Chapter 6

# Revision of Unit 1

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# 6A Multiple-choice questions

## Chapter 2: Investigating and comparing data distributions

**1** (VCAA-style question) In an experiment, researchers were interested in the effect of water on the growth of plants. They exposed groups of plants to three levels of water each day (5 – 10 mL, 15 – 20 mL, 25 – 30 mL), and then at the end of the experiment, classified the plants by size (as small, medium, large). The variables *levels of water* and *size* are:

**A** both ordinal variables
**B** a numerical variable and an ordinal variable respectively
**C** an ordinal variable and a numerical variable respectively
**D** an ordinal variable and a nominal variable respectively
**E** both numerical

**2** (VCAA-style question) For which of the following variables is a bar chart an appropriate display?

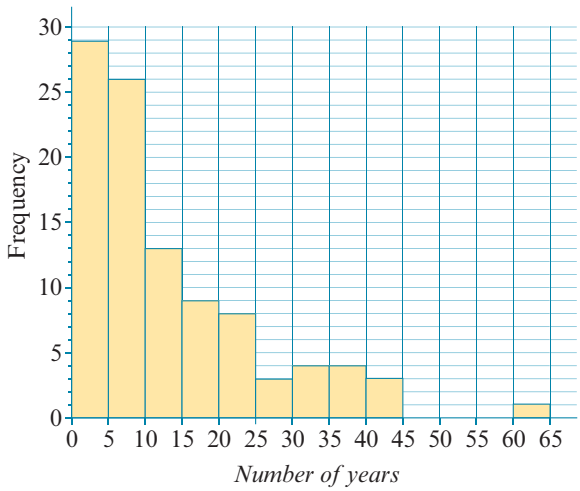
**A** Petrol consumption (km/litre)
**B** Distance between towns (km)
**C** Time to complete a puzzle (seconds)
**D** Price of houses ($)
**E** Coffee size (small, medium, large, jumbo)

**3** Which of the following displays are appropriate for numerical data:

**A** Dot plot and bar chart
**B** Dot plot and percentage bar chart
**C** Histogram and bar chart
**D** Histogram, dot plot and stem plot
**E** Stem plot, bar chart and percentage bar chart

*The following information relates to Questions 4 -7.*

A group of 100 people were asked how many years they have been doing their current type of work. Their responses are summarised in the following histogram.

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**4** The percentage of employees who have worked from 5 to less than 10 years is closest to:

**A** 29% **B** 26% **C** 55% **D** 13% **E** 39%

**5** (VCAA-style question) The median number of years employees have worked for their current employer is in the interval:

**A** 0 to less than 5 years **B** 5 to less than 10 years **C** 10 to less than 15 years
**D** 15 to less than 20 years **E** 20 to less than 25 years

**6** The modal interval for number of years worked for their current employer is:

**A** 0 to less than 5 years **B** 5 to less than 10 years **C** 10 to less than 15 years
**D** 15 to less than 20 years **E** 20 to less than 25 years

**7** (VCAA-style question) The shape of the histogram is:

**A** Positively skewed **B** Negatively skewed
**C** Symmetric **D** Bimodal
**E** Random

**8** (VCAA-style question) Ten people were asked how many cups of coffee they each drink a week, giving the following data:

7 5 10 2 5 7 5 14 21 0

The mean and standard deviation of this data are closest to:

**A** $\bar{x} = 7.6$ and $s = 5.8$ **B** $\bar{x} = 7.6$ and $s = 6.1$ **C** $\bar{x} = 5.8$ and $s = 5.5$
**D** $\bar{x} = 8.4$ and $s = 5.8$ **E** $\bar{x} = 8.4$ and $s = 5.5$

**9** A group of 500 people were asked how many hours per week they watched television. The five-number summary for the data collected is:

Min = 0 $Q_1 = 5$ $M = 18$ $Q_3 = 35$ Max = 92

The values of the lower and upper fences are:

**A** lower fence = 5 and upper fence = 35
**B** lower fence = 0 and upper fence = 92
**C** lower fence = −25 and upper fence = 65
**D** lower fence = −40 and upper fence = 80
**E** lower fence = 5 and upper fence = 80

**10** (VCAA-style question) Suppose the distribution of heart rates is approximately symmetric and bell shaped, with a mean of 80 beats per minute and a standard deviation of 10. Approximately what percentage of people have heart rates between 60 and 100?

**A** 50% **B** 68% **C** 95% **D** 99.7% **E** 100%

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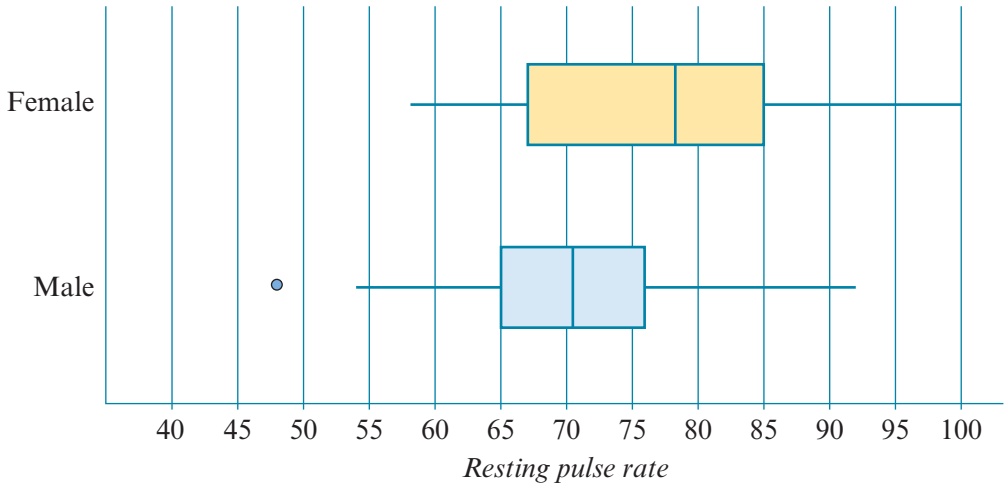
*The following information relates to Questions 11 and 12.*

The resting pulse rates in beats/min for a group of male students and a group of female students are summarised in the following boxplots.




