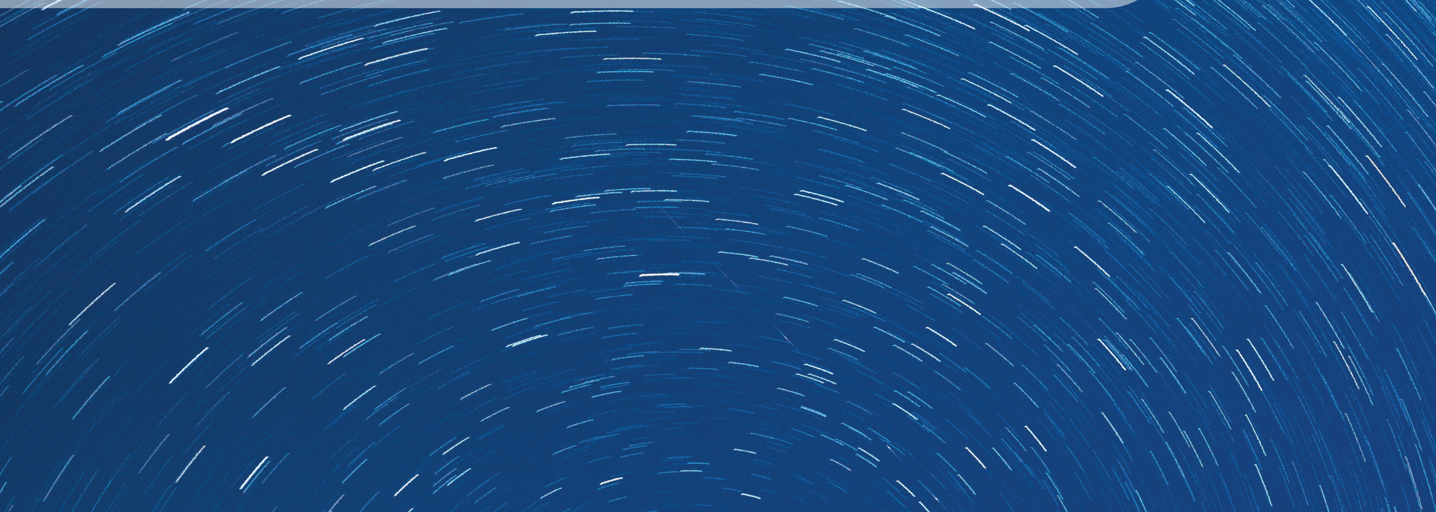
Chapter 5

Investigating and modelling time series



Chapter questions

- What is time series data?
- How do we construct a time series plot?
- How do we recognise features such as trend, seasonality and irregular fluctuations?
- How do we smooth a time series plot using moving means?
- How do we smooth a time series plot using moving medians?
- How do we calculate and interpret seasonal indices?
- How do we calculate and interpret a trend line?
- How do we make forecasts of future values?

In this chapter we will focus on a special case of numerical bivariate data, called **time series data**. In time series data the explanatory variable is always a measure of time (for example hour, day, month or year), and we are concerned with understanding how the response variable is changing over time.

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5A Time series data

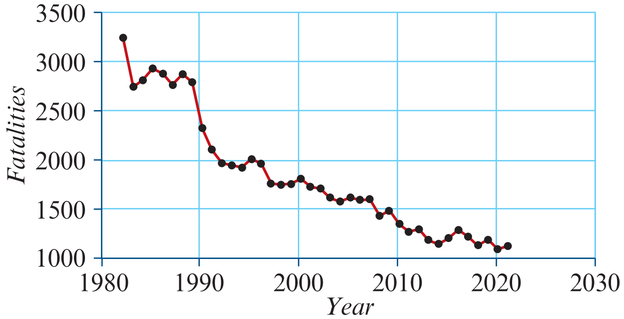
Learning intentions

- ▶ To be able to recognise time series data.
- ▶ To be able to construct a time series plot.
- ▶ To be able to recognise features in the plot such as trend, seasonality and irregular fluctuations.

When data concerned with a variable is collected, observed or recorded at successive intervals of time, it is referred to as **time series data**. An example of time series data is Annual road accident fatalities for Australia, 1982–2021, given in the following table.

Year	*Fatalities*	*Year*	*Fatalities*	*Year*	*Fatalities*	*Year*	*Fatalities*
1982	3252	1992	1974	2002	1715	2012	1300
1983	2755	1993	1953	2003	1621	2013	1187
1984	2822	1994	1928	2004	1583	2014	1150
1985	2941	1995	2017	2005	1627	2015	1209
1986	2888	1996	1970	2006	1598	2016	1293
1987	2772	1997	1767	2007	1603	2017	1225
1988	2887	1998	1755	2008	1437	2018	1135
1989	2801	1999	1764	2009	1491	2019	1195
1990	2331	2000	1817	2010	1353	2020	1095
1991	2113	2001	1737	2011	1277	2021	1127

Since time series data is just a special kind of two numerical variable example, where the explanatory variable is time, we will begin by drawing a scatterplot of the data. In this instance, the scatterplot is called a **time series plot**, with *time* always placed on the horizontal axis. A time series plot differs from a normal scatterplot in that, in general, the points will be joined by line segments in time order. An example of a time series plot, of the road accident fatality data, is given below.



Looking at the time series plot, we can readily see a clear trend of decreasing road fatalities, which is good news for drivers, as this provides some evidence that the many efforts being made to reduce the road toll across Australia have been effective.

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Constructing time series plots

As previously mentioned, time series data is a special case of bivariate numerical data, where the explanatory variable is time. Consider the variable *month*, which takes values such as January, February, March and so on. For the purpose of plotting and analysing time series data, we can consider the variable *month* as numerical, taking the values {1, 2, 3, . . . }. If we had monthly data for a two year period, then the variable *month* would take the values {1, 2, . . . , 24}. Whether the actual value of the variable is used in the plot (January, February, March, . . .) or its numerical equivalent (1, 2, 3, . . .) is used, both time series plots would be considered correct. We can use a similar approach for the variables *day*, or *quarter*.

Example 1 Constructing a time series plot

Maximum temperature was recorded each day for a week in a certain town. Construct a time series plot of the data.

Day	Mon	Tues	Wed	Thur	Fri	Sat	Sun
Temperature (°C)	20	21	25	36	34	25	26

Explanation

1 In a time series plot, time (day in this case) is always the explanatory variable (EV) and is plotted on the horizontal axis.

Solution

Day is the EV – this will label the horizontal axis.

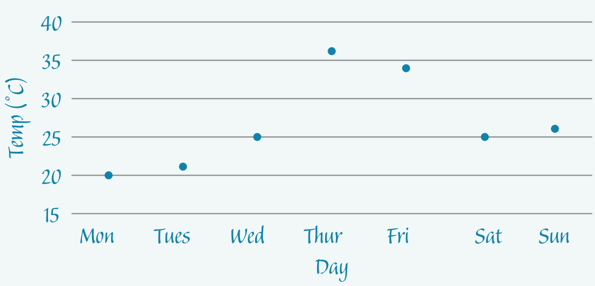
Temperature is the RV – this will label the vertical axis.

2 Determine the scales for each axis.

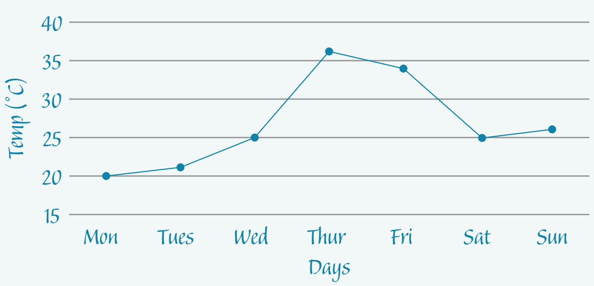
A horizontal scale from 0–7 with intervals of 1 for each day would be suitable.

Temperature ranges from 20–36. A vertical scale from 15–40 with intervals of 5 would be suitable.

3 Set up the axes, and then plot all seven data points as for a scatterplot.



4 Complete the graph, by joining consecutive data points with straight lines.



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Most real-world time series data come in the form of large data sets that are best plotted with the aid of a spreadsheet or statistical package. The availability of the data in electronic form via the internet greatly helps this process. However, in this chapter, most of the time series data sets are relatively small and can be readily plotted using a CAS calculator.

CAS 1: How to construct a time series using the TI-Nspire CAS

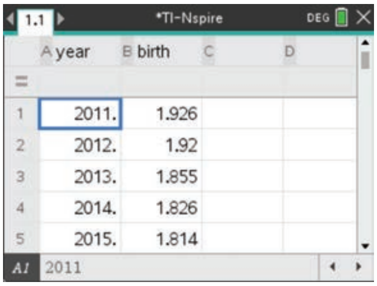
Construct a time series plot for the data presented below, which shows the birth rate in Australia (in births per woman) from 2011–2020.

year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
birth	1.926	1.920	1.855	1.826	1.814	1.752	1.741	1.740	1.657	1.580

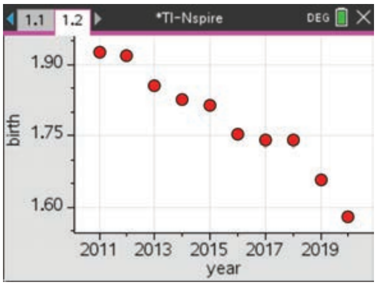
Steps

1 Start a new document by pressing ctrl + N.

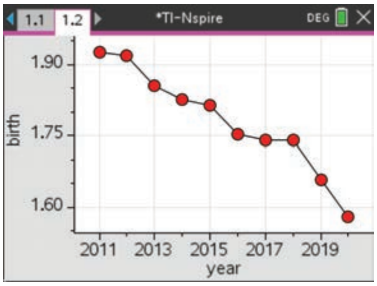
2 Select **Add Lists & Spreadsheet.** Enter the data into lists named *year* and *birth*.



3 Press ctrl + I and select **Add Data & Statistics**. Construct a scatterplot of *birth* against *year*. As is the case for a time series plot, *year* is the explanatory variable and *birth* the response variable.



4 To display as a connected time series plot, move the cursor to the main graph area and press ctrl + menu **>Connect Data Points**. Press enter.



CAS 1: How to construct a time series using the ClassPad

Construct a time series plot for the data presented below, which shows the birth rate in Australia (in births per woman) from 2011–2020.

