Chapter 8

Reducing balance loans, annuities and investments

Chapter objectives

- How can we combine both linear and geometric growth/decay?
- How do we model a compound interest investment where additional payments are made?
- How can recurrence relations be used to model reducing balance loans?
- How can recurrence relations be used to model annuities?
- What are amortisation tables and how can they be used?
- How can a finance solver be used to analyse reducing balance loans, annuities and investments with additional payments?
- What are interest-only loans?
- What are perpetuities?

Often loans and investments are more complex than described in the previous chapter. In particular, loans are often paid off through regular payments, investments may have additional contributions made throughout their life and interest rates may change. In this chapter, we analyse investments with additional payments, reducing balance loans, annuities, interest-only loans and perpetuities using our already developed tool of recurrence relations as well as new tools such as amortisation tables and the Finance Solver found on a CAS.

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8A Combining linear and geometric growth or decay to model compound interest investments with additions to the principal

Learning intentions

- To be able to generate a sequence from a recurrence relation that combines both geometric and linear growth or decay.
- To be able to model compound interest investments with additions to the principal.
- To be able to use a recurrence relation to analyse compound interest investments with additions to the principal.
- To be able to determine the annual interest rate from a recurrence relation.

In the previous chapter, recurrence relations were used to model financial situations with linear and geometric growth/decay such as simple and compound interest and the depreciation of assets. Recurrence relations can also be used to model situations that involve elements of both linear and geometric growth/decay.

There are several examples in finance that involve both geometric and linear growth or decay. For example, an investment with compound interest grows geometrically over time but might also have linear growth if regular additions are made. Alternatively, a personal loan may be paid off with regular payments rather than at the conclusion of the loan period.

In general, a recurrence relation of the form

$$V_0 = \text{starting value}, \qquad V_{n+1} = R \times V_n \pm D$$

can be used to model situations that involve both geometric and linear growth/decay.

Example 1 Generating a sequence from a recurrence relation of the form $V_{n+1} = R \times V_n \pm D$

Write down the first five terms of the sequence generated by the recurrence relation

$$V_0 = 3, \qquad V_{n+1} = 4V_n - 1$$

Explanation	Solution
1 Write down the starting value.	3
2 Apply the rule (multiply by 4, then subtract 1) to generate four more terms.	$3 \times 4 - 1 = 11$ $11 \times 4 - 1 = 43$ $43 \times 4 - 1 = 171$ $171 \times 4 - 1 = 683$
3 Write your answer.	The first five terms are 3, 11, 43, 171, 683

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Compound interest investments with regular additions to the principal

In the previous chapter, we considered compound interest investments. Here, we consider the option of adding to the investment by making additional payments on a regular basis. This is called an **annuity investment**.

Modelling compound interest investments with regular additions to the principal

Let V_n be the value of the compound interest investment (annuity investment) after n additional payments have been made. Then

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n + D$$

where D is the *additional payment* made, $R = 1 + \dfrac{r}{100 \times p}$ is the growth multiplier, r is the *annual interest rate* and p is the *number of compounding periods* per year.

To start, we recall compound interest investments without additional payments as in Chapter 7. Imagine Fred has \$5000 to invest in an account paying compound interest of 4% per annum, compounding annually. The starting value is $V_0 = 5000$. To find R, we use the formula $R = 1 + \dfrac{r}{100 \times p}$ where $r = 4$ and $p = 1$ to give $R = 1.04$. Thus, the recurrence relation is

$$V_0 = 5000, \qquad V_{n+1} = 1.04V_n$$

Now we can consider the possibility of regular additions to the principal.

Example 2 Modelling compound interest investments with additions to the principal (1)

Fred has saved \$5000 and invests this in a compound interest account paying 4% per annum, compounding yearly. He also adds an extra \$1000 each year.

Model this investment using a recurrence relation of the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n + D$$

where V_n is the value of the investment after n years.

Explanation	Solution
1 Write down the value of V_0 and D where D is the amount added each year.	$V_0 = 5000$ and $D = 1000$
2 Determine the value of R using $R = 1 + \dfrac{r}{100 \times p}$ where $r = 4$ and $p = 1$ because interest compounds annually.	$R = 1 + \dfrac{4}{100 \times 1} = 1.04$
3 Use the values of V_0, R and D to write down the recurrence relation.	$V_0 = 5000, \quad V_{n+1} = 1.04V_n + 1000$

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When interest compounds at intervals other than a year, we need to find the interest rate for the compounding period which is calculated based on the following:

- 12 equal months in every year (even though some months have different numbers of days)
- 4 quarters in every year (a quarter is equal to 3 months)
- 26 fortnights in a year (even though there are slightly more than this)
- 52 weeks in a year (even though there are slightly more than this)
- 365 days in a year (ignore the existence of leap years).

This gives us the value of p which we use in the formula

$$R = 1 + \frac{r}{100 \times p}$$

to find the growth multiplier, R.

Example 3 Modelling compound interest investments with additions to the principal (2)

Nor invests \$1200 and plans to add an extra \$50 each month. The account pays interest at a rate of 3% per annum, compounding monthly.

Model this investment using a recurrence relation of the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n + D$$

where V_n is the value of the investment after n months.

Explanation	Solution
1 Write down the value of V_0 and D where D is the amount added each month.	$V_0 = 1200$ and $D = 50$
2 Determine the value of R using the formula $R = 1 + \frac{r}{100 \times p}$ where $r = 3$ and $p = 12$ because interest compounds monthly.	$R = 1 + \frac{3}{100 \times 12} = 1.0025$
3 Use the values of V_0, R and D to write down the recurrence relation.	$V_0 = 1200, \quad V_{n+1} = 1.0025V_n + 50$

Once we have a recurrence relation, we can use it to determine the value of the investment after a given number of periods once interest has been paid and extra payments have been added to the principal. This value can be plotted on a graph so that we can see the impact of making additional payments over time.

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Example 4 Using a recurrence relation to analyse compound interest investments with additions to the principal

Albert has an investment that can be modelled by the recurrence relation

$$V_0 = 400, \qquad V_{n+1} = 1.005V_n + 30$$

where V_n is the value of the investment after n months.

a State the value of the initial investment.

b Determine the value of the investment after Albert has made three extra payments. Round your answer to the nearest cent.

c What will be the value of his investment after 6 months? Round your answer to the nearest cent.

d Plot the points for the value of the investment after 0, 1, 2 and 3 months on a graph.

Explanation

a Note that $V_0 = 400$.

b Perform the calculations.

c **i** Either continue performing the calculations or use your CAS by:

ii Type 400 and press enter (or EXE).

iii Type × 1.005 + 30 and press enter (or EXE) six more times.

ii Write your answer.

d Plot each of the points on the graph.

Solution

a The initial investment was $400.

b
$V_0 = \$400$
$V_1 = 1.005 \times 400 + 30 = \$432$
$V_2 = 1.005 \times 432 + 30 = \$464.16$
$V_3 = 1.005 \times 464.16 + 30 = \$496.48$

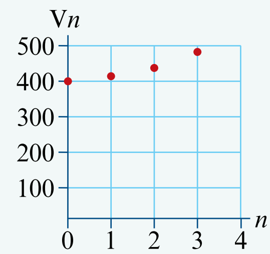
The value of Albert's investment is $496.48.

c

400	400
400 · 1.005 + 30	432
432 · 1.005 + 30	464.16
464.16 · 1.005 + 30	496.48
496.4808 · 1.005 + 30	528.96
528.9632 · 1.005 + 30	561.61
561.6180 · 1.005 + 30	594.42

The value of Albert's investment is $594.42.

d



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From the example above, it is clear that making additional payments on a regular basis causes the investment to increase more. In particular, making additional payments early on is beneficial as compound interest is earned on the additional payment for longer. An example of this type of investment might be saving for retirement.

Sometimes we are given a recurrence relation and asked to determine the annual interest rate. To do this, we use the formula $R = 1 + \frac{r}{100 \times p}$.

Example 5 Determining the annual interest rate from a recurrence relation

Determine the annual interest rates for each of the following investments.

a Consider an investment given by the recurrence relation

$$A_0 = 400, \qquad A_{n+1} = 1.005V_n + 30$$

where A_n is the value of the investment after n months.

b Consider an investment given by the recurrence relation

$$W_0 = 2000, \qquad W_{n+1} = 1.012V_n + 500$$

where W_n is the value of the investment after n quarters.

Explanation	Solution
a Solve $R = 1 + \frac{r}{100 \times p}$ for r where $R = 1.005$ and $p = 12$ because interest is compounded monthly.	Solve $1.005 = 1 + \frac{r}{100 \times 12}$. $r = 6$ Thus, the annual interest rate is 6%.
b Solve $R = 1 + \frac{r}{100 \times p}$ for r where $R = 1.012$ and $p = 4$ because interest is compounded quarterly.	Solve $1.012 = 1 + \frac{r}{100 \times 4}$. $r = 4.8$ Thus, the annual interest rate is 4.8%.

Exercise 8A

Generating a sequence using a recurrence relation

Example 1

1 Write down the first five terms of the sequences generated by the following recurrence relations.

a $A_0 = 2, \qquad A_{n+1} = 2A_n + 1$

b $B_0 = 50, \qquad B_{n+1} = 2B_n - 10$

c $C_0 = 128, \qquad C_{n+1} = 0.5C_n + 32$

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Modelling compound interest investments with additions to the principal

Example 2

2 Molly has saved \$500 and plans to add an extra \$100 per year to an investment account immediately after the interest payment is calculated. The account pays 3% per annum, compounding annually.
Let V_n is the value of the investment after n years.

- **a** State the initial value of the investment, V_0.
- **b** State the amount added each year, D.
- **c** Determine the value of the growth multiplier, R.
- **d** Model this investment using a recurrence relation of the form

$$V_0 = \text{ the principal}, \quad V_{n+1} = RV_n + D$$

3 Jane has already saved \$300 000 and plans to add an extra \$50 000 per year to an investment account immediately after the interest payment is calculated. The account pays interest of 5.2% per annum, compounding annually.
Let V_n is the value of the investment after n years.

- **a** State the initial value of the investment, V_0.
- **b** State the amount added each year, D.
- **c** Determine the value of the growth multiplier, R.
- **d** Model this investment using a recurrence relation of the form

$$V_0 = \text{ the principal}, \quad V_{n+1} = RV_n + D$$

Example 3

4 Henry invests \$3500 and plans to add an extra \$150 per month after the interest is calculated. The account pays interest of 3.6% per annum, compounding monthly.
Let V_n be the value of the investment after n months.

- **a** Determine the value of the growth multiplier, R.
- **b** Model this investment using a recurrence relation of the form

$$V_0 = \text{ the principal}, \quad V_{n+1} = RV_n + D$$

- **c** What is the value of the investment after two months? Round your answer to the nearest cent.

5 Lois invests \$1700 and plans to add an extra \$100 per quarter. The account pays interest of 3.2% per annum, compounding quarterly.
Let V_n be the value of the investment after n quarters.

- **a** Model this investment using a recurrence relation of the form

$$V_0 = \text{ the principal}, \quad V_{n+1} = RV_n + D$$

- **b** What is the value of the investment after six quarters? Round your answer to the nearest cent.

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6 Sarah invests \$1500 at 7.3% per annum, compounding daily. She plans to add an extra \$4 to her investment each day, immediately after the interest is calculated.
Let V_n be the value of the investment after n days.
Write down a recurrence relation to model Sarah's investment.

7 Rachel invests \$24 000 at 6% per annum, compounding monthly. She plans to add an extra \$500 to her investment each month.
Let V_n be the value of the investment after n months.
Write down a recurrence relation to model Rachel's investment and determine the value of the investment after six months. Round your answer to the nearest cent.

Using a recurrence relation to model and analyse an investment with additions to the principal

Example 4 **8** A compound interest investment with regular yearly additions to the principal can be modelled by the recurrence relation

$$V_0 = 2000, \qquad V_{n+1} = 1.08V_n + 1000$$

where V_n is the value of the investment after n years.

a What is the principal of this investment?
b How much is added to the principal each year?
c Use your calculator to determine the balance of the investment after 2 years. Round your answer to the nearest cent.
d Plot the value of the investment after 0, 1 and 2 years on a graph.

9 A compound interest investment with regular quarterly additions to the principal can be modelled by the recurrence relation

$$V_0 = 20\,000, \qquad V_{n+1} = 1.025V_n + 2000$$

where V_n is the value of the investment after n quarters.

a What is the principal of this investment?
b How much is added to the principal each quarter?
c Use your calculator to determine the balance of the investment after three quarterly payments have been made. Round your answer to the nearest cent.
d Plot the value of the investment after 0, 1, 2 and 3 quarters on a graph.

Example 5 **10** Consider the compound interest investment with regular annual additions to the principal given by the recurrence relation

$$V_0 = 2000, \qquad V_{n+1} = 1.08V_n + 1000$$

where V_n is the value of the investment after n years.
Determine the annual interest rate for the investment.

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11 Consider the compound interest investment with regular quarterly additions to the principal given by the recurrence relation:

$$V_0 = 20\,000, \qquad V_{n+1} = 1.025V_n + 2000$$

where V_n is the value of the investment after n quarters.
Determine the annual interest rate for the investment.

Exam 1 style questions

12 The value of an annuity investment, in dollars, after n years, V_n, can be modelled by the recurrence relation shown below

$$V_0 = 54\,000, \quad V_{n+1} = 1.0055V_n + 1500$$

What is the value of the regular payment added to the principal of this annuity investment?

A \$55 **B** \$297 **C** \$1500 **D** \$1797 **E** \$5400

13 The value of an annuity investment, in dollars, after n quarters, V_n, can be modelled by the recurrence relation shown below

$$V_0 = 36\,000, \quad V_{n+1} = 1.008V_n + 200$$

The increase in the value of this investment in the third quarter is closest to

A \$200.00
B \$495.84
C \$499.81
D \$1475.74
E \$37 475.74

14 Consider the following five recurrence relations representing the value of an asset after n years, V_n.

$$V_0 = 10\,000, \qquad V_{n+1} = V_n + 1500$$
$$V_0 = 10\,000, \qquad V_{n+1} = V_n - 1500$$
$$V_0 = 10\,000, \qquad V_{n+1} = 1.15V_n - 1500$$
$$V_0 = 10\,000, \qquad V_{n+1} = 1.125V_n - 1500$$
$$V_0 = 10\,000, \qquad V_{n+1} = 1.25V_n - 1500$$

How many of these recurrence relations indicate that the value of an asset is depreciating?

A 0 **B** 1 **C** 2 **D** 3 **E** 4

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8B Using recurrence relations to analyse and model reducing balance loans and annuities

Learning intentions

- To be able to model a reducing balance loan with a recurrence relation.
- To be able to use a recurrence relation to analyse a reducing balance loan.
- To be able to model an annuity with a recurrence relation.
- To be able to use a recurrence relation to analyse an annuity.

Reducing balance loans

When money is borrowed from a bank, the borrower usually makes regular payments to reduce the amount owed, rather than waiting until the end of the loan to repay the balance. This kind of loan is called a **reducing balance loan**.

Modelling reducing balance loans

Let V_n be the *balance* of the loan after n payments have been made. Then

$$V_0 = \text{the principal}, \quad V_{n+1} = RV_n - D$$

where D is the *additional payment* made, $R = 1 + \dfrac{r}{100 \times p}$ is the growth multiplier, r is the *annual interest rate* and p is the *number of compounding periods* per year.

Example 6 Modelling a reducing balance loan with a recurrence relation (1)

Flora borrows \$8000 at an interest rate of 13% per annum, compounding annually. She makes yearly payments of \$2100.

