Chapter 10

Measurement, scale and similarity

Chapter questions

- How do we make useful approximate values for measurements?
- What is Pythagoras' theorem?
- How do we use Pythagoras' theorem?
- How do we find the perimeter of a shape?
- How do we find the area of a shape?
- What is a composite shape?
- How do we find the volume of an object?
- How do we find the surface area of a shape?
- What does it mean when we say that two figures are similar?
- What are the tests for similarity for triangles?
- How do we know whether two solids are similar?
- How can we use a scale factor to make a similar but larger object?

Measurement explores length, area, volume and capacity. Approximate and more concise ways of writing numbers are reviewed. Pythagoras' theorem is used to find lengths within two-dimensional and three-dimensional objects. Perimeters and areas of various shapes are investigated, including similar shapes, and we explore using scale factor to compare objects of similar shapes but different sizes.

ISBN 978-1-009-11034-1

10A Approximations, decimal places and significant figures

Learning intentions

- To be able to round numbers to the required accuracy.
- To be able to express numbers in scientific notation.
- To be able to round numbers to the required significant figures.

Approximations are useful when it is not practical to give exact numerical values. Some numbers are too long (e.g. 0.573 128 9 or 107 000 000 000) to work with, and they are rounded to make calculations easier. Some situations do not require an exact answer, and a stated degree of accuracy is often sufficient.

Rules for rounding

Rules for rounding

1. Look at the value of the digit to the right of the specified digit.
2. If the value is 5, 6, 7, 8 or 9, *round the specified digit up*.
3. If the value is 0, 1, 2, 3 or 4, *leave the specified digit unchanged*.

Example 1 Rounding to the nearest thousand

Round 34 867 to the nearest thousand.

Explanation	Solution
1 Look at the first digit to the right of the thousands. It is an 8.	⇓ 34 867
2 As it is 5 or more, increase the thousands digit by one. So the 4 becomes a 5. The digits to the right all become zero. Write your answer. Note: 34 867 is closer to 35 000 than 34 000.	35 000

Now try this 1 Rounding to the nearest thousand (Example 1)

Round 57 642 to the nearest thousand.

Decimal places

23.798 is a decimal number with three digits after the decimal point. The first digit after the decimal point (7) is the first (or one) decimal place. Depending on the required accuracy, we round to one decimal place, two decimal places, etc.

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Example 2 Rounding to a number of decimal places

Round 94.738 295 to two decimal places.

Explanation	Solution
1 For two decimal places, count two places to the right of the decimal point and look at the digit to the right (8).	94.738 295
2 As 8 is '5 or more', increase the digit of the second decimal place by one. (3 becomes 4) Write your answer.	= 94.74 (to two decimal places)

Now try this 2 Rounding to a number of decimal places (Example 2)

Round 43.632 697 to two decimal places.

Scientific notation (standard form)

When we work with very large or very small numbers, we often use *scientific notation*, also called *standard form*.

To write a number in scientific notation, we express it as a number between 1 and 10, multiplied by a power of 10. More precisely, we use a number greater than or equal to 1, and less than 10.
For example, 134.7 written in scientific notation is 1.347×10^2.
Similarly, 0.0823 written in scientific notation is 8.23×10^{-2}.

Example 3 Writing a number in scientific notation

Write the following numbers in scientific notation.

a 7 800 000 **b** 0.000 000 5

Explanation	Solution
a **1** Place a decimal point to the right of the first non-zero digit.	7.800 000
2 Count the number of places the decimal point needs to move and whether it is to the left or right.	 Decimal point needs to move 6 places to the right from 7.8 to make 7 800 000.
3 To move the decimal point 6 places to the right, we need to multiply by 10^6. Write your answer.	$7\,800\,000 = 7.8 \times 10^6$

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b **1** Place a decimal point after the first non-zero digit.

5.0

2 Count the number of places the decimal point needs to move and whether it is to the left or right.

7 places
0.0 0 0 0 0 0 5

Decimal point needs to move 7 places to the left from 5.0 to make 0.000 000 5

3 To move the decimal point 7 places to the left, we need to multiply by 10^{-7}. Write your answer.

$0.000\,000\,5 = 5.0 \times 10^{-7}$

Now try this 3 — Writing a number in scientific notation (Example 3)

Write the following numbers in scientific notation.

a 670 000

b 0.000 006

Example 4 — Writing a scientific notation number as a basic numeral

Write the following scientific notation numbers as basic numerals.

a 3.576×10^{7}

b 7.9×10^{-5}

Explanation / Solution

a **1** Multiplying 3.576 by 10^{7} means that the decimal point needs to be moved 7 places to the right.

3.576×10^{7}

7 places
3.5 7 6 0 0 0 0 × 10^{7}

2 Move the decimal place 7 places to the right and write your answer. Zeroes will need to be added as placeholders.

$= 35\,760\,000$

b **1** Multiplying 7.9 by 10^{-5} means that the decimal point needs to be moved 5 places to the left.

7.9×10^{-5}

5 places
0.0 0 0 0 7 9 × 10^{-5}

2 Move the decimal place 5 places to the left and write your answer.

$= 0.000\,079$

Now try this 4 — Writing a scientific notation number as a basic numeral (Example 4)

Write the following scientific notation numbers as basic numerals.

a 4.231×10^{6}

b 8.2×10^{-4}

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All digits that appear in scientific notation are regarded as **significant figures**.

Significant figures

When a football match attendance is reported to be 64 000 people, it means that when measured in thousands, the crowd was estimated to be closer to 64 000 than 63 000 or 65 000. The figures 6 and 4 are called significant because the two figures make a claim to being accurate to the nearest thousand. We say 64 000 is accurate to two *significant figures*. Whereas 64 208 makes a claim of accuracy involving the thousands, hundreds, tens and units to describe the crowd size and is said to be accurate to 5 significant figures: 6, 4, 2, 0 and 8.

When asked to write a number to a required number of significant figures, the many zeroes in numbers such as 26 508 000 000 and 0.000 000 076 can cause confusion. As a first step, write the number in scientific notation. Then round to the required significant figures.

$26\,508\,000\,000 = 2.6508 \times 10^{10}$ is 5 significant figures.

2.651×10^{10} is 4 significant figures.

2.65×10^{10} is 3 significant figures.

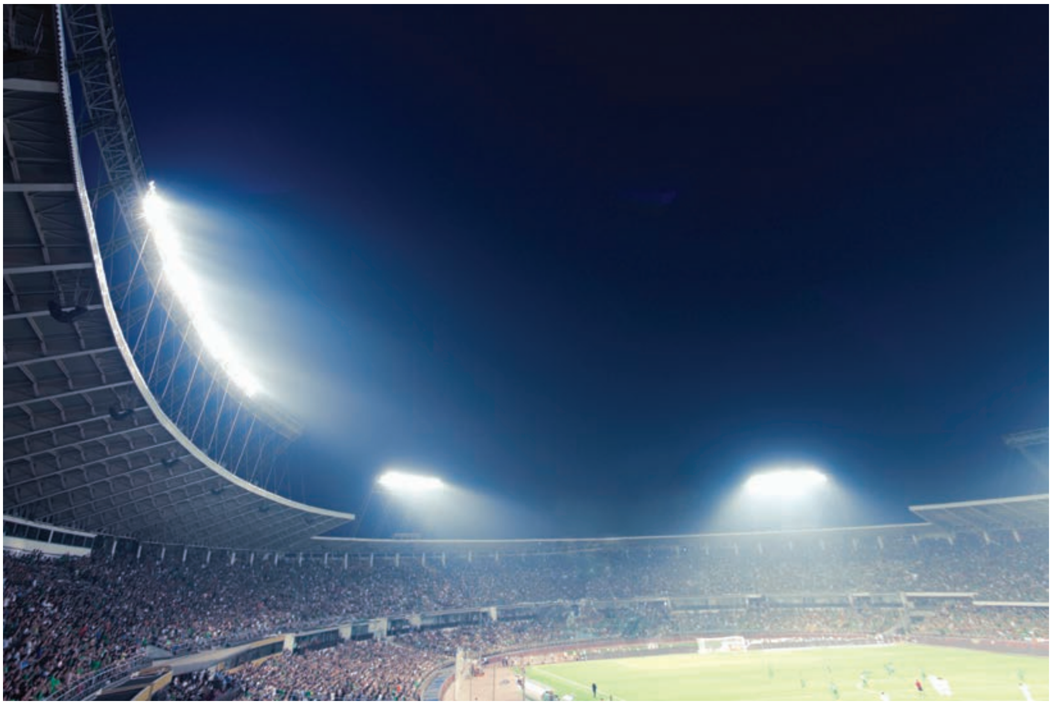
2.7×10^{10} is 2 significant figures.

3×10^{10} is 1 significant figure.

Similarly,

$0.000\,000\,076 = 7.6 \times 10^{-8}$ is 2 significant figures.

8×10^{-8} is 1 significant figure.



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Example 5 Rounding to a certain number of significant figures

Write each number in scientific notation, then round to two significant figures.

a 9764.809 4 **b** 0.000 004 716 8

Explanation	Solution
a 1 To write in scientific notation, put a decimal point after the first non-zero digit, and multiply by the required power of 10.	$9.764\,809\,4 \times 10^3$
2 To round to the second significant figure, check if the third digit is greater than 5.	The third digit (6) is greater than 5, so the second digit must be increased by 1 (so changes from 7 to 8).
3 Round to two significant figures. Write your answer.	$9.8 \times 10^3 = 9\,800$
b 1 To write in scientific notation, put a decimal point after the first non-zero digit, and multiply by the required power of 10.	$4.716\,8 \times 10^{-6}$
2 To round to the second significant figure, check if the third digit is greater than 5.	The third digit is not greater than 5, so the second digit is unchanged.
3 Round to two significant figures. Write your answer.	$4.7 \times 10^{-6} = 0.0000047$

Now try this 5 Rounding to a certain number of significant figures (Example 5)

Round to 3 significant figures.

a 57.892 607 **b** 0.000 471 68

Section Summary

- When rounding a number, look at the value of the digit to the right of the specified digit. Round the specified digit up if the following digit is 5 or more, and leave the specified digit unchanged if the following digit is less than 5.
- To determine the number of **decimal places** in a number, count the digits after the decimal point.
- A number written in **scientific notation** is expressed as a number between 1 and 10, multiplied by a power of 10.
- To write a number to the required number of **significant figures**, write the number in scientific notation then round to the required number of significant figures.

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Exercise 10A

Building understanding

1 By counting the number of places after the decimal point, state how many decimal places the following numbers have?

a 3.473 **b** 40.15 **c** 678.098
d 6.02 **e** 0.0005

2 Round the following to the nearest whole number.

a 3.8 **b** 12.1 **c** 67.02 **d** 556.73

3 How many places to the right does the decimal point need to move to change the following scientific notation numbers to basic numerals?

a 6.43×10^3 **b** 4.01×10^5 **c** 7.1×10^4

4 How many places to the left does the decimal point need to move to change the following scientific notation numbers to basic numerals?

a 8.14×10^{-3} **b** 5.01×10^{-1} **c** 6.2×10^{-2}

Developing understanding

Example 1

5 Round to the nearest hundred.

a 482 **b** 46 770 **c** 79 399 **d** 313.4

6 Round to the nearest dollar.

a \$ 689.79 **b** \$20.45 **c** \$927.58 **d** \$13.50

Example 2

7 Use a calculator to find answers to the following. Give each answer to the number of decimal places indicated in the brackets.

a 3.185×0.49 (2) **b** $0.064 \div 2.536$ (3)
c 0.474×0.0693 (2) **d** $12.943 \div 6.876$ (4)

8 Calculate the following to two decimal places.

a $\sqrt{7^2 + 14^2}$ **b** $\sqrt{3.9^2 + 2.6^2}$ **c** $\sqrt{48.71^2 - 29^2}$

Example 3

9 Write these numbers in scientific notation.

a 792 000 **b** 14 600 000 **c** 500 000 000 000 **d** 0.000 009 8
e 0.145 697 **f** 0.000 000 000 06 **g** 2 679 886 **h** 0.0087

Example 4

10 Write these scientific notation numbers as basic numerals.

a 5.3467×10^4 **b** 3.8×10^6 **c** 7.89×10^5 **d** 9.21×10^{-3}
e 1.03×10^{-7} **f** 2.907×10^6 **g** 3.8×10^{-12} **h** 2.1×10^{10}

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11 Express the following approximate numbers using scientific notation.

a The mass of the Earth is 6 000 000 000 000 000 000 000 000 kilograms.
b The circumference of the Earth is 40 000 000 metres.
c The diameter of an atom is 0.000 000 000 1 metre.
d The radius of the Earth's orbit around the Sun is 150 000 000 kilometres.

Example 5

12 Write the following to the number of significant figures indicated in each of the brackets.

a 4.8736 (2) **b** 0.078 74 (3)
c 1506.862 (5) **d** 5.523 (1)

13 Calculate the following and give your answer to the number of significant figures indicated in each of the brackets.

a $4.3968 \times 0.000\,743\,8$ (2) **b** $0.611\,35 \div 4.1119$ (5)
c $3.4572 \div 0.0109$ (3) **d** $50\,042 \times 0.0067$ (3)

Testing understanding

14 Round 35.8997 to three decimal places.

15 Which is the smallest number?

A 7.87×10^{-1} **B** 2.0×10^{0} **C** 0.5×10^{-1}
D 3.21×10^{-3} **E** 0.00067×10^{3}

16 Write the following numbers:

i to the number of significant figures indicated in the brackets.
ii to the number of decimal places indicated in the brackets.

a 421.389 (2) **b** 64.031 (3) **c** 5090.0493 (3)
d 70.549 (2) **e** 0.4573 (2) **f** 0.405 (2)

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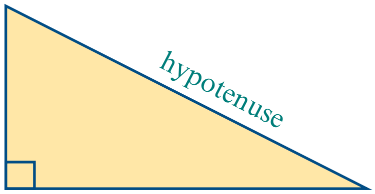
10B Pythagoras' theorem

Learning intentions

- To be able to find the length of an unknown side in a right-angled triangle using Pythagoras' theorem.
- To be able to find the length of an unknown side in a three-dimensional diagram.

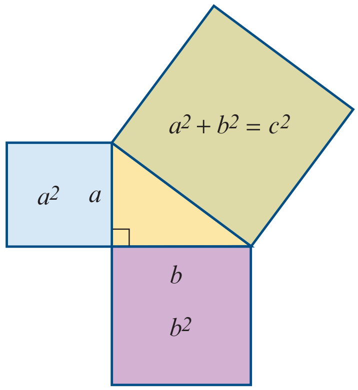
Pythagoras' theorem is a relationship connecting the side lengths of a right-angled triangle. In a right-angled triangle, the side *opposite* the **right angle** is called the **hypotenuse**. The hypotenuse is always the longest side of a right-angled triangle.



Pythagoras' theorem

Pythagoras' theorem states that, for any right-angled triangle, the sum of the areas of the squares of the two shorter sides (a and b) equals the area of the square of the hypotenuse (c).

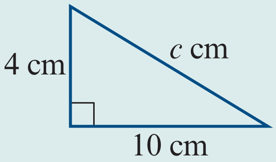
$$a^2 + b^2 = c^2$$



Pythagoras' theorem can be used to find the length of a side in a right-angled triangle when the lengths of the other two sides are known.

Example 6 **Using Pythagoras' theorem to calculate the length of the hypotenuse**

Calculate the length of the hypotenuse in the triangle opposite to two decimal places.



Explanation	Solution
1 Write Pythagoras' theorem.	$a^2 + b^2 = c^2$
2 Substitute known values.	$4^2 + 10^2 = c^2$
3 Take the square root of both sides, then evaluate.	$c = \sqrt{4^2 + 10^2}$ $= 10.770\ldots$
4 Write your answer to two decimal places, with correct units.	The length of the hypotenuse is 10.77 cm.

Hint: To ensure that you get a decimal answer, set your calculator to approximate or decimal mode. (See the Appendix, which is available through the Interactive Textbook.)

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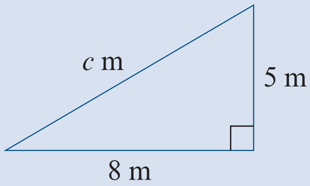
Now try this 6 Using Pythagoras' theorem to calculate the length of the hypotenuse (Example 6)

Find the length of the hypotenuse to two decimal places.



Hint 1 The hypotenuse is opposite the right angle.

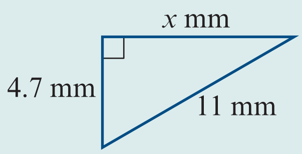
Hint 2 Work to at least three decimal places, then round your answer to two decimal places.

Hint 3 Answer using the correct units.

Pythagoras' theorem can also be rearranged to find sides other than the hypotenuse.

Example 7 Using Pythagoras' theorem to calculate the length of an unknown side in a right-angled triangle

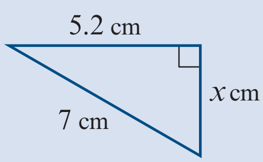
Calculate the length of the unknown side, x, in the triangle opposite to one decimal place.



Explanation	Solution
1 Write Pythagoras' theorem.	$a^2 + b^2 = c^2$
2 Substitute known values and the given variable.	$x^2 + 4.7^2 = 11^2$
3 Rearrange the formula to make x the subject, then evaluate.	$x = \sqrt{11^2 - 4.7^2}$ $= 9.945\ldots$
4 Write your answer to one decimal place, with correct units.	The length of x is 9.9 mm to one decimal place.

Now try this 7 Using Pythagoras' theorem to calculate the length of an unknown side in a right-angled triangle (Example 7)

Find the length of the unknown side to one decimal place.



Hint 1 Notice that the unknown is not the hypotenuse in this question.

Hint 2 Work to at least two decimal places, then round your answer to one decimal place.

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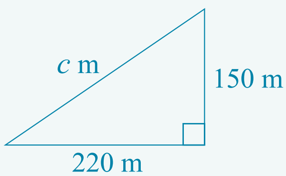
Pythagoras' theorem can be used to solve many practical problems.

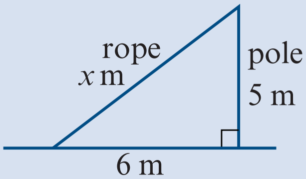
Example 8 Using Pythagoras' theorem to solve a practical problem

A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 220 m from a landing pad. Find the direct distance of the helicopter from the landing pad to two decimal places.

Explanation	Solution
1 Draw a diagram to show which distance is to be found.	
2 Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
3 Substitute known values.	$c^2 = 150^2 + 220^2$
4 Take the square root of both sides, then evaluate.	$c = \sqrt{150^2 + 220^2}$ $= 266.270\ldots$
5 Write your answer to two decimal places, with correct units.	The helicopter is 266.27 m from the landing pad.

Now try this 8 Using Pythagoras' theorem to solve a practical problem (Example 8)

A rope tied to the top of a 5 m pole is secured to the ground by a peg, 6 m from the base of the pole. What is the length of the rope to one decimal place?



