to one decimal place.

a 50°F **b** 0°F **c** 212°F **d** 92°F

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11 The formula for calculating simple interest is:

$$I = \frac{PRT}{100}$$

where P is the principal (amount invested or borrowed), R is the interest rate per annum and T is the time (in years). In the following questions, give your answers to the nearest cent (to two decimal places).

a Frank borrows \$5000 at 12% for 4 years. How much interest will he pay?

b Henry invests \$8500 for 3 years with an interest rate of 7.9%. How much interest will he earn?

12 In Australian football, a goal is worth 6 points and a behind is worth 1 point. The total number of points, P, is given by:

$$P = 6 \times \text{number of goals} + \text{number of behinds}$$

a Find the number of points if:

i 2 goals and 3 behinds are kicked **ii** 8 goals and 20 behinds are kicked.

b In a match, Redteam scores 4 goals and 2 behinds and Greenteam scores 3 goals and 10 behinds. Which team wins the match?

13 The circumference of a circle is given by:

$$C = 2\pi r$$

where r is the radius. Complete the table of values to show the circumferences of circles with radii from 0 to 1 cm in intervals of 0.1 cm. Give your answers to three decimal places.

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C (cm)	0	0.628	1.257	1.885							

14 A phone bill is calculated using the formula:

$$C = 40 + 0.18n$$

where C is the total cost and n represents the number of calls made. Complete the table of values to show the cost for 50, 60, 70, . . . 130 calls.

n	50	60	70	80	90	100	110	120	130
C(\$)	49	50.80	52.60						

15 The amount of energy (E) in kilojoules expended by an adult male of mass (M), at rest, can be estimated using the formula:

$$E = 110 + 9M$$

Complete the table of values in intervals of 5 kg for males of mass 60–120 kg to show the corresponding values of E.

M (kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
E (kJ)	650	695											

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16 Anita has \$10 000 that she wishes to invest at a rate of 4.5% per annum. She wants to know how much interest she will earn after 1, 2, 3, ...10 years. Using the formula:

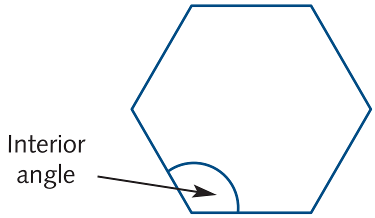
$$I = \frac{PRT}{100}$$

where P is the principal and R is the interest rate (%), construct a table of values, and use a calculator to find how much interest, I, she will have after $T = 1, 2, \ldots 10$ years.

17 The sum, S, of the interior angles of a polygon with n sides is given by the formula:

$$S = 90(2n - 4)$$

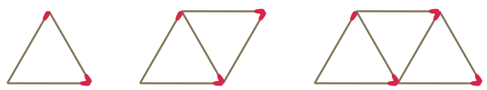
Construct a table of values showing the sum of the interior angles of polygons with 3 to 10 sides.



n	3	4	5					
S	180°	360°						

Testing understanding

18 The number of matchsticks used for each shape below follows the pattern 3, 5, 7, . . .



The rule for finding the number of matches used in this sequence is:

number of matches $= a + (n - 1)d$

where a is the number of matches in the first shape ($a = 3$), d is the number of extra matches used for each shape ($d = 2$) and n is the shape number.
Find the number of matches in the:

a 6th shape (so, $n = 6$) **b** 11th shape **c** 50th shape

19 Suggested cooking times for roasting x kilograms of different types of meat are given in the table.

Meat type	Minutes/kilogram
Chicken (well done)	45 min/kg + 20 mins
Lamb (medium)	55 min/kg + 25 mins
Lamb (well done)	65 min/kg + 30 mins
Beef (medium)	55 min/kg + 20 mins
Beef (well done)	65 min/kg + 30 mins

a How long, to the nearest minute, will it take to cook:

i a 2 kg chicken? **ii** 2.25 kg of beef (well done)?
iii a piece of lamb weighing 2.4 kg (well done)? **iv** 2.5 kg of beef (medium)?

b At what time should you put a 2 kg leg of lamb into the oven to have served medium at 7:30 p.m.?

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5B Solving linear equations and developing formulas

Learning intentions

- To be able to solve linear equations with one unknown.
- To be able to set up linear equations.

Solving linear equations

Practical applications of mathematics often involve the need to be able to solve **linear equations**. An equation is a mathematical statement that says that two things are equal. For example, these are all equations:

$x - 3 = 5$ $\quad 2w - 5 = 17$ $\quad 3m = 24$

Linear equations come in many different forms in mathematics but are easy to recognise because the powers on the unknown values are always 1. For example:

- $m - 4 = 8$ is a linear equation, with unknown value m
- $3x = 18$ is a linear equation, with unknown value x
- $4y - 3 = 17$ is a linear equation, with unknown value y
- $a + b = 0$ is a linear equation, with two unknown values, a and b
- $x^2 + 3 = 9$ is *not* a linear equation (the power of x is 2, not 1), with unknown value x
- $c = 16 - d^2$ is *not* a linear equation (the power of d is 2), with two unknowns, c and d.

The process of finding the unknown value is called solving the equation. When solving an equation, opposite (or inverse) operations are used so that the unknown value to be solved is the only term remaining on one side of the equation. Opposite operations are indicated in the table below.

Operation	$+$	$-$	$\times$	$\div$
Opposite operation	$-$	$+$	$\div$	$\times$

Remember: The equation must remain balanced. To balance an equation, add or subtract the same number to or from both sides of the equation or multiply or divide both sides of the equation by the same number.

Example 4 Solving a linear equation by hand

Solve the following linear equations:

a $x + 6 = 10$ **b** $3y = 18$ **c** $4(x - 3) = 24$

Explanation	Solution
a The equation needs to be 'undone,' leaving the unknown value by itself on one side of the equation.	
1 Write the equation.	$x + 6 = 10$

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2 Subtract 6 from both sides of the equation. This is the opposite process to adding 6.

$$x + 6 - 6 = 10 - 6$$
$$\therefore x = 4$$

3 Check your answer by substituting the found value for x into the original equation. If each side gives the same value, the solution is correct.

$$\begin{aligned} \text{LHS} &= x + 6 \\ &= 4 + 6 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

b **1** Write the equation.

$$3y = 18$$

2 The opposite process of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.

$$\frac{3y}{3} = \frac{18}{3}$$
$$\therefore y = 6$$

3 Check that the solution is correct by substituting $y = 6$ into the original equation.

$$\begin{aligned} \text{LHS} &= 3y \\ &= 3 \times 6 \\ &= 18 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

c **Method 1**

1 Write the equation.

$$4(x - 3) = 24$$

2 Expand the brackets.

$$4x - 12 = 24$$

3 Add 12 to both sides of the equation.

$$4x - 12 + 12 = 24 + 12$$
$$4x = 36$$

4 Divide both sides by 4.

$$\frac{4x}{4} = \frac{36}{4}$$
$$\therefore x = 9$$

5 Check that the solution is correct by substituting $x = 9$ into the original equation (see 4 below).

Method 2

1 Write the equation.

$$4(x - 3) = 24$$

2 Divide both sides by 4.

$$\frac{4(x - 3)}{4} = \frac{24}{4}$$
$$x - 3 = 6$$

3 Add 3 to both sides of the equation.

$$x - 3 + 3 = 6 + 3$$
$$\therefore x = 9$$

4 Check that the solution is correct by substituting $x = 9$ into the orginal equation.

$$\begin{aligned} \text{LHS} &= 4(x - 3) \\ &= 4(9 - 3) \\ &= 4 \times 6 = 24 = \text{RHS} \end{aligned}$$

$\therefore$ Solution is correct.

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Now try this 4 Solving a linear equation by hand (Example 4)

Solve the equation $2x - 6 = 10$.

Hint 1 Add 6 to both sides of the equation.

Hint 2 Divide by 2.

Note: All of the above linear equations can be solved using the **solve(** command on a CAS calculator.

Example 5 Solving a linear equation using a CAS calculator

Solve the equation $-4 - 5b = 8$.

Explanation

1 Use the **solve(** command on your CAS calculator to solve for b, as shown opposite.

Note: 1. Set the mode of your calculator to Approximate (or press (ctrl + enter)) (TI-Nspire) or Decimal (ClassPad) before using **solve(**.
2. Ensure that you use the variable b on CAS.

Solution

solve $(-4 - 5b = 8, b)$ $\quad b = -2.4$

Now try this 5 Solving a linear equation using a CAS calculator (Example 5)

Solve the equation $-5 - 2x = 15$.

Hint 1 Use the **solve(** function on a CAS calculator.

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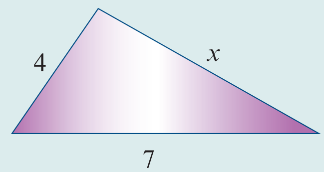
Developing a formula: setting up linear equations with one unknown

In many practical problems, we often need to set up a linear equation before finding the solution to a problem. Some practical examples are given below, showing how a linear equation is set up and then solved.

Example 6 Setting up a linear equation

Find an equation for the perimeter of this triangle.

Note: Perimeter is the distance around the outside of a shape.



Explanation

1. Choose a variable to represent the perimeter.
2. Add up all the sides of the triangle and let them equal the perimeter, P.
3. Write your answer.

Solution

Let P be the perimeter.

$P = 4 + 7 + x$

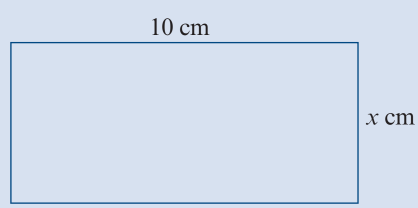
$P = 11 + x$

The required equation is:

$P = 11 + x$

Now try this 6 Setting up a linear equation (Example 6)

Find an equation for the perimeter of this rectangle.



Hint 1 Choose a variable to represent the perimeter.

Hint 2 Add up all the sides of the rectangle and let them equal the perimeter.

Hint 3 $x + x = 2x$

Example 7 Setting up and solving a linear equation

If 11 is added to a certain number, the result is 25. Find the number.

Explanation

1. Choose a variable to represent the number.
2. Using the information, write an equation.
3. Solve the equation by subtracting 11 from both sides of the equation.
4. Write your answer.

Solution

Let n be the number.

$n + 11 = 25$

$n + 11 - 11 = 25 - 11$

$\therefore n = 14$

The required number is 14.

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Now try this 7 **Setting up and solving a linear equation (Example 7)**

If 7 is added to a certain number, the result is 49. Find the number.

Hint 1 Choose a variable to represent the number.
Hint 2 Use the information to write an equation.
Hint 3 Solve the equation.

Example 8 **Setting up and solving a linear equation**

A car rental company has a fixed charge of \$110 plus \$84 per day for the hire of a car. The Brown family have budgeted \$650 for the hire of a car during their family holiday. For how many days can they hire a car?

Explanation	Solution
1 Choose a variable (d) for the number of days that the car is hired for. Use the information to write an equation.	Let d be the number of days that the car is hired for. $110 + 84d = 650$
2 Solve the equation. First, subtract 110 from both sides of the equation. Then divide both sides of the equation by 84.	$110 + 84d - 110 = 650 - 110$ $84d = 540$ $\frac{84d}{84} = \frac{540}{84}$ $\therefore d = 6.428$
3 Write your answer in terms of complete days. Car hire works on a daily rate, so 6.428 days is not an option. We therefore round down to 6 days to ensure that the Brown family stays within their budget of \$650.	The Brown family could hire a car for 6 days.

Now try this 8 **Setting up and solving a linear equation (Example 8)**

A 12-seater van can be hired for a fixed charge of \$56 plus \$80 per day. Chris and his friends have budgeted \$500 for the hire of a 12-seater van during their holiday. For how many days can they hire a 12-seater van?

Hint 1 Choose a variable for the number of days that the van is hired for.
Hint 2 Write an equation for the cost, using the given information.
Hint 3 Solve the equation.
Hint 4 Give your answer in whole days.

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Section Summary

- A **linear equation** is an equation whose unknown values are always to the power of one.
- To **solve** a linear equation means to find the value of the unknown variable.
- **Linear equations** can be solved by hand or by using a CAS calculator.
- **Linear equations** can be used to solve practical problems.

Skill-sheet

Exercise 5B

Building understanding

Example 4

1 Solve the following linear equations.

a $x + 6 = 15$ **b** $y + 11 = 26$ **c** $m - 5 = 1$
d $m - 5 = -7$ **e** $6 + e = 9$ **f** $-n + 5 = 1$

2 Solve the following linear equations.

a $5x = 15$ **b** $3g = 27$ **c** $6j = -24$
d $\frac{r}{3} = 4$ **e** $\frac{t}{-2} = 6$ **f** $\frac{h}{-8} = -5$

3 For the linear equation, $2a + 15 = 27$:

a What is the first step in solving this equation?
b What is the second step?
c What is the solution to this equation?

4 For the linear equation, $\frac{y}{4} - 10 = 0$:

a What is the first step in solving this equation?
b What is the second step?
c What is the solution to this equation?

Developing understanding

5 Solve the following linear equations.

a $v + 7 = 2$ **b** $9 - k = 2$ **c** $3 - a = -5$
d $-5b = -25$ **e** $13 = 3r - 11$ **f** $\frac{x + 1}{3} = 2$

6 Solve the following linear equations. (You do not necessarily have to multiply brackets out first.)

a $2(y - 1) = 6$ **b** $8(x - 4) = 56$ **c** $3(g + 2) = 12$
d $3(4x - 5) = 21$ **e** $8(2x + 1) = 16$ **f** $3(5m - 2) = 12$
g $\frac{2(a - 3)}{5} = 6$ **h** $\frac{4(r + 2)}{6} = 10$

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7 Solve these equations by first ensuring that the unknown variable is on one side of the equation.

a $2x = x + 5$ **b** $2a + 1 = a + 4$ **c** $4b - 10 = 2b + 8$
d $7 - 5y = 3y - 17$ **e** $3(x + 5) - 4 = x + 11$ **f** $6(c + 2) = 2(c - 2)$
g $2f + 3 = 2 - 3(f + 3)$ **h** $5(1 - 3y) - 2(10 - y) = -10y$

Example 5

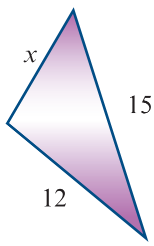
8 Solve the following linear equations using a CAS calculator. Give answers to one decimal place where appropriate.

a $3a + 5 = 11$ **b** $4b + 3 = 27$ **c** $2w + 5 = 9$
d $7c - 2 = 12$ **e** $3y - 5 = 16$ **f** $4f - 1 = 7$
g $3 + 2h = 13$ **h** $2 + 3k = 6$ **i** $-4(g - 4) = -18$
j $\dfrac{2(s-6)}{7} = 4$ **k** $\dfrac{5(t+1)}{2} = 8$ **l** $\dfrac{-4(y-5)}{5} = 2.4$
m $2(x - 3) + 4(x + 7) = 10$ **n** $5(g + 4) - 6(g - 7) = 25$ **o** $5(p + 4) = 25 + (7 - p)$

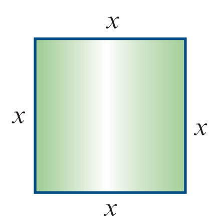
Example 6

9 Find an expression for the perimeter, P, for each of the following shapes.

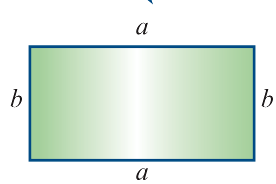
a



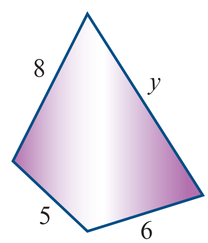
b



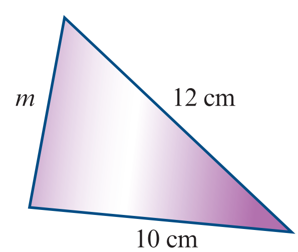
c



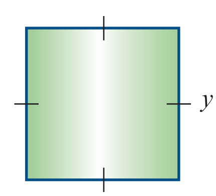
d



10 **a** Write an expression for the perimeter of this triangle.
b If the perimeter, P, of the triangle is 30 cm, solve the equation to find the value of m.



11 **a** Write an equation for the perimeter of this square.
b If the perimeter is 52 cm, what is the length of one side?



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Example 7 **12** Seven is added to a number and the result is 15.

a Write an equation, using n to represent the number.

b Solve the equation for n.

13 Five is added to twice a number and the result is 17. What is the number?

14 When a number is doubled and 15 is subtracted, the result is 103. Find the number.

15 A bicycle hire company charges a fixed cost of \$20 plus \$15 per hour of hire.

a Define a variable to use for the time. Remember to specify the units.

b Write a formula showing the cost, C, of hiring a bicycle, using your variable for time from part **a**.

c Find the cost of hiring a bicycle for:

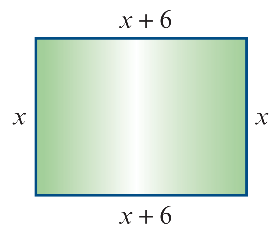
i 2 hours **ii** 5 hours

16 The perimeter of a rectangle is 84 cm. The length of the rectangle is 6 cm longer than the width, as shown in the diagram.

a Write an expression for the perimeter, P, of the rectangle.

b Find the value of x.

c Find the lengths of the sides of the rectangle.



Example 8 **17** Year 11 students want to run a social. The cost of hiring a band is \$820 and they are selling tickets at \$15 per person. The profit, P, is found by subtracting the band hire cost from the money raised from selling tickets. The students would like to make a profit of \$350. Use the information to write an equation, and then solve the equation to find how many tickets they need to sell.

Testing understanding

18 The cost for printing cards at the Stamping Printing Company is \$60 plus \$2.50 per card. Kate paid \$122.50 to print invitations for her party. How many invitations were printed?

19 A raffle prize of \$1000 is divided between Anne and Barry, so that Anne receives 3 times as much as Barry. How much does each person receive?

20 Bruce cycles x kilometres then walks half as far as he cycles. If the total distance covered is 45 km, find the value of x.

21 Amy and Ben live 17.2 km apart. They cycle to meet each other. Ben travels at 12 km/h and Amy travels at 10 km/h.

a How long (to the nearest minute) until they meet each other?

b What distances, to one decimal place, have they both travelled?

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5C Developing a formula: setting up and solving an equation in two unknowns

Learning intentions

- To be able to set up and solve linear equations with two unknowns.