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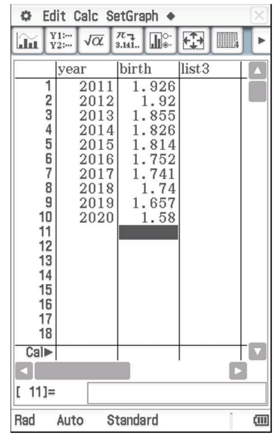
year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
birth	1.926	1.920	1.855	1.826	1.814	1.752	1.741	1.740	1.657	1.580

Steps

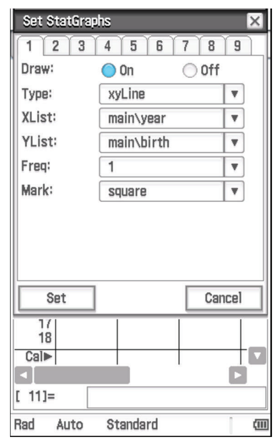
1 Open the **Statistics** application and enter the data into the columns named *year* and *birth*. Your screen should look like the one shown.



2 Tap to open the **Set StatGraphs** dialog box and complete as follows.

- **Draw:** select **On**.
- **Type:** select **xyLine** (▼).
- **XList:** select **main/year** (▼).
- **YList:** select **main/birth** (▼).
- **Freq:** leave as **1**.
- **Mark:** leave as **square**.

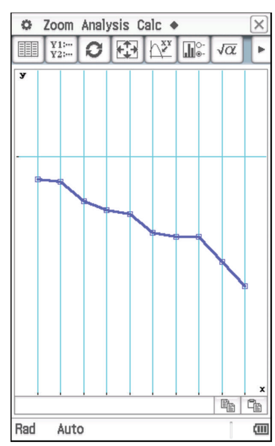
Tap **Set** to confirm your selections.



3 Tap in the toolbar at the top of the screen to display the time series plot in the bottom half of the screen.

To obtain a full-screen display, tap from the icon panel.

Tap from the toolbar, and use ▶ and ◀ to move from point to point to read values from the plot.



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Looking for patterns in time series plots

The features we look for in a time series are:

- trend
- cycles
- seasonality
- structural change
- possible outliers
- irregular (random) fluctuations.

We would always expect to see irregular or random fluctuations in a time series, and it is common to see one or more of the other features as well.

Trend

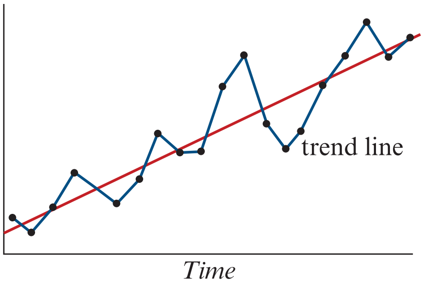
Examining a time series plot we can often see a general upward or downward movement over time. This indicates a long-term change over time that we call a trend.

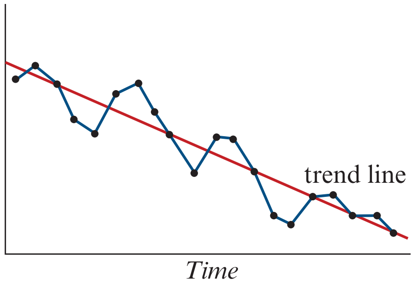
Trend

The tendency for values in a time series to generally increase or decrease over a significant period of time is called a **trend**.

One way of identifying trends on a time series graph is to draw a line that ignores the fluctuations, but which reflects the overall increasing or decreasing nature of the plot. These lines are called **trend lines**.

Trend lines have been drawn on the time series plots below to indicate an **increasing** trend (line slopes upwards) and a **decreasing** trend (line slopes downwards).





Sometimes, different trends are apparent in a time series for different time periods.

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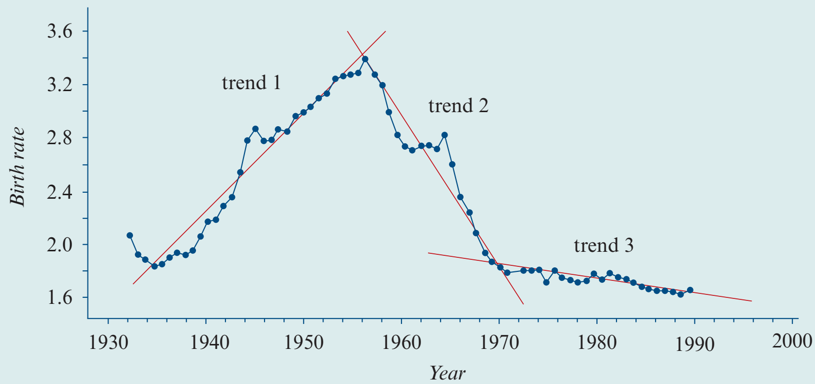
Example 2 Identifying trends

Consider the time series plot of the Australian annual birth rates over the years from 1931 to 1990, shown below. Comment on the trend shown in the plot.



Solution

There are three distinct trends, which can be seen by drawing trend lines on the plot. Each of these trends can be explained by changing socioeconomic circumstances.

Trend 1: Between 1940 and 1961 the birth rate in Australia grew quite dramatically. Those in the armed services came home from the Second World War, and the economy grew quickly. This rapid increase in the Australian birth rate during this period is known as the 'Baby Boom'.

Trend 2: From about 1962 until 1980 the birth rate declined very rapidly. Birth control methods became more effective, and women started to think more about careers. This period is sometimes referred to as the 'Baby Bust'.

Trend 3: During the 1980s, and beyond, the birth rate continued to decline slowly for a complex range of social and economic reasons.

Cycles

The term cycle refers to variations in time series that in general last longer than a year. These variations may not be of a regular height and they may not repeat at regular intervals.

Cycles

Cycles are recurrent movements in a time series, generally over a period greater than one year.

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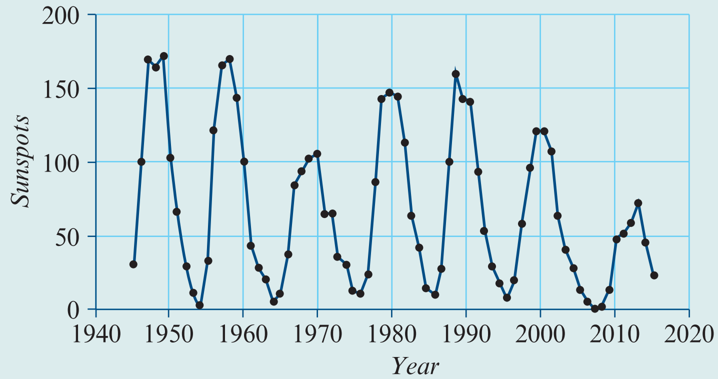
Example 3 Identifying cycles

Sunspots are darker, cooler area on the surface of the sun. The following plot shows the sunspot activity for the period 1945 to 2016. Comment on the cycles shown in the plot.



Solution

The recurrent pattern in the number of sunspots can be seen clearly from the time series plot. Looking at the plot the years of lowest sunspot activity look to be at approximately 1954, 1964, 1975, 1986, 1996, 2008.

Many business indicators, such as interest rates or unemployment figures, also vary in cycles, but their periods are usually less regular.

Seasonality

Cycles with calendar-related periods of one year or less are of special interest and are referred to as **seasonality**.

Seasonality

Seasonality is present when there is a periodic movement in a time series that is related to a calendar-related period – for example a year, a month or a week.

Seasonal movements tend to be more predictable than other time series features, and occur because of variations in the weather, such as ice-cream sales increasing in the summer, or institutional factors, like the increase in the number of unemployed people at the end of the school year.

Example 4 Identifying seasonality

The plot below shows the total percentage of hotel rooms occupied in Australia by quarter, over the years 2012–2016. Comment on the seasonality shown in the plot.

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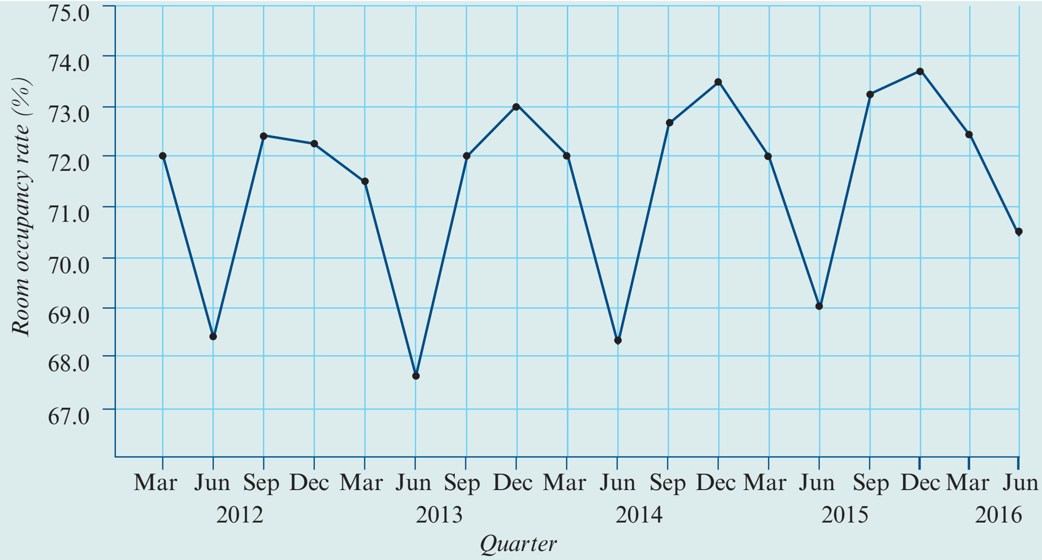

Solution

The regular peaks and troughs in the plot that occur at the same time each year signal the presence of **seasonality**. In this case, the demand for accommodation is at its lowest in the June quarter and highest in the December quarter.

This time series plot reveals both seasonality and trend in the demand for hotel rooms. The upward sloping trend line signals the presence of a general increasing **trend**. This tells us that, even though demand for accommodation has fluctuated from month to month, demand for hotel accommodation has increased over time.

Structural change

A **structural change** in a time series is a sudden change in the pattern of the time series at a point in time.

Structural change

Structural change is present when there is a sudden change in the established pattern of a time series plot.

Example 5 Identifying structural change

The time series plot below shows the power bill for a rental house (in kWh) for the 12 months of a year. Comment on any structural change in the plot.

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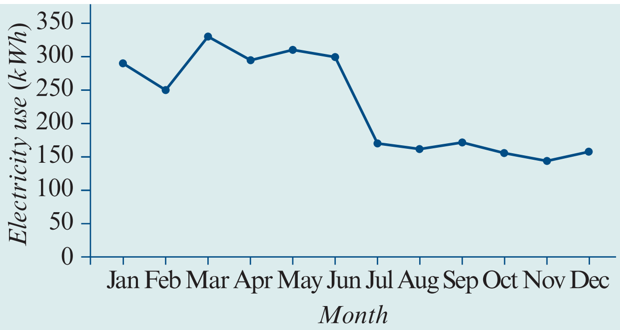


Solution

The plot reveals an abrupt change in power usage in June to July. During this period, monthly power use suddenly decreases from around 300 kWh per month from January to June to around 175 kWh for the rest of the year. This is an example of structural change that can probably be explained by a change in circumstances, for example, from a family with children to a person living alone.