Pythagoras' theorem in three dimensions

When solving three-dimensional problems, it is essential to carefully draw diagrams. In general, to find lengths in solid figures, we must first identify the correct right-angled triangle in the plane containing the unknown side. Remember, a plane is a flat surface, such as the cover of a book or a tabletop.

Once it has been identified, the right-angled triangle should be drawn separately from the solid figure, displaying as much information as possible.

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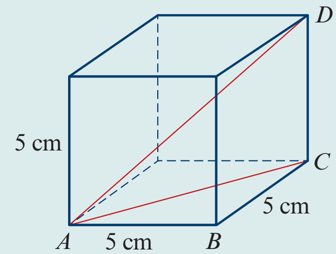
Example 9 Using Pythagoras' theorem in three dimensions

The cube in the diagram on the right has side lengths of 5 cm.

Find the length:

a AC to one decimal place.

b AD to one decimal place.



Explanation

a **1** Locate the relevant right-angled triangle in the diagram. Draw the right-angled triangle ABC that contains AC, and then mark in the known side lengths.

2 Using Pythagoras' theorem, calculate the length AC. Retain the lengthy decimal in your calculator for use in part **b**.

3 Write your answer with correct units and to one decimal place.

b **1** Locate the relevant right-angled triangle in the diagram.

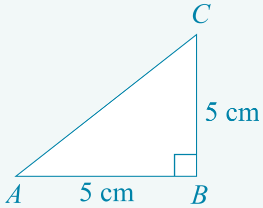
2 Draw the right-angled triangle ACD that contains AD, and mark in the known side lengths. From part **a**, put $AC = 7.07$ cm, working with at least one more decimal place than the answer requires.

3 Using Pythagoras' theorem, calculate the length AD.

4 Write your answer with correct units and to one decimal place.

Solution

a



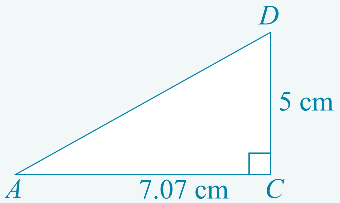
$(AC)^2 = (AB)^2 + (BC)^2$

$\therefore AC = \sqrt{5^2 + 5^2}$

$= 7.071\ldots$

The length AC is 7.1 cm to one decimal place.

b



$(AD)^2 = (AC)^2 + (CD)^2$

$\therefore AD = \sqrt{7.07^2 + 5^2}$

$= 8.659\ldots$

The length AD is 8.7 cm to one decimal place.

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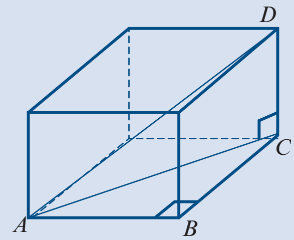
Now try this 9 Using Pythagoras' theorem in three dimensions (Example 9)

In the box shown, $AB = 8$ cm, $BC = 9$ cm and $CD = 6$ cm. Find to one decimal place:

a The length AC.

b The length AD.



Hint 1 In three-dimensional diagrams, right angles often appear distorted.

Hint 2 Identify the right-angled triangles, and draw them separately as they would appear, flat on the page.

Hint 3 Check: The diagonal to the opposite corner through a box with sides a, b and c is given by $\sqrt{a^2 + b^2 + c^2}$.

Section Summary

- In a right-angled triangle, the **hypotenuse** is the longest side and is opposite the right angle.
- Pythagoras' theorem states that, in any right-angled triangle:

$$c^2 = a^2 + b^2$$

  where c is the length of the hypotenuse and the other two sides have lengths of a and b.
- Given the lengths of any two sides in a right-angled triangle, Pythagoras' theorem can be used to find the length of the third side.
- Pythagoras' theorem can be used to find a distance in a three-dimensional figure. Draw a separate diagram for each triangle with a required unknown side.

Skill-sheet

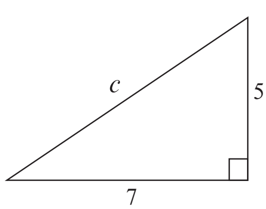
Exercise 10B

Building understanding

1 **a** Write Pythagoras' theorem for a right-angled triangle with sides a, b and c, where c is the hypotenuse.

b Complete: Let $a = 5$ and $b = ...$

c Substitute the values of a and b into the rule for Pythagoras' theorem from part **a**.



d Find the value of $a^2 + b^2$, then take the square root to find the value of c to one decimal place.

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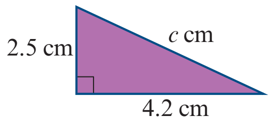
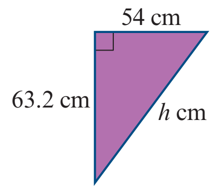
2 Find the length of the unknown side in each of these triangles to one decimal place.

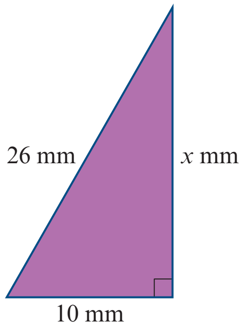
a



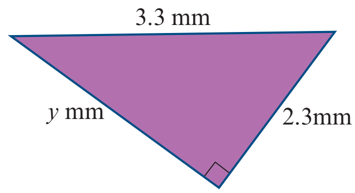
b



c



d

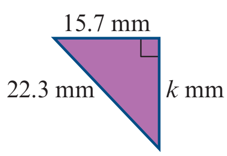


Developing understanding

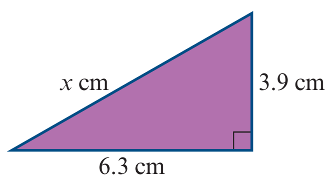
Example 6
Example 7

3 Determine the length of the unknown side in each of these triangles to one decimal place.

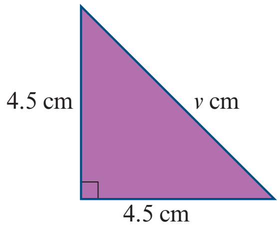
a



b



c



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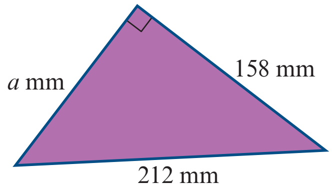
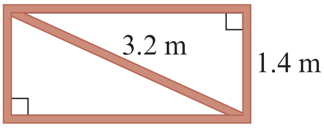


4 A farm gate that is 1.4 m high is supported by a diagonal bar of length 3.2 m. Find the width of the gate to one decimal place.



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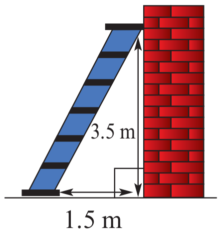
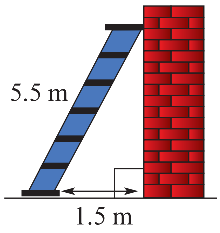
Example 8

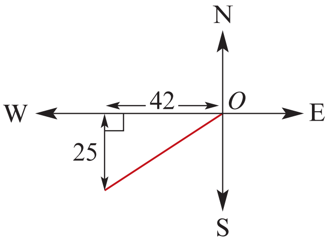
5 A ladder rests against a brick wall, as shown in the diagram on the right. The base of the ladder is 1.5 m from the wall, and the top reaches 3.5 m up the wall. Find the length of the ladder to one decimal place.



6 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is from the ground to one decimal place.

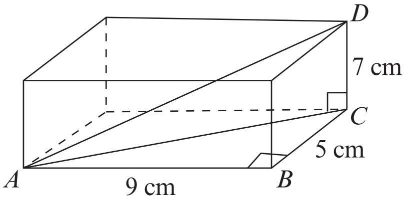


7 A ship sails 42 km due west and then 25 km due south. How far is the ship from its starting point? Answer to two decimal places.



8 A flying fox on a school camp starts from a tower 25 m high and finishes on the ground, 100 metres from the base of the tower. What is the distance travelled along the flying fox to the nearest metre?

9 Follow the steps below to find the length of the diagonal AD in the given diagram to one decimal place.



a The right-angled triangle ABC has sides AC, AB and BC. Draw the right-angled triangle ABC, then write Pythagoras' theorem for the sides AC, AB and BC.

b Substitute the values for sides AB and BC into your equation from part **a**.

c Calculate the value of $AB^2 + BC^2$, then take the square root to find the length AC. Keep the value in your calculator to use in part **e**.

The right-angled triangle ACD has sides AC, CD and AD.

d Draw the right-angled triangle ACD, then write Pythagoras' theorem for the sides AD, AC and CD.

e Substitute the values for sides AC and CD into your equation from part **c**.

f Calculate the value of $AC^2 + CD^2$, then take the square root to find the length AD to one decimal place.

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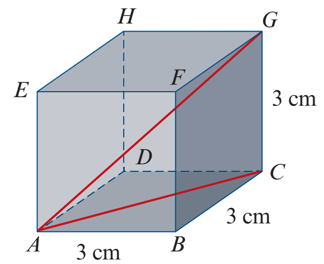
Example 9 **10** The cube shown in the diagram has sides of 3 cm. Find the length of:

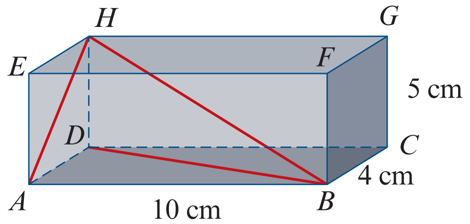
a AC to three decimal places

b AG to two decimal places.



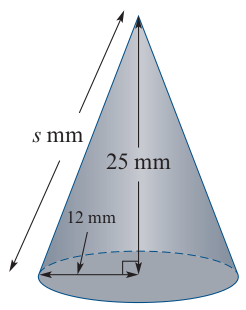
11 For this cuboid, calculate, to two decimal places, the lengths:

a DB **b** BH **c** AH

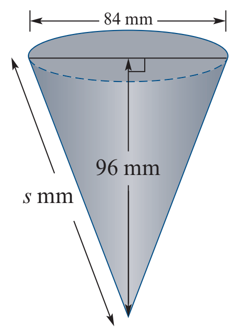


12 Find the sloping height, s, of each of the following cones to two decimal places.

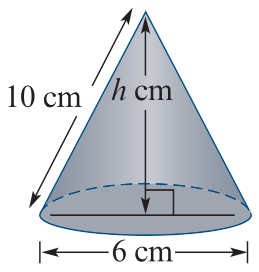
a



b



13 The slant height of this circular cone is 10 cm and the diameter of its base is 6 cm. Calculate the height of the cone to two decimal places.

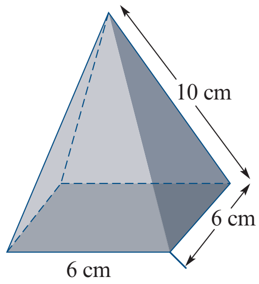


14 For each of the following square-based pyramids, find to one decimal place:

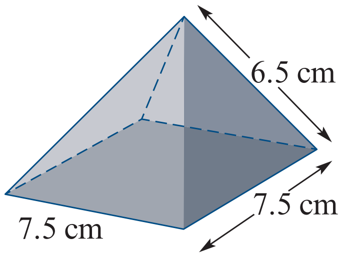
i the length of the diagonal on the base

ii the height of the pyramid.

a



b



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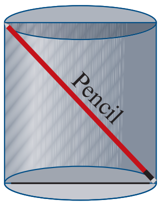
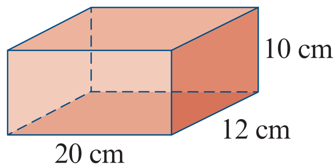
15 Find the length of the longest pencil that will fit inside a cylinder with height 15 cm and a diameter of 8 cm.



16 Chris wants to use a rectangular pencil box. What is the length of the longest pencil that would fit inside the box shown on the right? (Answer to the nearest centimetre.)

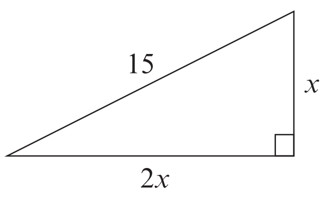


17 A broomstick is 145 cm long. Would it be able to fit in a cupboard measuring 45 cm by 50 cm and height 140 cm?

18 In the primate enclosure at the zoo, a rope is to be attached from the bottom corner of the enclosure to the opposite top corner for the monkeys to swing and climb on. If the enclosure measures 8 m by 10 m by 12 m, what is the length of the rope? Give your answer to two decimal places.

Testing understanding

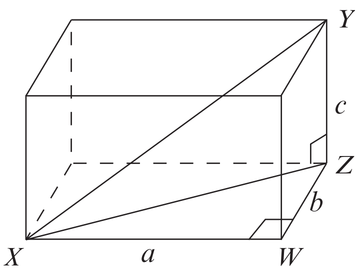
19 Find the value of x to one decimal place.



20 **a** In right-angled triangle WXZ, find an expression for $(XZ)^2$ in terms of a and b.

b In right-angled triangle XYZ, find an expression for $(XY)^2$ in terms of $(XZ)^2$ and $(YZ)^2$.

c Hence, find an expression for the length XY in terms of a, b and c.



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10C Perimeter and area

Learning intentions

- To be able to determine the perimeters and areas of regular shapes, such as rectangles, parallelograms, trapeziums and triangles.
- To be able to find the perimeter and area of composite shapes.
- To be able to find the circumference and area of a circle when given its radius.

Mensuration is a part of mathematics that looks at the measurement of length, area and volume. It comes from the Latin word *mensura*, which means 'measure'.

Perimeters of regular shapes

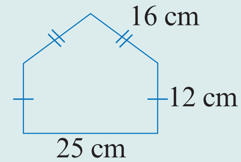
Perimeter

The **perimeter** of a two-dimensional shape is the total distance around its edge.

Example 10 Finding the perimeter of a shape

Find the perimeter of the shape shown.

The same dashes indicate sides of the same length.



Explanation

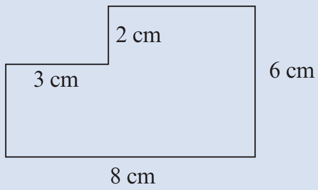
To find the perimeter, add up all the side lengths of the shape.

Solution

$$\text{Perimeter} = 25 + 12 + 12 + 16 + 16$$
$$= 81 \text{ cm}$$

Now try this 10 Finding the perimeter of a shape (Example 10)

By first finding the unknown lengths, find the perimeter of the given shape.



Hint 1 Where a length is not shown, look at the opposite side of the rectangle.

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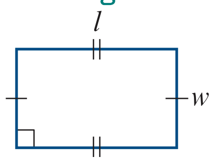
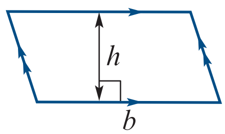
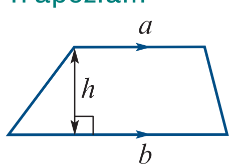
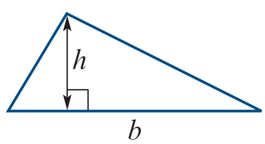
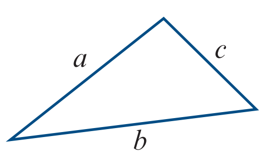
Areas of regular shapes

Area

The **area** of a shape is a measure of the region enclosed by its boundaries.

When calculating area, the answer will be in *square units*, i.e. mm^2, cm^2, m^2, km^2.

Shape	Area	Perimeter
Rectangle 	$A = lw$	$P = 2l + 2w$ or $P = 2(l + w)$
Parallelogram 	$A = bh$	Sum of four sides
Trapezium 	$A = \frac{1}{2}(a + b)h$	Sum of four sides
Triangle 	$A = \frac{1}{2}bh$	Sum of three sides
Heron's formula for finding the area of a triangle with three side lengths known. 	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$ (s is the semi-perimeter)	$P = a + b + c$

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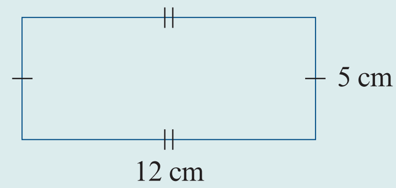
Example 11 Finding the perimeter of a rectangle

Find the perimeter of the rectangle shown.



Explanation

1. Since the shape is a rectangle, use the formula $P = 2l + 2w$.
2. Substitute length and width values into the formula. Evaluate.
3. Give your answer with correct units.

Solution

$$P = 2l + 2w$$
$$= 2 \times 12 + 2 \times 5$$
$$= 34\text{ cm}$$

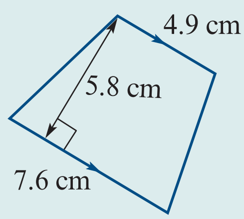
The perimeter is 34 cm.

Now try this 11 Finding the perimeter of a square (Example 11)

Find the perimeter of a square with sides of 3 m.

Example 12 Finding the area of a shape

Find the area of the given shape.



Explanation

1. Since the shape is a trapezium, use the formula $A = \frac{1}{2}(a + b)h$.
2. Substitute the values for a, b and h.
3. Evaluate.
4. Give your answer with correct units.

Solution

$$A = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(4.9 + 7.6)5.8$$
$$= 36.25\text{ cm}^2$$

The area of the shape is 36.25 cm^2.

Now try this 12 Finding the area of a shape (Example 12)

Find the area of a trapezium with a distance of 8 cm between the two parallel sides that have lengths of 10 cm and 14 cm.

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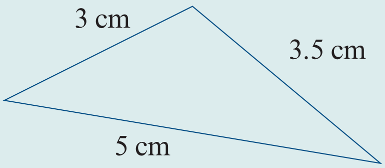
Example 13 Finding the area of a triangle using Heron's formula

Find the area of the following triangle. Give your answer to two decimal places.



Explanation	Solution
1 Since the height of the triangle is not given, you need to use Heron's formula as the three side lengths are known.	
2 Write down Heron's formula.	$A = \sqrt{s(s-a)(s-b)(s-c)}$
3 Find the perimeter of the triangle by adding the three side lengths.	$P = 3 + 3.5 + 5$ $= 11.5$
4 Divide the perimeter by 2 to find s, the semi-perimeter.	$s = \frac{11.5}{2}$ $= 5.75$
5 Substitute the value for s into Heron's formula to find the area of the triangle.	$A = \sqrt{5.75(5.75-3)(5.75-3.5)(5.75-5)}$ $= 5.16562\ldots$
6 Give your answer to two decimal places and with correct units.	The area of the triangle is 5.17 cm^2 to two decimal places.

Now try this 13 Finding the area of a triangle using Heron's formula (Example 13)

Find the area of a triangle with sides of 4 m, 5 m and 6 m to one decimal place.

The formulas for area and perimeter can be applied to many practical situations.

Example 14 Finding the area and perimeter in a practical problem

A display board for a classroom measures 150 cm by 90 cm.

a If ribbon costs $0.55 per metre, how much will it cost to add a ribbon border around the display board?

b The display board is to be covered with yellow paper. What is the area to be covered? Give you answer in m^2 to two decimal places.