Construct a recurrence relation to model this loan, in the form

$$V_0 = \text{the principal}, \quad V_{n+1} = RV_n - D$$

where V_n is the balance of the loan after n years.

Explanation	Solution
1 State V_0 and D.	$V_0 = 8000$ and $D = 2100$
2 Determine the value of R using the formula $R = 1 + \dfrac{r}{100 \times p}$, where $r = 13$ and $p = 1$.	$R = 1 + \dfrac{13}{100 \times 1} = 1.13$
3 Use the values of V_0, R and D to write down the recurrence relation.	$V_0 = 8000, \quad V_{n+1} = 1.13V_n - 2100$

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Example 7 Modelling a reducing balance loan with a recurrence relation (2)

Alyssa borrows \$1000 at an interest rate of 15% per annum, compounding monthly. She makes monthly payments of \$250.

Construct a recurrence relation to model this loan, in the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

where V_n is the balance of the loan after n months.

Explanation	Solution
1 State V_0 and D.	$V_0 = 1000$ and $D = 250$
2 Determine the value of R using the formula $R = 1 + \dfrac{r}{100 \times p}$ where $r = 15$ and $p = 12$.	$R = 1 + \dfrac{15}{100 \times 12} = 1.0125$
3 Use the values of V_0, R and D to write down the recurrence relation.	$V_0 = 1000, \quad V_{n+1} = 1.0125V_n - 250$

Once we have a recurrence relation, we can use it to determine things such as the balance of a loan after a given number of payments.

Example 8 Using a recurrence relation to analyse a reducing balance loan

Alyssa's loan can be modelled by the recurrence relation:

$$V_0 = 1000, \qquad V_{n+1} = 1.0125V_n - 257.85$$

a Use your calculator to find the balance of the loan after four payments.

b Find the balance of the loan after two payments have been made. Round your answer to the nearest cent.

Explanation

a **i** Write down the recurrence relation.

ii Type '1000' and press 'enter' or 'EXE'.

iii Type '× 1.0125-257.85' and press 'enter' (or EXE) 4 times to obtain the screen opposite.

Solution

$V_0 = 1000, V_{n+1} = 1.0125V_n - 257.85$

1000	1000
1000 ·1.0125 – 257.85	754.65
754.65·1.0125 – 257.85	506.23
506.2331·1.0125 – 257.85	254.71
254.7110·1.0125 – 257.85	0.044927
|	

Balance \$0.04 (to the nearest cent).

b Read the third line of the calculator.

\$506.23 (to the nearest cent)

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Annuities

An **annuity** is an investment where compound interest is earned and money is withdrawn from the investment by the individual in the form of regular payments. The calculations used to model the values of reducing balance loans and annuities are identical. The value of the annuity represents how much money is left in the investment.

An annuity can be modelled with a recurrence relation. Once we have a recurrence relation, we can use it to determine things such as the value of the annuity after a given number of payments have been received.

Modelling an annuity

Let V_n be the value of the annuity after n payments have been made. Then

$$V_0 = \text{principal}, \qquad V_{n+1} = RV_n - D$$

where D is the *payment* that has been made, $R = 1 + \dfrac{r}{100 \times p}$ is the growth multiplier, r is the *annual interest rate* and p is the *number of compounding periods* per year.

Example 9 Modelling an annuity with a recurrence relation

Reza invests \$12 000 in an annuity that earns interest at the rate of 6% per annum, compounding monthly, providing him with a monthly income of \$2035.

a Model this annuity using a recurrence relation of the form

$$V_0 = \text{the principal}, \quad V_{n+1} = RV_n - D$$

where V_n is the value of the annuity after n months.

b Use your calculator to find the value of the annuity after the first four months. Round your answer to the nearest cent.

Explanation	Solution
a **i** State the value of V_0 and D.	$V_0 = 12\,000$ and $D = 2035$
ii Determine the value of R using the formula $R = 1 + \dfrac{r}{100 \times p}$.	$R = 1 + \dfrac{6}{100 \times 12} = 1.005$
iii Use the values of V_0, R and D to write down the recurrence relation.	$V_0 = 12\,000, \quad V_{n+1} = 1.005V_n - 2035$
b **i** Type **12000** and press enter or EXE. **ii** Type **× 1.005-2035** and press enter or EXE four times to obtain the screen opposite.	See calculator screen below

	12000.
12000. · 1.005 − 2035	10025.
10025. · 1.005 − 2035	8040.125.
8040.125. · 1.005 − 2035	6045.326.
6045.326. · 1.005 − 2035	4040.55.

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Skill-sheet

Exercise 8B

Modelling reducing balance loans with recurrence relations

Example 6

1 Brooke borrows \$5000 at an interest rate of 5.4% per annum, compounding annually. The loan will be repaid by making annual payments of \$1400.
Construct a recurrence relation to model this loan, in the form:

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

where V_n be the balance of the loan after n years.

Example 7

2 Jackson borrows \$2000 at an interest rate of 6% per annum, compounding monthly. The loan will be repaid by making monthly payments of \$339.
Let V_n be the balance of the loan after n months.

a State V_0 and D.

b Determine the value of R, using the formula $R = 1 + \dfrac{r}{100 \times p}$.

c Use the values of V_0, R and D to write down the recurrence relation in the form:

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

3 Benjamin borrows \$10 000 at an interest rate of 12% per annum, compounding quarterly. The loan will be repaid with quarterly payments of \$2600.
Let B_n be the balance of the loan after n quarters.

a Model this loan using a recurrence relation of the form:

$$B_0 = \text{the principal}, \qquad B_{n+1} = RB_n - D$$

b Use the recurrence relation to determine the balance of the loan after two payments have been made.

4 Write a recurrence relation to model a loan of \$3500 borrowed at 4.8% per annum, compounding monthly, with payments of \$280 per month.
Let V_n be the balance of the loan after n months.

5 Write a recurrence relation to model a loan of \$150 000 borrowed at 3.64% per annum, compounding fortnightly, with payments of \$650 per fortnight.
Let V_n be the balance of the loan after n fortnights.

6 Consider a loan of \$235 000 borrowed at 3.65% per annum, compounding daily, with payments of \$150 per day.
Let V_n be the balance of the loan after n days.

a Write a recurrence relation to model this loan.

b Find the value of the loan after 3 days. Round your answer to the nearest cent.

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Using a recurrence relation to analyse a reducing balance loan

Example 8

7 A reducing balance loan is modelled by the recurrence relation

$$V_0 = 2500, \qquad V_{n+1} = 1.08V_n - 626$$

where V_n is the balance of the loan after n years.

a State the initial balance of the reducing balance loan.

b State the payment that is made each year.

c Determine the annual interest rate, r, using $1.08 = 1 + \frac{r}{100 \times p}$.

d Use your calculator to determine the balance of the loan after three years. Round your answer to the nearest cent.

8 A reducing balance loan can be modelled by the recurrence relation:

$$V_0 = 5000, \qquad V_{n+1} = 1.01V_n - 865$$

where V_n is the balance of the loan after n months.

a State the initial balance of the reducing balance loan.

b State the payment that is made each month.

c Determine the annual interest rate, r.

d Find the balance of the loan after two payments have been made. Round your answer to the nearest cent.

Modelling and analysing an annuity with a recurrence relation

Example 9

9 Mark invests $20 000 in an annuity paying interest at the rate of 7.2% per annum, compounding annually. He receives a payment of $3375 each year until the annuity is exhausted.

Let V_n be the value of the annuity after n years.

a State the value of V_0 and D.

b Determine the value of the growth multiplier, R.

c Use your values of V_0, D and R to model this annuity using a recurrence relation of the form:

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

10 Sandra invests $750 000 in an annuity paying interest at the rate of 5.4% per annum, compounding monthly. She receives a payment of $4100 per month until the annuity is exhausted.

Let V_n be the value of the annuity after n payments have been received.

a State the value of V_0 and D.

b Determine the value of the growth multiplier, R.

c Use your values of V_0, D and R to model this annuity using a recurrence relation of the form:

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

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11 Helen invests \$40 000 in an annuity paying interest at the rate of 6% per annum, compounding quarterly. She receives a payment of \$10 380 each quarter. Let V_n be the balance of the loan after n quarters.

- **a** Model this loan using a recurrence relation of the form:

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

- **b** Use the recurrence relation to determine the balance of the annuity after 3 quarters. Round your answer to the nearest cent.

12 An annuity is modelled by the recurrence relation

$$V_0 = 5000, \qquad V_{n+1} = 1.01V_n - 1030$$

where V_n is the balance of the annuity after n monthly payments have been received.

- **a** State the initial balance of the annuity.
- **b** State the payment that is received each month.
- **c** Determine the annual interest rate, r, using $R = 1 + \dfrac{r}{100 \times p}$.
- **d** Use your calculator to determine the balance of the annuity after three payments have been received. Round your answer to the nearest cent.
- **e** How much will the annuity pay out in the first three months?

13 An annuity can be modelled by the recurrence relation

$$V_0 = 6000, \qquad V_{n+1} = 1.005V_n - 1500$$

where V_n is the balance of the annuity after n payments have been made.

- **a** Use your calculator to determine the balance of the annuity after the two payments have been received.
- **b** Assuming that payments are made quarterly, how much will the annuity pay out in the first year?

14 Jeff invests \$1 000 000 in an annuity and receives a regular monthly payment. The balance of the annuity, in dollars, after n months, A_n, can be modelled by a recurrence relation of the form

$$A_0 = 1\,000\,000, \qquad A_{n+1} = 1.0024A_n - 4000$$

- **a** State the initial balance of the annuity.
- **b** State the payment that Jeff receives each month.
- **c** Calculate the annual compound interest rate.
- **d** Calculate the balance of this annuity after two months.

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15 Esme invests \$100 000 in an annuity and receives a regular monthly payment. The balance of the annuity, in dollars, after n months, E_n, can be modelled by the recurrence relation

$$E_0 = 100\,000, \qquad E_{n+1} = 1.0055E_n - 18\,400$$

- **a** What monthly payment does Esme receive?
- **b** Find the annual interest rate for this annuity.
- **c** At some point in the future, the annuity will have a balance that is lower than the monthly payment amount. What is the balance of the annuity when it **first** falls below the monthly payment amount? Round your answer to the nearest cent.

Exam 1 style questions

16 Matthew would like to purchase a new home. He will establish a loan for \$640 000 with interest charged at the rate of 4.2% per annum, compounding monthly.
Each month, Matthew will pay \$3946.05.
Let V_n be the value of Matthew's loan, in dollars, after n months.
A recurrence relation that models the value of V_n is

- **A** $V_0 = 640\,000, \qquad V_{n+1} = 1.0035V_n$
- **B** $V_0 = 640\,000, \qquad V_{n+1} = 1.042V_n$
- **C** $V_0 = 640\,000, \qquad V_{n+1} = 1.042V_n - 3946.05$
- **D** $V_0 = 640\,000, \qquad V_{n+1} = 1.0035V_n - 3946.05$
- **E** $V_0 = 640\,000, \qquad V_{n+1} = 1.0035V_n + 3946.05$

17 Tim invests \$3800 in an annuity and receives a regular monthly payment of \$480. The balance of the annuity, in dollars, after n months, T_n, can be modelled by a recurrence relation of the form

$$T_0 = 3800, \quad T_{n+1} = 1.002T_n - 480$$

The balance of the annuity after three months is closest to

A \$3327 **B** \$3328 **C** \$2854 **D** \$2379 **E** \$2380

18 Suzanne invests \$9200 in an annuity at 4.2% per annum, compounding monthly.
Suzanne receives a regular monthly payment of \$620.
The amount of interest earned in the second month is closest to

A \$30 **B** \$588 **C** \$590 **D** \$8612 **E** \$8022

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8C Amortisation tables

Learning intentions

- To be able to apply the amortisation process.
- To be able to construct an amortisation table.
- To be able to analyse an amortisation table for a reducing balance loan.
- To be able to read and interpret an amortisation table for an annuity to find the interest rate.
- To be able to interpret and construct an amortisation table for a compound interest investment with additions to the principal.

Amortisation tables provide additional information for each period, rather than just the balance after each payment.

Amortisation tables for reducing balance loans

Loans that are repaid by making regular payments until the balance of the loan is zero are called **amortising** loans. In an amortising loan, part of each of the regular payments goes towards paying the interest owed on the unpaid balance of the loan with the remainder used to reduce the principal of the loan (the amount borrowed).

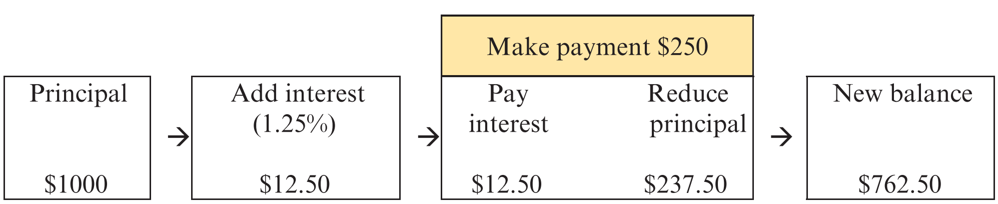
For example, consider Alyssa's loan from Example 7. Interest on the $1000 loan was charged at the rate of 15% per year and the loan was to be repaid with monthly payments of $250. The recurrence relation was given as

$$V_0 = 1000, \qquad V_{n+1} = 1.0125V_n - 250$$

To determine where the first payment goes:

- Calculate the interest charged ($p = 12$): $\frac{15\%}{12} = 1.25\%$. Thus, $1000 \times 1.25\% = \$12.50$
- Calculate the **principal reduction**: $\$250 - \$12.50 = \$237.50$
- Calculate the new balance: $\$1000 - \$237.50 = \$762.50$

This process is shown in the following diagram:



We can also represent this information in table format, showing the impact of a payment, interest and the subsequent reduction of the principal to give a new balance.

Payment number	Payment	Interest	Principal reduction	Balance
1	250.00	12.50	237.50	762.50

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When this analysis is repeated, the results can be summarised in an **amortisation table**. The amortisation table shows all of the details that explain how the new balance was calculated. Note that the first line shows the initial value of the loan as the balance when no payments have been received.

The first three payments for Alyssa's loan are shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	1000.00
1	250.00	12.50	237.50	762.50
2	250.00	9.53	240.47	522.03
3	250.00	6.53	243.47	278.56

Example 10 Applying the amortisation process

Flora borrows \$20 000 at an interest rate of 8% per annum, compounding annually. She makes annual payments of \$2500.

a State the principal of the loan.
b Calculate the initial interest charged on the principal.
c Determine the impact of the first annual payment to find the principal reduction.
d Calculate the new balance.
e Complete the row in the table below with your calculations.

Payment number	Payment	Interest	Principal reduction	Balance
1	2500.00			

Explanation

a Read the principal from the question or recurrence relation.

Solution

The principal is \$20 000.

b Calculate the interest paid.

Interest paid = 8% of \$20 000 = \$1600

c Principal reduction = payment − interest.

Principal reduction = 2500 − 1600
= \$900

d New balance = balance owing − principal reduction

New balance = 20 000 − 900 = \$19 100

e Place each of the numbers from the calculations into the relevant boxes.

Interest	Principal reduction	Balance
1600.00	900.00	19 100.00

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Constructing an amortisation table for a reducing balance loan

1. Interest paid $= \frac{r}{100 \times p} \times$ previous balance, where r is the annual interest rate and p is the number of compounding periods each year.
2. Principal reduction = payment − interest
3. New balance = (previous) balance − reduction in balance

Example 11 Constructing an amortisation table for a reducing balance loan

Flora borrows $20 000 at an interest rate of 8% per annum, compounding annually. She makes annual payments of $2500.