It is often necessary to develop formulas so that problems can be solved. Constructing a formula is similar to developing an equation from a description.

Example 9 Setting up and solving a linear equation in two unknowns

Sausage rolls cost \$1.30 each and party pies cost 75 cents each.

a Construct a formula for finding the cost, C dollars, of buying x sausage rolls and y party pies.

b Find the cost of 12 sausage rolls and 24 party pies.

Explanation	Solution
a 1 Work out a formula using x. One sausage roll costs \$1.30. Two sausage rolls cost $2 \times \$1.30 = \$2.60$. Three sausage rolls cost $3 \times \$1.30 = \$3.90$ *etc*. Write a formula using x.	x sausage rolls cost $x \times 1.30 = 1.3x$
2 Work out a formula using y. One party pie costs \$0.75. Two party pies cost $2 \times \$0.75 = \$1.50$ Three party pies cost $3 \times \$0.75 = \$2.25$ *etc*. Write a formula using y.	y party pies cost $y \times 0.75 = 0.75y$
3 Combine to get a formula for total cost, C.	$C = 1.3x + 0.75y$
b 1 Write the formula for C.	$C = 1.3x + 0.75y$
2 Substitute $x = 12$ and $y = 24$ into the formula.	$C = 1.3 \times 12 + 0.75 \times 24$
3 Evaluate.	$C = 33.6$
4 Give your answer in dollars and cents.	The total cost for 12 sausage rolls and 24 party pies is \$33.60.

Now try this 9 Setting up and solving a linear equation in two unknowns (Example 9)

Lemon tarts cost \$3.50 each and apple crumble tarts cost \$4.75 each.

a Construct a formula for finding the cost, C dollars, of buying x lemon tarts and y apple crumble tarts.

b Find the cost of 10 lemon tarts and 15 apple crumble tarts.

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Hint 1 Write out a rule for the cost of x lemon tarts.
Hint 2 Write out a rule for the cost of y apple crumble tarts.
Hint 3 Combine the two rules to give the total cost.
Hint 4 Substitute values for x and y in the formula.

Section Summary

- Practical problems can be solved by developing **formulas**.

Skill-sheet

Exercise 5C

Building understanding

Example 9

1 At a French patisserie, a baguette costs \$4.50 and a large family quiche costs \$14.30.
  a State the cost of buying x baguettes.
  b State the cost of buying y family quiches.
  c Construct a formula for the cost, \$$C$, of x baguettes and y family quiches.

2 The cost, \$$C$, of buying x muffins and y cookies is $C = 4x + 2.5y$.
  a How much does it cost to buy a single muffin?
  b How much does it cost to buy one cookie?
  c How much does it cost to buy 8 muffins and 4 cookies?

Developing understanding

3 Balloons cost 50 cents each and streamers costs 20 cents each.
  a Construct a formula for the cost, \$$C$, of x balloons and y streamers.
  b Find the cost of 25 balloons and 20 streamers.

4 Tickets to a concert cost \$40 for adults and \$25 for children.
  a Construct a formula for the total amount, \$$C$, paid by x adults and y children.
  b How much money altogether was paid by 150 adults and 315 children?

5 At the football canteen, chocolate bars cost \$1.60 and muesli bars cost \$1.40.
  a Construct a formula to show the total money, \$$C$, made by selling x chocolate bars and y muesli bars.
  b How much money would be made if 55 chocolate bars and 38 muesli bars were sold?

6 At the bread shop, custard tarts cost \$1.75 and iced doughnuts \$0.70 cents.
  a Construct a formula to show the total cost, \$$C$, if x custard tarts and y iced doughnuts are purchased.
  b On Monday morning, Mary bought 25 custard tarts and 12 iced doughnuts. How much did it cost her?

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7 At the beach cafe, Marion takes orders for coffee and milkshakes. A cup of coffee costs \$3.50 and a milkshake costs \$5.00.

a Let x = number of coffees ordered and y = number of milkshakes ordered. Using x (coffee) and y (milkshakes), write a formula showing the cost, \$$C$, of the number of coffees and milkshakes ordered.

b Marion took orders for 52 cups of coffee and 26 milkshakes. How much money did this make?

8 Joe sells budgerigars for \$30 and parrots for \$60.

a Write a formula showing the money, \$$C$, made by selling x budgerigars and y parrots.

b Joe sold 28 parrots and 60 budgerigars. How much money did he make?

Testing understanding

9 James has been saving fifty-cent and twenty-cent pieces.

a If James has x fifty-cent pieces and y twenty-cent pieces, write a formula to show the number, N, of coins that James has.

b Write a formula to show the value, V dollars, of James's collection.

c When James counts his coins, he has 45 fifty-cent pieces and 77 twenty-cent pieces. How much money does he have in total?

5D Drawing straight-line graphs and finding their slope

Learning intentions

- To be able to draw a straight-line graph.
- To be able to find the slope of a straight line.

Plotting straight-line graphs

Relations defined by equations such as:

$$y = 1 + 2x \qquad y = 3x - 2 \qquad y = 10 - 5x \qquad y = 6x$$

are linear relations, and they generate straight-line graphs.

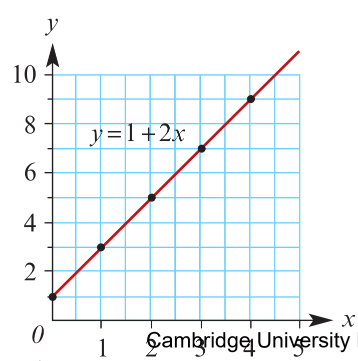
For example, consider the relation $y = 1 + 2x$. To plot this graph, we can form a table.

x	0	1	2	3	4
y	1	3	5	7	9

We can then plot the values from the table on a set of axes, as shown opposite.

The points appear to lie on a straight line.

A ruler can then be used to draw in this straight line to give the graph of $y = 1 + 2x$.



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Example 10 Constructing a graph from a table of values

Plot the graph of $y = 8 - 2x$ by forming a table of values of y, using $x = 0, 1, 2, 3, 4$.

Explanation

1 Set up the table of values.
When $x = 0$, $y = 8 - 2 \times 0 = 8$.
When $x = 1$, $y = 8 - 2 \times 1 = 6$,
and so on.

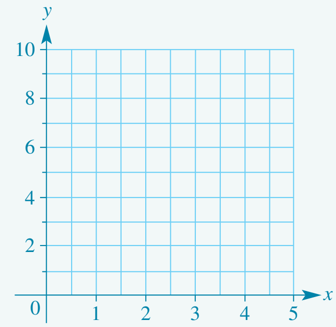
2 Draw, label and scale a set of axes to cover all values.

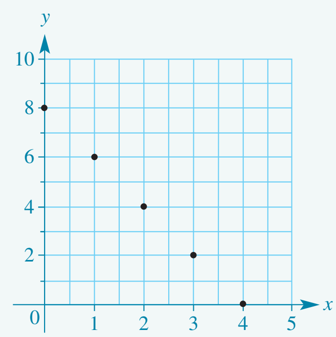
3 Plot the values in the table on the graph by marking with a dot (●). The first point is (0, 8). The second point is (1, 6), and so on.

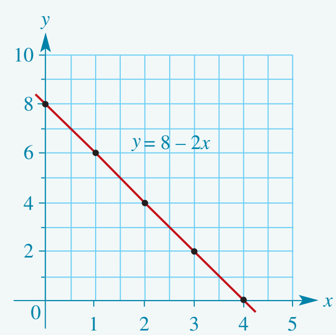
4 The points appear to lie on a straight line. Use a ruler to draw in the straight line. Label the line $y = 8 - 2x$.

Solution

x	0	1	2	3	4
y	8	6	4	2	0







Now try this 10 Constructing a graph from a table of values (Example 10)

Plot the graph of $y = 9 - 3x$ by forming a table of values of y, using $x = 0, 1, 2, 3$.

Hint 1 Draw up a table of values.

Hint 2 Plot points on a scaled set of axes and connect with a ruler.

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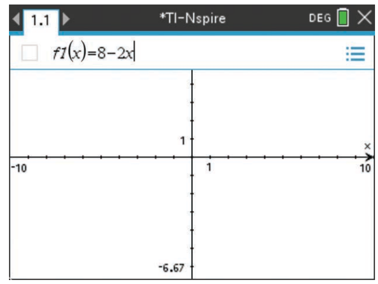
How to draw a straight-line graph and show a table of values using the TI-Nspire

Use a CAS calculator to draw the graph $y = 8 - 2x$ and show a table of values.

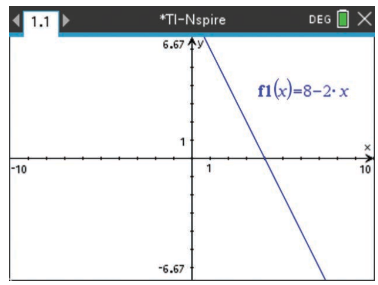
Steps

1 Start a new document (ctrl + N) and select **Add Graphs**.

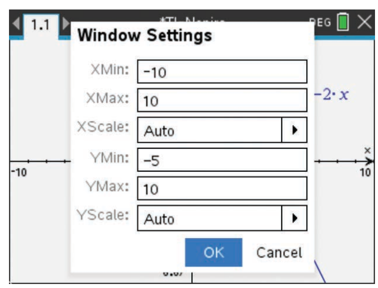


2 Type in the equation as shown. Note that f1(x) represents the y. Press enter to obtain the graph on the right.

Hint: If the function entry line is not visible, press tab.

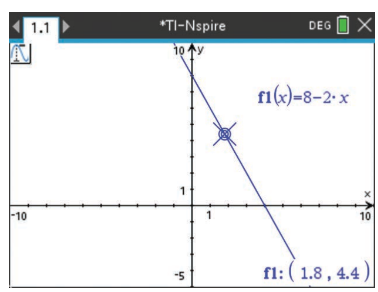


3 Change the window setting to see the key features of the graph. Use menu>**Window/Zoom**>**Window Settings** and edit as shown. Use the tab key to move between the entry lines. Press enter when finished editing the settings. The re-scaled graph is shown on the right.

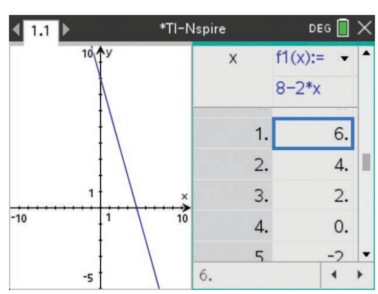


4 To show values on the graph, use menu>**Trace**>**Graph Trace** and then use the ◀▶ arrows to move along the graph.

Note: Press esc to exit the **GraphTrace** tool.



5 To show a table of values, press ctrl + T. Use the ▲▼ arrows to scroll through the values in the table.



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How to draw a straight-line graph and show a table of values using the ClassPad

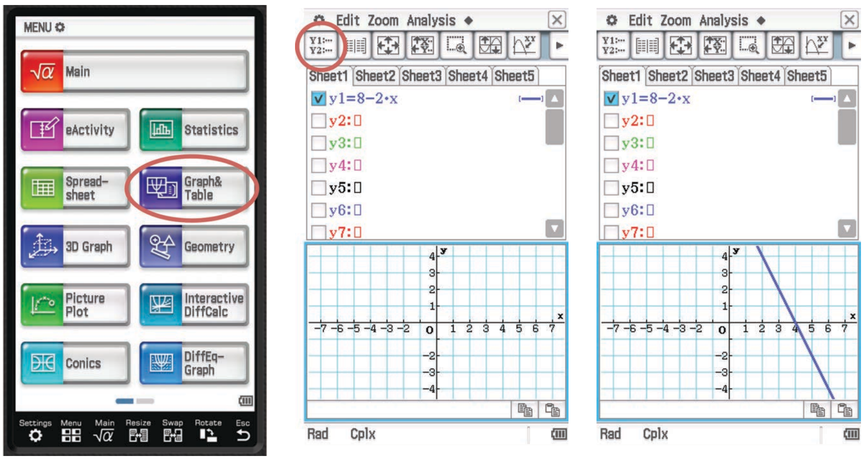
Use a CAS calculator to draw the graph of $y = 8 - 2x$ and show a table of values.

Steps

1 Open the **Graphs and Table** application.

2 Enter the equation into the graph editor window by typing **8 – 2*x***. Tick the box and press EXE.

3 Tap the icon to plot the graph.

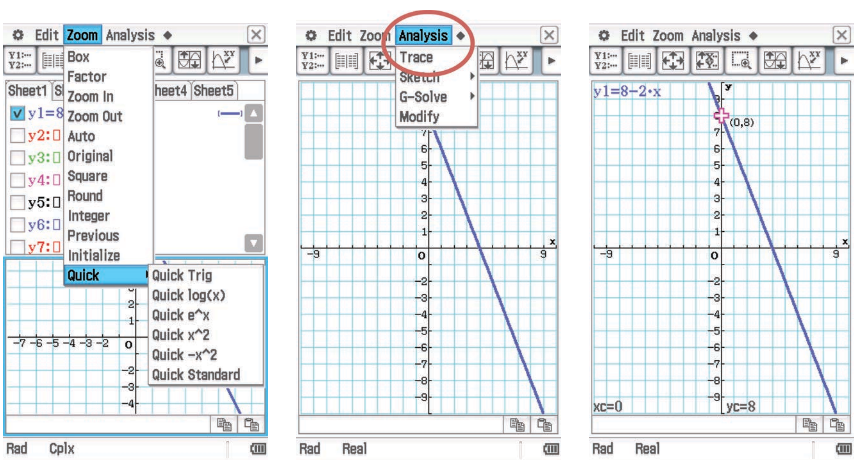


4 To adjust the graph screen, go to **Zoom > Quick > Quick Standard**. Quick Standard changes the window settings to [-10, 10] in the x and y directions.

5 Tap resize from the toolbar to fill the screen with the graph window.

6 Select **Analysis>Trace** to place a cursor on the graph. The coordinates of the point will be displayed at the location of the cursor. E.g. (0, 8).

7 Use the cursor key to move the cursor along the line.



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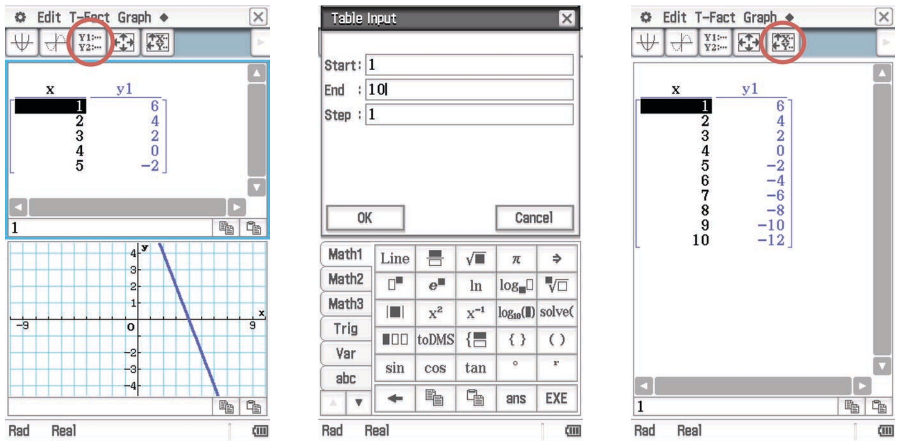
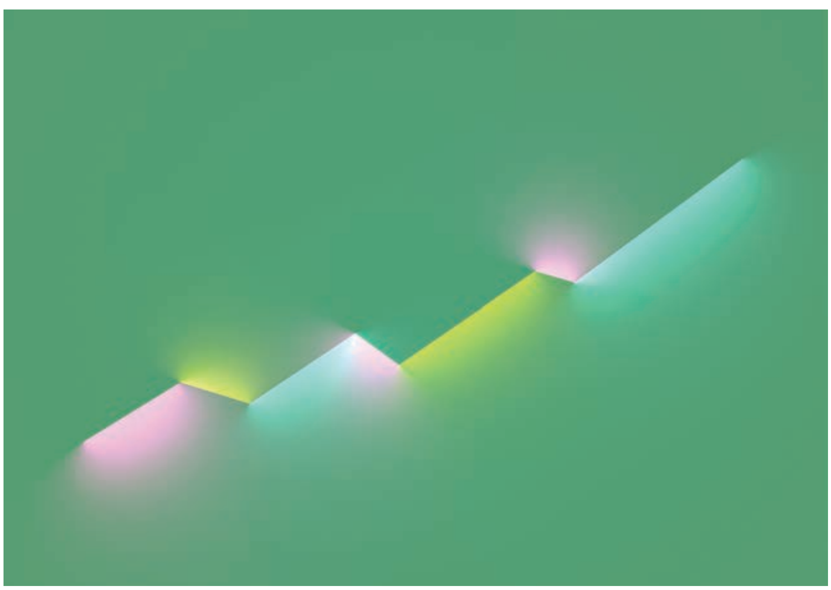
8 Tap the icon from the toolbar to display a table of values.

9 Tap the icon from the toolbar to open the **Table Input** dialog box. The values displayed in the table can be adjusted by changing the values in this window.



10 Tap resize from the toolbar to fill the screen with the table window.



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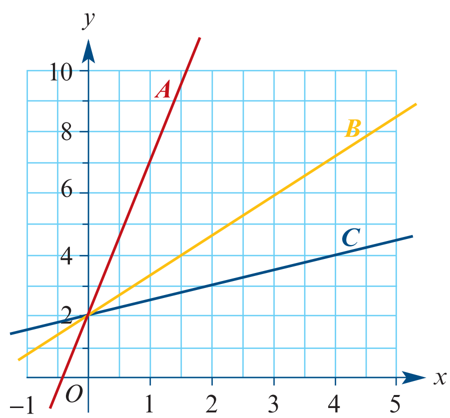
Positive and negative slopes of a straight line

One thing that makes one straight-line graph look different from another is its steepness or **slope**. Another name for slope is **gradient**[1].

For example, the three straight lines on the graph below all cut the y-axis at $y = 2$, but they have quite different slopes.



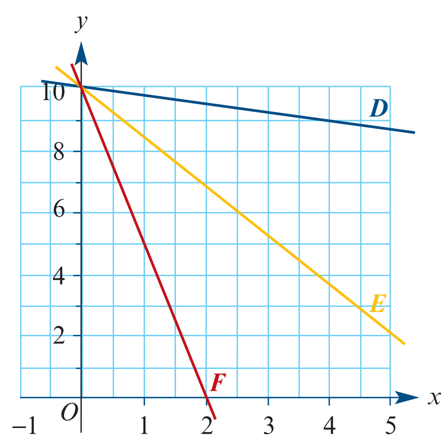
Line A has the steepest slope while Line C has the gentlest slope. Line B has a slope somewhere in between.