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Explanation	Solution
a **1** To find the length of ribbon required, we need to work out the perimeter of the display board. The display board is a rectangle, so use the formula: $P = 2l + 2w$	$P = 2l + 2w$
2 Substitute $l = 150$ and $w = 90$ and evaluate.	$P = 2(150) + 2(90)$ $= 480$ cm of ribbon.
3 To convert from centimetres to metres, divide by 100.	$P = 480 \div 100$ $= 4.8$ m
4 To find the cost of the ribbon, multiply the length of the ribbon by \$0.55. Write your answer.	$4.8 \times 0.55 = \$2.64$ The cost of ribbon is \$2.64.
b **1** To find the area, use $A = lw$.	$A = lw$
2 Substitute $l = 150$ and $w = 90$ and evaluate.	$= 150 \times 90$ $= 13\,500$ cm^2
3 Convert your answer to m^2 by dividing by $(100 \times 100 = 10\,000)$.	$A = 13\,500 \div 10\,000$ $= 1.35$
4 Write your answer with correct units.	Area to be covered is 1.35 m^2.

Now try this 14 **Finding the area and perimeter in a practical problem (Example 14)**

The paint in a 2-litre can covers 6 m^2. How many litres will be needed to paint a wall which is 3 m high and 9 m long?

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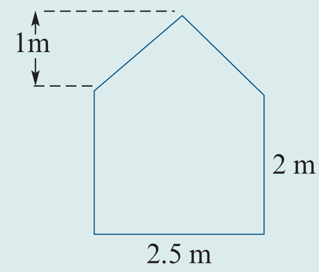
Composite shapes

Composite shapes

A composite shape is a shape that is made up of two or more basic shapes.

Example 15 **Finding the perimeter and area of a composite shape in a practical problem**

A gable window at a reception venue is to have LED lights around its perimeter (but not along the bottom of the window). The window is 2.5 m wide and the height of the room is 2 m. The height of the gable is 1 m, as shown in the diagram.



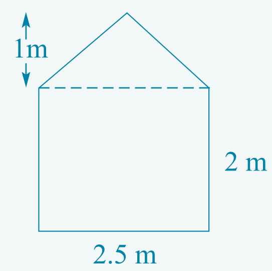
a Calculate the length of LED lights needed to two decimal places.

b The glass in the window needs to be replaced. Find the total area of the window to two decimal places.

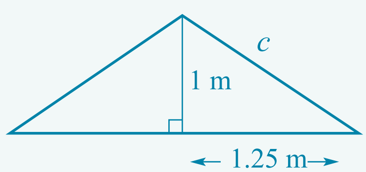
Explanation

Solution

a The window is made of two shapes: a rectangle and a triangle.



1 First find the length of the slant edge of the triangle.
Draw a diagram and label the slant edge as c.



Use Pythagoras' theorem to find c.
Note: The length of the base of the triangle is 1.25 m ($\frac{1}{2}$ of 2.5 m).

$c^2 = 1^2 + 1.25^2$

$\therefore c = \sqrt{1^2 + 1.25^2}$

$c = 1.6007\ldots$

$c = 1.600$ m

2 Add all the outside edges of the window, but do not include the bottom length.

$2 + 2 + 1.600 + 1.600 =$
7.20 to two decimal places.

3 Write your answer with correct units.

Require 7.20 m of LED lights.

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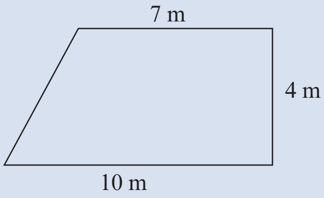
b **1** To find the total area of the window, first find the area of the rectangle by using the formula $A = bh$.	$A = bh$
2 Substitute the values for b and h.	$= 2.5 \times 2$
3 Evaluate and write your answer with correct units.	$= 5 \text{ m}^2$
4 Find the area of the triangle by using the formula $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh$
5 Substitute the values for b and h.	$= \frac{1}{2} \times 2.5 \times 1$
6 Evaluate and write your answer with correct units.	$= 1.25 \text{ m}^2$
7 To find the total area of the window, add the area of the rectangle and the area of the triangle.	Total area = area of rectangle + area of triangle $= 6.25 \text{ m}^2$
8 Give your answer with correct units to two decimal places.	Total area of window is 6.25 m^2 to two decimal places.

Now try this 15 **Finding the perimeter and area of a composite shape in a practical problem (Example 15)**

A courtyard is being renovated and the landscape gardener needs to order the materials required.

a Find the area of grass that will be needed for the courtyard.

b What will be the total length of wooden border required?



Hint 1 To find the length of the sloping edge, consider it as part of a right-angled triangle.

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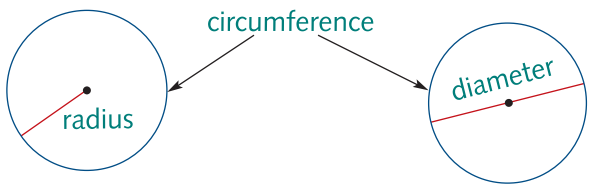
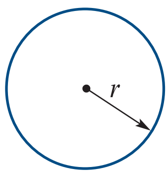
The circumference and area of a circle

The perimeter of a circle is also known as the **circumference** (C) of the circle.



The area and the circumference of a circle are given by the following formulas.

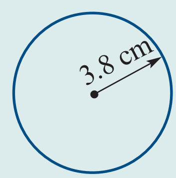
	Area	Circumference
Circle 	$A = \pi r^2$ where r is the **radius**	$C = 2\pi r$ or $C = \pi d$ where d is the **diameter**

Example 16 **Finding the circumference and area of a circle**

For the circle shown, find:

a the circumference to one decimal place

b the area to one decimal place.



Explanation	Solution
a 1 For the circumference, use the formula $C = 2\pi r$.	$C = 2\pi r$
2 Substitute $r = 3.8$ and evaluate.	$= 2\pi \times 3.8$ $= 23.876\ldots$
3 Give your answer to one decimal place and with correct units.	The circumference of the circle is 23.9 cm to one decimal place.
b 1 To find the area of the circle, use the formula $A = \pi r^2$.	$A = \pi r^2$
2 Substitute $r = 3.8$ and evaluate.	$= \pi \times 3.8^2$ $= 45.364\ldots$
3 Give your answer to one decimal place and with correct units.	The area of the circle is 45.4 cm^2 to one decimal place.

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Now try this 16 Finding the circumference and area of a circle (Example 16)

A circle has a radius of 12 metres. Find to one decimal place:

a the circumference of the circle

b the area of the circle.

Hint 1 Take care to give the correct units for the area. This will depend on the units for the radius that were used in the question.

Section Summary

- The perimeter of a two-dimensional shape is the total distance around its edge.
- The areas of common shapes can be found using the formulas:

Rectangle	Area $= lw$	l = length, w = width
Parallelogram	Area $= bh$	b = base, h = height
Trapezium	Area $= \frac{1}{2}(a+b)h$	a, b are the parallel sides, h = height
Triangle	Area $= \frac{1}{2}bh$	b = base, h = height

When the three sides a, b and c are known, use Heron's formula:

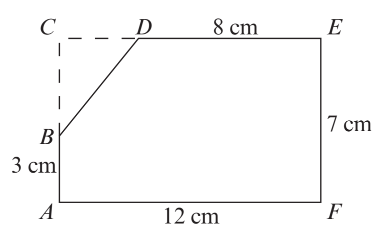
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{1}{2}(a+b+c)$$

- The circumference of a circle is given by $C = 2\pi r$, where r is the radius and π is a constant.
- The area of a circle can be calculated using $A = \pi r^2$, where r is the radius.

Exercise 10C

Building understanding

1 Follow the steps to find the area of the shape $ABDEF$ to one decimal place.



a Find the area of the rectangle $ACEF$.

b Find the length of the side BC and the side CD.

c Find the area of the triangle BCD.

d Find the area $ABDEF$ by subtracting the answer to part **c** from the answer to part **a**.

2 A circle has a radius of 7 cm. Substitute $r = 7$ and complete the following:

a Find, to two decimal places, the circumference.

$$\begin{aligned} \text{Use } C &= 2\pi r \\ &= 2\pi(...) \\ &= ... \text{ cm} \end{aligned}$$

b Find, to two decimal places, the area of the circle.

$$\begin{aligned} \text{Use } A &= \pi r^2 \\ &= \pi(...)^2 \\ &= ... \text{ cm}^2 \end{aligned}$$

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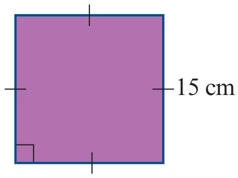
Developing understanding

Example 10
Example 11

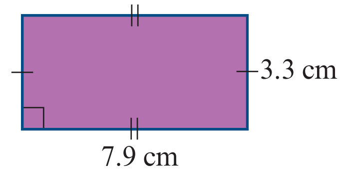
3 For each of the following shapes, find to one decimal place:

i the perimeter **ii** the area

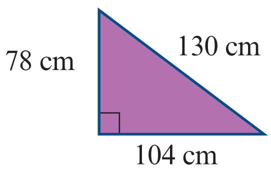
a



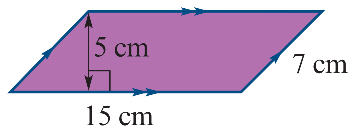
b



c



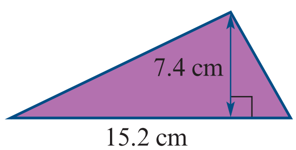
d



Example 12
Example 13

4 Find the areas of the given shapes to one decimal place where appropriate.

a



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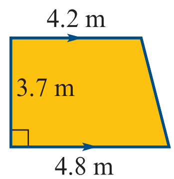

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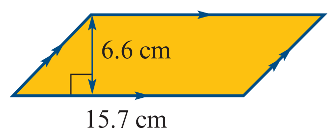

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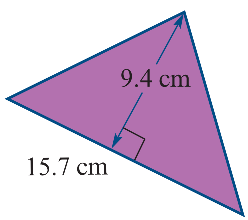

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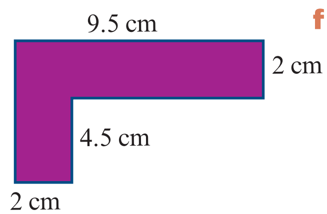

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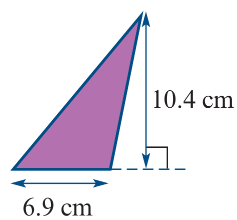

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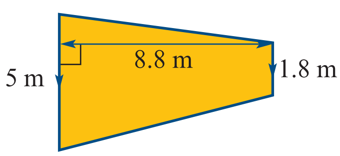

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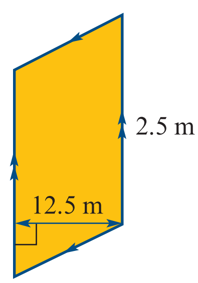
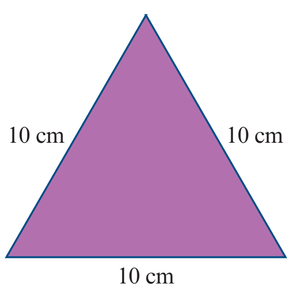


i



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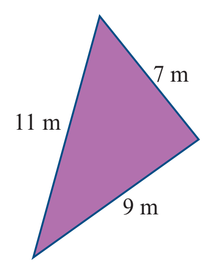

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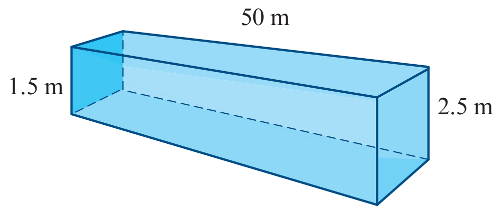
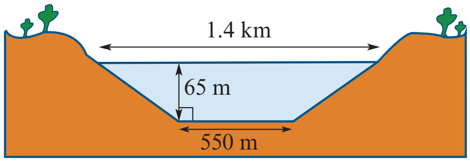
5 A 50 m swimming pool increases in depth from 1.5 m at the shallow end to 2.5 m at the deep end, as shown in the diagram (*not* to scale). Calculate the area of a side wall of the pool.



6 A dam wall is built across a valley that is 550 m wide at its base and 1.4 km wide at its top, as shown in the diagram (*not* to scale). The wall is 65 m deep. Calculate the area of the dam wall.

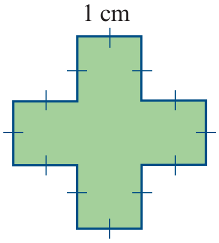


Example 14 **7** Ray wants to tile a rectangular area measuring 1.6 m by 4 m outside her holiday house. The tiles that she wishes to use are 40 cm by 40 cm. How many tiles will she need?

8 One litre of paint covers 9 m^2. How much paint is needed to paint a wall measuring 3 m by 12 m?

9 Find the area of the following composite shapes.

a



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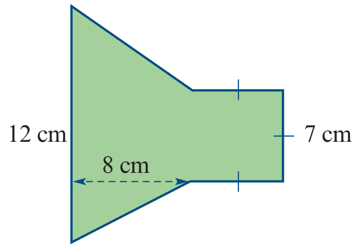
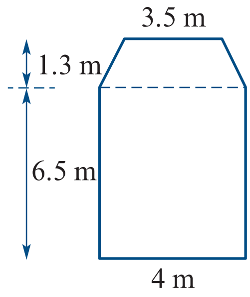


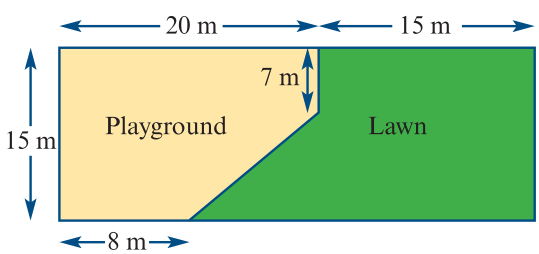
10 A driveway, as shown in the diagram, is to be paved. What is the area of the driveway to two decimal places?



Example 15 **11** A local council plans to fence a rectangular piece of land to make a children's playground and a lawn, as shown. (Not drawn to scale.)

a What is the area of the children's playground?

b What is the area of the lawn?



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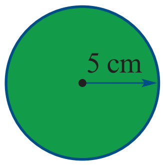
Example 16

12 For each of the following circles, find:

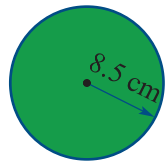
i the circumference to one decimal place

ii the area to one decimal place.

a



b

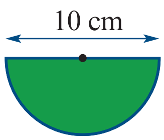


13 For each of the following shapes, find:

i the perimeter to two decimal places

ii the area to two decimal places.

a



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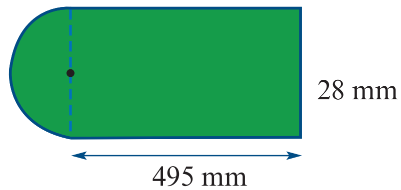

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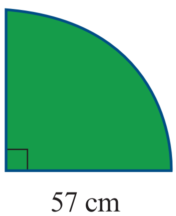

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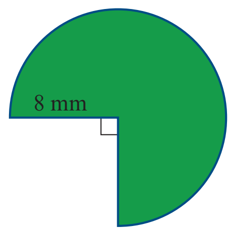
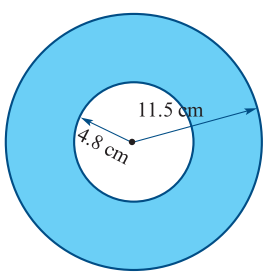


14 Find the shaded areas in the following diagrams to one decimal place.

a



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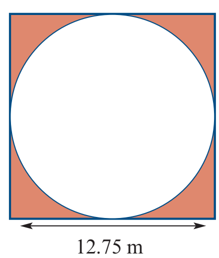

15 A fence needs to be built around a sports field that has two parallel straight sides, 400 m long, and semicircular ends with a diameter of 80 m.

a What length of fencing, to two decimal places, is required?

b What area will be enclosed by the fencing to two decimal places?

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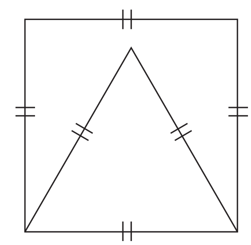
16 Three juggling rings cut from a thin sheet are to be painted. The diameter of the outer circle of the ring is 25 cm and the diameter of the inside circle is 20 cm. If both sides of the three rings are to be painted, what is the total area to be painted? (Ignore the inside and outside edges.) Round your answer to the nearest cm^2.

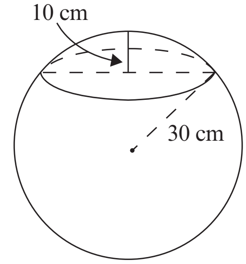
17 A path 1.2 m wide surrounds a circular garden bed whose diameter is 7 metres. What is the area of the path? Give the answer to two decimal places.

Testing understanding

18 An equilateral triangle with sides of 12 cm fits inside a square which also has sides of 12 cm.
What fraction of the square does the equilateral triangle occupy? Answer to three decimal places.



19 A sphere with a radius of 30 cm is sliced horizontally, 10 cm below the top of the sphere.
What is the circumference of the circle formed?
Answer to one decimal place.



10D Length of an arc and area of a sector

Learning intentions

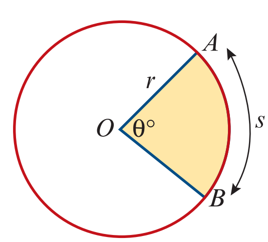
- To be able to find the length of an arc.
- To be able to find the area of a sector.

Length of an arc

Recall that the circumference, C, of a circle of radius r is given by $C = 2\pi r$. The fraction of the circumference will be $\frac{\theta}{360}$.

Therefore, the length, s, of an arc that subtends an angle of $\theta°$ at the centre is:

$$s = \frac{\theta}{360} \times 2\pi r$$



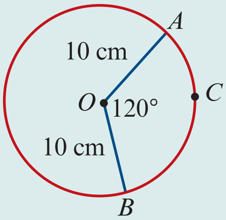
Length of an arc

The length, s, of an arc of a circle with a radius, r, that subtends an angle of $\theta°$ at the centre is given by:

$$s = \frac{\pi r \theta}{180}$$

Example 17 Calculating the length of an arc

In this circle with a centre at point O and a radius length of 10 cm, the angle subtended at O by arc ACB has a magnitude of 120°. Find the length of the arc, ACB, to one decimal place.



Explanation

1 Write down the formula.

2 Substitute $\theta = 120°$ and $r = 10$.

Solution

$$s = \frac{\pi r \theta}{180}$$

$$s = \frac{\pi \times 10 \times 120}{180} = \frac{20\pi}{3}$$

≈ 20.9 cm (to one decimal place)

Now try this 17 Calculating the length of an arc (Example 17)

Find the length of an arc that subtends an angle of 52° at the centre of a circle with a radius of 18 cm. Answer to one decimal place.

Area of a sector

If $\angle AOB = \theta°$, the area of the sector is a fraction of the area of the circle.

The area, A, of a circle of radius r is given by $A = \pi r^2$.

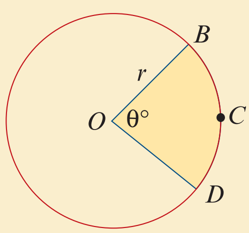
Minor and major sectors

In the diagram below with circle centre O, the yellow region is a **minor sector** and the unshaded region is a **major sector**.