Structural change is also displayed in the birth rate time series plot we saw earlier. This revealed three quite distinct trends during the period 1931–1990. These reflect significant external events (like a war) or changes in social and economic circumstances.

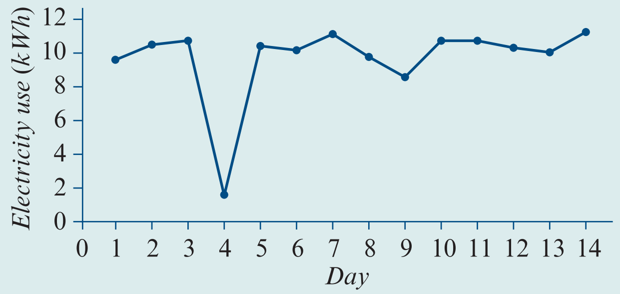
Outliers

Outliers

Outliers are individual values that stand out from the general body of data.

Example 6 Identifying outliers

The time series plot below shows the daily power bill for a house (in kWh) for a fortnight. Comment on any outliers in the plot.



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Solution

For this household, daily electricity use follows a regular pattern that, although fluctuating, averages about 10 kWh per day. In terms of daily power use, day 4 is a clear outlier, with less than 2 kWh of electricity used. A follow-up investigation found that, on this day, the house was without power for 18 hours due to a storm, so much less power was used than normal.

Irregular (random) fluctuations

Irregular (random) fluctuations

Irregular (random) fluctuations include all the variations in a time series that we cannot reasonably attribute to systematic changes like trend, cycles, seasonality and structural change or an outlier. There will **always** be irregular, random variation present in any real world time series data.

There can be many sources of irregular fluctuations, mostly unknown. A general characteristic of these fluctuations is that they are unpredictable.

One of the aims of time series analysis is to develop techniques to identify regular patterns in time series plots that are often obscured by irregular fluctuations. One of these techniques is smoothing, which you will meet in the next section.

Identifying patterns in time series plots

The features we look for in a time series are:

- trend
- cycles
- seasonality
- structural change
- possible outliers
- irregular (random) fluctuations.

Trend is present when there is a *long-term* upward or downward movement in a time series.

Cycles are present when there is a periodic movement in a time series. The period is the time it takes for one complete up and down movement in the time series plot. In practice, this term is reserved for periods greater than 1 year.

Seasonality is present when there is a periodic movement in a time series that has a calendar related period – for example a year, a month or a week.

Structural change is present when there is a sudden change in the established pattern of a time series plot.

Outliers are present when there are individual values that stand out from the data.

Irregular (random) fluctuations are always present in any real-world time series plot. They include all the unexplained variations in a time series.

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Exercise 5A

Constructing a time series plot

Note: A CAS calculator may be used to construct the time series plots. You may assign numerical values to the values of the time variable where convenient to do so.

Example 1

1 Construct a time series plot to display the following data.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Sales	2	23	35	31	45	23	67	70

2 Researchers recorded the number of penguins present on a remote island each month for 12 months. Construct a time series plot of the data.

Month	*Number of penguins*	*Month*	*Number of penguins*
January	449	July	180
February	214	August	241
March	170	September	311
April	265	October	499
May	434	November	598
June	102	December	674

3 The following table shows the maximum temperature in Melbourne during one week in March. Construct a time series plot of the data.

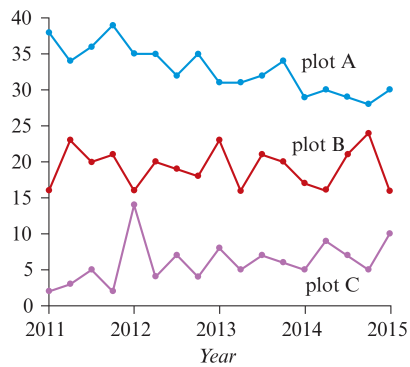
Day	Mon	Tues	Wed	Thur	Fri	Sat	Sun
Temperature (°C)	24.0	24.2	17.4	17.7	18.3	19.5	17.4

Identifying key features in a time series plot

Example 2

4 Complete the table below by indicating which of the listed features are present in each of the time series plots shown below.

Feature	Plot		
	A	B	C
Irr fluctuations			
Increasing trend			
Decreasing trend			



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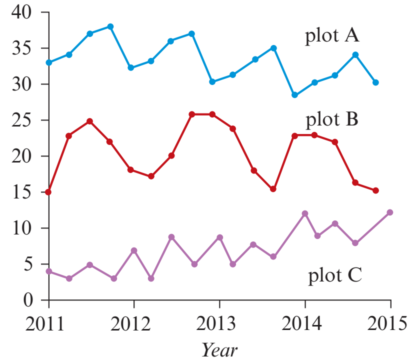
Example 3
Example 4

5 Complete the table below by indicating which of the listed features are present in each of the time series plots shown below.

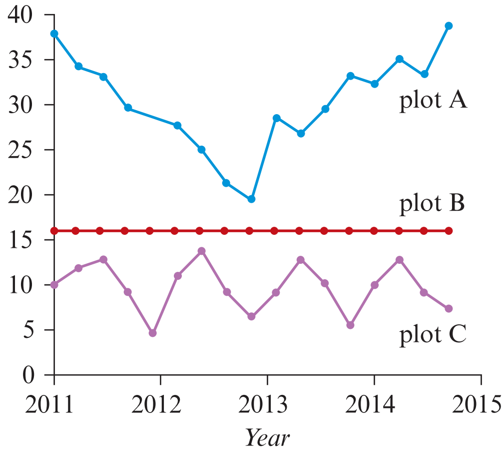
Feature	Plot A	Plot B	Plot C
Irr fluctuations			
Increasing trend			
Decreasing trend			
Cycles			
Seasonality			



Example 5

6 Complete the table below by indicating which of the listed features are present in each of the time series plots shown below.

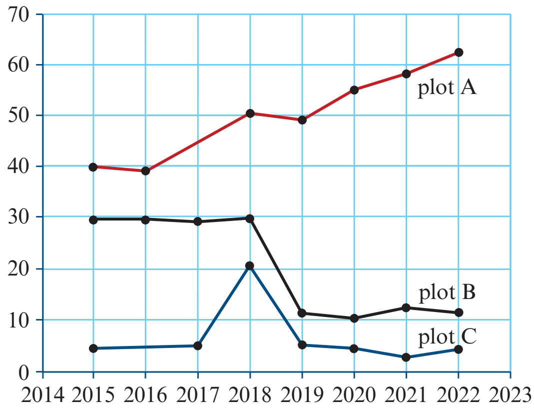
Feature	Plot A	Plot B	Plot C
Irr fluctuations			
Struc change			
Increasing trend			
Decreasing trend			
Seasonality			



Example 6

7 Complete the table below by indicating which of the listed features are present in each of the time series plots shown below.

Feature	Plot A	Plot B	Plot C
Irr fluctuations			
Struc change			
Increasing trend			
Decreasing trend			
Outliers			



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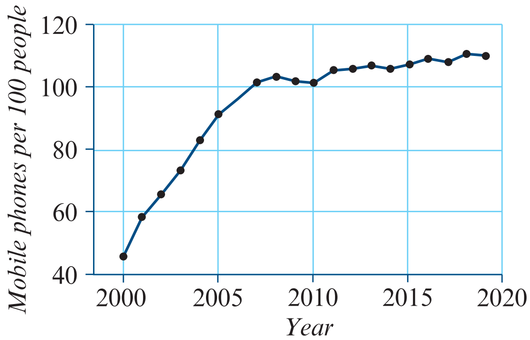
Describing time series plots

8 The time series plot for the number of mobile phones per 100 people from 2000–2019 is shown to the right. Describe the features of the time series plot.



9 The data below shows the population (in millions) in Australia over the period 2012–2021.

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Population	22.73	23.13	23.48	23.82	24.19	24.60	24.98	25.37	25.69	25.97

a Construct a time series plot of the data.

b Describe the features of the plot.

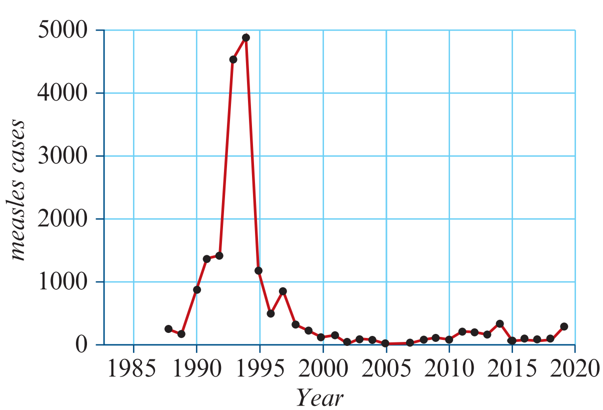
10 The table below shows the motor vehicle theft rate per 100 000 cars in Australia from 2003 to 2018.

Year	2003	2004	2005	2006	2007	2008	2009	2010
Theft rate	500.9	442.4	398.3	367.2	337.6	320.0	274.2	214.8
Year	2011	2012	2013	2014	2015	2016	2017	2018
Theft rate	220.0	228.4	204.2	191.0	194.5	231.0	213.3	214.1

a Construct a time series plot of the data.

b Describe the features of the plot.

11 The time series plot below shows the number of measles cases reported in Australia from 1988 to 2019. Describe the features of the plot.



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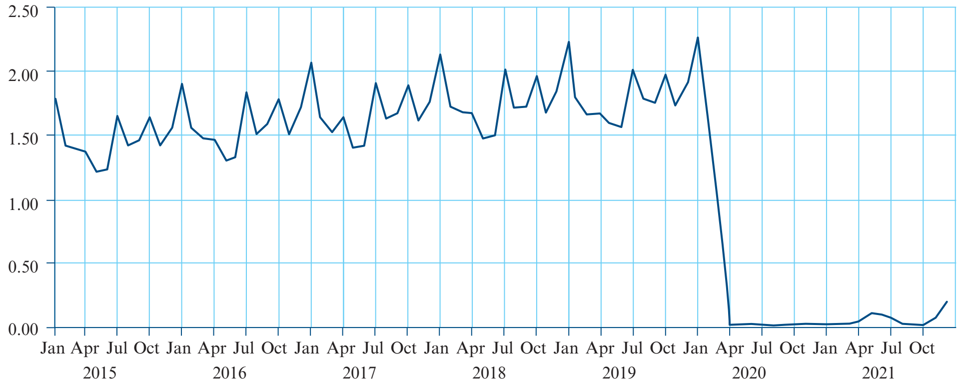
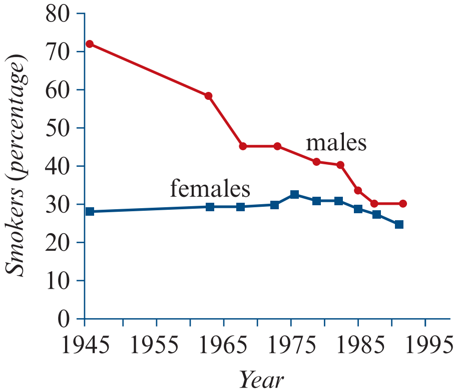
12 The time series plot below shows the number of overseas arrivals (millions of people per month) in Australia from November 2011 until December 2021. Describe the features of the plot.



