Chapter 12

Revision of Unit 2

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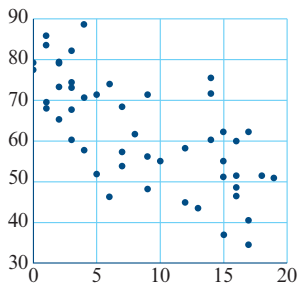
12A Multiple-choice questions

Chapter 7: Investigating relationships between two numerical variables

1 For which one of the following pairs of variables would it be appropriate to construct a scatterplot?

A *computer preference* (apple, PC) and *phone preference* (iPhone, android)
B *test score* and *year level* (Year11, Year 12)
C *interest in politics* (very interested, interested, not interested) and *age* in years
D *time spent on social media*, in minutes, and *age group* (less than 18, 18-25, over 25)
E *age*, in years, and *reaction time*, in seconds

2 For the scatterplot shown, the association between the variables is best described as:

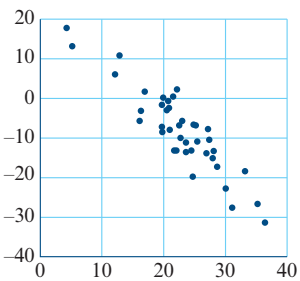


A weak linear negative
B strong linear negative
C no association
D weak linear positive
E strong linear positive

3 (VCAA-type question) A designer of luxury handbags discovered that there was a positive association between the price of their handbags and demand for the handbags. Given this information, it can be concluded that:

A there is a linear association between the price of the handbags and demand.
B demand for the handbags is not related to their price.
C demand for the handbags tends to increase as the price of the handbags increases.
D demand for the handbags tends to decrease as the price of the handbags increases.
E demand for the handbags tends to increase as the price of the handbags decreases.

4 For the scatterplot shown, the value of the Pearson's correlation coefficient, r, is closest to:



A -0.82 **B** -0.51 **C** 0.15
D 0.53 **E** 0.77

5 A correlation coefficient of $r = 0.64$ would classify a linear relationship as:

A weak, positive **B** weak, negative **C** moderate, positive
D moderate, negative **E** moderately weak

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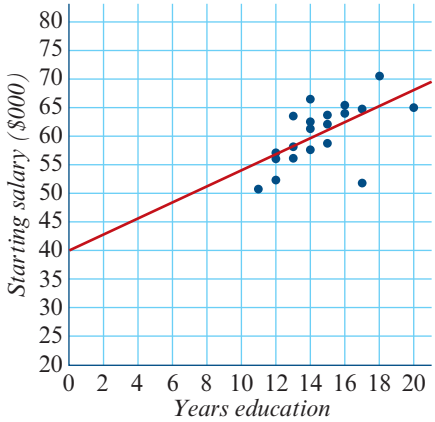
6 (VCAA-type question) The following scatterplot shows the association between the number of years employees in a company had spent in formal education (*years education*) and their *starting salary*, in $000s. The least squares regression line is also shown on the scatterplot.



The equation of this line is:

A *years education* = 40 + 1.4 × *starting salary*

B *starting salary* = 1.4 + 40 × *years education*

C *years education* = 40 + 0.714 × *starting salary*

D *expenditure* = 0.714 − 40 × *years education*

E *starting salary* = 40 + 1.4 × *years education*

7 (VCAA-type question) The equation of the least squares regression line, $y = a + bx$, when:

$r = -0.600$ $s_x = 3.20$ $s_y = 6.40$ $\bar{x} = 48.7$ $\bar{y} = 63.3$

is given by:

A $y = 4.86 - 1.20x$ **B** $y = 77.9 - 0.3x$ **C** $y = 1.2 + 122x$

D $y = 122 - 1.20x$ **E** $y = 0.3 - 77.9x$

The following information relates to Questions 8 and 9.

The *age*, in years, and percentage *body fat* for a group of 8 people is given in the following table.

age	58	72	67	43	51	52	25	35
body fat (%)	20.1	26.1	25.8	19.5	14.1	27.0	6.1	4.1

8 The value of Pearson's correlation coefficient, r, for these data is closest to:

A 0.4 **B** 0.6 **C** 0.7 **D** 0.8 **E** 0.9

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9 The least squares regression line that enables *body fat* to be predicted from *age* is closest to:

A $age = 23.9 + 1.49 \times body\ fat$

B $body\ fat = -6.40 + 0.481 \times body\ fat$

C $age = 6.40 + 0.481 \times body\ fat$

D $body\ fat = 23.9 + 1.49 \times age$

E $body\ fat = -6.40 + 0.846 \times body\ fat$

The following information relates to Questions 10 and 11.

Data from a large group of males was used to determine the equation of a regression line that enables *weight*, in kilograms, to be predicted from percentage *body fat* as follows:

$$weight = 61.32 + 1.057 \times body fat$$

10 Using this equation, the weight of an individual with a *body fat* of 20% is predicted to be:

A 61.5 kg **B** 78.4 kg **C** 82.5 kg **D** 125.1 kg **E** 1251.1 kg

11 From the equation of the regression line, it can be concluded that on average there is:

A an increase of 1 kg in *weight* for each 1.057 percentage increase in *body fat*.

B a decrease of 1 kg in *weight* for each 1 percentage increase in *body fat*.

C a decrease of 1 kg in *weight* for each 1 percentage increase in *body fat*.

D an increase of 61.32 kg in *weight* for each 1 percentage point increase in *body fat*.

E a decrease of 1.057 kg in *weight* for each 1 percentage decrease in *body fat*.