Area of a sector

The area, A, of a sector of a circle with a radius, r, where the arc (BCD) of the sector subtends an angle of $\theta°$ at the centre is given by:

$$A = \frac{\pi r^2 \theta}{360}$$



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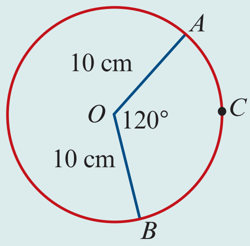
Example 18 Calculating the area of a sector

In this circle with a centre at point O and a radius length of 10 cm, the angle subtended at O by arc ACB has magnitude 120°.



Find:

a the area of the minor sector AOB

b the area of the major sector AOB.

Give your answer to two decimal places.

Explanation	Solution
a 1 Write down the formula.	$A = \frac{\pi r^2 \theta}{360}$
2 Substitute $\theta = 120°$ and $r = 10$.	$A = \frac{\pi \times 100 \times 120}{360}$ $= \frac{100\pi}{3}$ $\approx 104.72\ \text{cm}^2$
b 1 Write down the formula.	$A = \frac{\pi r^2 \theta}{360}$
2 Substitute $\theta = 240°$ and $r = 10$. Note: The angle is 240°, which is 360° minus 120°.	$A = \frac{\pi \times 100 \times 240}{360}$ $= \frac{200\pi}{3}$ $\approx 209.44\ \text{cm}^2$

Now try this 18 Calculating the area of a sector (Example 18)

Find the area of a sector that subtends an angle of 73° at the centre of a circle with a radius of 34 cm. Answer to one decimal place.

Hint 1 Take care to give the correct units for the area. This will depend on the units for the radius that were used in the question.

Section Summary

- The length, s, of an arc of a circle with a radius, r, that subtends an angle of $\theta°$ at the centre is given by:

$$s = \frac{\pi r \theta}{180}$$

- The area, A, of a sector of a circle with a radius, r, where the arc of the sector subtends an angle of $\theta°$ at the centre is given by:

$$A = \frac{\pi r^2 \theta}{360}$$

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Skill-sheet

Exercise 10D

Building understanding

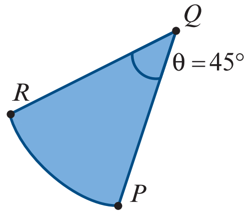
1 What fraction of a circle is each sector?

a Angle at the centre is 90°.
b Angle at the centre is 270°.
c Angle at the centre is 30°.
d Angle at the centre is 120°.
e Angle at the centre is 60°.
f Angle at the centre is 150°.

Developing understanding

2 Find the arc length of each sector. The radius is 10 cm. Answer to two decimal places.

a



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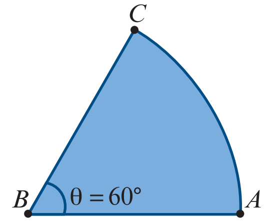

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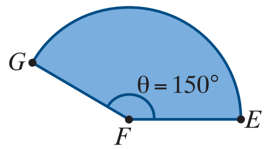

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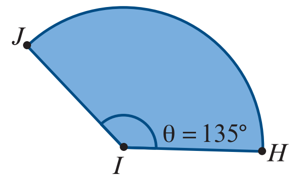


Example 17

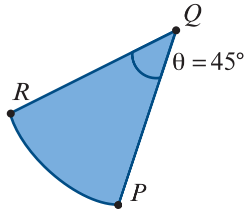
3 Findthe arc length of the following arcs where the radius of the circle (given in cm) and the angle subtended at the centre are given. Give your answer to two decimal places.

a $r = 15,\ \theta = 50°$
b $r = 20,\ \theta = 15°$
c $r = 30,\ \theta = 150°$
d $r = 16,\ \theta = 135°$
e $r = 40,\ \theta = 175°$
f $r = 30,\ \theta = 210°$

4 Find the perimeter of a sector that makes an angle of 58° in a circle with a radius of 3 metres. Answer to one decimal place.

5 Find the area of each of the following sectors, to one decimal place. The radius of the circle is 10 cm.

a



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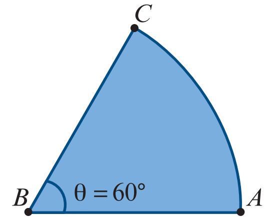

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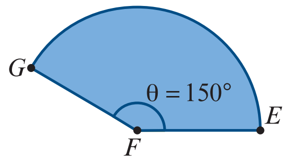
c



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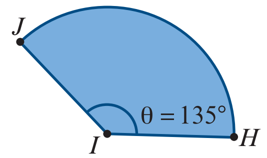

Example 18

6 Find the area of each sector where the radius of the circle (given in cm) and the angle subtended at the centre are given. Give your answer to two decimal places.

a $r = 10,\ \theta = 150°$
b $r = 40,\ \theta = 35°$
c $r = 45,\ \theta = 150°$
d $r = 16,\ \theta = 300°$
e $r = 50,\ \theta = 108°$
f $r = 30,\ \theta = 210°$

7 Find, to two decimal places, the size of the angle subtended at the centre of a circle of radius length 30 cm, by:

a an arc of length 50 cm
b an arc of length 25 cm.

Testing understanding

8 Find the area of the region between an equilateral triangle of side length 10 cm and the circle that passes through the three vertices of the triangle.

9 A person stands on level ground, 60 m from the nearest point of a cylindrical tank of radius length 20 m. Calculate to two decimal places:

a the circumference of the tank
b the percentage of the circumference that is visible to the person when viewed from ground level.

10 The minute hand of a large clock is 4 m long.

a How far does the tip of the minute hand move between 12:10 p.m. and 12:35 p.m?
b What is the area covered by the minute hand between 12:10 p.m. and 12:35 p.m?

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10E Volume

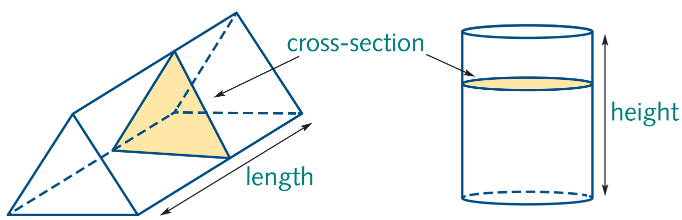
Learning intentions

- To be able to determine the volumes of rectangular prisms, triangular prisms, square prisms, cylinders and cones.
- To be able to find the capacity of three-dimensional containers.

Volume

Volume is the amount of space occupied by a three-dimensional object.

Prisms and cylinders are three-dimensional objects that have a uniform cross-section along their entire length. The volume of a prism or cylinder is found by using its **cross-sectional area**.

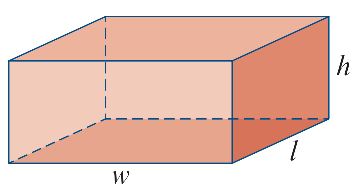
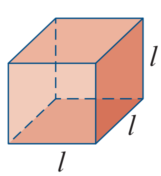


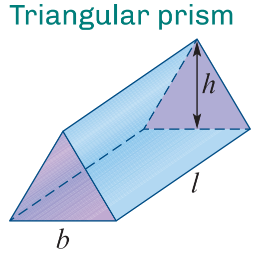
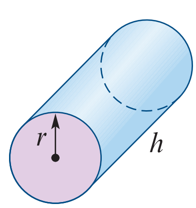
For prisms and cylinders:

$$\text{volume} = \text{area of cross-section} \times \text{height (or length)}$$

When calculating volume, the answer will be in *cubic units*, i.e. mm^3, cm^3, m^3.

The formulas for the volumes of regular prisms and a cylinder are given in the table below.

Shape	Volume
Rectangular prism (cuboid) 	$V = lwh$
Square prism (cube) 	$V = l^3$

Shape	Volume
Triangular prism 	$V = \frac{1}{2}bhl$
Cylinder 	$V = \pi r^2 h$

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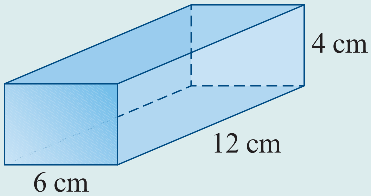
Example 19 Finding the volume of a cuboid

Find the volume of the cuboid shown.



Explanation

1. Use the formula $V = lwh$.
2. Substitute in $l = 12$, $w = 6$ and $h = 4$.
3. Evaluate.
4. Give your answer with correct units.

Solution

$V = lwh$
$= 12 \times 6 \times 4$
$= 288 \text{ cm}^3$

The volume of the cuboid is 288 cm^3.

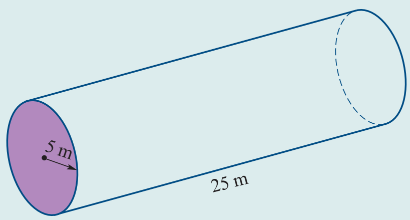
Now try this 19 Finding the volume of a cuboid (Example 19)

A cuboid has a length of 4 metres, a width of 3 metres and a height of 2 metres. Find its volume.

Hint 1 Take care to give the correct units for volume. The units for volume will depend on the units for length that were used.

Example 20 Finding the volume of a cylinder

Find the volume of this cylinder in cubic metres. Give your answer to two decimal places.



Explanation

1. Use the formula $V = \pi r^2 h$.
2. Substitute in $r = 5$ and $h = 25$ and evaluate.
3. Write your answer to two decimal places and with correct units.

Solution

$V = \pi r^2 h.$
$= \pi \times 5^2 \times 25$
$= 1963.495\ldots$

The volume of the cylinder is 1963.50 m^3, to two decimal places.

Now try this 20 Finding the volume of a cylinder (Example 20)

A cylinder has a radius of 18 cm and a height of 24 cm. Determine its volume to one decimal place.

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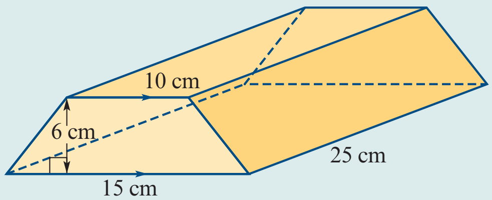
Example 21 Finding the volume of a three-dimensional object

Find the volume of the three-dimensional object shown.



Explanation

Strategy: To find the volume, find the area of the light-orange shaded cross-section and multiply it by the length of the shape.

1 Find the area of the cross-section, which is a trapezium. Use the formula $A = \frac{1}{2}(a + b)h$.

Substitute in $a = 10, b = 15$ and $h = 6$ and evaluate.

Solution

$$\text{Area of trapezium} = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(10 + 15)6$$
$$= 75 \text{ cm}^2$$

2 To find the volume, multiply the area of the cross-section by the length of the shape (25 cm).

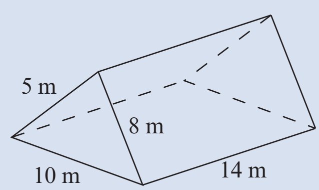
$$V = \text{area of cross-section} \times \text{length}$$
$$= 75 \times 25$$
$$= 1875 \text{ cm}^3$$

3 Give your answer with correct units.

The volume of the shape is 1875 cm^3.

Now try this 21 Finding the volume of a three-dimensional object (Example 21)

Find the volume of the triangular prism shown.
Give your answer to one decimal place.



Hint 1 The volume of a prism is the cross-section area times the length.

Hint 2 What rule is used to find the area of a triangle when you know the length of the three sides?

Hint 3 Work to at least two decimal places and round your answer to one decimal place.

Capacity

Capacity

Capacity is the amount of substance that an object can hold.

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The difference between volume and capacity is that volume is the space available whilst capacity is the amount of substance that fills the volume.

For example:

- a cube that measures 1 metre on each side has a volume of one cubic metre (m^3) and is able to hold 1000 litres (L) (capacity)
- a bucket of volume 7000 cm^3 can hold 7000 mL (or 7 L) of water.

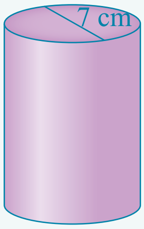
Volume to capacity conversion

The following conversions are useful to remember.

$$1 \text{ m}^3 = 1000 \text{ litres (L)}$$
$$1 \text{ cm}^3 = 1 \text{ millilitre (mL)}$$
$$1000 \text{ cm}^3 = 1 \text{ litre (L)}$$

Example 22 Finding the capacity of a cylinder

A drink container is in the shape of a cylinder. How many litres of water can it hold if the height of the cylinder is 20 cm and the diameter is 7 cm? Give your answer to two decimal places.

Explanation	Solution
1 Draw a diagram of the cylinder.	
2 Use the formula for finding the volume of a cylinder: $V = \pi r^2 h$.	$V = \pi r^2 h$
3 Since the diameter is 7 cm then the radius is 3.5 cm. Substitute $h = 20$ and $r = 3.5$.	$V = \pi \times 3.5^2 \times 20$
4 Evaluate to find the volume of the cylinder.	$V = 769.6902\ldots$ The volume of the cylinder is 769.69 cm^3
5 As there are 1000 cm^3 in a litre, divide the volume by 1000 to convert to litres.	$\frac{769.69}{1000} = 0.76969\ldots$
6 Give your answer to two decimal places and with correct units.	Cylinder has capacity of 0.77 litres to two decimal places.

Now try this 22 Finding the capacity of a cylinder (Example 22)

A cylindrical fuel can has a radius of 9 cm and a height of 30 cm. How many litres of fuel could it hold? Give your answer to one decimal place.

Hint 1 First find the volume of the cylinder.

Hint 2 Convert cubic centimetres into litres.

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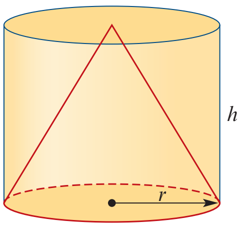
Volume of a cone

Calculating the volume of a cone is not required by the Study Design; however, it is often considered when studying the volume of solids.

A cone can fit inside a cylinder, as shown in the diagram. The cone occupies one-third of the volume of the cylinder containing it. Therefore, the formula for finding the volume of a cone is:

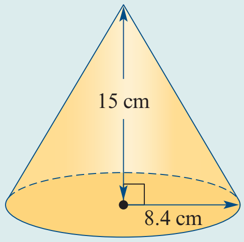
$$\text{volume of cone} = \tfrac{1}{3} \times \text{volume of its cylinder}$$
$$\text{volume of cone} = \tfrac{1}{3} \times \text{ area of base} \times \text{height}$$
$$V = \tfrac{1}{3}\pi r^2 h$$



The cone in the diagram is called a right circular cone because a line drawn from the centre of the circular base to the vertex at the top of the cone is perpendicular to the base.

Example 23 Finding the volume of a cone

Find the volume of this right circular cone.
Give your answer to two decimal places.



Explanation	Solution
1 Use the formula for the volume of a cone, $V = \frac{1}{3}\pi r^2 h$.	$V = \dfrac{1}{3}\pi r^2 h$
2 Substitute $r = 8.4$ and $h = 15$ and evaluate.	$= \dfrac{1}{3}\pi(8.4)^2 \times 15$ $= 1108.353\ldots$
3 Give your answer to two decimal places and with correct units.	The volume of the cone is 1108.35 cm^3 to two decimal places.

Now try this 23 Finding the volume of a cone (Example 23)

Find the volume of a cone with a height of 3 m and a base with a radius of 2.5 m.
Give your answer to one decimal place.

Hint 1 Work to at least two decimal places then round your answer to one decimal place.

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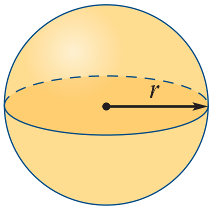
Volume of a sphere

The volume of a sphere of radius r can be found by using the formula:

$$V = \frac{4}{3}\pi r^3$$



Example 24 Finding the volume of a sphere

Find the volume of this sphere, giving your answer to two decimal places.

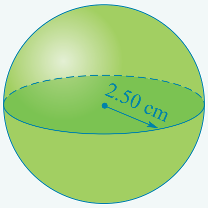
Explanation

1 Use the formula: $V = \frac{4}{3}\pi r^3$.

2 Substitute $r = 2.5$ and evaluate.

3 Give your answer to two decimal places and with correct units.

Solution



$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times 2.5^3$$
$$= 65.449\ldots$$

The volume of the sphere is $65.45\ \text{cm}^3$.

Now try this 24 Finding the volume of a sphere (Example 24)

Find the volume of a sphere with a radius of 45 cm. Answer to one decimal place.

Hint 1 Work to at least two decimal places and round your answer so it is to one decimal place.

Section Summary

- Volume is the amount of space occupied by a three-dimensional object.
- The volumes of common objects can be found using the formulas:

Rectangular prism	$V = lwh$	l = length, w = width, h = height
Triangular prism	$V = \frac{1}{2}bhl$	b = base, h = height, l = length
Cylinder	$V = \pi r^2 h$	r = radius, h = height
Cone	$V = \frac{1}{3}\pi r^2 h$	r = radius of the base, h = height
Sphere	$V = \frac{4}{3}\pi r^3$	r = radius.

- Capacity is the amount of substance an object can hold.

  Volume can be converted into capacity using:

$$1\ \text{m}^3 = 1000 \text{ litres (L)}$$
$$1\ \text{cm}^3 = 1 \text{ millilitre (mL)}$$
$$1000\ \text{cm}^3 = 1 \text{ litre (L)}$$

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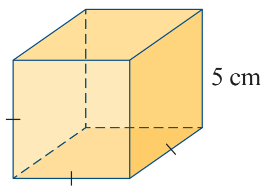
Exercise 10E

Building understanding

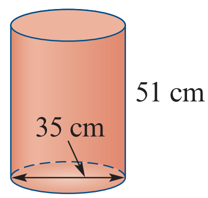
Example 19–21

1 Find the volumes of the following solids. Give your answers to one decimal place where appropriate.

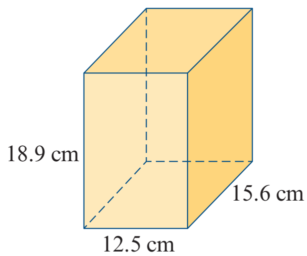
a



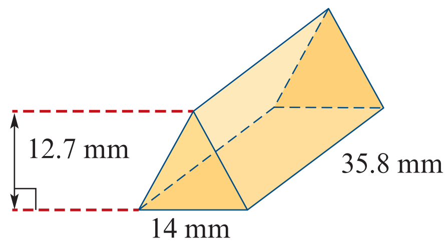
b



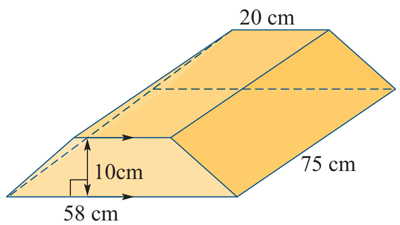
c



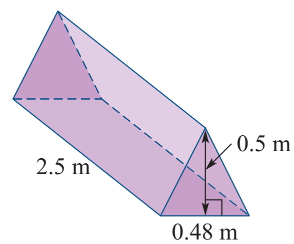
d



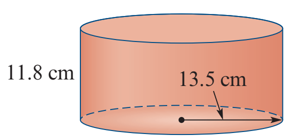
e



f



g



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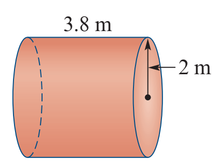
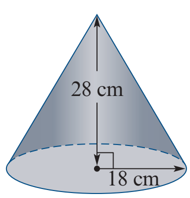


Example 23

i



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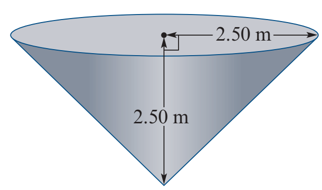
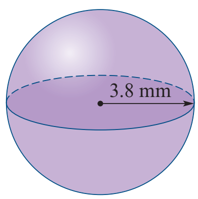
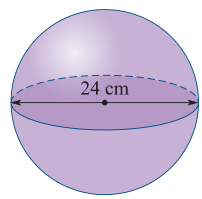


Example 24

k



l



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Developing understanding

2 A cylindrical plastic container is 15 cm high, and its circular end surfaces each have a radius of 3 cm. What is its volume, to the nearest cm^3?