**11** (VCAA-style question) Which of the following statements is not true?

**A** About 50% of male students have pulse rates greater than 70 beats/min.

**B** About 25% of female students have pulse rates more than 85 beats/min.

**C** More than 75% of female students have pulse rates higher than, at most, 25% of male students.

**D** The lowest pulse rate for males is about 54 beats/min.

**E** The interquartile range of the pulse rates for males is about 11.

**12** Using the median as a measure of the average pulse rate, which of the following can be concluded from the boxplots?

**A** The pulse rates of female students are lower on average and more variable than the pulse rates of male students.

**B** The pulse rates of female students are lower on average and less variable than the pulse rates of male students.

**C** The pulse rates of female students are higher on average and more variable than the pulse rates of male students.

**D** The pulse rates of female students are higher on average and less variable than the pulse rates of male students.

**E** The pulse rates of female students are higher on average and about the same in variability as the pulse rates of male students.

**13** (VCAA-style question) The distribution of the length (in mm) of a species of earth worm is bell shaped, with a mean of 75.5 mm and a standard deviation of 1.5 mm.

The percentage of the worms with a length less than 72.5 mm is closest to:

**A** 2.5% **B** 5% **C** 16% **D** 84% **E** 95%

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## Chapter 3: Sequences and finance

**14** (VCAA-style question) The sequence generated by the recurrence relation $V_0 = 6,\ V_{n+1} = V_n + 3$ is

**A** 6, 9, 46, 136, 406, . . . **B** 6, 9, 12, 15, 18, . . . **C** 6, 2, −2, −6, −10, . . .
**D** 6, 14, 44, 134, 401, . . . **E** 6, −14, 44, −134, . . .

**15** (VCAA-style question) The sequence generated by the recurrence relation $V_0 = 6,\ V_{n+1} = V_n - 4$ is

**A** 6, 16, 46, 136, 406, . . . **B** 6, 9, 12, 15, 18, . . . **C** 6, 2, −2, −6, −10, . . .
**D** 6, 14, 44, 134, 401, . . . **E** 6, −14, 44, −134, 404, . . .

**16** (VCAA-style question) Which of the following could be the first five terms, $t_0, t_1, t_2, t_3, t_4$, of an arithmetic sequence?

**A** 2, 4, 2, 4, 2 **B** 1, 10, 100, 1000, 10000 **C** −189, −89, 11, 111, 211
**D** 1, 4, 9, 16, 25 **E** 4, 4, 6, 6, 8

**17** Which of the following is not an arithmetic sequence?

**A** 11, 2, −8, −19, . . . **B** 4, 7, 10, 13, . . . **C** 57, 51, 45, 39, . . .
**D** −3, −5, −7, −9, . . . **E** 1, 2, 3, 4, . . .

**18** (VCAA-style question) Brian initially has two camelias in his garden. Every week, he will plant three more camelias.
A recurrence model for the number of camelias in Brian's backyard after $n$ weeks is

**A** $T_0 = 2,\ T_{n+1} = 3T_n$ **B** $T_0 = 2,\ T_{n+1} = 3T_n + 3$ **C** $T_0 = 2,\ T_{n+1} = T_n + 3$
**D** $T_0 = 2,\ T_{n+1} = T_n - 3$ **E** $T_0 = 2,\ T_{n+1} = 3T_n - 3$

**19** (VCAA-style question) Lee invests $40 000 with a bank. She will be paid simple interest at the rate of 5.1% per annum. If $V_n$ is the value of Lee's investment after $n$ years, a recurrence model for Lee's investment is

**A** $V_0 = 40\,000,\ V_{n+1} = V_n + 5.1$ **B** $V_0 = 40\,000,\ V_{n+1} = 5.1V_n$
**C** $V_0 = 40\,000,\ V_{n+1} = 0.051V_n + 102$ **D** $V_0 = 40\,000,\ V_{n+1} = V_n + 2040$
**E** $V_0 = 40\,000,\ V_{n+1} = 5.1V_n + 2000$

**20** A sequence is generated from the recurrence relation $V_0 = 50,\ V_{n+1} = V_n - 20$.
The rule for the value of the term $V_n$ is

**A** $V_n = 50n - 20$ **B** $V_n = 50 - 20n$ **C** $V_n = 50n$
**D** $V_n = 50 + 20n$ **E** $V_n = 50n - 20$

**21** (VCAA-style question) A computer is depreciated using a flat-rate depreciation method. It was purchased for $3200 and depreciates at the rate of 15% per annum.
The value of the computer after 4 years is

**A** $1280 **B** $1920 **C** $1760 **D** $2720 **E** $5120

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**22** For the arithmetic sequence with $t_0 = 39, t_1 = 36, t_2 = 33, \ldots$, the term $t_8$ is equal to

**A** 14 **B** 15 **C** 16 **D** 24 **E** 26

**23** For an arithmetic sequence with terms $t_0, t_1, t_2, \ldots$, it is known that $t_1 = 26$ and $t_5 = 10$. The term $t_3$ is equal to:

**A** 14 **B** 18 **C** 32 **D** 40 **E** 52

**24** (VCAA-style question) A recurrence relation is defined by: $t_{n+1} - t_n = -7$ with $t_0 = 3$. The term $t_n$ is equal to

**A** $3 + 7n$ **B** $3 + 6n$ **C** $-7 + 3n$ **D** $-3n + 7$ **E** $3 - 7n$

**25** Which one of the following is not a geometric sequence?

**A** $2, 30, 450, 6750 \ldots$ **B** $-3, 9, -27, 81, -243, 729, -2187 \ldots$

**C** $7, 21, 55, 165, \ldots$ **D** $1, 3, 9, 27, 81 \ldots$

**E** $60, 12, 2.4, 0.48, 0.096, \ldots$

**26** (VCAA-style question) A car purchased on 1 June 2022 loses value at a depreciation rate of 20% per year. The original purchase price was \$25 000. The value of the car, in dollars, on 1 June 2028 will be closest to

**A** \$7000 **B** \$8000 **C** \$9000 **D** \$10 000 **E** \$11 000

**27** (VCAA-style question) The following is a geometric sequence: 2000, 500, 125, 31.25, ... The common ratio, $R$, is equal to

**A** $-500$ **B** 0.25 **C** 0.5 **D** 0.75 **E** $-0.5$

**28** (VCAA-style question) The present value of an investment (to the nearest dollar) is \$72 000. It was reached by investing \$$P$ at a compound interest rate of 3% for 4 years. The value of $P$ was?

