Chapter 11

Applications of trigonometry

Chapter questions

- How are sin θ, cos θ and tan θ defined using a right-angled triangle?
- How can the trigonometric ratios be used to find the side lengths or angles in right-angled triangles?
- What is meant by an angle of elevation or an angle of depression?
- How are three-figure bearings measured?
- How can the sine and cosine rules be used to solve triangles which are not right-angled?
- What are the three rules that are used to find the area of a triangle?

Trigonometry can be used to solve many practical problems. How high is that tree? What is the height of the mountain we can see in the distance? What is the exact location of the fire that has just been seen by fire spotters? How wide is the lake? What is the area of this irregular-shaped paddock?

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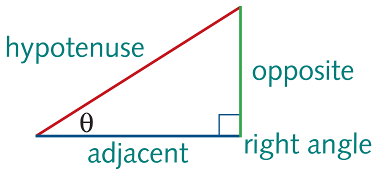
11A Trigonometry basics

Learning intentions

- To be able to name the sides of a right-angled triangle.
- To be able to know the definitions of the trigonometric ratios.
- To be able to use a CAS calculator to find the value of a trigonometric ratio for a given angle.

Although you are likely to have studied some trigonometry, it may be helpful to review a few basic ideas.

Naming the sides of a right-angled triangle

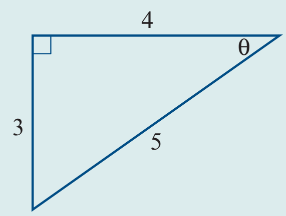


- The **hypotenuse** is the longest side of the right-angled triangle and is always opposite the right angle (90°).
- The **opposite** side is directly opposite the angle θ.
- The **adjacent** side is beside the angle θ, but it is not the hypotenuse. It runs from θ to the right angle.

The opposite and adjacent sides are located in relation to the position of angle θ.
If θ was in the other corner, the sides would have to swap their labels.
The letter θ is the Greek letter *theta*. It is commonly used to label an angle.

Example 1 Identifying the sides of a right-angled triangle

Give the lengths of the hypotenuse, the opposite side and the adjacent side in the triangle shown.



Explanation	Solution
The hypotenuse is opposite the right angle.	The hypotenuse: $h = 5$
The opposite side is opposite the angle θ.	The opposite side: $o = 3$
The adjacent side is between θ and the right-angle.	The adjacent side: $a = 4$

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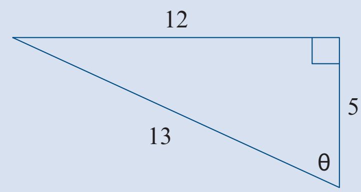
Now try this 1 **Identifying the sides of a right-angled triangle (Example 1)**

Refer to the diagram to answer the questions below.



a What is the name of the side that is 12 units long?

b Name the side that is 5 units long.

c Give the name of the side that is 13 units long.

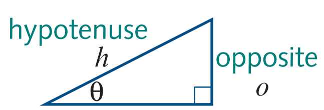
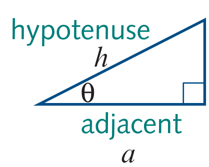
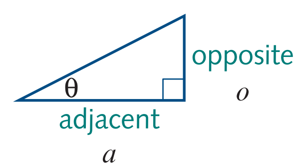
Hint 1 It is opposite the angle θ.

Hint 2 It is between the angle θ and the right angle.

Hint 3 It is opposite the right angle.

The trigonometric ratios

The **trigonometric ratios**, **sin** θ, **cos** θ and **tan** θ, can be defined in terms of the sides of a right-angled triangle.

 	 	
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{o}{h}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{a}{h}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{o}{a}$

"SOH — CAH — TOA"

This mnemonic, **SOH-CAH-TOA**, is often used by students to help them remember the rule for each trigonometric ratio.

In this mnemonic:

- **SOH** reminds us that **S**ine equals **O**pposite over **H**ypotenuse
- **CAH** reminds us that **C**osine equals **A**djacent over **H**ypotenuse
- **TOA** reminds us that **T**an equals **O**pposite over **A**djacent.

Or you may prefer:

'**S**ir **O**liver's **H**orse **C**ame **A**mbling **H**ome **T**o **O**liver's **A**rms'

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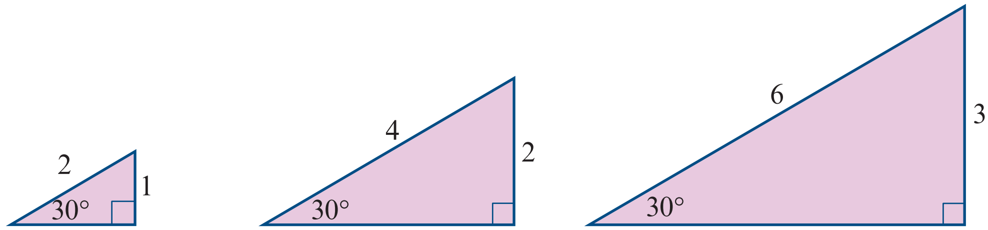
The meaning of the trigonometric ratios

Using a calculator we find, for example, that $\sin 30^\circ = 0.5$. This means that in *all* right-angled triangles with an angle of 30°, the length of the side opposite the 30° divided by the length of the hypotenuse is always 0.5.



$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} = 0.5$ $\quad\frac{\text{opposite}}{\text{hypotenuse}} = \frac{2}{4} = 0.5$ $\quad\frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{6} = 0.5$

Try drawing any right-angled triangle with an angle of 30° and check that the ratio:

$$\frac{\text{opposite}}{\text{hypotenuse}} = 0.5$$

Similarly, for *any* right-angled triangle with an angle of 30°, the ratios cos 30° and tan 30° always have the same values:

$$\cos 30^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ is always } \frac{\sqrt{3}}{2} = 0.8660 \text{ (to four decimal places)}$$

$$\tan 30^\circ = \frac{\text{opposite}}{\text{adjacent}} \text{ is always } \frac{1}{\sqrt{3}} = 0.5774 \text{ (to four decimal places).}$$

A calculator gives the value of each trigonometric ratio for any angle entered.

Using your CAS calculator to evaluate trigonometric ratios

Warning!

Make sure that your calculator is set in DEGREE mode before attempting the example on the following page.

See the Appendix, which can be accessed online through the Interactive Textbook.

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Example 2 Finding the values of trigonometric ratios

Use your graphics calculator to find, to four decimal places, the value of:

a sin 49° **b** cos 16° **c** tan 27.3°

Explanation

1 For the **TI-Nspire CAS**, ensure that the mode is set in **Degree** and **Approximate (Decimal)**. Refer to Appendix to set mode.

2 In a Calculator page, press [trig], select **sin** and type 49.

3 Repeat for **b** and **c** as shown on the calculator screen.
Optional: you can add a degree symbol from the [αβ°] palette if desired. This will override any mode settings.

4 Write your answer to four decimal places.

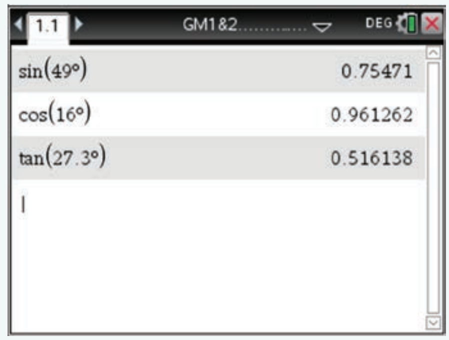
5 For **ClassPad**, in the Main application ensure that the status bar is set to **Decimal** and **Degree** mode.

6 To enter and evaluate the expression:
- Display the **keyboard**
- In the Trig palette select [sin]
- Type [4][9][°][)]
- Press [EXE]

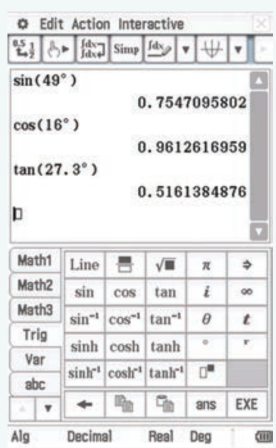
7 Repeat for **b** and **c** as shown on the calculator screen.

8 Write your answer to four decimal places.

Solution



a sin (49°) = 0.7547
b cos (16°) = 0.9613
c tan (27.3°) = 0.5161



a sin (49°) = 0.7547
b cos (16°) = 0.9613
c tan (27.3°) = 0.5161

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Now try this 2 Finding the values of trigonometric ratios (Example 2)

Find the values of the trigonometric ratios to three decimal places:

a $\tan 28°$ **b** $\cos 43°$ **c** $\sin 62.8°$

Hint 1 Make sure your calculator is in Degree mode.

Hint 2 Choose the required trigonometric button on your CAS calculator.

Hint 3 Type the required angle and press **enter** or **EXE**.

Hint 4 Look at the fourth decimal place. If it is 5 or larger, increase the third decimal place by 1.

Section Summary

- In a right-angled triangle:
  The hypotenuse is the longest side and is opposite the right-angle
  The opposite side is directly opposite the angle θ
  The adjacent side runs from θ to the right-angle.
- The trigonometric ratios are:
  $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Skill-sheet

Exercise 11A

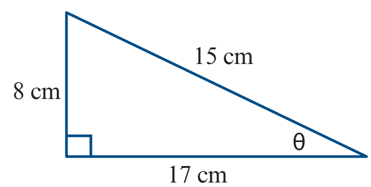
Building understanding

1 Use the position of the angle θ to name each side.

a Name the side that is 8 cm long.

b Write the name of the side that is 15 cm long.

c What is the name of the side that is 17 cm long?

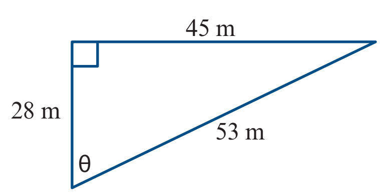


2 Name each side by using the position of the angle θ.

a Write the name of the side that is 45 m long.

b What is the name of the side that is 28 m long?

c Name the side that is 53 m long.

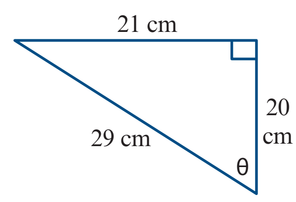


3 Using the sides given in the diagram, write the trigonometric ratio for each of the following:

a $\sin\theta$

b $\cos\theta$

c $\tan\theta$



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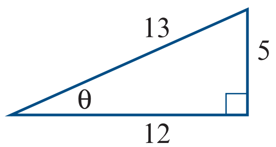
Developing understanding

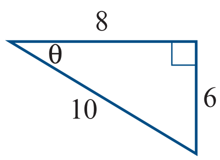
Example 1

4 State the values of the hypotenuse, the opposite side and the adjacent side in each triangle.

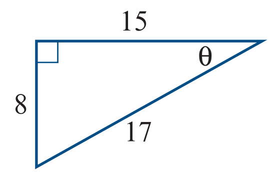
a



b



c



Example 2

5 Write the ratios for sin θ, cos θ and tan θ for each triangle in Question **4**.

6 Find the values of the following trigonometric ratios to four decimal places.

a sin 27° **b** cos 43° **c** tan 62° **d** cos 79°
e tan 14° **f** sin 81° **g** cos 17° **h** tan 48°

Testing understanding

Use Pythagoras' theorem to find the answers as fractions.

7 Given $\cos\theta = \frac{20}{29}$, find sin θ.

8 Use $\sin\theta = \frac{9}{41}$ to find tan θ.

11B Finding an unknown side in a right-angled triangle

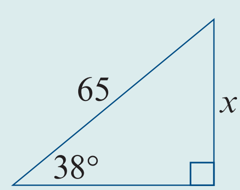
Learning intentions

- ▶ To be able to choose the required trigonometric ratio when finding an unknown side of a right-angled triangle.
- ▶ To be able to substitute in values and solve the required equation to find the length of the unknown side.

The trigonometric ratios can be used to find unknown sides in a right-angled triangle, given an angle and one side. When the unknown side is in the numerator (top) of the trigonometric ratio, proceed as follows.

Example 3 **Finding an unknown side in a right-angled triangle**

Find the length of the unknown side, x, in the triangle shown to two decimal places.



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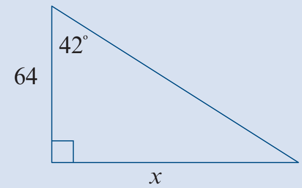
Explanation	Solution
1 The sides involved are the opposite and the hypotenuse, so use sin θ.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
2 Substitute in the known values.	$\sin 38^\circ = \dfrac{x}{65}$
3 Multiply both sides of the equation by 65 to obtain an expression for x. Use a calculator to evaluate.	$65 \times \sin 38^\circ = x$ $x = 65 \times \sin 38^\circ$ $= 40.017\ldots$
4 Write answer to 2 decimal places.	$x = 40.02$

Now try this 3 Finding an unknown side in a right-angled triangle (Example 3)

Find the length of the unknown side, x, in the triangle shown to one decimal place.



Hint 1 What is the position of the unknown side, looking from the given angle?

Hint 2 What position name should be given to the side that is 64 units long?

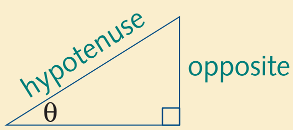
Hint 3 What trigonometric ratio uses the position names of the 'x' and the '64' side?

Finding an unknown side in a right-angled triangle

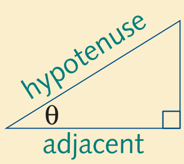
1 Draw the triangle and write in the given angle and side. Label the unknown side as x.

2 Use the trigonometric ratio that includes the given side and the unknown side.

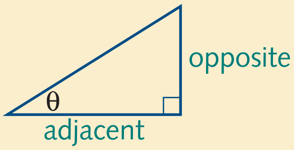
a For the opposite and the hypotenuse, use $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$



b For the adjacent and the hypotenuse, use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



c For the opposite and the adjacent, use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$



3 Rearrange the equation to make x the subject.

4 Use your calculator to find the value of x to the required number of decimal places.

An extra step is needed when the unknown side is in the denominator (at the bottom) of the trigonometric ratio, as in the example on the following page.

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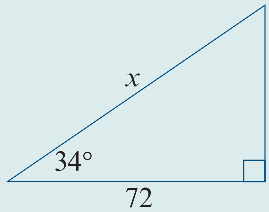
Example 4 Finding an unknown side which is in the denominator of the trig ratio

Find the value of x in the triangle shown to two decimal places.



Explanation

1. The sides involved are the adjacent and the hypotenuse, so use cos θ.
2. Substitute in the known values.
3. Multiply both sides by x.
4. Divide both sides by cos 34° to obtain an expression for x. Use a calculator to evaluate.
5. Write your answer to two decimal places.

Solution

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 34^\circ = \frac{72}{x}$$

$$x\cos 34^\circ = 72$$

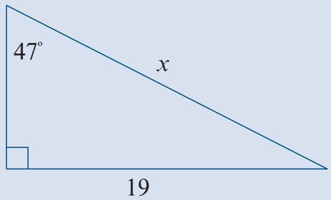
$$x = \frac{72}{\cos 34^\circ}$$

$$= 86.847\ldots$$

$$x = 86.85$$

Now try this 4 Finding an unknown side which is in the denominator of the trig ratio (Example 4)

Find the length of the unknown side, x, in the triangle shown to one decimal place.



Hint 1 What are the position names of side x and the side that is 19 units long?

Hint 2 Choose the trigonometric ratio that uses the position names of the two sides involved.

Hint 3 Write the trigonometric equation for the given angle and sides.

Hint 4 Multiply both sides of the equation by x. Now what do you need to divide both sides by?

Section Summary

To find an unknown side in a right-angled triangle:

- Use the position of the given angle to name the given side and the required side.
- Write an equation using the trigonometric ratio that uses the given and required sides.
- Substitute the given values and solve the equation to find the unknown side to the required decimal places.

Skill-sheet

Exercise 11B

Building understanding

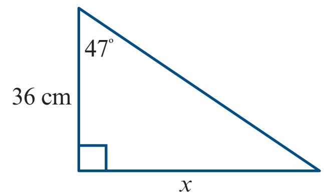
1 **a** State the name of the side that is 36 cm long.

b What is the name of the unknown side, x?

c Write the trigonometric ratio rule that uses the names of the sides in parts **a** and **b**.

d Substitute the value of the angle and the known side into the rule. Call the unknown side x.

e Solve the equation by multiplying both sides by the denominator and using your CAS calculator to find the value of x to two decimal places.



2 **a** Give the name of the side that is 19 cm long.

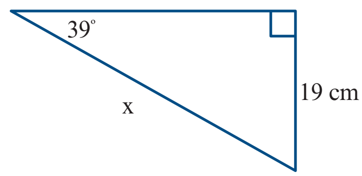
b State the name of the unknown side, x.

c Write the trigonometric ratio rule that uses the names of the sides in parts **a** and **b**.

d Substitute the value of the angle and the known side into the rule. Call the unknown side x.

e Because the unknown, x, is the denominator, two steps are needed. Multiply both sides by x, then divide both sides by $\sin 39°$, to make x the subject.

f Use your CAS calculator to find x to one decimal place.



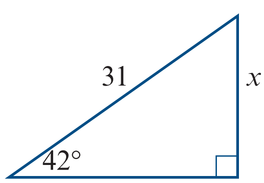
Developing understanding

Example 3

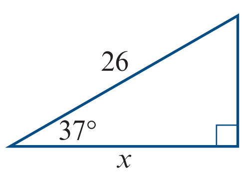
3 In each right-angled triangle below:

- decide whether the $\sin \theta$, $\cos \theta$ or $\tan \theta$ ratio should be used
- then find the unknown side, x, to two decimal places.