Construct an amortisation table for Flora's reducing balance loan for the first three payments.

Solution

Repeat the calculations from Example 10, rounding all numbers to the nearest cent.

Once the new balance has been calculated, repeat the process for the first three payments.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	20 000.00
1	2500.00	1600.00	900.00	19 100.00
2	2500.00	1528.00	972.00	18 128.00
3	2500.00	1450.24	1049.76	17 078.24

Sometimes we are asked to fill in gaps of a given amortisation table.

Example 12 Analysing an amortisation table for a reducing balance loan

A business borrows $10 000 at a rate of 8% per annum, compounding quarterly. The loan is to be repaid by making quarterly payments of $2700.00. The amortisation table for this loan is shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	10 000.00
1	2700.00		2500.00	7500.00
2	2700.00	150.00		4950.00
3	2700.00	99.00	2601.00	

a Calculate the interest paid on the initial balance.

b Calculate the principal reduction from the second payment.

c Calculate the balance of the loan after payment 3 has been made.

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Explanation	Solution
a Use $\frac{r}{100 \times p}$, where $r = 8$ and $p = 4$ since interest is calculated quarterly. Interest paid = 2% × unpaid balance	Interest paid $= \frac{8}{100 \times 4} \times 10\,000 = \$200$
Alternatively, note that \$2700 is paid and the principal was reduced by \$250.	Or, Interest paid = \$2700 – \$2500 = \$200
b Principal reduction = payment – interest	Principal reduction = 2700.00 – 150.00 = \$2550.00
c New Balance = balance owing – principal reduction	Balance of the loan after 3 payments = 4950 – 2601 = \$2349

Amortisation tables for annuities

An amortisation table for an annuity is very similar to one for a reducing balance loan. Each row shows the payment number, the payment received, the interest earned, the principal reduction and the balance of the annuity after each payment has been received.

Example 13 Analysing an amortisation table for an annuity to find the interest rate

Consider the following amortisation table for an annuity after 3 monthly payments.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	12 000.00
1	2200.00	60.00	2140.00	9860.00
2	2200.00	49.30	2150.70	7709.30
3	2200.00	*A*	*B*	5547.85

a State the principal of the annuity and the amount of interest paid in the first month.

b Calculate the monthly interest rate.

c Find the value of *A* and *B*.

Explanation	Solution
a Read off from the table.	Principal: \$12 000, Interest: \$60
b Calculate: $\frac{\text{Interest}}{\text{Principal}} \times 100$	$\frac{60}{12000} \times 100 = 0.5\%$ per month
c *A* is the interest due on \$7709.30	A: $\frac{0.5}{100} \times 7709.30 = 38.55$
B is the principal reduction after the third payment.	B: 2200 – 38.55 = 2161.45

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Amortisation tables for compound interest investments with additions to the principal

An amortisation table for a compound interest investment with additions to the principal is similar to the previous examples but here, the payment **increases** the balance of the principal further.

Example 14 **Interpreting and constructing an amortisation table for a compound interest investment with additional payments**

Consider the following amortisation table for a compound interest investment with monthly additions to the principal. Assume that interest compounds monthly.

Payment number	Payment	Interest	Principal increase	Balance
0	0.00	0.00	0.00	1200.00
1	50.00	3.00	53.00	1253.00
2	50.00	3.13	53.13	1306.13
3	50.00	3.27	53.27	1359.40

Complete two additional lines for the table corresponding to payment 4 and payment 5.

Solution

Begin by calculating the monthly interest rate $\frac{3}{1200} \times 100 = 0.25\%$.

Now we can calculate the line associated with payment 4. The interest paid is calculated on the balance \$1359.40:

$$\text{Interest} = 0.25\% \times 1359.40 = \$3.40$$

The principal increases by the interest and the additional payment:

$$\text{Principal increase} = \text{interest} + \text{payment} = 3.40 + 50 = \$53.40$$

Thus, the new balance becomes:

$$\text{New balance} = \text{previous balance} + \text{principal increase} = 1359.40 + 53.40 = \$1412.80$$

Repeating gives the following two lines of the table.

Payment number	Payment	Interest	Principal increase	Balance
4	50.00	3.40	53.40	1412.80
5	50.00	3.53	53.53	1466.33

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Exercise 8C

Applying the amortisation process and constructing an amortisation table for a reducing balance loan

Example 10
Example 11

1 Walter borrows $14 000 at an interest rate of 11% per annum, compounding annually. He makes annual repayments of $1800 per year.

a State the principal of the loan.

b Calculate the interest charged on the principal in the first year.

c Determine the impact of the first annual payment to find the principal reduction.

d Calculate the new balance after the first year.

e Complete the row in an amortisation table corresponding to payment 1.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	14 000.00
1	1800.00			

f Complete the next two rows of the amortisation table corresponding to payment 2 and 3 for Walter.

2 Ellie borrows $12 000 at an interest rate of 6% per annum, compounding monthly. She makes regular repayments of $300 per month.

a State the principal of the loan.

b Calculate $\frac{r}{100 \times p}$ where r is the annual interest rate and p is the number of compounding periods each year.

c Calculate the interest charged on the principal in the first month.

d Determine the impact of the first monthly payment to find the principal reduction.

e Calculate the new balance after the first month.

f Complete the row in an amortisation table corresponding to payment 1.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	12 000.00
1	300.00			

g Complete the next two rows of the amortisation table corresponding to payment 2 and 3 for Ellie.

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3 Anna borrows \$36 000 at an interest rate of 8% per annum, compounding quarterly. She makes regular repayments of \$1000 per quarter.

a State the principal of the loan.

b Calculate $\frac{r}{100 \times p}$ where r is the annual interest rate and p is the number of compounding periods each year.

c Construct an amortisation table corresponding to the first three payments for the loan.

Reading and interpreting an amortisation table for a reducing balance loan

Example 12

4 A student borrows \$2000 at an interest rate of 12% per annum, compounding monthly. The student makes monthly payments of \$345.
The amortisation table for this loan after 5 payments is shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	2000.00
1	345.00	20.00	325.00	1675.00
2	345.00	A	328.25	1346.75
3	345.00	13.47	331.53	1015.22
4	345.00	10.15	B	680.37
5	345.00	6.80	338.20	C

a Calculate the monthly interest rate.

b Determine the values of A, B and C.

5 The amortisation table for a loan with quarterly payments is shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	4000.00
1	557.65	100.00	457.65	3542.35
2	557.65	88.65	469.09	3073.26
3	557.65	76.83	480.82	2592.44
4	557.65	A	492.84	2099.60
5	557.65	52.49	B	1594.44
6	557.65	39.86	517.79	C
7	557.65	D	E	545.92

a State the principal and interest paid in the first quarter.

b Calculate the quarterly interest rate.

c Determine the values of A, B, C, D and E.

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6 Ada has a reducing balance loan with an interest rate of 3.6% per annum, compounding monthly. She makes monthly payments of \$1800 as shown in the amortisation table below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	460 000.00
1	1800.00	1380.00	420.00	459 580.00
2	1800.00	1378.74	*A*	459 158.74
3	1800.00			*B*

Calculate the value of *A* and *B*.

Analysing an amortisation table for an annuity to find the interest rate

Example 13

7 A student invested \$6000 in an annuity paying an interest rate of 3% per annum, compounding monthly.
She receives a monthly payment of \$508 as per the amortisation table shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	6000.00
1	508.00	15.00	493.00	5507.00
2	508.00	13.77	494.23	5012.77
3	508.00	*A*	*B*	*C*

a Reading from the table, determine:

i the interest earned when payment 1 is received,

ii the monthly interest rate.

b Calculate the values of *A*, *B* and *C*.

Interpreting and constructing an amortisation table for a compound investment with additional payments

Example 14

8 The amortisation table below charts the growth of an investment which compounds monthly and has regular additions made to the balance each month.

Payment number	Payment	Interest	Principal increase	Balance
0	0.00	0.00	0.00	5000.00
1	100.00	50.00	150.00	5150.00
2	100.00	51.50	151.50	5301.50
3	100.00	*A*	*B*	*C*

a Calculate the monthly interest rate of the investment.

b Determine the values of *A*, *B* and *C*.

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Exam 1 style questions

9 Edith invested \$400 000 in an annuity that provides an annual payment of \$51 801.82
Interest is calculated annually.
The first five lines of the amortisation table are shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	400 000.00
1	51 801.82	20 000.00	31 801.82	368 198.18
2	51 801.82	18 409.91	33 391.91	334 806.27
3	51 801.82	16 740.31		299 744.76
4	51 801.82	14 987.24	36 814.58	262 930.18

The principal reduction associated with payment number 3 is

A \$31 801.82 **B** \$33 391.82 **C** \$35 061.50 **D** \$35 061.51 **E** \$36 814.58

10 Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	200 000.00
1	3000.00	1800.00	1200.00	198 800.00
2	3000.00	1789.20	1210.80	197 589.20
3	3000.00	11778.30	1221.70	196 367.50

The annual interest rate for this loan is 3.6%.
Interest is calculated immediately before each payment.
For this loan, the repayments are made

A weekly **B** fortnightly **C** monthly **D** quarterly **E** yearly

11 Four lines of an amortisation table for an annuity investment are shown below.
The interest rate for this investment remains constant, but the amount of the additional payment may vary.

Payment number	Payment	Interest	Principal increase	Balance
1	50.00	50.00	100.00	10 100.00
2	50.00	50.50	100.50	10 200.50
3	50.00	51.00	101.00	10 301.50
4				10 553.01

The balance of the investment after payment number 4 is \$10 533.01.
The value of payment number 4 is

A \$50 **B** \$100 **C** \$150 **D** \$200 **E** \$250

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8D Analysing financial situations using amortisation tables

Learning intentions

- ▶ To be able to find the final payment in a reducing balance loan and an annuity.
- ▶ To be able to find the total payment made/received and the total interest paid/earned.
- ▶ To be able to plot points using an amortisation table for a reducing balance loan, an annuity and a compound interest investment with additional payments.

Finding the final payment in a reducing balance loan and an annuity

A reducing balance loan must be paid off at some point in time and likewise, an annuity will run out after a period of time. In both cases, it is possible that the final payment might need to be different from the regular payment so that the final balance can reach zero.

Example 15 Finding the final payment for a reducing balance loan or annuity

Consider the following amortisation table for a reducing balance loan of \$20 000 with an interest rate of 8% per annum, compounding annually. Regular payments of \$5009.12 are made for the first four years as shown in the amortisation table.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	20 000.00
1	5009.12	1600.00	3409.12	16 590.88
2	5009.12	1327.27	3681.85	12 909.03
3	5009.12	1032.72	3976.40	8932.63
4	5009.12	714.61	4294.51	4638.12

Calculate the final payment required in the fifth year to pay off the loan fully.

Solution

To find the final payment, we first need to calculate the interest on \$4638.12:

$$\text{Interest} = \frac{8}{100} \times \$4638.12 = \$371.05$$

Thus, the final payment can be calculated by adding the balance that is still due and the interest.

$$\begin{aligned}\text{Final payment} &= \text{Balance} + \text{interest}\\ &= 4638.12 + 371.05\\ &= 5009.17\end{aligned}$$

Thus, the final payment is \$5009.17.

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Finding the total payment made/received and total interest paid/earned

Once we have a completed amortisation table, we can find the total interest paid/earned and the total payment made/received for a reducing balance loan, an annuity or a compound interest investment with additional payments.

Example 16 Finding the total payment made and total interest paid

Consider the following amortisation table for a reducing balance loan of $10 000 with an interest rate of 8% per annum, compounding quarterly. Three quarterly payments of $2626 are made.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	10 000.00
1	2626.00	200.00	2426.00	7574.00
2	2626.00	151.48	2474.52	5099.48
3	2626.00	101.99	2524.01	2575.47
4	A	B	C	0.00

a Complete the amortisation table corresponding to payment four such that the final payment ensures that the balance is 0.

b Calculate the total payment made for the loan.

c Calculate the total interest paid on the loan.

Explanation

a Follow the previous example by first finding the interest applied to the loan (B), the principal reduction (C) and the final adjusted payment (A).

Solution

B: Interest $= \dfrac{8}{100 \times 4} \times \$2575.47 = \$51.51$

C: Principal reduction $= \$2575.47$

A: Final payment $= 2575.47 + 51.51$

$= 2626.98$

Payment	Interest	Principal reduction
2626.98	51.51	2575.47

b Add up all of the payments made over the four quarters.

Payments $= 2626 \times 3 + 2626.98 = \$10\ 504.98$

c Subtract the principal from the total payments.
Alternatively, we can add up the interest column.

Interest $= 10\ 504.98 - 10\ 000 = \$504.98$
OR
Interest $= 200 + 151.48 + 101.99 + 51.51 = \$504.98$

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Plotting points from an amortisation table

The values in an amortisation table can be plotted to give a picture of the impact of regular payments.

Example 17 **Plotting from an amortisation table**

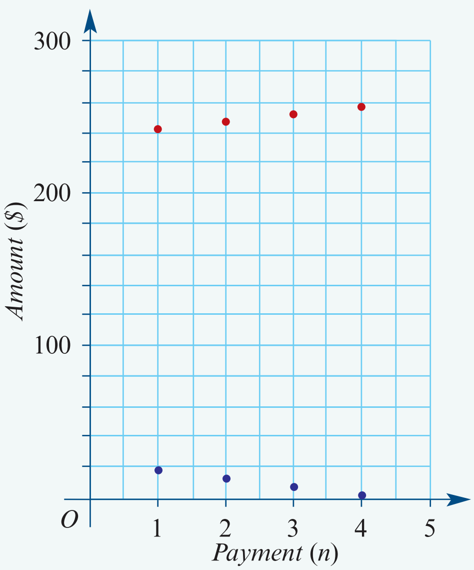
Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	1000.00
1	257.85	12.50	245.35	754.65
2	257.85	9.43	248.42	506.23
3	257.85	6.33	251.52	254.71
4	257.89	3.18	254.71	0.00

Plot a graph of the interest and principal reduction on the same graph.

Solution

For this loan, we can plot a graph for each payment period to show that the amount of interest paid each payment (blue dots) declines while the amount of principal paid increases (red dots).



Note that for a loan, the graph above shows how the amount of interest paid for each payment (blue dots) decreases with the payment number, while the amount of principal paid off increases (red dots). This is because the balance is decreasing and so the interest is being calculated on a lower balance each period.

For a compound interest investment, we would expect to see the interest earned each period increase because the balance increases each time a payment is made.

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Exercise 8D

Finding the final payment in a reducing balance loan and an annuity

Example 15

1 Consider the following amortisation table for a reducing balance loan of $2000 with an interest rate of 4% per annum, compounding annually. Regular payments of $550 are made for the first three years as shown in the amortisation table.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	2000.00
1	550.00	80.00	470.00	1530.00
2	550.00	61.20	488.80	1041.20
3	550.00	41.65	508.35	532.85
4				0.00

Calculate the value of the fourth payment to ensure the loan is repaid in full.

2 Consider the following amortisation table for a reducing balance loan of $5000 with an interest rate of 4.8% per annum, compounding monthly. Regular payments of $1262.50 are made for the first three months as shown in the amortisation table.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	5000.00
1	1262.50	20.00	1242.50	3757.50
2	1262.50	15.03	1247.47	2510.03
3	1262.50	10.04	1252.46	1257.57
4				0.00

Calculate the value of the fourth payment to ensure the loan is repaid in full.