13 **a** The time series plot shown shows the smoking rates (%) of Australian males and females over the period 1945–92.

i Describe any trends in the time series plot.

ii Did the *difference* in smoking rates increase or decrease over the period 1945–92?



b The table below shows the smoking rates for females and males aged 15 years at several time points from 2000–2018 (smoking rate data is not collected every year).

Year	2000	2005	2007	2010	2011	2012	2013	2018	2014	2015	2018
Female	22.4	18.9	19.9	17.9	15.4	16.6	14.4	15.6	13.5	14.5	13.6
Male	26.7	22.9	24.9	22.9	19.1	21.9	18.0	20.7	17.0	19.7	18.7

i Use a CAS calculator to construct time series plots of the male and female data.

ii Describe any trends in the time series plot.

iii Did the difference in smoking rates change over the period 2000–2018?

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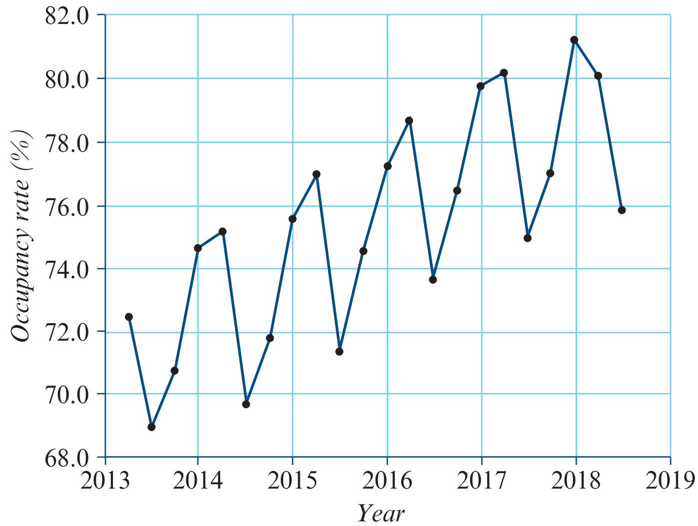
Exam 1 style questions

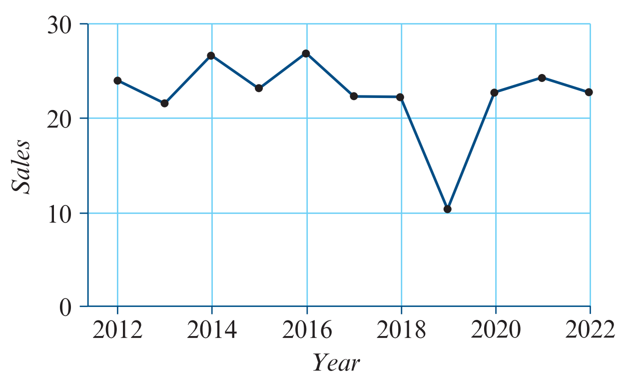
14 The time series plot below shows the quarterly room occupancy rate for a chain of hotels over the years 2013 to 2019.



The time series plot is best described as having

A seasonality only
B irregular fluctuations only
C seasonality with irregular fluctuations
D an increasing trend with irregular fluctuations
E an increasing trend with seasonality and irregular fluctuations

15 The time series plot below shows the annual sales (in $ millions) for a car sales company.



The time series plot is best described as having

A seasonality only
B irregular fluctuations only
C seasonality with irregular fluctuations
D irregular fluctuations with an outlier
E seasonality with an outlier

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5B Smoothing a time series using moving means

Learning intentions

- ▶ To be able to smooth a time series plot using moving means.
- ▶ To be able to know when and how to use centring when smoothing.

A time series plot can incorporate many of the sources of variation previously mentioned: trend, cycles, seasonality, structural change, outliers and irregular fluctuations. One effect of the irregular fluctuations and seasonality can be to obscure an underlying trend. The technique of **smoothing** can sometimes be used to overcome this problem.

In this section we consider **moving mean smoothing**, which involves replacing individual data points in the time series with the mean of the data point and some adjacent points. The simplest method is to smooth over a small odd number of data points – for example, three or five, but any number of points can be used.

The three-moving mean

To use **three-moving mean smoothing**, replace each data value with the mean of that value and the one on each side. That is, if y_1, y_2 and y_3 are sequential data values, then:

$$\text{smoothed } y_2 = \frac{y_1 + y_2 + y_3}{3}$$

The first and last points in the time series do not have values on each side, so they are omitted.

The five-moving mean

To use **five-moving mean smoothing**, replace each data value with the mean of that value and the two values on each side. That is, if y_1, y_2, y_3, y_4, y_5 are sequential data values, then:

$$\text{smoothed } y_3 = \frac{y_1 + y_2 + y_3 + y_4 + y_5}{5}$$

The first two and last two points in the time series do not have two values on each side, so they are omitted.

These definitions can be readily extended for moving means involving more points.

Example 7 Three- and five-moving mean smoothing

The table below gives the temperature (°C) recorded at a weather station at 9.00 a.m. each day for a week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

a Calculate the three-moving mean smoothed temperature for Tuesday.

b Calculate the five-moving mean smoothed temperature for Thursday.

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Explanation	Solution
a 1 Write down the three temperatures centred on Tuesday.	18.1, 24.8, 26.4
2 Find their mean and write down your answer.	$\text{Mean} = \frac{(18.1 + 24.8 + 26.4)}{3} = 23.1$ The three-moving mean smoothed temperature for Tuesday is 23.1°C.
b 1 Write down the five temperatures centred on Thursday	24.8, 26.4, 13.9, 12.7, 14.2
2 Find their mean and write down your answer.	$\text{Mean} = \frac{(24.8 + 26.4 + 13.9 + 12.7 + 14.2)}{5} = 18.4$ The five-moving mean smoothed temperature for Thursday is 18.4°C.

The next step is to extend these computations to smooth all terms in the time series.

Example 8 Three- and five-moving mean smoothing of a time series

The following table gives the number of births per month over a calendar year in a country hospital. Use the three-moving mean and the five-moving mean methods, rounded to one decimal place, to complete the table.

Solution

Complete the calculations as shown below.

Month	*Number of births*	3-moving mean	5-moving mean
January	10		
February	12	$\frac{10+12+6}{3} = 9.3$	
March	6	$\frac{12+6+5}{3} = 7.7$	$\frac{10+12+6+5+22}{5} = 11.0$
April	5	$\frac{6+5+22}{3} = 11.0$	$\frac{12+6+5+22+18}{5} = 12.6$
May	22	$\frac{5+22+18}{3} = 15.0$	$\frac{6+5+22+18+13}{5} = 12.8$
June	18	$\frac{22+18+13}{3} = 17.7$	$\frac{5+22+18+13+7}{5} = 13.0$
July	13	$\frac{18+13+7}{3} = 12.7$	$\frac{22+18+13+7+9}{5} = 13.8$
August	7	$\frac{13+7+9}{3} = 9.7$	$\frac{18+13+7+9+10}{5} = 11.4$
September	9	$\frac{7+9+10}{3} = 8.7$	$\frac{13+7+9+10+8}{5} = 9.4$
October	10	$\frac{9+10+8}{3} = 9.0$	$\frac{7+9+10+8+15}{5} = 9.8$
November	8	$\frac{10+8+15}{3} = 11.0$	
December	15		

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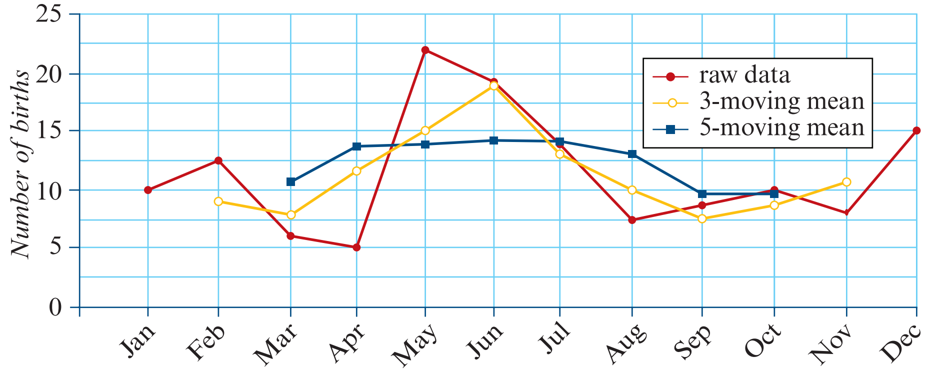
The result of this smoothing can be seen in the plot below, which shows the raw data, the data smoothed with a three-moving means and the data smoothed with a five-moving means.



Note: In the process of smoothing, **data points are lost** at the beginning and end of the time series.

Two observations can be made from this plot:

1. Five-moving mean smoothing is more effective in reducing the irregular fluctuations than three-mean smoothing.
2. The five-moving mean smoothed plot shows that there is no clear trend although the raw data suggest that there might be an increasing trend.

Moving mean smoothing with centring

If we smooth over an even number of points, we run into a problem. The centre of the set of points is not at a time point belonging to the original series. Usually, we solve this problem by using a process called **centring**.

Centring

Smoothing with centring involves taking a two-moving mean of the already smoothed values so that they line up with the original data values. Smoothing with centring is only required when smoothing using an **even** number of data values, for example 2-moving mean smoothing, or 4-moving mean smoothing.

We will illustrate the process by finding the two-moving mean, centred on Tuesday, for the daily temperature data opposite.

Day	*Temperature*
Monday	18.1
Tuesday	24.8
Wednesday	26.4

It is straightforward to calculate a series of two-moving means for this data by calculating the mean for Monday and Tuesday, followed by the mean for Tuesday and Wednesday. However, as we can see in the diagram below, these means do not align with a particular day, but lie between days. To solve this problem find the average of these two means, as shown in the following diagram.