In all cases, the lines have **positive slopes**; that is, they rise from left to right.

Similarly, the three straight lines on the graph opposite all cut the y-axis at $y = 10$, but they have quite different slopes.

In this case, Line D has the gentlest slope while Line F has the steepest slope. Line E has a slope somewhere in between.

In all cases, the lines have **negative slopes**; that is, they fall from left to right.



[1] Note: For linear graphs, the terms slope and gradient mean the same thing. However, when dealing with practical applications of linear graphs and, most particularly, in statistics applications, the word 'slope' is preferred. For this reason, and to be consistent with the VCE General Mathematics curricula, this is the term used throughout this book.

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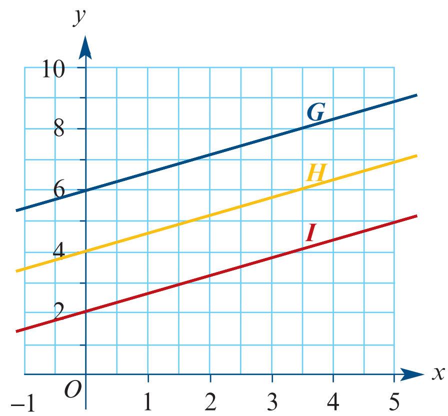
By contrast, the three straight lines, G, H and I, on the graph opposite, cut the y-axis at different points, but they all have the *same* slope.



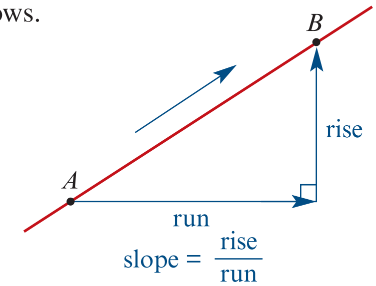
Calculating the slope

When talking about the **slope of a straight line**, we want to be able to do more than say that it has a gentle positive slope. We would like to be able to give the slope a value that reflects this fact. We do this by defining the slope of a line as follows.

First, two points, A and B, on the line are chosen. As we go from A to B along the line, we move:

- up by a distance called the **rise**
- and across by a distance called the **run**.

The slope is found by dividing the rise by the run.



$$\textbf{slope} = \frac{\textbf{rise}}{\textbf{run}}$$

Example 11 Finding the slope of a line from a graph: positive slope

Find the slope of the line through the points (1, 4) and (4, 8).

Explanation

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

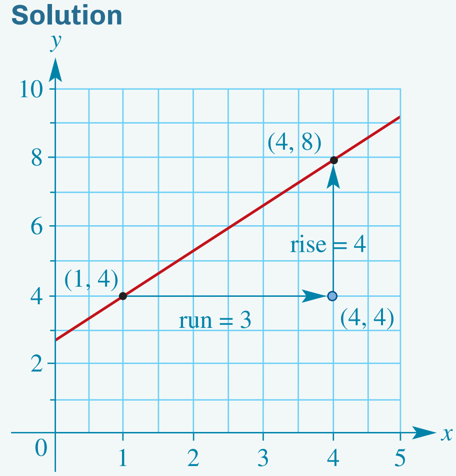
$$\text{rise} = 8 - 4 = 4$$

$$\text{run} = 4 - 1 = 3$$

$$\therefore \text{ slope} = \frac{4}{3} = 1.33 \text{ (to 2 decimal places)}$$

Note: To find the 'rise,' look at the y-coordinates.
To find the 'run,' look at the x-coordinates.

Solution



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Now try this 11 Finding the slope of a line from a graph: positive slope (Example 11)

Find the slope of the line through the points (2, 6) and (3, 9).

Hint 1 By sketching a graph, find the values for the rise and the run.

Hint 2 Divide the rise by the run.

Example 12 Finding the slope of a line from a graph: negative slope

Find the slope of the line through the points (0, 10) and (4, 2).

Explanation

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

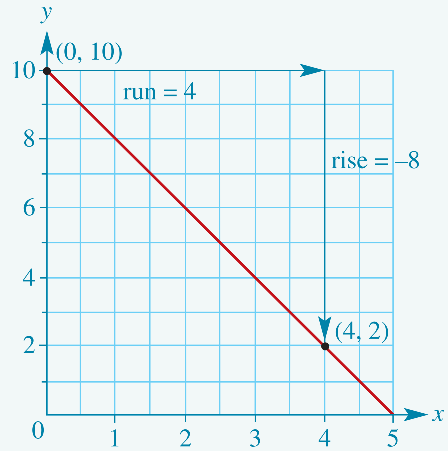
$$\text{rise} = 2 - 10 = -8$$

$$\text{run} = 4 - 0 = 4$$

$$\therefore\ \text{slope} = \frac{-8}{4} = -2$$

Note: In this example, we have a negative 'rise' which represents a 'fall'.

Solution



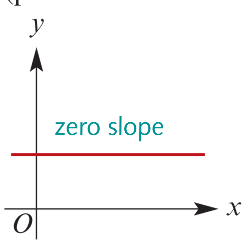
Now try this 12 Finding the slope of a line from a graph: negative slope (Example 12)

Find the slope of the line through the points (1, 4) and (5, 2).

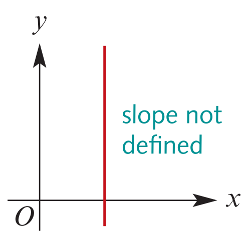
Hint 1 By sketching a graph, find the values for the rise and the run.

Hint 2 Divide the rise by the run.

A straight-line graph that is horizontal (parallel to the x-axis) has a slope of **zero**.



A straight-line graph that is vertical (parallel to the y-axis) has a slope that is **undefined**.



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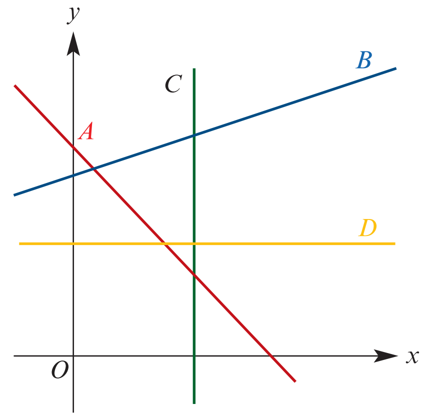
Section Summary

- The graph of a linear equation is a straight line.
- The **slope** of a straight-line graph tells us the steepness of the graph.
- The **slope** of a graph can be **positive**, **negative**, **zero** or **undefined**.
- A **positive slope** rises from left to right.
- A **negative slope** falls from left to right.
- The slope of a straight-line graph that is horizontal (parallel to the *x*-axis) is **zero**.
- The slope of a straight-line graph that is vertical (parallel to the *y*-axis) is **undefined**.

Exercise 5D

Building understanding

1 Identify the slope of each of the straight-line graphs *A*, *B*, *C* and *D* as: positive, negative, zero, or undefined.



2 Complete the table below for the equation $y = 3 + 2x$.

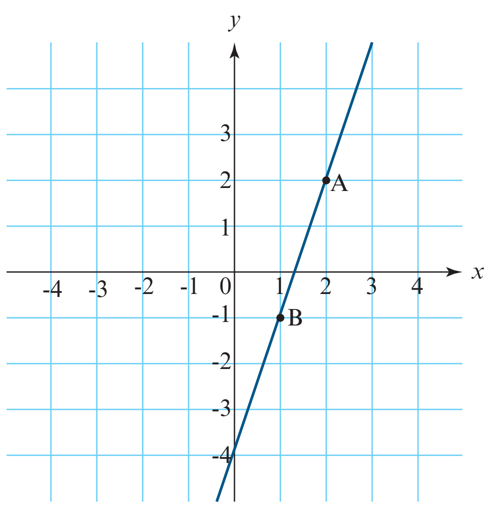
x	-2	-1	0	1	2	3
y	-1				7	

3 Consider the following graph.

a State the rise from point B to point A.

b State the run from point B to point A.

c Using your answers from part **a** and **b**, state the slope of the line.



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Developing understanding

Example 10

4 Plot the graph of the linear equations below by first completing a table of values for y when $x = 0, 1, 2, 3, 4$.

a $y = 2 + x$

b $y = 1 + 2x$

c $y = 10 - x$

d $y = 9 - 2x$

5 Use your CAS calculator to plot a graph and generate a table of values of y for $x = 0, 1, 2, 3, 4$.

a $y = 4 + x$

b $y = 2 + 3x$

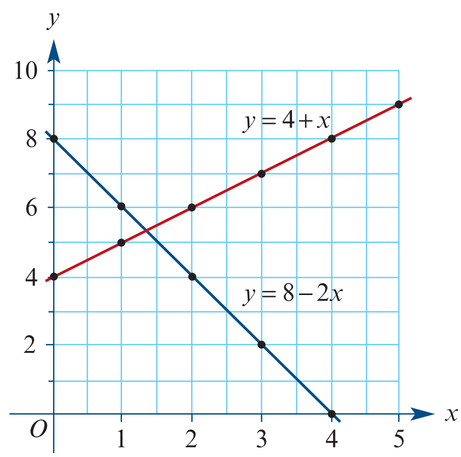
c $y = 10 + 5x$

d $y = 100 - 5x$

6 Two straight-line graphs, $y = 4 + x$ and $y = 8 - 2x$, are plotted as shown opposite.

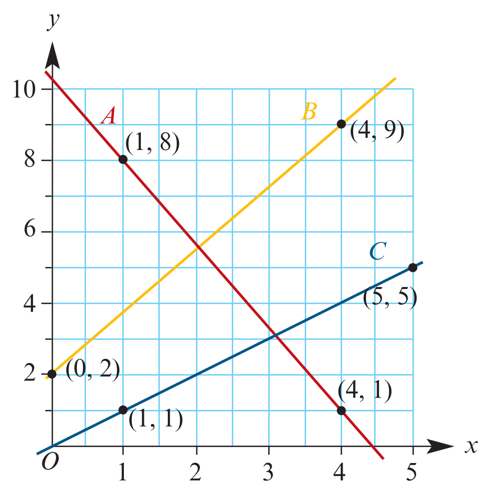
a Reading from the graph of $y = 4 + x$, determine the missing coordinates: (0, ?), (2, ?), (?, 7), (?, 9).

b Reading from the graph of $y = 8 - 2x$, determine the missing coordinates: (0, ?), (1, ?), (?, 4), (?, 2).



Example 11

7 Find the slope, to two decimal places, of each of the lines (A, B, C) shown on the graph below.



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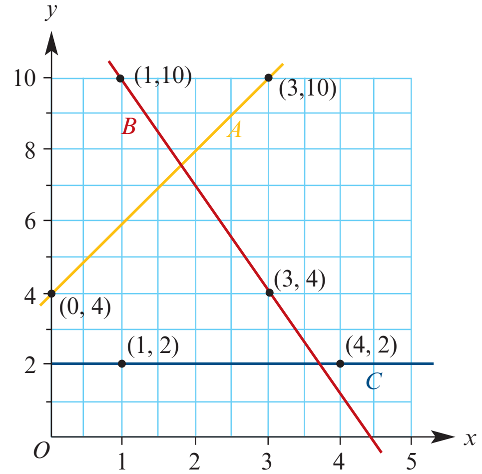
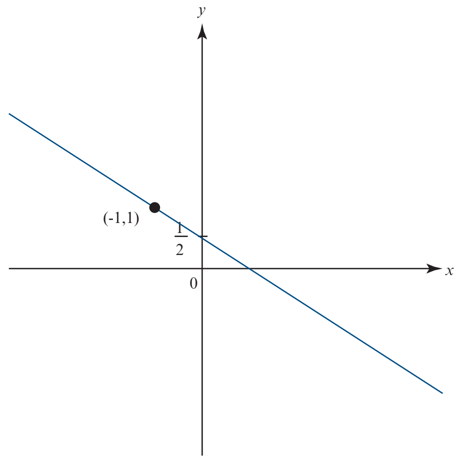
Example 12

8 Find the slope of each of the lines (A, B, C) shown on the graph below.



Testing understanding

9 Find the slope of the graph.



10 Consider the following rules:

- $y = 3x$
- $y = 3x + 1$
- $y = 3x + 2$

a Sketch each line.

b Find the slope of each line.

c What do you notice about the lines and your answer to part **b**?

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5E Equations of straight lines

Learning intentions

- To be able to use the formula for finding the slope of a straight line.
- To be able to find the y-intercept and slope from a straight-line equation.

A formula for finding the slope of a line

While the 'rise/run' method for finding the slope of a line will always work, some people prefer to use a formula for calculating the slope. The formula is derived as follows.

Label the coordinates of point A: (x_1, y_1).

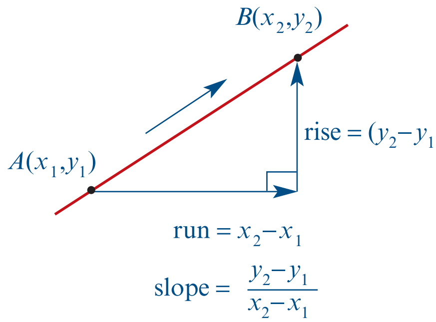
Label the coordinates of point B: (x_2, y_2).

By definition: slope $= \dfrac{\text{rise}}{\text{run}}$.

From the diagram:

$$\text{rise} = y_2 - y_1$$

$$\text{run} = x_2 - x_1$$



The slope of a line can be found by substituting in the formula:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 13 Finding the slope of a line using the formula for the slope

Find the slope of the line that passes through the points (1, 7) and (4, 2) using the formula for the slope of a line. Give your answer to two decimal places.

Explanation

Use

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

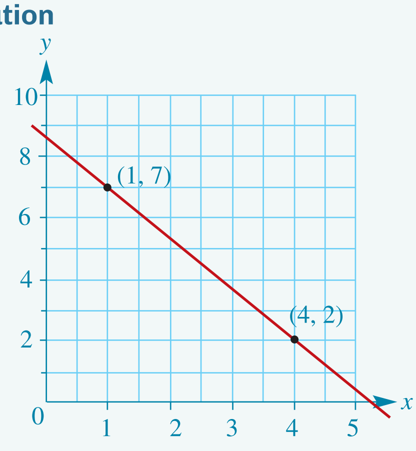
Let $(x_1, y_1) = (1, 7)$ and $(x_2, y_2) = (4, 2)$.

$$\text{slope} = \frac{2-7}{4-1}$$

$$= -1.67 \text{ (to 2 d.p.)}$$

Note: To use this formula it does not matter which point you call (x_1, y_1) and which point you call (x_2, y_2). The rule still works.

Solution



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Now try this 13 Finding the slope of a line using the formula for the slope (Example 13)

Find the slope of the line through the points (2, 8) and (6, 3) using the formula for the slope of a line. Give your answer to two decimal places.

Hint 1 Write down the formula for the slope.

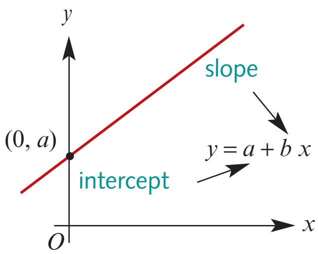
Hint 2 Substitute your values into the formula.

The intercept-slope form of the equation of a straight line

We can write the equation* of a straight line in the form: $y = a + bx$.

We call $y = a + bx$ the **intercept-slope form** of the equation of a straight line because:

- a = the ***y*-intercept** of the graph (i.e, when $x = 0$)
- b = the **slope** of the graph.



The intercept–slope form of the equation of a straight line is useful in modelling relationships in many practical situations. It is also the form used in **bivariate** (two-variable) statistics.

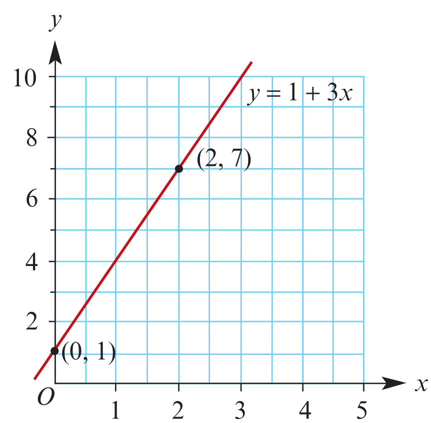
An example of the equation of a straight line written in intercept–slope form is $y = 1 + 3x$.

Its graph is shown opposite.

From the graph we see that the:

$$y\text{-intercept} = 1$$

$$\text{slope} = \frac{7-1}{2-0} = \frac{6}{2} = 3$$



That is:

- the y-intercept corresponds to the (*constant*) term in the equation (intercept = 1)
- the slope is given by the *coefficient of x* in the equation (slope = 3).

* Note: You may be used to writing the equation of a straight line as $y = mx+c$. However, when we are using a straight-line graph to model (represent) real world phenomena, we tend to reverse the order of the terms and use 'a' for the intercept and 'b' for the slope (rather than 'c' and 'm') and write the equation as $y = a + bx$. This is particularly true in performing statistical computations where your calculator will use 'a' for slope and 'b' for the y-intercept. You will see this later in the book, so it is worth making the change now. This is also the language used in the VCE written examinations.

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Intercept–slope form of the equation of a straight line

When we write the equation of a straight line in the form:

$$y = a + bx$$

we are using the **intercept-slope form** of the equation of a straight line.

a = the y-intercept of the graph (where the graph cuts the y-axis)

b = the slope of the graph

Example 14 Finding the y- intercept and slope of a line from its equation

Write down the y-intercept and slope of each of the straight-line graphs defined by the following equations.

a $y = -6 + 9x$ **b** $y = 10 - 5x$ **c** $y = -2x$ **d** $y - 4x = 5$

Explanation	Solution
For each equation:	
1 Write the equation. If it is not in intercept–slope form, rearrange the equation.	**a** $y = -6 + 9x$ y-intercept = -6 slope = 9
2 Write down the y-intercept and slope. When the equation is in intercept–slope form, $y = a + bx$, the value of: a = the y-intercept (the constant term) b = the slope (the coefficient of x).	**b** $y = 10 - 5x$ y-intercept = 10 slope = -5 **c** $y = -2x$ or $y = 0 - 2x$ y-intercept = 0 slope = -2 **d** $y - 4x = 5$ or $y = 5 + 4x$ y-intercept = 5 slope = 4

Now try this 14 Finding the y-intercept and slope of a line from its equation (Example 14)

Write down the y-intercept and slope of the following straight-line graphs.

a $y = 4 - 2x$ **b** $y = 3x$ **c** $y - 3x = 6$

Hint 1 Rearrange equations, if necessary, into intercept-slope form.