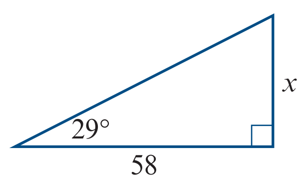
a



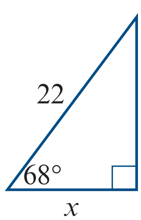
b



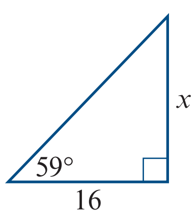
c



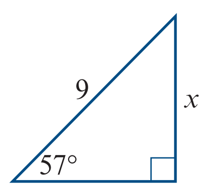
d



e



f



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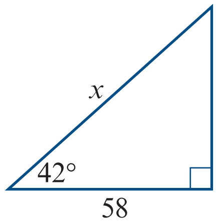
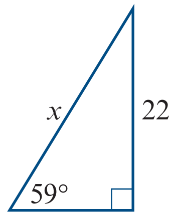
Example 4

4 Find the unknown side, x, in each right-angled triangle below to two decimal places.

a



b



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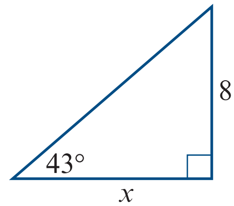
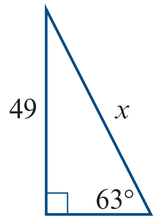


d



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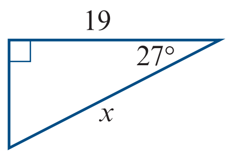

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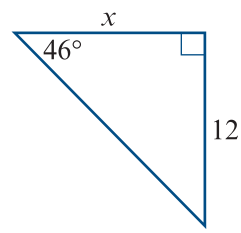
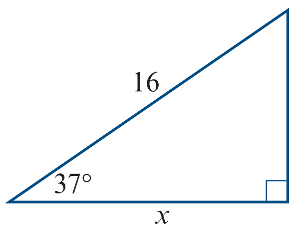


5 Find the length of the unknown side shown in each triangle to one decimal place.

a



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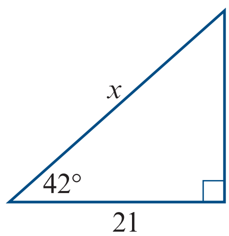

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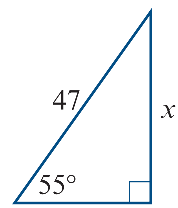
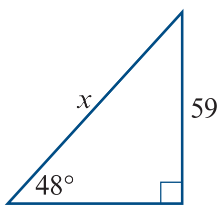


d



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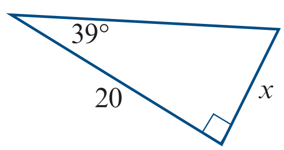

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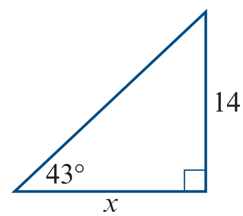
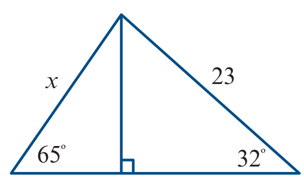
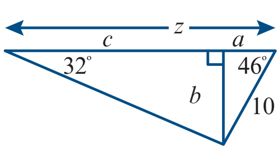


Testing understanding

6 Find the length of the unknown side, x, in the diagram shown.
Give your answer to one decimal place.



7 Find the length, z, to one decimal place.



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11C Finding an angle in a right-angled triangle

Learning intentions

- ► To be able to use a CAS calculator to find an angle when given the value of its trigonometric ratio.
- ► To be able to find the required angle in a right-angled triangle when given two sides of the triangle.

Finding an angle from a trigonometric ratio value

Before we look at how to find an unknown angle in a right-angled triangle, it will be useful to see how to find the angle when we know the value of the trigonometric ratio.

Suppose a friend told you that they found the sine value of a particular angle to be 0.8480, and challenged you to find out the mystery angle that had been used.

This is equivalent to saying:

$\sin\theta = 0.8480$, find the value of angle θ.

To do this, you need to work backwards from 0.8480 by undoing the sine operation to get back to the angle used. It is as if we have to find reverse gear to undo the effect of the sine function.

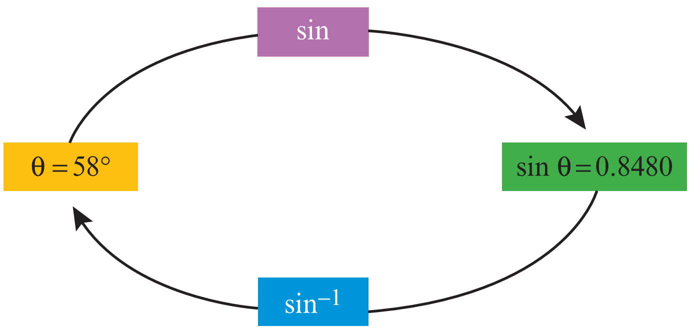
The reverse gear for sine is called the inverse of sine, written $\sin^{-1}$. The superscript, -1, is not a power. It is just saying, let us undo, or take one step backwards from using, the sine function.

The step to find θ when $\sin\theta = 0.8480$ can be written as:

$$\sin^{-1}(0.8480) = \theta$$

This process is summarised in the following diagram.

- The top arrow in the diagram corresponds to: given θ, find $\sin\theta$. We use the sine function on our calculator to do this by entering $\sin 58°$ into a calculator to obtain the answer, 0.8480.
- The bottom arrow in the diagram corresponds to: given $\sin\theta = 0.8480$, find θ. We use the $\sin^{-1}$ function on our calculator to do this by entering $\sin^{-1}(0.8480)$ to obtain the answer, 58°.



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Similarly:

- The inverse of cosine, written as $\cos^{-1}$, is used to find θ when, for example, $\cos\theta = 0.5$.
- The inverse of tangent, written as $\tan^{-1}$, is used to find θ when, for example, $\tan\theta = 1.67$.

You will learn how to use the $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ functions of your calculator in the following example.

Example 5 Finding an angle from a trigonometric ratio

Find the angle, θ, to one decimal place, given:

a $\sin\theta = 0.8480$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 1.67$

Explanation	Solution
a We need to find $\sin^{-1}(0.8480)$.	
1 For **TI-Nspire CAS**, press [trig], select $\sin^{-1}$, then press [0] [.] [8] [4] [8] [0] [enter].	$\sin^{-1}(0.848)$ 57.9948
2 For **ClassPad**, tap [$\sin^{-1}$] [0] [.] [8] [4] [8] [0] [)] [EXE].	
3 Write your answer to one decimal place.	$\theta = 58.0°$
b We need to find $\cos^{-1}(0.5)$.	
1 For **TI-Nspire CAS**, press [trig], select $\cos^{-1}$, then press [0] [.] [5] [enter].	$\cos^{-1}(0.5)$ 60
2 For **ClassPad**, tap [$\cos^{-1}$] [0] [.] [5] [)] [EXE].	
3 Write your answer to one decimal place.	$\theta = 60°$
c We need to find $\tan^{-1}(1.67)$.	
1 For **TI-Nspire CAS**, press [trig], select $\tan^{-1}$, then press [1] [.] [6] [7] [enter].	$\tan^{-1}(1.67)$ 59.0867
2 For **ClassPad**, tap [$\tan^{-1}$] [1] [.] [6] [7] [)] [EXE].	
3 Write your answer to one decimal place.	$\theta = 59.1°$

Now try this 5 Finding an angle from a trigonometric ratio (Example 5)

Find the angle, θ, to two decimal places, given:

a $\cos\theta = 0.6847$ **b** $\tan\theta = 7.5509$ **c** $\sin\theta = 0.2169$

Hint 1 In each question, select the required inverse function: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on your CAS calculator.

Hint 2 Then enter the given decimal value of the trigonometric function, and press **enter** or **EXE** to find the required angle.

Getting the language right

The language we use when finding an angle from a trig ratio is difficult when you first meet it. The samples below are based on the results of Example **5**.

- When you see:

  $\sin(58°) = 0.8480$

  think: 'the sine of the angle 58° equals 0.8480'.

- When you see:

  $\sin^{-1}(0.8480) = 58°$

  think: 'the angle whose sine is 0.8480 equals 58°'.

- When you see:

  $\cos(60°) = 0.5$

  think: 'the cosine of the angle 60° equals 0.5'.

- When you see:

  $\cos^{-1}(0.5) = 60°$

  think: 'the angle whose cosine is 0.5 equals 60°'.

- When you see:

  $\tan(59.1°) = 1.67$

  think: 'the tan of the angle 59.1° equals 1.67'.

- When you see:

  $\tan^{-1}(1.67) = 59.1°$

  think: 'the angle whose tan is 1.67 equals 59.1°'.

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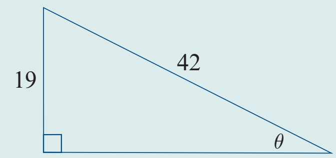
Finding an angle given two sides

Example 6 **Find an angle, given 2 sides in a right-angled triangle**

Find the angle, θ, in the right-angled triangle shown to one decimal place.

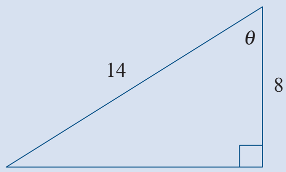


Explanation	Solution
1 The sides involved are the opposite and the hypotenuse, so use sin θ.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
2 Substitute in the known values.	$\sin\theta = \dfrac{19}{42}$
3 Write the equation to find an expression for θ. Use a calculator to evaluate.	$\theta = \sin^{-1}\left(\dfrac{19}{42}\right) = 26.896\ldots$
4 Write your answer to one decimal place.	$\theta = 26.9°$

The three angles in a triangle add to 180°. As the right angle is 90°, the other two angles must add to make up the remaining 90°. When one angle has been found, just subtract it from 90° to find the other angle. In Example **6**, the other angle must be 90° − 26.9° = 63.1°.

Now try this 6 **Find an angle, given 2 sides in a right-angled triangle (Example 6)**

Find the angle, θ, in the triangle shown to two decimal places.



Hint 1 What are the position names of the sides that are 8 and 14 units long?

Hint 2 Which of: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ can use the sides from Hint 1 to find the angle θ?

Finding an angle in a right-angled triangle

1. Draw the triangle with the given sides shown. Label the unknown angle as θ.
2. Use the trigonometric ratio that includes the two known sides.
   - If given the opposite and hypotenuse, use $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
   - If given the adjacent and hypotenuse, use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
   - If given the opposite and adjacent, use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
3. Divide the side lengths to find the value of the trigonometric ratio.
4. Use the appropriate inverse function key to find the angle, θ.

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Section Summary

To find an unknown angle:

- Use the position of the required angle to name the two given sides.
- Write an equation using the trigonometric ratio with the required angle and the two given sides.
- Substitute the given values and solve the equation to find the unknown angle, to the required number of decimal places.

Skill-sheet

Exercise 11C

Building understanding

1 Give answers to one decimal place.

a Given $\cos\theta = 0.4867$, use $\cos^{-1}(0.4867)$ to find the value of θ.

b Given $\tan\theta = 0.6384$, use $\tan^{-1}(0.6348)$ to find the value of θ.

c Given $\sin\theta = 0.3928$, use $\sin^{-1}(0.3928)$ to find the value of θ.

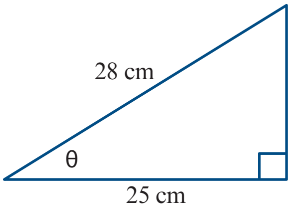
2 To find the value of θ, answer the following questions. Give θ to one decimal place.

a State the name of the side that is 28 cm long.

b Write the name of the side that is 25 cm long.

c Write the trigonometric ratio rule that uses the names of the two sides in parts **a** and **b**.

d Substitute the values of the sides into the rule.



e To find θ, use the inverse cosine, $\cos^{-1}$, feature on your CAS calculator to evaluate $\cos^{-1}(\frac{25}{28})$.

Developing understanding

Example 5

3 Find the unknown angle, θ, to one decimal place.

a $\sin\theta = 0.4817$

b $\cos\theta = 0.6275$

c $\tan\theta = 0.8666$

d $\sin\theta = 0.5000$

e $\tan\theta = 1.0000$

f $\cos\theta = 0.7071$

g $\sin\theta = 0.8660$

h $\tan\theta = 2.500$

i $\cos\theta = 0.8383$

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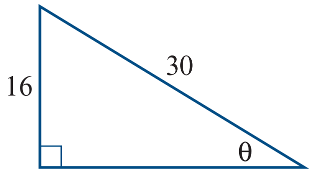
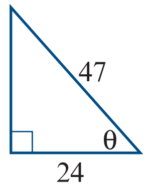
Example 6

4 Find the unknown angle, θ, in each triangle to one decimal place.

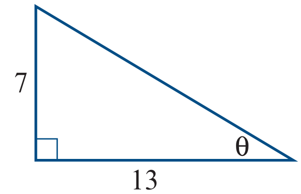
a Use $\sin^{-1}$ for this triangle.



b Use $\cos^{-1}$ for this triangle.



c Use $\tan^{-1}$ for this triangle.



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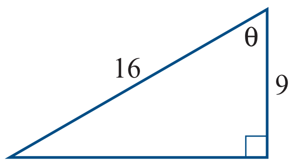

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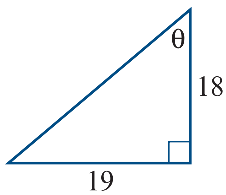

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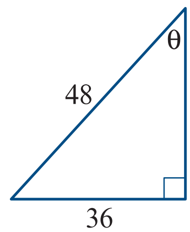

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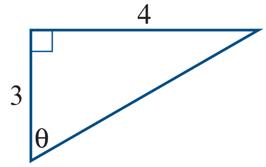

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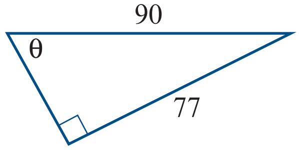

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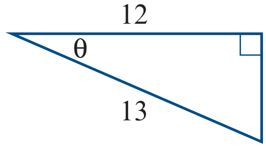
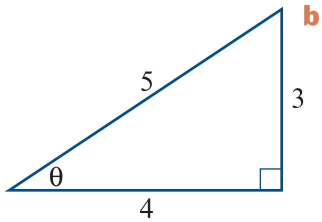
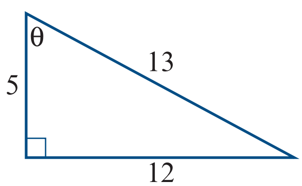


5 Find the value of θ in each triangle to one decimal place.

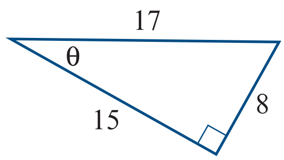
a



b

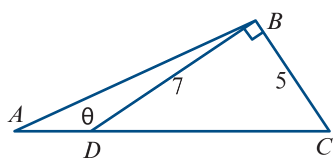


c

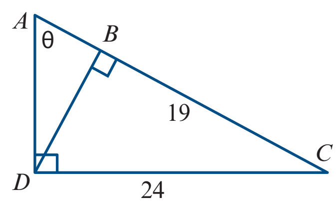


Testing understanding

6 Find the unknown angle, θ, to one decimal place.



7 Find the unknown angle, θ, to one decimal place.



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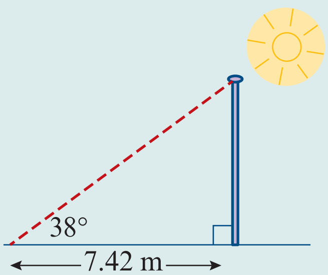
11D Applications of right-angled triangles

Learning intentions

- To be able to draw clearly labelled diagrams of practical situations, showing the given sides and angles.
- To be able to set up and solve equations to find unknown sides and angles.

Example 7 Application requiring a length

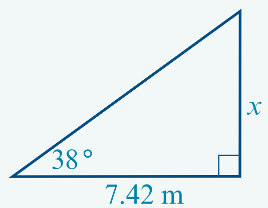
A flagpole casts a shadow 7.42 m long. The sun's rays make an angle of 38° with the level ground. Find the height of the flagpole to two decimal places.



Explanation

1. Draw a diagram showing the right-angled triangle. Include all the known details and label the unknown side as x.

Solution



2. The opposite and adjacent sides are involved, so use tan θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

3. Substitute in the known values.

$$\tan 38^\circ = \frac{x}{7.42}$$

4. Multiply both sides by 7.42.

$$7.42 \times \tan 38^\circ = x$$

5. Use your calculator to find the value of x.

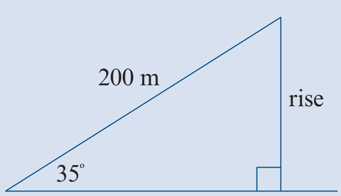
$$x = 5.797\ldots$$

6. Write your answer to two decimal places.

The height of the flagpole is 5.80 m.

Now try this 7 Application requiring a length (Example 7)

A person walked 200 m up a slope of 35°. How much did they rise vertically? Answer to the nearest metre.



Hint 1 What are the position names of the sloping side and the rise?

Hint 2 Now use the trigonometric ratio that uses those position names.

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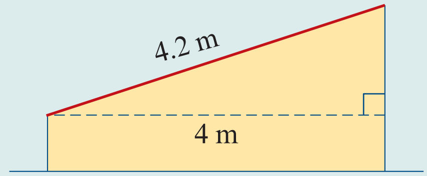
Example 8 Application requiring an angle

A sloping roof uses sheets of corrugated iron, 4.2 m long, on a shed, 4 m wide. There is no overlap of the roof past the sides of the walls. Find the angle that the roof makes with the horizontal to one decimal place.



Explanation

1 Draw a diagram showing the right-angled triangle. Include all known details and label the required angle.

2 The adjacent and hypotenuse are involved, so use cos θ.

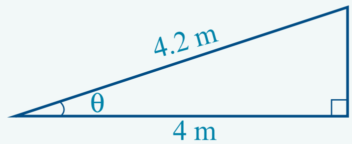
3 Substitute in the known values.

4 Write the equation to find θ.

5 Use your calculator to find the value of θ.

6 Write your answer to one decimal place.

Solution



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{4}{4.2}$$

$$\theta = \cos^{-1}\left(\frac{4}{4.2}\right)$$

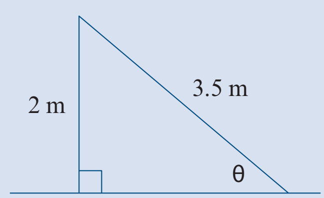
$\cos^{-1}\left(\frac{4}{4.2}\right)$	17.7528

The roof makes an angle of 17.8° with the horizontal.

Remember: Always evaluate a mathematical expression as a whole, rather than breaking it into several smaller calculations. Rounding-off errors accumulate as more approximate answers are fed into the calculations.

Now try this 8 Application requiring an angle (Example 8)

A vertical ladder of a children's slide is 2 m in height. The length of the slide is 3.5 m. Find the unknown angle, θ, that the slide makes with the level ground to the nearest degree.



Hint 1 What are the position names of the given lengths?

Hint 2 Which of: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ uses the sides named in Hint 1?

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Section Summary

To solve applications questions:

- Draw a clearly labelled diagram showing the given values and a symbol for the required value.
- Use the position of the angle to choose the required trigonometric equation.
- Substitute the given values and solve the equation to the required number of decimal places.

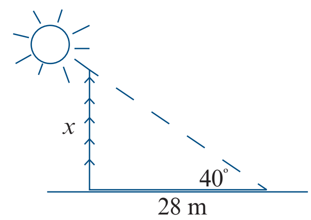
Skill-sheet

Exercise 11D

Building understanding

Answer to one decimal place.

1 A tree casts a shadow that is 28 m long, making an angle of 40° with the horizontal ground. To find the height, x, of the tree, answer the questions below.



a Name the 28 m side.

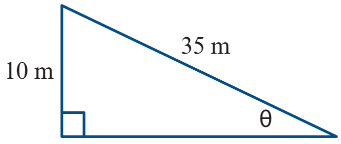
b Write the name of the side x.

c Write the trigonometric ratio rule that uses the sides named in parts **a** and **b**.

d Substitute the known values of the angle and a side into the equation. Call the unknown side x.

e Multiply both sides of the equation by the value of the denominator to get x by itself. Use your CAS calculator to find the value of x.

2 The starting ramp for a ski race is 35 m long and 10 m high. Find the angle the ramp makes with the horizontal by answering the questions below.



a State the name of the side that is 35 m long.