12 In an investigation of the association between hearing test scores and age, the equation of the least squares line fitted to the data was:

$$hearing\ test\ score = 48.9034 - 0.0428681 \times age$$

When the coefficients in the equation are rounded to four significant figures, the equation becomes:

A $hearing\ test\ score = 48.9034 - 0.0429 \times age$

B $hearing\ test\ score = 48.9 - 0.0429 \times age$

C $hearing\ test\ score = 48.90 - 0.04287 \times age$

D $hearing\ test\ score = 48.903 - 0.0429 \times age$

E $hearing\ test\ score = 48.900 - 0.04287 \times age$

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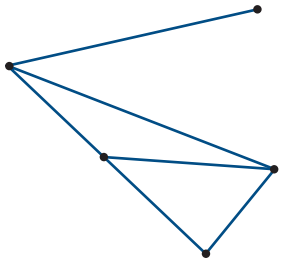
Chapter 8: Graphs and networks

13 (VCAA-type question) A graph has 6 vertices. The minimum number of edges required for this graph to be connected is:

A 3 **B** 4 **C** 5 **D** 6 **E** 7

14 (VCAA-type question) The sum of the degrees of the graph shown is:

A 8
B 9
C 10
D 11
E 12

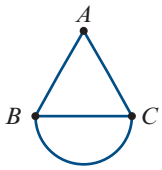


15 (VCAA-type question) A tree with 10 vertices will have:

A 8 edges **B** 9 edges **C** 10 edges **D** 11 edges **E** 12 edges

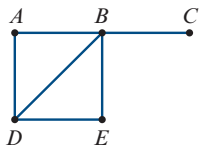
16 For the graph shown, the number of paths of length 3 from vertex A to vertex A is:

A 0 **B** 1 **C** 2 **D** 3 **E** 4



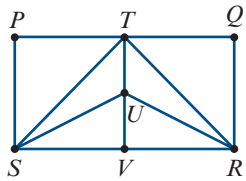
17 The graph shown does not have an Eulerian circuit.
The addition of which edge will ensure that the graph has an Eulerian circuit?

A AB **B** AC **C** AE
D DE **E** DC



18 (VCAA-type question)
For the graph shown opposite, which of the following is a Hamiltonian cycle.

A $T - P - S - V - R - U - Q - T$
B $T - Q - R - T - P - S - V - U - T$
C $R - V - S - P - T - Q - R - U$
D $U - V - S - P - T - Q - R$
E $U - V - S - P - T - Q - R - U$



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19 A graph has four vertices, A, B, C and D, and the adjacency matrix of this graph is:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & 3 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{pmatrix} \end{array}$$

This graph has an Eulerian trail that begins and ends at which of the following pairs of vertices?

A A and B **B** A and C **C** A and D
D B and C **E** B and D

20 (VCAA-type question) A connected planar graph has 8 vertices and 8 faces. The number of edges for this graph is:

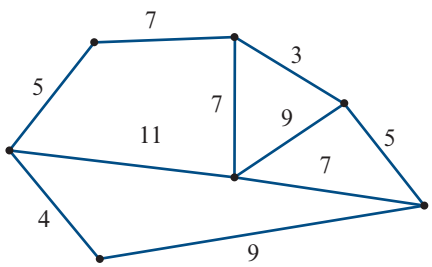
A 10 **B** 11 **C** 12 **D** 13 **E** 14

21 A connected planar graph that has 6 vertices could have:

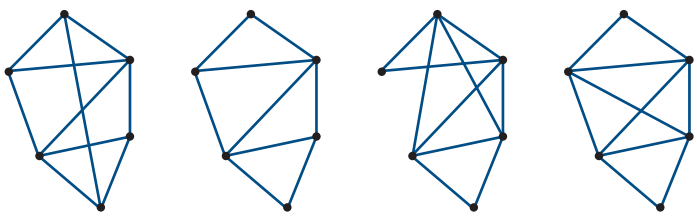
A 4 edges and 4 faces **B** 7 edges and 5 faces **C** 11 edges and 6 faces
D 10 edges and 6 faces **E** 8 edges and 3 faces

22 The total weight on the minimum spanning tree for the graph is:

A 25 **B** 27 **C** 31
D 33 **E** 34



23 (VCAA-type question) How many of the following graphs are non-planar?



A 0 **B** 1 **C** 2 **D** 3 **E** 4

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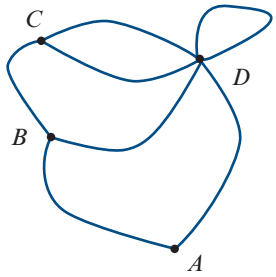
24 (VCAA-type question) Consider the following graph.



An adjacency matrix that could be used to represent the graph is:

A
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 \end{bmatrix} \end{array}$$

B
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix} \end{array}$$

D
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \end{array}$$

E
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \end{array}$$

Chapter 9: Variation

25 If $m \propto n$ and $m = 9$ when $n = 4$, then the constant of variation, k, equals:

A $\frac{9}{4}$ **B** 13 **C** 36 **D** $\frac{4}{9}$ **E** 5

26 If $y \propto \frac{1}{x}$ and $y = 14$ when $x = 2$, the value of y when $x = 7$ is:

A $\frac{1}{4}$ **B** 4 **C** 9 **D** 19 **E** 49

27 If $x \propto \frac{1}{y}$ and y is multiplied by 5, then x will be:

A decreased by 5 **B** increased by 5 **C** multiplied by 5
D divided by 5 **E** none of these

28 The area of a triangle varies directly as the base length, provided the height of the triangle is constant. If the area equals 14 when the base length is 2.4, then the base length (correct to three decimal places) when the area is 18 will equal:

A 3.086 **B** 5.000 **C** 6.400 **D** 9.600 **E** 0.324

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29 (VCAA-type question) The area of ground that can be covered by a bag of mulch is inversely proportional to the depth at which the mulch is spread. If a bag of mulch covers 1.25 m^2 at a depth of 2 cm, then approximately how much area could be covered at a depth of 3 cm?