3 Consider the following amortisation table for an annuity of $3000 with an interest rate of 5% per annum, compounding annually. Regular payments of $693 are received for the first four months as shown in the amortisation table.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	3000.00
1	693.00	150.00	543.00	2457.00
2	693.00	122.85	570.15	1886.85
3	693.00	94.34	598.66	1288.19
4	693.00	64.41	628.59	659.60

Calculate the value of the fifth payment to ensure the annuity is completely exhausted.

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4 Charlie invests $4500 into an annuity with an interest rate of 5.4% per annum, compounding monthly. He receives monthly payments of $760 for five months. Calculate the value of the sixth payment that Charlie receives to ensure the annuity is completely exhausted.

Finding the total payment made/received and the total interest paid/earned

Example 16

5 Consider the following amortisation table for an investment of $11 000 with an interest rate of 4.8% per annum, compounding quarterly. Regular quarterly payments of $1200 are added each quarter as shown below for the first four quarters.

Payment number	Payment	Interest	Principal increase	Balance
0	0.00	0.00	0.00	11 000.00
1	1200.00	132.00	1332.00	12 332.00
2	1200.00	147.98	1347.98	13 679.98
3	1200.00	164.16	1364.16	15 044.14
4	1200.00	A	B	16 424.67

a Find the value of A.

b Find the value of B.

c Find the total interest earned on the investment in the first four quarters.

6 Consider the following amortisation table for a reducing balance loan of $4000 with an interest rate of 6% per annum, compounding monthly. A monthly payment of $344.14 is made for the first 11 months.

Payment number	Payment	Interest	Principal reduction	Balance
9	344.14	6.81	337.33	1023.71
10	344.14	5.12	339.02	684.69
11	344.14	3.42	340.72	343.97
12	A	B	C	0.00

a State the value of A, B and C in the amortisation table corresponding to payment twelve such that the final payment ensures that the balance is 0.

b Calculate the total payment made for the loan.

c Calculate the total interest paid on the loan.

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7 Consider the final two lines of the amortisation table for an annuity of \$30 000 with an interest rate of 3.6% per annum, compounding quarterly. A quarterly payment of \$3903.50 is made for the first 7 quarters.

Payment number	Payment	Interest	Principal reduction	Balance
7	3903.50	69.32	3834.18	3868.37
8	A	B	C	0.00

a State the value of A, B and C in the amortisation table corresponding to payment eight such that the final payment ensures that the balance is 0.
b Calculate the total payments received from the annuity.
c Calculate the total interest paid on the loan.

8 Tania invests \$12 000 in an annuity with an interest rate of 6.6% per annum, compounding monthly. She receives regular monthly payments of \$3040 per month for three months followed by a final payment in the fourth month. Calculate the total payment and the interest for the annuity.

Plotting points from an amortisation table

Example 16 9 Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	2000.00
1	345.14	20.00	325.14	1674.86
2	345.14	16.75	328.39	1346.47
3	345.14	13.46	331.68	1014.79
4	345.14	10.15	334.99	679.80
5	345.14	6.80	338.34	341.46
6	344.87	3.41	341.46	0.00

Plot a graph of the interest and principal reduction on the same graph for the first six payments.

10 The amortisation table below charts the growth of a compound interest investment with regular additions made to the principal each month.

Payment number	Payment	Interest	Principal increase	Balance
0	0.00	0.00	0.00	5000.00
1	100.00	50.00	150.00	5150.00
2	100.00	51.50	151.50	5301.50
3	100.00	53.02	153.02	5454.52

Plot a graph of the interest and principal increase on the same graph.

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Exam 1 style questions

11 Francesca invests $6000 into an annuity with an interest rate of 5.8% per annum, compounding quarterly.
She receives a quarterly payment of $1554.13 for the first three quarters and then a final payment in the fourth quarter.
After three quarters, the value of the annuity is $1534.48.
The payment that Francesca receives in the fourth quarter is

A $0.04 **B** $2.60 **C** $1534.48 **D** $1554.13 **E** $1556.73

12 Ned borrows $15 000 at an interest rate of 6% per annum, compounding monthly. He makes regular monthly payments of $3796.99 for three months followed by a final payment in the fourth month.
The total interest that Ned pays on the loan is closest to

A $56 **B** $75 **C** $188 **D** $300 **E** $900

8E Using a finance solver to find the balance and final payment

Learning intentions

- ► To be able to use the finance solver to find the value of a compounding interest investment with additional payments.
- ► To be able to use the finance solver to find the balance and final payment of a reducing balance loan.
- ► To be able to use the finance solver to find the balance of an annuity.

While the techniques used so far in this chapter are useful for performing a small number of calculations, they are tedious over a long period. For example, a typical home loan may involve monthly payments over 30 years. CAS calculators have a **Finance Solver** that allow for larger calculations to be performed with ease.

ISBN 978-1-009-11041-9

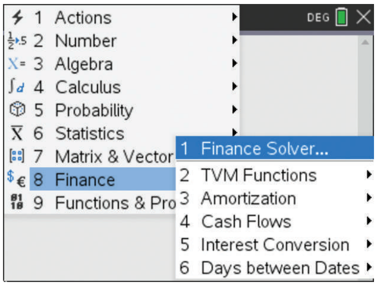
Using the Finance Solver on the TI-Nspire CAS

Steps

1 Press ctrl + N

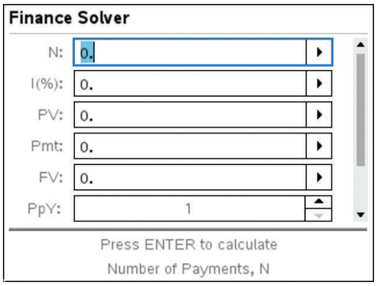
2 Select **Add Calculator**.

Press menu>**Finance**> **Finance Solver**.



3 To use Finance Solver you need to know the meaning of each of its symbols.

- **N** is the total number of payments.
- **I(%)** is the annual interest rate.
- **PV** is the present value of the loan/investment.
- **Pmt** is the amount paid at each payment.

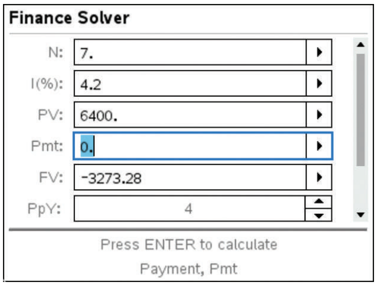


- **FV** is the future value of the loan/investment.
- **PpY** is the number of payments per year.
- **CpY** is the number of times the interest is compounded per year. (It is the same as **PpY**.)
- **PmtAt** is used to indicate whether the interest is compounded at the end or at the beginning of the time period. Ensure this is set at **END**.

Note: Use tab or ▼ to move down boxes, press ▲ to move up. For **PpY** and **CpY** press tab to move down to the next entry box.

4 When using Finance Solver to solve loan and investment problems, there will be one unknown quantity. To find its value, move the cursor to its entry field and press enter to solve.

In the example shown, pressing enter will solve for **Pmt**.



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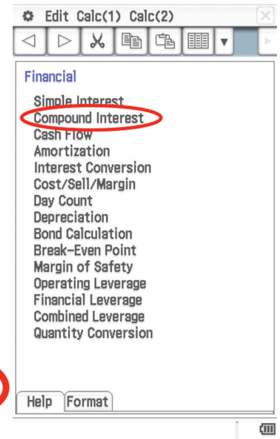
Using the Finance Solver on the Casio ClassPad

Steps

1 Tap **Financial** from the main menu screen.

2 Select the compound interest solver by tapping on **Compound Interest** from the solver screen.



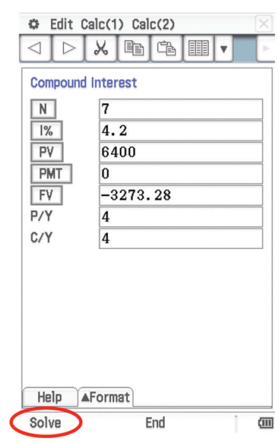
3 To use Finance Solver you need to know the meaning of each of its symbols.

- **N** is the total number of payments.
- **I%** is the annual interest rate.
- **PV** is the present value of the loan or investment.
- **PMT** is the amount paid at each payment.
- **FV** is the future value of the loan or investment.
- **P/Y** is the number of payments per year.
- **C/Y** is the number of times interest is compounded per year. (It is the same as **P/Y**.)

4 Tap **Format** and confirm that the setting for 'Odd Period' is set to 'off' and 'Payment Date' is set to 'End of period'.

5 When using Finance Solver to solve loan problems, there will be one unknown quantity. To find its value, tap its entry field and tap **Solve**.

In the example shown, tapping **Solve** will solve for **Pmt**.



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Using a financial solver to analyse a compound interest investment with regular additions to the principal

A finance solver is a powerful computation tool. However, you have to be very careful in the way you enter information because it needs to know which way the money is flowing. It does this by following a **sign convention**.

In general terms:

- if you **receive** money, or someone owes you money, we treat this as a **positive** (+ve)
- if you **pay out** money or you owe someone money, we treat this as a **negative** (−ve).

Recall that a compound interest investment with regular additions to the principal is an investment where the balance increases through both the interest earned and the additional payments.

Using Finance solver for a compound interest investment with regular additions to the principal

In finance solver:

- **PV:** Negative: you make an investment by giving the bank some money.
- **PMT:** Negative: you make regular payments to the bank.
- **FV:** Positive: after the payment is made and the investment matures, the bank will give you the money.

Example 18 Determining the value of an investment with regular additions made to the principal using a financial solver

Lars invests $500 000 at 5.5% per annum, compounding monthly. He makes a regular deposit of $500 per month into the account. What is the value of his investment after 5 years? Round your answer to the nearest cent.

Explanation

1 Open Finance Solver and enter the information below, as shown opposite.
- **N**: 60 (5 years)
- **I%**: 5.5
- **PV**: –500 000 (you give this to the bank)
- **PMT**: –500 (you give this to the bank)
- **FV**: to be determined
- **Pp/Y**: 12 payments per year
- **Cp/Y**: 12 compounding periods per year

2 Solve for FV and write your answer, rounding to the nearest cent. Note that this is positive as the bank will give this money to you.

Solution

N:	60
I%:	5.5
PV:	–500000
Pmt or PMT:	–500
FV:	692292.297...
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

After 5 years, Lars' investment will be worth $692 292.30.

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Using finance solver for a reducing balance loan

Recall that for a reducing balance loan, you receive money from the bank (in the form of the loan) and then make payments until you no longer owe the bank any money. The sign convention for a reducing balance loan is summarised below.

Finance solver for a reducing balance loan

In finance solver:

- **PV:** Positive: the bank gives you money through a loan.
- **PMT:** Negative: you repay the loan by making regular repayments to the bank.
- **FV:** Negative, zero or positive: after the payment is made:
  - you still owe the bank money (FV negative),
  - the loan is fully repaid (FV zero), or
  - you have overpaid your loan and the bank now owes you money (FV positive).

Example 19 Determining the balance and final payment of a reducing balance loan after a given number of payments

Andrew borrows \$20 000 at an interest rate of 7.25% per annum, compounding monthly. This loan will be repaid over 4 years with regular payments of \$481.25 each month for 47 months followed by a final payment to fully repay the loan.

a How much does Andrew owe after 3 years? Round your answer to the nearest cent.

b What is the final payment amount that Andrew must make to fully repay the loan within 4 years (48 months)? Round your answer to the nearest cent.

Explanation

a 1 Open Finance Solver and enter the following:

- **N**: 36 (number of months in 3 years)
- **I%**: 7.25 (annual interest rate)
- **PV**: 20000 (positive to indicate that this is money received by Andrew from the lender)
- **Pmt** or **PMT**: −481.25 (negative as Andrew is giving this back to the lender)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solution

N:	36
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

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2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press to enter solve.
- *ClassPad:* Tap on the **FV** entry box and tap 'Solve'. The amount –5554.3626 . . . now appears in the **FV** entry box.

Note: A negative FV indicates that Andrew will still owe the lender money after the payment has been made.

N:	36
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	-5554.3626
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer, correct to the nearest cent

Andrew owes $5554.36.

b **1** Enter the information below.

- **N**: 48 (number of months in 4 years)
- **I%**: 7.25 (annual interest rate)
- **PV**: 20000
- **Pmt** or **PMT** (the payment amount is negative): −481.25
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	48
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press enter to solve.
- *ClassPad:* Tap on the **FV** entry box and tap **Solve** The amount 0.1079. . . (11 cents) now appears in the **FV** entry box.

N:	48
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	0.107924
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Since FV is positive (+11 cents), the bank owes Andrew 11 cents so we **subtract** this from the regular payment.

3 Write your answer.

Final payment
= $481.25 – $0.11
= $481.14

Andrew's final payment will be $481.14.

Using a finance solver to analyse an annuity

Recall that an annuity is when an individual puts money into an account and then receives payments from the bank.

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Finance solver for an annuity

In finance solver:

- **PV:** Negative: you buy an annuity by giving the bank some money.
- **PMT:** Positive: you receive regular payments from the bank.
- **FV:** Positive or zero: after the payment is made:
  - the bank still owes you money (FV positive),
  - the annuity is fully paid out (FV zero)

Note: An annuity should never have a negative FV as a bank would never overpay the individual.

Example 20 Determining the balance of an annuity using a finance solver

Charlie invests $300 000 into an annuity, paying 5% interest per annum, compounding monthly. Over the next ten years, Charlie receives a payment of $3182 per month from the annuity for each month except the final month.

a Find the value of the annuity after five years. Round your answer to the nearest cent.

b Find the final payment from the annuity. Round your answer to the nearest cent.

Explanation

a **1** Open Finance Solver and enter the following:

- **N**: 60 (number of monthly payments in 5 years)
- **I%**: 5.00 (annual interest rate)
- **PV**: −300000 (negative to indicate that this is money paid by Charlie to the bank)
- **Pmt** or **PMT**: 3182 (positive to indicate that the bank is paying back to Charlie)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press to [enter] solve.
- *ClassPad:* Tap on the **FV** entry box and tap ‘Solve’. The amount 168612.24795 . . . now appears in the **FV** entry box.

Note: A positive FV indicates that Charlie is still owed money from the annuity.

3 Write your answer, rounding to the nearest cent.

Solution

N:	60
I%:	5.00
PV:	−300000
Pmt or PMT:	3182
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

N:	60
I%:	5.00
PV:	−300000
Pmt or PMT:	3182
FV:	168612.247951
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

The balance of the annuity is $168 612.25

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b **1** Find the value of the annuity after 120 payments. Enter the information below, as shown opposite.

- **N**: 120 (number of monthly payments in 10 years)
- **I%**: 5.00
- **PV**: −300000
- **Pmt** or **PMT**: 3182
- **Pp/Y**: 12
- **Cp/Y**: 12

N:	120
I%:	5.00
PV:	−300000
Pmt or PMT:	3182
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press [enter] to solve.
- *ClassPad:* Tap on the **FV** entry box and tap **Solve**

The amount −5.36 (−$5.36) now appears in the **FV** entry box.

Note: The FV is negative ($5.36). This means that Charlie owes the annuity $5.36. To compensate, Charlie's final payment will be decreased by $5.36.

N:	120
I%:	5.00
PV:	−300000
Pmt or PMT:	3182
FV:	−5.36388
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Final payment
= $3182 – $5.36
= $3176.64

3 Write your answer.

Charlie's final payment will be $3176.64.

Skill-sheet

Exercise 8E

Determining the value of an investment with regular additions made to the principal using a financial solver

Example 18

1 Wanda invested $20 000 at 7.1% per annum, compounding annually. She makes a regular deposit of $6000 per year into the account.