Hint 2 Remember in the intercept-slope form, $y = a + bx$, a is the y-intercept and b is the slope.

Example 15 Writing the equation of a straight line given its y-intercept and slope

Write down the equations of the straight lines with the following y-intercepts and slopes.

a y-intercept = 9 slope = 6 **b** y-intercept = 2 slope = -5

c y-intercept = -3 slope = 2

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Explanation

The equation of a straight line is $y = a + bx$. In this equation, a = y-intercept and b = slope.
Form an equation by inserting the given values of the y-intercept and the slope for a and b in the standard equation $y = a + bx$.

Solution

a y-intercept = 9 slope = 6
equation : $y = 9 + 6x$

b y-intercept = 2 slope = −5
equation : $y = 2 + (-5)x$
or $y = 2 - 5x$

c y-intercept = −3 slope = 2
equation : $y = -3 + 2x$
or $y = 2x - 3$

Now try this 15 Writing the equation of a straight line given its y-intercept and slope (Example 15)

Write down the equations of the straight lines with the following y-intercepts and slopes.

a y-intercept = 5 slope = 3

b y-intercept = 6 slope = −2

c y-intercept = −4 slope = 5

Hint 1 Write down the equation of a straight line in **intercept-slope** form.
Hint 2 Substitute in values for the y-intercept and the slope.

Sketching straight-line graphs

Since only two points are needed to draw a straight line, all we need to do is find two points on the graph and then draw a line passing through these two points. When the equation of a straight line is written in intercept–slope form, one point on the graph is immediately available: the y-intercept. A second point can then be quickly calculated by substituting a suitable value of x into the equation.

When we draw a graph in this manner, we call it a **sketch graph**.

Example 16 Sketching a straight-line graph from its equation

Sketch the graph of $y = 8 + 2x$.

Explanation

1 Write the equation of the line.

2 As the equation is in intercept–slope form, the y-intercept is given by the constant term. Write it down.

3 Find a second point on the graph. Choose an x-value (not 0) that makes the calculation easy: $x = 5$ would be suitable.

Solution

$y = 8 + 2x$

y-intercept = 8

When $x = 5$, $y = 8 + 2(5) = 18$
$\therefore$ (5, 18) is a point on the line.

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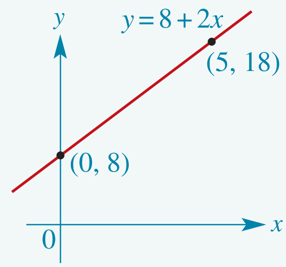
4 To sketch the graph:
- draw a set of labelled axes
- mark in the two points with coordinates
- draw a straight line through the points
- label the line with its equation.



Now try this 16 — Sketching a straight-line graph from its equation (Example 16)

Sketch the graph of $y = 5 + 3x$.

Hint 1 Find the y-intercept.

Hint 2 Choose a value for x to find another point on the graph.

Section Summary

- The formula for finding the **slope** of a line is $\dfrac{y_2 - y_1}{x_2 - x_1}$.
- The **intercept-slope form** of a straight-line equation is $y = a + bx$ where a is the **y-intercept** and b is the **slope**.

Skill-sheet

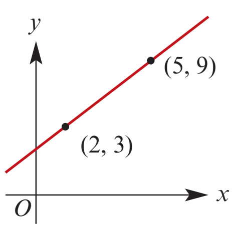
Exercise 5E

Building understanding

Example 13

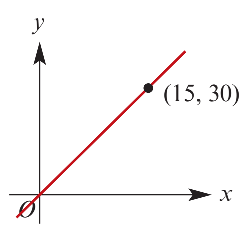
1 Use the formula, slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the slope of each of the lines shown below. An uncompleted formula is under each graph to assist you.

a



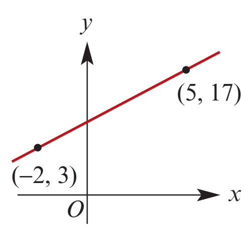
slope $= \dfrac{9 - \square}{5 - \square}$

b



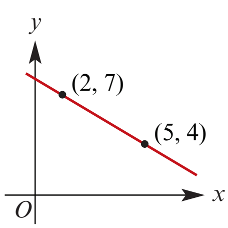
slope $= \dfrac{30 - \square}{\square - 0}$

c



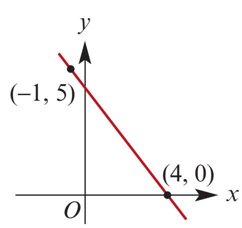
slope $= \dfrac{\square - \square}{\square - \square}$

d



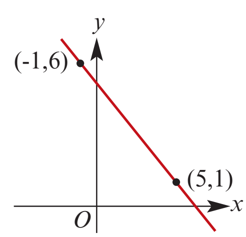
slope $= \dfrac{7 - \square}{2 - \square}$

e



slope $= \dfrac{5 - \square}{\square - 4}$

f



slope $= \dfrac{\square - \square}{\square - \square}$

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Developing understanding

Example 14

2 Write down the y-intercepts and slopes of the straight lines with the following equations.

a $y = 5 + 2x$ **b** $y = 6 - 3x$ **c** $y = 15 - 5x$ **d** $y = 3x$

3 Find the y-intercept and the slope by first rearranging the equations to make y the subject.

a $y + 3x = 10$ **b** $4y + 8x = -20$ **c** $x = y - 4$ **d** $x = 2y - 6$
e $2x - y = 5$ **f** $y - 5x = 10$ **g** $2.5x + 2.5y = 25$ **h** $y - 2x = 0$
i $y + 3x - 6 = 0$ **j** $y = 3$ **k** $4x - 5y - 8 = 7$ **l** $2y - 8 = 2(3x - 6)$

Example 15

4 Write down the equation of the straight line that has:

a y-intercept = 2, slope = 5 **b** y-intercept = 5, slope = 10
c y-intercept = −2, slope = 4 **d** y-intercept = 0, slope = −3
e y-intercept = −2, slope = 0 **f** y-intercept = 1.8, slope = −0.4
g y-intercept = 2.9, slope = −2 **h** y-intercept = −1.5, slope = −0.5

Example 16

5 Sketch the graphs of the straight lines with the following equations, clearly showing the y-intercepts and the coordinates of one other point.

a $y = 5 + 2x$ **b** $y = 5 + 5x$ **c** $y = 20 - 2x$
d $y = -10 + 10x$ **e** $y = 4x$ **f** $y = 16 - 2x$

Testing understanding

6 Sketch the graphs of the following straight lines.

a $y - x = 3$ **b** $y + 2x = 1$ **c** $y - 3x - 4 = 0$

5F Finding the equation of a straight-line graph

Learning intentions

- ▶ To be able to use the y-intercept and the slope to find the equation of a straight line.
- ▶ To be able to use two points on a graph to find the equation of a straight line.

Using the intercept and slope to find the equation of a straight line

We have learned how to construct a straight-line graph from its equation. We can also determine the equation from a graph. In particular, if the graph shows the y-intercept, it is a relatively straightforward procedure.

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Finding the equation of a straight-line graph from its intercept and slope

To find the equation of a straight line in intercept–slope form ($y = a + bx$) from its graph:

1 identify the y-intercept (a)

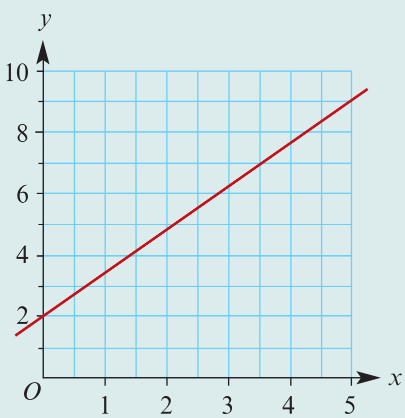
2 use two points on the graph to find the slope (b)

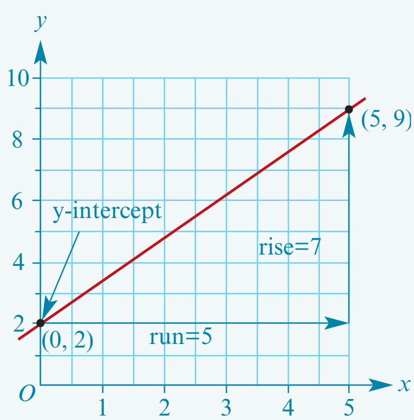
3 substitute these two values into the standard equation $y = a + bx$.

Note: This method *only works* when the graph *scale includes* $x = 0$.

Example 17 Finding the equation of a line: intercept–slope method

Determine the equation of the straight-line graph shown opposite.



Explanation	Solution
1 Write the general equation of a line in intercept–slope form.	$y = a + bx$
2 Read the y-intercept from the graph. **3** Find the slope using two well-defined points on the line, for example, (0, 2) and (5, 9).	 y-intercept = 2 so $a = 2$ $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{7}{5} = 1.4$ so $b = 1.4$
4 Substitute the values of a and b into the equation.	$y = 2 + 1.4x$
5 Write your answer.	$y = 2 + 1.4x$ is the equation of the line.

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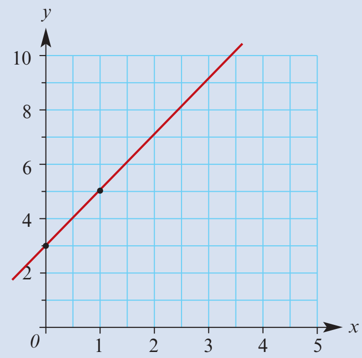
Now try this 17 **Finding the equation of a line: intercept–slope method (Example 17)**

Determine the equation of this straight-line graph.



Hint 1 Find the y-intercept.

Hint 2 Find the slope using two points on the line.

Hint 3 Substitute the values for a and b into the straight-line equation $y = a + bx$.

Using two points on a graph to find the equation of a straight line

Unfortunately, not all straight-line graphs show the y-intercept. When this happens, we have to use the two-point method for finding the equation of the line.

Finding the equation of a straight-line graph using two points

The general equation of a straight-line graph is $y = a + bx$.

1. Use the coordinates of two points on the line to determine the slope (b).
2. Substitute this value for the slope into the equation. There is now only one unknown, a.
3. Substitute the coordinates of one of the two points on the line into this new equation and solve for the unknown (a).
4. Substitute the values of a and b into the general equation, $y = a + bx$, to obtain the equation of the straight line.

Note: This method works in *all* circumstances.

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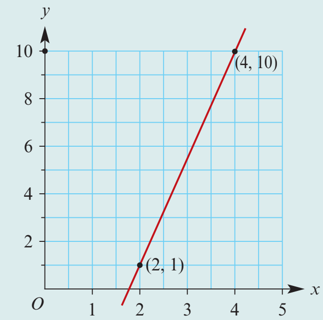
Example 18 **Finding the equation of a straight line using two points on the graph**

Find the equation of the line that passes through the points (2, 1) and (4, 10).



Explanation	Solution
1 Write down the general equation of a straight-line graph.	$y = a + bx$
2 Use the coordinates of the two points on the line to find the slope (b).	$\text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{10-1}{4-2} = \dfrac{9}{2} = 4.5$ so $b = 4.5$
3 Substitute the value of b into the general equation.	$y = a + 4.5x$
4 To find the value of a, substitute the coordinates of one of the points on the line (either will do) and solve for a.	Using the point (2, 1): $1 = a + 4.5 \times 2$ $1 = a + 9$ $a = -8$
5 Substitute the values of a and b into the general equation, $y = a + bx$, to find the equation of the line.	Thus, the equation of the line is: $y = -8 + 4.5x$

Now try this 18 **Finding the equation of a straight line using two points on the graph (Example 18)**

Find the equation of the line that passes through the points (2, 4) and (3, 10).

Hint 1 Write down the general equation of a straight line.
Hint 2 Find the slope of the line (b-value).
Hint 3 Substitute b and either of the given coordinates into the general equation.
Hint 4 Solve for a.

Finding the equation of a straight-line graph from two points using a CAS calculator

While the intercept–slope method of finding the equation of a line from its graph is relatively quick and easy to apply, using the two-point method to find the equation of a line can be time consuming.

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An alternative to using either of these methods is to use the line-fitting facility of your CAS calculator. You will meet this method again when you study the topic 'Investigating relationships between two numerical variables' later in the year.

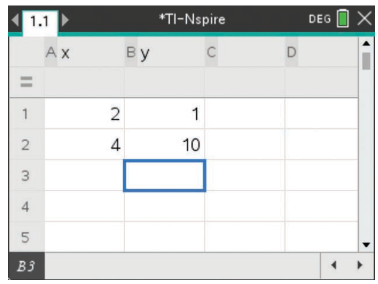
How to find the equation of a line from two points using the TI-Nspire

Find the equation of the line that passes through the two points (2, 1) and (4, 10).

Steps

1 Write the coordinates of the two points. Label one point A, the other B.

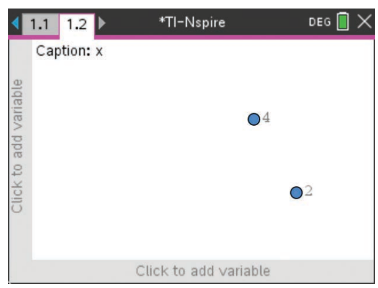
2 Start a new document (ctrl + doc▼ and select **Add Lists & Spreadsheet**.
Enter the coordinate values into lists named ***x*** and ***y***.



3 Plot the two points on a scatterplot. Press ctrl + N and select **Add Data & Statistics**.

(or press on and arrow to the Data & Statistics icon and press enter)

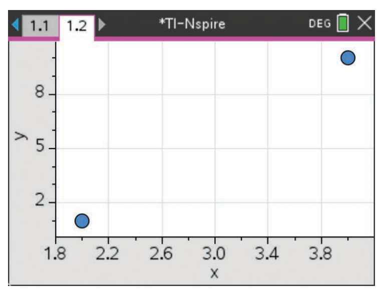
Note: A random display of dots will appear – this is to indicate list data is available for plotting. It is not a statistical plot.



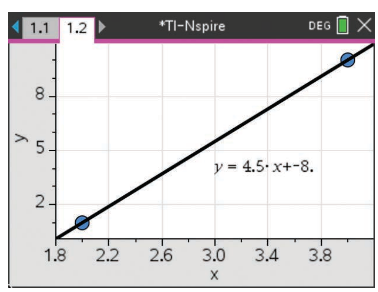
4 To construct a scatterplot:

a Press tab and select the variable ***x*** from the list. Press enter to paste the variable ***x*** to the ***x***-axis.

b Press tab again and select the variable ***y*** from the list. Press enter to paste the variable ***y*** to the ***y***-axis to generate the required scatter plot.



5 Use the **Regression** command to plot a line through the two points and determine its equation.
Press menu **>Analyze>Regression>Show Linear (a+bx)** and enter to complete the task.
Correct to one decimal place, the equation of the line is: $y = -8.0 + 4.5x$.



6 Write your answer. The equation of the line is $y = -8 + 4.5x$.

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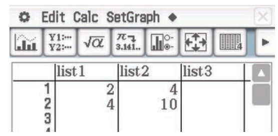
How to find the equation of a line from two points using the ClassPad

Find the equation of the line that passes through the two points (2, 4) and (4, 10).

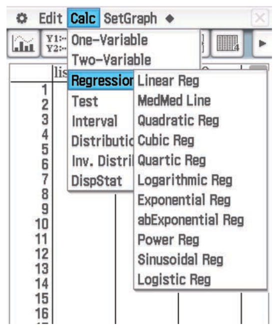
Steps

1 Open the **Statistics** application and enter the coordinate values into the lists as shown.

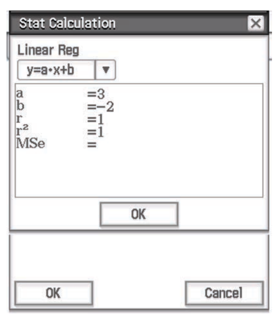


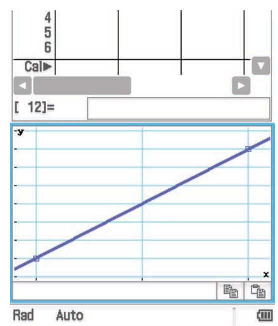
2 To find the equation of the line $y = ax + b$ that passes through the two points:

- Select **Calc** from the menu bar
- Select **Regression** and **Linear Reg**
- Ensure that the **Set Calculation** dialog box is set as shown
- Press OK.



3 The results are given in a **Stat Calculation** dialog box.





The equation of the line is $y = -2 + 3x$.

Note: Tapping **OK** will automatically display the graph window with the line drawn through the two points. This confirms that the line passes through the two points.

Section Summary

- The equation of a straight line can be found using the y-intercept and the slope.
- The equation of a straight line can be found using two points on the line.
- The equation of a straight line can be found using a CAS calculator.

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Skill-sheet

Exercise 5F

Building understanding

The graphs for Questions 1-3 are shown below Question 3.

1 For Graph 1 below:

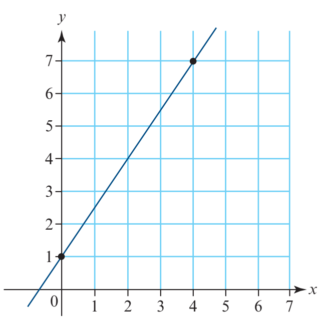
- **a** Find the y-intercept.
- **b** Find the slope of the line using the formula for the slope.
- **c** State the equation of the straight line in the form $y = a + bx$.

2 For Graph 2 below:

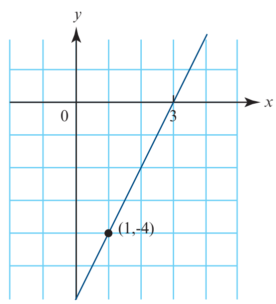
- **a** Find the slope of the line using the formula for the slope.
- **b** Find the y-intercept (a) by substituting the slope for b and the point (3, 0) into $y = a + bx$.

3 For Graph 3 below:

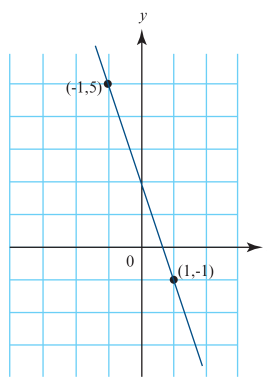
- **a** Find the slope of the line using the formula.
- **b** Find the y-intercept by substituting the slope for b and the point $(-1, 5)$ into $y = a + bx$.