**A** \$63 741 **B** \$63 971 **C** \$64 008 **D** \$65 392 **E** \$81 036

## Chapter 4: Matrices

*Use matrices A and B in Questions 29 to 32.*

$$A = \begin{bmatrix} 3 & -5 \\ 1 & 8 \\ 2 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & -2 \\ -1 & 0 \\ 3 & 7 \end{bmatrix}$$

**29** The order of matrix $A$ is

**A** 6 **B** $2 \times 3$ **C** $3 \times 2$ **D** $2, 3$ **E** $3, 2$

**30** The element $a_{12}$ is

**A** 3 **B** $-5$ **C** 1 **D** 8 **E** 2

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**31** The matrix $A + B$ is

**A** $\begin{bmatrix} 8 & -3 \\ 2 & 8 \\ 5 & 3 \end{bmatrix}$ **B** $\begin{bmatrix} 8 & -3 \\ 2 & 8 \\ 5 & 11 \end{bmatrix}$ **C** $\begin{bmatrix} 8 & -7 \\ 2 & 8 \\ 5 & 3 \end{bmatrix}$ **D** $\begin{bmatrix} 8 & -7 \\ 0 & 8 \\ 5 & 3 \end{bmatrix}$ **E** $\begin{bmatrix} 8 & -3 \\ 0 & 8 \\ 5 & 3 \end{bmatrix}$

**32** The matrix $2A - B$ is

**A** $\begin{bmatrix} 1 & -8 \\ 1 & 16 \\ 1 & -15 \end{bmatrix}$ **B** $\begin{bmatrix} 1 & -6 \\ 1 & 16 \\ 1 & -15 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & -6 \\ 3 & 16 \\ 1 & -15 \end{bmatrix}$ **D** $\begin{bmatrix} 1 & -8 \\ 3 & 16 \\ 1 & -15 \end{bmatrix}$ **E** $\begin{bmatrix} 1 & -7 \\ 2 & 8 \\ -1 & -11 \end{bmatrix}$

*Use the matrices E, F, G, H in Questions 33 to 35.*

$$E = \begin{bmatrix} 4 & 5 & 2 \\ 0 & 9 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} \quad G = \begin{bmatrix} -2 & 8 \end{bmatrix} \quad H = \begin{bmatrix} 3 & 2 \\ -1 & 7 \end{bmatrix}$$

**33** Matrix multiplication is not defined for

**A** $EF$ **B** $GH$ **C** $GE$ **D** $HE$ **E** $EH$

**34** The order of matrix $FG$ is

**A** $1 \times 1$ **B** $1 \times 2$ **C** $3 \times 2$ **D** 6 **E** 5

**35** The inverse of matrix $H$ is

**A** $\begin{bmatrix} 7 & 2 \\ 1 & 3 \end{bmatrix}$ **B** $\frac{1}{23}\begin{bmatrix} 7 & -2 \\ 1 & 3 \end{bmatrix}$ **C** $\begin{bmatrix} 7 & -2 \\ 1 & 3 \end{bmatrix}$ **D** $\frac{1}{23}\begin{bmatrix} 7 & 2 \\ 1 & 3 \end{bmatrix}$ **E** $23\begin{bmatrix} 7 & -2 \\ 1 & 3 \end{bmatrix}$

**36** Three students were asked the number of pets they each had. The following matrix shows the number of pets owned by the three friends.

| | *Dogs* | *Cats* | *Fish* |
|---|---|---|---|
| *Daniel* | 2 | 1 | 0 |
| *Eloise* | 1 | 0 | 4 |
| *Freddy* | 1 | 3 | 0 |

The number of pets owned by Freddy's family is

**A** 1 **B** 2 **C** 3 **D** 4 **E** 5

**37** The matrix opposite shows the number of different social media applications used by three friends: Manesh ($M$), Nikita ($N$) and Paolo ($P$).
The total number of ways that Manesh directly connects to each of his friends is:

| | $M$ | $N$ | $P$ |
|---|---|---|---|
| $M$ | 0 | 3 | 2 |
| $N$ | 3 | 0 | 4 |
| $P$ | 2 | 4 | 0 |

**A** 1 **B** 2 **C** 3 **D** 4 **E** 5

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**38** (VCAA-style question) The matrix below shows how five people, Anna ($A$), Ben ($B$), Charis ($C$), Damien ($D$) and Emma ($E$), can communicate with each other. A '1' in the matrix indicates that the sender (row) can send a message directly to the receiver (column).

| Sender | Receiver: $A$ | $B$ | $C$ | $D$ | $E$ |
|---|---|---|---|---|---|
| $A$ | 0 | 1 | 1 | 0 | 0 |
| $B$ | 1 | 0 | 1 | 0 | 0 |
| $C$ | 1 | 1 | 0 | 1 | 1 |
| $D$ | 0 | 0 | 1 | 0 | 1 |
| $E$ | 0 | 0 | 1 | 1 | 0 |

Damien wishes to send a message to Anna. Which one of the following shows the order through which the message could be sent?

**A** $D - C - A$ **B** $D - E - A$

**C** $D - C - E - B - A$ **D** $D - E - B - A$

**E** $D - C - E - A$

**39** (VCAA-style question)

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} v \\ w \\ x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 3 \\ 8 \end{bmatrix}$$

Which of the following systems of simultaneous linear equations best represents the matrix equation above?