A 0.42 m^2 **B** 0.63 m^2 **C** 0.83 m^2 **D** 1.88 m^2 **E** 3.75 m^2

30 (VCAA-type question) The given table of values follows the rule: $y = kx^2 + c$.

x	1	2	3	4
y	4.5	9	16.5	27

The values of k and c respectively are:

A 1 and 5 **B** 1.5 and 3 **C** 2 and 11 **D** 3 and 6 **E** 5 and 1.5

31 The given table of values follows the rule: $y = \frac{k}{x} + c$.

x	1	2	4	5
y	7.5	4.5	3	2.7

The values of k and c respectively are:

A 1 and 6.5 **B** 2 and 3 **C** 2 and 5 **D** 6 and 1.5 **E** 8 and 2.5

32 (VCAA-type question) The following data can be modelled by $y = k\log_{10}(x) + c$.

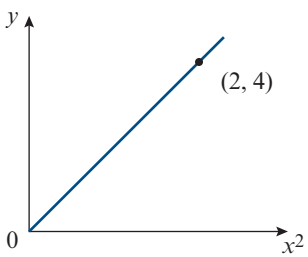
x	1	10	100	1000
y	5	105	205	305

The values of k and c respectively are:

A 1 and 5 **B** 1 and 105 **C** 10 and 15 **D** 100 and 5 **E** 300 and 50

33 (VCAA-type question) The rule connecting y and x as shown in the graph is:

A $y = 2x$ **B** $y = 2x^2$ **C** $y = 2\sqrt{x}$
D $y = \frac{1}{2}x$ **E** $y = x^2 + 2$



34 The pressure, P, of a given quantity of gas is inversely proportional to its volume, V, as long as the temperature remains constant. If the pressure is 80 when the volume is 60, the value of P when $V = 80$ is:

A 60 **B** 4800 **C** 20 **D** 100 **E** $106\frac{2}{3}$

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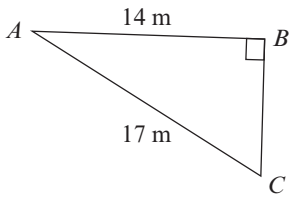
Chapter 10: Measurement, scale and similarity

35 (VCAA-type question) When rounded to 3 significant figures, 3978.6249 is:

A 3.97×10^2 **B** 3.98×10^2 **C** 3978.624
D 3978.625 **E** 3.98×10^3

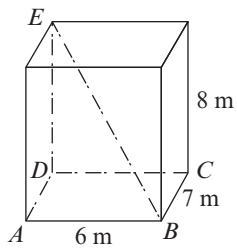
36 (VCAA-type question) A right-angled triangle, ABC, has side lengths $AB = 14$ m and $AC = 17$ m, as shown to the right. The length of BC, in metres, is closest to:

A 3.0 **B** 9.0 **C** 9.6
D 22.0 **E** 31.0



37 The distance, BE, from the corner, B, of the cuboid shown, directly to the far corner, E, is closest to:

A 9 m **B** 11 m **C** 12 m
D 17 m **E** 21 m

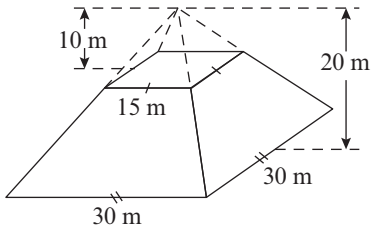


38 (VCAA-type question) The capacity of a petrol can that is 45 cm high with a diameter of 35 cm is closest to:

A 5 litres **B** 43 litres **C** 43 000 cm^3
D 173 litres **E** 173 000 cm^3

39 (VCAA-type question) A pyramid with an original height of 20 m has a small pyramid, 10 m high, removed from its peak, as shown in the diagram . The square base of the original pyramid has a length of 30 m, while the small pyramid has a base with a length of 15 m.
The volume remaining, in m^3, is closest to:

A 2250 **B** 5250 **C** 6000
D 7875 **E** 15 750



40 The volume of a hemisphere with a diameter of 50 cm is closest to:

A 7854 cm^3 **B** 31 416 cm^3 **C** 32 725 cm^3 **D** 65 450 cm^3 **E** 261 799 cm^3

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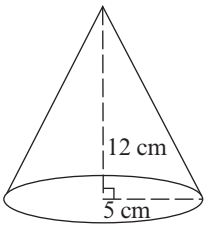
41 (VCAA-type question)

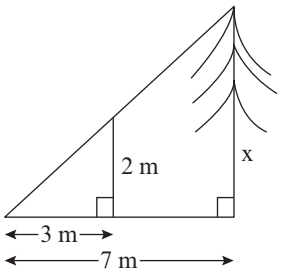
A cone with a base radius of 5 cm and a height of 12 cm is shown in the diagram to the right.
Its surface area, in square centimetres, is closest to:



A 267.0 **B** 282.7 **C** 314.2
D 340.3 **E** 534.1

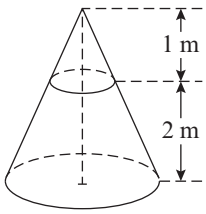
42 (VCAA-type question)

The tree in the diagram to the right cast a shadow that was 7 metres long. A person 2 m tall cast a shadow 3 m long.
The height, x m, of the tree was closest to:



A 4.7 **B** 6.0 **C** 10.5 **D** 14.0 **E** 21.0

43 A cone that was one third the height of a large cone was sliced off the top of the large cone. What fraction of the large cone's volume remained?

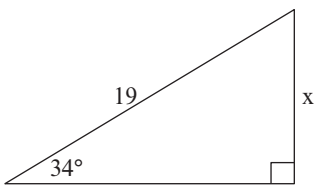


A $\frac{8}{27}$ **B** $\frac{2}{3}$ **C** $\frac{19}{27}$
D $\frac{8}{9}$ **E** $\frac{26}{27}$

Chapter 11: Applications of trigonometry

Choose an answer, correct to one decimal place, unless otherwise asked.