3 What is the volume, to the nearest cm^3, of a rectangular box with dimensions 5.5 cm by 7.5 cm by 12.5 cm?

Example 23

4 Find the volume, to two decimal places, of the cones with the following dimensions.

- **a** Base radius 3.50 cm, height 12 cm
- **b** Base radius 7.90 m, height 10.80 m
- **c** Base diameter 6.60 cm, height 9.03 cm
- **d** Base diameter 13.52 cm, height 30.98 cm

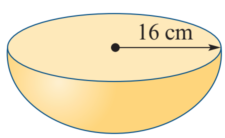
Example 24

5 Find the volumes, to two decimal places, of the following balls.

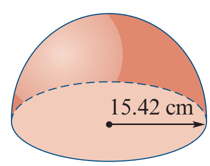
- **a** Tennis ball, radius 3.5 cm
- **b** Basketball, radius 14 cm
- **c** Golf ball, radius, 2 cm

6 Find the volumes, to two decimal places, of the following hemispheres.

a



b



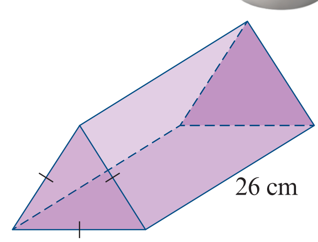
Example 22

7 How many litres of water does a fish tank with dimensions 50 cm by 20 cm by 24 cm hold when full?

8 **a** What is the volume, to two decimal places, of a cylindrical paint tin with height 33 cm and diameter 28 cm?

b How many litres of paint would fill this paint tin? Give your answer to the nearest litre.

9 A chocolate bar is made in the shape of an equilateral triangular prism. What is the volume of the chocolate bar if the length is 26 cm and the side length of the triangle is 4.5 cm? Give your answer to the nearest cm^3.



10 What volume of crushed ice will fill a snow cone level to the top if the snow cone has a top radius of 5 cm and a height of 15 cm? Give your answer to the nearest cm^3.

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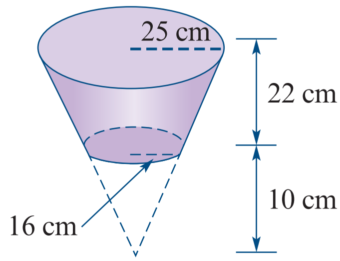
11 What is the volume, to two decimal places, of a cone with height 2.6 m and diameter of 3.4 m?

12 How many litres of water can be poured into a cone with a diameter of 2.8 cm and a height of 10 cm?

13 A solid figure is *truncated* when a portion of the bottom is cut and removed. Find the volume, to two decimal places, of the truncated cone shown in the diagram.



14 A plastic cone of height 30 cm and diameter 10 cm is truncated to make a rain gauge. The rain gauge has a height of 25 cm. What is its capacity? Give your answer to the nearest millilitre.

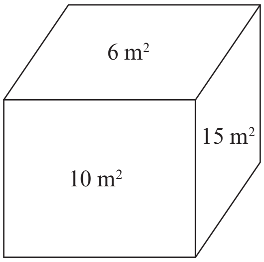
15 Len wants to serve punch at Christmas time in his new hemispherical punch bowl with diameter of 38 cm. How many litres of punch could be served, given that 1 millilitre (mL) is the amount of fluid that fills 1 cm^3? Answer to the nearest litre.

Testing understanding

16 The area of each face of the rectangular prism is shown.

a Find the volume of the rectangular prism.

b If the area of each face shown was X m^2, Y m^2 and Z m^2, what would be the volume of the rectangular prism?



17 A flat-bottomed silo for grain storage was constructed as a cylinder with a cone on top. The cylinder has a circumference of 53.4 m and a height of 10.8 m. The total height of the silo is 15.3 m. What is the volume of the silo? Give your answer to the nearest m^3.

18 A cylindrical water tank with a base of 250 cm^2 had enough water in it so that when a sphere with a volume of 500 cm^3 was lowered into the tank it was completely submerged in the water. How much did the water level rise?

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10F Volume of a pyramid

Learning intentions

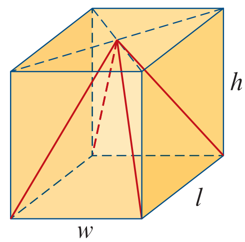
- To be able to calculate the volume of a square pyramid given its height and the width of its base.
- To be able to calculate the volume of a pyramid given its height and the area of its base.

A square pyramid can fit inside a prism, as shown in the diagram. The pyramid occupies one third of the volume of the prism containing it. The formula for finding the volume of a pyramid is therefore:

$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{volume of its prism}$$

$$\text{volume of pyramid} = \tfrac{1}{3} \times \text{area of base} \times \text{height}$$

$$V = \tfrac{1}{3}lwh$$

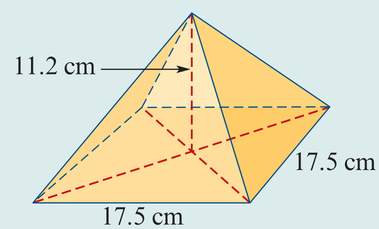


For the volume of pyramids with various-shaped bases:

$$V = \tfrac{1}{3} \times \text{area of base} \times \text{height}$$

Example 25 Finding the volume of a square pyramid

Find the volume of a square pyramid of height 11.2 cm and base length of 17.5 cm. Give your answer to two decimal places.



Explanation	Solution
1 Use the formula: $V = \frac{1}{3} \times \text{area of base} \times \text{height}$	$V = \frac{1}{3} \times \text{area of base} \times \text{height}$
2 Substitute the values for the area of the base (in this example, the base is a square) and height of the pyramid, and evaluate.	$= \frac{1}{3} \times 17.5^2 \times 11.2$ $= 1143.333\ldots$
3 Give your answer to two decimal places and with correct units.	The volume of the pyramid is 1143.33 cm^3.

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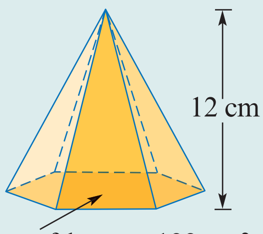
Now try this 25 **Finding the volume of a square pyramid (Example 25)**

Determine the volume of a square pyramid that has a height of 54 m and the sides of its base are 38 m long.

Example 26 **Finding the volume of a hexagonal pyramid**

Find the volume of this hexagonal pyramid that has a base of area 122 cm^2 and a height of 12 cm.



Explanation

1 Use the formula:
$V = \frac{1}{3} \times \text{area of base} \times \text{height}$

2 Substitute the values for the area of the base (122 cm^2) and height (12 cm) and evaluate.

3 Give your answer with correct units.

Solution

$V = \frac{1}{3} \times \text{area of base} \times \text{height}$

$= \frac{1}{3} \times 122 \times 12$

$= 488 \text{ cm}^3$

The volume is 488 cm^3.

Now try this 26 **Finding the volume of a hexagonal pyramid (Example 26)**

A hexagonal pyramid has a base area of 320 m^2 and a height of 24 m. Find its volume.

Section Summary

- The volume of a pyramid is given by the formula:

$$V = \tfrac{1}{3} \times \text{area of base} \times \text{height}$$

- For a square or rectangular-based pyramid, the formula becomes:

$V = \frac{1}{3}lwh$ $\quad$ l = side length, w = width, h = height

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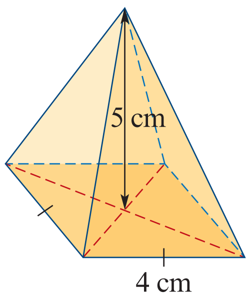
Exercise 10F

Building understanding

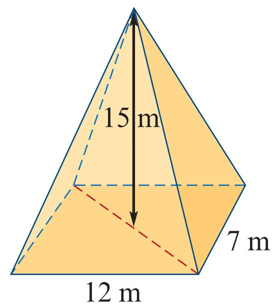
Example 25
Example 26

1 Find the volumes of the following right pyramids to two decimal places.

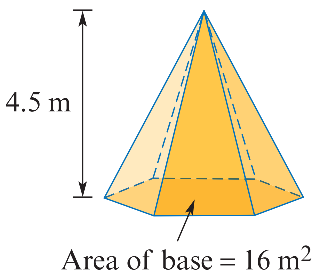
a



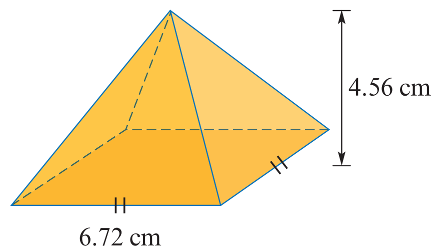
b



c



d

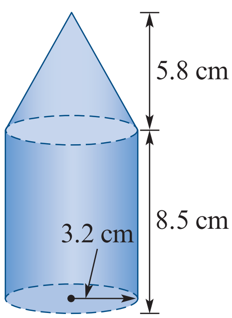


Developing understanding

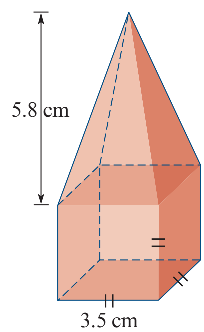
2 The first true pyramid in Egypt is known as the Red Pyramid. It has a square base approximately 220 m long and is about 105 m high. What is its volume?

3 Find the volumes of these composite objects to one decimal place.

a

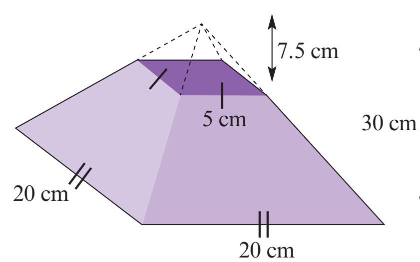


b



Testing understanding

4 Calculate the volume of the following truncated pyramid to one decimal place.



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10G Surface area

Learning intentions

- To be able to find the surface area of objects with plane surfaces, such as prisms, cuboids and pyramids.
- To be able to find the surface area of objects with curved surfaces, such as cylinders, cones and spheres.

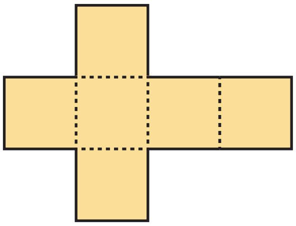
To find the **surface area (*SA*)** of a solid, you need to find the area of each of the surfaces of the solid and then add them all together.

Solids with plane faces (prisms and pyramids)

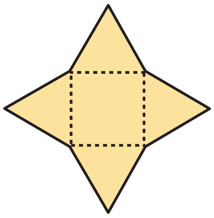
It is often useful to draw the **net** of a solid to ensure that all sides have been added.

A **net** is a flat diagram consisting of the plane faces of a polyhedron, arranged so that the diagram may be folded to form the solid.

For example: The net of a cube and of a square pyramid are shown below.



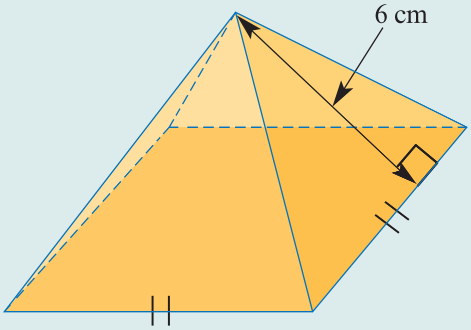
Net of a cube



Net of a square pyramid

Example 27 Finding the surface area of a pyramid

Find the surface area of this square-based pyramid.



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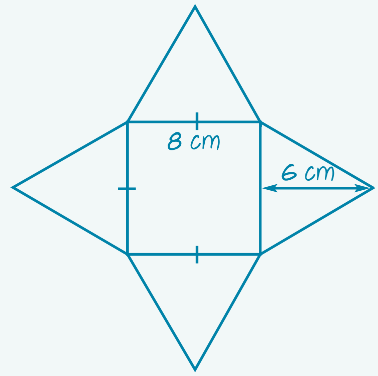
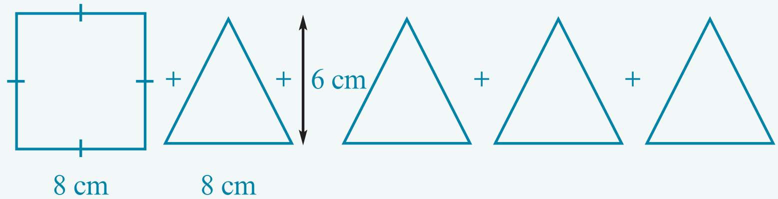
Explanation

Solution

1 Draw a net of the square pyramid. Note that the net is made up of one square and four identical triangles, as shown right.





2 Write down the formula for the surface area, using the net as a guide, and evaluate.

Surface area = area of 

$= 8 \times 8 + 4 \times (\frac{1}{2} \times 8 \times 6)$

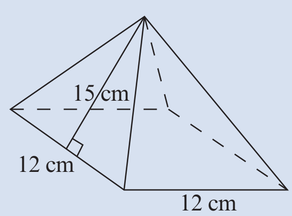
$= 160$

The surface area of the square pyramid is $160\ cm^2$.

Note: To find the area of the square, multiply the length by the width (8×8). To find the area of the triangles, use $A = \frac{1}{2}bh$, where b is 8 and h is 6.

Now try this 27 Finding the surface area of a pyramid (Example 27)

Find the surface area of the square-based pyramid shown.



Hint 1 Use the units of length given to decide what are the correct units of volume.

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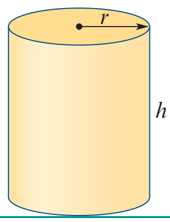
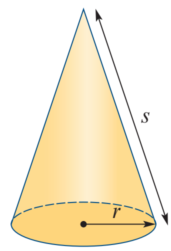
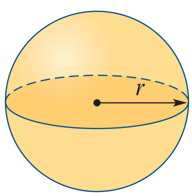
Solids with curved surfaces (cylinder, cone, sphere)

Calculating the surface area of a cone is not required by the Study Design; however, it is often considered when studying the surface area of solids.

For some special objects, such as the cylinder, cone and sphere, formulas to calculate the surface area can be developed. The formulas for the surface area of a cylinder, cone and sphere are given below.

Shape	Surface area
Cylinder 	$SA = 2\pi r^2 + 2\pi rh$

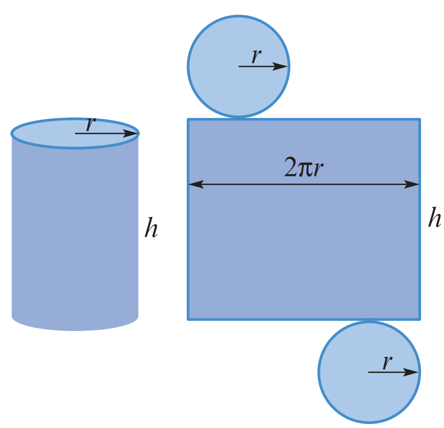
Shape	Surface area
Cone 	$SA = \pi r^2 + \pi rs$

Shape	Surface area
Sphere 	$SA = 4\pi r^2$

To develop the formula for the surface area of a cylinder, we first draw a net, as shown.

The **total surface area** of a cylinder can therefore be found using:

$$SA = \text{area of ends} + \text{area of curved surface}$$
$$= \text{area of 2 circles} + \text{area of rectangle}$$
$$= 2\pi r^2 + 2\pi rh$$



Note: $2\pi r$ is the circumference of the circle, and this is the side length of the rectangle.

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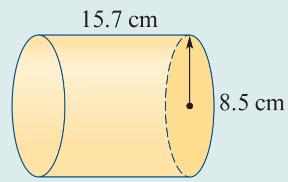
Example 28 Finding the surface area of a cylinder

Find the surface area of this cylinder to one decimal place.



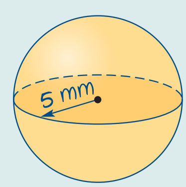
Explanation	Solution
1 Use the formula for the surface area of a cylinder.	$SA = 2\pi r^2 + 2\pi rh$
2 Substitute $r = 8.5$ and $h = 15.7$ and evaluate.	$= 2\pi(8.5)^2 + 2\pi \times 8.5 \times 15.7$ $= 1292.451\ldots$
3 Give your answer to one decimal place and with correct units.	The surface area of the cylinder is 1292.5 cm^2 to one decimal place.

Now try this 28 Finding the surface area of a cylinder (Example 28)

Find the surface area of a cylinder with a height of 45 m and a radius of 30 m to one decimal place.

Example 29 Finding the surface area of a sphere

Find the surface area of a sphere with radius 5 mm to two decimal places.



Explanation	Solution
1 Use the formula $SA = 4\pi r^2$.	$SA = 4\pi r^2$
2 Substitute $r = 5$ and evaluate.	$= 4\pi \times 5^2$ $= 314.159\ldots$
3 Give your answer to two decimal places and with correct units.	The surface area of the sphere is 314.16 mm^2.

Now try this 29 Finding the surface area of a sphere (Example 29)

Find the surface area of a sphere with a radius of 20 cm to one decimal place.

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Section Summary

- Solids with plane faces, such as prisms, cuboids and pyramids, can be represented by their net as a way of calculating the total surface area.
- The surface areas of solids with curved surfaces can be found using the formulas:

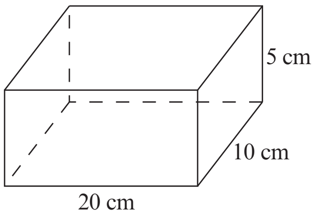
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	r = radius, h = height
Cone	$SA = \pi r^2 + \pi rs$	r = radius, s = sloping (or slant) edge
Sphere	$SA = 4\pi r^2$	r = radius

Skill-sheet

Exercise 10G

Building understanding

1 To find the surface area of a cuboid (box), its net could be drawn as a useful aid. Alternatively, you could directly calculate the different faces.



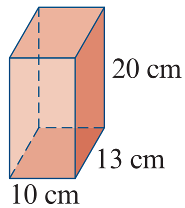
a Find the total area of the top and bottom faces.
b What is the total area of the two side faces?
c Find the total area of the front and back faces.
d Calculate the total surface area.