**A**

$$\begin{aligned} v + 2x + y &= 5 \\ 2v + w &= 13 \\ w + 2x + y &= 3 \\ v + x + 2y &= 8 \end{aligned}$$

**B**

$$\begin{aligned} v + 2w + y &= 5 \\ w + x &= 13 \\ 2v + 2x + y &= 3 \\ v + x + 2y &= 8 \end{aligned}$$

**C**

$$\begin{aligned} v &= 5 \\ w &= 13 \\ x &= 3 \\ y &= 8 \end{aligned}$$

**D**

$$\begin{aligned} v + 2w + x &= 5 \\ 2v + w &= 13 \\ v + 2w + x &= 3 \\ v + w + 2x &= 8 \end{aligned}$$

**E**

$$\begin{aligned} v &= 5 \\ w &= 3 \\ x &= -1 \\ y &= 12 \end{aligned}$$

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## Chapter 5: Linear relations and modelling

**40** If $a = 2$, $b = 3$ and $c = 5$, then $ab - c$ is:

**A** 0 **B** 1 **C** 18 **D** 27 **E** 28

**41** The solution to $2x + 3 = 12$ is:

**A** 2.4 **B** 4.5 **C** 7.5 **D** 9 **E** 18

**42** The equation of a straight line is $y = 3x - 7$. When $x = 5$, $y$ is:

**A** $-7$ **B** 5 **C** 8 **D** 15 **E** 28

**43** A music band charges a fixed fee of \$400, plus \$150 per hour of music.
The equation that represents the total amount, \$$C$, charged for $t$ hours of music is:

**A** $C = 150t$ **B** $C = 400t$ **C** $C = 550t$
**D** $C = 400 + 150t$ **E** $C = 150 + 400t$

**44** The slope and the $y$-intercept of the straight line $y = 3 - 2x$ are:

**A** slope = 1, $y$-intercept = 3 **B** slope = 2, $y$-intercept = 3
**C** slope = $-2$, $y$-intercept = 3 **D** slope = 3 $y$-intercept = 2
**E** slope = 3 $y$-intercept = $-2$

**45** The solution to the simultaneous equations:

$$2x - 3y = 5$$
$$7x + 8y = 3$$

is:

**A** $(-0.78, 1.32)$ **B** $(1, -1)$ **C** $(5, 3)$ **D** $(1.32, -0.78)$ **E** $(2.5, 2.7)$

**46** If 5 times a number added to 71 gives 101, then the number is:

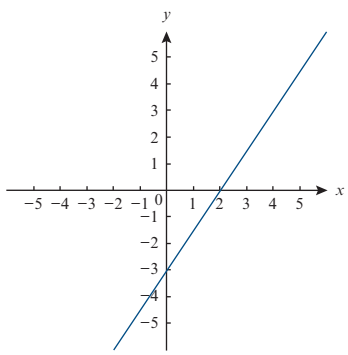
**A** 3 **B** 6 **C** 25 **D** $34\frac{2}{5}$ **E** 53

**47** The slope of the line passing through the points $(3, -1)$ and $(4, 1)$ is:

**A** $-2$ **B** 0 **C** $\frac{1}{2}$ **D** 2 **E** $\frac{7}{2}$

**48** The equation of the graph shown is:

**A** $y = -3 - 2x$
**B** $y = -2 - 3x$
**C** $y = -3 + 2x$
**D** $y = 1.5x - 3$
**E** $y = 3 - 2x$

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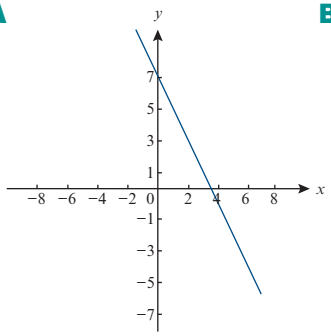
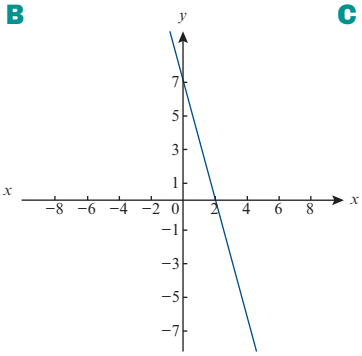
**49** The graph of $y = 7 - 2x$ is:

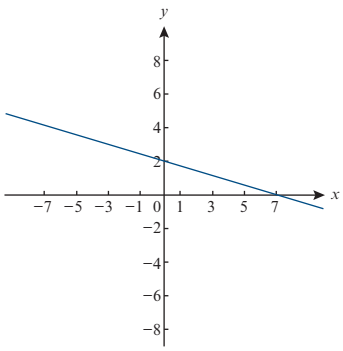
**A**



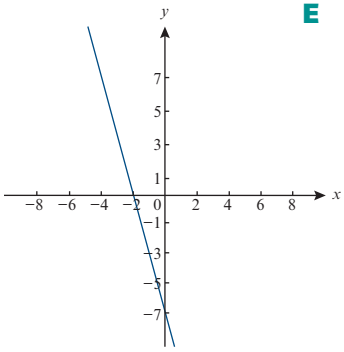

**B**



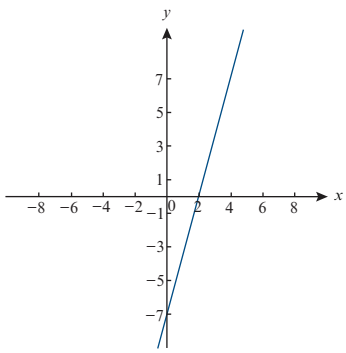

**C**




**D**




**E**



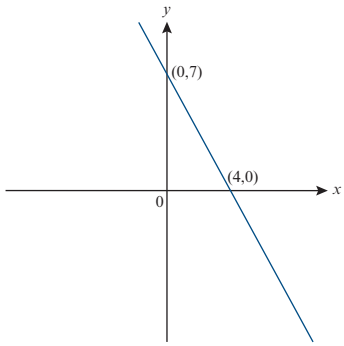

**50** The solution to the following simultaneous equations is:

$$3x + 6y = 60$$
$$x + 9y = 69$$

**A** $x = 6, y = 6$ **B** $x = 6, y = 7$ **C** $x = 7, y = 6$ **D** $x = 3, y = 6$ **E** $x = 1, y = 9$

**51** The graph shows a line intersecting the $x$-axis at (4, 0) and the $y$-axis at (0, 7).
The gradient of the line is:

**A** $-\frac{4}{7}$ **B** $-\frac{7}{4}$ **C** $\frac{4}{7}$ **D** $\frac{7}{4}$ **E** 3

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**52** (VCAA-style question) A sports store is having a sale. Each soccer ball costs $15 and each basketball costs $30. Chloe plans to restock the sporting equipment at her school. She buys 8 balls for a total of $165. The number of basketballs that Chloe buys is:

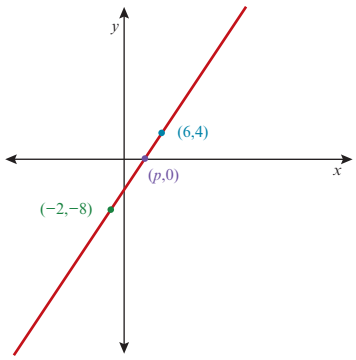
**A** 1 **B** 2 **C** 3 **D** 4 **E** 5

**53** (VCAA-style question) Sebastian makes and sells postcards. Each postcard costs 70 cents to make and sells for $3.50. Sebastian also has a fixed cost for making a batch of postcards. On Saturday, Sebastian sold 62 postcards for a profit of $153.60. Sebastian's fixed cost for making a batch of postcards is:

**A** $ 20 **B** $ 100 **C** $ 173.60 **D** $ 4123 **E** $ 4276.60

*Use the following information to answer Questions 54 and 55.*

The graph below shows a straight line that passes through the points (6,4), (−2, −8) and ($p$,0).