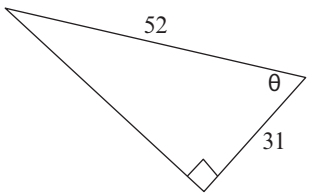
44 The length of the unknown side, x, is:



A 10.6 **B** 12.8 **C** 15.8
D 19.6 **E** 34.0

45 The angle, θ, is:

A 30.8° **B** 36.6° **C** 54.3° **D** 53.4° **E** 59.2°



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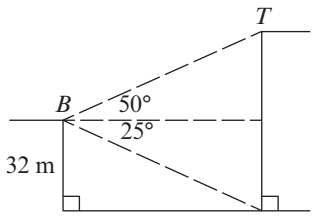
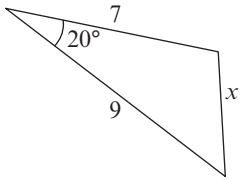
46 From the top of a building, B, that is 32 m tall, the angle of elevation of the top of a nearby building, T, was 50°. The angle of depression of the base of the building was 25°. The height of the nearby building, to the nearest metre, was:

A 77 m **B** 80 m **C** 82 m **D** 101 m **E** 114 m



47 In the triangle shown, the value of x is:

A 3.4 **B** 5.6 **C** 5.7 **D** 11.0 **E** 15.8

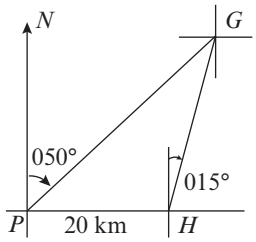


48 (VCAA-type question)

From a port, P, a ship, G, was sighted on a bearing of 050°. A ship, H, 20 km east of port P, reported ship G on a bearing of 015°.

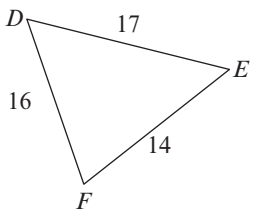
The distance, to the nearest kilometre, between the two ships was:

A 13 km **B** 17 km **C** 18 km **D** 22 km **E** 24 km



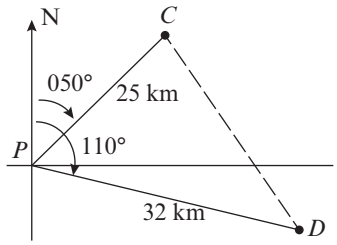
49 (VCAA-type question) In the triangle shown, angle DEF is:

A 34.6° **B** 48.8° **C** 61.0° **D** 61.2° **E** 72.0°



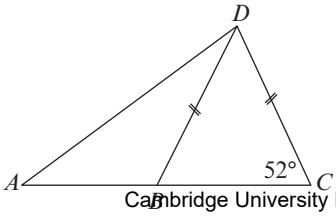
50 (VCAA-type question) Ship C travelled 25 km on a bearing of 050° from port P. Meanwhile, ship D sailed 32 km from port P on a bearing of 110°. The distance, to the nearest kilometre, between the two ships was:

A 20 km **B** 27 km **C** 29 km **D** 31 km **E** 49 km



51 (VCAA-type question) In the given triangle, the angle ABD is:

A 38° **B** 52° **C** 76° **D** 128° **E** 138°



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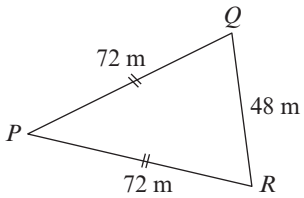
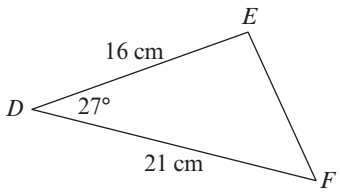
52 (VCAA-type question) The area of the triangle shown is closest to:

A 432 m^2 **B** 748 m^2 **C** 864 m^2
D 1629 m^2 **E** 1728 m^2



53 (VCAA-type question) In triangle DEF, side DE is 16 cm long and side DF is 21 cm. The angle, EDF, is 27°. The area of the triangle is closest to:

A 76 cm^2 **B** 150 cm^2 **C** 153 cm^2
D 168 cm^2 **E** 336 cm^2



12B Written-response questions

Chapter 7: Investigating relationships between two numerical variables

1 The table shows the number of times a student revised in the two-week period before the mathematics exam (*revised*) and their exam score (*exam score*) for a group of Year 11 students.

revised	10	9	9	8	7	7	6	5	4	3	0
exam score	85	85	83	84	82	80	75	60	72	64	60

a Which is the explanatory variable and which is the response variable?
b Construct a scatterplot of this data.
c Determine the value of the Pearson correlation coefficient, r, for this data.
d Describe the relationship between *exam score* and *revised* in terms of direction, form and strength.
e Determine the equation for the least squares regression line, and write it down in terms of the variables *exam score* and *revised*.
f Interpret the slope and intercept of the regression line in the context of the problem.
g Use your equation to predict the exam score for a student who revised 12 times in the two weeks before the exam.
h In making this prediction, are you interpolating or extrapolating?

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2 A teacher is concerned that students who spend a lot of time playing video games do not spend enough time reading. The following table shows the data she collected from a group of 10 of her students, who recorded the number of hours they spent reading (*reading*) and the number of hours they spent playing computer games (*games*) in one week.

reading	10	4	7	8	6	3	4	1	10	8
games	7	15	13	15	8	20	10	21	0	2

a Construct a scatterplot of these data, with *games* as the explanatory variable and *reading* as the response variable.

b Determine the correlation coefficient, r, and give your answer to four decimal places.

c Describe the association between *reading* and *games* in terms of direction, form and strength.