- **a** State whether the **PV** is positive or negative.
- **b** State whether the **PMT** is positive or negative.
- **c** Find the value of the investment after 10 years. Round your answer to the nearest cent.
- **d** Find the value of the investment after 30 years. Round your answer to the nearest cent.

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2 Ingrid invested $20 000 at 4.9% per annum, compounding monthly. She makes a regular deposit of $380 per month into the account.

a Find the value of the investment after 5 months. Round your answer to the nearest cent.

b Find the value of the investment after 3 years. Round your answer to the nearest cent.

Determining the balance of a reducing balance loan using a financial calculator

Example 19

3 Barry borrows $8000 at an interest rate of 4.5% per annum, compounding monthly. This loan will be repaid with regular payments of $350 per month, followed by a final payment.

a State whether the **PV** is positive or negative.

b State whether the **PMT** is positive or negative.

c How much does Barry owe after six months? Round your answer to the nearest cent.

4 Suzanne borrows $25 000 at an interest rate of 7.8% per annum, compounding monthly. She repays the loan with regular payments of $1200 per month and then a final payment to bring the balance to zero.

a How much does Suzanne owe after 3 months? Round your answer to the nearest cent.

b How much does Suzanne owe after 1 year? Round your answer to the nearest cent.

5 Rachel borrows $240 000 at an interest rate of 8.3% per annum, compounding quarterly. She makes 119 regular payments of $5442.90 each quarter followed by a final payment. Rachel repays the loan over thirty years.

a How much does Rachel owe after 6 years? Round your answer to the nearest cent.

b What is the final payment that Rachel must make to fully repay the loan in 30 years? Round your answer to the nearest cent.

6 David borrows $50 000 for a new car at an interest rate of 4.6%, compounding weekly. He repays the loan over 3 years with regular payments of $343.27 per week except for the final payment.

a How much does David owe after 1 year? Round your answer to the nearest cent.

b What is the final payment that David must make to fully repay the loan in three years? Round your answer to the nearest cent.

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Determining the balance of an annuity using a financial calculator

Example 20

7 Kazou invests \$50 000 into an annuity, paying 6.1% per annum, compounding annually. Kazou receives \$6825.61 per year for nine years then a final payment so that the annuity lasts exactly 10 years.

- **a** State whether the **PV** is positive or negative.
- **b** Find the value of the annuity after five years. Round your answer to the nearest cent.
- **c** What is the final payment made to Kazou so that the value of the annuity is zero after 10 years? Round your answer to the nearest cent.

8 Eliza invests \$20 000 into an annuity, paying 7.2% per annum, compounding monthly. The annuity regularly pays \$1732.37 per month, for eleven months followed by a final payment to exhaust the annuity.

- **a** Find the value of the annuity after three months. Round your answer to the nearest cent.
- **b** What is the final payment made to Eliza so that the value of the annuity is zero after 12 months? Round your answer to the nearest cent.

9 Ezra is going backpacking around Europe and has invested \$15 000 into an annuity for this trip. The annuity pays 6.8% per annum, compounding weekly for one year. The annuity pays \$298.57 per week for each week except for the final payment.

- **a** Find the value of the annuity after twenty-six weeks. Round your answer to the nearest cent.
- **b** What is the final payment made to Ezra so that the value of the annuity is zero after 1 year? Round your answer to the nearest cent.

Exam 1 style questions

10 Josie invests \$3000 in an account that pays interest at the rate of 2.8% per annum, compounding monthly. She makes an additional payment of \$200 each month.
The value of the investment, correct to the nearest cent, after 6 years is

A \$4249.26 **B** \$4249.27 **C** \$12 112.32 **D** \$19 208.55 **E** \$19 208.56

11 Bronwyn borrows \$450 000 to buy an apartment.
The interest rate for this loan was 4.24% per annum, compounding monthly for 20 years.
Bronwyn makes regular monthly payments of \$2784 each month except for her final payment. To pay out her loan fully in 20 years, her final payment is

A \$58.57 **B** \$2725.42 **C** \$2725.43 **D** \$2784.00 **E** \$2842.57

12 Benjamin invests \$75 000 in an annuity, paying 7.3% per annum, compounding monthly.
Benjamin receives a payment of \$2326 each month from the annuity.
The value of the annuity, correct to the nearest cent after 2 years is

A \$26842.05 **B** \$26842.06 **C** \$69356.55 **D** \$69356.56 **E** \$85153.41

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8F Using a finance solver to find interest rates, time taken and regular payments

Learning intentions

- ▶ To be able to use the finance solver to find the interest rate or time taken for a compounding interest investment with additional payments.
- ▶ To be able to use the finance solver to find the regular payment for a reducing balance loan.
- ▶ To be able to use the finance solver to find the interest rate, time taken or regular payment for an annuity.

As well as finding the future value or balance of a loan, annuity or investment, we can also use the financial solver to find the interest rate, regular payment or length of the loan, annuity or investment.

Using finance solver for investments with additional payments to find interest rates and time taken

Recall that for an investment with compound interest and additional payments:

- **PV:** Negative: you make an investment by giving the bank some money.
- **PMT:** Negative: you make regular payments to the bank.
- **FV:** Positive: when the investment matures, the bank gives you the money.

Example 21 Finding the interest rate for an investment with additional payments

Mingjia puts $20 000 into a compound interest investment where interest compounds monthly. She adds $50 per month. She wants her investment to reach $40 000 in 10 years.

Find the annual interest rate required for this to occur. Round your answer to two decimal places.

Explanation

1 Open finance solver and enter the following:
- **N**: 120 (10 years)
- **PV**: −20000
- **PMT**: −50
- **FV**: 40000 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

2 Solve for **I**.

3 Write your answer.

Solution

N:	120
I%:	4.807676
PV:	−20000
Pmt or PMT:	−50
FV:	40000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Minjia would require an interest rate of 4.81% per annum.

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When using a financial solver, rounding is very important so it is always a good idea to check your answer in case you need to round up or down accordingly.

Example 22 Finding the regular monthly payment and time taken for an investment with additions to the principal

Winston puts \$20 000 into an investment, paying 5.1% interest per annum, compounding monthly.

a If Winston wants his investment to be worth at least \$40 000 in 5 years, what is the minimum he will need to add each month?

b If Winston invests \$1000 each month immediately after interest is calculated, what is the minimum number of months required for his investment to at least triple in value?

Explanation

a **1** Open finance solver and enter the following:

- **N**: 60 (5 years)
- **I%**: 5.1 (annual interest rate)
- **PV**: −20000
- **FV**: 40000 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

2 Solve for **Pmt** or **PMT**.

Note: The sign of Pmt or PMT is negative, because it is money that Winston invests.

3 Write your answer, noting that \$208.34 per month is insufficient as it gives a balance of \$39999.89…

b **1** Change the payment **Pmt** or **PMT** to −1000 and the **FV** to 60 000 and solve for **N**.

2 Write your answer, noting that 34 months has a **FV** of \$59598.147… so we need to round up

Solution

a

N:	60
I%:	5.1
PV:	−20000
Pmt or PMT:	−208.341646
FV:	40000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Pmt or PMT:	−208.34
FV:	−39999.8877

Winston will add \$208.35 each month to the investment.

b

N:	34.3211
I%:	5.1
PV:	−20000
Pmt or PMT:	−1000
FV:	60000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

The value of Winston's investment will take 35 months to triple.

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Analysing a reducing balance loan with financial solver

Recall that for a reducing balance loan:

- **PV:** Positive - the bank gives you money through a loan
- **PMT:** Negative - you repay the loan by making regular repayments
- **FV:** Negative, zero or positive - balance after the payment is made.

Example 23 Determining the payment amount, total repayment and total amount of interest paid for a reducing balance loan

Sipho borrows $10 000 to be repaid in 59 equal monthly payments followed by a 60th payment of less than one dollar more than the regular payment. Interest is charged at the rate of 8% per annum, compounding monthly.

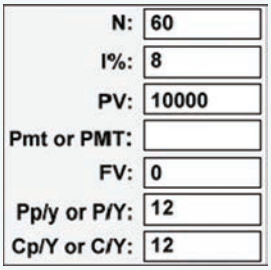
a Find the regular monthly payment amount. Round your answer to the nearest cent.
b Find the final payment. Round your answer to the nearest cent.
c Find the total of the repayments on the loan. Round your answer to the nearest cent.
d Find the total amount of interest paid. Round your answer to the nearest cent.

Explanation

a 1 Open Finance Solver and enter the following:

- **N**: 60 (number of monthly payments in 5 years, assuming 60 equal payments)
- **I%**: 8 (annual interest rate)
- **PV**: 10000
- **FV**: 0 (the balance will be zero when the loan is repaid)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solution

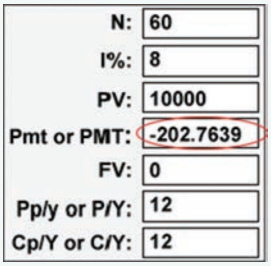


2 Solve for the unknown future value (Pmt or PMT). On the:

- *TI-Nspire:* Move the cursor to the **Pmt** entry box and press [enter] to solve.
- *ClassPad:* Tap on the **PMT** entry box and tap **Solve**.

The amount –202.7639… now appears in the **Pmt** or **PMT** entry box.

Note: The sign of the payment is negative to indicate that this is money Sipho is giving back to the lender.



3 Write your answer.

Sipho repays $202.76 as the regular payment.

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b To find the final payment:

1 Find the final value after 60 payments of \$202.76.

2 Since **FV** is −0.289, the final payment is 0.29 more than the regular payment.

Final payment
= 202.76 + 0.29
= 203.05
Final payment is \$203.05

c Total of repayments of the loan = 59 × regular payment + final payment

Total of repayments = 59 × 202.76 + 203.05 = \$12 165.89

d Total interest = total repayments − the principal

Interest paid = 12 165.89 − 10 000 = \$2165.89

Analysing an annuity with financial solver

Recall that for an annuity:

- **PV:** Negative: you buy an annuity by giving the bank some money.
- **PMT:** Positive: you receive regular payments from the bank.
- **FV:** Positive or zero: balance after the payment is made.

Example 24 Finding the interest rate, time taken and regular payment for an annuity

Joe invests \$200 000 into an annuity, with interest compounding monthly.

a What interest rate would allow Joe to withdraw \$2500 each month for 10 years? Round your answer to one decimal place.

b Assume the interest rate is 5% per annum and that Joe receives a regular monthly payment of \$3000. For how many months will Joe receive a regular payment?

c Assume that the interest rate is 5% per annum and that Joe wishes to be paid monthly payments for 10 years. How much will he regularly receive each month?

d If Joe receives the regular monthly payment found in part **c** for 119 months, what will his final payment be? Round your answer to the nearest cent.

Explanation

a **1** Open finance solver and enter the following:

- **N**: 120 (10 years)
- **PV**: −200000
- **PMT**: 2500
- **FV**: 0 (exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solve for I%.

Solution

N:	120
I%:	8.68922416
PV:	-200000
Pmt or PMT:	2500
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

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2 Write your answer.

Joe would require an interest rate of 8.7% per annum to make monthly withdrawals of $2500 for 10 years.

b **1** Change the payment **Pmt** or **PMT** to 3000 and solve for **N**.

N:	78.2639745
I%:	5
PV:	-200000
Pmt or PMT:	3000
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Write your answer, rounding down as we are only counting regular payments.

Joe will receive a regular payment for 78 months.

c **1** Open the finance solver on your calculator and enter the information below, as shown.

- **N**: 120 (10 years)
- **I%**: 5 (annual interest rate)
- **PV**: −200000
- **FV**: 0 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	120
I%:	5
PV:	-200000
Pmt or PMT:	
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for **Pmt** or **PMT**.

Note: The sign of Pmt or PMT is positive, because it is money received.

N:	120
I%:	5
PV:	-200000
Pmt or PMT:	2121.3103
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer.

Joe will receive $2121.31 each month from the annuity.

d Find the final value after 120 months of $2121.31.

N:	120
I%:	5
PV:	−200000
Pmt or PMT:	2121.31
FV:	0.047
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Since **FV** is 0.047…, the final payment is 0.05 more than the regular payment.

Final payment is $2121.36

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Skill-sheet

Exercise 8F

Analysing an investment with additional payments using a financial calculator

Example 21

1 Armaan puts \$15 000 into an investment that compounds annually.
Find the annual interest rate that would allow Armaan's investment to reach \$100 000 after 10 years if he invests an additional \$4500 each year for 10 years. Round your answer to two decimal places.

Example 22

2 Jemima puts \$30 000 into an investment that compounds monthly.

a What annual interest rate would allow Jemima's investment to double after 5 years if she invests an additional \$400 each month for 5 years? Round your answer to one decimal place.

Assume the investment pays 3.2% per annum, compounding monthly.

b **i** If she wants her investment to be worth at least \$40 000 in 1 year, what is the minimum she will need to add to the investment each month? Round your answer to the nearest cent.

Note: You will need to round your answer up so that it reaches \$40 000.

ii If she invests an additional monthly payment of at least \$1000, what is the minimum number of months that it will take for the investment to first reach \$100 000?

3 Kelven puts \$7500 into an investment, paying 4.7% per annum, compounding monthly. He makes regular additional contributions to the investment each month. After one year, Kelven's investment is worth \$13 991.15 to the nearest cent.

a Find the amount of Kelven's regular monthly payment. Round your answer correct to the nearest cent.

b **i** Find the amount that Kelven invested in the first year through monthly payments.

ii Find the increase in the value of the investment in the first year.

iii Hence, find how much interest was earned in the first year. Round your answer correct to the nearest cent.

c Given Kelven's monthly payment found in **a**, how many months will it take for Kelven's investment to be worth at least \$20 000?

Analysing a reducing balance loan using a financial calculator

Example 23

4 Dan arranges to make regular payments of \$450 per month followed by a single smaller final payment to repay a loan of \$20 000. Interest is charged at 9.5% per annum, compounding monthly.
Find the number of monthly payments required to pay out the loan.

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5 A building society offers $240 000 loans at an interest rate of 10.25% compounding monthly for a 30 year period.

a If payments are $2200 per month, calculate the amount still owing on the loan after 12 years. Round your answer to the nearest cent.

b If the loan has a regular monthly payment of $2150.64 for the first 359 payments, calculate:

i the final payment, rounding your answer to the nearest cent.

ii the total amount repaid, rounding your answer to the nearest cent.

iii the total amount of interest paid, rounding your answer to the nearest cent.

6 Rahul borrows $17 000 at an interest rate of 6.8% per annum, compounding monthly. Rahul wishes to pay off the reducing balance loan in 30 months by making equal payments for 29 months followed by a final payment that is as close to the regular payment as possible.

a Find the regular monthly payment. Round your answer to the nearest cent.

b Find the final payment of the loan. Round your answer to the nearest cent.

c Find the total of the repayments of the loan.

d Find the total amount of interest that Rahul has paid.

7 Cale borrows $140 000 at an interest rate of 8.6% per annum, compounding quarterly. Cale makes regular equal quarterly payments except for the final payment which is as close to the regular payment as possible.

a If Cale pays off the loan in 10 years with 39 regular quarterly payments of $5253.39:

i find the final payment. Round your answer to the nearest cent.

ii find the total amount that Cale repays.

b Rounding each of your answers to the nearest cent, if Cale pays off the loan in 15 years, find

i the regular quarterly payment.

ii the final payment.

iii the total cost of repaying the loan.

8 Lorenzo borrows $250 000 at 5.2% per annum, compounding fortnightly. Lorenzo makes 649 equal fortnightly payments followed by a final payment which is as close to the regular payment as possible. Find the total cost of repaying the loan. Round your answer to the nearest cent.