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Day	*Temperature*	*Two-moving means*	*Two-moving mean with centring*
Monday	18.1		
		$\frac{(18.1+24.8)}{2} = 21.45$	
Tuesday	24.8		$\frac{(21.45+25.6)}{2} = 23.525$
		$\frac{(24.8+26.4)}{2} = 25.60$	
Wednesday	26.4		

In practice, we do not have to draw such a diagram to perform these calculations. The purpose of doing so is to show how the centring process works. Calculating a two-moving mean with centring is illustrated in the following example.

Example 9 Two-moving mean smoothing with centring

The temperatures (°C) recorded at a weather station at 9 a.m. each day for a week are displayed in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

Calculate the two-moving mean smoothed temperature with centring for Tuesday.

Explanation

1 For two-mean smoothing with centring, write down the **three** data values centred on Tuesday (highlighted in red).

2 Calculate the mean of the first two values (mean 1). Calculate the mean of the second two values (mean 2).

3 The centred mean is then the average of mean 1 and mean 2.

4 Write down your answer, rounded to one decimal place.

Solution

18.1 24.8 26.4

$$\text{mean 1} = \frac{(18.1 + 24.8)}{2} = 21.45$$

$$\text{mean 2} = \frac{(24.8 + 26.4)}{2} = 25.60$$

$$\text{Centred mean} = \frac{(\text{mean 1} + \text{mean 2})}{2} = \frac{(21.45 + 25.60)}{2} = 23.525$$

The two-moving mean smoothed temperature for Tuesday is 23.5°C.

The process of smoothing with centring across more data values is the same as two-mean smoothing except that the means are determined in larger groups. This process is illustrated in the following example with groups of four and six.

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Example 10 Four- and six-moving mean smoothing with centring

The table below gives the temperature (°C) recorded at a weather station at 9.00 a.m. each day for a week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Temperature	18.1	24.8	26.4	13.9	12.7	14.2	24.9

a Calculate the four-moving mean smoothed temperature with centring for Thursday.

b Calculate the six-moving mean smoothed temperature with centring for Thursday.

Explanation / **Solution**

a **1** For four-mean smoothing with centring, write down the **five** data values centred on Thursday.

24.8 26.4 13.9 12.7 14.2

2 Calculate the mean of the first four values (mean 1) and the mean of the last four values (mean 2).

$$\text{mean } 1 = \frac{(24.8 + 26.4 + 13.9 + 12.7)}{4} = 19.45$$

$$\text{mean } 2 = \frac{(26.4 + 13.9 + 12.7 + 14.2)}{4} = 16.8$$

3 The centred mean is then the average of mean 1 and mean 2.

$$\text{centred mean} = \frac{(\text{mean } 1 + \text{mean } 2)}{2} = \frac{(19.45 + 16.8)}{2} = 18.125$$

4 Write down your answer.

The four-mean smoothed temperature centred on Thursday is 18.1 °C (to 1 d.p.).

b **1** For six-mean smoothing with centring, write down the **seven** data values centred on Thursday.

18.1 24.8 26.4 13.9 12.7 14.2 24.9

2 Calculate the mean of the first six values (mean 1) and the mean of the last six values (mean 2).

$$\text{mean } 1 = \frac{(18.1 + 24.8 + 26.4 + 13.9 + 12.7 + 14.2)}{6} = 18.35$$

$$\text{mean } 2 = \frac{(24.8 + 26.4 + 13.9 + 12.7 + 14.2 + 24.9)}{6} = 19.4833\ldots$$

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3 The centred mean is then the average of mean 1 and mean 2.	$\text{centred mean} = \dfrac{(\text{mean } 1 + \text{mean } 2)}{2}$ $= \dfrac{(18.35 + 19.483)}{2}$ $= 18.917$
4 Write down your answer.	The six-mean smoothed temperature centred on Thursday is 18.9 °C (to 1 d.p.).

The next step is to extend these computations to smooth all terms in the time series. This process is illustrated using four-moving mean smoothing in the following example. Setting up and using a table like the one shown in the example will help keep track of the process.

Example 11 Smoothing of a time series using four-mean smoothing with centring

The following table gives the number of births per month over a calendar year in a country hospital. Use the four moving mean with centring method to complete the table.

Solution

Complete the calculations as shown below.

Month	*Number of births*	4-moving mean	4-moving mean with centring
January	10		
February	12	$\frac{10 + 12 + 6 + 5}{4} = 8.25$	
March	6		$\frac{8.25 + 11.25}{2} = 9.75$
		$\frac{12 + 6 + 5 + 22}{4} = 11.25$	
April	5		$\frac{11.25 + 12.75}{2} = 12$
		$\frac{6 + 5 + 22 + 18}{4} = 12.75$	
May	22		$\frac{12.75 + 14.5}{2} = 13.625$
		$\frac{5 + 22 + 18 + 13}{4} = 14.5$	
June	18		$\frac{14.5 + 15}{2} = 14.75$
		$\frac{22 + 18 + 13 + 7}{4} = 15$	
July	13		$\frac{15 + 11.75}{2} = 13.375$
		$\frac{18 + 13 + 7 + 9}{4} = 11.75$	
August	7		$\frac{11.75 + 9.75}{2} = 10.75$
		$\frac{13 + 7 + 9 + 10}{4} = 9.75$	
September	9		$\frac{9.75 + 8.5}{2} = 9.125$
		$\frac{7 + 9 + 10 + 8}{4} = 8.5$	
October	10		$\frac{8.5 + 10.5}{2} = 9.5$
		$\frac{9 + 10 + 8 + 15}{4} = 10.5$	
November	8		
December	15		

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Skillsheet

Exercise 5B

Note: A CAS calculator may be used to construct the time series plots.

Calculating the smoothed values of an odd number of individual points

Example 7

1

t	1	2	3	4	5	6	7	8	9
y	5	2	5	3	1	0	2	3	0

For the time series data in the table above, find:

a the three-mean smoothed y-value for

i $t = 4$ **ii** $t = 6$ **iii** $t = 2$

b the five-mean smoothed y-value for

i $t = 3$ **ii** $t = 7$ **iii** $t = 4$

c the seven-mean smoothed y-value for

i $t = 4$ **ii** $t = 6$

d the nine-mean smoothed y-value for $t = 5$

2 The table below gives the temperature (°C) recorded at a weather station at 3.00 p.m. each day for a week.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temp (°C)	28.9	33.5	21.6	18.1	16.2	17.9	26.4

a Find the three-mean smoothed temperature for Wednesday.

b Find the five-mean smoothed temperature for Friday.

c Find the seven-mean smoothed temperature for Thursday.

Example 8

3 Complete the following table.

t	1	2	3	4	5	6	7	8	9
y	10	12	8	4	12	8	10	18	2
3-moving mean y	–								–
5-moving mean y	–	–						–	–

Smoothing and plotting a time series plot (odd number of points)

4 The maximum temperature of a city over a period of 10 days is given below.

Day	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	24	27	28	40	22	23	22	21	25	26
3-moving mean		26.3		30.0		22.3		22.7	24.0	
5-moving mean			28.2		27			23.4		

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a Construct a time series plot of the temperature data.

b Use the five-mean and seven-mean smoothing method to complete the table.

c Plot the smoothed temperature data and compare the plots.

5 The value of the Australian dollar in US dollars (exchange rate) over 10 days is given below.

Day	1	2	3	4	5	6	7	8	9	10
Exchange rate	0.743	0.754	0.737	0.751	0.724	0.724	0.712	0.735	0.716	0.711
3-moving mean		0.745	0.747		0.733			0.721	0.721	
5-moving mean				0.738	0.730		0.722			

a Construct a time series plot of the data.

b Use the three-mean and five-mean smoothing method to complete the table.

c Plot the smoothed exchange rate data and compare the plots.

Calculating the smoothed values of an even number of individual points

Example 9

6

t	1	2	3	4	5	6	7	8	9
y	5	2	5	3	1	0	2	3	0

For the time series data above, find the two-mean smoothed y-value centred at:

a $t = 3$

b $t = 8$

Example 10

7 Use the time series data in the table in Question 6 to find:

a the four-mean smoothed y-value centred at $t = 3$

b the four-mean smoothed y-value centred at $t = 6$

c the six-mean smoothed y-value centred at $t = 3$

d the six-mean smoothed y-value centred at $t = 6$

8 The table below gives the minimum daily temperature (°C) recorded at a weather station over a 10 day period.

Day	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	8.9	3.5	11.6	14.1	12.5	13.3	6.4	8.5	9.1	4.5

a Find the two-mean smoothed temperature with centring for Day 5.

b Find the four-mean smoothed temperature with centring for Day 5.

c Find the six-mean smoothed temperature with centring for Day 5.

Smoothing and plotting a time series plot (even number of points)

Example 11

9 The table below gives the number of complaints recorded at a customer service centre over a 12 month period.

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Month	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
Number of complaints	10	12	6	5	22	18	13	7	9	10	8	15
2-moving mean		10.0	7.3	9.5	16.8	17.8	12.8	9.0		9.3	10.3	

a Construct a time series plot of the data.

b Show that the two-mean smoothed value with centring for September is equal to 8.8 (rounded to one decimal place).

c Plot the smoothed data and compare the plots.

10 The table below gives the amount of rain (in mm) recorded each month at a weather station.

Month	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	21.4	40.5	52.3	42.1	58.9	79.9	81.5	54.3	50.0
4-moving mean			43.8	53.4		67.1	67.5		

a Construct a time series plot of the data.

b Show that the four-mean smoothed value with centring for August is equal to 62.0 (rounded to one decimal place).

c Plot the smoothed data and compare the plots.

Exam 1 style questions

11 Hay Lam records the number of emails (*emails*) he receives over a one-week period.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Emails	85	65	77				10

The numbers of *emails* he received on Thursday, Friday and Saturday are not shown.
The five-mean smoothed number of *emails* he received on Friday is 39.
The three-mean smoothed number of *emails* he received on Friday is:

A 36 **B** 39 **C** 40 **D** 42 **E** 45

12 The table shows the closing price (*price*) of a company's shares on the stock market over a 10 day period.

Day	1	2	3	4	5	6	7	8	9	10
Price($)	0.99	1.05	1.10	1.25	1.29	1.37	2.42	1.95	2.05	2.35

The six-mean smoothed with centring closing share *price* on Day 6 is closest to:

A $1.56 **B** $1.62 **C** $1.64 **D** $1.88 **E** $1.72

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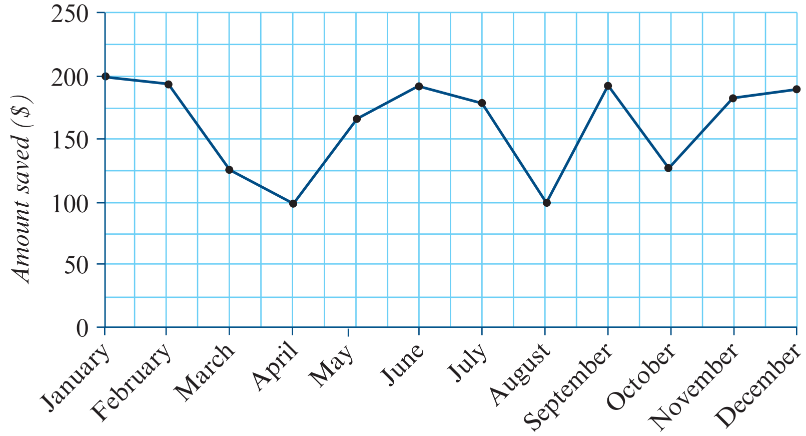
The following information relates to Questions 13 and 14

The time series plot below shows the amount that Arnold saved each month (in dollars) over a 12 month period.