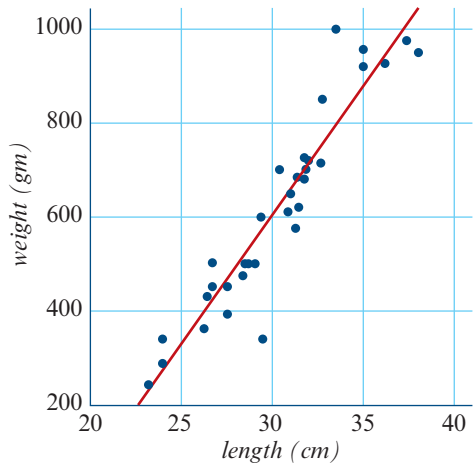
d Determine the equation for the least squares regression line and write it down in terms of the variables *reading* and *games*. Give coefficients to three significant figures.

e Interpret the slope and the intercept (if appropriate) of the regression line.

f Predict the number of hours a student who plays 10 hours of computer games would spend reading.

g How reliable is the prediction made in part **f**.

3 To investigate the association between the weight of a certain species of fish, in grams, (*weight*) and its length, in cm, (*length*), data was collected from a sample of 35 fish. The following scatterplot shows this data with the least squares line added.



a Which is the explanatory variable and which is the response variable?

b The value of the correlation coefficient, r, is 0.937. Describe the relationship between *weight* and *length* in terms of direction, form and strength.

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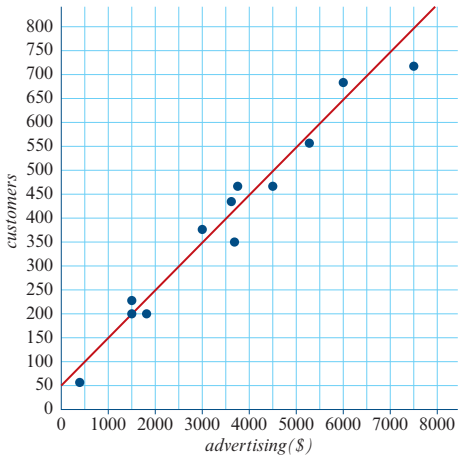
c It was determined from the data that the mean length of the sample of fish is 30.306 cm with a standard deviation of 3.594 cm, and the mean weight of the sample of fish is 617.829 grams with a standard deviation of 209.206 grams. Use this information to determine the equation of the least squares regression line, and write it down in terms of the variables *weight* and *length*. Write the values in the equation to two decimal places.

d Interpret the intercept and slope of the least squares regression line in terms of the variables in the study.

e Use your regression equation to predict the weight of a fish which is 50 cm long. Give your answer to the nearest gram.

f How reliable is the prediction made in part **e**?

4 Data was collected to investigate the association between the number of customers served at a coffee shop each week (*customers*) and the amount of money the shop owner spent in advertising (*advertising*), in dollars, over the previous week. The following scatterplot shows this data, with a line fitted by eye added.



a Which is the explanatory variable and which is the response variable?

b Describe the relationship between *customers* and *advertising* in terms of direction, form and strength.

c Use the scatterplot to determine the equation of the line shown, and write it down in terms of the variables *customers* and *advertising*. Write the values in the equation to two decimal places.

d How many customers would the coffee shop expect if they spend nothing on advertising the previous week?

e How much on average does the coffee shop need to spend on advertising to attract one additional customer?

f The coffee shop owner finds a new advertising option which promised to cost only $2 to attract each additional customer. Write down the equation which would summarise this relationship between *customers* and *advertising*.

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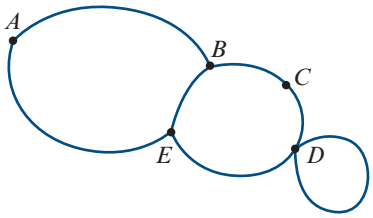
Chapter 8: Graphs and networks

5 The tracks in a park are shown in the diagram opposite.



a What is the degree of vertex D?

b What is the mathematical name for a walk that follows every edge without repeating an edge and returns to the starting point?

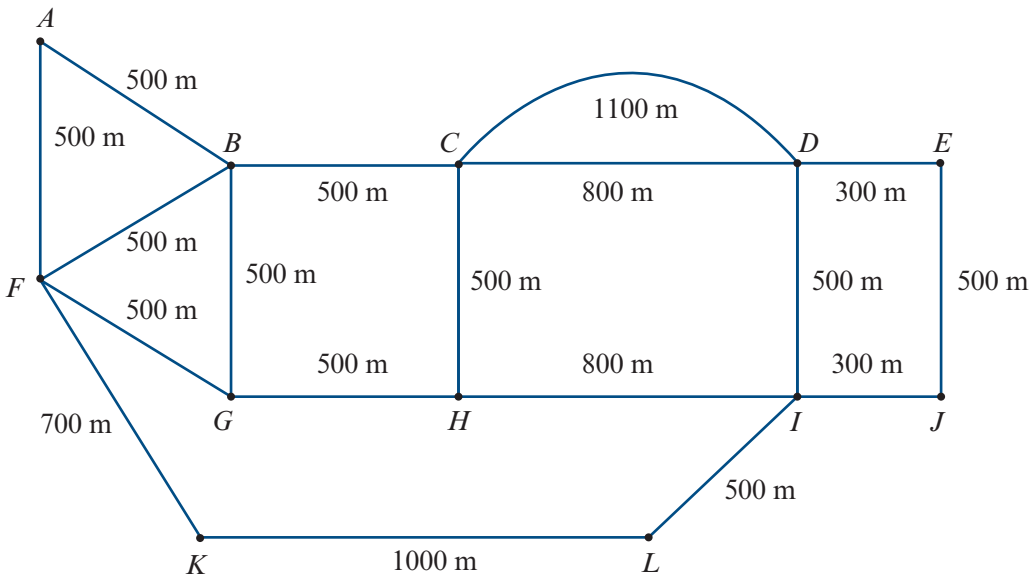
c How do you know the walk described in part **b** is not possible in the park?

d Which section of the track could be repeated to make it possible to return to the starting point and only walking on each of the other tracks once?

e An incomplete adjacency matrix for the network is shown opposite. Copy the adjacency matrix and enter the five missing elements in the spaces provided.