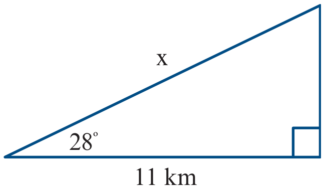
b Give the name of the side that is 10 m long?

c Write the trigonometric ratio rule that uses the sides named in parts **a** and **b**?

d Substitute the values of the side lengths into the equation.

e To find θ, use the inverse sine, $\sin^{-1}$, feature on your CAS calculator to evaluate $\sin^{-1}(\frac{10}{35})$.

3 A map showed a bushwalker had moved a horizontal distance of 11 km as she walked up a slope of 28°. To find the distance she had walked up the slope, answer the questions on the following page.



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a Give the name of the 11 km side and side x.

b State the trigonometric ratio rule that uses the sides in part **a**.

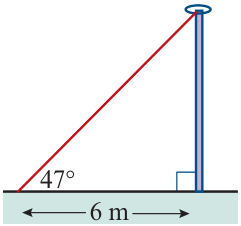
c Substitute the known value for the side and the angle into the equation.

d Multiply both sides by x. Divide both sides by $\cos 28°$. Use your calculator to find the value of x.

Developing understanding

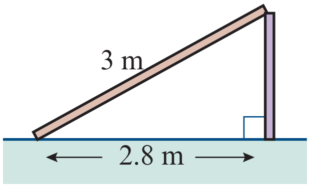
Example 7

4 A pole is supported by a wire that runs from the top of the pole to a point on the level ground, 6 m from the base of the pole. The wire makes an angle of 47° with the ground. Find the height of the pole to two decimal places.

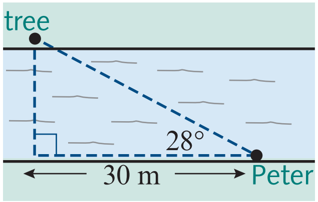


Example 8

5 A 3-metre log rests with one end on the top of a post and the other end on the level ground, 2.8 m from the base of the post. Find the angle the log makes with the ground to one decimal place.



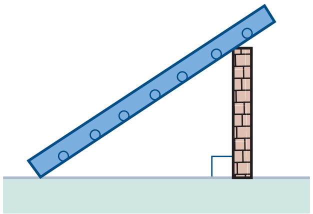
6 Peter noticed that a tree was directly opposite him on the far bank of the river. After he walked 30 m along his side of the river, he found that his line of sight to the tree made an angle of 28° with the riverbank. Find the width of the river, to the nearest metre.



7 A ladder rests on a wall that is 2 m high. The foot of the ladder is 3 m from the base of the wall on level ground.

a Copy the diagram and include the given information. Label as θ the angle the ladder makes with the ground.

b Find the angle the ladder makes with the ground to one decimal place.



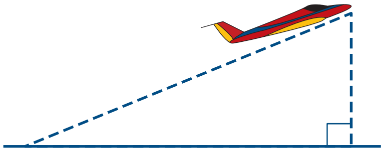
8 An aeroplane maintains a flight path of 17° with the horizontal after it takes off. It travels for 2 km along that flight path.

a Show the given and required information on a copy of the diagram.

b Find to two decimal places:

i the horizontal distance of the aeroplane from its take-off point

ii the height of the aeroplane above ground level.



9 A 3 m ladder rests against an internal wall. The foot of the ladder is 1 m from the base of the wall. Find the angle the ladder makes with the floor to one decimal place.

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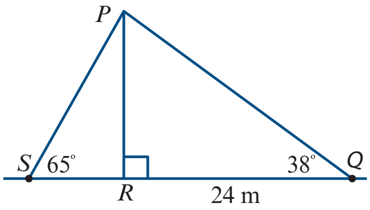
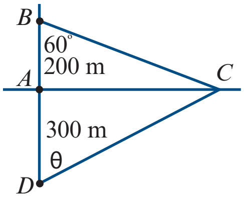
10 The entrance to a horizontal mining tunnel has collapsed, trapping the miners inside. The rescue team decide to drill a vertical escape shaft from a position 200 m further up the hill. If the hill slopes at 23° from the horizontal, how deep does the rescue shaft need to be to meet the horizontal tunnel? Answer to one decimal place.

Testing understanding

11 A cable, PQ, is secured 24 m from the base of the pole PR.
A second cable, SP, is required to make an angle of 65° with the horizontal ground.
Find the length of the cable, SP, needed to one decimal place.

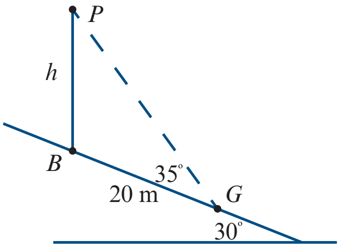


12 Two spotlights, B and D, are located at ground level, 200 m north and 300 m south of a highway, respectively. The spotlight at B directed its beam at an angle of 60° to BA, so that it shone on a parked car, C, on the highway.



At what angle, θ, to one decimal place, should the spotlight at D direct its beam to shine on the car?

13 A vertical pole on a slope of 30° is secured 20 m down the slope by a cable, PG, that makes an angle of 35° with the slope. Find the height of the pole to one decimal place.

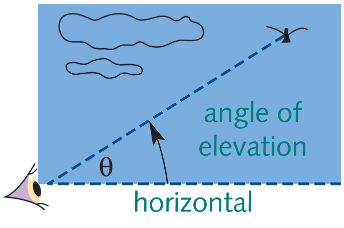


11E Angles of elevation and depression

Learning intentions

- To be able to identify and label the angles of elevation and depression in diagrams of practical situations.
- To be able to choose the appropriate trigonometric ratios and solve equations to find unknown sides and angles.

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal when you are looking *up* at something.



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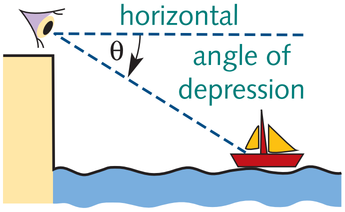
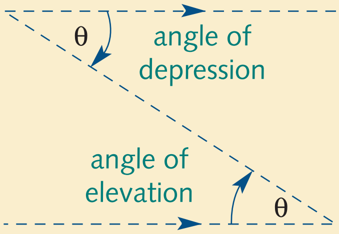
The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal when you are looking *down* at something.



Angles of elevation and depression

angle of elevation = angle of depression

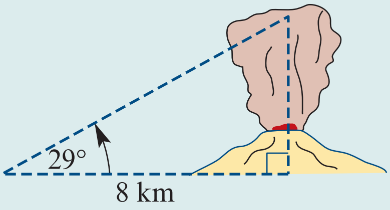
The diagram shows that the angle of elevation and the angle of depression are alternate angles ('Z' angles), so they are equal.

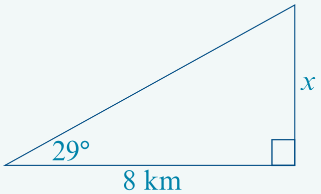


Applications of angles of elevation and depression

Example 9 Angle of elevation

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 29°. From her map she noted that the volcano was 8 km away. She calculated the height of the plume to be 4.4 km. Show how she might have done this. Give your answer to one decimal place.



Explanation	Solution
1 Draw a right-angled triangle showing the given information. Label the required height, x.	
2 The opposite and adjacent sides are involved so use tan θ.	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
3 Substitute in the known values.	$\tan 29° = \frac{x}{8}$
4 Multiply both sides by 8.	$8 \times \tan 29° = x$
5 Use your calculator to find the value of x.	$x = 4.434\ldots$
6 Write your answer to one decimal place.	The height of the ash plume was 4.4 km.

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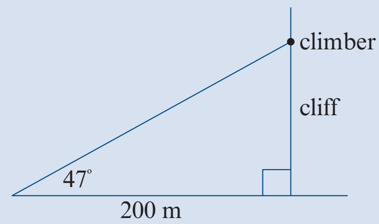
Now try this 9 Angle of elevation (Example 9)

The angle of elevation of a rock climber scaling a vertical cliff was 47°. The horizontal distance to the base of the cliff was 200 m. How high was the climber up the face of the cliff? Answer to the nearest metre.



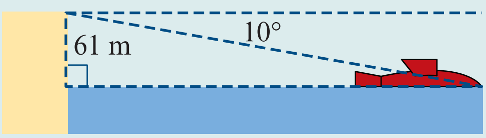
Hint 1 State the position names of the given length and the required length?

Hint 2 Write the trigonometric equation that uses the given angle and length and the required length.

Hint 3 Solve the equation to find the required length to the nearest metre.

Example 10 Angle of depression

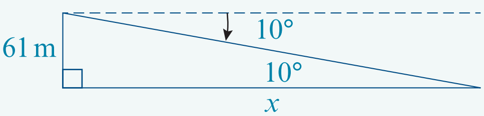
From the top of a cliff that was 61 m above sea level, Chen saw a capsized yacht. He estimated the angle of depression to be about 10°. How far was the yacht from the base of the cliff, to the nearest metre?



Explanation

1. Draw a diagram showing the given information. Label the required distance, x.

Solution



2. Mark in the angle at the yacht corner of the triangle. This is also 10° because it and the angle of depression are alternate (or 'Z') angles.

   Note: The angle between the cliff face and the line of sight is *not* 10°.

3. The opposite and adjacent sides are involved, so use tan θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

4. Substitute in the known values.

$$\tan 10° = \frac{61}{x}$$

5. Multiply both sides by x.

$$x \times \tan 10° = 61$$

6. Divide both sides by tan 10°.

$$x = \frac{61}{\tan 10°}$$

7. Do the division using your calculator.

$$x = 345.948\ldots$$

8. Write your answer to the nearest metre.

The yacht was 346 m from the base of the cliff.

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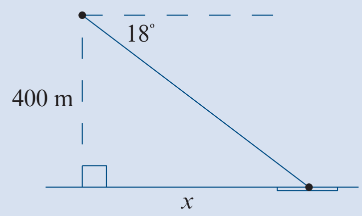
Now try this 10 Angle of depression (Example 10)

From a helicopter flying at a height of 400 m, the navigator sighted a landing platform at an angle of depression of 18°. Find the horizontal distance to the landing platform to the nearest metre.



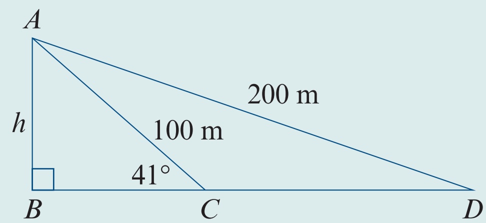
Hint 1 Find an angle inside the right-angled triangle.
Hint 2 What are the position names of the given and required distances?
Hint 3 Choose the relevant trigonometric ratio.
Hint 4 Write the equation with the given values and symbol for the unknown value.
Hint 5 Solve the equations to find the distance to the nearest metre.

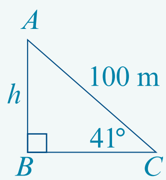
Example 11 Application with two right-angled triangles

A cable 100 m long makes an angle of elevation of 41° with the top of a tower.

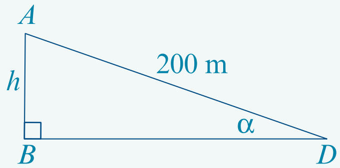
a Find the height, h, of the tower, to the nearest metre.

b Find the angle of elevation, to the nearest degree, that a cable that is 200 m long would make with the top of the tower.



Explanation	Solution
Strategy: Find h in triangle ABC, then use it to find the angle in triangle ABD.	
a 1 Draw triangle ABC, showing the given and required information.	
2 The opposite and hypotenuse are involved, so use sin θ.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
3 Substitute in the known values.	$\sin 41^\circ = \dfrac{h}{100}$
4 Multiply both sides by 100.	$h = 100 \times \sin 41^\circ$
5 Evaluate 100 sin(41°) using your calculator, and store the answer as the value of the variable h for later use.	$100 \cdot \sin(41^\circ) \rightarrow h$ 65.6059
6 Write your answer to the nearest metre.	The height of the tower is 66 m.

b **1** Draw triangle ABD, showing the given and required information.



2 The opposite and hypotenuse are involved, so use sin α.

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values. In part **a** we stored the height of the tower as h.

$$\sin \alpha = \frac{h}{200}$$

4 Write the equation to find α.

$$\alpha = \sin^{-1}\left(\frac{h}{200}\right)$$

5 Use your calculator to evaluate α.

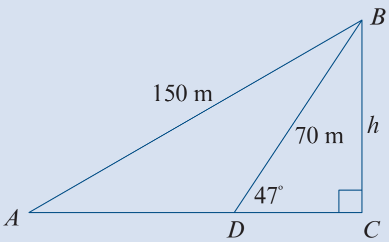
$100 \cdot \sin(41°) \to h$	65.6059
$\sin^{-1}\left(\frac{h}{200}\right)$	19.1492

6 Write your answer to the nearest degree.

The 200 m cable would have an angle of elevation of 19°.

Now try this 11 Application with two right-angled triangles (Example 11)

A wire that is 70 m long has secured the top, B, of a transmission tower to the point D on level ground. The angle of elevation looking from the point D to the top of the tower was 47°.



a Find the height of the tower, to the nearest metre.

Hint 1 What are the position names of the sides in triangle BCD?
Hint 2 Write and solve the equation for the appropriate trigonometric ratio.

b Find the angle of elevation, to the nearest degree, that a wire 150 m from point A to B, makes with the ground.

Hint 1 Write your answer from part **a** for the length BC onto triangle ABC, using at least two decimal places.
Hint 2 Choose the relevant trigonometric ratio and write the equation.
Hint 3 Use the appropriate inverse from: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ to find θ.

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Section Summary

- The **angle of elevation** is the angle from the horizontal through which you raise your line of sight to view the object.
- The **angle of depression** is the angle from the horizontal through which you lower your line of sight to view the object.

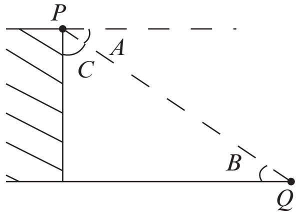
Skill-sheet

Exercise 11E

Building understanding

Answer to one decimal place.

1 The angle of depression, when looking at point Q from P, is 38°.



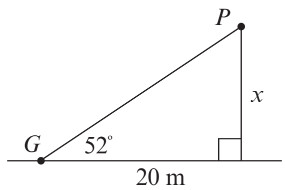
a State the value of angle A.

b Give angle B.

c Find angle C.

d What is the angle of elevation, looking from point Q to point P?

2 The angle of elevation of the top of a pole, P, viewed 20 m from the base of the pole, at G, is 52°.



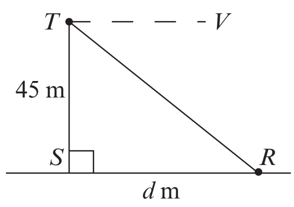
a Give the names for the side x and the 20 m distance.

b Write the trigonometric ratio rule which uses the names in part **a**.

c Substitute the known values of the angle and a side into the equation. Call the unknown side x.

d Solve the equation to one decimal place.

3 A spotlight at the top of a 45-m tower, T, makes an angle of depression of 40° as it shines on a rabbit, R. The rabbit is d metres from the base of the tower. Copy the diagram and write the 40° angle into its correct position.



a Is the angle of depression $\angle STR$, $\angle TRS$ or $\angle RTV$?

b State the trigonometric rule which uses the 45 m and d m sides.

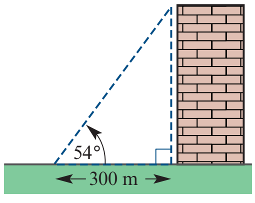
c Substitute the known values and unknown side, d, into the rule.

d Solve the equation, giving the value of d to one decimal place.

Developing understanding

Example 9

4 After walking 300 m away from the base of a tall building on level ground, Elise measured the angle of elevation to the top of the building to be 54°. Find the height of the building to the nearest metre.



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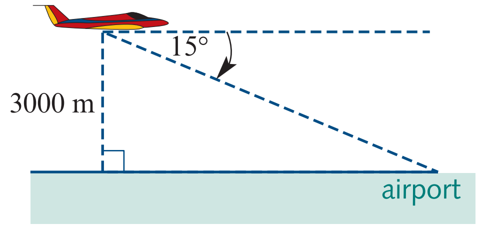
Example 10

5 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 15°. His altimeter showed that the aeroplane was at a height of 3000 m. Find the horizontal distance of the plane from the airport to the nearest metre.

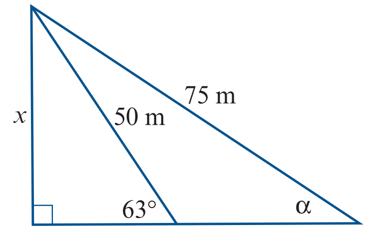


6 What would be the angle of elevation to the top of a radio transmitting tower that is 100 m tall and 400 m from the observer? Answer to the nearest degree.

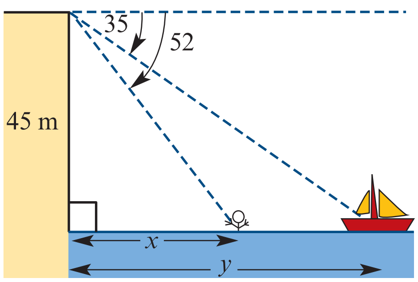
Example 11

7 **a** Find the unknown length, x, to one decimal place.

b Find the unknown angle, α, to the nearest degree.



8 From the top of a cliff that is 45 m high, an observer looking along an angle of depression of 52° could see a man swimming in the sea. The observer could also see a boat at an angle of depression of 35°. Calculate to the nearest metre:



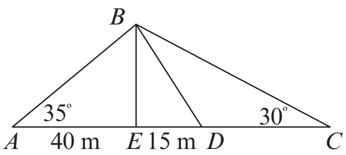
a the distance, x, of the man from the base of the cliff

b the distance, y, of the boat from the base of the cliff

c the distance from the man to the boat.

Testing understanding

9 The angle of elevation of the top of the pole BE is 35° when read from point A. The point A is 40 m from the base of the pole.



a Find the angle of elevation of the top of the pole when measured from point D, 15 m from the base of the pole.

b At what distance from D should a cable be secured so that it makes an angle of 30° with the horizontal ground?

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11F Bearings and navigation

Learning intentions

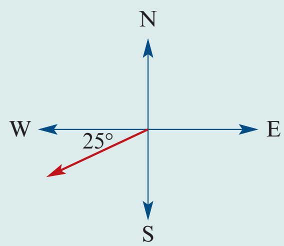
- To be able to use three-figure bearings to draw navigation and surveying diagrams.
- To be able to solve the appropriate equations to find unknown bearings and distances.

True bearings or three-figure bearings

A **true bearing** is the angle measured clockwise from north around to the required direction. True bearings are also called **three-figure bearings** because they are written using three numbers or figures. For example, 090° is the direction measured 90° clockwise from north, better known as east!

Example 12 Determining three-figure bearings

Give the three-figure bearing for the direction shown.