13 If he saved a total of $831 over the period from May to September, the five-mean smoothed amount that he saved in July is closest to:

A $277 **B** $190 **C** $182 **D** $152 **E** $166

14 If seven-mean smoothing is used to smooth this time series plot, the number of smoothed data points would be:

A 3 **B** 4 **C** 5 **D** 6 **E** 7

5C Smoothing a time series plot using moving medians

Learning intentions

- To be able to locate the median of a data set graphically.
- To be able to smooth a time series plot using moving medians.

Another simple and convenient method of smoothing a time series is to use **moving median smoothing**. The advantage of this method is that it can be done directly on the graph without needing to know the exact values of each data point.[1] However, before smoothing a time series plot graphically using moving medians we will first need to know how to locate medians graphically.

[1] Note that, in this course, median smoothing is restricted to smoothing over an odd number of points, so centring is not required.

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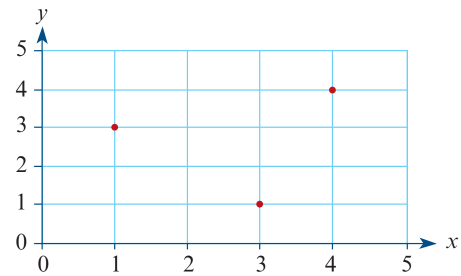
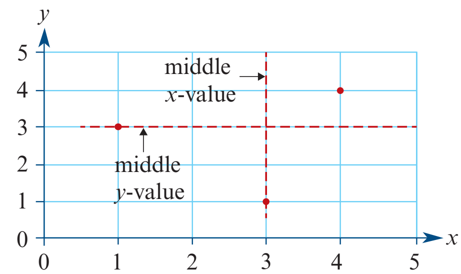
Locating medians graphically

The graph opposite shows three data points plotted on a set of coordinate axes. The task is to locate the median of these three points. The median will be a point somewhere on this set of coordinate axes. To locate this point we proceed as follows.



Step 1

Identify the middle data point moving in the x-direction. Draw a vertical line through this value as shown.



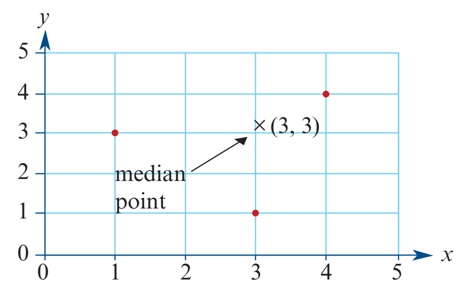
Step 2

Identify the middle data point moving in the y-direction. Draw a horizontal line through this value as shown.

Step 3

The median value is where the two lines intersect – in this case, at the point (3, 3).

Mark this point with a cross (×).

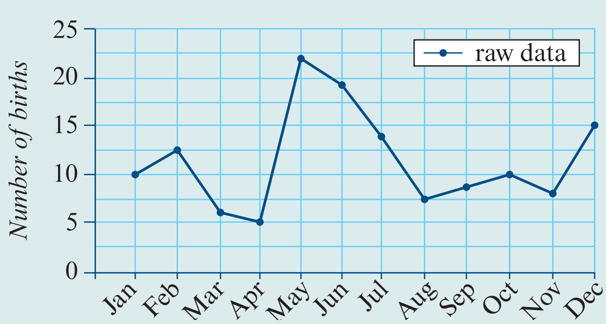


Smoothing a time series using moving median smoothing

The process of graphically smoothing a time series plot requires no more than repeating the above process for each group of three or five data points in the plot as required. The following worked examples demonstrate the process.

Example 12 **Three-moving median smoothing using a graphical approach**

Construct a three-median smoothed plot of the time series plot shown opposite.



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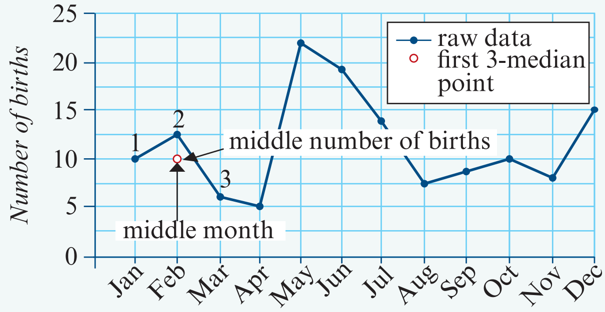
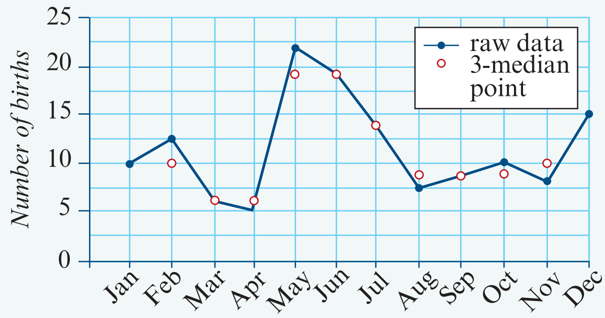
Explanation

1 Locate on the time series plot the median of the *first* three points (Jan, Feb, Mar).

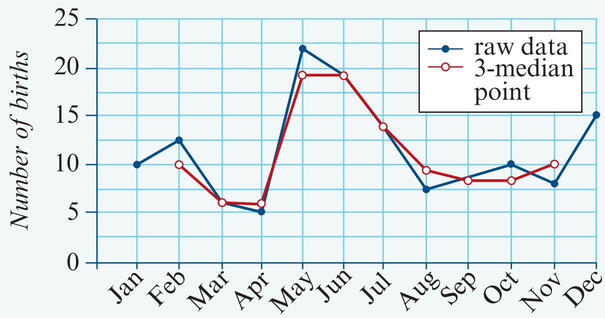
Solution



2 Continue this process by moving onto the next three points to be smoothed (Feb, Mar, Apr).
Mark their medians on the graph, and continue the process until you run out of groups of three.



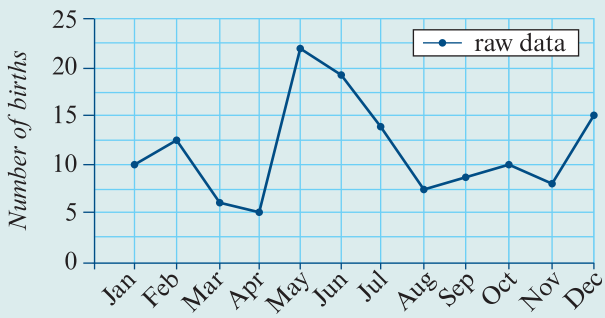
3 Join the median points with a line segment – see opposite.



Example 13 Five-moving median smoothing using a graphical approach

Construct a five-median smoothed plot of the time series plot shown opposite.

Note: The starting point for a median smoothing is a time series plot and you smooth directly onto the plot. Copies of the plots in this section can be accessed through the skillsheet icon in the Interactive Textbook.



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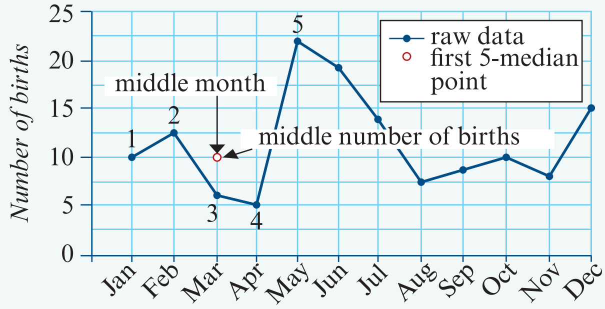
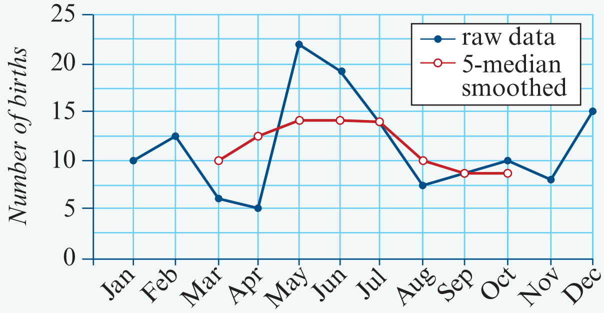
Explanation

1 Locate on the time series plot the median of the first five points (Jan, Feb, Mar, Apr, May), as shown.

Solution



2 Then move onto the next five points to be smoothed (Feb, Mar, Apr, May, Jun). Repeat the process until you run out of groups of five points. The five-median points are then joined up with line segments to give the final smoothed plot, as shown.



Note: The five-median smoothed plot is much smoother than the three-median smoothed plot.

Skill-sheet

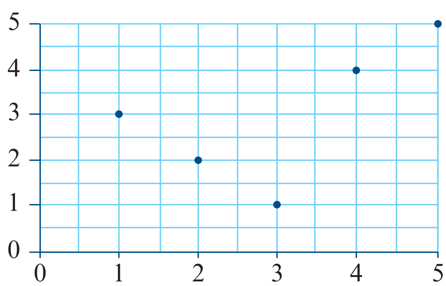
Exercise 5C

Note: Copies of all plots in this section can be accessed through the skillsheet icon in the Interactive Textbook.