	A	B	C	D	E
A			0	0	1
B		0	1	0	1
C	0	1	0	1	0
D	0	0	1		1
E	1	1	0		0

6 The network below shows the road layout in a new housing estate. The vertices represent the road intersections and the weighted edges represent roads and their lengths.



a Cars can enter the estate through gates located at either A or J. What is the shortest distance a car can travel if it enters the estate at gate A and leaves at gate J?

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b The post office is located at intersection G. The postie must travel along each road in the estate at least once.

i Can the postie start and finish their delivery round (which includes every road in the housing estate) at intersection G, without travelling along the same road more than once? If not, why not?

ii Can the postie start their delivery round at intersection G and finish at some other intersection in this network of roads, without travelling along the same road more than once? If so, where would they end the delivery round?

iii If the postie must start and finish their delivery round at the post office at G, what is the shortest distance they will cover and still travel along each road at least once?

c Broadband is to be provided to the estate by connecting cables from the 'exchange,' located at intersection L, to distribution nodes, located at each of the intersections in the estate. What is the shortest length of cable needed to complete this task?

Chapter 9: Variation

7 A stone falls from rest down a mine shaft. It falls d metres in t seconds, where d varies directly as the square of t. It falls 20 metres in the first 2 seconds.

a Find the rule for d in terms of t.

b If it takes 5 seconds to reach the bottom of the shaft:

i Find the depth of the shaft.

ii Find the time taken to reach halfway down the shaft.

c Sketch the graph of d against t, for t between 0 and 5 inclusive.

8 The effectiveness of a pain-killing drug is being tested by varying the dosage given (d mL) and recording both the time (t min) for the drug to take effect and the time (T min) before the effect of the drug wears off. The following data was recorded.

Dosage (d mL)	10	20	40
Time to take effect (t min)	60	30	15
Time to wear off (T min)	120	480	1920

a Establish the relationship between d and t. (Assume that t is inversely proportional to d.)

b Establish the relationship between d and T. (Assume that T is directly proportional to d^2.)

c A patient is given a dose of 30 mL. How long will it take for the drug to take effect, and how long before it wears off?

d If a second patient takes an amount of the drug which takes effect in 40 minutes, calculate the dosage taken.

e How long would it take before the effect of the dosage in part **d** wears off?

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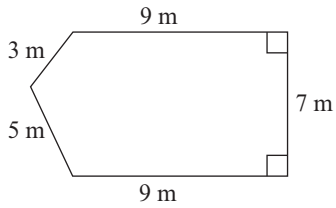
Chapter 10: Measurement, scale and similarity

Answer to one decimal place unless asked otherwise.

9 The diagram shows a plan for a children's sandpit.



a Find the total area of the sandpit.

b The area is to be filled with sand to a depth of 20 cm. Find the volume of sand needed.

c Calculate the cost of the sand if sand costs $60 per cubic metre.

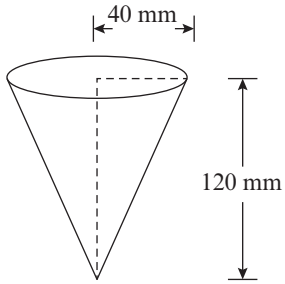
d Find the cost of timber for the border if it costs $9 per metre.

10 A landscape gardener has a 12 m and a 30 m length of flexible garden border edging. She intends to use the 12 m length for the inner circle of a garden bed border and the 30 m length for the outer circular border.

a What would be the radius of each circle?

b Find the area of the garden bed enclosed by the two circular borders.

11 An inverted cone is used to collect rainwater. The cone is 120 mm high with a base radius of 40 mm.



a If 12 mm of rain fell overnight, what volume of water would have been collected in the rain gauge? [Hint: The volume of water collected depends on the area of the mouth of the cone × rainfall.]

b The gauge was emptied and after the next day's rainfall it was found to be filled to half the depth of the cone. Calculate the volume of water in the rain gauge.

c What was the overnight rainfall?

12 A proposal for a rectangular park measures 8 cm by 10 cm on a map with a scale of 1:5000.

a Find the actual area proposed for the park in hectares. One hectare equals 10 000 m^2.

b What diameter should be used on the map to draw a circular pond with a diameter of 50 metres?

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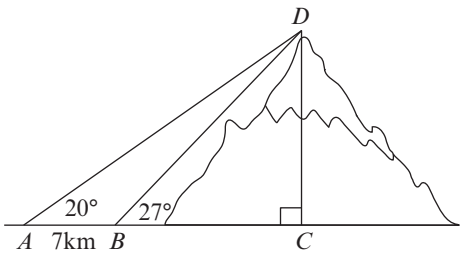
Chapter 11: Applications of trigonometry

Answer to one decimal place unless asked otherwise.

13 Suppose in your attempt to measure the height of Mount Everest from points A and B which are 7 km apart, you measured the angle of elevation of the summit to be 20° and 27°, respectively.



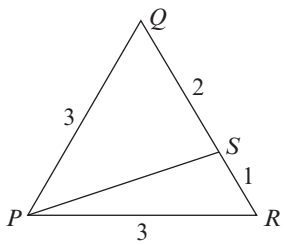
a Find the angle ABD to the nearest degree.

b Determine the angle ADB to the nearest degree.

c Calculate the distance AD, in metres, to two decimal places.

d Hence, determine the height CD to the nearest metre.

14 In equilateral triangle PQR, a line is drawn from P to S as shown.

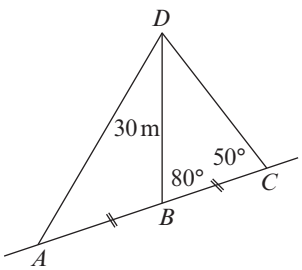


a Find the length PS.

b Find the angle QPS.

c Show how two different rules can be used to find the area of triangle PQS.

15 A vertical transmission tower, BD, is 30 m high and makes an angle of 80° with the sloping hillside. A cable, DC, makes an angle of 50° with the hillside.