Graph 1



Graph 2

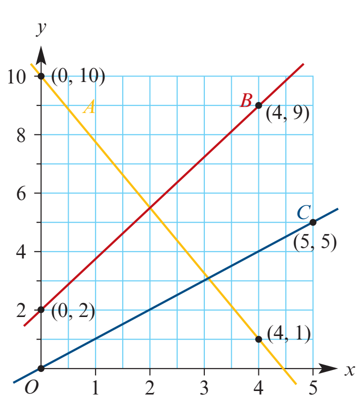


Graph 3

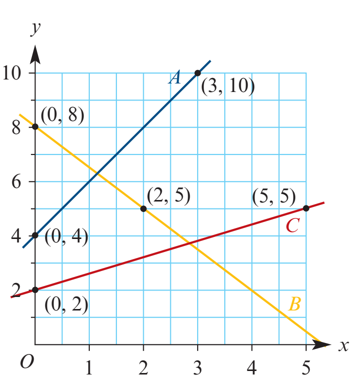
Developing understanding

Example 17

4 Find the equation of the lines (A, B, C) shown on the graph below. Write your answers in the form $y = a + bx$.

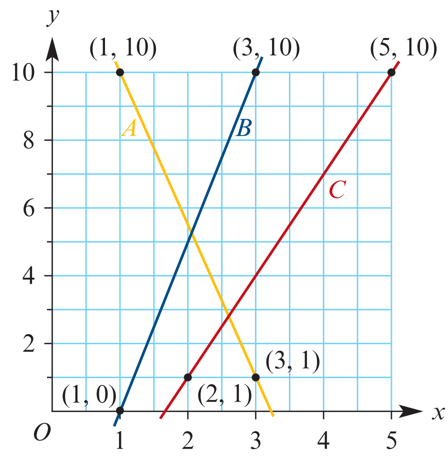


5 Find the equation of the lines (A, B, C) shown on the graph below. Write your answers in the form $y = a + bx$.

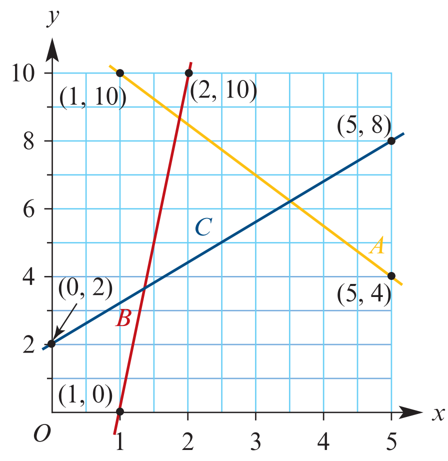


Example 18

6 Find the equation of the lines (A, B, C) on the graph below. Write your answers in the form $y = a + bx$.

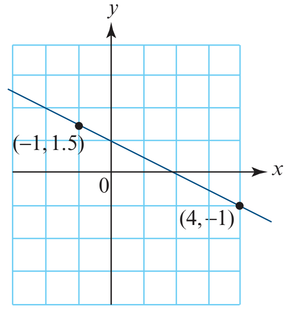


7 Use a CAS calculator to find the equation of each of the lines (A, B, C) on the graph below. Write your answers in the form $y = a + bx$.



Testing understanding

8 Find the equation of the straight-line graph shown.



9 A line passes through (0, 2) and (5, 9). Find the equation of the line in the form $y = a + bx$.

10 A line passes through (2, −4) and (−4, 8). Find the equation of the line in the form $y = a + bx$.

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5G Linear modelling

Learning intentions

- To be able to construct linear models.
- To be able to interpret and analyse linear models.

Many real-life relationships between two variables can be described mathematically by linear (straight-line) equations. This is called **linear modelling**.

These linear models can then be used to solve problems, such as finding the time taken to fill a partially filled swimming pool with water, estimating the depreciating value of a car over time or describing the growth of a plant over time.

Modelling plant growth with a linear equation

Some plants, such as tomato plants, grow remarkably quickly.

When first planted, the height of this plant was 5 cm.

The plant then grows at a constant rate of 6 cm per week for the next 10 weeks.

From this information, we can now construct a mathematical model that can be used to chart the growth of the plant over the following weeks and predict its height at any time during the first 10 weeks after planting.

Start　After 1 week　After 2 weeks

Constructing a linear model

Let h be the height of the plant (in cm).

Let t be the time (in weeks) after it was planted.

For a linear growth model we can write:

$$h = a + bt$$

where:

- a is the initial height of the plant; in this case, 5 cm (in graphical terms, the y-intercept)
- b is the constant rate at which the plant's height increases each week; in this case, 6 cm per week (in graphical terms, the slope of the line).

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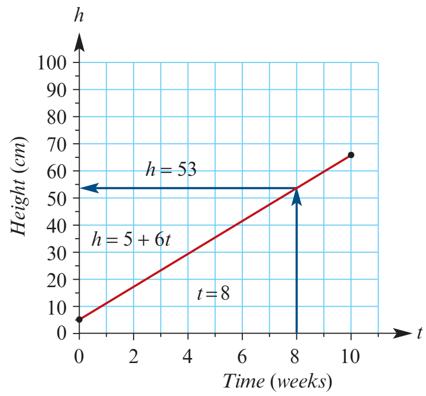
Thus we can write: $h = 5 + 6t$ for $0 \le t \le 10$ (see note below). The graph for this model is plotted here.



Three important features of the linear model $h = 5 + 6t$ for $0 \le t \le 10$ should be noted:

- The h-intercept gives the height of the plant at the start; that is, its height when $t = 0$. The plant was 5 cm tall when it was first planted.
- The *slope* of the graph gives the growth rate of the plant. The plant grows at a rate of 6 cm per week; that is, each week the height of the plant increases by 6 cm.
- The graph is only plotted for $0 \le t \le 10$. This is because the model is only valid for the time (10 weeks) when the plant is growing at the constant rate of 6 cm a week.

Note: The expression $0 \le t \le 10$ is included to indicate the range of the number of weeks for which the model is valid. In more formal language this would be called the **domain** of the model.

Using a linear model to make predictions

To use the mathematical model to make predictions, we simply substitute a value of t into the model and evaluate.

For example, after eight weeks' growth ($t = 8$), the model predicts the height of the plant to be:

$$h = 5 + 6(8) = 53 \text{ cm}$$

This value could also be read directly from the graph, as shown above.

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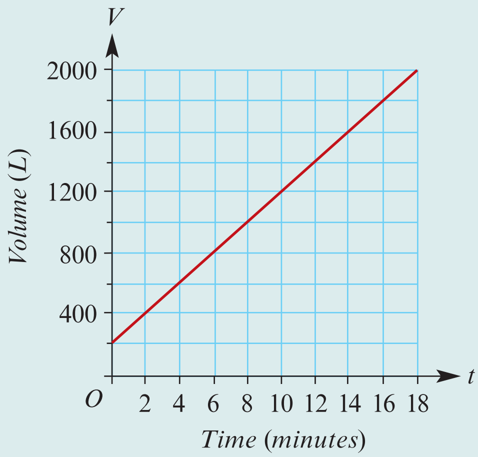
Interpreting and analysing the graphs of linear models

Example 19 Graphs of linear models with a positive slope

Water is pumped into a partially full tank. The graph gives the volume of water, V, (in litres) after t minutes.



a How much water is in the tank at the start ($t = 0$)?

b How much water is in the tank after 10 minutes ($t = 10$)?

c The tank holds 2000 L. How long does it take to fill?

d Find the equation of the line in terms of V and t.

e Use the equation to calculate the volume of water in the tank after 15 minutes.

f At what rate is the water pumped into the tank; that is, how many litres are pumped into the tank each minute?

Explanation	Solution
a Read from the graph (when $t = 0$, $V = 200$).	200 L of water are in the tank.
b Read from the graph (when $t = 10$, $V = 1200$).	1200 L of water are in the tank.
c Read from the graph (when $V = 2000$, $t = 18$).	It takes 18 minutes to fill the tank.
d The equation of the line is $V = a + bt$. a is the V-intercept. Read from the graph. b is the slope. Calculate using two points on the graph, say (0, 200) and (18, 2000). **Note:** You can use your calculator to find the equation of the line if you wish.	$V = a + bt$ $a = 200$ $b = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{2000 - 200}{18 - 0}$ $= 100$ $\therefore\ V = 200 + 100t \left(t \geq 0\right)$
e Substitute $t = 15$ into the equation. Evaluate.	$V = 200 + 100(15) = 1700$ L
f The rate at which water is pumped into the tank is given by the slope of the graph, 100 (from part **d**).	The rate is 100 L/min.

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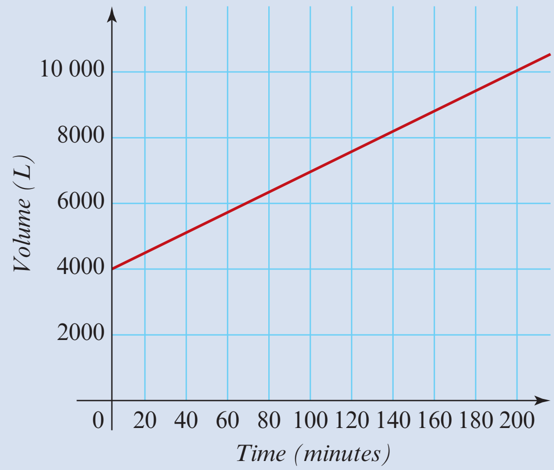
Now try this 19 **Graphs of linear models with a positive slope (Example 19)**

Petrol is pumped into a partially full storage tank. The graph shows the volume of petrol, V, (in litres) after t minutes.



a How much petrol is in the tank at the start ($t = 0$)?

b How much petrol is in the tank after 100 minutes ($t = 100$)?

c The tank holds 10 000 L. How long does it take to fill?

d Find the equation of the line in terms of V and t.

e Use the equation to calculate the volume of water in the tank after 150 minutes.

f At what rate is the water pumped into the tank; that is, how many litres are pumped into the tank each minute?

Hint 1 Read from the graph when $t = 0$.

Hint 2 Read from the graph when $t = 100$.

Hint 3 Read from the graph when $V = 10\,000$.

Hint 4 Find the slope of the graph and then write an equation for the line ($V = a + bt$) using the slope and V-intercept.

Hint 5 Substitute in $t = 150$.

Hint 6 The slope will tell you the rate.

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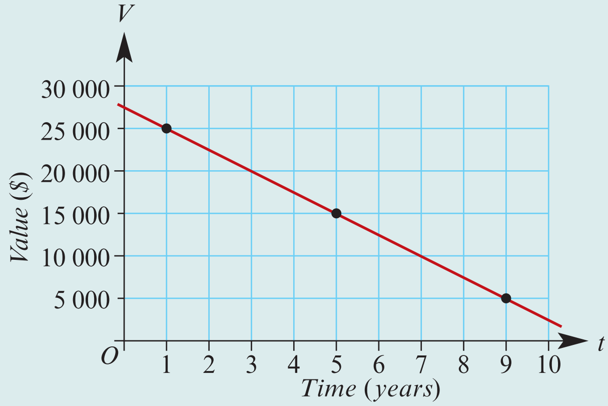
Example 20 Graphs of linear models with a negative slope

The value of new cars depreciates with time. The graph shows how the value, V, (in dollars) of a new car depreciates with time, t, (in years).



a What was the value of the car when it was new?

b What was the value of the car when it was 5 years old?

c Find the equation of the line in terms of V and t.

d At what rate does the value of the car depreciate with time; that is, by how much does its value decrease each year?

e When does the equation predict the car will have no (zero) value?

Explanation	Solution
a Read from the graph (when $t = 0, V = 27\,500$).	\$27 500 when the car was new
b Read from the graph (when $t = 5, V = 15\,000$).	\$15 000 when the car was 5 years old
c The equation of the line is $V = a + bt$. ■ a is the V-intercept. Read from the graph. ■ b is the slope. Calculate using two points on the graph, say (1, 25 000) and (9, 5000). **Note:** You can use your calculator to find the equation of the line if you wish.	$V = a + bt$ $a = 27\,500$ $b = \text{slope} = \dfrac{25\,000 - 5000}{1 - 9}$ $= -2500$ $\therefore\ V = 27\,500 - 2500t$ for $t \geq 0$
d The slope of the line is –2500, so the car depreciates in value by \$2500 per year.	\$2500 per year
e Substitute into the equation and solve for t.	$0 = 27\,500 - 2500t$ $2500t = 27\,500$ $\therefore t = \dfrac{27\,500}{2500} = 11$ years

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Now try this 20 Graphs of linear models with a negative slope (Example 20)

A car's value depreciates with time. Its value, V, over time, t, (in years) is shown on the graph.

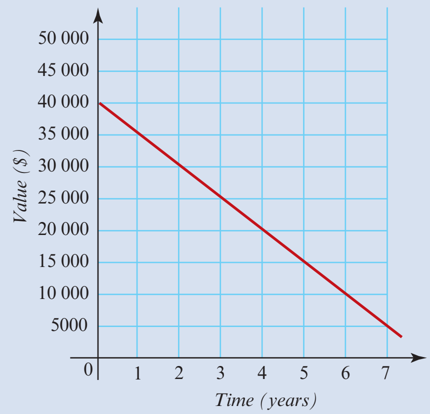
a What was the value of the car when it was new?

b What was the value of the car when it was 3 years old?

c Find the equation of the line in terms of V and t.

d At what rate does the value of the car depreciate with time?

e When does the equation predict the car will have no (zero) value?



Hint 1 Look at the V-intercept.

Hint 2 Find the slope of the graph.

Hint 3 Write an equation for the line in the form $V = a + bt$.

Hint 4 The slope tells you the rate.

Section Summary

- **Linear modelling** uses linear (straight-line) equations to describe relationships between variables.
- A linear model can be used to make predictions.

Exercise 5G

Building understanding

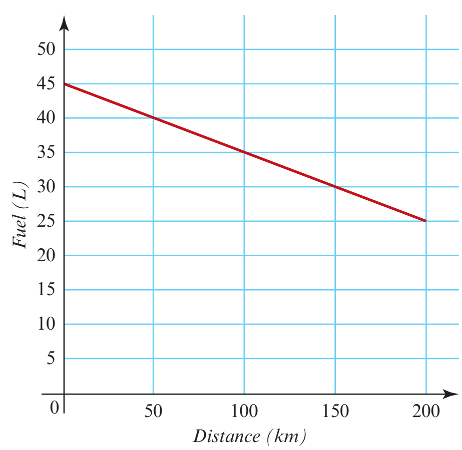
Example 19

1 The graph shows the amount of fuel in the tank of a car after driving for a distance, d, kilometres.

a State the amount of fuel in the tank initially, when $d = 0$.

b State the amount of fuel in the tank after travelling $d = 50$ kilometres.

c State the amount of fuel in the tank when $d = 200$ kilometres.



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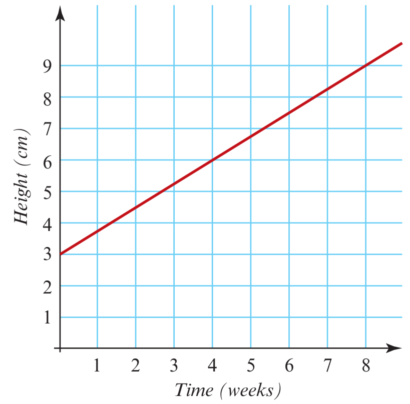
2 The graph shows the height, h, in cm, for the first 8 weeks' growth of a plant.



a What was the height of the plant initially, (at $t = 0$)?

b What was the height of the plant after four weeks, (at $t = 4$)?

c What was the height of the plant after 8 weeks?

d Find the slope of the straight line.

e Write down the linear model in terms of h and t.

Developing understanding

3 An empty 20-litre cylindrical beer keg is to be filled with beer at a constant rate of 5 litres per minute.
Let V be the volume of beer in the keg after t minutes.

a Write down a linear model in terms of V and t to represent this situation.
The beer keg is filled in 4 minutes.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the volume of beer in the keg after 3.2 minutes.

4 A home waste removal service charges \$80 to come to your property. It then charges \$120 for each cubic metre of waste it removes. The maximum amount of waste that can be removed in one visit is 8 cubic metres.
Let C be the total charge for removing w cubic metres of waste.

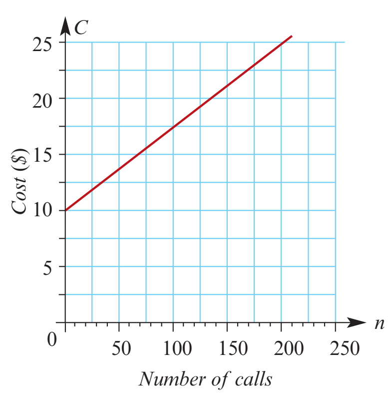
a Write down a linear model in terms of C and w to represent this situation.

b Sketch the graph showing the coordinates of the intercept and its end point.

c Use the model to predict the cost of removing 5 cubic metres of waste.

Example 19
Example 20

5 A phone company charges a monthly service fee plus the cost of calls. The graph shown gives the total monthly charge, C dollars, for making n calls. This includes the service fee.



a How much is the monthly service fee ($n = 0$)?

b How much does the company charge if you make 100 calls a month?

c Find the equation of the line in terms of C and n.

d Use the equation to calculate the cost of making 300 calls in a month.

e How much does the company charge per call?

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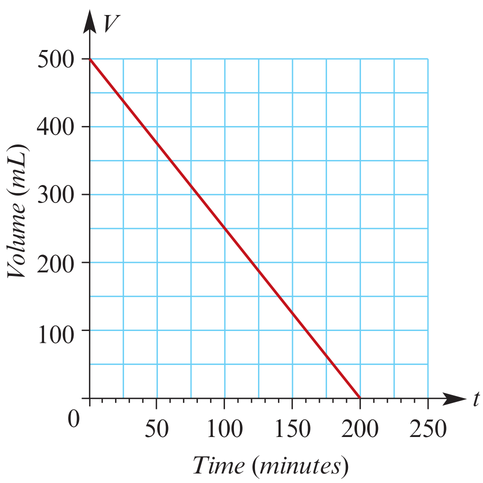
6 A motorist fills the tank of her car with unleaded petrol, which costs \$1.57 per litre. Her tank can hold a maximum of 60 litres of petrol. When she started filling her tank, there was already 7 litres in her tank.
Let C be the cost of adding v litres of petrol to the tank.