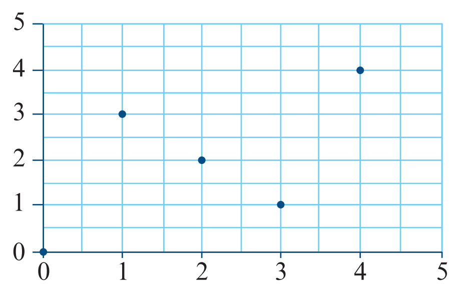
Locating the median of a set of data points graphically

1 Mark the location of the median point for each of the sets of data points below.

a



b



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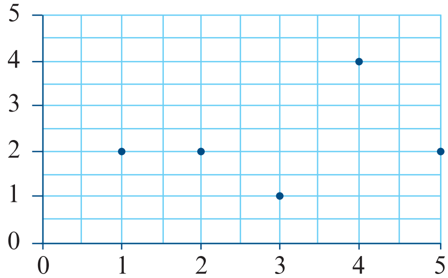
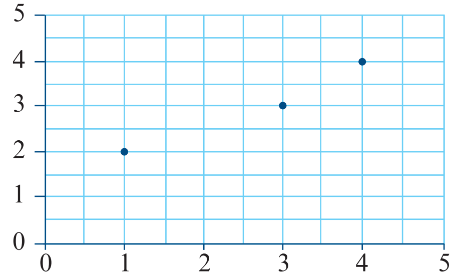
c



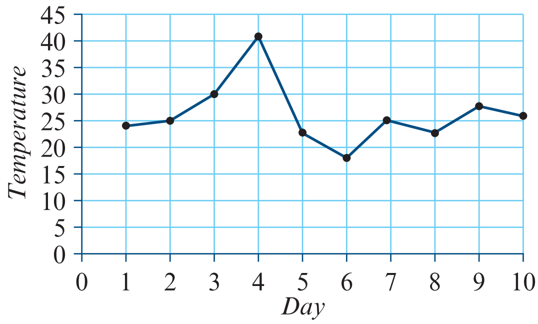
d



Smoothing a time series graphically

Example 12

2 The time series plot below shows the maximum daily temperatures (in °C) in a city over a period of 10 consecutive days.

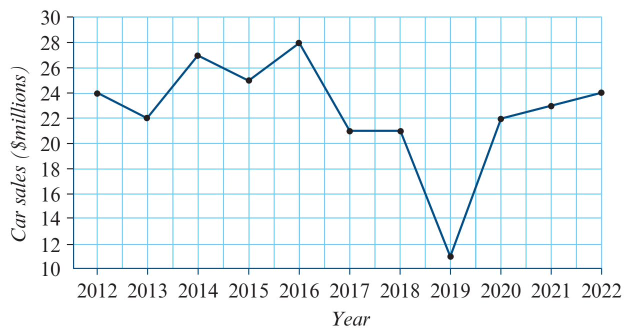


Use three-median smoothing to determine the smoothed temperature for:

a day 4

b day 8

3 The time series plot below shows the annual sales (in $ millions) for a car sales company. Use three-moving median smoothing to graphically smooth the plot and comment on the smoothed plot.



Example 13

4 Use the time series plot in Question 2 to find the five-median smoothed temperature for:

a day 4

b day 8

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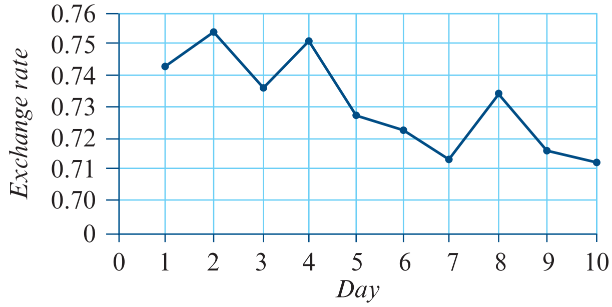
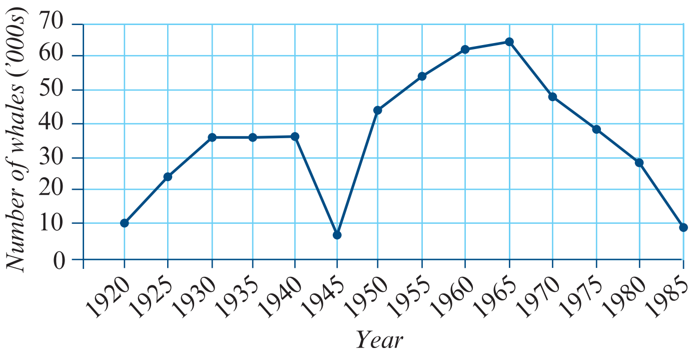
5 The time series plot opposite shows the value of the Australian dollar in US dollars (the exchange rate) over a period of 10 consecutive days in 2009.
Use five-moving median smoothing to graphically smooth the plot and comment on the smoothed plot.

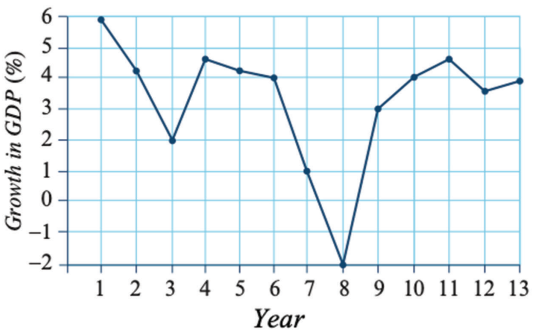


6 Use the graphical approach to smooth the time series plot below using:

a three-moving median smoothing **b** five-moving median smoothing.



7 The time series plot opposite shows the percentage growth of GDP (gross domestic product) over a 13-year period.



a Find the median value of the percentage growth in GDP over the 13 year period.

b Smooth the times series graph using:

i three-moving median smoothing **ii** five-moving median smoothing.

c What conclusions can be drawn about the variation in GDP growth from these smoothed time series plots?

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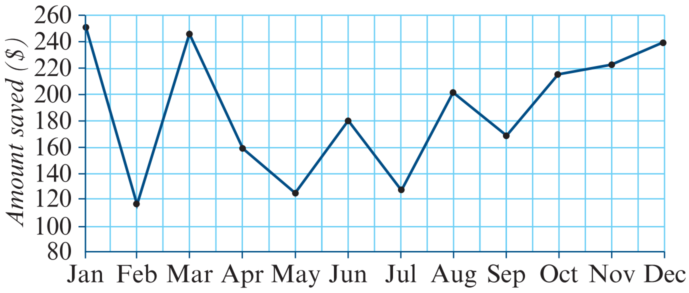
Exam 1 style questions

Use the following information to answer Questions 8 to 10

The time series plot below shows the amount that Lulu saved each month (to the nearest $) over a 12 month period.



8 During the year shown in the time series plot the median monthly amount Lulu saved is closest to:

A $180 **B** $155 **C** $130 **D** $190 **E** $200

9 The five-median smoothed amount saved by Lulu in July is closest to:

A $130 **B** $150 **C** $170 **D** $190 **E** $200

10 The nine-median smoothed amount saved by Lulu in August is closest to:

A $132 **B** $160 **C** $168 **D** $180 **E** $218

5D Seasonal indices

Learning intentions

- To be able to interpret the meaning of seasonal indices.
- To be able to seasonally adjust data using seasonal indices.
- To be able to calculate seasonal indices from time series data.

When the data is considered to have a seasonal component, it is often necessary to remove this component so any underlying trend is clearer. The process of removing the seasonal component is called **deseasonalising** the data. To do this we need to calculate **seasonal indices**. Seasonal indices tells us how a particular season (generally a day, month or quarter) compares to the average season.

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The concept of a seasonal index

Consider the (hypothetical) monthly seasonal indices for unemployment given in the table.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec	Total
1.1	1.2	1.1	1.0	0.95	0.95	0.9	0.9	0.85	0.85	1.1	1.1	12.0

Key fact 1

Seasonal indices are calculated so that their **average** is 1. This means that the sum of the seasonal indices equals the number of seasons. Thus, if the seasons are months, the seasonal indices add to 12. If the seasons are quarters, then the seasonal indices add to 4, and so on.

Key fact 2

Seasonal indices tell us how a particular season (generally a day, month or quarter) compares to the average season.

For example:

- seasonal index for unemployment for the month of February is 1.2 or 120%.
  This tells us that February unemployment figures tend to be 20% higher than the monthly average. Remember, the average seasonal index is 1 or 100%.
- seasonal index for August is 0.90 or 90%.
  This tells us that the August unemployment figures tend to be only 90% of the monthly average. Alternatively, August unemployment figures are 10% lower than the monthly average.

Example 14 Interpreting seasonal indices

Suppose that the seasonal indices (SI) for electricity usage in Esse's home are as shown in the table:

Summer	Autumn	Winter	Spring
1.16	0.94	1.26	0.64

a What does the seasonal index for Winter tell us?

b What does the seasonal index for Spring tell us?

Solution

a The seasonal index for Winter is 1.26. This tells us that Esse's electricity usage in Winter is typically 26% higher than the average season.

b The seasonal index for Spring is 0.64. This tells us that Esse's electricity usage in Spring is typically 36% lower than the average season.

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Using seasonal indices to seasonally adjust a time series

We can use seasonal indices to remove the seasonal component (**deseasonalise**) from a time series, or to put it back in (**reseasonalise**).When we do this we are said to **seasonally adjust** the data.

To calculate deseasonalised figures, each entry is divided by its seasonal index as follows.

Deseasonalising data

Time series data are deseasonalised using the relationship:

$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$

The rule for determining deseasonalised data values can also be used to reseasonalise data – that is, convert a deseasonalised value into an actual data value.

Reseasonalising data

Time series data are reseasonalised using the rule:

$$\text{actual figure} = \text{deseasonalised figure} \times \text{seasonal index}$$

Example 15 Using seasonal indices

The seasonal indices (SI) for cold drink sales for Imogen's kiosk are as shown in the table:

Summer	Autumn	Winter	Spring
1.75	0.66	0.46	1.13

a If the actual actual cold drink sales last summer totalled $21 653, what is the deseasonalised sales figure for that time period?

b If the deseasonalised cold drink sales last spring totalled $10 870, what were the actual sales for that time period?

Explanation	Solution
a To deseasonalise we divide by the seasonal index for Summer (1.75)	$\text{Deseasonalised sales} = \frac{21\,653}{1.75} = \$12\,373.14$
b To find the actual sales we multiply by the seasonal index for Spring (1.13).	$\text{Actual sales} = 10\,870 \times 1.13 = \$12\,283.10$

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Example 16 Using seasonal indices to determine percentage change required to correct for seasonality

Consider the table below which gives the seasonal indices for heater sales at a discount store:

Summer	Autumn	Winter	Spring
0.65	1.25	1.35	0.0.75

a By what percentage should the sales in summer be increased or decreased in order to deseasonalise the data? Give your answer as a percentage rounded to one decimal place.

b By what percentage should the sales in winter be increased or decreased in order to deseasonalise the data? Give your answer as a percentage rounded to one decimal place.

Explanation

a 1 Insert the seasonal index for summer into the rule

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

Solution

In general for summer:

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{0.65}$$

$$= \frac{1}{0.65} \times \text{actual sales}$$

$$= 1.538 \times \text{actual sales}$$

2 Convert 1.538 into a percentage increase or decrease. Write the answer in a sentence.

Multiplying the actual sales by 1.538 is the equivalent of increasing the actual sales by 53.8%.