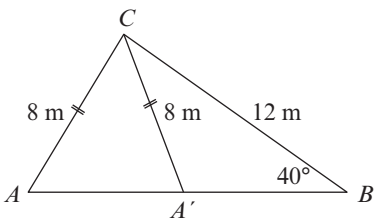
a Find the length of the cable, DC.

b What is the distance, BC, from the base of the tower to the point C where the cable is anchored to the ground?

c Find the length of cable AD, required to secure the cable at point A, where $AB = BC$.

d What acute angle will the cable, AD, make with the hillside?

16 In triangle ABC, we are given $B = 40°$, $b = 8$ m and $a = 12$ m. It is possible to draw two triangles, ABC and $A'BC$, using the given information.



a In triangle ABC, find angle CAB.

b Find angle $CA'B$.

c Give the values of the three angles in triangle ABC.

d State the values of the angles in triangle $A'BC$.

e Find the length of AB in triangle ABC.

f Find the length of $A'B$ in triangle $A'BC$.

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12C Investigations

1 **Cricket Captains** The table gives details of Australian Test Cricket Captains, based on all tests from 1930 until 2021.

Name	Years	Played	Won	Name	Years	Played	Won
W M Woodfull	1930–34	25	14	W M Lawry	1967–71	25	9
V Y Richardson	1935–36	5	4	B N Jarman	1968	1	0
D G Bradman	1936–48	24	15	I M Chappell	1970–75	30	15
W A Brown	1945–46	1	1	G S Chappell	1975–83	48	21
A L Hassett	1949–53	24	14	G N Yallop	1978–79	7	1
A R Morris	1951–55	2	0	K J Hughes	1978–85	28	4
I W Johnson	1954–57	17	7	A R Border	1984–94	93	32
R R Lindwall	1956–57	1	0	M A Taylor	1994–1999	50	26
I D Craig	1957–58	5	3	S R Waugh	1999–2004	57	41
R Benaud	1958–64	28	12	R Ponting	2004–2010	77	48
R N Harvey	1961	1	1	M Clarke	2010–2014	47	24
R B Simpson	1963–78	39	12	S Smith	2014–2018	34	18
B C Booth	1965–66	2	0	T Paine	2018–2021	23	11

a **i** Construct a stem plot and a boxplot for the number of tests played by the captains.

ii Describe the distribution of the number of tests captained in terms of shape, centre, spread and outliers, quoting the values of appropriate statistics.

iii Who is Australia's longest serving captain on the basis of this data?

b **i** Construct a scatterplot of matches won (*won*) against the matches played (*played*) by each captain. *Played* is the EV. Estimate the value of the correlation coefficient, r.

ii Describe the association between matches played and matches won in terms of strength, direction, form and outliers if any.

iii Determine the correlation coefficient, r, and compare it to your earlier estimate.

iv Fit a least squares regression line to the data and write its equation in terms of the variables *played* and *won*.

v Write down the slope of the least squares regression line and interpret it in terms of the variables *played* and *won*.

vi Based on your analyses, who would you suggest was Australia's most successful cricket captain?

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2 **Global warming** The table gives details of the average maximum yearly temperature (C°) (*maximum*) and the average minimum yearly temperature (C°) (*minimum*) in Melbourne for the years 1995 to 2020.

Year	Av Max Temp	Av Min Temp	Year	Av Max Temp	Av Min Temp
1995	18.5	9.4	2008	20.2	9.6
1996	18.6	9.0	2009	20.9	10.0
1997	20.1	9.6	2010	20.0	10.0
1998	19.7	9.2	2011	19.9	9.8
1999	20.3	9.9	2012	20.2	9.7
2000	20.2	10.0	2013	20.9	10.2
2001	20.1	9.9	2014	21.2	10.4
2002	20.4	9.5	2015	20.9	9.7
2003	20.1	9.7	2016	20.5	10.4
2004	19.6	9.6	2017	21.1	9.8
2005	20.6	9.8	2018	21.1	9.8
2006	20.2	9.2	2019	21.3	9.9
2007	21.3	10.4	2020	19.8	9.5

a **i** Construct parallel boxplots of average minimum and average maximum temperatures.

ii Use the boxplots to write a report comparing distributions in terms of shape, centre and spread.

b **i** Construct a scatterplot using *maximum* as the RV and *minimum* as the EV.

ii Describe the association between *maximum* and *minimum* in terms of direction, form and strength.

iii Determine the value of the correlation coefficent, r, between *maximum* and *minimum* temperatures.

iv Fit a least squares regression line which could be used to predict *maximum* from *minimum*.

v Interpret the intercept and slope of this regression line in terms of *maximum* and *minimum* temperatures.

c **i** Construct a scatterplot of *maximum* against *year*.

ii Describe the association between *maximum* and *year* in terms of direction, form and strength.

iii Fit a least squares regression line which could be used to predict *maximum* temperature from the *year*.

iv Interpret the intercept and slope of this regression line in terms of *maximum* temperature and *year*.

v Use the regression line to predict the average yearly max temp in 2025.

vi Are you interpolating or extrapolating when making this prediction?

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d **i** Construct a scatterplot of *minimum* against *year*.

ii Describe the association between *minimum* and *year* in terms of direction, form and strength.

iii Fit a least squares regression line which could be used to predict *minimum* temperature from the *year*.

iv Interpret the intercept and slope of this regression line in terms of *minimum* temperature and *year*.