**54** (VCAA-style question) The coordinates of another point on the line are:

**A** (4,0) **B** (5,3) **C** (7,5) **D** (12,13) **E** (15,15)

**55** (VCAA-style question) The value of $P$ is:

**A** 1 **B** 2 **C** 5/2 **D** 3 **E** 10/3

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# 6B Written-response questions

## Chapter 2: Investigating and comparing data distributions

**1** A group of 200 people was asked to describe their general happiness by choosing one of the responses: very happy, pretty happy, and not too happy. The data is summarised in the following frequency table:

| *General happiness* | Frequency | |
|---|---|---|
| | Number | % |
| Very happy | 53 | 26.5 |
| Pretty happy | | 59.0 |
| Not too happy | 29 | |
| Total | 200 | 100.0 |

**a** What type of data has been collected: categorical or numerical?

**b** Complete the frequency table and the percentage frequency table.

**c** Construct a percentage bar chart.

**d** Write a report summarising the responses, quoting appropriate percentages to support your conclusion.

**2** The following data gives daily rainfall, in mm, for a city for the 31 days of May:

0 4 0 1 2 0 0 6 1 9 10 0

0 1 2 3 2 1 0 1 1 0 0 0

2 8 9 1 1 7 3

**a** Construct a dot plot of the data.

**b** Determine the values of:

**i** the range

**ii** the median

**iii** the interquartile range.

**c** In what percentage of the days was there no rain?

**d** Construct a histogram of the rainfall data, starting at 0 and with interval widths of 2.

**3** Annual salaries data for a group of employees in a company were collected in 2016 and again in 2020.

**a** A five-number summary for the salaries in the Marketing team of the company in 2016 was as follows:

Min = \$29 000 $Q_1$ = \$49 000 $M$ = \$63 000 $Q_3$ = \$92 000 Max = \$133 000

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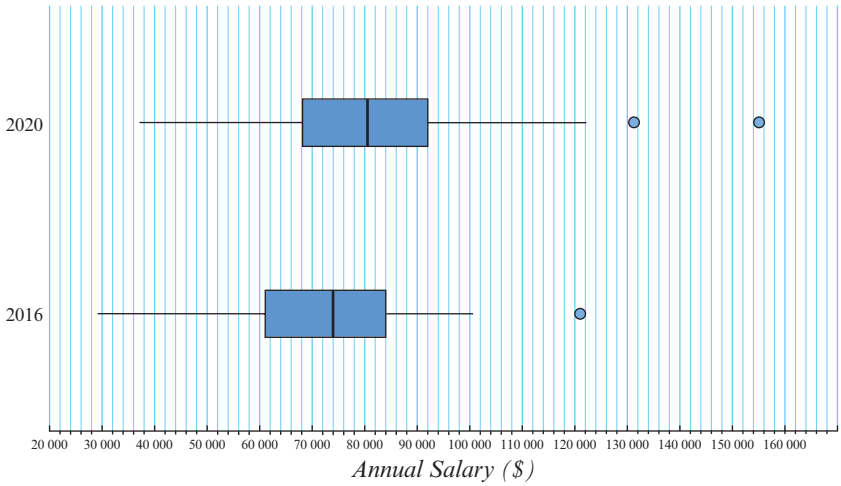
**i** Use the five-number summary to construct a boxplot.

**ii** Determine the values of the upper and lower fences, and use these to confirm that there are outliers.

**iii** What percentage of Marketing team employees earned more than $49 000?

**b** The boxplots below display the distribution of the salaries of all employees in 2016 and 2020:




**i** What is the median salary for each year?

**ii** Use the information from the boxplots to write a short report comparing the salaries at the company in 2016 and in 2020.

**4** The number of goals scored by each member of two basketball teams in the finals is as follows:

Team A

20 19 16 14 11 10 9 8 8 7 6 4 2 0 0

Team B

31 30 15 13 13 12 10 6 6 4 2 2 2 2 0

**a** Which of the variables, *team* or *score*, is a categorical variable?

**b** Construct back-to-back stem plots of this data, using the following stems:

**c** Determine the five-number summary for each team.

**d** Identify any outliers in the scores for each team.

**e** How do you think the mean score for Team A would compare to the mean score for Team B? Give a reason for your answer.

| Team A | *Score* | Team B |
|---|---|---|
| | 0 | |
| | 0 | |
| | 1 | |
| | 1 | |
| | 2 | |
| | 2 | |
| | 3 | |

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## Chapter 3: Sequences and finance

**5** The following recurrence relation can be used to model a simple interest investment of $2000, paying interest at the rate of 3.8% per annum.

$$V_0 = 2000, \qquad V_{n+1} = V_n + 76$$

In the recurrence relation, $V_n$ is the value of the investment after $n$ years.

**a** Use the recurrence relation to find the value of the investment after 1, 2 and 3 years.

**b** Use your calculator to determine how many years it takes for the value of the investment to first be more than $3000.

**c** Write down a recurrence relation model if $1500 was invested at the rate of 6.0% per annum.

**6** Jane and Kate each inherit $50 000 from a long-lost relative.

**a** Jane decides to loan her money to a friend, who agrees to pay her $750/month for 10 years.

**i** How much does the friend repay in total, including monthly repayments and the principal.

**ii** What is the equivalent simple interest rate per annum of this investment?

**b** Kate decides to invest her money in an investment account, which pays 6.75% compound interest per annum, compounding monthly.

**i** How much will Kate have in her investment account after 10 years?

**ii** Find the lowest annual interest rate, correct to 1 decimal place, that would make Kate's investment worth at least as much as Jane's investment after 10 years if interest compounds monthly.