9 Joan takes out a loan of $50 000 with an interest rate of 4.9% per annum, compounding monthly. She makes regular monthly payments for 23 months of $2191.33 followed by a single final payment. How much interest does Joan pay in total over the duration of the two year loan?

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Analysing an annuity using a financial calculator

Example 24

10 Olek invests \$100 000 into an annuity with interest compounding monthly.

a What is the smallest interest rate that would allow Olek to withdraw \$2500 each month for 4 years. Round your answer to two decimal places.

b Assume the interest rate is 6% and Olek wishes to be paid monthly payments for 4 years. How much will he receive each month as his regular payment? Assume that the final payment is as close as possible to the regular payment.

c Assume the interest rate is 6% and Olek receives a regular payment of \$2000. For how many months will he receive his full payment?

11 Sophia invests \$300 000 into an annuity, paying 4.3% interest per annum, compounding quarterly. She wishes to receive a payment of at least \$5000 every quarter. For how many quarters will Sophia receive at least \$5000?

12 Kai invests \$500 000 in an annuity. The annuity earns interest at the rate of 4.7% per annum, compounding monthly. The balance of Kai's annuity at the end of the first year of the investment is \$474 965.28.

a What monthly payment did Kai receive? Round your answer to the nearest cent.

b How much interest would Kai's annuity earn in the first year? Round your answer to the nearest cent.

Exam 1 style questions

13 Simone invests \$3000 in an account that pays interest at the rate of 3.1% per annum, compounding monthly. She makes an additional payment of \$250 each month.
The number of months that it will take the investment to reach a balance of at least \$30 000 is

A 40 **B** 41 **C** 91 **D** 92 **E** 93

14 Lachlan borrows \$480 000 to buy an apartment using a reducing balance loan that compounds monthly.
Lachlan makes regular monthly payments of \$3075.72 followed by a final payment of \$3075.53. If the loan is paid out fully in 20 years, the annual interest rate is closest to

A 0.3875% **B** 0.0465% **C** 4.65% **D** 11.55% **E** 14.67%

15 Audrey invests \$85 000 in an annuity, paying 6.3% per annum, compounding monthly. Audrey receives a regular monthly payment from the annuity.
If the value of the annuity after one year is \$71 983.41, the amount of interest earned in the first year is closest to

A \$1500 **B** \$4983 **C** \$5355 **D** \$13 017 **E** \$31 016

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8G Solving harder financial problems

Learning intentions

- To be able to find the value of an investment when the regular payment changes.
- To be able to analyse the impact of a change in the interest rate on a reducing balance loan, an annuity and an investment.

Sometimes the conditions of a reducing balance loan can change, requiring the regular repayment to increase or decrease for the loan to be repaid in full. Similarly, a change in the interest rate can also alter the payment received from an annuity or the balance of an investment with compound interest rates. A financial solver on the CAS calculator can help to solve for the regular payment or the new balance after a change has occurred.

Changing the regular payment to an investment

Sometimes an investor may want to change the regular additional payment to an investment. When this happens, we need to consider the investment in two parts. First, we consider the time before the change. Then, we consider the time after the change occurs.

Banks and other financial institutions do not round the value of an investment or loan until the investment is withdrawn or the loan is fully repaid. Thus, for any intermediate step, we will use unrounded values.

Example 25 Finding the value of an investment when the regular payment changes

Derek invests $50 000 into a compound interest investment paying 6.1% per annum, compounding annually. Derek invests an additional $8000 per year immediately after interest is calculated.

After five years, Derek increases his additional investment to $10 000 per year.

Calculate the value of Derek's investment after twelve years (in total).

Explanation

1 Open finance solver and enter the following:
- **N**: 5 (5 years before the change)
- **I%**: 6.1 (annual interest rate)
- **PV**: -50000 (value of initial investment)
- **PMT**: -8000 (additional amount added)
- **Pp/Y**: 1 (annual payment)
- **Cp/Y**: 1 (interest compounds annually)

Solve for **FV**.

Solution

N:	5
I%:	6.1
PV:	–50000
Pmt or PMT:	–8000
FV:	112414.364
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

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2 Change the following in finance solver:

- **N**: 7 (7 years after the change)
- **PV**: -112414.364. . . (copied from FV above)
- **PMT**: -10000 (additional amount added)

Solve for **FV**

N:	7
I%:	6.1
PV:	–112414.3641
Pmt or PMT:	–10000
FV:	254343.79745
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

3 Write your answer.

The value of Derek's investment is $254 343.80

Changing the interest rate

Example 26 Finding the final payment of a reducing balance loan when the interest rate changes

Adrian borrows $150 000 for 25 years at an interest rate of 6.8% per annum, compounding monthly.

For the first three years, Adrian repays $1041.11 each month.

After 3 years, the interest rate rises to 7.2% per annum. Adrian still wishes to pay off the loan in 25 years so makes 263 monthly payments of $1076.18 followed by a final payment.

Calculate the final payment to ensure the loan is fully repaid at the end of 25 years. Round your answer to the nearest cent.

Explanation

1 Open the finance solver on your calculator and enter the information below, as shown opposite.

- **N**: 36 (number of monthly payments in 3 years)
- **I%**: 6.8 (annual interest rate)
- **PV**: 150000 (initial value of loan)
- **Pmt**: −1041.11 (monthly repayments)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Note: You can enter **N** as 3 × 12 (3 years of monthly payments). The finance solver will calculate this as 36 for you.

Solution

N:	36
I%:	6.8
PV:	150000
Pmt or PMT:	-1041.11
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

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2 Solve for **FV**.

N:	36
I%:	6.8
PV:	150000
Pmt or PMT:	-1041.11
FV:	-142391.8359
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 If the loan is still to be repaid in 25 years, there are still 22 years left.
Change:

- **N** to 22 × 12 or 264 payments
- **I(%)** to 7.2 (the new interest rate)
- **PV** to 142391.83593707 (the balance after 3 years)
- **Pmt** to −1076.18

N:	264
I%:	7.2
PV:	-142391.8359
Pmt or PMT:	-1076.81
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

4 Solve for **FV**.

N:	264
I%:	7.2
PV:	-142391.8359
Pmt or PMT:	-1076.18
FV:	0.11735068
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

5 Since **FV** is 0.1173…, the final payment decreases by \$0.12.

Final payment =
\$1076.18 – \$0.12 = \$1076.06
Thus, the final payment will be \$1076.06

Note: It is important that you do not round prematurely or you will get the incorrect answer of \$1076.08

A similar analysis can be used for both annuities and investments with additions to the principal.

Exercise 8G

Changing the regular payment

Example 25

1 Danielle invests \$8000 into a compound interest investment paying 7.6% per annum, compounding annually.
She invests an additional \$1000 per year immediately after interest is calculated.
After five years, Danielle increases her additional investment to \$2000 per year.
Calculate the value of Danielle's investment after twelve years (in total). Round your answer to the nearest cent.

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2 Peta invests $20 000 into a compound interest investment paying 4.8% per annum, compounding monthly.
Peta invests an additional $200 per month immediately after interest is calculated.
After ten years, Peta increases her additional investment to $500 per month.
Caculate the value of Peta's investment after twenty years (in total). Round your answer to the nearest cent.

3 Jarrod opens an account with an initial balance of $0 that pays interest at a rate of 6% per annum, compounding monthly.
He makes monthly deposits of $500 to the account for 10 years.
After 10 years of making deposits, Jarrod withdraws the balance and places it in an annuity, also with an annual interest rate of 6%, compounding monthly. He withdraws $500 each month from the account.

a How much does he invest in the annuity after the initial 10 years?

b How much will remain in the annuity after 10 years? Round your answer to the nearest cent.

Reducing balance loans with changing conditions

Example 26

4 Julien borrows $35 000 for 20 years at an interest rate of 10.5% per annum, compounding monthly.
For four years, he pays $349.43 each month.
After four years, the interest rate rises to 13.75% per annum. Julien still wishes to pay off the loan in a total of 20 years so he makes 191 monthly payments of $418.66 followed by a final payment. For the loan to be fully repaid to the nearest cent, Julien's final repayment will be a smaller amount.
Calculate the final payment that Julien must make to repay the loan in 20 years. Round your answer to the nearest cent.

5 A couple negotiates a 25-year mortgage of $500 000 at a fixed rate of 7.5% per annum compounding monthly for the first seven years.
The monthly repayment amount of $3694.96 is paid each month for seven years.
After seven years, the interest rate rises to 8.5% per annum. The couple now pay $3959.44 each month.
Calculate the value of the loan after a further seven years at the higher interest rate. Round your answer to the nearest cent.

6 Zian borrows $750 000 for a new home at an interest rate of 8.5% per annum, compounding monthly.
For the first five years, he only pays the interest so the value of the loan remains at $750 000.

a Calculate Zian's monthly repayments.

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After five years the interest rate increases to 9.4%. Zian must now pay more each month in order to pay the loan in full within the original 30 years. He does this by making 299 regular monthly repayments followed by a final payment which is as close to the regular payment as possible.

b Calculate the new regular monthly payment amount Zian must make.

c Find the final payment.

d Calculate the total amount that Zian pays over 30 years.

e How much interest will Zian pay over the lifetime of the loan?

Changed conditions with annuities

7 Helen has \$80 000 to invest. She chooses an annuity that pays interest at the rate of 6.4% per annum, compounding monthly. Helen expects her investment to be fully exhausted after 15 years.
She receives a monthly payment of \$692.50 each month for two years.
After two years, the interest rate of Helen's investment was reduced to 6.2% per annum, compounding monthly.

a If Helen continues to withdraw \$692.50 each month, how many more months can she withdraw this regular amount?

b The final payment received from this annuity will be less than the regular repayments. Find the final payment that will exhaust the annuity.

8 Ethan invests \$125 000 into an annuity from which he receives a regular monthly payment of \$850. The interest rate for the annuity is 5.4% per annum, compounding monthly.

a Let V_n be the balance of the annuity after n monthly payments. Write a recurrence relation written in terms of V_0, V_{n+1} and V_n to model the value of this annuity from month to month.

b After two years, the interest rate for this annuity will fall to 4.1%.
So that Ethan will continue to receive a monthly payment of \$850 for the following 18 years, he will add an extra one-off amount to the annuity at this time.
Determine the minimum value of the one-off addition. Give your answer to the nearest dollar.

9 Sameep deposits \$150 000 into a savings account earning 6% per annum, compounding monthly, for 10 years. He makes no withdrawals or deposits during that time.

a Let S_n be the balance of Sameep's investment after n months. Write a recurrence relation to model this investment.

b What is the value of this account after 10 years? Round your answer to the nearest cent.

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After 10 years Sameep withdraws the money and invests the full amount into an annuity. He will require this investment to provide monthly withdrawals of \$2600.

c What is the minimum annual interest rate required if Sameep's investment is to be exhausted after 10 additional years? Round your answer to two decimal places.

Changed conditions on investments

10 Marcus' grandparents place \$2000 in an investment account that pays interest at a rate of 4% per annum, compounding annually. For 18 years from Marcus' birth, they contribute an additional \$1000 to the account each year.

a Find the balance of the investment after 18 years. Round your answer to the nearest cent.

When Marcus turns 18, the interest rate increases to 5% and his grandparents stop contributing.

b Give the balance of the account to the nearest dollar when Marcus turns 21.

11 When Jessica starts working, she sets up an investment account with an initial balance of \$1000. Each month she deposits \$200 into the account.
The account has an annual interest rate of 4.9% compounding monthly.

a Find the balance of the investment after one year. Round your answer to the nearest cent.

b How much interest has the investment earned in the first year? Round your answer to the nearest cent.

After three years, the interest rate increases to 6% per annum and Jessica will increase her monthly deposit to \$350 per month.

c Find the balance of Jessica's investment account after two years at the higher interest rate. Round your answer to the nearest cent.

d How many payments of \$350 will Jessica need to make until her investment first exceeds \$35 000?

Exam 1 style questions

12 Cherry borrowed \$500 000 to buy an apartment.
The interest rate for this loan was 4.31% per annum, compounding monthly.
Cherry paid \$4200 per month for the first two years.
After these two years, the interest rate changed. Cherry was able to pay off the loan in a further 8 years by paying \$5361.49 each month.
The interest rate, per annum, for the final 8 years of the loan was closest to

A 3.90% **B** 4.00% **C** 4.10% **D** 4.30% **E** 4.31%

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13 Thirty years ago, Irene invested a sum of money in an account earning interest at the rate of 3.1% per annum, compounding monthly.
After 10 years, the interest rate changed.
For the next twenty years, the account earned interest at the rate of 2.7% per annum, compounding monthly.
The balance of her account today is \$876 485.10.
The sum of money that Irene originally invested is closest to

A \$360 300 **B** \$375 000 **C** \$390 000 **D** \$511 100 **E** \$670 000

14 Calvin plans to retire from his work in 12 years' time and hopes to have \$800 000 in an annuity investment at that time.
The present value of this annuity investment is \$227 727.96, where the interest rate is 3.6% per annum, compounding monthly.
To make this investment grow faster, Calvin adds \$2500 at the end of each month.
Two years from now, Calvin expects the interest rate to fall to 3.3% per annum, compounding monthly, and to remain at this level until he retires.
When the interest rate changes, Calvin must change his monthly payment if he wishes to make his retirement goal.
The value of his new monthly payment will be closest to

A \$1950 **B** \$2500 **C** \$2560 **D** \$2600 **E** \$2630

8H Interest-only loans

Learning intentions

- To be able to find the regular payment amount for an interest-only loan with and without a financial solver.
- To be able to find the amount borrowed for an interest-only loan.
- To be able to find the interest rate for an interest-only loan.

In an **interest-only loan**, the borrower repays only the interest that is charged. As a result, the balance of the loan remains the same for the duration of the loan. To understand how this happens, consider a loan of \$1000 with an interest rate of 5% per annum, compounding yearly. The interest that is charged after 1 year will be 5% of \$1000, or \$50. If the borrower only repays \$50, the value of the loan will still be \$1000.

The recurrence relation $V_0 = 1000, \quad V_{n+1} = 1.05V_n - D$ can be used to model this loan. The table below shows the balance of the loan over a 4-year period for three different payment amounts: $D = 40, D = 50$ and $D = 60$.

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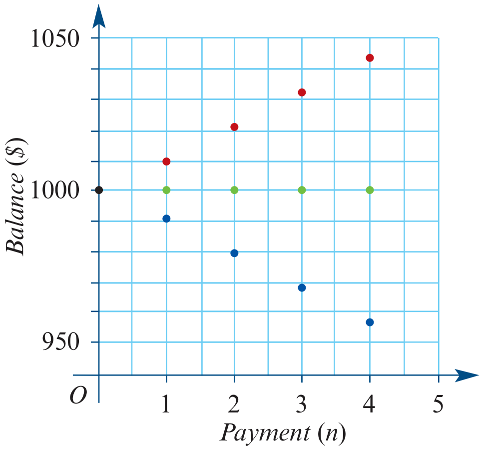
$D = 40$	$D = 50$	$D = 60$
$V_0 = 1000$, $V_{n+1} = 1.05V_n - 40$	$V_0 = 1000$, $V_{n+1} = 1.05V_n - 50$	$V_0 = 1000$, $V_{n+1} = 1.05V_n - 60$
$V_0 = 1000$ $V_1 = 1010$ $V_2 = 1020.50$ $V_3 = 1031.525$ $V_4 = 1043.101\ldots$	$V_0 = 1000$ $V_1 = 1000$ $V_2 = 1000$ $V_3 = 1000$ $V_4 = 1000$	$V_0 = 1000$ $V_1 = 990$ $V_2 = 979.50$ $V_3 = 968.475$ $V_4 = 956.898\ldots$
The amount owed keeps increasing.	The amount owed stays constant.	The amount owed keeps decreasing.