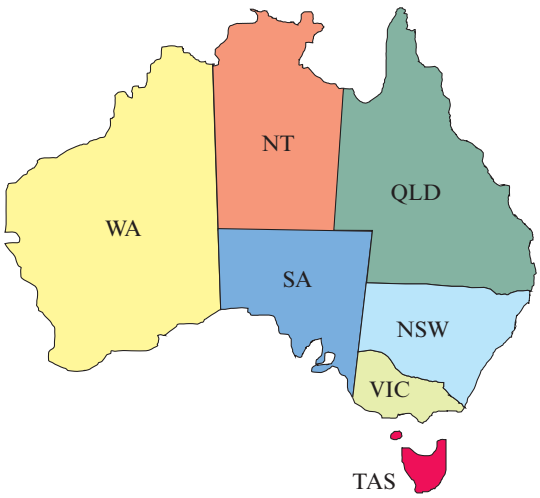
v Use the regression line to predict the average yearly minimum temperature in 2025.

vi Are you interpolating or extrapolating when making this prediction?

e Use the results of parts **c** and **d** to write a paragraph describing how the temperature in Melbourne has changed since 1995. Include a suggestion of other variables which could be included to extend the study.

3 Road Trip

Nick and Maria live in Melbourne and will soon have family visiting them from Greece. They would like to plan a trip around Australia, focusing on the major cities in each of the states and territories.



a On a previous trip, Nick and Maria travelled from Melbourne to Sydney to Brisbane, back to Sydney, across to Canberra and finally returning to Melbourne.

i Using vertices to represent each capital city and an edge for each direct route between a capital city, construct a network to show Nick and Maria's complete route.

ii Show that Euler's formula is verified for this network.

iii What is the sum of degrees of this network?

iv Are there any bridges? If yes, indicate the vertices they exist between. If no, give a reason for why there are none.

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v What type of walk can Nick and Maria's previous trip be described as? Justify your reasoning with reference to the vertices and edges of your network.

vi How many different spanning trees are possible for your network?

The following table includes the distances travelled and flight times for their previous trip.

From	To	Distance (km)	Time
Brisbane	Sydney	750	1 hour 30 minutes
Canberra	Melbourne	469	1 hour 5 minutes
Canberra	Sydney	236	45 minutes
Melbourne	Sydney	705	1 hour 20 minutes

vii Calculate the total distance travelled by Nick and Maria on their previous trip.

viii Calculate the total time Nick and Maria spent flying.

The following adjacency matrix summarises all domestic flights between the capital cities of Australia. A '1' is used to represent a direct flight between the cities and '0' to represent no direct flight. The first letter of each capital city is used to represent each city, e.g. M = Melbourne, A = Adelaide, B = Brisbane etc.

$$\begin{array}{c} \begin{matrix} A & B & C & D & H & M & P & S \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ H \\ M \\ P \\ S \end{matrix} \end{array}$$

b i Plan three potential journeys Nick and Maria could take their relatives on if they intend to visit all capital cities of Australia, starting and ending their route in Melbourne.

ii Create a network that illustrates all three potential routes.

iii Is your network planar? Justify your reasoning with a calculation.

iv Draw three different spanning trees for this network.

v If they could start their journey in any city, which ones could they start in if they must end in the same city they start? Write the sequence of some potential journeys.

Because Nick and Maria can work remotely, they do not need to end their journey in their home city, Melbourne. They do not intend on travelling to every city anymore and plan to end their journey in Perth.

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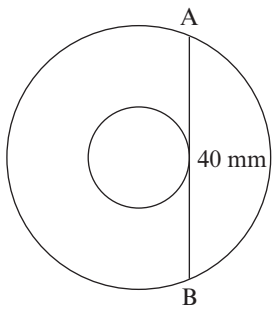
c **i** If this new journey must include three other cities between Melbourne and Perth, suggest three new routes they could take, and create a network that includes all three routes between Melbourne and Perth. Use the matrix above to help identify flight paths between the capital cities.

ii Research the distance or time of flights between cities and calculate the shortest or cheapest journey they could take.

4 Gold Medal You have won a gold medal in the form of an annulus. You can design your medal with any sizes for the inner and outer circle. The only condition is that the chord, AB, that is a tangent to the inner circle, must be exactly 40 mm long.

a Design your own gold annulus. Provide a clearly labelled diagram. Describe the steps you used when designing an annulus that satisfied the required condition, and show all of the calculations involved.



b State the inner radius, r, and the outer radius, R, of the annulus you have designed, showing the calculations used.

c Calculate the area of the gold annulus you have designed.

d Design another annulus that meets the requirement that the chord which is a tangent to the inner circle is 40 mm long, but with different values for the inner and outer radii. Provide a clearly labelled diagram and working.

e Calculate the area of the gold annulus you designed in part **d**.

f Comment on the results of your investigation so far. What conclusion are you inclined to make? Provide further evidence to support or confirm your conclusion.

g What would be the area of an annulus if the chord was c mm long? Does this result confirm the results of your earlier calculations?

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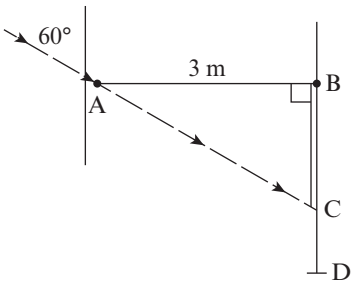
5 **Awning angles** A three-metre awning, AB, is intended to provide some shade, BC, for the clothes in a display window, BD.
Give answers to one decimal place.

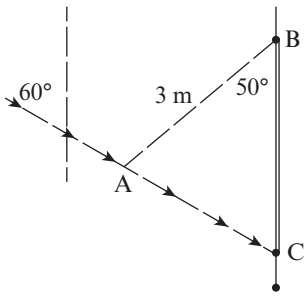
a At the time of interest, the morning sun makes an angle of 60° from the vertical.



i Find the length of the shade, BC.

The awning is hinged at point B.

ii What is the length of shade, BC, when the awning is adjusted so the angle, ABC, is 50°?



b Assume the angle the sun makes with the vertical is 60°. For simplicity, use θ for the angle ABC. Your goal is to find the value of θ that would give the maximum length of shade.
Several methods are possible. Support your answer with clear and logical working.

c What is the rule connecting the angle of the sun with the value of θ for maximum shade?

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