**7** The following recurrence relation can be used to model the depreciation of a computer with purchase price $2500 and annual depreciation of $400.

$$V_0 = 2500, \qquad V_{n+1} = V_n - 400$$

In the recurrence relation, $V_n$ is the value of the computer after $n$ years.

**a** Use the recurrence relation to find the value of the computer after 1, 2 and 3 years.

**b** Use your calculator to determine how many years it takes for the value of the computer to be less than $1000.

**c** Write down a recurrence relation model if the computer was purchased for $1800 and depreciated at $350 per annum.

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**8** A factory has a tall chimney stack from which a pollutant gas is emitted at the rate of 1500 kilograms per day. New technology has been developed that enables the emissions to be reduced in stages to a minimum of 200 kilograms per day.

**a** Using one method of installation, the emissions will be reduced by a constant amount each day until the minimum emission of 200 kilograms per day is reached. Consider the case where the emissions are reduced by 130 kilograms each day. The installation will be completed after 12 days, and from the 13th day, the emissions will be 200 kilograms per day. Let $E_i$ be the emissions after $i$ days of the installation process.

**i** Use this information to complete the table below.

| Day($i$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $E_i$ | 1500 | 1370 | 1240 | | | | | | | | |

**ii** Write the recurrence relation in terms of $E_i$ to describe the reduction in emissions for $0 \leq i \leq 12$.

**iii** Write down the rule for $E_n$ in terms of $n$, where $0 \leq n \leq 12$.

**b** Now suppose that the installation is to be completed by the end of the fifth day ($K_5$), so that from the sixth day, the emissions will be 200 kilograms per day. ($K_i$ is the emissions after $i$ days of the new installation process) .

**i** By what constant amount must the emissions be reduced each day during the installation period?

**ii** Draw a graph for the sequence for $0 \leq i \leq 5$.

**iii** Write the recurrence relation in terms of $K_i$ to describe the reduction in emissions for $0 \leq i \leq 5$.

**iv** Write down the rule for $K_n$ in terms of $n$, where $0 \leq n \leq 5$.

**9** The number of rabbits on a remote island is causing concern. Scientists sent to investigate the problem find that, on their arrival, there are 360 000 rabbits on the island.

**a** If the rabbit population on the island increases at a constant rate of 12 000 rabbits per week:

**i** How many rabbits will be on the island one week after the scientists' arrival?

**ii** Write down a rule for the number of rabbits on the island, recorded at $n$ weeks after the arrival of the scientists.

**iii** How many weeks would it take for this rabbit population to grow to 600 000?

**iv** Calculate the constant weekly increase that would lead to a rabbit population of one million on this island, recorded at 40 weeks after the scientists' arrival.

*Continued*

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**b** To help reduce the rabbit population, the scientists, upon their arrival on the island, introduced a viral infection into the rabbit population. The scientists kept a record of the number of rabbits killed each week by the virus, with the following results for the first three weeks:

| Week number ($n$) | 1 | 2 | 3 |
|---|---|---|---|
| Number of rabbits killed by virus | 512 | 768 | 1152 |

The scientists found that the number of rabbits killed each week by the virus followed a geometric sequence

**i** Show that the common ratio, $R$, for the sequence is 1.5.

**ii** Calculate the number of rabbits that die from the virus in Week 5.

**iii** Calculate the total number of rabbits that die from the virus during the first six weeks of the scientists' stay on the island.

## Chapter 4: Matrices

**10** Mary and Derek are fitness enthusiasts. Each week, they record how far they run, cycle and swim in km. The information for last week is recorded in matrix $F$.

$$F = \begin{matrix} \\ Mary \\ Derek \end{matrix} \begin{matrix} \begin{matrix} Run & Cycle & Swim \end{matrix} \\ \begin{bmatrix} 38 & 85 & 4 \\ 12 & 130 & 0 \end{bmatrix} \end{matrix}$$

**a** How many kilometres did Mary do in total last week?

**b** What is the total distance that Mary and Derek ran last week?

**c** Mary cycles to and from work each day (Monday to Friday) and then goes for a longer ride on the weekends. If her workplace is 7 km from her home, how far does she cycle on the weekend?

**11** A sushi shop records the sales for two stores (A and B) of Handrolls, Sashimi packs and Maki packs in matrix $S$. The prices were recorded in the prices matrix, $P$.

$$S = \begin{matrix} \\ A \\ B \end{matrix} \begin{matrix} \begin{matrix} Handrolls & Sashimi & Maki \end{matrix} \\ \begin{bmatrix} 60 & 15 & 8 \\ 45 & 21 & 10 \end{bmatrix} \end{matrix} \qquad P = \begin{matrix} \\ Handrolls \\ Sashimi \\ Maki \end{matrix} \begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{bmatrix} 4 \\ 15 \\ 10 \end{bmatrix} \end{matrix}$$

**a** How many Handrolls were sold by Shop $B$?

**b** What is the selling price of the Sashimi Packs?

**c** Calculate the matrix product $SP$.

**d** What information is contained in matrix $SP$?

**e** Which shop had the larger income from its sales? How much were its takings?

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## Chapter 5: Linear relations and modelling

**12** A shop sells fruit in two types of gift boxes: standard and deluxe. Each standard box contains 1 kg of peaches and 2 kg of apples, and each deluxe box contains 2 kg of peaches and 1.5 kg of apples. On one particular day, the shop sold 12 kg of peaches and 14 kg of apples in gift boxes. How many of each kind of box were sold on the day?

**13** A gardener charges a fixed fee of $50 plus $15 for each hour of work.

**a** Using $t$ for time (in hours), write a formula for the total cost, $C$, of $t$ hours of work.

**b** Sketch a graph of the cost, $C$.

**c** Martha paid the gardener $76.25. How many hours did the gardener work?

**14** A person's total body water - the amount of water in their body - is dependent on their gender, age, height and weight.
Total body water (TBW) for males and females can be found by using the following formulas:

$$\underset{\text{(litres)}}{\textit{Male TBW}} = 2.447 - 0.09516 \times \underset{\text{(years)}}{\textit{age}} + 0.1074 \times \underset{\text{(cm)}}{\textit{height}} + 0.3362 \times \underset{\text{(kg)}}{\textit{weight}}$$

$$\underset{\text{(litres)}}{\textit{Female TBW}} = -2.097 + 0.1069 \times \underset{\text{(cm)}}{\textit{height}} + 0.2466 \times \underset{\text{(kg)}}{\textit{weight}}$$

**a** What is the TBW for a female of height 175 cm and weight 62 kg? Give your answer correct to two decimal places.

**b** Calculate the TBW, correct to two decimal places, for a 45-year-old male of height 184 cm and weight 87 kg.

**c** A healthy 27-year-old female has a TBW of 32 and weighs 62 kg. What is her height, to the nearest cm?

**d** What would be the TBW, correct to two decimal places, for a 78-year-old man of height 174 cm and weight 80 kg?

**e** Over a period of a week, the 78-year-old man's weight rapidly increases to 95 kg. What is his new TBW, correct to two decimal places?