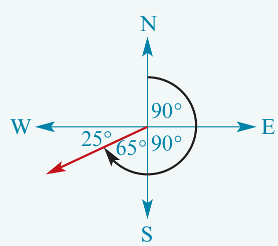


Explanation

1 Calculate the total angles swept out clockwise from north.
There is an angle of 90° between each of the four points of the compass.

2 Write your answer.

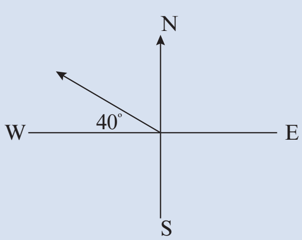
Solution



The angle from north
$= 90° + 90° + 65° = 245°$
or $270° - 25° = 245°$
The three-figure bearing is 245°.

Now try this 12 Determining three-figure bearings (Example 12)

Give the three-figure bearing for the direction shown.



Hint 1 From the north direction, sweep clockwise to the required direction.
Hint 2 The angle swept out is three right angles plus 40°.

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Navigation problems

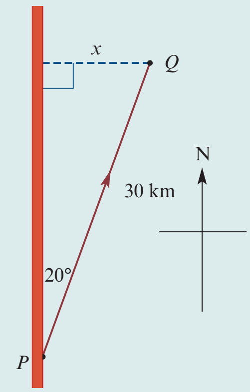
Navigation problems usually involve a consideration of not only the *direction* of travel, given as a bearing, but also the *distance* travelled.

Example 13 **Navigating using a three-figure bearing**

A group of bushwalkers leave point P, which is on a road that runs north–south, and walk for 30 km on a bearing 020° to reach point Q.

a What is the shortest distance, x, from Q back to the road to one decimal place?

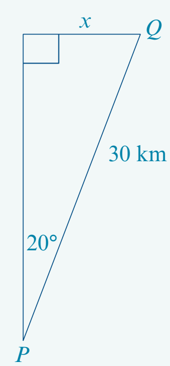
b Looking from point Q, what would be the three-figure bearing of their starting point?



Explanation

a **1** Show the given and required information in a right-angled triangle.

Solution



2 The opposite and hypotenuse are involved, so use sin θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

3 Substitute in the known values.

$$\sin 20^\circ = \frac{x}{30}$$

4 Multiply both sides by 30.

$$30 \times \sin 20^\circ = x$$

5 Find the value of x using your calculator.

$$x = 10.260\ldots$$

6 Write your answer to one decimal place.

The shortest distance to the road is 10.3 km.

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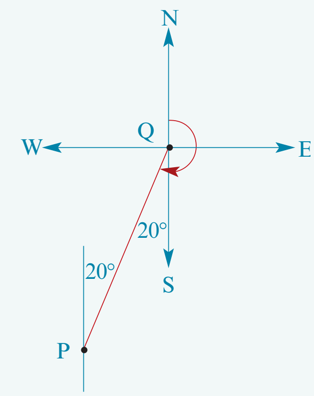
b **1** Draw the compass points at Q. Enter the alternate angle 20°.

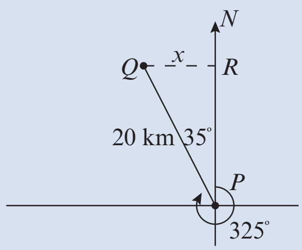


2 Standing at Q, add all the angles when facing north and then turning clockwise to look at P. This gives the three-figure bearing of P when looking from Q.

The angle from north is
$180° + 20° = 200°$
The three-figure bearing is 200°.

Now try this 13 Navigating using a three-figure bearing (Example 13)

A car was travelling north along a highway but then left the highway at the point P and travelled across the desert, heading on a bearing of 325°. The car broke down at point Q after travelling 20 km.



a Find the shortest distance, x, for the driver to walk to the highway at R to one decimal place.

Hint 1 Use the position names of the two sides involved to decide which trigonometric ratio to use.

Hint 2 Write the appropriate equation and solve to find x.

b From the point Q, what three-figure bearing should the driver take if, instead, he decided to walk back to P?

Hint 1 Draw the points of the compass at Q and use the alternate angles rule.

Hint 2 Turning clockwise from north to face P, what angle was swept out?

Section Summary

- The **three-figure bearing** is the angle swept clockwise from north to the required direction.

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Skill-sheet

Exercise 11F

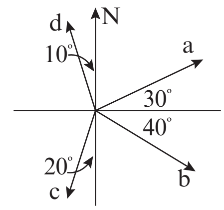
Building understanding

1 Give the three-figure bearings for the compass directions.

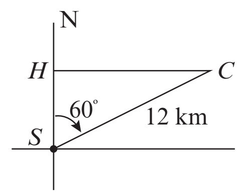
a North **b** East
c South **d** West

2 Give the three-figure bearings for each of the directions shown.

a Direction **a**
b Direction **b**
c Direction **c**
d Direction **d**



3 A car stopped at S on a highway that pointed in a North-South direction. The car then travelled for 12 km along a dirt road with a bearing of 060° and broke down at C. The driver needed to know the distance, x, he would have to walk to reach the highway at H.

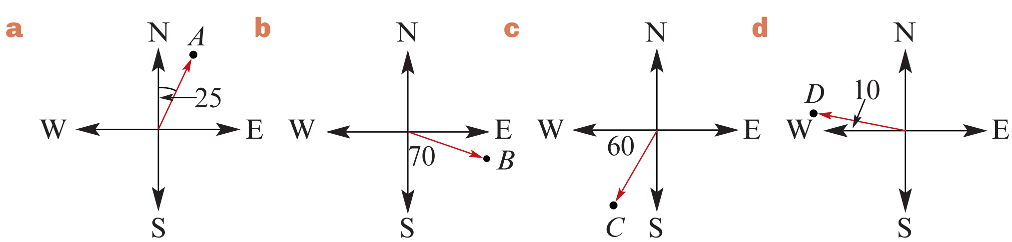


a In triangle SHC, give the names for the 12 km side and side HC.
b Write the trigonometric ratio rule that uses the names of the sides in part **a**.
c Substitute the value of the angle and the known side into the rule. Call the unknown side x.
d Solve the equation by multiplying both sides by the value of the denominator. Use your CAS calculator to find the value of x to one decimal place. As this is a practical problem, answer by responding in the context of the question.

Developing understanding

Example 12

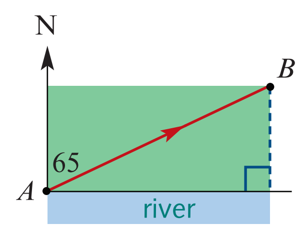
4 State the three-figure bearing of each of the points A, B, C and D.



Example 13

5 Kirra camped overnight at point A beside a river that ran east–west. She walked on a bearing of 065° for 18 km to point B.

a What angle did her direction make with the river?
b What is the shortest distance from B to the river to two decimal places?



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6 A ship sailed 3 km west, then 2 km south.

a Give its three-figure bearings from an observer who stayed at its starting point to the nearest degree.

b For a person on the ship, what would be the three-figure bearings, looking back to the starting point?

7 A ship left port, P, and sailed 20 km on a bearing of 230°. It then sailed north for 30 km to reach point C. Give the following distances to one decimal place and directions to the nearest degree.

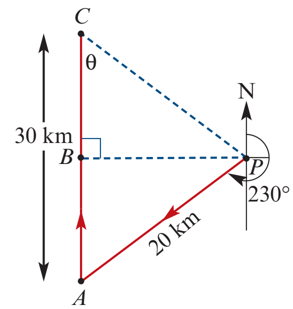
a Find the distance AB.

b Find the distance BP.

c Find the distance BC.

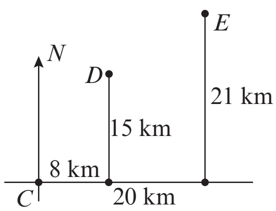
d Find the angle θ at point C.

e State the three-figure bearing and distance of the port, P, from the ship at C.



Testing understanding

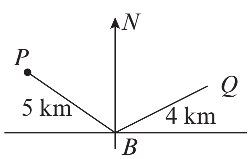
8 A bushwalker left the campsite, C, and walked 8 km east, then 15 km north to reach the point D.
A friend walked 20 km east, then 21 km north to the point E.
Find the bearing of point E from D to the nearest degree.



9 A surveyor walked 5 km on a bearing of 310° from a base camp, B, to reach point P. She then returned to camp B and walked 4 km on a bearing of 060° to the point Q.

a Find the bearing she had to walk from P to B.

b What bearing would be needed to return to B from Q?



11G The sine rule

Learning intentions

In a non-right-angled triangle

- To be able to use the sine rule to find an unknown angle, given two sides and an opposite angle.
- To be able to use the sine rule to find an unknown side, given two angles and a side.
- To be able to find the required angles and sides when the given information fits two possible triangles.

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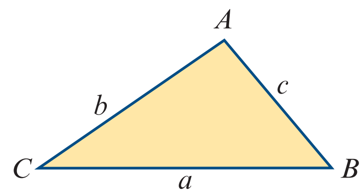
Standard triangle notation

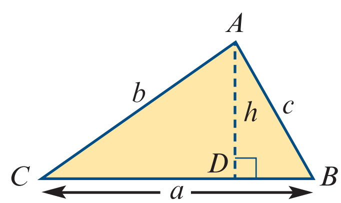
The convention for labelling a non-right-angled triangle is to use the upper case letters, A, B, and C, for the angles at each corner. The sides are named using lower case letters so that side a is opposite angle A, and so on.



This notation is used for the sine rule and cosine rule. Both rules can be used to find angles and sides in triangles that do not have a right angle.

How to derive the sine rule

In triangle ABC, show the height, h, of the triangle by drawing a perpendicular line from D on the base of the triangle to A.



In triangle *ADC*, $\sin C = \frac{h}{b}$

So $h = b \times \sin C$

In triangle *ABD*, $\sin B = \frac{h}{c}$

So $h = c \times \sin B$

We can make the two rules for h equal to each other. $b \times \sin C = c \times \sin B$

Divide both sides by $\sin C$. $b = \frac{c \times \sin B}{\sin C}$

Divide both sides by $\sin B$. $\frac{b}{\sin B} = \frac{c}{\sin C}$

If the triangle was redrawn with side c as the base, then using similar steps we would get: $\frac{a}{\sin A} = \frac{b}{\sin B}$

We can combine the two rules as shown in the following box.

The sine rule

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **sine rule** is used to find the sides and angles in a non-right-angled triangle when given:

- two sides and an angle opposite one of the given sides
- two angles and one side.

Note: If neither of the two given angles is opposite the given side, find the third angle using $A + B + C = 180°$.

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The sine rule can take the form of any of these three possible equations:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

Each equation has two sides and two angles opposite those sides. If we know three of the parts, we can find the fourth.

So if we know two angles and a side opposite one of the angles, we can find the side opposite the other angle. Similarly, if we know two sides and an angle opposite one of those sides, we can find the angle opposite the other side.

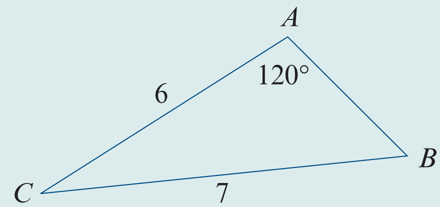
Although we have expressed the sine rule using a triangle *ABC*, for any triangle, such as *PQR*, the pattern of fractions consisting of 'side / sine of angle' pairs would appear as:

$$\frac{p}{\sin P} = \frac{q}{\sin Q} \qquad \frac{q}{\sin Q} = \frac{r}{\sin R} \qquad \frac{p}{\sin P} = \frac{r}{\sin R}$$

Using the sine rule

Example 14 Using the sine rule, given two sides and an opposite angle

Find angle *B* in the triangle shown to one decimal place.



Explanation	Solution
1 We have the pairs $a = 7$ and $A = 120°$, $b = 6$ and $B = ?$ with only *B* unknown. So use $\frac{a}{\sin A} = \frac{b}{\sin B}$.	$\frac{a}{\sin A} = \frac{b}{\sin B}$
2 Substitute in the known values.	$\frac{7}{\sin 120°} = \frac{6}{\sin B}$
3 Cross-multiply.	$7 \times \sin B = 6 \times \sin 120°$
4 Divide both sides by 7.	$\sin B = \frac{6 \times \sin 120°}{7}$
5 Write the equation to find angle *B*.	$B = \sin^{-1}\left(\frac{6 \times \sin 120°}{7}\right)$
6 Use your calculator to evaluate the expression for *B*.	$B = 47.928\ldots°$
7 Write your answer to one decimal place.	Angle *B* is 47.9°.

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When an angle, such as B, is unknown, the fractions on each side of the sine rule can be flipped as the first step.
For example:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Then just multiply both sides by b to find $\sin B$, and solve.

In Example **14**, now that we know that $A = 120°$ and $B = 47.9°$, we can use the fact that the angles in a triangle add to 180° to find C.

$$\begin{aligned} A + B + C &= 180° \\ 120° + 47.9° + C &= 180° \\ 167.9° + C &= 180° \\ C &= 180° - 167.9° = 12.1° \end{aligned}$$

As we now know that $A = 120°, a = 7$ and $C = 12.1°$, we can find side c using:

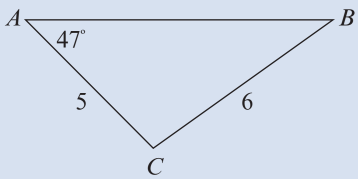
$$\frac{a}{\sin A} = \frac{\text{c}}{\sin C}$$

The steps are similar to those in the example.

Finding all the angles and sides of a triangle is called solving the triangle.

Now try this 14 **Using the sine rule, given two sides and an opposite angle (Example 14)**

Find angle B to one decimal place.

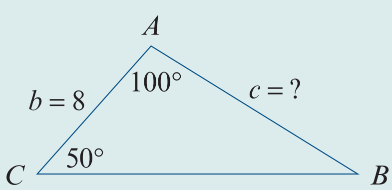


Hint 1 Write the sine rule using the letter names for the given information and the required angle.

Hint 2 Substitute in the known values and solve to find angle B.

Example 15 **Using the sine rule, given two angles and one side**

Find side c in the triangle shown to one decimal place.



Explanation

1 Find the angle opposite the given side by using $A + B + C = 180°$

Solution

$$\begin{aligned} A + B + C &= 180° \\ 100° + B + 50° &= 180° \\ B + 150° &= 180° \\ B &= 30° \end{aligned}$$

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2 We have the pairs $b = 8$ and $B = 30°, c = ?$ and $C = 50°$ with only c unknown. So use $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

3 Substitute in the known values.

$$\frac{8}{\sin 30°} = \frac{c}{\sin 50°}$$

4 Multiply both sides by $\sin 50°$.

$$c = \frac{8 \times \sin 50°}{\sin 30°}$$

5 Use your calculator to find c.

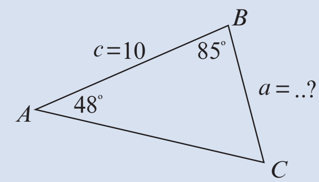
$$c = 12.256\ldots$$

6 Write your answer to one decimal place.

Side c is 12.3 units long.

Now try this 15 **Using the sine rule, given two angles and one side (Example 15)**

Find side a to one decimal place.



Hint 1 Find angle C, using $A + B + C = 180°$.

Hint 2 Write the sine rule using the letter names for the known information and the required side.

Hint 3 Substitute in the known values and solve to find side a.

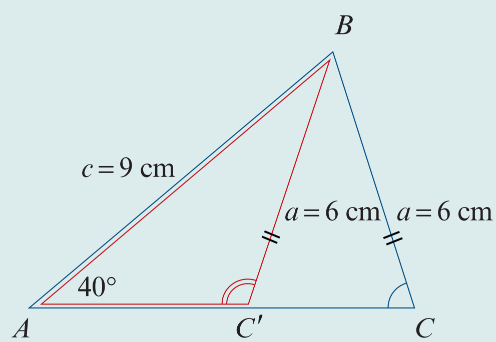
Ambiguous case

Sometimes, two triangles can be drawn to fit the given information. This can happen when you are given two sides and an angle *not* between the two given sides. The solution strategy uses the sine rule and the fact that the angles at the base of an isosceles triangle are equal.

Example 16 **Ambiguous case using the sine rule**

In triangle ABC, $A = 40°$, $c = 9$ cm and $a = 6$ cm.

Side c is drawn for 9 cm at 40° to the base. From vertex B, side a must be 6 cm long when it meets the base of the triangle. When side a is measured out with a compass, it can cross the base in two possible places - C and C'.



There are two possible triangles. ABC drawn in blue and ABC' drawn in red.

Find the two possible values for angle C, shown as $\angle BCA$ and $\angle BC'A$ in the diagram. Give the answers to two decimal places.

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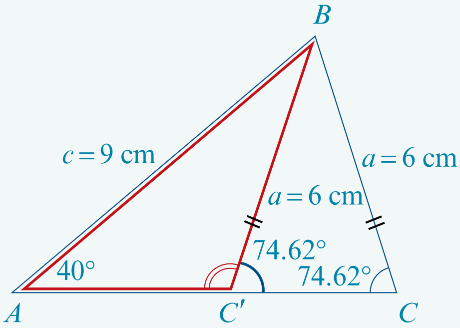
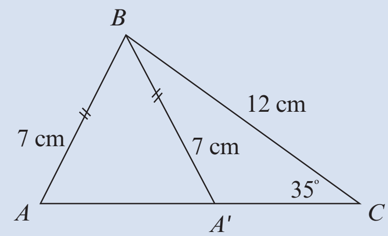
Explanation	Solution
1 Using the sine rule, we need two angle-side pairs with only one unknown. The unknown is angle C. Both sides of the sine rule were flipped to make sin C a numerator.	$\frac{\sin C}{c} = \frac{\sin A}{a}$ $\frac{\sin C}{9} = \frac{\sin 40^\circ}{6}$ $\sin C = \frac{9 \times \sin 40^\circ}{6}$ $C = \sin^{-1}\left(\frac{9 \times \sin 40^\circ}{6}\right)$ $C = 74.62^\circ$
2 Clearly $\angle BC'A$ is greater than 90°, so it is not the value of angle C just calculated.	So $\angle BCA = 74.62^\circ$ 
3 The two angles at the base of the isosceles triangle $C'BC$ are equal.	$\angle BC'C = \angle BCC'$ So $\angle BC'C = 74.62^\circ$
4 Two angles on a straight line add to 180°.	$\angle BC'A + 74.62^\circ = 180^\circ$ $\angle BC'A = 180^\circ - 74.62^\circ = 105.38^\circ$ The possible values for angle C are 74.62° and 105.38°

Now try this 16 Ambiguous case using the sine rule (Example 16)

In triangle ABC, $C = 35^\circ$, $a = 12$ cm and $c = 7$ cm.

Side a was drawn for 12 cm at 35° to the base. From the vertex B, side c must be 7 cm long when it meets the base of the triangle. When side c is measured out with a compass, it can cross the base in two possible places, A and A'.



There are two possible triangles: CBA and CBA'.

Find the two possible values for angle A, seen as $\angle BAC$ and $\angle BA'C$ in the diagram. Answer to two decimal places.