- **a** Write down a linear model in terms of C and v to represent this situation.
- **b** Sketch the graph of C against v, showing the coordinates of the intercept and its end point.
- **c** Use the model to predict the cost of filling the tank of her car with petrol.

7 The graph opposite, shows the volume of saline solution, V mL, remaining in the reservoir of a saline drip after t minutes.

- **a** How much saline solution was in the reservoir at the start?
- **b** How much saline solution remains in the reservoir after 40 minutes? Read the result from the graph.
- **c** How long does it take for the reservoir to empty?
- **d** Find the equation of the line in terms of V and t.
- **e** Use the equation to calculate the amount of saline solution in the reservoir after 115 minutes.



- **f** At what rate (in mL/minute) is the saline solution flowing out of the drip?

Testing understanding

8 A swimming pool when full contains 10 000 litres of water. Due to a leak, it loses on average 200 litres of water per day.
Let V be the volume of water remaining in the pool after t days.

- **a** Write down a linear model in terms of V and t to represent this situation.
- **b** The pool continues to leak. How long will it take to empty the pool?
- **c** Sketch the graph showing the coordinates of the intercept and its end point.
- **d** Use the model to predict the volume of water left in the pool after 30 days.

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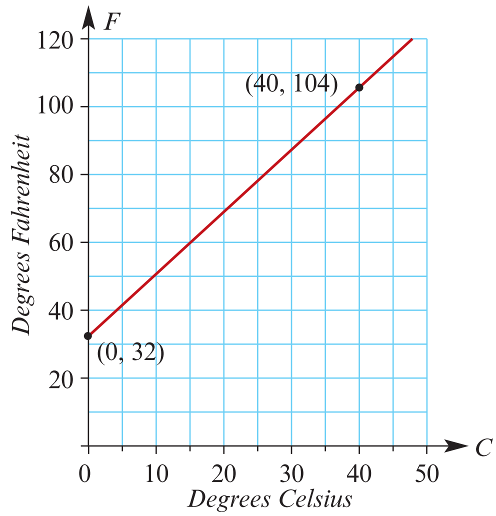
9 The graph, opposite, can be used to convert temperatures in degrees Celsius (C) to temperatures in degrees Fahrenheit (F).



a Find the equation of the line in terms of F and C.

b Use the equation to predict the temperature in degrees Fahrenheit when the temperature in degrees Celsius is:

i 50°C **ii** 150°C **iii** −40°C

c Complete the following sentence by filling in the box.
When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by $\boxed{}$ degrees.

5H Solving simultaneous equations

Learning intentions

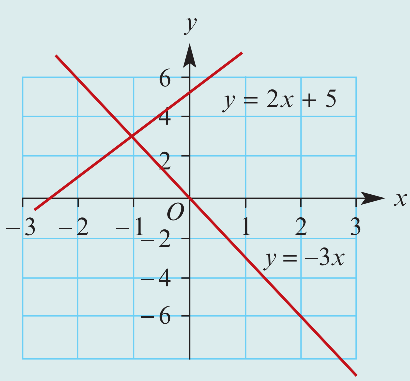
- ▶ To be able to find the intersection point of two lines.
- ▶ To be able to solve simultaneous equations.

Finding the point of intersection of two linear graphs

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and then reading off the coordinates at the point of intersection. When we find the *point of intersection*, we are said to be **solving the equations simultaneously**.

Example 21 Finding the point of intersection of two linear graphs

The graphs of $y = 2x + 5$ and $y = -3x$ are shown. Write their point of intersection.



Solution

From the graph, it can be seen that the point of intersection is $(-1, 3)$.

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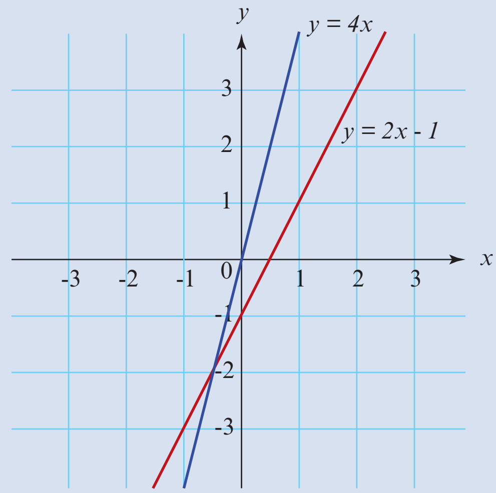
Now try this 21 Finding the point of intersection of two linear graphs (Example 21)

The graphs of $y = 2x - 1$ and $y = 4x$ are shown. Write their point of intersection.



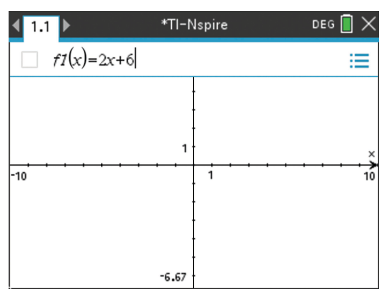
A CAS calculator can also be used to find the point of intersection.

How to find the point of intersection of two linear graphs using the TI-Nspire

Use a CAS calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

1 Start a new document (ctrl + N) and select **Add Graphs**.

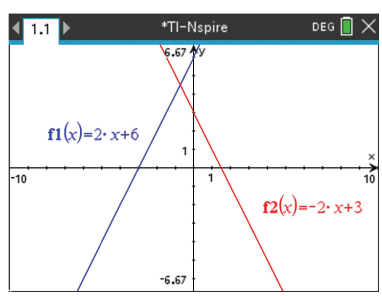


2 Type in the first equation as shown. Note that $f1(x)$ represents the y. Press ▼ and the edit line will change to $f2(x)$ and the first graph will be plotted. Type in the second equation and press enter to plot the second graph.

Hint: If the entry line is not visible, press tab.

Hint: To see all entered equations, move the cursor onto the and press del.

Note: To change window settings, press menu**>Window/Zoom>Window Settings** and change to suit. Press enter when finished.



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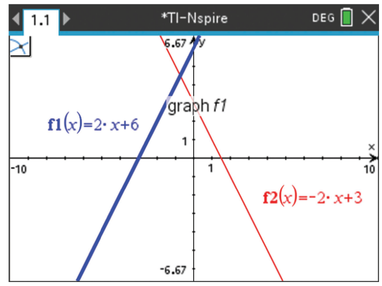
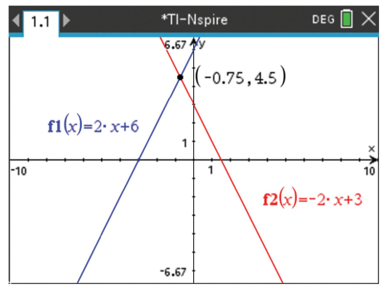
3 To find the point of intersection, press menu **>Geometry>Points & Lines>Intersection Point(s)**.
Move the cursor to one of the graphs until it flashes, press , then move to the other graph and press . The solution will appear.
Alternatively, use menu **>Analyze Graph>Intersection**.



4 Press enter to display the solution on the screen.
The coordinates of the point of intersection are $x = -0.75$ and $y = 4.5$.

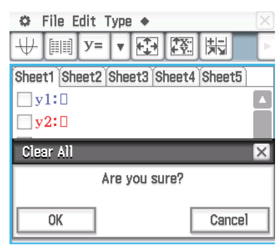


How to find the point of intersection of two linear graphs using the ClassPad

Use a CAS calculator to find the point of intersection of the simultaneous equations $y = 2x + 6$ and $y = -2x + 3$.

Steps

1 Open the built-in **Graphs and Tables** application. Tapping Menu from the icon panel (just below the touch screen) will display the Application menu if it is not already visible.



2 If there are any equations from previous questions, go to **Edit Clear all** and tap **OK**.

3 Enter the equations into the graph editor window. Tick the boxes.
Tap the icon to plot the graphs.

4 To adjust the graph window, tap **Zoom, Quick Standard**. Alternatively, tap the icon and complete the **View Window** dialog box.

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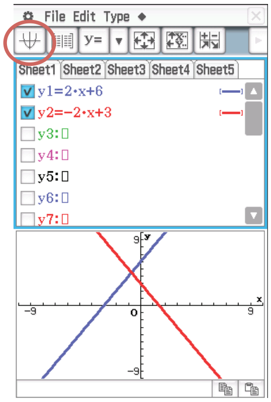
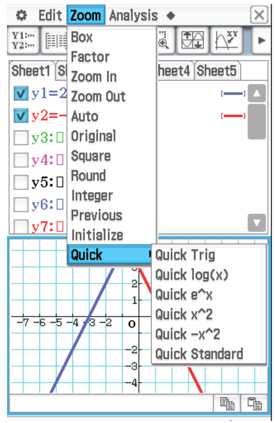
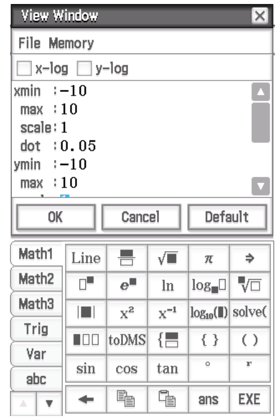
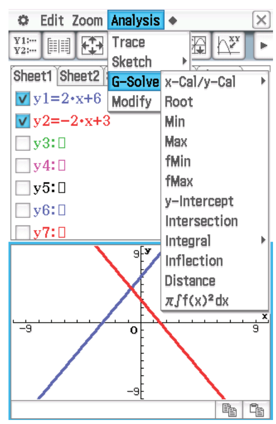
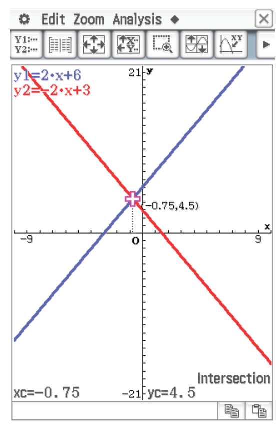






5 Solve by finding the point of intersection. Select **Analysis** > **G-solve**> **Intersection**. Tap Resize to view graph only. The required solution is displayed on the screen: $x = -0.75$ and $y = 4.5$.





Solving simultaneous linear equations using a CAS calculator

Another way of solving simultaneous equations, rather than drawing the graphs on a CAS calculator, is to use the **solve simultaneous functions** on the CAS calculator.

How to solve a pair of simultaneous linear equations algebraically using the TI-Nspire

Solve the following pair of simultaneous equations:

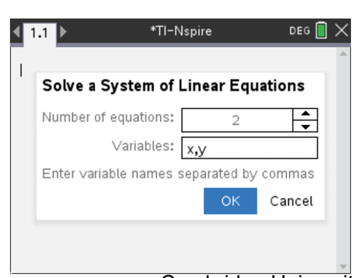
$$24x + 12y = 36$$

$$45x + 30y = 90$$

Steps

1 Start a new document and select **Add Calculator**.

2 Press menu **>Algebra>Solve System of Equations>Solve System of Linear Equations**. Complete the pop-up screen as shown.



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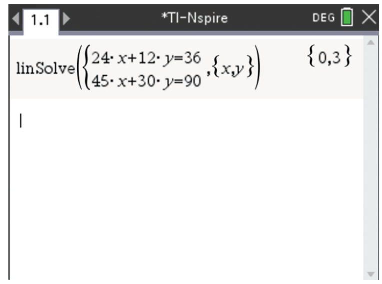
(The default settings are for two equations with variables x & y). A simultaneous equation template will be pasted to the screen.

3 Enter the equations as shown into the template. Use the tab key to move between entry boxes.

4 Press enter to display the solution, which is interpreted as $x = 0$ and $y = 3$.



5 Write your answer. $x = 0,\ y = 3$

How to solve a pair of simultaneous linear equations algebraically using the ClassPad

Solve the following pair of simultaneous equations:

$$24x + 12y = 36$$

$$45x + 30y = 90$$

Steps

1 Open the built-in **Main** application.

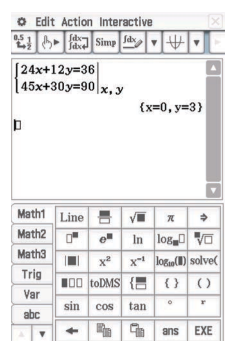
a Press Keyboard on the front of the calculator to display the built-in keyboard.

b Tap the simultaneous equations icon:

c Enter the information

$$\begin{cases} 24x + 12y = 36 \\ 45x + 30y = 90 \end{cases}\Bigg|_{x,\ y}$$

2 Press EXE to display the solution, $x = 0$ and $y = 3$.



3 Write your answer. $x = 0,\ y = 3$

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Section Summary

- When two straight lines intersect and the point of intersection is found, we are solving the equations **simultaneously**.
- **Simultaneous equations** can be solved graphically or by using a CAS calculator.

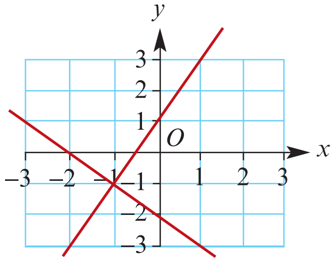
Exercise 5H

Building understanding

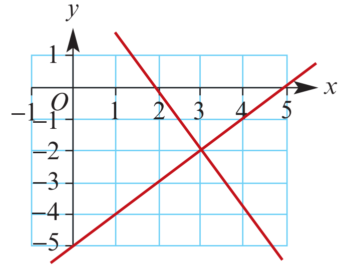
Example 21

1 For each of the following graphs, state the value of x where the two straight lines intersect.

a

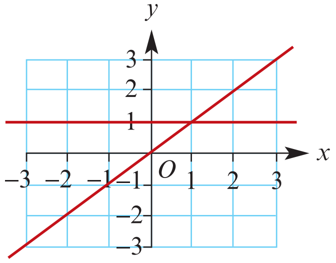


b

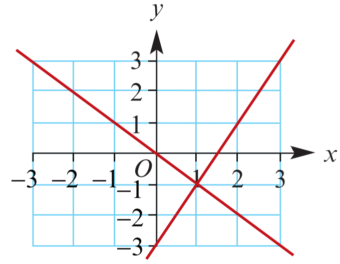


2 State the point of intersection for each of these pairs of straight lines.

a



b



Developing understanding

3 Using a CAS calculator, find the point of intersection of each of these pairs of lines.

a $y = x - 6$ and $y = -2x$ **b** $y = x + 5$ and $y = -x - 1$

c $y = 3x - 2$ and $y = 4 - x$ **d** $x - y = 5$ and $y = 2$

e $x + 2y = 6$ and $y = 3 - x$ **f** $2x + y = 7$ and $y - 3x = 2$

g $3x + 2y = -4$ and $y = x - 3$ **h** $y = 4x - 3$ and $y = 3x + 4$

Testing understanding

4 Solve the simultaneous equations, to one decimal place, using a CAS calculator.

a $2x + 5y = 3$
$x + y = 3$

b $3x + 2y = 5.5$
$2x - y = -1$

c $3x - 8y = 13$
$-2x - 3y = 8$

d $2m - n = 1$
$2n + m = 8$

e $15x - 4y = 6$
$-2y + 9x = 5$

f $2.9x - 0.6y = 4.8$
$4.8x + 3.1y = 5.6$

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5I Practical applications of simultaneous equations

Learning intentions

- To solve practical problems using simultaneous equations.

Simultaneous equations can be used to solve problems in real situations. It is important to define the unknown quantities with appropriate variables before setting up the equations.

Example 22 Using simultaneous equations to solve a practical problem

Mark buys 3 roses and 2 gardenias for \$15.50. Peter buys 5 roses and 3 gardenias for \$24.50. How much did each type of flower cost?

Explanation

Strategy: Using the information given, set up a pair of simultaneous equations to solve.

1 Choose appropriate variables to represent the cost of roses and the cost of gardenias.

2 Write equations using the information given in the question.

3 Use your CAS calculator to solve the two simultaneous equations.

4 Write down the solutions.

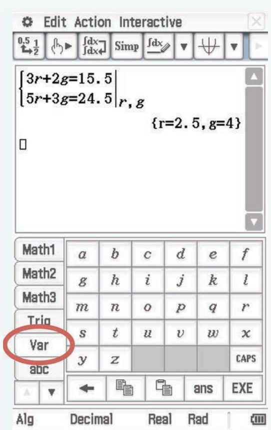
5 Check by substituting $r = 2.5$ and $g = 4$ into equation (2).

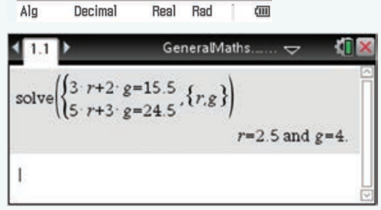
6 Write your answer with the correct units.

Solution

Let r be the cost of a rose and g be the cost of a gardenia.

$3r + 2g = 15.5 \quad (1)$
$5r + 3g = 24.5 \quad (2)$





$r = 2.50$ and $g = 4$

$\text{LHS} = 5(2.5) + 3(4)$
$\quad = 12.5 + 12 = 24.5 = \text{RHS}$

Roses cost \$2.50 each and gardenias cost \$4 each.

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Now try this 22 **Using simultaneous equations to solve a practical problem (Example 22)**

Ian bought 6 plain croissants and 6 chocolate croissants for \$46.20. Anne bought 5 plain croissants and 3 chocolate croissants for \$30.10. How much was each type of croissant?

Hint 1 Choose appropriate variables to represent the cost of plain croissants and chocolate croissants.

Hint 2 Write 2 equations using information from the question.

Hint 3 Use a CAS calculator to solve the simultaneous equations.

Example 23 **Using simultaneous equations to solve a practical problem**

The perimeter of a rectangle is 48 cm. If the length of the rectangle is three times the width, determine its dimensions.

Explanation	Solution
Strategy: Using the information given, set up a pair of simultaneous equations to solve.	
1 Choose appropriate variables to represent the dimensions of width and length.	Let w = width and l = length
2 Write two equations from the information given in the question. Label the equations as (1) and (2). Remember: The perimeter of a rectangle is the distance around the outside and can be found using $2w + 2l$. Note: If the length, l, of a rectangle is three times its width, w, then this can be written as $l = 3w$.	$2w + 2l = 48 \quad (1)$ $l = 3w \quad (2)$
3 Use your CAS calculator to solve the two simultaneous equations.	See Example 22.
4 Give your answer in the correct units.	The dimensions of the rectangle are width 6 cm and length 18 cm.