**f** Construct a table showing the TBW for a 22-year-old male of height 185 cm, with weights in increments of 5 kg from 60–120 kg. Give your answers correct to two decimal places.

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# 6C Investigations

## Statistics investigation

**1** To investigate the age of parents at the birth of their first child, a hospital recorded the ages of the mothers and fathers for the first 40 babies born in the hospital for each of the years 1970, 1990 and 2010. The data is given below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1970 Mother | | | | | | | | | |
| 23 | 22 | 33 | 19 | 19 | 26 | 20 | 15 | 26 | 17 |
| 18 | 31 | 24 | 20 | 29 | 28 | 25 | 45 | 28 | 22 |
| 1970 Father | | | | | | | | | |
| 29 | 15 | 39 | 29 | 22 | 35 | 32 | 26 | 37 | 29 |
| 25 | 31 | 20 | 34 | 28 | 22 | 33 | 25 | 34 | 46 |
| 1990 Mother | | | | | | | | | |
| 28 | 14 | 38 | 28 | 21 | 34 | 31 | 25 | 36 | 28 |
| 24 | 30 | 19 | 33 | 27 | 21 | 32 | 24 | 33 | 45 |
| 1990 Father | | | | | | | | | |
| 31 | 27 | 46 | 31 | 26 | 28 | 30 | 27 | 43 | 37 |
| 39 | 22 | 27 | 35 | 31 | 29 | 32 | 27 | 38 | 35 |
| 2010 Mother | | | | | | | | | |
| 30 | 26 | 45 | 32 | 25 | 27 | 29 | 26 | 42 | 36 |
| 38 | 21 | 26 | 34 | 37 | 28 | 28 | 37 | 37 | 34 |
| 2010 Father | | | | | | | | | |
| 37 | 31 | 39 | 36 | 21 | 34 | 34 | 23 | 17 | 37 |
| 23 | 33 | 31 | 32 | 24 | 39 | 45 | 30 | 35 | 34 |

**a** **i** Construct parallel boxplots of the mothers' and fathers' ages in 1970.

**ii** Determine the median and interquartile range for the ages of the mothers and fathers in 1970.

**iii** Based on the analyses in part **ai** and part **aii**, write a report comparing the ages of mothers and fathers in 1970.

**b** **i** Construct parallel boxplots of the mothers' and fathers' ages in 1990.

**ii** Determine the median and interquartile range for the ages of the mothers and fathers in 1990.

**iii** Based on the analyses in part **bi** and part **bii**, write a report comparing the ages of mothers and fathers in 1990.

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**c** **i** Construct parallel boxplots of the mothers' and fathers' ages in 2010.

**ii** Determine the median and interquartile range for the ages of the mothers and fathers in 2010.

**iii** Based on the analyses in part **ci** and part **cii**, write a report comparing the ages of mothers and fathers in 2010.

**d** Use your previous analyses to describe the changes in mothers' ages and fathers' ages over time.

**2** Does Year 12 require more study time than Year 11? Carry out a statistical investigation to answer this question. The statistical investigation process is the principal means of problem-solving and modelling in statistics, and it encompasses the following steps which you should follow:

- Pose the question – Decide what variables would allow you to address the problem. Options could include the number of hours each week students spend on their homework, or the number of homework sessions per week for each year level.
- Data – Collect the data from a sample of students at each of the year levels, or obtain it if it already exists. The sample size should be large enough to analyse: at least 20 students at each year level would be desirable.
- Analyse – Summarise and display the data to answer the question posed. Here, the analysis methods would include summary statistics and boxplots.
- Conclusion – Interpret the results and communicate what has been learned in a written report.

## Matrices investigation - Encoding and decoding

**3** In the past, commonly used codes replaced each letter of the alphabet with a randomly chosen number or symbol. The intended recipient could use a list of the changes to change each number back to a letter. The weakness in this type of code is that, in the English language, *E* is the most frequently occurring letter, followed by *T* and then *A*. A table of frequencies for letters can be used to replace numbers occurring with about the same frequency, and hence, to break the code.

Matrices can be used to encode words so that each letter does *not* have the same number throughout the coded message. This makes the message extremely difficult to decode, without knowing the secret encoding matrix. We will use this simple table with the letters numbered in order, but use a matrix to encode the final message.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

| O | P | Q | R | S | T | U | V | W | X | Y | Z | space |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |

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## Example 1

Encode the message: 'Meet me,' then show how to decode the encoded message. Use the encoding matrix, $C$, where:

$$C = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

### Explanation

#### Encoding the message

**a** Write the letters in a $2 \times 4$ matrix, $M$.

**b** Use the table on the opposite page to replace each letter with its number.
Do all of the following steps using your graphics calculator.

**c** Multiply the message matrix, $M$, by the secret encoding matrix, $C$.
Notice that the three Es now have three different numbers in this encoded message matrix, $CM$.

#### Decoding the message

**a** Work out the *inverse* of the secret encoding matrix. Use a calculator.

**b** Multiply the encoded message matrix, $CM$, by the inverse of the encoding matrix, $C^{-1}$, to return to the message matrix, $M$.