Hint 1 In triangle ABC, use the sine rule to find angle A, known as $\angle BAC$.

Hint 2 Find $\angle BA'A$ and hence $\angle BA'C$, which is angle A'.

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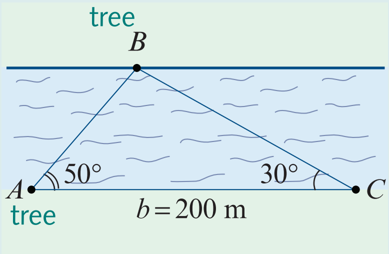
Example 17 Application of the sine rule

Leo wants to tie a rope from a tree at point A to a tree at point B on the other side of the river. He needs to know the length of rope required.

When he stood at A, he saw the tree at B at an angle of 50° with the riverbank. After walking 200 metres east to C, the tree was seen at an angle of 30° with the riverbank.



Find the length of rope required to reach from A to B to two decimal places.

Explanation

1 To use the sine rule, we need two angle-side pairs with only one item unknown. The unknown is the length of the rope, side c. Angle $C = 30^\circ$ is given.

Solution: So one part of the sine rule equation will be: $\dfrac{c}{\sin C}$

2 We know side $b = 200$ and need to find angle B to use the sine rule equation:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

3 Use $A + B + C = 180^\circ$ to find angle B.

$$A + B + C = 180^\circ$$
$$50^\circ + B + 30^\circ = 180^\circ$$
$$B = 100^\circ$$

4 We have the pairs:
$b = 200$ and $B = 100^\circ$
$c = ?$ and $C = 30^\circ$
with only c unknown.

So use $\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

5 Substitute in the known values.

$$\frac{c}{\sin 30^\circ} = \frac{200}{\sin 100^\circ}$$

6 Multiply both sides by $\sin 30^\circ$.

$$c = \frac{200 \times \sin 30^\circ}{\sin 100^\circ}$$

7 Use your calculator to find c.

$$c = 101.542\ldots$$

8 Write your answer to two decimal places.

The rope must be 101.54 m long.

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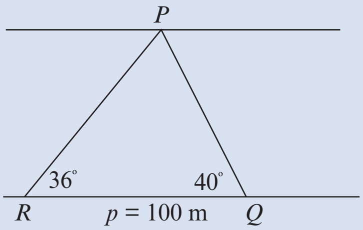
Now try this 17 Application of the sine rule (Example 17)

Engineers needed to construct a bridge across a canyon from P on the edge of one side to Q on the edge of the other side. The diagram is the view looking down on the parallel sides of the canyon and the proposed position of the bridge, PQ.

From point R on one side of the canyon, a surveyor sighted post P on the other side at an angle of 36° to the edge of the canyon. Moving 100 m to Q, she sighted point P at an angle of 40° to the edge.

Find the required length of the bridge to two decimal places.



Hint 1 Find angle P.

Hint 2 Use the sine rule to find PQ $(=r)$.

Tips for solving trigonometry problems

- Always make a rough sketch in pencil as you read the details of a problem. You may need to make changes as you read more, but it is very helpful to have a sketch to guide your understanding.
- In any triangle, the longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
- When labelling the sides and angles of a triangle, make sure the name of a side is opposite the angle with the same letter. For example, side c is opposite angle C.
- When you have found a solution, re-read the question and check that your answer fits well with the given information and your diagram.
- Round answers for each part to the required decimal places. Keep more decimal places when the results are used in further calculations. Otherwise, rounding off errors accumulate.

Section Summary

- The sine rule can take the form of any of these three possible equations:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}$$

- Each equation consists of two sides and two angles opposite those sides. If three parts are known, the sine rule can be used to find the fourth unknown part.
- In the ambiguous case, two possible triangles can be drawn from the given information. Draw the two triangles within one diagram, and use the fact that the base angles of an isosceles triangle are equal.

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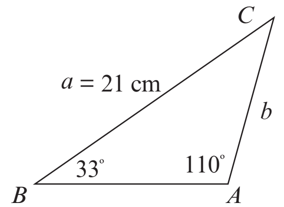
Skill-sheet

Exercise 11G

Building understanding

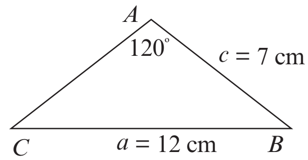
1 **a** For a triangle, ABC, write the three possible sine rule equations.
b For a triangle, PQR, write the three possible sine rule equations.

2 In triangle ABC, $A = 110°$, $a = 21$ cm and $B = 33°$.



a To find side b, which form of the sine rule should be used?
b Substitute the known values into the equation.
c Solve the equation to find side b to one decimal place.

3 In triangle ABC, $A = 120°$, $a = 12$ cm and $c = 7$ cm.



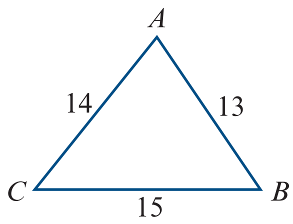
a To find angle C, which form of the sine rule should be used?
b Substitute the known values into the equation.
c When the unknown, such as $\sin C$, is in the denominator, the equation is easier to solve if each fraction is flipped. Flip each fraction.
d Solve the equation to find angle C to one decimal place.

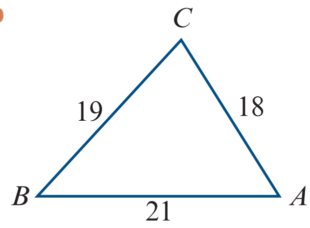
Developing understanding

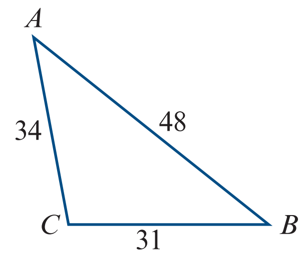
In this exercise, calculate lengths correct to two decimal places and angles to one decimal place, where necessary.

Basic principles

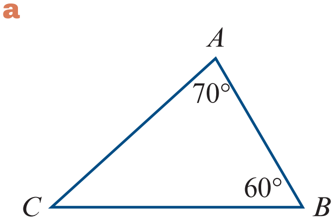
4 In each triangle, state the lengths of sides a, b and c.

a


b


c


5 Find the value of the unknown angle in each triangle. Use $A + B + C = 180°$.

a


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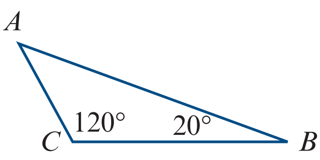

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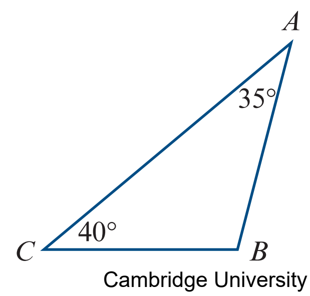

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6 In each of the following, a student was using the sine rule to find an unknown part of a triangle but was unable to complete the final steps of the solution. Find the unknown value by completing each problem. For ambiguous cases, find one possible value.

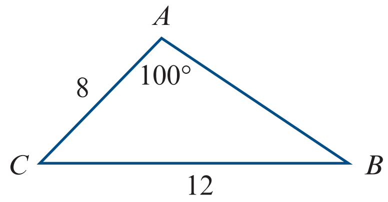
a $\frac{a}{\sin 40^\circ} = \frac{8}{\sin 60^\circ}$ **b** $\frac{b}{\sin 50^\circ} = \frac{15}{\sin 72^\circ}$ **c** $\frac{c}{\sin 110^\circ} = \frac{24}{\sin 30^\circ}$

d $\frac{17}{\sin A} = \frac{16}{\sin 70^\circ}$ **e** $\frac{26}{\sin B} = \frac{37}{\sin 95^\circ}$ **f** $\frac{21}{\sin C} = \frac{47}{\sin 115^\circ}$

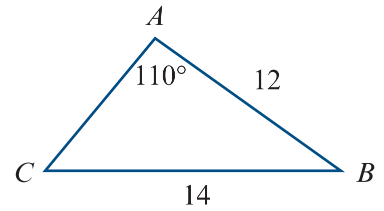
Using the sine rule to find angles

Example 14

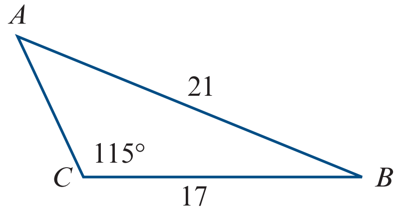
7 a Find angle B.



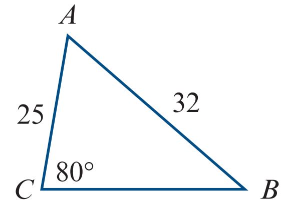
b Find angle C.



c Find angle A.



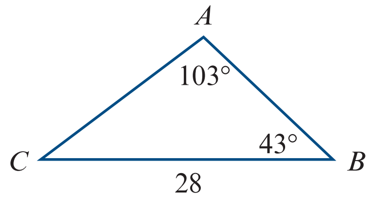
d Find angle B.



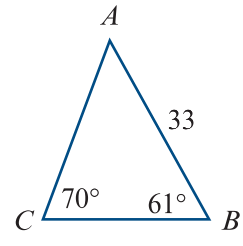
Using the sine rule to find sides

Example 15

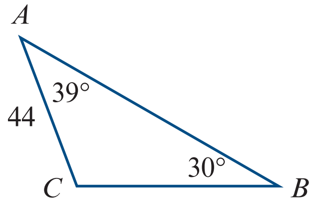
8 a Find side b.



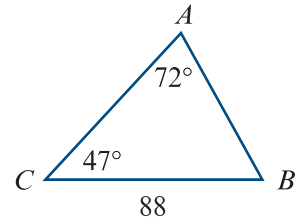
b Find side b.



c Find side a.



d Find side c.



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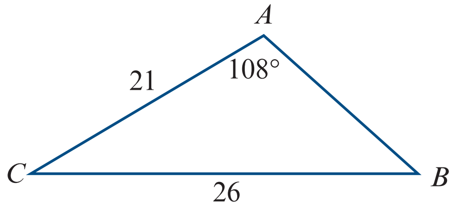
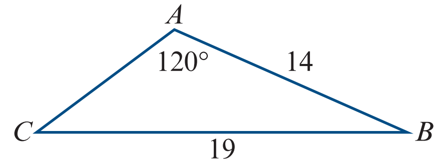
Solving triangles using the sine rule

9 Solve (find all the unknown sides and angles of) the following triangles.

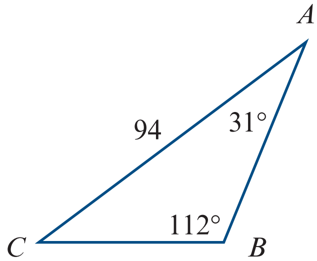
a



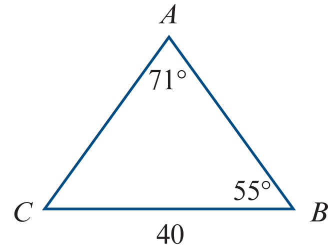
b



c



d



10 In the triangle ABC, $A = 105°$, $B = 39°$ and $a = 60$. Find side b.

11 In the triangle ABC, $A = 112°$, $a = 65$ and $c = 48$. Find angle C.

12 In the triangle PQR, $Q = 50°$, $R = 45°$ and $p = 70$. Find side r.

13 In the triangle ABC, $B = 59°$, $C = 74°$ and $c = 41$. Find sides a and b and angle A.

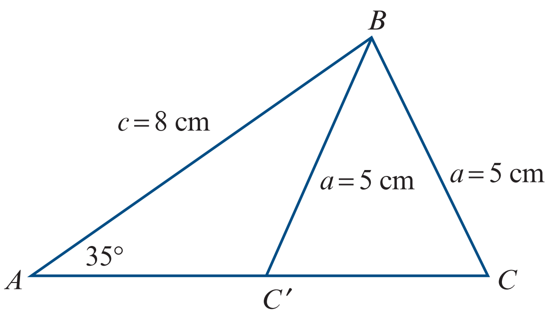
14 In the triangle ABC, $a = 60$, $b = 100$ and $B = 130°$. Find angles A and C and side c.

15 In the triangle PQR, $P = 130°$, $Q = 30°$ and $r = 69$. Find sides p and q and angle R.

The ambiguous case of the sine rule

Example 16

16 In triangle ABC, $A = 35°$, $c = 8$ cm and $a = 5$ cm.
Two triangles, ABC and ABC', can be drawn using the given information.
Give the angles to two decimal places.



a Use the sine rule to find $\angle BCA$ in triangle ABC.

b Use isosceles triangle $C'BC$ to find $\angle BC'C$.

c Find $\angle AC'B$ by using the rule for two angles on a straight line.

d Give the possible values for C and C', the angles opposite side c.

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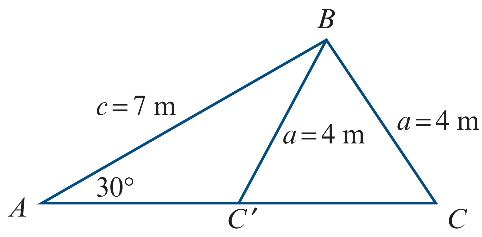
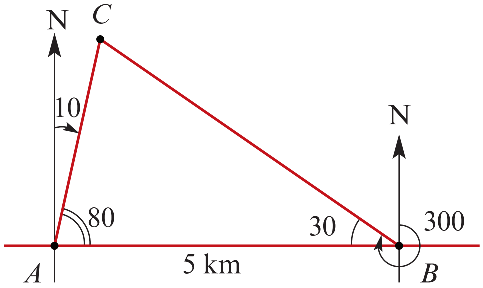
17 In triangle ABC, $A = 30°$, $c = 7$ m and $a = 4$ m.
Find the two possible values for angle C, shown as $\angle BCA$ and $\angle BC'A$ in the diagram. Give the angles to two decimal places.



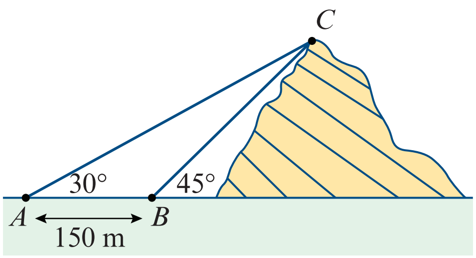
Applications

Example 17

18 A fire-spotter, located in a tower at A, saw a fire in the direction 010°. Five kilometres to the east of A, another fire-spotter at B saw the fire in the direction 300°.
Find the distance of the fire from each tower.



19 A surveyor standing at point A measured the angle of elevation to the top of the mountain as 30°. She moved 150 m closer to the mountain and, at point B, measured the angle of elevation to the top of the mountain as 45°.
There is a proposal to have a strong cable from point A to the top of the mountain to carry tourists in a cable car. What is the length of the required cable?



20 A naval officer sighted the smoke of a volcanic island on a bearing of 044°. A navigator on another ship 25 km due east of the first ship saw the smoke on a bearing of 342°.

a Find the distance of each ship from the volcano.

b If the ship closest to the volcano can travel at 15 km/h, how long will it take to reach the volcano?

21 An air-traffic controller at airport A received a distress call from an aeroplane low on fuel. The bearing of the aeroplane from A was 070°. From airport B, 80 km north of airport A, the bearing of the aeroplane was 120°.

a Which airport was closest for the aeroplane?

b Find the distance to the closest airport.

c The co-pilot estimates fuel consumption to be 1525 litres per 100 km. The fuel gauge reads 1400 litres. Is there enough fuel to reach the destination?

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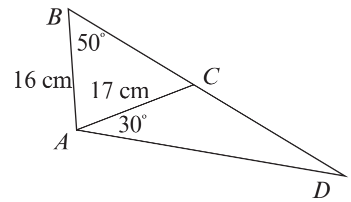
Testing understanding

22 Find the length AD to one decimal place.



23 Decide which of the descriptions given is for:

i a possible triangle **ii** an impossible triangle **iii** an ambiguous case

a $A = 30°$, $a = 4.5$ and $c = 10$ **b** $C = 40°$, $b = 12$ and $c = 8.5$

c $B = 50°$, $a = 8$ and $b = 9$

11H The cosine rule

Learning intentions

In a non-right-angled triangle

- To be able to use the cosine rule to find the unknown side when given two sides and the angle between them.
- To be able to use the cosine rule to find an angle in a triangle when given the three sides.
- To be able to identify when the sine rule or the cosine rule should be used.

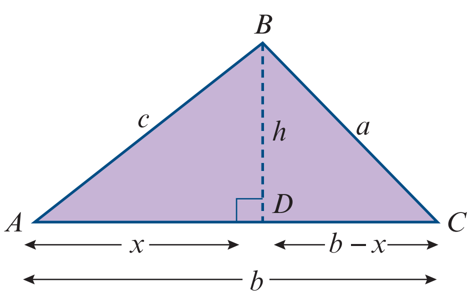
The **cosine rule** can be used to find the length of a side in any non-right-angled triangle when two sides and the angle between them are known. When you know the three sides of a triangle, the cosine rule can be used to find any angle.

How to derive the cosine rule

In the triangle ABC, show the height, h, of the triangle by drawing a line perpendicular from D on the base of the triangle to B.

Let $AD = x$

As $AC = b$, then $DC = b - x$.



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In triangle *ABD*, $\cos A = \frac{x}{c}$

Multiply both sides by *c*. $x = c\cos A$ (1)

Using Pythagoras' theorem in triangle *ABD*. $x^2 + h^2 = c^2$ (2)

Using Pythagoras' theorem in triangle *CBD*. $(b - x)^2 + h^2 = a^2$

Expand (multiply out) the squared bracket. $b^2 - 2bx + x^2 + h^2 = a^2$

Use (1) to replace x with $c\cos A$. $b^2 - 2bc\cos A + x^2 + h^2 = a^2$

Use (2) to replace $x^2 + h^2$ with c^2. $b^2 - 2bc\cos A + c^2 = a^2$

Reverse and rearrange the equation. $a^2 = b^2 + c^2 - 2bc\cos A$

Repeating these steps with side *c* as the base, we get: $b^2 = a^2 + c^2 - 2ac\cos B$

Repeating these steps with side *a* as the base, we get: $c^2 = a^2 + b^2 - 2ab\cos C$

The three versions of the cosine rule can be rearranged to give rules for $\cos A$, $\cos B$, and $\cos C$.

The cosine rule

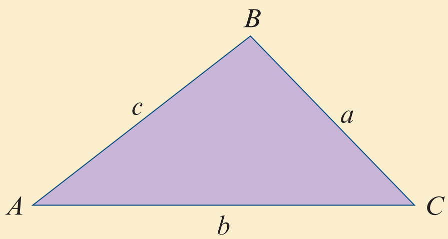
The cosine rule in any triangle, *ABC*:

- when given two sides and the angle between them, the third side can be found using one of the equations:

  $a^2 = b^2 + c^2 - 2bc\cos A$

  $b^2 = a^2 + c^2 - 2ac\cos B$

  $c^2 = a^2 + b^2 - 2ab\cos C$



- when given three sides, any angle can be found using one of the following rearrangements of the cosine rule:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

In triangles using different letters, the cosine rule follows the same pattern. For example, in triangle *PQR*:

$$p^2 = q^2 + r^2 - 2qr\cos P$$

$$q^2 = p^2 + r^2 - 2pr\cos Q$$

$$r^2 = p^2 + q^2 - 2pq\cos R$$

"The square of one side equals the sum of the squares of the other sides, minus twice their product, times the cosine of the angle between them."