Developing understanding

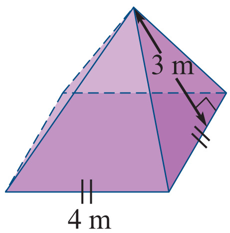
Example 27

2 Find the surface areas of these prisms and pyramids to one decimal place.

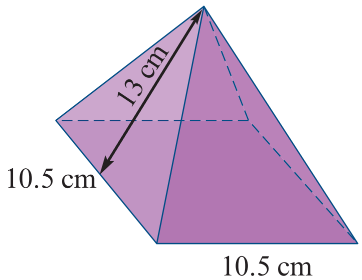
a



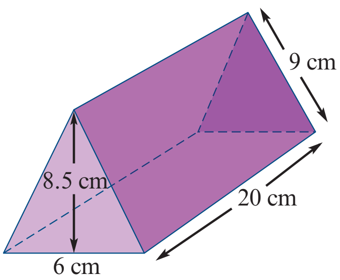
b



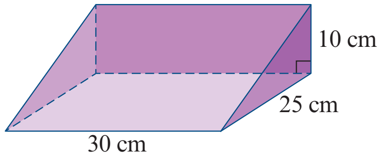
c



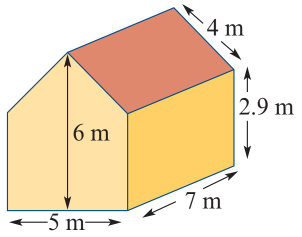
d



e



f



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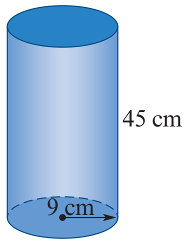
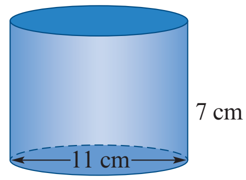
Example 28
Example 29

3 Find the surface area of each of these solids with curved surfaces to two decimal places.

a



b



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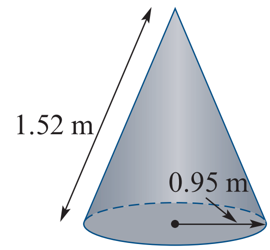

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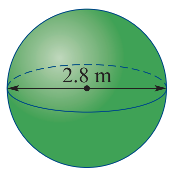
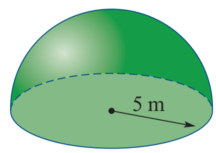
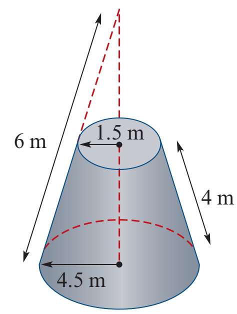


e



f



4 A tennis ball has a radius of 3.5 cm. A manufacturer wants to provide sufficient material to cover 100 tennis balls. What area of material is required? Give your answer to the nearest cm^2.

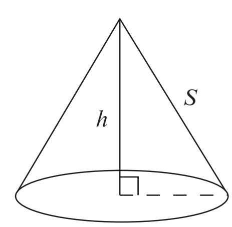
5 A set of 10 conical paper hats are covered with material. The height of a hat is 35 cm and the diameter is 19 cm.

a What amount of material, in m^2, will be needed? Give your answer to two decimal places.

b Tinsel is to be placed around the base of the hats. How much tinsel, to the nearest metre, is required?

Testing understanding

6 When the curved surface of a cone was cut along the sloping edge, S, and laid flat on the table, it formed a semicircle with a radius of 20 cm.
Find the height h of the cone to one decimal place.



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10H Similar figures

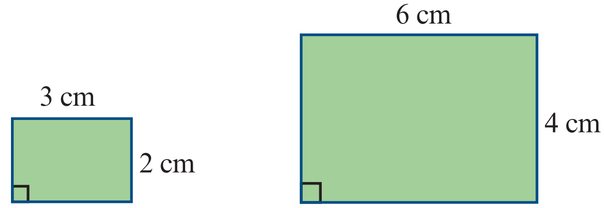
Learning intentions

- To be able to determine when shapes are similar and find their scale factor.
- To be able to find the scale factor for the areas of similar shapes.
- To be able to use similar triangles to find an unknown value.

Shapes that are similar have the same shape but are different sizes. The three frogs below are **similar figures**.

Polygons (closed plane figures with straight sides), like the rectangles in the diagram below, are similar if:

- corresponding angles are equal
- corresponding sides are proportional (which means that each pair of corresponding side lengths are in the same *ratio*).



For example, the two rectangles above are similar because their corresponding angles are equal and their side lengths are in the same ratio.

Ratio of side lengths = 6 : 3 which simplifies to 2 : 1

Ratio of side lengths = 4 : 2 which simplifies to 2 : 1

The ratios could be written as 3 : 6 and 2 : 4, simplifying to 1 : 2, as long as the measurements are read in the same order from one diagram to the next.
Ratios are sometimes written as fractions, such as $\frac{1}{2}$ for 1 : 2.

When we enlarge or reduce a shape by a **scale factor**, the *original* and the *image* are similar.

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Scale factor

When determining the **scale factor** of similar shapes, the numerator is a length of the second shape and the denominator is the corresponding length from the first (or original) shape.

In the previous diagram,

$$\textbf{scale factor} = \frac{a\ length\ of\ the\ second\ shape}{corresponding\ length\ of\ the\ first\ shape} = \frac{6}{3} = 2$$

Rectangle A has been enlarged by a scale factor, $k = 2$, to give rectangle B.

We say that rectangle A has been *scaled up* to give rectangle B.

We can also compare the rectangles' areas.

Area of rectangle A = 6 cm^2

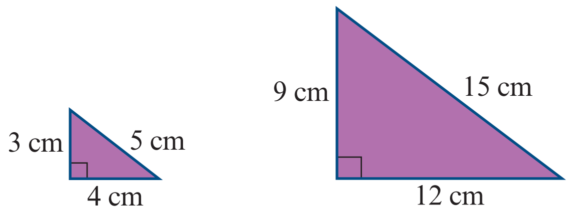
Area of rectangle B = 24 cm^2

The area of rectangle A has been enlarged by a scale factor, $k^2 = 4$, to give rectangle B.

We notice that, as the lengths are enlarged by a scale factor of 2, the area is enlarged by a scale factor of $2^2 = 4$.

Area scale factor

When all the lengths are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.



The two triangles above are similar because their corresponding side lengths are in the same ratio.

Ratio of side lengths = 3 : 9 or 1 : 3

Ratio of side lengths = 4 : 12 or 1 : 3

Ratio of side lengths = 5 : 15 or 1 : 3

$$\text{Scale factor, } k = \frac{a\ length\ of\ the\ second\ shape}{corresponding\ length\ of\ the\ first\ shape} = \frac{15}{5} = 3$$

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We would expect the area scale factor, k^2, or the ratio of the triangles' areas, to be 9 (= 3^2).

$$\text{Area of small triangle} = 6\ \text{cm}^2$$

$$\text{Area of large triangle} = 54\ \text{cm}^2$$

$$\text{Area scale factor, } k^2 = \text{Ratio of areas} = \frac{54}{6} = 9.$$

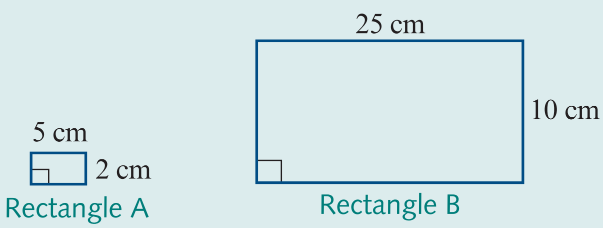
Shapes can be scaled up or scaled down. When a shape is made larger, it is *scaled up*, and when it is made smaller, it is *scaled down*.

Example 30 Finding the scale factor of length and area

The rectangles shown are similar.

a Find the scale factor of their side lengths.

b Find the scale factor of their areas.



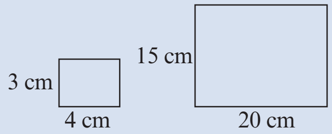
Explanation	Solution
a 1 To find the scale factor, put a length of the second shape as the numerator and the corresponding length from the first (or original) shape as the denominator.	$\frac{25}{5} = 5$
2 The small rectangle has been scaled up by a scale factor of 5. Write your answer.	The scale factor of the side lengths is 5.
b 1 Since the lengths are multiplied by a scale factor of 5, the area will be multiplied by a scale factor of 5^2.	$5^2 = 25$
2 The area of the small rectangle has been scaled up by a scale factor of 25. Write your answer.	The scale factor of the areas is 25.

Now try this 30 Finding the scale factor of length and area (Example 30)

For the similar rectangles shown:

a Find the scale factor of their sides.

b Find the scale factor of their areas.



Hint 1 The scale factor of their sides is found by dividing the length of a side in the second shape by the length of the corresponding side in the first shape.

Hint 2 The area scale factor is the length scale factor squared.

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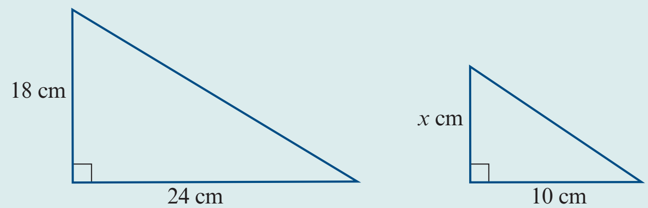
When scaling down from a larger figure to a smaller figure, the scale factor will always be less than one. This is because the numerator is always a length of the second shape, which in this case is smaller than the denominator (the corresponding length of the first shape).

Example 31 Using similar triangles to find unknown values

The following two triangles are similar. Find the value of x.

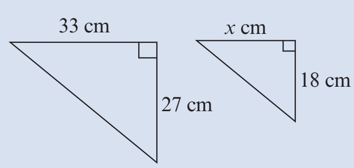


Explanation	Solution
1 Since the triangles are similar, their corresponding side lengths are in the same ratio.	$\frac{x}{18} = \frac{10}{24}$
2 Solve for x. Multiply both sides by 18.	$\frac{x}{18} \times 18 = \frac{10}{24} \times 18$
3 Write your answer using correct units.	$x = 7.5$ cm

Now try this 31 Using similar triangles to find unknown values (Example 31)

Two similar triangles are shown.

Find the value of x.



Hint 1 Since the triangles are similar, the ratio of their corresponding side lengths will be equal. So x over 33 is equal to ... over ... .

Hint 2 Now solve for x.

Section Summary

- Similar figures have the same shape but different sizes.
- Similar figures have:
  - corresponding angles that are equal
  - corresponding pairs of sides that are in the same proportion.
- The *scale factor* is the length of a side in the second shape divided by the length of the corresponding side in the first shape.
- The scale factor can be used to find an unknown length in similar triangles.
- The scale factor for area is the ratio of the areas of the similar shapes.
- A scale factor of k for lengths implies a scale factor of k^2 for areas.

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Exercise 10H

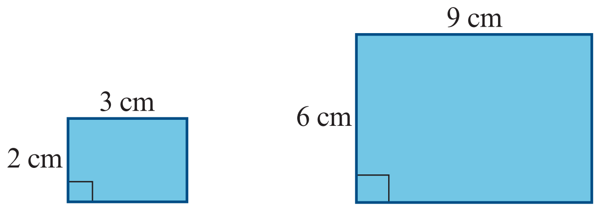
Building understanding

Example 30

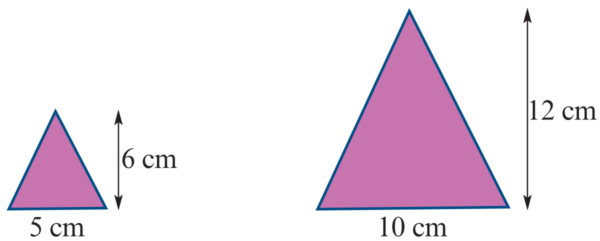
1 The following pairs of figures are similar. For each pair, find:

i the scale factor of their side lengths **ii** the scale factor of their areas.

a



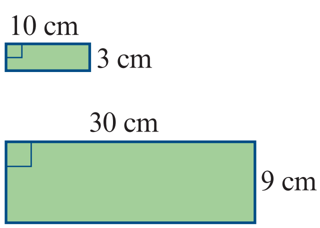
b



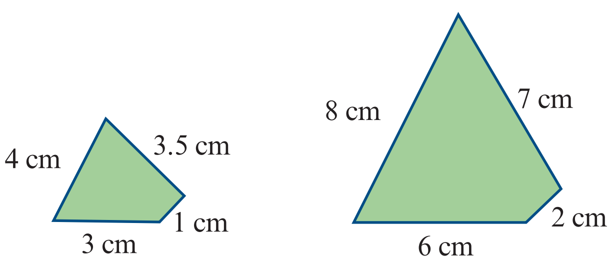
Developing understanding

2 Which of the following pairs of figures are similar? For those that are similar, find the scale factor of their sides.

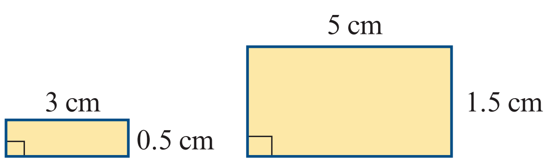
a



b



c



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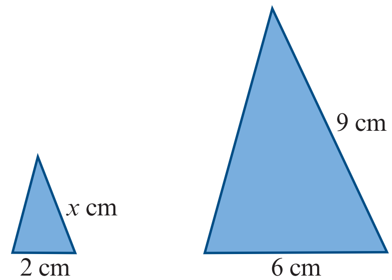
Example 31

3 The following triangles are similar.

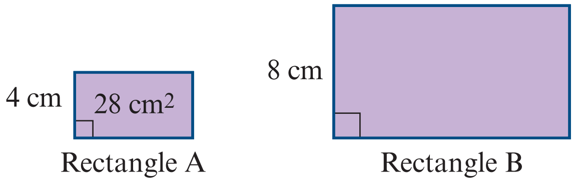
a Find the scale factor.

b Find the value of x.

c Find the ratio of their areas.



4 The two rectangles shown right are similar. The area of rectangle A is 28 cm^2. Find the area of rectangle B.



5 A photo is 12 cm by 8 cm. It is to be enlarged and then framed. If the dimensions are tripled, what will be the area of the new photo?

6 What is the scale factor if a photo has been enlarged from 15 cm by 9 cm, to 25 cm by 15 cm? Give your answer to two decimal places.

Testing understanding

7 A scale on a map is 1 : 500 000.

a What is the actual distance between two towns if the distance on the map is 7.2 cm? Give your answer in kilometres.

b If the actual distance between two landmarks is 15 km, find the distance represented on the map in cm.

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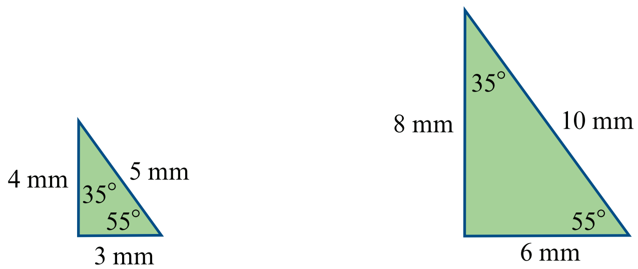
10I Similar triangles

Learning intentions

- To be able to use the tests for similar triangles to determine if triangles are similar.

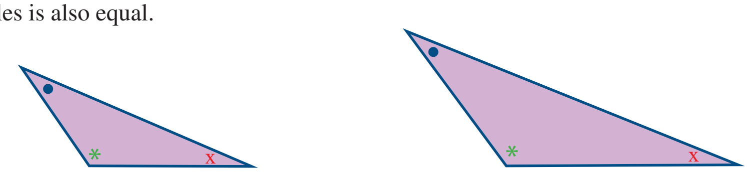
In mathematics, two **triangles** are said to be **similar** if they have the same shape. As in the previous section, this means that corresponding angles are equal and the lengths of corresponding sides are in the same ratio.

For example, these two triangles are similar.

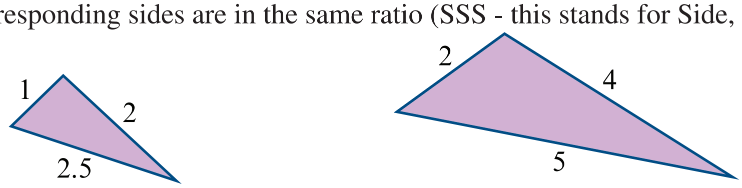


Two triangles can be tested for similarity by considering the following necessary conditions.

- Corresponding angles are equal (AA - this stands for Angle, Angle).
  Remember: If two pairs of corresponding angles are equal, then the third pair of corresponding angles is also equal.

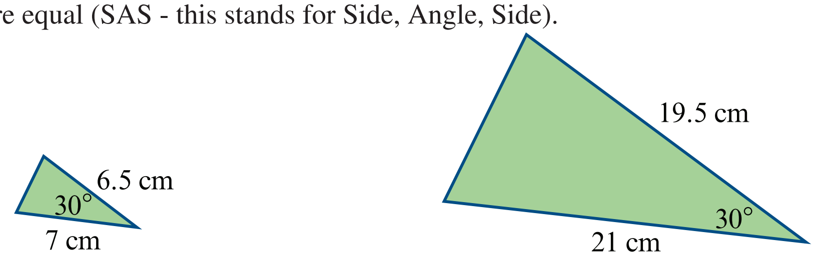


- Corresponding sides are in the same ratio (SSS - this stands for Side, Side, Side).



$$\frac{5}{2.5} = \frac{4}{2} = \frac{2}{1} = 2$$

- Two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS - this stands for Side, Angle, Side).



$$\frac{19.5}{6.5} = \frac{21}{7} = 3$$

Both triangles have an included corresponding angle of 30°.

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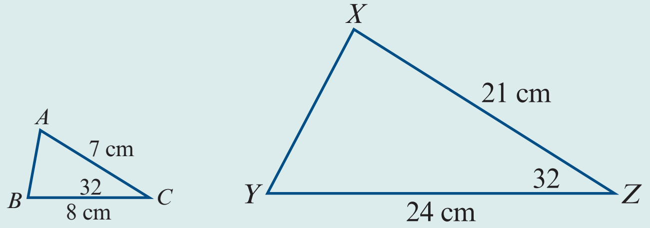
Example 32 Checking if triangles are similar

Explain why triangle *ABC* is similar to triangle *XYZ*.



Explanation

1. Compare corresponding side ratios:
   AC and *XZ*
   BC and *YZ*.
2. Triangles *ABC* and *XYZ* have an included corresponding angle.
3. Write an explanation as to why the two triangles are similar.

Solution

$$\frac{XZ}{AC} = \frac{21}{7} = \frac{3}{1}$$

$$\frac{YZ}{BC} = \frac{24}{8} = \frac{3}{1}$$

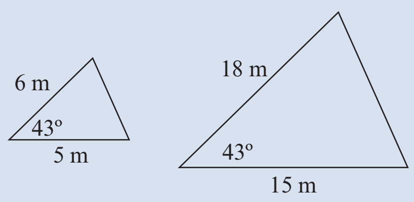
The angle 32° is included and corresponding.