Now try this 23 **Using simultaneous equations to solve a practical problem (Example 23)**

The perimeter of a rectangle is 64 cm. If the length of one side of the rectangle is four times its width, determine its dimensions.

Hint 1 Choose appropriate variables to represent the length and the width of the rectangle.

Hint 2 Write two equations using information from the question.

Hint 3 Use a CAS calculator to solve the simultaneous equations.

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Example 24 Using simultaneous equations to solve a practical problem

Tickets for a movie cost \$19.50 for adults and \$14.50 for children. Two hundred tickets were sold, giving a total of \$3265. How many children's tickets were sold?

Explanation	Solution
Strategy: Using the information given, set up a pair of simultaneous equations to solve.	
1 Choose appropriate variables to represent the number of adult tickets sold and the number of children's tickets sold.	Let a be the number of adult tickets sold and c be the number of children's tickets sold.
2 Write two equations using the information given in the question. Note: The total number of adult and child tickets is 200, which means that $a + c = 200$.	$19.5a + 14.5c = 3265 \quad (1)$ $a + c = 200 \quad (2)$
3 Use your CAS calculator to solve the two simultaneous equations.	See Example 22.
4 Write down your solution.	127 children's tickets were sold.

Now try this 24 Using simultaneous equations to solve a practical problem (Example 24)

Tickets to a football game cost \$26.95 for adults and \$6.00 for children. In one morning, 498 tickets were sold, giving a total of \$9692. How many adult tickets were sold?

Hint 1 Choose appropriate variables to represent the cost of an adult ticket and the cost of a child's ticket.

Hint 2 Write two equations using information from the question, and solve with CAS.

Section Summary

- **Simultaneous equations** can be used to solve practical problems.

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Exercise 5I

Building understanding

Example 22

1 Jessica bought 5 crayons and 6 pencils for \$12.75, and Tom bought 7 crayons and 3 pencils for \$13.80.
Using c for crayon and p for pencil, complete the following to form a pair of simultaneous equations to solve.
$\square c + \square p = 12.75$
$\square c + \square p = 13.80$

2 Peter buys 50 litres of petrol and 5 litres of motor oil for \$109. His brother Anthony buys 75 litres of petrol and 5 litres of motor oil for \$146.

a Using p for petrol and m for motor oil, complete the following to form a pair of simultaneous equations to solve.
$\square p + \square m = 109$
$\square p + \square m = 146$

b How much does a litre of petrol cost?

c How much does a litre of motor oil cost?

3 Six oranges and ten bananas cost \$7.10. Three oranges and eight bananas cost \$4.60.

a Write down a pair of simultaneous equations to solve.

b How much does an orange cost?

c How much does a banana cost?

Developing understanding

4 The weight of a box of nails and a box of screws is 2.5 kg. Four boxes of nails and a box of screws weigh 7 kg. Determine the weight of each.

5 An enclosure at a wildlife sanctuary contains wombats and emus. If the number of heads totals 28 and the number of legs totals 88, determine the number of each species present.

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Example 23

6 The perimeter of a rectangle is 36 cm. If the length of the rectangle is twice its width, determine its dimensions.

7 The sum of two numbers x and y is 52. The difference between the two numbers is 8. Find the values of x and y.

8 The sum of two numbers is 35 and their difference is 19. Find the numbers.

9 Bruce is 4 years older than Michelle. If their combined age is 70, determine their individual ages.

10 A boy is 6 years older than his sister. In three years' time he will be twice her age. What are their present ages?

11 A chocolate thickshake costs \$2 more than a fruit smoothie. Jack pays \$27 for 3 chocolate thickshakes and 4 fruit smoothies. What is the cost of

a a chocolate thickshake?

b a fruit smoothie?

12 In 4 years' time, a mother will be three times as old as her son. Four years ago she was five times as old as her son. Find their present ages.

Example 24

13 The registration fees for a mathematics competition are \$1.20 for students aged 8–12 years and \$2 for students 13 years and over. One hundred and twenty-five students have already registered and an amount of \$188.40 has been collected in fees. How many students between the ages of 8 and 12 have registered for the competition?

14 A computer company produces 2 laptop models: standard and deluxe. The standard laptop requires 3 hours to manufacture and 2 hours to assemble. The deluxe model requires $5\frac{1}{2}$ hours to manufacture and $1\frac{1}{2}$ hours to assemble. The company allows 250 hours for manufacturing and 80 hours for assembly over a limited period. How many of each model can be made in the time available?

15 A chemical manufacturer wishes to obtain 700 litres of a 24% acid solution by mixing a 40% solution with a 15% solution. How many litres of each solution should be used?

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Testing understanding

16 In a hockey club there are 5% more boys than there are girls. If there is a total of 246 members in the club, what is the number of boys and the number of girls?

17 The owner of a service station sells unleaded petrol for \$1.42 per litre and diesel fuel for \$1.54 per litre. In five days he sold a total of 10 000 litres and made \$14 495. How many litres of each type of petrol did he sell? Give your answer to the nearest litre.

18 James had \$30 000 to invest. He chose to invest part of it at 5% and the other part at 8%. Overall he earned \$2100 in interest. How much did he invest at each rate?

19 The perimeter of a rectangle is 120 metres. The length is one and a half times the width. Calculate the width and length.

5J Piecewise linear graphs

Learning intentions

- ► To be able to construct and analyse piecewise linear graphs.
- ► To be able to construct and analyse step graphs.

Sometimes a situation requires two linear graphs to obtain a suitable model. The graphs we use to model such situations are called **piecewise linear graphs**.

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Example 25 Constructing a piecewise linear graph model

The amount, C dollars, charged to supply and deliver x m^3 of crushed rock is given by the equations:

$$C = 50 + 40x \qquad (0 \le x < 3)$$

$$C = 80 + 30x \qquad (3 \le x \le 8)$$

a Use the appropriate equation to determine the cost to supply and deliver the following amounts of crushed rock.

i 2.5 m^3 **ii** 3 m^3 **iii** 6 m^3

b Use the equations to construct a piecewise linear graph for $0 \le x \le 8$.

Explanation

a **1** Write the equations.

2 Then, in each case:

- choose the appropriate equation.
- substitute the value of x and evaluate.
- write down your answer.

b The graph has two line segments.

1 Determine the coordinates of the end points of both lines.

2 Draw a set of labelled axes and mark in the points with their coordinates.

3 Join up the end points of each line segment with a straight line.

4 Label each line segment with its equation.

Solution

a

$C = 50 + 40x \qquad (0 \le x < 3)$

$C = 80 + 30x \qquad (3 \le x \le 8)$

i When $x = 2.5$
$C = 50 + 40(2.5) = 150$
Cost for 2.5 m^3 of crushed rock is \$150.

ii When $x = 3$
$C = 80 + 30(3) = 170$
Cost for 3 m^3 of crushed rock is \$170.

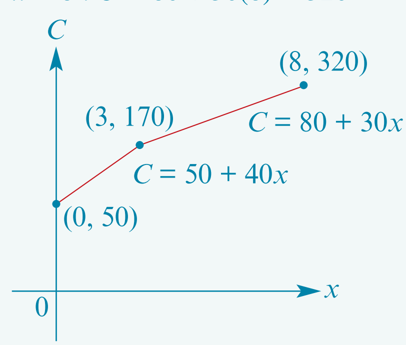
iii When $x = 6$,
$C = 80 + 30(6) = 260$
Cost for 6 m^3 of crushed rock is \$260.

b $x = 0 : C = 50 + 40(0) = 50$

$x = 3 : C = 50 + 40(3) = 170$

$x = 3 : C = 80 + 30(3) = 170$

$x = 8 : C = 80 + 30(8) = 320$



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Now try this 25 **Constructing a piecewise linear graph model (Example 25)**

The amount, C dollars, charged to supply and deliver x m^3 of sand is given by the equations:

$$C = 20 + 30x \qquad (0 \le x < 4)$$

$$C = 60 + 20x \qquad (4 \le x \le 10)$$

a Use the appropriate equation to determine the cost to supply and deliver the following amounts of sand.

i $3.5\ m^3$ **ii** $4\ m^3$ **iii** $8\ m^3$

b Use the equations to construct a piecewise linear graph for $0 \le x \le 10$.

Hint 1 Decide whether 3.5, 4 and 8 are in the $(0 \le x < 4)$ or $(4 \le x \le 10)$ interval. This will tell you in which equation to substitute $x = 3.5$, $x = 4$ and $x = 8$.

Hint 2 Sketch the two graphs on the same axes. They will join up at $x = 4$.

Hint 3 Label each line segment.

Step graphs

A **step graph** is a type of piecewise graph that uses line segments where each line segment is horizontal.

Example 26 **Constructing a step graph**

Sophie invests \$10 000, which earns compound interest of 12% per annum. The interest is calculated at the end of each year and added to the amount invested; i.e. \$10 000 is invested at 12% per annum compound interest.

The amount of money she has in the account for the first 5 years is shown in the table. Sketch the graph of the amount in the account against the year.

Time period (years)	Amount of interest earned (to the nearest dollar)	Total amount
0–1	0	10 000
1–2	1200	11 200
2–3	1344	12 544
3–4	1505	14 049
4–5	1686	15 735

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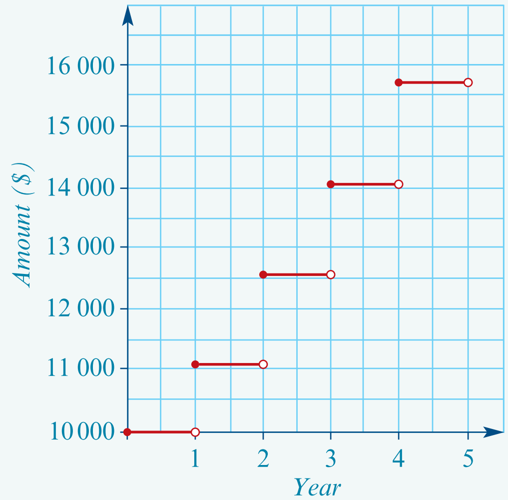
Explanation

For each time interval, draw the appropriate line segment. In this case, ●——○ indicates that change takes place at the end of the year.

Solution



Section Summary

- A piecewise linear graph uses two or more linear graphs.
- A step graph uses line segments where each line segment is horizontal.

Exercise 5J

Building understanding

1 Consider the piecewise linear equation:

$$C = 90 + 10x \qquad (0 \le x < 4)$$
$$C = 50 + 20x \qquad (4 \le x < 8)$$

Which equation would you use to find C if:

a $x = 2$

b $x = 6$

2 Consider the piecewise linear equation:

$$D = 45 - 5t \qquad (0 \le t < 3)$$
$$D = 90 - 20t \qquad (3 \le t < 8)$$

Which equation would you use to find D if:

a $t = 2$

b $t = 3$

c $t = 7$

3 Sketch the following piecewise equation.

$$y = 1 + 3x \qquad (0 \le x < 3)$$
$$y = 4 + 2x \qquad (3 \le x \le 6)$$

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Developing understanding

Example 25

4 An empty tank is being filled from a mountain spring. For the first 30 minutes, the equation giving the volume, V, of water in the tank (in litres) at time, t minutes, is:

$$V = 15t \quad (0 \le t \le 30)$$

After 30 minutes, the flow from the spring slows down. For the next 70 minutes, the equation giving the volume of water in the tank at time, t, is given by the equation:

$$V = 150 + 10t \quad (30 < t \le 100)$$

a Use the appropriate equation to determine the volume of water in the tank after:

i 20 minutes **ii** 30 minutes **iii** 60 minutes **iv** 100 minutes.

b Use the equations to construct a piecewise linear graph for $0 \le t \le 100$.

5 A multistorey car park has tariffs as shown. Sketch a step graph showing this information.

First 2 hours	\$5.00
2–3 hours (more than 2, less than or equal to 3)	\$7.50
3–4 hours (more than 3, less than or equal to 4)	\$11.00
4–8 hours (more than 4, less than or equal to 8)	\$22.00

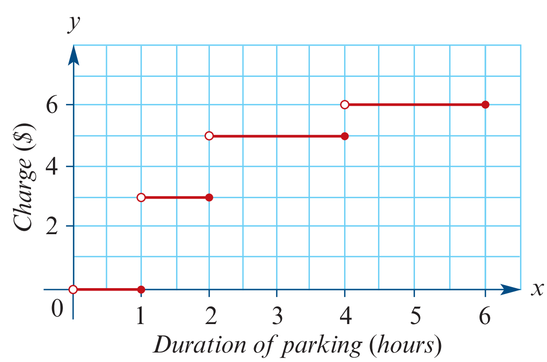
Example 26

6 This step graph shows the charges for a market car park.

a How much does it cost to park for 40 minutes?

b How much does it cost to park for 2 hours?

c How much does it cost to park for 3 hours?



Testing understanding

7 A National Park has an entrance fee of \$20 and charges \$10 per person for a guided tour with a group of 1 to 6 people. For a group of 7-10 people, the cost remains constant at \$80.

a Complete the following piecewise linear equation where C is the cost (\$) and n is the number of people.

$$C = \boxed{\quad\quad}, \qquad \left(1 \le n \le \boxed{\quad\quad}\right)$$

$$C = \boxed{\quad\quad}, \qquad \left(\boxed{\quad\quad} \le n \le \boxed{\quad\quad}\right)$$

b Sketch a graph of the piecewise linear equation.

c Find the cost for

i a family of 4 **ii** 10 people

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Key ideas and chapter summary

Assign-
ment

Formula

A **formula** is a mathematical relationship connecting two or more variables.

Linear equation

A **linear equation** is an equation whose unknown values are always to the power of 1.

Slope of a straight-line graph

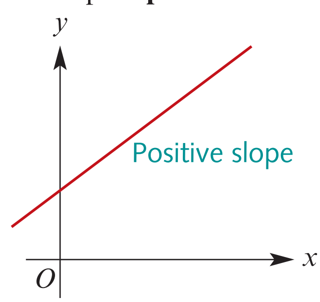
The **slope** of a straight-line graph is defined to be:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

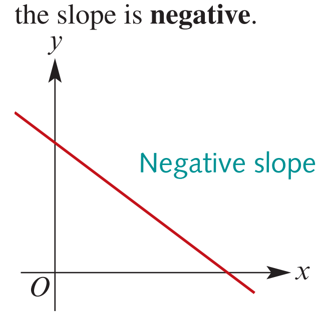
where (x_1, y_1) and (x_2, y_2) are two points on the line.

Positive and negative slope

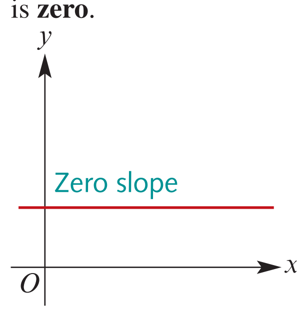
If the line rises to the right, the slope is **positive**.



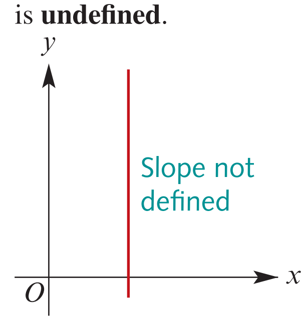
If the line falls to the right, the slope is **negative**.



If the line is horizontal, the slope is **zero**.



If the line is vertical, the slope is **undefined**.



Equation of a straight-line graph: the intercept–slope form

The equation of a straight line can take several forms.
The **intercept–slope form** is:

$$y = a + bx$$

where a is the **y-intercept** and b is the **slope** of the line.

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Linear model

A **linear model** has a linear equation or relation of the form:

$$y = a + bx \quad \text{where} \quad c \leq x \leq d$$

where a, b, c and d are constants.

Simultaneous equations

Two straight lines will always intersect, unless they are parallel. At the point of intersection the two lines will have the same coordinates. When we find the point of intersection, we are solving the equations simultaneously. **Simultaneous equations** can be solved graphically, algebraically or by using a CAS calculator.

Example:

$$3x + 2y = 6$$

$$4x - 5y = 12$$

are a pair of simultaneous equations.

Piecewise linear graphs

Piecewise linear graphs are used in practical situations where more than one linear equation is needed to model the relationship between two variables.

Step graphs

A **step graph** is a particular type of piecewise linear graph made of horizontal intervals or 'steps'.

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Review

Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

5A **1** I can substitute values into linear equations and formulas. ☐

e.g. The cost, C, of hiring a trailer is $C = 10t + 50$, where t is the time in hours. How much will it cost to hire the trailer for 6 hours?

5A **2** I can construct tables of values from given formulas. ☐

e.g. The weekly wage, W, of a car salesperson is given by $W = 790 + 40n$, where n is the number of cars sold. Construct a table of values to show how much their weekly wage will be if they sell 5 to10 cars.

5B **3** I can solve linear equations. ☐

e.g. Solve the linear equation $2x - 5 = 25$ for x.

5B **4** I can use linear equations to solve practical problems. ☐

e.g. A plumber charges \$100 up front and \$60 for each hour, h, that they work. How much do they earn if they work for 5 hours?

5C **5** I can develop formulas from descriptions. ☐

e.g. A sausage roll costs \$2.90 and a pie costs \$2.50. Write a formula showing the cost, C, of x sausage rolls and y pies.

5D **6** I can draw a straight-line graph using a CAS calculator or from constructing a table of values. ☐

e.g. Draw the graph of $y = -2x + 5$.

5E **7** I can find the slope of a straight line given two points on the line. ☐

e.g. Find the slope of the line that goes through the points $(1, 8)$ and $(4, 2)$.

5E **8** I can find the intercept and slope of a straight-line graph from its equation and from its graph. ☐

e.g. What is the y-intercept and the slope of the straight line $y = -7 + 2x$?

5F **9** I can find the equation of a straight line given the slope and the y-intercept. ☐

e.g. Find the equation of the line with a slope of 3, going through the point $(0, 5)$.

5F **10** I can find the equation of a straight line given two points on the graph. ☐

e.g. Find the equation of the line that goes through the points $(3, -1)$ and $(-2, 7)$.

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5G **11** I can construct a linear model to represent a practical situation using a linear equation or a straight-line graph. ☐

e.g. The height, h, of a plant is 70 cm tall when it is first planted. For the next 6 months, it grows 3 cm every month. Write down a linear model for the situation.

5H **12** I can solve simultaneous equations. ☐

e.g. Solve the simultaneous equations $5x - 2y = 17$ and $3x + 4y = 20$. Give your answer to two decimal places.