If the angle is 90°, a right-angled triangle is formed and Pythagoras' theorem results.

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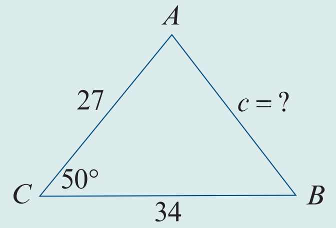
Using the cosine rule

Example 18 Using the cosine rule, given two sides and the angle between them

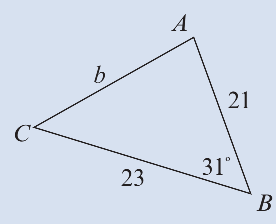
Find side c, to two decimal places, in the triangle shown.



Explanation	Solution
1 Write down the given values and the required unknown value.	$a = 34, b = 27, c = ?, C = 50°$
2 We are given two sides and the angle between them. To find side c, use $c^2 = a^2 + b^2 - 2ab\cos C$	$c^2 = a^2 + b^2 - 2ab\cos C$
3 Substitute the given values into the rule.	$c^2 = 34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°$
4 Take the square root of both sides.	$c = \sqrt{(34^2 + 27^2 - 2 \times 34 \times 27 \times \cos 50°)}$
5 Use your calculator to find c.	$c = 26.548\ldots$
6 Write your answer to two decimal places.	The length of side c is 26.55 units.

Now try this 18 Using the cosine rule, given two sides and the angle between them (Example 18)

Find side b to two decimal places.



Hint 1 Write the letter names for the given values and the required unknown.
Hint 2 Choose a form of the cosine rule for b^2.
Hint 3 Substitute in the known values and solve to find side b to two decimal places.

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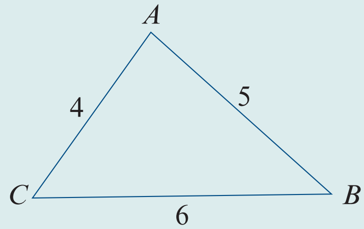
Example 19 Using the cosine rule to find an angle, given three sides

Find the largest angle, to one decimal place, in the triangle shown.



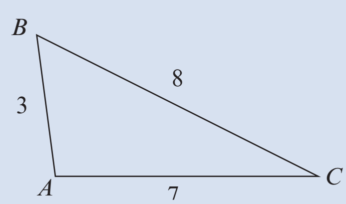
Explanation	Solution
1 Write down the given values.	$a = 6, b = 4, c = 5$
2 The largest angle is always opposite the largest side, so find angle A.	$A = ?$
3 We are given three sides. To find angle A use: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
4 Substitute the given values into the rule.	$\cos A = \dfrac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}$
5 Write the equation to find angle A.	$A = \cos^{-1}\left(\dfrac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5}\right)$
6 Use your calculator to evaluate the expression for A. Make sure that your calculator is in DEGREE mode. Tip: Wrap all the terms in the numerator (top) within brackets. Also put brackets around all of the terms in the denominator (bottom).	$A = 82.819\ldots^\circ$
7 Write your answer to one decimal place.	The largest angle is 82.8°.

When finding an angle, such as A, a negative value for $\cos A$ indicates that:

$$90^\circ < A < 180^\circ$$

Now try this 19 Using the cosine rule to find an angle, given three sides (Example 19)

Find the smallest angle to one decimal place.



Hint 1 The smallest angle is opposite the smallest side.
Hint 2 Write a form of the cosine rule with $\cos C$.
Hint 3 Substitute in the known values and solve to find angle C to one decimal place.

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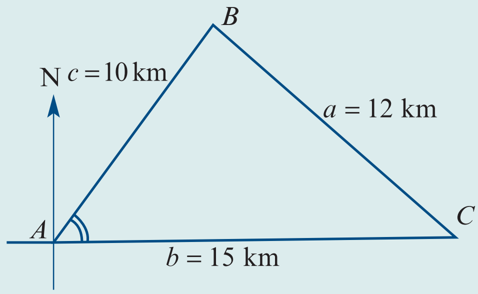
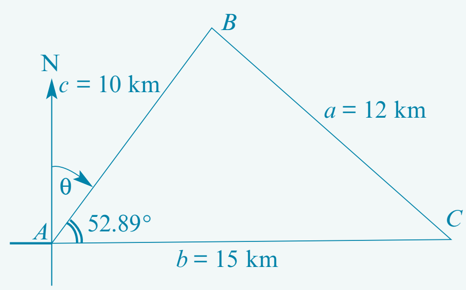
Example 20 Application of the cosine rule: finding an angle and a bearing

A yacht left point A and sailed 15 km east to point C. Another yacht also started at point A and sailed 10 km to point B, as shown in the diagram. The distance between points B and C is 12 km.



a What was the angle between their directions as they left point A? Give the angle to two decimal places.

b Find the bearing of point B from the starting point, A, to the nearest degree.

Explanation	Solution
a **1** Write the given values.	$a = 12, b = 15, c = 10$
2 Write the form of the cosine rule for the required angle, A.	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
3 Substitute the given values into the rule.	$\cos A = \dfrac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}$
4 Write the equation to find angle A.	$A = \cos^{-1}\left(\dfrac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10}\right)$
5 Use your calculator to evaluate the expression for A.	$A = 52.891°$
6 Give the answer to two decimal places.	The angle was 52.89°.
b **1** The bearing, θ, of point B from the starting point, A, is measured clockwise from north.	
2 Consider the angles in the right angle at point A.	$\theta + 52.89° = 90°$
3 Find the value of θ.	$\theta = 90° - 52.89°$ $= 37.11°$
4 Write your answer.	The bearing of point B from point A is 037°.

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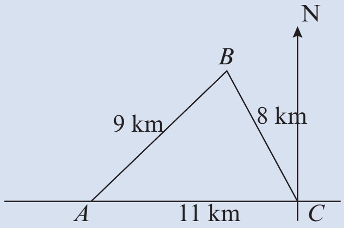
Now try this 20 **Application of the cosine rule: finding an angle and a bearing (Example 20)**

A bushwalker left their camp at point C and walked 8 km to point B, as shown in the diagram. A friend walked 11 km to point A, a distance of 9 km from B.



a What was the angle between their directions as they left C? Answer to one decimal place.

Hint 1 Write the form of the cosine rule with $\cos C$, and substitute in the known values.

Hint 2 Solve to find angle C.

b What was the bearing of point B from their starting point, C, to the nearest degree?

Hint 1 Add the angles swept out as you sweep clockwise from north until you face point B.

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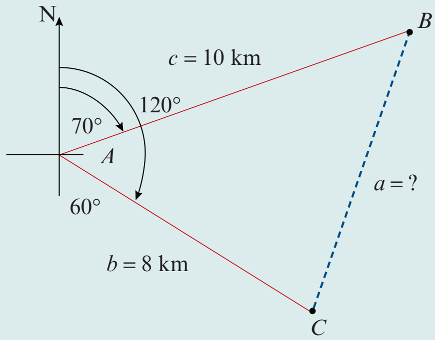
Example 21 Application of the cosine rule involving bearings

A bushwalker left his base camp and walked 10 km in the direction 070°.

His friend also left the base camp but walked 8 km in the direction 120°.

a Find the angle between their paths.

b How far apart were they when they stopped walking? Give your answer to two decimal places.

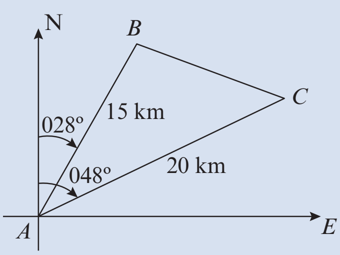


Explanation	Solution
a **1** Angles lying on a straight line add to 180°.	$60° + A + 70° = 180°$ $A + 130° = 180°$ $A = 50°$
2 Write your answer.	The angle between their paths was 50°.
b **1** Write down the known values and the required unknown value.	$a =?\ b = 8,\ c = 10,\ A = 50$
2 We have two sides and the angle between them. To find side a, use $a^2 = b^2 + c^2 - 2bc\cos A$.	$a^2 = b^2 + c^2 - 2bc\cos A$
3 Substitute in the known values.	$a^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50°$
4 Take the square root of both sides.	$a = \sqrt{(8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 50°)}$
5 Use a calculator to find the value of a.	$a = 7.820\ldots$
6 Answer to two decimal places.	Distance between them was 7.82 km.

Now try this 21 Application of the cosine rule involving bearings (Example 21)

A sailor sailed 15 km in a bearing of 028° from the port at A and stopped at B.

Her friend sailed 20 km from port A on a bearing of 048° and stopped at the point C.



a Find the angle between their courses.

Hint 1 What was the difference in the angles swept out from north?

b How far apart were they when they stopped? Answer to two decimal places.

Hint 1 Use the form of the cosine rule for a^2.

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Section Summary

The cosine rule can be used in a triangle, ABC, to find unknown sides.

- To find an unknown side when given two sides and an included angle, use the equation for the unknown side:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

- To find an unknown angle when given three sides, use the equation for the unknown angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- In a triangle with other lettering, use the pattern:
  The square of one side equals the sum of the squares of the other two sides, minus twice their product, times the cosine of the angle between them.

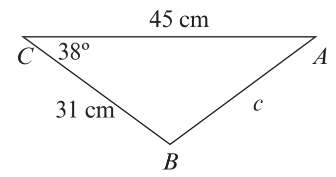
Skill-sheet

Exercise 11H

Building understanding

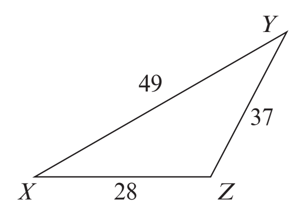
1 **a** For the triangle ABC, write the three possible forms of the cosine rule.
b Write the three possible forms of the cosine rule for the triangle XYZ.

2 In triangle ABC, $C = 38°$, $a = 31$ cm and $b = 45$ cm.



a To find side c, which form of the cosine rule should be used?
b Substitute the known values into the equation.
c Solve the equation to find side c to one decimal place.

3 In triangle XYZ, $x = 37$, $y = 28$ and $z = 49$.



a To find angle Y, which form of the cosine rule should be used?
b Substitute the known values into the equation.
c Tidy up the equation after evaluating the squares and calculating the product.
d Add 3626 $\cos Y$ to both sides. Subtract 784 from both sides. Divide both sides by 3626 to get an equation for $\cos Y$.
e Use the inverse cosine, $\cos^{-1}$, feature on your CAS calculator to find angle Y to one decimal place. Alternatively, after part **b**, use the Solve command on your CAS calculator to find angle Y.

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Developing understanding

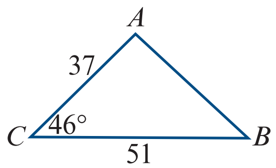
In this exercise, calculate lengths to two decimal places and angles to one decimal place.

Using the cosine rule to find sides

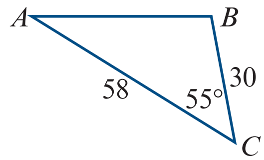
Example 18

4 Find the unknown side in each triangle.

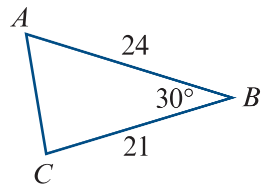
a



b



c



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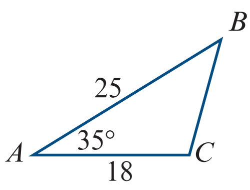
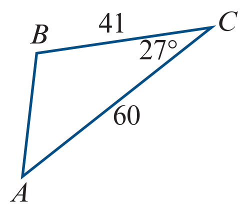


e



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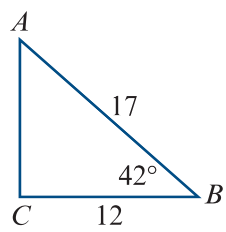


5 In the triangle ABC, $a = 27, b = 22$ and $C = 40°$. Find side c.

6 In the triangle ABC, $a = 18, c = 15$ and $B = 110°$. Find side b.

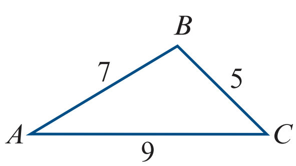
7 In the triangle ABC, $b = 42, c = 38$ and $A = 80°$. Find side a.

Using the cosine rule to find angles

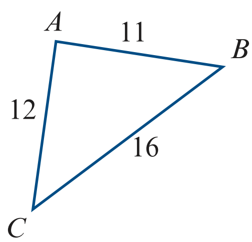
Example 19

8 Find angle A in each triangle.

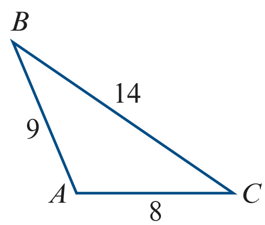
a



b



c



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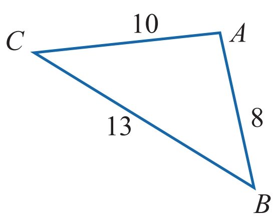

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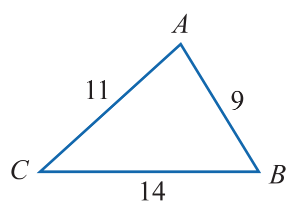

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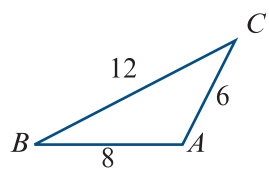

9 In the triangle ABC, $a = 31, b = 47$ and $c = 52$. Find angle B.

10 In the triangle RST, $r = 66$, $s = 29$ and $t = 48$. Find angle T.

11 Find the smallest angle in the triangle ABC, with $a = 120, b = 90$ and $c = 105$.

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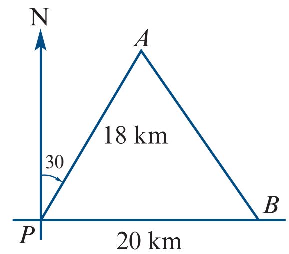
Applications

Example 20

12 A farm has a triangular shape with fences of 5 km, 7 km and 9 km in length. Find the size of the smallest angle between the fences. The smallest angle is always opposite the smallest side.

Example 21

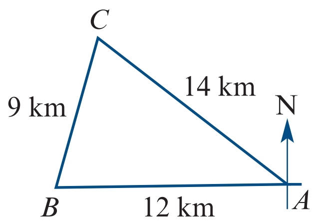
13 A ship left the port, P, and sailed 18 km on a bearing of 030° to point A. Another ship left port P and sailed 20 km east to point B. Find the distance from A to B to one decimal place.



14 A bushwalker walked 12 km west, from point A to point B. Her friend walked 14 km, from point A to point C, as shown in the diagram. The distance from B to C is 9 km.

a Find the angle at A, between the paths taken by the bushwalkers, to one decimal place.

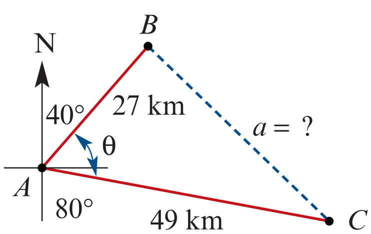
b What is the bearing of point C from A? Give the bearing to the nearest degree.



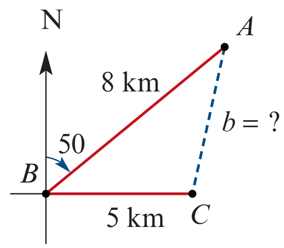
15 A ship left port A and travelled 27 km on a bearing of 040° to reach point B. Another ship left the same port and travelled 49 km on a bearing of 100° to arrive at point C.

a Find the unknown angle, θ, between the directions of the two ships.

b How far apart were the two ships when they stopped?

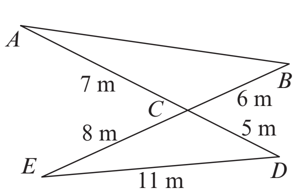


16 A battleship, B, detected a submarine, A, on a bearing of 050° and at a distance of 8 km. A cargo ship, C, was 5 km due east of the battleship. How far was the submarine from the cargo ship?



Testing understanding

17 Find distance AB to one decimal place.



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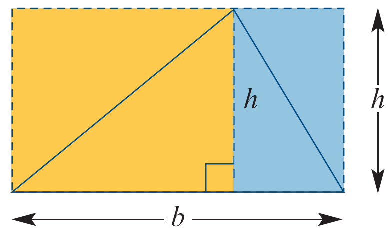
11I The area of a triangle

Learning intentions

- To be able to identify from the given information which of the three area rules should be used to find the area of the triangle.

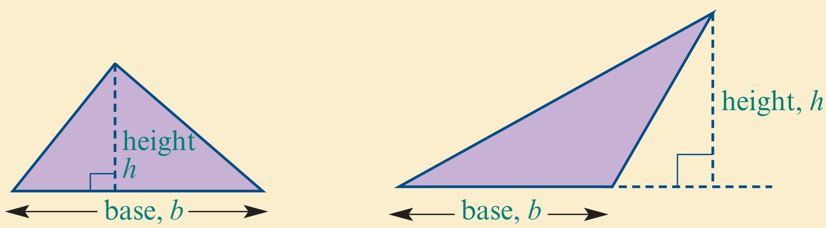
Area of a triangle = $\frac{1}{2}$ × base × height

From the diagram, we see that the area of a triangle with a base, b, and height, h, is equal to half the area of the rectangle, $b \times h$, that it fits within.



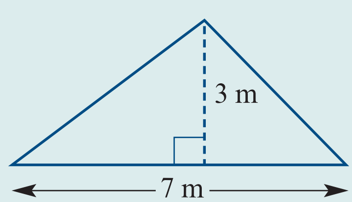
Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times b \times h$$



Example 22 Finding the area of a triangle using $\frac{1}{2}$ × base × height

Find the area of the triangle shown to one decimal place.



Explanation	Solution
1 As we are given values for the base and height of the triangle, use Area $= \frac{1}{2} \times \text{base} \times \text{height}$	Base, $b = 7$ Height, $h = 3$ Area of triangle $= \frac{1}{2} \times b \times h$

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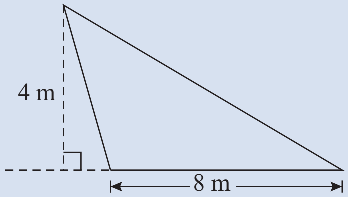
2 Substitute the given values.	$= \frac{1}{2} \times 7 \times 3$
3 Evaluate.	$= 10.5\ \text{m}^2$
4 Write your answer.	The area of the triangle is $10.5\ \text{m}^2$

Now try this 22 **Finding the area of a triangle using $\frac{1}{2}$ × base × height (Example 22)**

Find the area of the triangle shown to one decimal place.