**c** Use the table above to replace each number with its letter.

### Solution

$$M = \begin{bmatrix} \text{M} & \text{E} & \text{E} & \text{T} \\ \text{M} & \text{E} & & \end{bmatrix}$$

$$M = \begin{bmatrix} 13 & 5 & 5 & 20 \\ 27 & 13 & 5 & 27 \end{bmatrix}$$

$$CM = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 13 & 5 & 5 & 20 \\ 27 & 13 & 5 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 107 & 49 & 25 & 121 \\ 67 & 31 & 15 & 74 \end{bmatrix}$$

The inverse of $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ is $C^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.

$$C^{-1} \times CM = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 107 & 49 & 25 & 121 \\ 67 & 31 & 15 & 74 \end{bmatrix}$$

$$M = \begin{bmatrix} 13 & 5 & 5 & 20 \\ 27 & 13 & 5 & 27 \end{bmatrix}$$

$$M = \begin{bmatrix} \text{M} & \text{E} & \text{E} & \text{T} \\ \text{M} & \text{E} & & \end{bmatrix}$$

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A $2 \times 2$ matrix can be used to encode any information written as a matrix with two rows. Credit card numbers consisting of 16 digits can be written into a $2 \times 8$ matrix and encoded using a $2 \times 2$ matrix.
Choose a $2 \times 2$ encoding matrix with small positive numbers so that the numbers in the encoded message do not get too large.

| A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

| O | P | Q | R | S | T | U | V | W | X | Y | Z | space |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |

**a** Use the table for swapping letters with numbers, and use matrix $C$ to *encode* each of the following messages.

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

**i** $\begin{bmatrix} F & B & I & \\ K & N & O & W \end{bmatrix}$ **ii** $\begin{bmatrix} M & A & P & \\ L & O & S & T \end{bmatrix}$

**iii** $\begin{bmatrix} A & P & E & \\ F & A & C & E \end{bmatrix}$ **iv** $\begin{bmatrix} F & I & N & D \\ & T & O & M \end{bmatrix}$

**v** $\begin{bmatrix} N & O & & G & U & A & R & D \\ & T & O & N & I & G & H & T \end{bmatrix}$ **vi** $\begin{bmatrix} M & E & E & T & & A & N & N \\ A & T & & J & O & H & N & S \end{bmatrix}$

**b** Use the letters-to-numbers table and the *inverse* of matrix $S$ to *decode* the following messages.

$$S = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

**i** $\begin{bmatrix} 10 & 27 & 33 & 62 \\ 19 & 45 & 53 & 97 \end{bmatrix}$ **ii** $\begin{bmatrix} 28 & 41 & 52 & 59 \\ 47 & 62 & 85 & 91 \end{bmatrix}$

**iii** $\begin{bmatrix} 20 & 39 & 63 & 59 \\ 34 & 66 & 101 & 91 \end{bmatrix}$ **iv** $\begin{bmatrix} 37 & 65 & 29 & 79 \\ 56 & 109 & 44 & 131 \end{bmatrix}$

**v** $\begin{bmatrix} 22 & 45 & 48 & 67 & 73 & 15 & 36 & 59 \\ 40 & 75 & 82 & 114 & 119 & 23 & 57 & 91 \end{bmatrix}$ **vi** $\begin{bmatrix} 26 & 51 & 30 & 35 & 58 & 23 & 39 & 58 \\ 46 & 84 & 55 & 66 & 89 & 37 & 59 & 89 \end{bmatrix}$

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**c** Use the encoding matrix, $B$, to do the following.

$$B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

**i** Encode the credit card number written into this $2 \times 8$ matrix:

$$\begin{bmatrix} 3 & 1 & 4 & 7 & 2 & 3 & 8 & 1 \\ 6 & 0 & 5 & 8 & 9 & 3 & 0 & 7 \end{bmatrix}$$

**ii** Decode the encoded credit card number received in this matrix:

$$\begin{bmatrix} 7 & 10 & 8 & 13 & 12 & 0 & 12 & 12 \\ 8 & 19 & 9 & 19 & 20 & 0 & 16 & 21 \end{bmatrix}$$

**d** Make up an encoded message to fit within a $2 \times 8$ matrix. Give a classmate the $2 \times 2$ encoding matrix, and see whether they can decode it.

**e** Work with a matrix with 3 rows, and use a $3 \times 3$ matrix to encode.

## Financial mathematics investigation

**4** One of the first big purchases you are likely to make is buying your first car. How much can you afford to pay for that car, and how will you go about financing the purchase?

In this investigation, you are going to use available resources to determine the best strategy. You will need to investigate each of the following questions.

**a** What can you afford?

Assuming that you will need to finance the car, what can you afford to repay each week or fortnight? You will need to consider your likely salary, as well as your other living costs. Some of the major banks will give advice regarding this amount and include 'affordability' calculators on their websites.

**b** How should you finance the car?

Compare some different forms of finance (such as variable interest personal loans, fixed interest personal loans, credit cards), some different financial institutions, as well as some of the financing options offered directly by the car dealerships to determine your best finance option.

**c** What car should you buy?

Cars often depreciate in value very quickly, especially if they are purchased new. In the worst-case scenario, you can end up owing more money on a car than its current market value! Compare the depreciation of two or three different brands of car when purchased new, and at various stages over the period of time for which you have decided to finance it. How much will the car be worth when it is finally paid for?

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**5** Leonne is repaying $250 000 at $200 a month. She also decides to pay back $2500 at the end of the first year, $5000 at the end of the second, $7500 at the end of the third and so on.

**a** Use a spreadsheet to give a table of the amount owing each year until there is zero owing. The first three years and the spreadsheet rules are given here.

| n | An |
|---|---|
| 0 | 250000 |
| 1 | 245100 |
| 2 | 237700 |
| 3 | 227800 |

| n | An |
|---|---|
| 0 | 250000 |
| 1 | =B2-2400-2500*A3 |
| =A3+1 | =B3-2400-2500*A4 |
| =A4+1 | =B4-2400-2500*A5 |

**b** Now consider what happens when Leonne is also paying 1.5% per annum interest on the amount owing each year. Use your spreadsheet to calculate the amount owing each year. How much longer does it take to repay the loan?

**c** Experiment to find the maximum interest which is payable, and for her to be able to pay the $250 000 back in less than 10 years if she uses the same repayment method.

## Linear relations and modelling investigation

**6** As part of its urban renewal strategy, Camtown Council makes $\frac{1}{4}$ hectare of land available for building middle-income homes. The project manager decides to build 10 houses on blocks of varying sizes.

There are five small blocks, three medium-sized blocks and two large blocks. The medium-sized blocks are 100 $m^2$ larger than the small blocks, and the large blocks are 200 $m^2$ larger than the medium-sized ones.

What are the sizes of the blocks?

Note: 1 hectare = 10 000 $m^2$

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