They have two pairs of corresponding sides in the same ratio and the included corresponding angles are equal. (SAS)

Now try this 32 Checking if triangles are similar (Example 32)

Show that the smaller triangle is similar to the larger triangle.

State the rule which was used to decide that they are similar.



Hint 1 Find the side length ratios.

Section Summary

Two triangles are similar if:

- Corresponding angles are equal, AA OR
- Corresponding sides are in the same ratio, SSS OR
- Two pairs of corresponding sides are in the same ratio and the included angles are equal, SAS.

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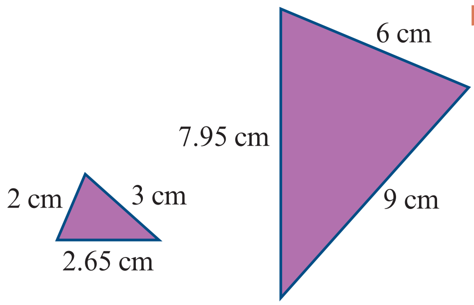
Exercise 10I

Building understanding

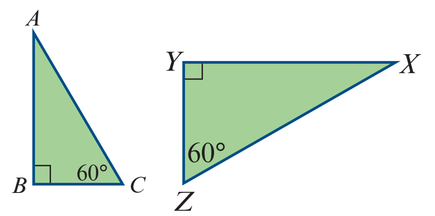
Example 32

1 Three pairs of similar triangles are shown below. Explain why each pair of triangles are similar.

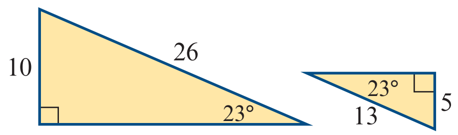
a



b



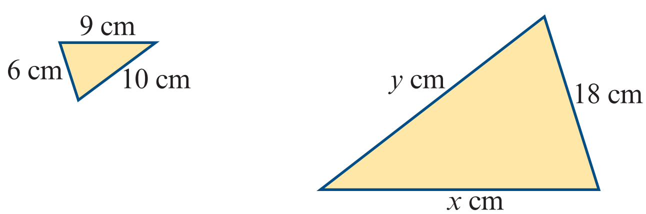
c



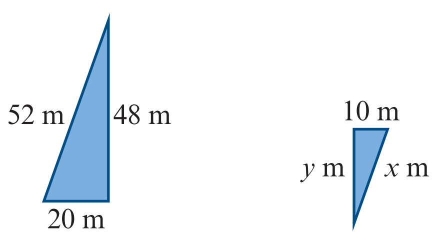
Developing understanding

2 Calculate the missing dimensions, marked x and y, in these pairs of similar triangles by first finding the scale factor.

a



b



3 A triangle with sides 5 cm, 4 cm and 8 cm is similar to a larger triangle with a longest side of 56 cm.

a Find the scale factor.

b Find the lengths of the larger triangle's other two sides.

c Find the perimeter of the larger triangle.

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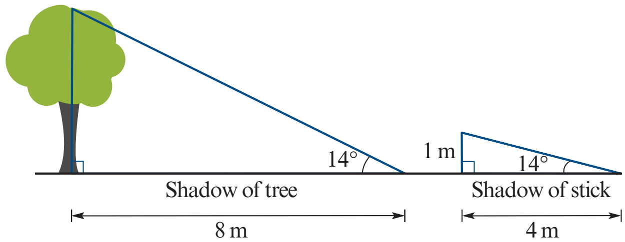
4 A tree and a one metre vertical stick cast their shadows at a particular time in the day. The shadow lengths are shown in the diagram below (*not* drawn to scale).

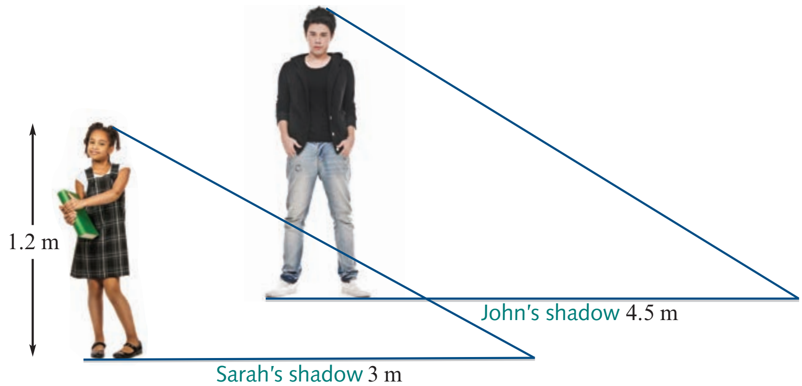
a Give reasons why the two triangles shown are similar.

b Find the scale factor for the side lengths of the triangles.

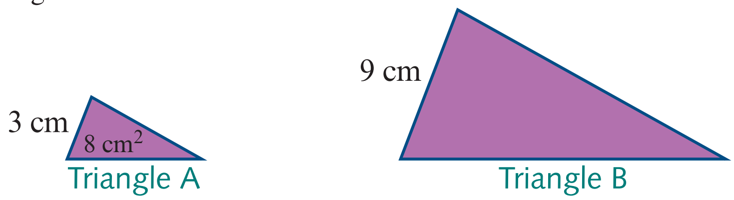
c Find the height of the tree.



5 John and his younger sister, Sarah, are standing side by side. Sarah is 1.2 m tall and casts a shadow 3 m long. How tall is John if his shadow is 4.5 m long?

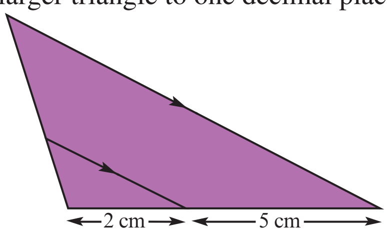


6 The area of triangle A is 8 cm^2. Triangle B is similar to triangle A. What is the area of triangle B?



Testing understanding

7 Given that the area of the small triangle in the following diagram is 2.4 cm^2, find the area of the larger triangle to one decimal place.



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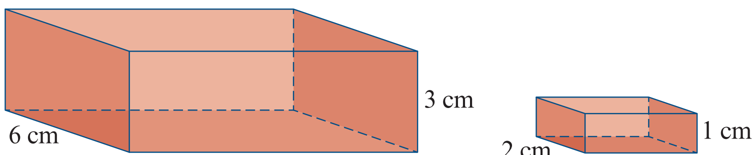
10J Similar solids

Learning intentions

- To be able to determine when two solids are similar, and to calculate their scale factor.

Two solids are similar if they have the same shape, and the ratios of their corresponding linear dimensions are equal.

Cuboids



Cuboid A Cuboid B

The two cuboids are similar because:

- they are the same shape (both are cuboids)
- the ratios of the corresponding dimensions are the same.

$$\frac{\text{length of cuboid B}}{\text{length of cuboid A}} = \frac{\text{width of cuboid B}}{\text{width of cuboid A}} = \frac{\text{height of cuboid B}}{\text{height of cuboid A}}$$

$$\frac{2}{6} = \frac{3}{9} = \frac{1}{3} = \frac{1}{3}$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{2 \times 3 \times 1}{6 \times 9 \times 3} = \frac{6}{162} = \frac{1}{27} = \frac{1}{3^3}$$

As the length dimensions are reduced by a scale factor of $\frac{1}{3}$, the volume is reduced by a scale factor of $\frac{1}{3^3} = \frac{1}{27}$.

Scaling volumes

When all the dimensions are multiplied by a scale factor of k, the volume is multiplied by a scale factor of k^3.

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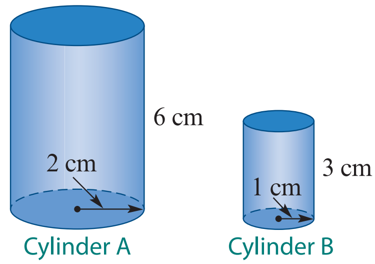
Cylinders

These two cylinders are similar because:

- they are the same shape (both are cylinders)
- the ratios of the corresponding dimensions are the same.

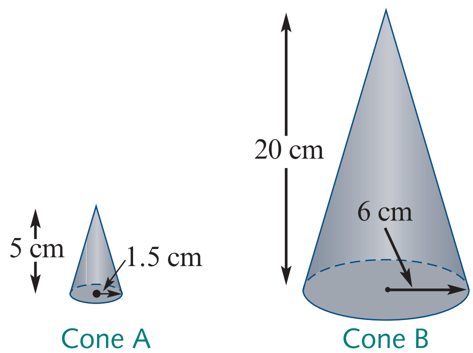
$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{\text{height of cylinder B}}{\text{height of cylinder A}} = \frac{\text{radius of cylinder B}}{\text{radius of cylinder A}}$$



$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\pi \times 1^2 \times 3}{\pi \times 2^2 \times 6} = \frac{3}{24} = \frac{1}{8} = \frac{1}{2^3}$$

Cones



These two cones are similar because:

- they are the same shape (both are cones)
- the ratios of the corresponding dimensions are the same.

$$\frac{20}{5} = \frac{6}{1.5} = 4$$

$$k = \frac{\text{height of cone B}}{\text{height of cone A}} = \frac{\text{radius of cone B}}{\text{radius of cone A}} = 4$$

$$\text{Volume scale factor, } k^3 = \text{Ratio of volumes} = \frac{\frac{1}{3} \times \pi \times 6^2 \times 20}{\frac{1}{3} \times \pi \times 1.5^2 \times 5}$$

$$= 64 = 4^3$$

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Example 33 Comparing volumes of similar solids

Two solids are similar such that the larger one has all of its dimensions three times that of the smaller solid. How many times larger is the larger solid's volume?

Explanation	Solution
1 Since all of the larger solid's dimensions are 3 times those of the smaller solid, the volume will be 3^3 times larger. Evaluate 3^3.	$3^3 = 27$
2 Write your answer.	The larger solid's volume is 27 times the volume of the smaller solid.

Now try this 33 Comparing volumes of similar solids (Example 33)

A solid has dimensions seven times those of a smaller similar solid. How many times is the volume of the larger solid greater than the volume of the smaller solid?

Hint 1 A scale factor of k for lengths implies a scale factor of k^3 for volumes.

Section Summary

- Similar solids have the same shape and a constant ratio for their corresponding sides.
- Lengths with a scale factor of k imply that there is a scale factor of k^3 for the volumes.

Skill-sheet

Exercise 10J

Building understanding

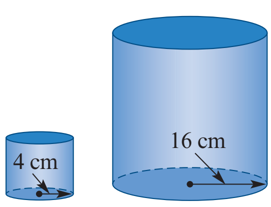
Example 33

1 Two cylindrical water tanks are similar such that the height of the larger tank is 3 times the height of the smaller tank. How many times larger is the volume of the larger tank compared to the volume of the smaller tank?

2 Two cylinders are similar and have radii of 4 cm and 16 cm, respectively.

a What is the ratio of their heights?

b What is the ratio of their volumes?



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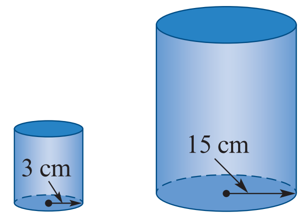
Developing understanding

3 Find the ratio of the volumes of two cuboids whose sides are in the ratio $\frac{3}{1}$.

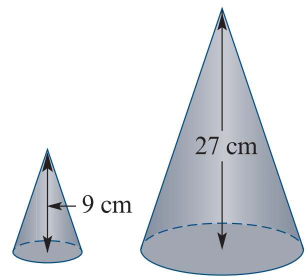
4 The radii of the bases of two similar cylinders are in the ratio $\frac{5}{1}$. The height of the larger cylinder is 45 cm. Calculate:



a the height of the smaller cylinder

b the ratio of the volumes of the two cylinders.

5 Two similar cones are shown at right. The ratio of their heights is $\frac{3}{1}$.



a Determine whether the smaller cone has been scaled up or down to give the larger cone.

b What is the volume scale factor?

c The volume of the smaller cone is 120 cm^3. Find the volume of the larger cone.

6 The radii of the bases of two similar cylinders are in the ratio 3 : 4. The height of the larger cylinder is 8 cm. Calculate:

a the height of the smaller cylinder

b the ratios of the volumes of the two cylinders.

Testing understanding

7 A pyramid has a square base of side 4 cm and a volume of 16 cm^3. Calculate:

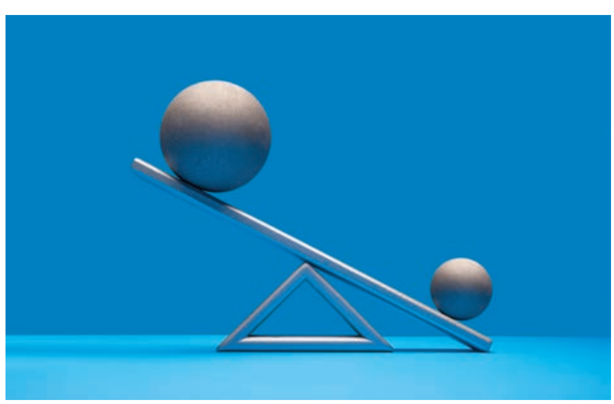
a the height of the pyramid

b the height and the length of the side of the base of a similar pyramid with a volume of 1024 cm^3.

8 Two spheres have diameters of 12 cm and 6 cm respectively. Calculate:

a the ratio of their surface areas

b the ratio of their volumes.



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Key ideas and chapter summary

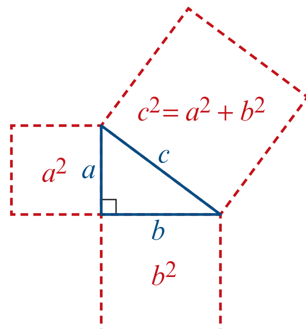
Assign-
ment

Scientific notation — To write a number in scientific notation, express it as a number between 1 and 10, multiplied by a power of 10.

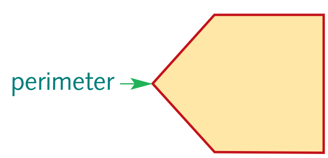
Rounding — If the number following the specified digit is 5 or more, than round the specified digit up. If the following number is less than 5, then leave the specified digit unchanged.
e.g. 5.417 rounded to two decimal places is 5.42 (number after the 1 is 7, so round up).

Significant figures — To write a number to the required number of **significant figures**, write the number in scientific notation, then round to the required number of significant figures.

Pythagoras' theorem — **Pythagoras' theorem** states that:
For any right-angled triangle, the sum of the areas of the squares of the two shorter sides (a and b) equals the area of the square of the hypotenuse (c): $c^2 = a^2 + b^2$



Perimeter (P) — **Perimeter** is the distance around the edge of a two-dimensional shape.



Perimeter of rectangle — $P = 2l + 2w$

Circumference (C) — **Circumference** is the perimeter of a circle: $C = 2\pi r$.

Length of an arc — The length, s, of an arc of a circle with a radius, r, that subtends an angle of $\theta°$ at the centre is given by:

$$s = \frac{\pi r \theta}{180}$$

Area (A) — **Area** is the measure of the region enclosed by the boundaries of a two-dimensional shape.

Area of a sector — The area, A, of a sector of a circle with a radius, r, where the arc of the sector subtends an angle of $\theta°$ at the centre is given by:

$$A = \frac{\pi r^2 \theta}{360}$$

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Area formulas

Area of rectangle = lw

Area of parallelogram = bh

Area of triangle = $\frac{1}{2}bh$

Area of trapezium = $\frac{1}{2}(a+b)h$

Area of circle = $\pi \times r^2$

Heron's formula

Area of triangle = $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$ and a, b and c are the sides of the triangle.

Volume (V)

Volume is the amount of space occupied by a 3-dimensional object.

- For prisms and cylinders,
  Volume = area of cross-section × height
- For pyramids and cones,
  Volume = $\frac{1}{3}$ × area of base × height

Volume formulas

Volume of cube = l^3

Volume of cuboid = lwh

Volume of triangular prism = $\frac{1}{2}bhl$

Volume of cylinder = $\pi r^2 h$

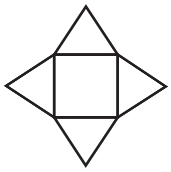
Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of pyramid = $\frac{1}{3}lwh$

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area (SA)

Surface area is the total of the areas of all the surfaces of a solid. When finding surface area, it is often useful to draw the net of the shape.



Surface area formulas

Surface area of cylinder = $2\pi r^2 + 2\pi rh$

Surface area of cone = $\pi r^2 + \pi rs$

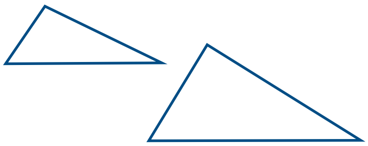
Surface area of sphere = $4\pi r^2$

Similar figures or solids

Similar figures or solids are the same shape but different sizes. Corresponding sides are in the same ratio.

Similar triangles

Triangles are shown to be **similar** if:



- corresponding angles are similar (AA)
- corresponding sides are in the same ratio (SSS)
- two pairs of corresponding sides are in the same ratio and the included corresponding angles are equal (SAS).

Ratios of area and volume for similar shapes

When all the dimensions of similar shapes are multiplied by a scale factor of k, the areas are multiplied by a scale factor of k^2 and the volumes are multiplied by a scale factor of k^3.

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Review

Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

10A **1** I can round numbers to a specific number of decimal places. ☐

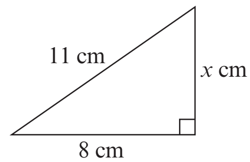
e.g. Round 307.509 to 2 decimal places.

10A **2** I can round numbers to a specific number of significant figures. ☐

e.g. Round 307.509 to 2 significant figures.

10B **3** I can find the length of an unknown side in a right-angled triangle. ☐

e.g. Find the length of the unknown side, x, to one decimal place.



10B **4** I can find the length of an unknown side in a three-dimensional object using Pythagoras' theorem. ☐

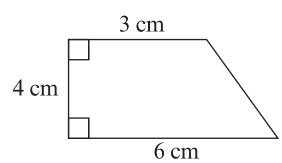
e.g. A cube has sides of 10 cm. Find the length of the diagonal from one corner, through the centre of the cube, to the opposite corner. Give your answer to one decimal place.

10C **5** I can determine the perimeters and areas of regular shapes, such as rectangles, parallelograms, trapeziums and triangles. ☐

e.g. Find the perimeter and area of a triangle with sides of 6 cm, 7 cm and 8 cm to the nearest whole number.

10C **6** I can find the perimeter and area of composite shapes. ☐

e.g. Find the perimeter and area of the trapezium shown.



10C **7** I can find the circumference and area of a circle when given its radius. ☐

e.g. Find the circumference and area of a circle with a radius of 7 cm, to one decimal place.

10D **8** I can find the length of an arc. ☐

e.g. An arc on a circle with a radius of 10 m, subtends an angle of 76° at the centre of the circle. Find the length of the arc to one decimal place.

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10D **9** I can find the area of a sector. ☐

e.g. A sector of a circle with a radius of 10 m, subtends an angle of 76° at the centre of the circle. Find the area of the sector to one decimal place.