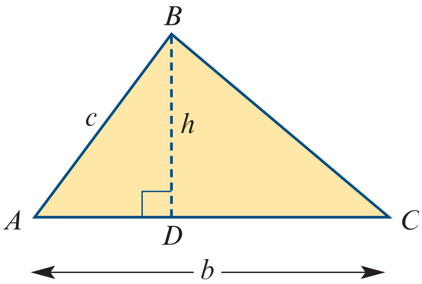
Hint 1 Use: area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$.

Hint 2 Round the answer to one decimal place and give the correct units for area.

Area of a triangle = $\frac{1}{2} bc \sin A$

In triangle ABD,

$$\sin A = \frac{h}{c}$$
$$h = c \times \sin A$$



So we can replace h with $c \times \sin A$ in the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$
$$\text{Area of a triangle} = \frac{1}{2} \times b \times c \times \sin A$$

Similarly, using side c or a for the base, we can make a complete set of three rules:

Area of a triangle

$$\text{Area of a triangle} = \tfrac{1}{2}\, bc \sin A$$
$$\text{Area of a triangle} = \tfrac{1}{2}\, ac \sin B$$
$$\text{Area of a triangle} = \tfrac{1}{2}\, ab \sin C$$

Notice that each version of the rule follows the pattern:

Area of a triangle = $\frac{1}{2}$ × (product of two sides) × *sin*(angle between those two sides)

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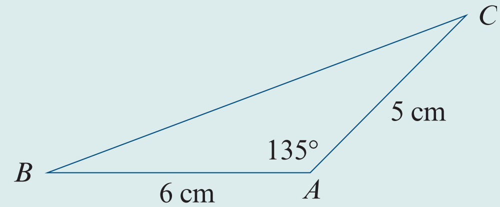
Example 23 Finding the area of a triangle using $\frac{1}{2}bc \sin A$

Find the area of the triangle shown to one decimal place.



Explanation

1 We are given two sides, b and c, and the angle, A, between them, so use:
Area of a triangle $= \frac{1}{2} bc \sin A$

2 Substitute values for b, c and A into the rule.

3 Use your calculator to find the area.

4 Write your answer to one decimal place.

Solution

$b = 5, c = 6, A = 135°$

Area of triangle $= \frac{1}{2}bc \sin A$

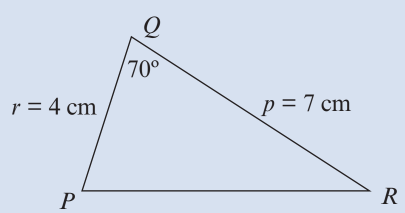
$= \frac{1}{2} \times 5 \times 6 \times \sin 135°$

$= 10.606\ldots$

The area of the triangle is $10.6\ \text{cm}^2$.

Now try this 23 Finding the area of a triangle using $\frac{1}{2}bc \sin A$ (Example 23)

Find the area of the triangle shown to one decimal place.



Hint 1 The pattern of the area rule needed is:

Area of a triangle $= \frac{1}{2} \times$ (product of two sides) $\times$ sin(angle between the two sides)

Heron's rule for the area of a triangle

Heron's rule can be used to find the area of any triangle when we know the lengths of the three sides.

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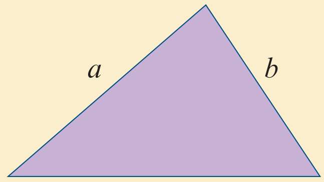
Heron's rule for the area of a triangle

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

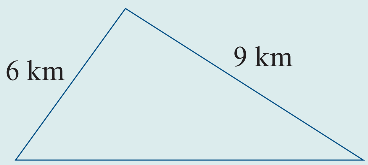
where $s = \frac{1}{2}(a+b+c)$

The variable s is called the *semi-perimeter* because it is equal to half the sum of the sides.



Example 24 Finding the area of a triangle using Heron's formula

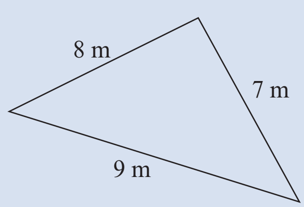
The boundary fences of a farm are shown in the diagram. Find the area of the farm to the nearest square kilometre.



Explanation	Solution
1 As we are given the three sides of the triangle, use Heron's formula. Start by finding s, the semi-perimeter.	Let $a = 6, b = 9, c = 11$ $s = \frac{1}{2}(a+b+c)$ $= \frac{1}{2}(6+9+11) = 13$
2 Write Heron's formula.	Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
3 Substitute the values of s, a, b and c into Heron's formula.	$= \sqrt{13(13-6)(13-9)(13-11)}$ $= \sqrt{13 \times 7 \times 4 \times 2}$
4 Use your calculator to find the area.	$= 26.981\ldots$
5 Write your answer.	The area of the farm, to the nearest square kilometre, is 27 km^2.

Now try this 24 Finding the area of a triangle using Heron's formula (Example 24)

Find the area of the triangle shown to one decimal place.



Hint 1 Calculate the semi-perimeter using
$$s = \frac{1}{2}(a+b+c).$$

Hint 2 Write Heron's formula and substitute in the values for s, a, b and c.

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Section Summary

There are three rules for finding the area of a triangle, ABC:

- When given the length of the base and the perpendicular height, use:

$$Area = \tfrac{1}{2}\,base \times height$$

- When given two sides and the angle between those sides, use the given information to choose from:

$$Area = \tfrac{1}{2}\,bc\sin A$$
$$Area = \tfrac{1}{2}\,ac\sin B$$
$$Area = \tfrac{1}{2}\,ab\sin C$$

- When given the three sides of the triangle, use Heron's formula:

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$
$$where\ s = \tfrac{1}{2}(a+b+c)$$

Skill-sheet

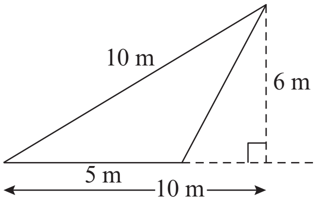
Exercise 11I

In this exercise, calculate areas to one decimal place, where necessary.

Building understanding

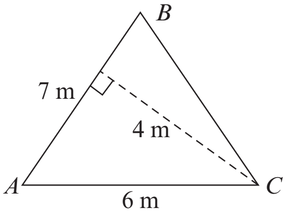
1 Finding the area of a triangle using:
Area = $\dfrac{1}{2}$ base × height.

a Give the perpendicular height.
b What is the length of the base?
c Find the area of the triangle.



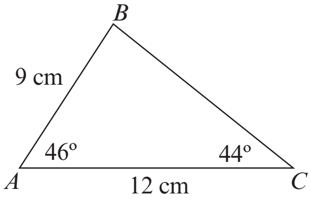
2 Find the area of triangle ABC. The base is not always the horizontal line. In this case, the perpendicular height is measured from the base, AB.

a State the perpendicular height.
b What is the length of the base?
c Find the area.



3 Find the area of the triangle using:
Area = $\dfrac{1}{2}\,bc \times \sin\theta$
where θ is the angle between the two given sides.

a Which angle should be used? Why?
b Find the area to one decimal place.



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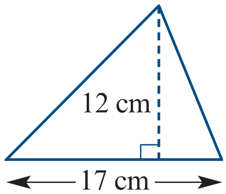
Developing understanding

Finding areas using $\frac{1}{2}$ × base × height

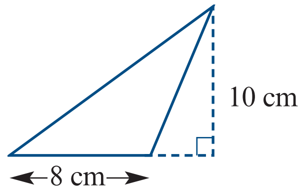
Example 22

4 Find the area of each triangle.

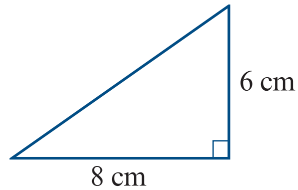
a



b



c

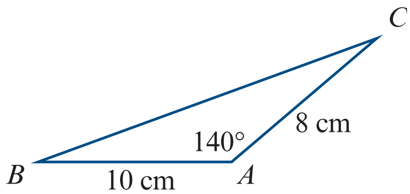


Finding areas using $\frac{1}{2}$ *bc* sin *A*

Example 23

5 Find the areas of the triangles shown.

a



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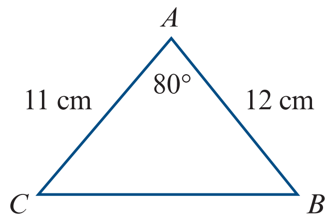

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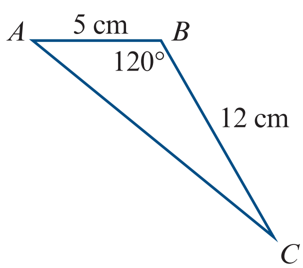

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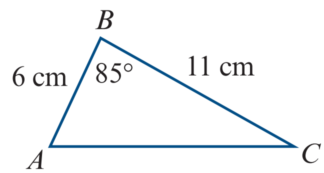
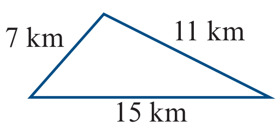


Finding areas using Heron's formula

Example 24

6 Find the area of each triangle.

a



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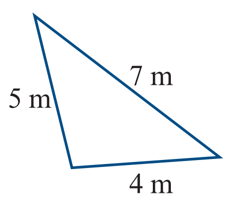

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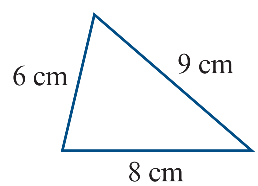

Mixed problems

7 For each triangle on the following page, choose the rule for finding its area from:

i $\frac{1}{2}$ *base* × *height*

ii $\frac{1}{2}\, bc \sin A$

iii $\frac{1}{2}\, ac \sin B$

iv $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

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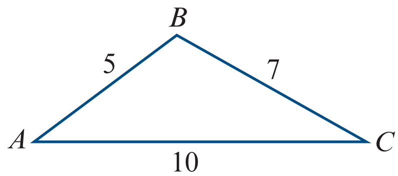
a



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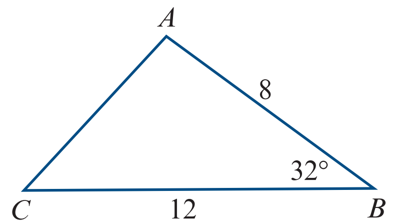

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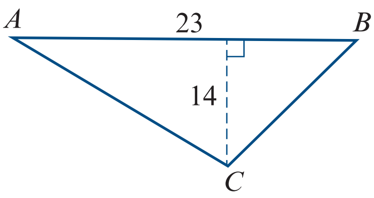

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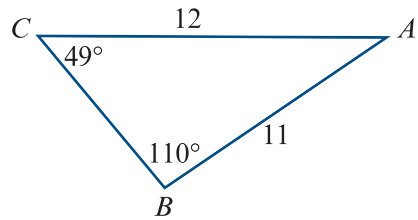
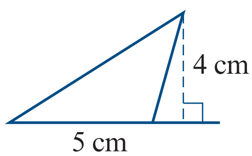


8 Find the area of each triangle shown.

a



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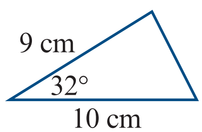

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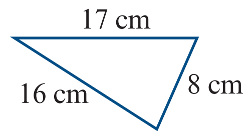

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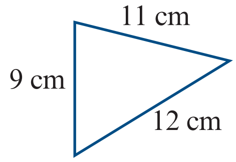

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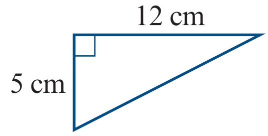

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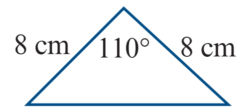


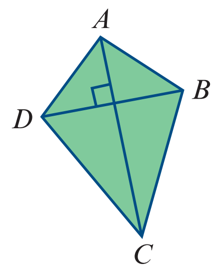
9 Find the area of a triangle with a base of 28 cm and a height of 16 cm.

10 Find the area of triangle *RST* with side *r* (42 cm), side *s* (57 cm) and angle *T* (70°).

11 Find the area of a triangle with sides of 16 km, 19 km and 23 km.

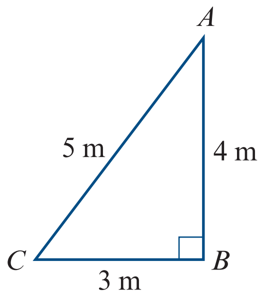
Applications

12 The kite shown is made using two sticks, *AC* and *DB*.
The length of *AC* is 100 cm and the length of *DB* is 70 cm.
Find the area of the kite.

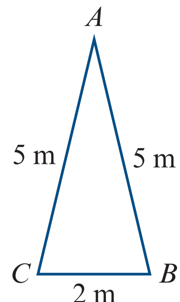


13 Three students, *A*, *B* and *C*, stretched a rope loop that was 12 m long into different triangular shapes. Find the area of each shape.

a



b



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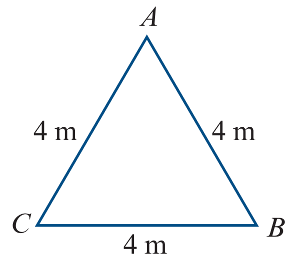

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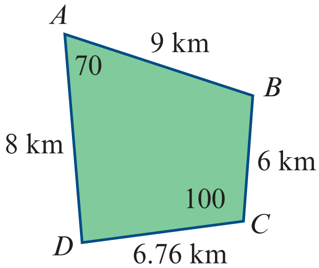
14 A farmer needs to know the area of her property with the boundary fences as shown. Give answers to two decimal places.

Hint: Draw a line from B to D to divide the property into two triangles.

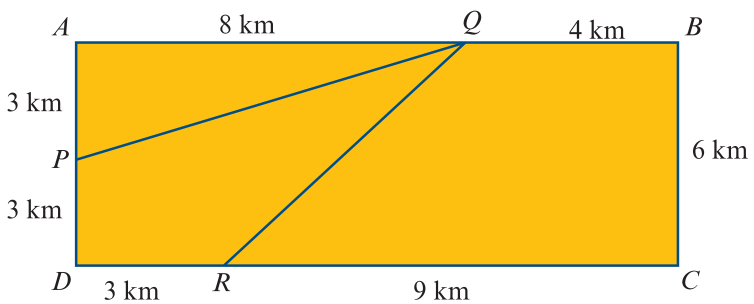


a Find the area of triangle ABD.

b Find the area of triangle BCD.

c State the total area of the property.

15 A large rectangular area of land, $ABCD$ in the diagram, has been subdivided into three regions as shown.



a Find the area of:

i region PAQ **ii** region $QBCR$ **iii** region $PQRD$.

b Find the size of angle PQR to one decimal place.

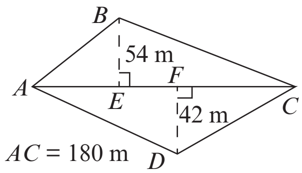
Testing understanding

16 A technique that surveyors use to find the area of an irregular four-sided shape is to measure the length, AC, joining opposite corners, then the lengths of lines perpendicular to AC to the other corners.

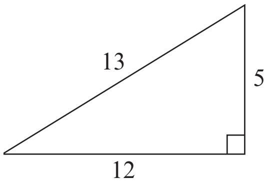
The measurements for the lengths were:

$AC = 180$ m, $BE = 54$ m and $DF = 42$ m.

Find the area $ABCD$ to one decimal place.

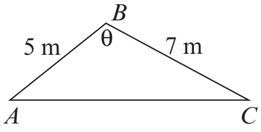


17 Show how the three different area rules can be used to find the area of the triangle shown.



18 Triangle ABC has sides of 5 m and 7 m. Angle θ is between the two given sides.

Find the angle, θ, that would give the maximum area for triangle ABC.



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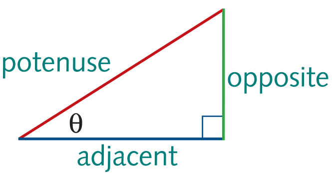
Review

Key ideas and chapter summary

Assign-ment

Naming the sides of a right-angled triangle

The **hypotenuse** is the longest side and is always opposite the right angle (90°).
The **opposite** side is directly opposite the angle θ (the angle being considered).



The **adjacent** side is beside angle θ and runs from θ to the right angle.

Trigonometric ratios

The **trigonometric ratios** are sin θ, cos θ and tan θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

SOH-CAH-TOA

This helps you to remember the trigonometric ratio rules.

Degree mode

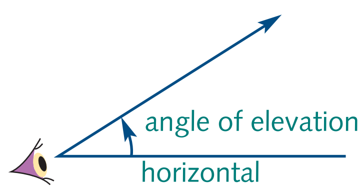
Make sure your calculator is in DEGREE mode when doing calculations with trigonometric ratios.

Applications of right-angled triangles

Always draw well-labelled diagrams, showing all known sides and angles. Also label any sides or angles that need to be found.

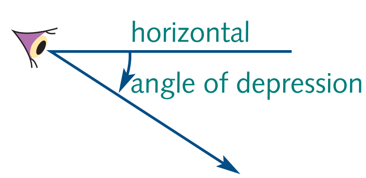
Angle of elevation

The **angle of elevation** is the angle through which you *raise* your line of sight from the horizontal, looking *up* at something.



Angle of depression

The **angle of depression** is the angle through which you *lower* your line of sight from the horizontal, looking *down* at something.

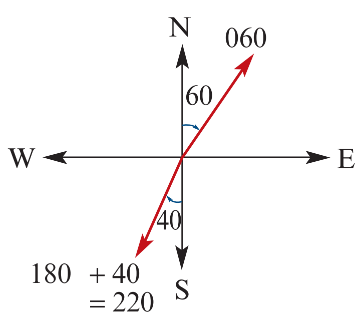


Angle of elevation = angle of depression

The angles of elevation and depression are alternate ('Z') angles, so they are equal.

Three-figure bearings

Three-figure bearings are measured clockwise from North and always have three digits, e.g. 060°, 220°.



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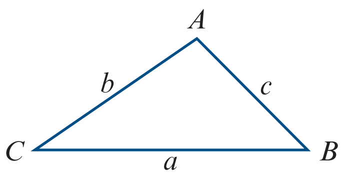
Distance, speed and time

Navigation problems often involve distance, speed and time, as well as direction.

Distance travelled = time taken × speed

Labelling a non-right-angled triangle

Side a is always opposite angle A, and so on.



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use the **sine rule** when given:

- two sides and an angle opposite one of those sides
- two angles and one side.

If neither angle is opposite the given side, find the third angle using $A + B + C = 180°$.

Ambiguous case of the sine rule

The ambiguous case of the sine rule occurs when it is possible to draw two different triangles that both fit the given information.

Cosine rule

The **cosine rule** has three versions. When given two sides and the angle between them, use the rule that starts with the required side:

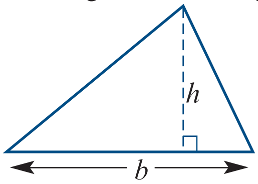
$$a^2 = b^2 + c^2 - 2bc\cos A$$

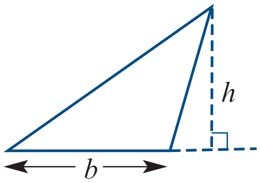
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

- Use the formula: area of triangle $= \frac{1}{2} \times b \times h$, if the base and height of the triangle are known:





- Use the formula: area of triangle $= \frac{1}{2} \times bc\sin A$, if two sides and the angle between them are known.
- Use Heron's formula if the lengths a, b and c, of the three sides of the triangle are known.