We can plot these balances against the payment number.

If the periodic payments on this loan are smaller than \$50 (e.g. $D = 40$), the amount owed will increase over time. The balance of the loan is shown as red dots.

If the periodic payments on this loan are larger than \$50 (e.g. $D = 60$), the amount owed will decrease over time. The balance of the loan is shown as blue dots.

If the periodic payments on this loan are exactly \$50, then the amount owed on the loan will always be \$1000. The balance of the loan is shown as green dots.



A loan where the balance stays constant is called an **interest-only loan** and is commonly used for investment purposes.

Modelling interest-only loans

Let V_n be the value of the interest-only loan after n payments have been made. Then

$$V_0 = \text{principal}, \quad V_{n+1} = RV_n - D$$

where $R = 1 + \dfrac{r}{100 \times p}$ is the growth multiplier, r is the annual interest rate, p is the number of compounding periods per year and D is the regular payment per compounding period which is equal to the interest charged, given by

$$D = \frac{r}{100 \times p} \times V_0$$

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Example 27 Finding the regular payment for an interest-only loan

Jane borrows \$50 000 to buy some shares. Jane negotiates an interest-only loan at an interest rate of 9% per annum, compounding monthly. What is the monthly amount Jane will be required to pay?

Explanation

Calculation method

Use the rule $D = \frac{r}{100 \times p} \times V_0$.

1 V_0 is the amount borrowed = \$50 000

2 Calculate the interest payable where $r = 9$ and $p = 12$.

3 Evaluate the rule for these values and write your answer.

Solution

$V_0 = 50\,000$

$D = \frac{r}{100 \times p} \times V_0$

$D = \frac{9}{100 \times 12} \times 50\,000$

$D = 375$

Jane will need to repay \$375 every month on this interest-only loan.

Finance solver method

Consider one compounding period because all compounding periods will be identical.

1 Open Finance Solver and enter the following.

- **N**: 1 (one compounding period)
- **I%**: 9 (annual interest rate)
- **PV**: 50000
- **FV**: −50000 (the amount owing will be the same after one payment)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	1
I%:	9
PV:	50000
Pmt or PMT:	
FV:	–50000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (Pmt or PMT). On the:

- *TI-Nspire:* Move the cursor to the **Pmt** entry box and press [enter] to solve.
- *ClassPad:* Tap on the **PMT** entry box and tap **Solve**.

The amount –375 now appears in the **Pmt** or **PMT** entry box.

N:	1
I%:	9
PV:	50000
Pmt or PMT:	–375
FV:	–50000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

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Example 28 Finding the amount borrowed for an interest-only loan

A loan at 6% per annum, compounding monthly, requires payments of \$440 each month.

If the loan is an interest-only loan, what is the principal?

Explanation	Solution
1 Use formula $D = \frac{r}{100 \times p} \times V_0$, where $D = 440$, $r = 6$, $p = 12$.	Solving for the principal: $440 = \frac{6}{100 \times 12} \times V_0$ $V_0 = 88\ 000$
2 Write the answer.	The principal is \$88 000.

Example 29 Finding the interest rate for an interest-only loan

An interest-only loan of \$1 000 000 requires quarterly payments of \$4000.

What is the annual interest rate on the loan?

Explanation	Solution
1 Use formula $D = \frac{r}{100 \times p} \times V_0$, where $D = 4000$, $p = 4$, $V_0 = 1\ 000\ 000$.	Solving for r: $4000 = \frac{r}{100 \times 4} \times 1\ 000\ 000$ $r = 1.6$
2 Write the answer.	The annual interest rate is 1.6%

Exercise 8H

Finding the regular payment for an interest-only loan

Example 27

1 Georgia borrows \$100 000 to buy an investment property. If the interest on the loan is 7.2% per annum, compounding monthly, find her monthly payment on an interest-only loan.

2 In order to invest in the stockmarket, Jamie takes out an interest-only loan of \$50 000. If the interest on the loan is 8.4% per annum compounding monthly, find his monthly payment amount.

3 Robert takes out an interest-only loan for \$220 000 at an interest rate of 5.46% per annum, compounding fortnightly. Find the fortnightly payment.

4 Frannie borrows \$180 000 at an interest rate of 4.95% per annum, compounding quarterly. If Frannie only pays the interest, find the total payments made over a five year period.

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5 Jackson takes out an interest-only loan of $30 000 from the bank to buy a painting. He hopes to resell it at a profit in 12 months' time. The interest on the loan is 9.25% per annum, compounding monthly. He makes monthly payments on the loan.

a Find the total amount that Jackson pays in 12 months.

b How much will he need to sell the painting for in order not to lose money?

6 Ric takes out an interest-only loan of $600 000 to buy an investment property. The interest on the loan is 5.11% per year, compounding monthly.

a Calculate Ric's monthly repayments if he only pays the interest.

b Ric sells the property after 10 years. Calculate the total interest paid on the loan.

c How much must Ric sell the property for if he wishes to make a profit of at least $100 000?

7 Mindy borrows $35 000 for 20 years at 6.24% per annum, compounding monthly. For the first five years, Mindy pays interest only.

a Calculate the monthly repayments that Mindy makes for the first two years.

b State the balance of the loan after five years.

c For the next 179 months, Mindy pays $300 per month followed by a smaller payment to fully repay the loan. Find this final repayment. Round your answer to the nearest cent.

d Find the total amount that Mindy paid for the duration of the 20 year loan.

Find the amount borrowed for an interest-free loan

Example 28

8 An interest-only loan with an interest rate of 5.3% per annum, compounding annually, requires annual payments of $2120. What is the principal?

9 An interest-only loan with an interest rate of 6.6% per annum, compounding monthly, requires a monthly payment of $88. What is the principal?

10 Yianni took out an interest-only loan with an interest rate of 4.2% per annum, compounding monthly. Over a two year period, Yianni paid $2352 in total. Find the principal of the loan.

Find the interest rate for an interest-free loan

Example 29

11 An interest-only loan of $4000 requires annual payments of $116. What is the annual interest rate?

12 An interest-only loan of $12 000 compounds monthly and requires monthly payments of $36. What is the annual interest rate?

13 Leo takes out an interest-only loan of $35 000 which compounds monthly and requires monthly payments. Over a two year period, Leo pays a total of $3360. What is the annual interest rate?

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14 Svetlana borrows \$320 000 on an interest-only loan at an interest rate of 4.92% per annum, compounding monthly for the first 5 years. Following this, the interest rate changes.

a Calculate the monthly repayments that Svetlana makes for the first five years.

b Calculate the total amount that Svetlana paid during the first five years.

For the next five years, Svetlana pays \$86 400 on the interest-only loan.

c State the total interest that she paid during the second half of the loan.

d Calculate the monthly repayment that she made during the second half of the loan.

e Hence, find the annual interest rate during the second half of the loan.

Exam 1 style questions

15 Matthew would like to purchase a new home. He establishes a 20 year loan for \$310 000 with interest charged at the rate of 3.84% per annum, compounding monthly. Each month, Matthew will only pay the interest charged for that month.
After three years, the amount that Matthew will owe is

A \$274 288 **B** \$277 222 **C** \$310 000 **D** \$345 712 **E** \$418 350

16 Eve borrowed \$780 000 to buy a house.
The interest rate for this loan was 4.82% per annum, compounding monthly.
A scheduled monthly repayment that allowed Eve to fully repay the loan in 20 years was determined.
Eve decided to pay interest-only for the first two years.
After these two years, the interest rate changed. Eve was still able to pay off the loan in the 20 years by paying the original monthly repayment amount each month.
The new interest rate of the loan was closest to

A 3.9% **B** 4.0% **C** 4.1% **D** 4.2% **E** 4.3%

17 Jason takes out an interest-only loan for five years.
The value of Jason's interest-only loan, V_n, after n months, can be modelled by the recurrence relation

$$V_0 = 56\,000, \qquad V_{n+1} = 1.0034V_n - D$$

The total interest paid on the loan over the five years is

A \$4.08 **B** \$190.40 **C** \$952 **D** \$11 424 **E** \$56 000

8I Perpetuities

Learning intentions

- ► To be able to calculate the regular payment from a perpetuity.
- ► To be able to calculate the investment required to establish a perpetuity.
- ► To be able to calculate the interest rate of a perpetuity.

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Recall that an annuity involves money being deposited in an investment and then withdrawn over time in the form of regular payments. In our earlier analysis, we considered the case where the withdrawals were made to exhaust the annuity over a given time frame. That is, the value of the annuity eventually reached zero.

If the regular payments are *smaller* than the interest received, the annuity will continue to grow. If the payments received are exactly the same as the interest earned in each compounding period, the annuity will maintain its value indefinitely. This type of annuity is called a **perpetuity** and the payments that are equal to the interest earned can be made forever (or in Perpetuity). Perpetuities have the same relationship to annuities as interest-only loans have to reducing balance loans.

Modelling perpetuities

Let V_n be the value of the perpetuity after n payments have been made. Then

$$V_0 = \text{principal}, \qquad V_{n+1} = RV_n - D$$

where $R = 1 + \frac{r}{100 \times p}$ is the growth multiplier, r is the annual interest rate, p is the number of compounding periods per year and D is the regular payment per compounding period which is equal to the interest earned, given by

$$D = \frac{r}{100 \times p} \times V_0$$

Example 30 Calculating the payment from a perpetuity

Elizabeth invests her superannuation payout of $500 000 into a perpetuity that will provide a monthly income.

If the interest rate for the perpetuity is 6% per annum, what monthly payment will Elizabeth receive?

Explanation	Solution
1 Find the monthly interest earned.	$D = \frac{r}{100 \times p} \times V_0$ $= \frac{6}{100 \times 12} \times 500000$ $= 2500$
2 Write your answer, rounding as required.	Elizabeth will receive $2500 every month from her investment.

Example 31 Calculating the investment required to establish a perpetuity

Calculate how much money will need to be invested in a perpetuity account, earning interest of 4.8% per annum compounding monthly, if $300 will be withdrawn every month.

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Explanation

1 Use the rule $D = \frac{r}{100 \times p} \times V_0$ to write down an equation that can be solved for V_0.

Solution

$$300 = \frac{4.8}{100 \times 12} \times V_0$$
$$V_0 = \frac{300}{0.004}$$
$$= 75000$$

2 Write your answer.

$75 000 will need to be invested to establish the perpetuity investment.

Problems involving perpetuities can also be solved using a financial calculator.

Example 32 Calculating the interest rate of a perpetuity

A university mathematics faculty has $30 000 to invest. It intends to award an annual mathematics prize of $1500 with the interest earned from investing this money in a perpetuity.

What is the minimum interest rate that will allow this prize to be awarded indefinitely?

Explanation

We will consider just one compounding period because all compounding periods will be identical.

Calculation method

1 Use the rule $D = \frac{r}{100 \times p} \times V_0$ and solve the equation for r.

Solution

$$1500 = \frac{r}{100 \times 1} \times 30\,000$$
$$1500 = r \times 300$$
$$r = \frac{1500}{300} = 5$$

2 Write your answer.

The minimum annual interest rate to award this prize indefinitely is 5%.

Financial solver

1 Open Finance Solver and enter the following.

- **N**: 1 (one payment)
- **PV**: –30 000
- **Pmt or PMT**: 1500 (prize is $1500 each year)

N:	1
I%:	
PV:	-30000
Pmt or PMT:	1500
FV:	30000
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

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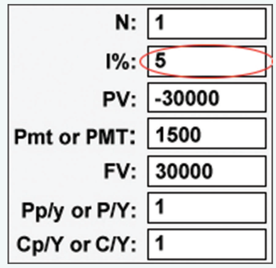
- **FV**: 30 000 (the balance will be the same after each payment)
- **Pp/Y**: 1 (yearly payment)
- **Cp/Y**: 1 (interest compounds yearly)

2 Solve for the unknown interest rate (**I%**). On the:

- *TI-Nspire:* Move the cursor to the **I%** entry box and press enter to solve.
- *ClassPad:* Tap on the **I%** entry box and tap **Solve** The amount **5** now appears in the **I%** entry box.



3 Write your answer, rounding as required.

The minimum annual interest rate to award this prize indefinitely is 5%.

Exercise 8I

Calculating the payment from a perpetuity

Example 30

1 Suzie invests her inheritance of \$642 000 in a perpetuity that pays 6.1% per annum compounding quarterly.

a What quarterly payment does she receive?

b After five quarterly payments, how much money remains invested in the perpetuity?

c After 10 quarterly payments, how much money remains invested in the perpetuity?

2 Craig wins \$1 000 000 in a lottery and decides to place it in a perpetuity that pays 5.76% per annum interest, compounding monthly.

a What monthly payment does he receive?

b How much interest does he earn in the first year?

3 Donna sold her cafe business for \$720 000 and invested this amount in a perpetuity. The perpetuity earns interest at a rate of 3.6% per annum. Interest is calculated and paid monthly.

a What monthly payment will Donna receive from this investment?

b After three years, the interest rate for the perpetuity increases. Describe whether Donna's monthly payment will increase, decrease or stay the same.

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Calculating the investment required to establish a perpetuity

Example 31

4 Geoff wishes to set up a fund so that every year $2500 is donated to the RSPCA in his name.
If the interest on his initial investment averages 2.5% per annum, compounding annually, how much should he invest?

5 Barbara wishes to start a scholarship that will reward the top mathematics student each quarter with a $600 prize.
If the interest on the initial investment averages 4.8% per annum, compounding quarterly, how much should be invested?

6 Omar inherits $920 000 and splits the money between a perpetuity and an annuity investment.
The perpetuity pays $2340 each month based on an interest rate of 5.2% per annum that compounds monthly.
The annuity investment has an interest rate of 4.8% per annum that compounds monthly.

- **a** Calculate how much Omar invested in the perpetuity.
- **b** State how much is initially invested in the annuity investment.
- **c** Omar realises that he only needs $2000 from his perpetuity each month and so he adds $340 as an aditional payment into the annuity investment each month. Find the value of the annuity investment after three years.

Calculating the interest rate of a perpetuity

Example 32

7 If Sandra has $80 000 to invest, what is the minimum interest rate she requires to provide an annual donation of $2400 indefinitely into the future if interest is compounding annually?

8 Benjamin has $12 000 to invest in a perpetuity to provide a prize of $750 each year. What is the minimum interest rate that he requires in order to pay the prize in perpetuity if interest compounds annually?

9 On retiring from work, Tyson received a superannuation payout of $694 400.
If Tyson invests the money in a perpetuity, he would then receive $3645.60 each month for the rest of his life. At what annual percentage rate is interest earned by this perpetuity?

Analysis of perpetuities

10 Marco invests $350 000 in a perpetuity from which he will receive a regular monthly payment of $1487.50.
The perpetuity earns interest at the rate of 5.1% per annum.

- **a** Determine the total amount, in dollars, that Marco will receive after one year of monthly payments.

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b Write down the value of the perpetuity after Marco has received one year of monthly payments.

c Let M_n be the value of Marco's perpetuity after n months. Write down a recurrence relation in terms of M_0, M_{n+1} and M_n, that would model the value of this perpetuity over time.