10E **10** I can find the volumes of rectangular prisms, triangular prisms, square prisms and cylinders. ☐

e.g. A crystal has a triangular cross-section with a base length of 44 mm and a height of 28 mm.The crystal is 52 mm long. Find its volume.

10E **11** I can find the capacity of three-dimensional containers. ☐

e.g. A rectangular can has a base area of 600 cm^2 and a height of 45 cm. How many litres of petrol could it hold?

10E **12** I can calculate the volume of a cone given its height and radius. ☐

e.g. Find the volume of a cone, 30 cm high and with a base radius of 18 cm, to one decimal place.

10E **13** I can calculate the volumes of spheres and hemispheres when given their radii. ☐

e.g. Find the volume of a hemisphere with a radius of 15 cm to one decimal place.

10F **14** I can calculate the volume of a square or a hexagonal pyramid given its height and the width (or area) of its base. ☐

e.g. Determine the volume of a pyramid with a height of 48 m and a square base with sides of 58 m.

10F **15** I can determine the surface area of objects with plane surfaces, such as prisms, cuboids and pyramids. ☐

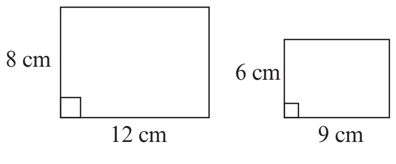
e.g. Determine the surface area of a rectangular prism with sides of 2, 3 and 4 metres.

10G **16** I can find the surface area of objects with curved surfaces, such as cylinders, cones and spheres. ☐

e.g. Find the surface area of a cone with a radius of 26cm and a sloping edge of 39 cm to one decimal place.

10H **17** I can determine when shapes are similar and find their scale factor. ☐

e.g. Show that the shapes are similar and find the scale factor.



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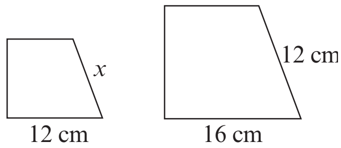
Review

10H **18** I can find the scale factor for the areas of similar shapes. ☐

e.g. Give the scale factor for the areas in the previous question.

10H **19** I can use the scale factor to find unknown values. ☐

e.g. Find the value of x for these similar shapes.



10I **20** I can use the appropriate tests to determine if triangles are similar. ☐

e.g. Two triangles have sides of 3, 5, 6 and 6, 10, 12. Determine if they are similar, giving a reason.

10J **21** I can determine when two solids are similar and calculate their scale factor. ☐

e.g. One cuboid has sides of 6, 9 and 12, while the other has sides of 10, 15 and 20. Determine if they are similar, and if so, calculate their scale factor.

Multiple-choice questions

1 3.895 rounded to two decimal places is:

A 3.8 **B** 3.89 **C** 3.9 **D** 3.99 **E** 4.0

2 4679 rounded to the nearest hundred is:

A 4600 **B** 4670 **C** 4680 **D** 4700 **E** 5000

3 5.21×10^5 is the same as:

A 0.000 052 1 **B** 260.50 **C** 52 105 **D** 521 000 **E** 52 100 000

4 0.0048 written in scientific notation is:

A 4.8×10^{-4} **B** 4.8×10^{-3} **C** 48×10^{-3} **D** 48×10^{-2} **E** 4.8×10^{3}

5 28 037.2 rounded to two significant figures is:

A 7.2 **B** 20 000.2 **C** 20 007 **D** 28 000 **E** 28 000.2

6 0.030 69 rounded to two significant figures is:

A 0.000 69 **B** 0.03 **C** 0.0306 **D** 0.0307 **E** 0.031

7 Which one of these numbers does *not* have exactly three significant figures?

A 0.0572 **B** 12.0 **C** 60.40 **D** 30 700 **E** 333 000

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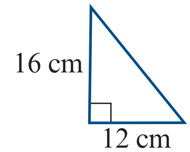
8 The three side measurements of five different triangles are given below. Which one is a right-angled triangle?

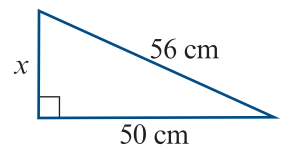
A 1, 2, 3 **B** 4, 5, 12 **C** 9, 11, 15 **D** 10, 10, 15 **E** 15, 20, 25

9 The length of the hypotenuse for the triangle shown is:



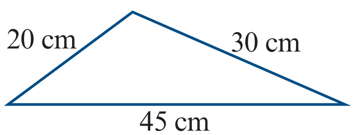
A 7.46 cm **B** 10.58 cm
C 20 cm **D** 28 cm
E 400 cm

10 The value of x in the triangle shown is:



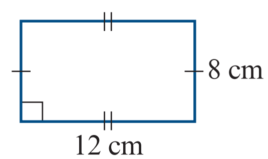
A 6 cm **B** 25.22 cm
C 75.07 cm **D** 116 cm
E 636 cm

11 The perimeter of the triangle shown is:



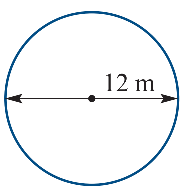
A 50 cm **B** 90 cm
C 95 cm **D** 95 cm^2
E 450 cm

12 The perimeter of the rectangle shown is:



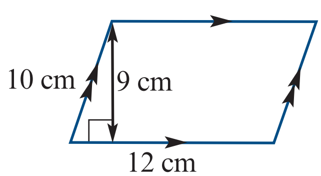
A 20 cm **B** 28 cm
C 32 cm **D** 40 cm
E 96 cm

13 The circumference of a circle with diameter 12 m is closest to:



A 18.85 m **B** 37.70 m
C 113.10 m **D** 118.44 m
E 453.29 m

14 The area of the shape shown is:



A 44 cm^2 **B** 90 cm^2
C 108 cm^2 **D** 120 cm^2
E 180 cm^2

15 The area of a circle with radius 3 cm is closest to:

A 9.42 cm^2 **B** 18.85 cm^2 **C** 28.27 cm^2 **D** 31.42 cm^2 **E** 113.10 cm^2

16 The length of an arc that subtends an angle of 49° on a circle of radius 5.4 m is closest to:

A 0.46 m **B** 2.3 m **C** 4.6 m **D** 9.2 m **E** 23 m

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Review

17 The area of a sector that subtends an angle of 110° on a circle of radius 34 cm is closest to:

A 1109 cm^2 **B** 1110 cm^2 **C** 1119 cm^2 **D** 1900 cm^2 **E** 1901 cm^2

18 The volume of a cube with side length 5 cm is:

A 30 cm^3 **B** 60 cm^3 **C** 125 cm^3 **D** 150 cm^3 **E** 625 cm^3

19 The volume of a box with length 11 cm, width 5 cm and height 6 cm is:

A 22 cm^3 **B** 44 cm^3 **C** 302 cm^3 **D** 330 cm^3 **E** 1650 cm^3

20 The volume of a sphere with radius 16 mm is closest to:

A 67.02 mm^3 **B** 268.08 mm^3 **C** 1072.33 mm^3
D 3217 mm^3 **E** 17 157.28 mm^3

21 The volume of a cone with base diameter 12 cm and height 8 cm is closest to:

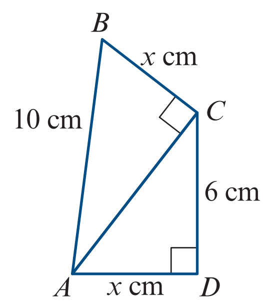
A 301.59 cm^3 **B** 904.78 cm^3 **C** 1206.37 cm^3
D 1809.56 cm^3 **E** 3619.11 cm^3

22 The volume of a cylinder with radius 3 m and height 4 m is closest to:

A 12 m^3 **B** 12.57 m^3 **C** 37.70 m^3
D 113.10 m^3 **E** 452.39 m^3

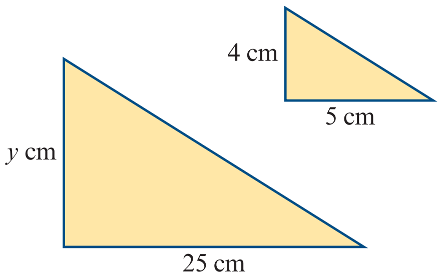
23 In the diagram, angles *ACB* and *ADC* are right angles. If *BC* and *AD* each have a length of x cm, then x is closest to:

A 4 **B** 5 **C** 5.66
D 7.07 **E** 8.25



24 The two triangles shown are similar.
The value of y is:

A 9 cm **B** 16 cm **C** 20 cm
D 21 cm **E** 24 cm



25 The diameter of a large sphere is 4 times the diameter of a smaller sphere. It follows that the ratio of the volume of the larger sphere to the volume of the smaller sphere is:

A 4 : 1 **B** 8 : 1 **C** 16 : 1 **D** 32 : 1 **E** 64 : 1

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Short-answer questions

1 Write each of the following in scientific notation.

a 2945 **b** 0.057 **c** 369 000 **d** 850.9

2 Write the basic numeral for each of the following.

a 7.5×10^3 **b** 1.07×10^{-3} **c** 4.56×10^{-1}

3 Write the following to the number of significant figures indicated in the brackets.

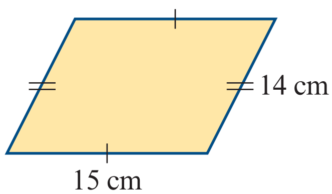
a 8.916 (2) **b** 0.0589 (2) **c** 809 (1)

4 Write the following to the number of decimal places indicated in the brackets.

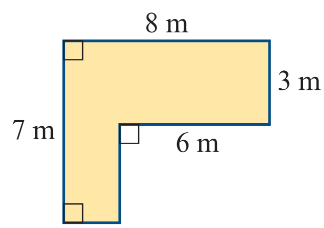
a 7.145 (2) **b** 598.241 (1) **c** 4.0789 (3)

5 Find the perimeters of these shapes.

a



b

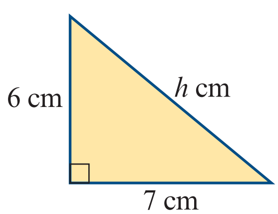


6 Find the perimeter of a square with side length 9 m.

7 Find the perimeter of a rectangle with length 24 cm and width 10 cm.

8 Find the lengths of the unknown sides, to two decimal places, in the following triangles.

a



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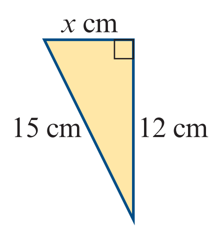
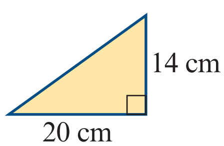


9 Find the areas of the following shapes.

a



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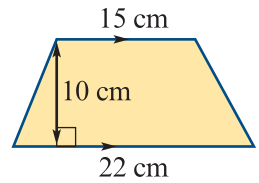

10 Find the surface area of a cube with side length 2.5 m.

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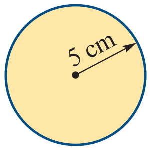
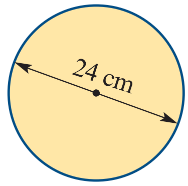
11 Find the circumferences of the following circles to two decimal places.

a



b



12 Find the areas of the circles in Question **11** to two decimal places.

13 A soup can has a diameter of 7 cm and a height of 13.5 cm.

- **a** How much metal, to two decimal places, is needed to make the can?
- **b** A paper label is made for the outside cylindrical shape of the can. How much paper, in m^2, is needed for 100 cans? Give your answer to two decimal places.
- **c** What is the capacity of one can, in litres, to two decimal places?

14 A circular swimming pool has a diameter of 4.5 m and a depth of 2 m. How much water will the pool hold to the nearest litre?

15 The radius of the Earth is approximately 6400 km. Calculate:

- **a** the surface area in square kilometres
- **b** the volume to four significant figures.

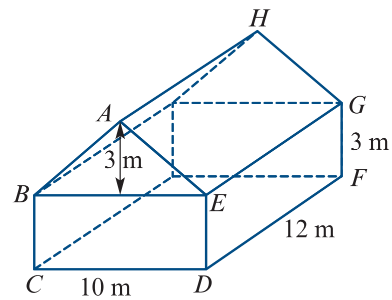
16 The diameter of the base of an oilcan in the shape of a cone is 12 cm and its height is 10 cm. Find:

- **a** its volume in square centimetres to two decimal places
- **b** its capacity to the nearest millilitre.

17 A pyramid with a square base of side length 8 m has a height of 3 m. Find the length of a sloping edge to one decimal place.

18 For the solid shown on the right, find to two decimal places:

- **a** the area of rectangle *BCDE*
- **b** the area of triangle *ABE*
- **c** the length *AE*
- **d** the area of rectangle *AEGH*
- **e** the surface area.



19 Find the volume of a rectangular prism with length 3.5 m, width 3.4 m and height 2.8 m.

20 You are given a circle of radius *r*. The radius increases by a scale factor of 2. By what factor does the area of the circle increase?

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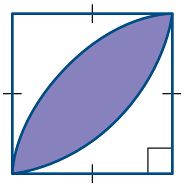
21 You are given a circle of diameter d. The diameter decreases by a scale factor of $\frac{1}{2}$. By how much does the area of the circle decrease?

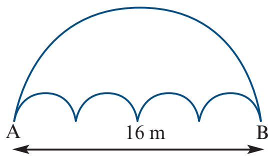
22 For the shaded region, find to two decimal places:

a the perimeter

b the area.



23 Which is the shorter path from A to B? Is it along the four semicircles or along the larger semicircle? Give reasons for your answer.

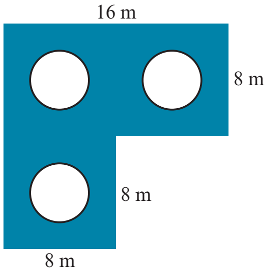


Written-response questions

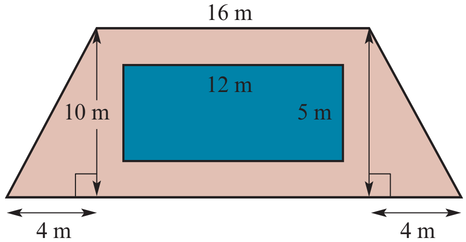
1 A lawn has three circular flowerbeds in it, as shown in the diagram. Each flowerbed has a radius of 2 m. A gardener has to mow the lawn and use a whipper-snipper to trim all the edges. Calculate:

a the area to be mown

b the length of the edges to be trimmed. Give your answer to two decimal places.



2 Chris and Gayle decide to build a swimming pool on their new housing block. The pool will measure 12 m by 5 m and it will be surrounded by timber decking in a trapezium shape. A safety fence will surround the decking. The design layout of the pool and surrounding area is shown in the diagram.



a What length of fencing is required? Give your answer to two decimal places.

b What area of timber decking is required?

c The pool has a constant depth of 2 m. What is the volume of the pool?

d The interior of the pool is to be painted white. What surface area is to be painted?

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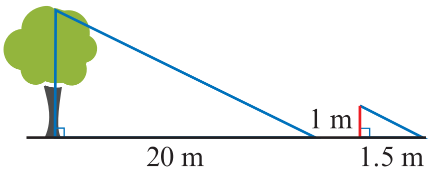
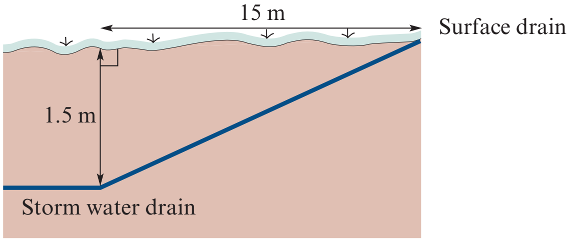
3 A biologist studying gum trees wanted to calculate the height of a particular tree. She placed a one metre ruler on the ground which cast a shadow on the ground measuring 1.5 m. The gum tree cast a shadow of 20 m, as shown in the diagram below (*not* to scale). Calculate the height of the tree. Give your answer to two decimal places.



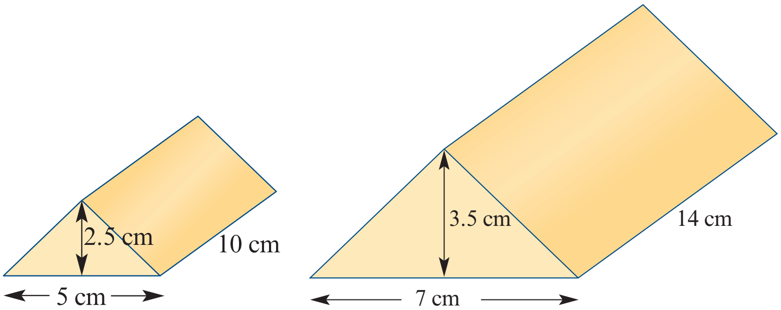
4 A builder is digging a trench for a cylindrical water pipe. From a drain at ground level, the water pipe goes 1.5 m deep where it joins a storm water drain. The horizontal distance from the surface drain to the storm water drain is 15 m, as indicated in the diagram below (*not* to scale).



a Calculate the length of water pipe required to connect the surface drain to the storm water drain to two decimal places.

b If the radius of the water pipe is 20 cm, what is the volume of the water pipe? Give your answer to two decimal places.

5 Two similar triangular prisms are shown below.



a Find the ratio of their surface areas.

b Find the ratio of their volumes.

c What is the volume of the smaller prism to the nearest cm^3?

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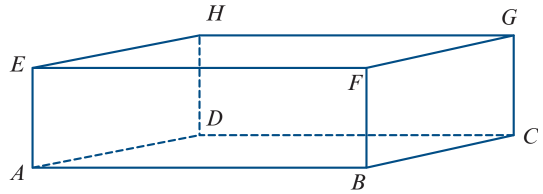
6 The length of a rectangular prism is eight times its height. The width is four times the height. The length of the diagonal between two opposite vertices (A and G) is 36 cm.
Find the volume of the prism.



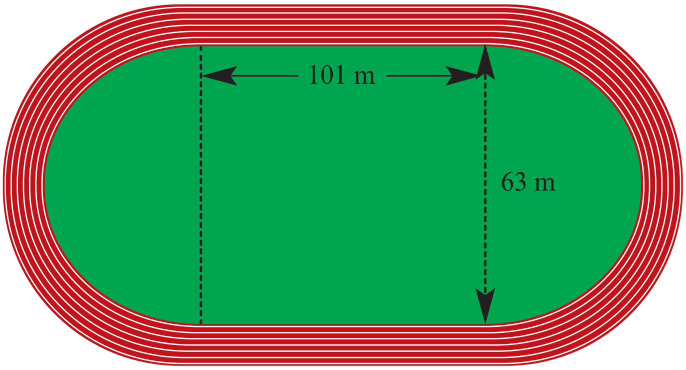
7 The volume of a cone of height 28.4 cm is 420 cm^3. Find the height of a similar cone whose volume is 120 cm^3 to two decimal places.

8 An athletics track is made up of a straight stretch of 101 m and two semicircles on the ends, as shown in the diagram. There are 6 lanes, each one metre wide.

a What is the total distance, to the nearest metre, around the inside lane?

b If 6 athletes run around the track keeping to their own lane, how far, to the nearest metre, would each athlete run?

c Draw a diagram and indicate at which point each runner should start so that they all run the same distance.



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