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \tfrac{1}{2}(a+b+c)$$

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Review

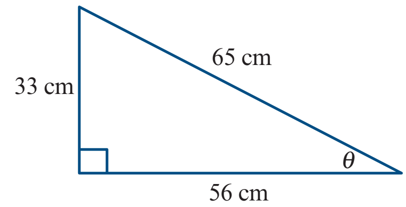
Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

11A **1** I can name the sides of a right-angled triangle. ☐

e.g. Name the sides 33 cm, 56 cm and 65 cm long.



11A **2** I can use the definitions of trigonometric ratios. ☐

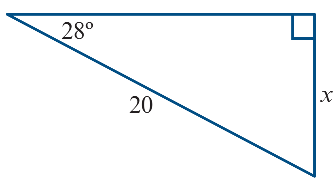
e.g. Using the diagram in the previous question, state the values of the trigonometric ratios for: $\cos\theta$, $\sin\theta$ and $\tan\theta$.

11A **3** I can use a CAS calculator to find the value of a trigonometric ratio for a given angle. ☐

e.g. Find $\cos 27°$, $\sin 58°$ and $\tan 73°$ to four decimal places.

11A **4** I can choose the required trigonometric ratio rule when finding an unknown side in a right-angled triangle. ☐

e.g. Name the trigonometric ratio needed to find x.



11B **5** I can substitute in values and solve the required equation to find the unknown side. ☐

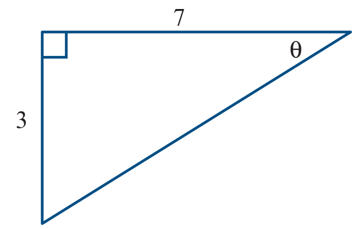
e.g. Use the triangle in the previous question to find side x to one decimal place.

11C **6** I can use a CAS calculator to find the required angle when given the value of its trigonometric ratio. ☐

Find θ, to one decimal place, when $\cos\theta = 0.7431$.

11C **7** I can find the required angle in a right-angled triangle given two sides of the triangle. ☐

e.g. Find angle θ to one decimal place.



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Review

11D	8	I can draw clearly labelled diagrams of practical situations, showing the given sides and angles.	☐

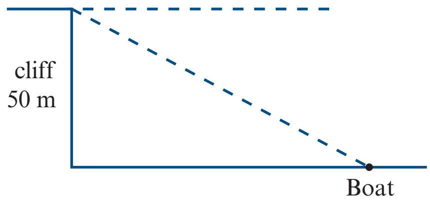
e.g. A tree casts a shadow that is 23 metres long when the sun's rays make an angle of 34° with the horizontal ground. Draw a clearly labelled diagram.

11D	9	I can set up and solve equations to find unknown sides and angles.	☐

e.g. Find the height of the tree described in the previous question to one decimal place.

11E	10	I can identify and label the angles of elevation and depression in diagrams of practical situations.	☐

e.g. From the height of a 50 m cliff, a boat is seen at an angle of depression of 28°. Write the information into the diagram shown.



11E	11	I can choose the appropriate trigonometric ratios and solve equations to find unknown sides and angles.	☐

e.g. In the situation described above, find the distance of the boat from the base of the cliff to the nearest metre.

11F	12	I can use three-figure bearings to draw navigation and surveying diagrams.	☐

e.g. A surveyor stopped on a highway pointing North and then walked on a bearing of 050° for 3 km. Show this in a clearly labelled diagram.

11F	13	I can solve the appropriate equations to find unknown bearings and distances.	☐

e.g. In the situation described above, what is the shortest distance the surveyor needs to walk to reach the highway to one decimal place.

11G	14	I can use the sine rule to find an unknown angle, given two sides and an opposite angle.	☐

e.g. In triangle ABC, $B = 115°$, $b = 27$ m and $c = 24$ m. Find angle C to one decimal place.

11G	15	I can use the sine rule to find an unknown side, given 2 angles and a side.	☐

e.g. In triangle ABC, $A = 30°$, $C = 110°$ and $c = 49$ km. Find side a to nearest km.

11G	16	I can find the required angles and sides when given information that fits two possible triangles.	☐

e.g. In triangle ABC, $A = 35°$, $a = 15$ and $c = 20$. Find the two possible values for angle C to one decimal place.

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11H **17** I can identify when the sine rule or the cosine rule should be used. ☐

e.g. In triangle ABC, $a = 14$, $b = 17$ and $c = 16$. Which rule should be used to find angle A?

11H **18** I can use the cosine rule to find the unknown side when given two sides and the angle between them. ☐

e.g. In triangle ABC, $B = 70°$, $a = 8$ and $c = 10$. Find side b to one decimal place.

11H **19** I can find the required angle in a triangle when given the three sides. ☐

e.g. In triangle ABC, $a = 21$, $b = 23$ and $c = 26$. Find angle C to one decimal place.

11I **20** I can use the given information to decide which area rule should be used. ☐

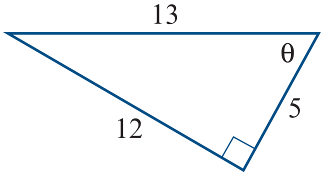
e.g. In triangle ABC, $a = 20$, $b = 23$ and $c = 27$. Name the rule that should be used to find the area.

11I **21** I can use the appropriate rule to find the area of a given triangle. ☐

e.g. In triangle ABC, $A = 47°$, $b = 29$ cm and $c = 31$ cm. Find the area of triangle ABC to one decimal place.

Multiple-choice questions

1 In the triangle shown, $\sin \theta$ equals:

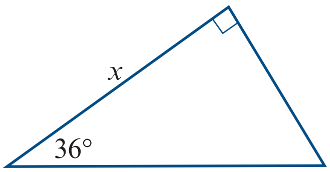


A $\frac{5}{13}$ **B** $\frac{5}{12}$

C $\frac{12}{13}$ **D** $\frac{13}{12}$

E $\frac{12}{5}$

2 The unknown length, x, is given by:

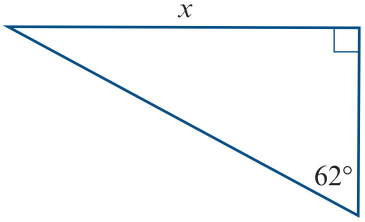


A $24 \sin 36°$ **B** $24 \tan 36°$

C $24 \cos 36°$ **D** $\frac{\sin 36°}{24}$

E $\frac{\cos 36°}{24}$

3 To find length x we should use:



A $17 \sin 62°$ **B** $17 \tan 62°$

C $17 \cos 62°$ **D** $\frac{\tan 62°}{17}$

E $\frac{\sin 62°}{17}$

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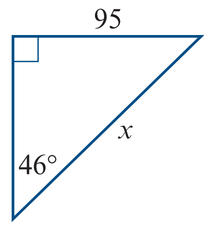
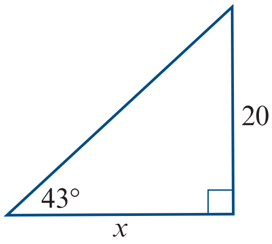
4 The unknown side, x, is given by:



A $95\tan 46^\circ$ **B** $\dfrac{95}{\cos 46^\circ}$ **C** $\dfrac{\sin 46^\circ}{96}$

D $95\sin 46^\circ$ **E** $\dfrac{95}{\sin 46^\circ}$

5 To find side x we need to calculate:

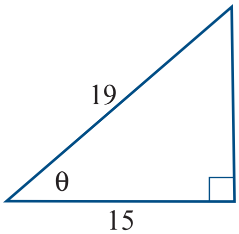


A $\dfrac{\tan 43^\circ}{20}$ **B** $\dfrac{20}{\tan 43^\circ}$

C $20\tan 43^\circ$ **D** $20\cos 43^\circ$

E $20\sin 43^\circ$

6 To find angle θ we need to use:

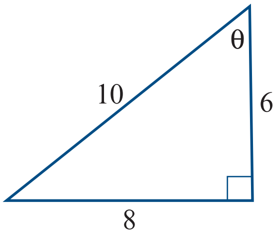


A $\cos^{-1}\left(\dfrac{15}{19}\right)$ **B** $\cos\left(\dfrac{15}{19}\right)$

C $\sin^{-1}\left(\dfrac{15}{19}\right)$ **D** $15\sin(19)$

E $19\cos(15)$

7 The unknown angle, θ, to one decimal place, is:

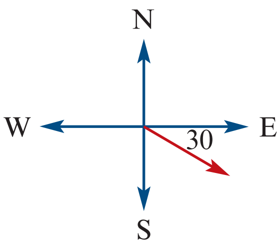


A 36.9° **B** 38.7°

C 51.3° **D** 53.1°

E 53.3°

8 The direction shown has the three-figure bearing:

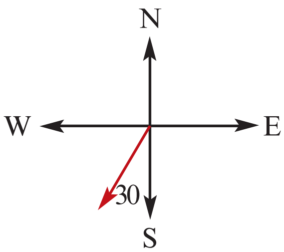


A 030° **B** 060°

C 120° **D** 210°

E 330°

9 The direction shown could be described as the three-figure bearing:



A −030° **B** 030°

C 060° **D** 120°

E 210°

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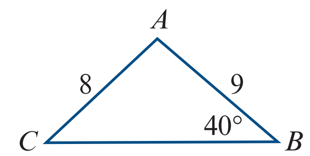
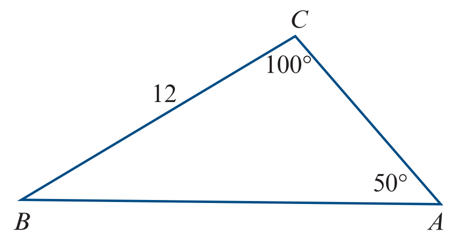
10 In this triangle, angle C equals:

A $34.8°$ **B** $46.3°$
C $53.9°$ **D** $55.2°$
E $86.1°$



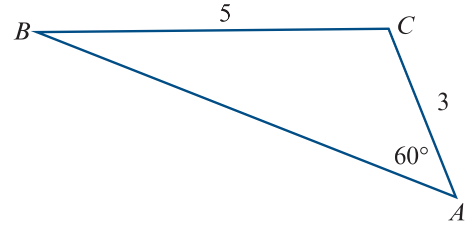
11 To find length c in triangle ABC we should use:

A $\dfrac{12\sin 100°}{\sin 30°}$ **B** $\dfrac{12\sin 50°}{\sin 100°}$
C $\dfrac{\sin 50°}{12\sin 100°}$ **D** $\dfrac{12\sin 100°}{\sin 50°}$
E $\dfrac{\sin 100°}{12\sin 50°}$



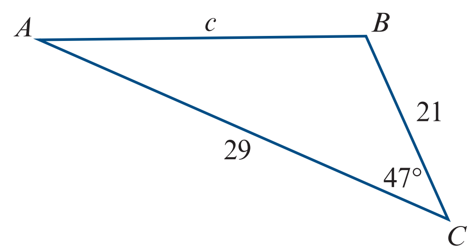
12 In triangle ABC, $\sin B$ equals:

A $\dfrac{3}{5}$ **B** $\dfrac{3\sin 60°}{5}$
C $\dfrac{3}{5\sin 60°}$ **D** $\dfrac{5\sin 60°}{3}$
E $\dfrac{5}{3\sin 60°}$



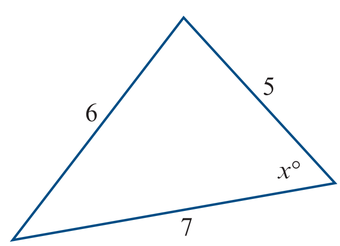
13 Which expression should be used to find length c in triangle ABC?

A $\dfrac{1}{2}(21)(29)\cos 47°$
B $\cos^{-1}\left(\dfrac{21}{29}\right)$
C $\sqrt{21^2 + 29^2}$
D $21^2 + 29^2 - 2(21)(29)\cos 47°$
E $\sqrt{21^2 + 29^2 - 2(21)(29)\cos 47°}$



14 For the given triangle, the value of $\cos x$ is given by:

A $\dfrac{6^2 - 7^2 - 5^2}{2(7)(5)}$ **B** $\dfrac{7^2 + 5^2 - 6^2}{2(7)(5)}$
C $\dfrac{5}{7}$ **D** $\dfrac{7^2 - 5^2 - 6^2}{2(5)(6)}$
E $\dfrac{5^2 - 6^2 - 7^2}{2(5)(6)}$



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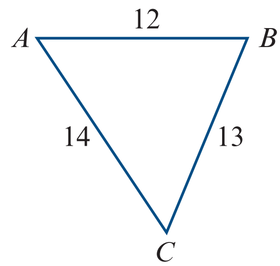
15 To find angle C we should use the rule:



A $\cos C = \dfrac{adjacent}{hypotenuse}$

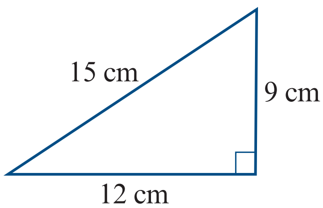
B $\sin C = \dfrac{opposite}{hypotenuse}$

C $\cos C = \dfrac{a^2 + c^2 - b^2}{2ac}$

D $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

E $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

16 The area of the triangle shown is:

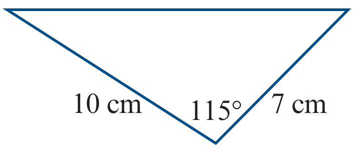


A 36 cm^2 **B** 54 cm^2

C 67.5 cm^2 **D** 90 cm^2

E 108 cm^2

17 The area of the triangle shown, to two decimal places, is:

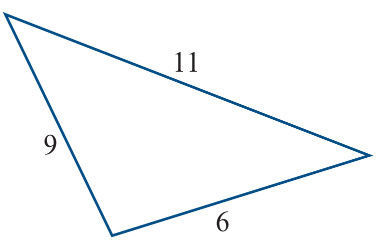


A 14.79 cm^2 **B** 31.72 cm^2

C 33.09 cm^2 **D** 35.00 cm^2

E 70.00 cm^2

18 The area of the triangle shown is given by:



A $26(26 - 6)(26 - 9)(26 - 11)$

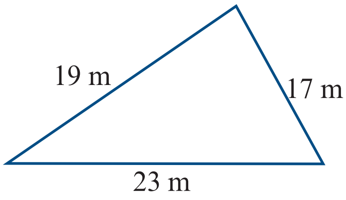
B $\sqrt{26(26 - 6)(26 - 9)(26 - 11)}$

C $\sqrt{13(13 - 6)(13 - 9)(13 - 11)}$

D $\sqrt{6^2 + 9^2 + 11^2}$

E $13(13 - 6)(13 - 9)(13 - 11)$

19 The area of the triangle shown, to one decimal place, is:



A 29.5 m^2 **B** 158.6 m^2 **C** 161.5 m^2

D 195.5 m^2 **E** 218.5 m^2

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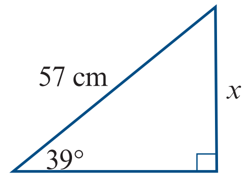
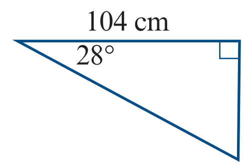
Review

Short-answer questions

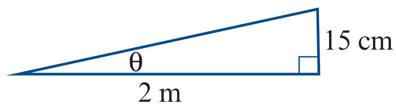
1 Find the length of x to two decimal places.



2 Find the length of the hypotenuse to two decimal places.

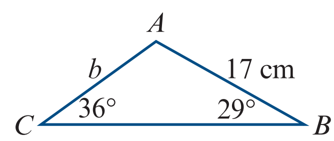


3 A road rises 15 cm for every 2 m travelled horizontally.
Find the angle of slope θ to the nearest degree.

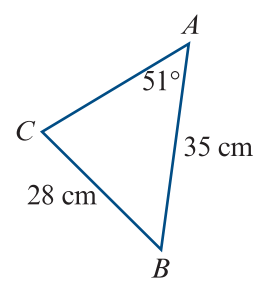


4 **a** Find the sides of a right-angled triangle for which $\cos\theta = \frac{72}{97}$ and $\tan\theta = \frac{65}{72}$.
b Hence, find $\sin\theta$.

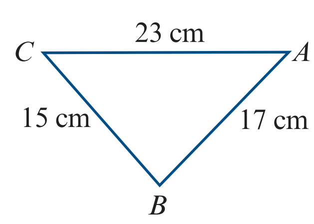
5 Find the length of side b to two decimal places.



6 Find two possible values for angle C to one decimal place.



7 Find the smallest angle in the triangle shown to one decimal place.



8 A car travelled 30 km east, then travelled 25 km on a bearing of 070°. How far was the car from its starting point? Answer to two decimal places.

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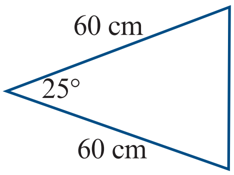
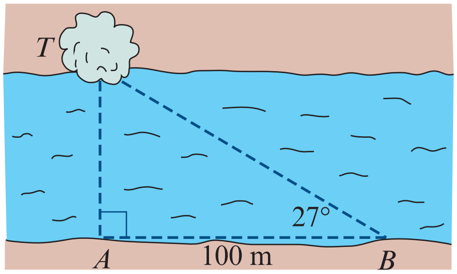
9 A pennant flag is to have the dimensions shown. What area of cloth will be needed for the flag? Answer to one decimal place.



10 Find the area of an equilateral triangle with sides of 8 m to one decimal place.

Written-response questions

1 Tim was standing at point A when he saw a tree, T, directly opposite him on the far bank of the river. He walked 100 m along the riverbank to point B and noticed that his line of sight to the tree made an angle of 27° with the riverbank. Answer the following to two decimal places.



a How wide was the river?

b What is the distance from point B to the tree?

Standing at B, Tim measured the angle of elevation to the top of the tree to be 18°.

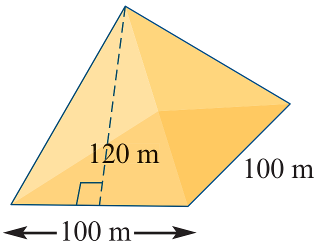
c Make a clearly labelled diagram showing distance TB, the height of the tree and the angle of elevation, then find the height of the tree.

2 A yacht, P, left port and sailed 45 km on a bearing of 290°.
Another yacht, Q, left the same port but sailed for 54 km on a bearing of 040°.

a What was the angle between their directions?

b How far apart were they at that stage (to two decimal places)?

3 The pyramid shown has a square base with sides of 100 m. The line down the middle of each side is 120 m long.



a Find the total surface area of the pyramid.
(As the pyramid rests on the ground, the area of its base is not part of its surface area.)

b If 1 kg of gold can be rolled flat to cover 0.5 m^2 of surface area, how much gold would be needed to cover the surface of the pyramid?

c At today's prices, 1 kg of gold costs $62 500. How much would it cost to cover the pyramid with gold?

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