11 Zihan invests $200 000 in a perpetuity from which he will receive a regular payment. There are two options available:

- Option A: Earn interest at a rate of 3.6% per annum, compounding monthly with a regular monthly payment.
- Option B: Earn interest at a rate of 3.8% per annum, compounding annually with a regular annual payment.

a Calculate the monthly payment from Option A.

b Calculate the annual payment from Option B.

c Determine which option pays the most over one year.

d Let Z_n be the value of the perpetuity after n payments that pays the most over the course of a year. Write down a recurrence relation in terms of Z_0, Z_{n+1} and Z_n, that would model the value of this perpetuity over time.

Exam 1 style questions

12 Which of the following recurrence relations could be used to model the value of a perpetuity investment, P_n, after n months?

A $P_0 = 100\,000$, $P_{n+1} = 1.005P_n + 500$

B $P_0 = 100\,000$, $P_{n+1} = 1.005P_n - 500$

C $P_0 = 100\,000$, $P_{n+1} = 0.005P_n - 500$

D $P_0 = 200\,000$, $P_{n+1} = 1.003P_n + 600$

E $P_0 = 200\,000$, $P_{n+1} = 1.103P_n - 600$

13 Aaliyah invests $120 000 in a perpetuity from which she will receive a regular monthly payment. The perpetuity has a compound interest rate of 5.2% per annum and compounds monthly. The amount that Aaliyah will receive from the perpetuity in the first two years is closest to

A $520 **B** $624 **C** $6240 **D** $12 480 **E** $149 760

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Key ideas and chapter summary

Assign-ment

Reducing balance loan	A **reducing balance loan** is a loan that attracts compound interest but is reduced in value by making regular payments. Each payment partly pays the interest that has been added and partly reduces the value of the loan.
Annuity	An **annuity** is an investment that earns compound interest and from which regular payments are made.
Amortisation	An **amortising loan** is one that is paid back with periodic payments. An amortising investment is one that is exhausted by regular withdrawals. **Amortisation of reducing balance loans** tracks the distribution of each periodic payment, in terms of the interest paid and the reduction in the value of the loan. **Amortisation of an annuity** tracks the source of each withdrawal, in terms of the interest earned and the reduction in the value of the investment.
Amortisation table	An **amortisation table** shows the amortisation (payment) of all or part of a reducing balance loan or annuity. It has columns for the payment number, the payment amount, the interest paid or earned, the principal reduction or increase and the balance after the payment has been made.
Finance Solver	**Finance Solver** is a function on a CAS calculator that performs financial calculations. It can be used to determine any of the principal, interest rate, periodic payment, future value or number of payments given all of the other values.
Interest-only loan	An **interest-only loan** is a loan where the regular payments made are equal in value to the interest charged. Interest-only loans have the same value after each payment is made.
Perpetuity	A **perpetuity** is an annuity where the regular payments or withdrawals are the same as the interest earned. The value of a perpetuity remains constant.

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Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills. ☑

8A **1** I can generate a sequence from a recurrence relation that combines both geometric and linear growth or decay. ☐

See Example 1, and Exercise 8A Question 1

8A **2** I can model compound interest investments with additions to the principal. ☐

See Example 2 and 3, and Exercise 8A Question 2 and 4

8A **3** I can use a recurrence relation to analyse compound interest investments with additions to the principal. ☐

See Example 4, and Exercise 8A Question 8

8A **4** I can determine the annual interest rate from a recurrence relation. ☐

See Example 5, and Exercise 8A Question 10

8B **5** I can model a reducing balance loan with a recurrence relation. ☐

See Example 6 and 7, and Exercise 8B Question 1 and 2

8B **6** I can use a recurrence relation to analyse a reducing balance loan. ☐

See Example 8, and Exercise 8B Question 7

8B **7** I can model an annuity with a recurrence relation. ☐

See Example 9, and Exercise 8B Question 9

8C **8** I can apply the amortisation process. ☐

See Example 10, and Exercise 8C Question 1

8C **9** I can construct an amortisation table for a reducing balance loan. ☐

See Example 11, and Exercise 8C Question 1

8C **10** I can analyse an amortisation table for a reducing balance loan. ☐

See Example 12, and Exercise 8C Question 4

8C **11** I can analyse an amortisation table for an annuity to find the interest rate. ☐

See Example 13, and Exercise 8C Question 7

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8C	12	I can interpret and construct an amortisation table for a compound interest investment with additional payments. See Example 14, and Exercise 8C Question 8	☐
8D	13	I can find the final payment for a reducing balance loan or annuity. See Example 15, and Exercise 8D Question 1	☐
8D	14	I can find the total payment made and the total interest paid. See Example 16, and Exercise 8D Question 5	☐
8D	15	I can plot from an amortisation table. See Example 17, and Exercise 8D Question 9	☐
8E	16	I can determine the value of an investment with regular additions made to the principal using a financial solver. See Example 18, and Exercise 8E Question 1	☐
8E	17	I can determine the balance and final payment of a reducing balance loan after a given number of payments. See Example 19, and Exercise 8E Question 3	☐
8E	18	I can determine the balance of an annuity using a finance solver. See Example 20, and Exercise 8E Question 7	☐
8F	19	I can find the interest rate for an investment with additional payments. See Example 21, and Exercise 8F Question 1	☐
8F	20	I can find the regular monthly payment and the time taken for an investment with additions to the principal. See Example 22, and Exercise 8F Question 2	☐
8F	21	I can determine the payment amount, total cost and total amount of interest paid for a reducing balance loan. See Example 23, and Exercise 8F Question 5	☐
8F	22	I can find the interest rate, time taken and regular payment for an annuity. See Example 24, and Exercise 8F Question 10	☐
8G	23	I can find the value of an investment when the regular payment changes. See Example 25, and Exercise 8G Question 1	☐
8G	24	I can analyse the impact of a change in the interest rate on a reducing balance loan, an annuity and an investment. See Example 26, and Exercise 8G Question 4, 8 and 10	☐

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Review

8H	25	I can find the repayment amount for an interest-only loan with and without finance solver.	☐
		See Example 27, and Exercise 8H Question 1	
8H	26	I can find the amount borrowed for an interest-only loan.	☐
		See Example 28, and Exercise 8H Question 8	
8H	27	I can find the interest rate for an interest-only loan.	☐
		See Example 29, and Exercise 8H Question 11	
8I	28	I can calculate the payment from a perpetuity.	☐
		See Example 30, and Exercise 8H Question 1	
8I	29	I can calculate the investment required to establish a perpetuity.	☐
		See Example 31, and Exercise 8H Question 4	
8I	30	I can calculate the interest rate of a perpetuity.	☐
		See Example 32, and Exercise 8H Question 7	

Multiple-choice questions

1 An investment of \$18 000, earning compound interest at the rate of 6.8% per annum, compounding yearly, and with regular additions of \$2500 every year can be modelled with a recurrence relation. If V_n is the value of the investment after n years, the recurrence relation is

A $V_0 = 18000, V_{n+1} = 1.006V_n - 2500$
B $V_0 = 2500, V_{n+1} = 1.068V_n - 18\,000$
C $V_0 = 18\,000, V_{n+1} = 1.068V_n + 2500$
D $V_0 = 18000, V_{n+1} = 1.068V_n - 2500$
E $V_0 = 2500, V_{n+1} = 1.006V_n - 18000$

2 Let V_n be the value of an investment after n months. The investment is modelled by the recurrence relation $V_0 = 25\,000,\ V_{n+1} = 1.007V_n - 400$. The annual interest rate for this investment is

A 0.084% **B** 0.7% **C** 2.8% **D** 8.4% **E** 36.4%

3 The value of an annuity investment, in dollars, after n months, V_n, can be modelled by the recurrence relation shown below

$$V_0 = 20\,000, \quad V_{n+1} = 1.045V_n + 500$$

The increase in the value of this investment in the second month is closest to

A \$500
B \$900
C \$1100
D \$1500
E \$2300

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Questions 4 and 5 relate to the following information.

A loan of \$28 000 is charged interest at the rate of 6.4% per annum, compounding monthly. It is repaid with regular monthly payments of \$1200.

4 Correct to the nearest cent, the value of the loan after 5 months is

A \$21 611.35 **B** \$22 690.33 **C** \$23 763.59 **D** \$24 831.16 **E** \$31 363.91

5 Following 24 regular payments of \$1200, a final payment is made to fully repay the loan. The final payment on the loan, correct to the nearest cent, will be

A \$1125.41 **B** \$1131.41 **C** \$1175.20 **D** \$1181.47 **E** \$1200

6 A loan of \$6000 is to be repaid in full by 11 quarterly payments followed by a final payment that is as close to the regular payment as possible. Interest at 10% per annum is calculated on the remaining balance each quarter. The regular quarterly payment that is required to pay out the loan is closest to

A \$527.50 **B** \$573.10 **C** \$584.92 **D** \$600 **E** \$630.64

7 Paula borrows \$12 000 from a bank, to be repaid over 5 years. Interest of 12% per annum is charged monthly on the amount of money owed. If Paula makes regular monthly payments of \$266.90, then the amount she owes at the end of the second year is closest to

A \$2880 **B** \$5590 **C** \$6410 **D** \$8040 **E** \$9120

8 Ayush invests \$12 000 in an annuity from which he receives a regular monthly payment of \$239. The annuity earns interest of 7.2% per annum, compounding monthly. The balance of the annuity after three months is closest to

A \$11 495 **B** \$11 496 **C** \$11 665 **D** \$12 938 **E** \$12 939

9 James invests \$50 000 in an annuity from which he receives a regular monthly payment of \$925.30.
The balance of the annuity, in dollars, after n months, J_n, can be modelled by a recurrence relation of the form

$$J_0 = 50\,000, \qquad J_{n+1} = 1.0035J_n - 925.30$$

The balance of the annuity after six months is closest to

A \$45 289 **B** \$45 458 **C** \$45 459 **D** \$56 659 **E** \$56 660

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Questions 10–13 refer to the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	40000.00
1	400.00	160.00	240.00	39760.00
2	400.00	159.04	240.96	39519.04
3	400.00	158.08	241.92	39277.12

10 The principal of this loan is

A \$20 000 **B** \$30 000 **C** \$40 000 **D** \$50 000 **E** \$60 000

11 The periodic payment amount on this loan is

A \$80 **B** \$160 **C** \$240 **D** \$400 **E** \$560

12 The principal reduction of the loan from the third payment is

A \$158.08 **B** \$159.04 **C** \$240 **D** \$240.96 **E** \$241.92

13 Assuming that payments are made monthly and interest compounds monthly, the annual interest rate on the loan is

A 0.4% **B** 0.48% **C** 4% **D** 4.8% **E** 16%

14 Amir borrows \$1500 in a reducing balance loan at a rate of 3.6% per annum, compounding quarterly.
She makes regular repayments of \$383.45 each quarter for three quarters followed by a final payment.
To pay out her loan fully, her final payment is

A \$0.01 **B** \$0.10 **C** \$383.35 **D** \$383.45 **E** \$383.55

15 Monthly withdrawals of \$220 are made from an account that has an opening balance of \$35 300, invested at 7% per annum, compounding monthly. The balance of the account after 1 year is closest to

A \$32 660 **B** \$33 500 **C** \$35 125 **D** \$35 211 **E** \$40 578

16 Tilly invests \$5000 in an account that pays interest at the rate of 3.9% per annum, compounding annually.
She makes an additional payment of \$1200 each year.
The number of years that it will take the investment to first reach a balance of \$20 000 is

A 1 **B** 2 **C** 9 **D** 10 **E** 13

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17 Twenty years ago, Oscar invested \$65 000 in an account earning interest at the rate of 2.8% per annum, compounding monthly.
After 10 years, he made a one-off payment of \$20 000 to the account.
For the next 10 years, the account earned interest at the rate of 3.2% per annum, compounding monthly. The balance of the account today is closest to

A \$65 000 **B** \$85 975.39 **C** \$105 975.39 **D** \$113 719.51 **E** \$145 879.51

18 The monthly payment on an interest-only loan of \$175 000, at an interest rate of 5.9% per annum, compounding monthly, is closest to

A \$198 **B** \$397 **C** \$860 **D** \$1117 **E** \$2581

19 A scholarship will be set up to provide an annual prize of \$400 to the best Mathematics student in a school. The scholarship is paid for by investing an amount of money into a perpetuity, paying interest of 3.4% per annum, compounding annually. The amount that needs to be invested to provide this scholarship is closest to

A \$400 **B** \$800 **C** \$1176 **D** \$11 764 **E** \$11 765

20 Pham invests \$74 000 from which he will receive a regular monthly payment.
The perpetuity has a compound interest rate of 4.8% per annum and compounds monthly. The amount that Pham will receive from the perpetuity in the first two years is closest to

A \$296 **B** \$3233 **C** \$3552 **D** \$7104 **E** \$77 756.64

Written response questions

1 Josie is considering borrowing \$250 000 to buy a house. A home loan at her bank will charge interest at the rate of 4.8% per annum, compounding monthly. Josie will make monthly payments of \$1800.
Let V_n be the value of Josie's loan after n months.

- **a** Write down a recurrence model for the value of Josie's loan after n months.
- **b** After 12 months, how much would Josie owe on this loan? Round your answer to the nearest cent.
- **c** After how many months is the loan first below \$200,000?
- **d** If Josie chose an interest-only loan for the first year:
  - **i** what would her monthly payments be?
  - **ii** how much interest in total would she pay in the first year?
  - **iii** how much would she owe after the 12th payment?

2 Samantha inherited \$150 000 from her aunt. She decides to invest this money into an account paying 6.25% per annum interest, compounding monthly.

- **a** If Samantha deposited her money into a perpetuity, what monthly payment would she receive?
- **b** If Samantha deposited her money into an annuity and withdrew \$1000 per month, how much would she have in the account after 1 year?

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c If Samantha deposited her money into an annuity and withdrew \$2000 per month, after how many months is the investment first below \$100,000?

d If Samantha deposited her money into an annuity and withdrew \$4000 per month for each month until the final month:

i How many regular payments of \$4000 would she receive?

ii What would be the value of her last withdrawal?

3 A loan of \$10 000 is to be repaid over 5 years with 19 equal quarterly payments of \$656.72 followed by a final payment. Interest is charged at the rate of 11% per annum compounding quarterly.
Find:

a the final payment. Round your answer to the nearest cent.

b the sum of all repayments, to the nearest dollar.

c the total amount of interest paid, to the nearest dollar.

4 The Andersons were offered a \$24 800 loan to pay for a new car. Their loan is to be repaid in equal monthly payments of \$750, except for the last month when less than this will be required to fully pay out the loan. The interest rate is 10.8% per annum, compounding monthly.

a Find the number of months needed to repay this loan.

b Calculate the amount of the final payment. Round your answer to the nearest cent.

c Calculate the total interest that is paid on the loan.

5 Elsa borrowed \$100 000 at 9.6% per annum, compounding quarterly. The loan was to be repaid over 25 years with 99 equal quarterly payments followed by a final payment that is as close to the regular payment as possible.

a How much of the first quarterly payment went towards paying off the principal?

b Elsa inherits some money and decides to terminate the loan after 10 years by paying what is owing in a lump sum. How much will this lump sum be?

6 Helene won \$750 000 in a lottery. She decides to place the money in an investment account that pays 4.5% per annum interest, compounding monthly.

a How much will Helene have in the investment account after 10 years?

b After 10 years, Helene withdraws the money from the investment account and places it in an annuity. The annuity pays 3.5% per annum, compounding monthly. Helene receives \$6000 per month from the annuity. For how many months will she receive \$6000?

c Helene's accountant suggests that rather than purchase an annuity she places the money in a perpetuity so that she will be able to leave some money to her grandchildren. If she places \$1,100,000 into a perpetuity that pays 3.6% per annum compounding monthly, how much is the monthly payment that Helene will receive?

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