5I **13** I can use simultaneous equations to solve practical problems. ☐

e.g. Six croissants and four chocolate eclairs cost \$45.20 and five croissants and eight chocolate eclairs cost \$62.40. What is the price of a croissant?

5J **14** I can use piecewise linear graphs that model a practical situation. ☐

e.g. A car park is open for 10 hours a day. The cost, C, for the time, t, is given by the following piecewise equation.

$$C = 12t \qquad (0 \le t < 4)$$

$$C = 8 + 10t \qquad (4 \le t \le 10)$$

Find the cost for parking your car for 8 hours.

5J **15** I can draw and interpret step graphs. ☐

e.g. Draw a step graph to show the cost of hiring a chain saw. The cost is defined by:

One hour or less	\$40
More than 1 hour but less than or equal to 2 hours	\$65
More than 2 hours but less than or equal to 3 hours	\$90
More than 3 hours but less than or equal to 4 hours	\$115

Multiple-choice questions

1 If $a = 4$, then $3a + 5 =$

A 12 **B** 17 **C** 27 **D** 34 **E** 39

2 If $b = 1$, then $2b - 9 =$

A -11 **B** -7 **C** 12 **D** 13 **E** 21

3 If $C = 50t + 14$ and $t = 8$, then $C =$

A 72 **B** 414 **C** 512 **D** 522 **E** 1100

4 If $P = 2L + 2W$, then the value of P when $L = 6$ and $W = 2$ is:

A 12 **B** 14 **C** 16 **D** 30 **E** 48

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5 If $x = -2$, $y = 3$ and $z = 7$, then $\dfrac{z - x}{y} =$

A -3 **B** $-\dfrac{5}{3}$ **C** $\dfrac{5}{3}$ **D** 3 **E** 9

6 If $a = 2$, $b = 5$, $c = 6$ and $d = 10$, then $bd - ac =$

A 7 **B** 24 **C** 38 **D** 39 **E** 484

7 The solution to $4x = 24$ is:

A $x = 2$ **B** $x = 6$ **C** $x = 8$ **D** $x = 20$ **E** $x = 96$

8 The solution to $\dfrac{x}{3} = -8$ is:

A $x = -38$ **B** $x = -24$ **C** $x = -\dfrac{8}{3}$ **D** $x = \dfrac{8}{3}$ **E** $x = 24$

9 The solution to $2v + 5 = 11$ is:

A $v = 3$ **B** $v = 6$ **C** $v = 8$ **D** $v = 16$ **E** $v = 17$

10 The solution to $3k - 5 = -14$ is:

A $k = -6.3$ **B** $k = -3$ **C** $k = 3$ **D** $k = 19$ **E** $k = 115.67$

11 The cost of hiring a car for a day is \$60 plus 0.25c per kilometre. Michelle travels 750 kilometres. Her total cost is:

A \$187.50 **B** \$188.10 **C** \$247.50 **D** \$810 **E** \$18810

12 Given $v = u + at$ and $v = 11.6$ when $u = 6.5$ and $a = 3.7$, the value of t, to two decimal places, is:

A 1.37 **B** 1.378 **C** 1.38 **D** 4.89 **E** 9.84

13 The solution to the pair of simultaneous equations:

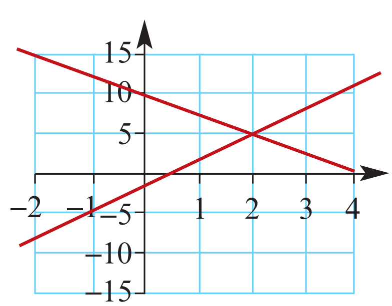
$y = 5x$

$y = 2x + 6$ is:

A $(-2, 0)$ **B** $(-1, -5)$ **C** $(3, 0)$ **D** $(2, 10)$ **E** $(5, 2)$

14 The point of intersection of the lines shown in the diagram is:

A $(5, 2)$ **B** $(0, 0)$ **C** $(0, 9)$
D $(2, 5)$ **E** $(4, 10)$



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15 The solution to the pair of simultaneous equations:

$2x + 3y = -6$

$x + 3y = 0$ is:

A $(-6, -2)$ **B** $(6, 2)$ **C** $(2, 3)$ **D** $(-2, 6)$ **E** $(-6, 2)$

16 The equation of a straight line is $y = 4 + 3x$. When $x = 2$, y is:

A 2 **B** 3 **C** 4 **D** 6 **E** 10

17 The equation of a straight line is $y = 5 + 4x$. The y-intercept is:

A 2 **B** 3 **C** 4 **D** 5 **E** 20

18 The equation of a straight line is $y = 10 - 3x$. The slope is:

A -3 **B** 0 **C** 3 **D** 7 **E** 10

19 The equation of a straight line is $y - 2x = 3$. The slope is:

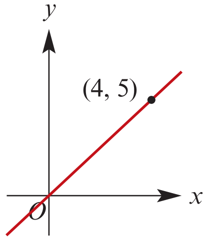
A -3 **B** -2 **C** 0 **D** 2 **E** 3

20 The slope of the line passing through the points (5, 8) and (9, 5) is:

A -1.3 **B** -1 **C** -0.75 **D** 0.75 **E** 1.3

21 The graph of $y = 4 + 5x$ is:

A



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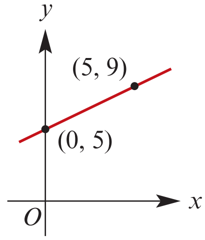

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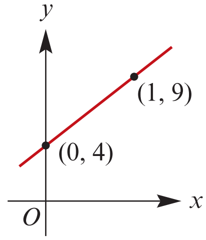

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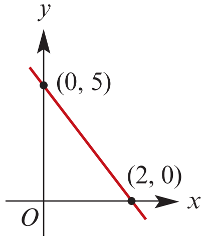

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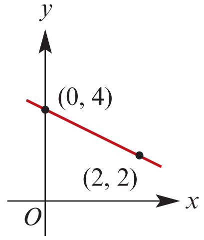

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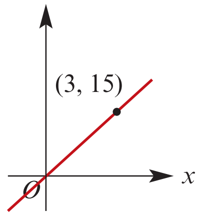
22 The graph of $y = 15 - 3x$ is:

A



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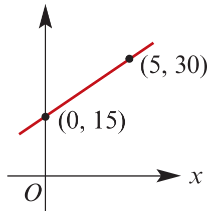

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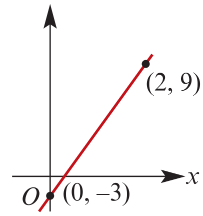

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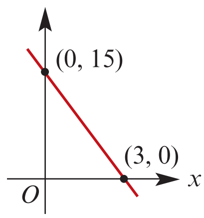

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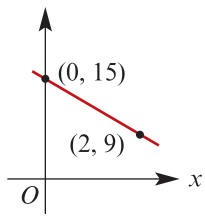
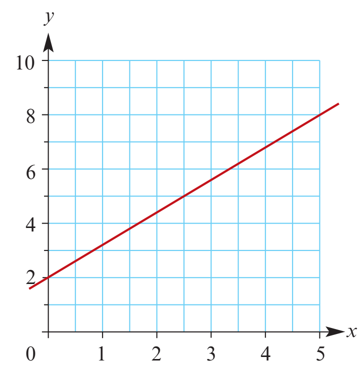


Questions 23 and 24 relate to the following graph.



23 The y-intercept is:

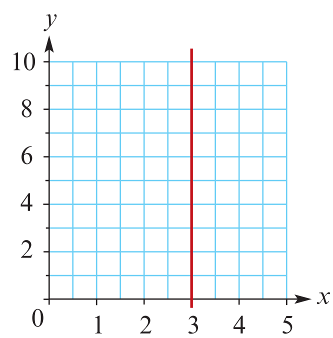
A -2 **B** 0 **C** 2 **D** 5 **E** 8

24 The slope is:

A 1.6 **B** 1.2 **C** 2 **D** 5 **E** 8

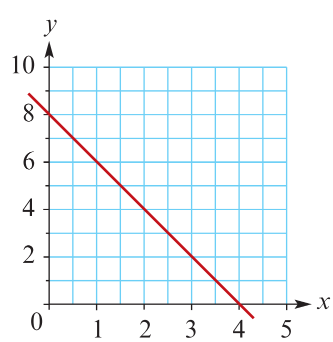
25 The slope of the line in the graph, shown opposite, is:

A negative
B zero
C positive
D three
E undefined



26 The equation of the graph, shown opposite, is:

A $y = -2 + 8x$
B $y = 4 - 2x$
C $y = 8 - 2x$
D $y = 4 + 2x$
E $y = 8 + 2x$



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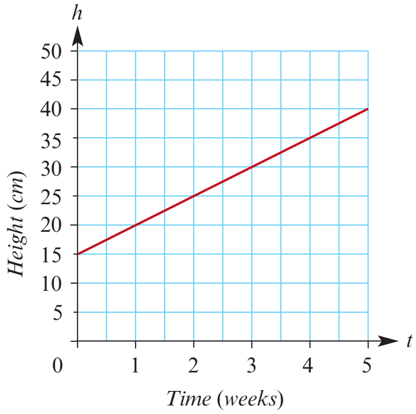
Review

27 Which of the following points lies on the line $y = -5 + 10x$?

A (1, −5) **B** (1, 5) **C** (1, 15) **D** (2, 20) **E** (2, 23)

28 The graph opposite shows the height of a small tree, h, as it increases with time, t. Its growth rate is closest to:

A 1 cm/week
B 3 cm/week
C 5 cm/week
D 8 cm/week
E 15 cm/week



Short-answer questions

1 Solve the following equations for x.

a $x + 5 = 15$ **b** $x - 7 = 4$ **c** $16 + x = 24$
d $9 - x = 3$ **e** $2x + 8 = 10$ **f** $3x - 4 = 17$
g $x + 4 = -2$ **h** $3 - x = -8$ **i** $6x + 8 = 26$
j $3x - 4 = 5$ **k** $\frac{x}{5} = 3$ **l** $\frac{x}{-2} = 12$

2 If $P = 2l + 2b$, find P if:

a $l = 12$ and $b = 8$ **b** $l = 40$ and $b = 25$.

3 If $A = \frac{1}{2}bh$, find A if:

a $b = 6$ and $h = 10$ **b** $b = 12$ and $h = 9$.

4 The formula for finding the circumference of a circle is given by $C = 2\pi r$, where r is the radius. Find the circumference of a circle with radius 15 cm, to two decimal places.

5 For the equation $y = 33x - 56$, construct a table of values for x in intervals of 5 from −20 to 25.

a For what value of x is $y = 274$?
b When $y = -221$, what value is x?

6 I think of a number, double it and add 4. If the result is 6, what is the original number?

7 Four less than three times a number is 11. What is the number?

8 Find the point of intersection of the following pairs of lines.

a $y = x + 2$ and $y = 6 - 3x$ **b** $y = x - 3$ and $2x - y = 7$
c $x + y = 6$ and $2x - y = 9$

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Review

9 Solve the following pairs of simultaneous equations.

a $y = 5x - 2$ and $2x + y = 12$

b $x + 2y = 8$ and $3x - 2y = 4$

c $2p - q = 12$ and $p + q = 3$

d $3p + 5q = 25$ and $2p - q = 8$

e $3p + 2q = 8$ and $p - 2q = 0$

10 Plot the graphs of these linear relations by hand.

a $y = 2 + 5x$

b $y = 12 - x$

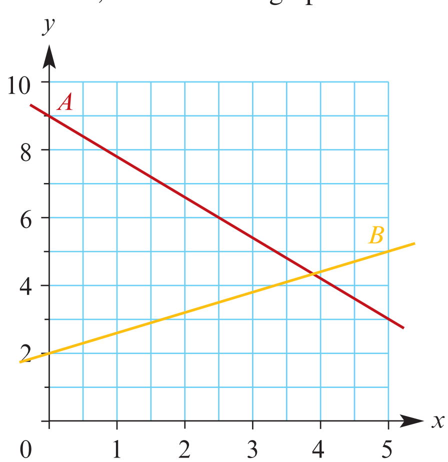
c $y = -2 + 4x$

11 A linear model for the amount C, in dollars, charged to deliver w cubic metres of builders' sand is given by $C = 95 + 110w$, for $0 \leq w \leq 7$.

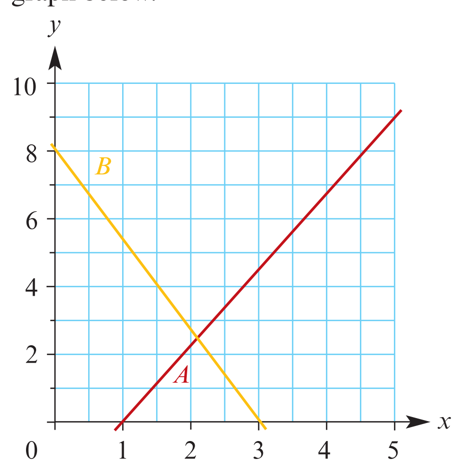
a Use the model to determine the total cost of delivering 6 cubic metres of sand.

b When the initial cost of \$95 is paid, what is the cost for each additional cubic metre of builders' sand?

12 Find the slope of each of the lines A and B, shown on the graph below.



13 Find the slope of each of the lines A and B (to two decimal places) shown on the graph below.



Written-response questions

1 The cost, C, of hiring a boat is given by $C = 25 + 8h$, where h represents hours.

a What is the cost if the boat is hired for 4 hours?

b For how many hours was the boat hired if the cost was \$81?

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2 A phone bill is calculated using the formula $C = 25 + 0.50n$, where n is the number of calls made.

a Complete the table of values below for values of n from 60 to 160.

n	60	70	80	90	100	110	120	130	140	150	160
C											

b What is the cost of making 160 phone calls?

3 An electrician charges \$80 up front and \$45 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b How much would a 3-hour job cost?

4 Two families went to the theatre. The first family bought tickets for 3 adults and 5 children and paid \$73.50. The second family bought tickets for 2 adults and 3 children and paid \$46.50.

a Write down two simultaneous equations that could be used to solve the problem.

b What was the cost of an adult's ticket?

c What was the cost of a child's ticket?

5 The perimeter of a rectangle is 10 times the width. The length is 9 metres more than the width. Find the width of the rectangle.

6 A secondary school offers three languages: French, Indonesian and Japanese. There are 105 students in Year 9. Each student studies one language. The Indonesian class has two-thirds the number of students that the French class has, and the Japanese class has five-sixths the number of students of the French class. How many students study each language?

7 A new piece of machinery is purchased by a business for \$300 000. Its value is then depreciated each month using the graph below.

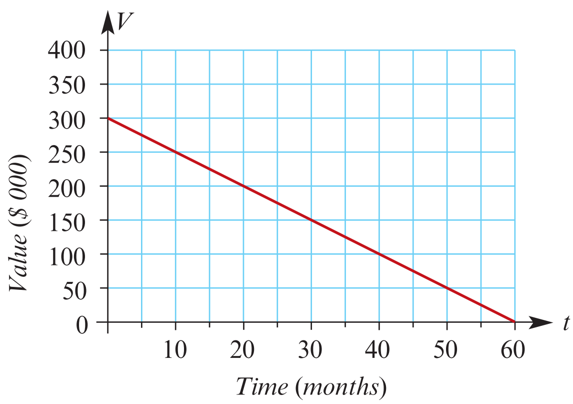
a What is the value of the machine after 20 months?

b When does the line predict that the machine has no value?

c Find the equation of the line in terms of value, V, (in thousands) and time, t.

d Use the equation to predict the value of the machine after 3 years.

e By how much does the machine depreciate in value each month?



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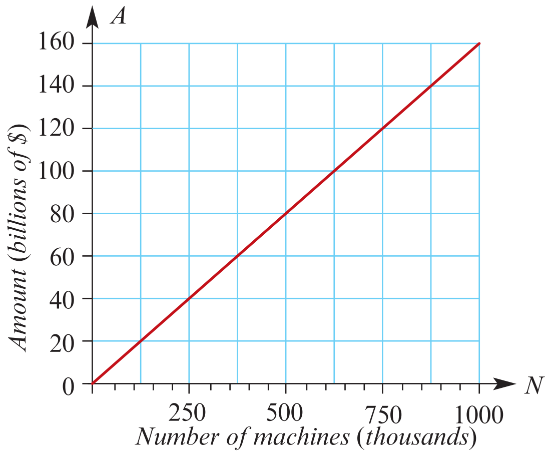
Review

8 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph charts this increase.

a What was the amount of money transacted through ATMs when there were 500 000 machines?

b Find the equation of the line in terms of amount of money transacted, A, and number of ATMs, N. (Leave A in billions and N in thousands).

c Use the equation to predict the amount transacted when there were 600 000 machines.



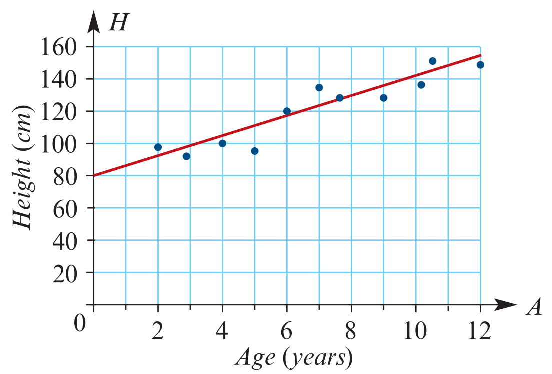
d If the same rule applies, how much money is predicted to be transacted through ATM machines when there are 1 500 000 machines?

e By how much does the amount of money transacted through ATMs increase with each 1000 extra ATMs?

9 The heights, H, of a number of children are shown, plotted against age, A. Also shown is a line of good fit.

a Find the equation of the line of good fit in terms of H and A.

b Use the equation to predict the height of a child, aged 3 (to the nearest cm).



c Complete the following sentence by filling in the box.
The equation of the line of good fit tells us that, each year, children's heights increase by ☐ cm.

10 To conserve water, one charging system increases the amount people pay as the amount of water used increases. The charging system is modelled by:

$$C = 5 + 0.4x \ (0 \le x < 30) \qquad C = -31 + 1.6x \ (x \ge 30)$$

C is the charge, in dollars, and x is the amount of water used, in kilolitres (kL).

a Use the appropriate equation to determine the charge for using:

i 20 kL **ii** 30 kL **iii** 50 kL

b How much does a kilolitre of water cost when you use:

i less than 30 kL? **ii** more than 30 kL?

c Use the equations to construct a piecewise graph for $0 \le x \le 50$.

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