To correct for seasonality, the actual sales should be increased by 53.8%.

b 1 Insert the seasonal index for winter into the rule

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

In general for winter:

$$\text{deseseasonalised sales} = \frac{\text{actual sales}}{1.35}$$

$$= \frac{1}{1.35} \times \text{actual sales}$$

$$= 0.741 \times \text{actual sales}$$

2 Convert 0.741 into a percentage increase or decrease. Write the answer in a sentence.

Multiplying the actual sales by 0.741 is the equivalent of decreasing the actual sales by (100%-74.1%) = 25.9%.

To correct for seasonality, the actual sales should be increased by 25.9%.

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Calculating seasonal indices

To complete this section, we will describe how to calculate a seasonal index. We will start by using only one year's data to illustrate the basic ideas and then move onto a more realistic example where several years' data are involved.

Example 17 Calculating seasonal indices (1 year's data)

Mikki runs a shop and she wishes to determine quarterly seasonal indices for the number of customers to her shop based on last year's figures which are shown in the table opposite.

Summer	Autumn	Winter	Spring
920	1085	1241	446

Explanation

1 The seasons are quarters. Write the formula in terms of quarters.

Solution

$$\text{Seasonal index} = \frac{\text{value for season}}{\text{seasonal average}}$$

2 Find the quarterly average for the year.

$$\text{Quarterly average} = \frac{920 + 1085 + 1241 + 446}{4}$$
$$= 923$$

3 The seasonal index (SI) for each quarter is the ratio of that quarter's sales to the average quarter.

$$SI_{Summer} = \frac{920}{923} = 0.997$$

$$SI_{Autumn} = \frac{1085}{923} = 1.176$$

$$SI_{Winter} = \frac{1241}{923} = 1.345$$

$$SI_{Spring} = \frac{446}{923} = 0.483$$

4 Check that the seasonal indices sum to 4 (the number of seasons). The slight difference is due to rounding.

Check: $0.997 + 1.176 + 1.345 + 0.483 = 4.001$

5 Write out your answers as a table of the seasonal indices.

Seasonal indices

Summer	Autumn	Winter	Spring
0.997	1.176	1.345	0.483

The next example illustrates how seasonal indices are calculated with 3 years' data. While the process looks more complicated, we just repeat what we did in the previous example three times and average the results for each year at the end.

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Example 18 Calculating seasonal indices (several years' data)

Suppose that Mikki has 3 years of data, as shown. Use the data to calculate seasonal indices, rounded to two decimal places.

Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

Solution

The strategy is as follows:

- Calculate the seasonal indices for years 1, 2 and 3 separately. As we already have the seasonal indices for year 1 in the previous example we will save ourselves some time by simply quoting the result.
- Average the three sets of seasonal indices to obtain a single set of seasonal indices.

Explanation

1 Write down the result for year 1.

Solution

Year 1 seasonal indices:

Summer	Autumn	Winter	Spring
0.997	1.176	1.345	0.483

2 Now calculate the seasonal indices for year 2.

a The seasons are quarters. Write the formula in terms of quarters.

$$\text{Seasonal index} = \frac{\text{value for quarter}}{\text{quarterly average}}$$

b Find the quarterly average for the year.

$$\text{Quarterly average} = \frac{1035 + 1180 + 1356 + 541}{4} = 1028$$

c Work out the seasonal index (SI) for each time period.

$$SI_{Summer} = \frac{1035}{1028} = 1.007$$

$$SI_{Autumn} = \frac{1180}{1028} = 1.148$$

$$SI_{Winter} = \frac{1356}{1028} = 1.319$$

$$SI_{Spring} = \frac{541}{1028} = 0.526$$

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d Check that the seasonal indices sum to 4.

Check: 1.007 + 1.148 + 1.319 + 0.526 = 4.000

e Write out your answers as a table of the seasonal indices.

Year 2 seasonal indices:

Summer	Autumn	Winter	Spring
1.007	1.148	1.319	0.526

3 Now calculate the seasonal indices for year 3.

a Find the quarterly average for the year.

$$\text{Quarterly average} = \frac{1299 + 1324 + 1450 + 659}{4} = 1183$$

b Work out the seasonal index (SI) for each time period.

$$SI_{Summer} = \frac{1299}{1183} = 1.098$$
$$SI_{Autumn} = \frac{1324}{1183} = 1.119$$
$$SI_{Winter} = \frac{1450}{1183} = 1.226$$
$$SI_{Spring} = \frac{659}{1183} = 0.557$$

c Check that the seasonal indices sum to 4.

Check: 1.098 + 1.119 + 1.226 + 0.557 = 4.000

d Write out your answers as a table of the seasonal indices.

Year 3 seasonal indices:

Summer	Autumn	Winter	Spring
1.098	1.119	1.226	0.557

4 Find the 3-year averaged seasonal indices by averaging the seasonal indices for each season.

Final seasonal indices:

$$S_{Summer} = \frac{0.997 + 1.007 + 1.098}{3} = 1.03$$
$$S_{Autumn} = \frac{1.176 + 1.148 + 1.119}{3} = 1.15$$
$$S_{Winter} = \frac{1.345 + 1.319 + 1.226}{3} = 1.30$$
$$S_{Spring} = \frac{0.483 + 0.526 + 0.557}{3} = 0.52$$

5 Check that the seasonal indices sum to 4.

Check: 1.03 + 1.15 + 1.30 + 0.52 = 4.00

6 Write out your answers as a table of the seasonal indices.

Summer	Autumn	Winter	Spring
1.03	1.15	1.30	0.52

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Interpreting the seasonal indices

Having calculated these seasonal indices, what do they tell about the previous situation?

The seasonal index of:

- 1.03 tells us that in summer, customer numbers are typically 3% above average.
- 1.15 tells us that in autumn, customer numbers are typically 15% above average.
- 1.30 tells us that in winter, customer numbers are typically 30% above average.
- 0.52 tells us that in spring, customer numbers are typically 48% below average.

Using seasonal indices to deseasonalise a time series

Once we have determined the seasonal indices we can use the rule for deseasonalising the time series introduced earlier on this section

$$\text{deseasonalised figure} = \frac{\text{actual figure}}{\text{seasonal index}}$$

to deseasonalise the data.

Example 19 Deseasonalising a time series

The quarterly sales figures for Mikki's shop over a 3-year period are given below.

Year	Summer	Autumn	Winter	Spring
1	920	1085	1241	446
2	1035	1180	1356	541
3	1299	1324	1450	659

Use the seasonal indices shown to deseasonalise these sales figures. Write answers rounded to the nearest whole number.

Summer	Autumn	Winter	Spring
1.03	1.15	1.30	0.52

Explanation

1 To deseasonalise each sales figure in the table, divide by the appropriate seasonal index.
For example, for summer, divide the figures in the 'Summer' column by 1.03.
Round results to the nearest whole number.

2 Repeat for the other seasons.

Solution

Deseasonalised Summer sales:

Year 1: $\frac{920}{1.03} = 893$

Year 2: $\frac{1035}{1.03} = 1005$

Year 3: $\frac{1299}{1.03} = 1261$

Deseasonalised sales figures

Year	Summer	Autumn	Winter	Spring
1	893	943	955	858
2	1005	1026	1043	1040
3	1261	1151	1115	1267

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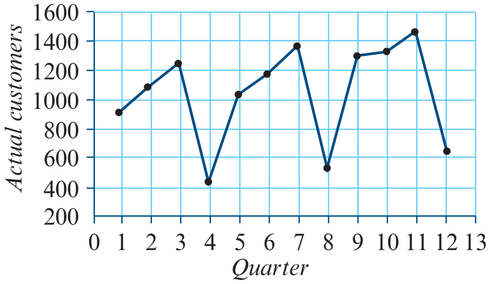
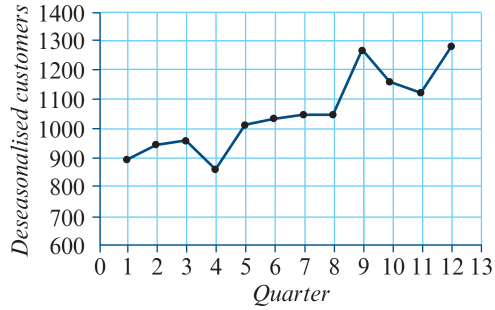
Why deseasonalise?

The purpose of removing the seasonality component of a time series is generally so that any trend in the time series is clearer. Consider again the actual customer data, and the deseasonalised customer data from Example 18, both of which are shown in the following time series plots.





It is hard to see from the first plot whether there has been any growth in Mikki's business, but the deseasonalised plot reveals revealed a clear underlying trend in the data.

It is common to deseasonalise time series data before you fit a trend line. We will consider this further in the next section.

Skill-sheet

Exercise 5D

Basic skills and interpretation

Use the following information to answer Questions 1 to 4.

The table below shows the monthly sales figures (in $'000s) and seasonal indices (for January to November) for a product produced by the U-beaut company.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ($'000s)	9.6	10.5	8.6		7.1	6.0	5.4		6.4	7.2	8.3	7.4
Seasonal index	1.2	1.3	1.1	1.0	1.0	0.9	0.8	0.7	0.9	1.0	1.1	

Example 14

1 **a** Find the seasonal index for December.

b Interpret the seasonal index for February.

c Interpret the seasonal index for September.

Example 15

2 **a** Find the deseasonalised sales figure (in $'000s) for March, giving your answer rounded to one decimal place.

b Find the deseasonalised sales figure (in $'000s) for June, giving your answer rounded to one decimal place.

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3 **a** The deseasonalised sales figure (in \$'000s) for August is 5.6. Find the actual sales (in \$'000s), giving your answer rounded to one decimal place.

b The deseasonalised sales figure (in \$'000s) for April is 6.9. Find the actual sales (in \$'000s), giving your answer rounded to one decimal place.

Example 16

4 **a** By what percentage should the sales in August be increased or decreased in order to correct for seasonality? Give your answer as a percentage rounded to one decimal place.

b By what percentage should the sales in February be increased or decreased in order to correct for seasonality? Give your answer as a percentage rounded to one decimal place.

5 The table below shows the quarterly newspaper sales (in \$'000s) of a corner store. Also shown are the seasonal indices for newspaper sales for the first, second and third quarters.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Sales		1060	1868	1642
Seasonal index	0.8	0.7	1.3	

a Find the seasonal index for quarter 4.

b Find the deseasonalised sales (in \$'000s) for quarter 2.

c Find the deseasonalised sales (in \$'000s) for quarter 3.

d The deseasonalised sales (in \$'000s) for quarter 1 are 1256. Find the actual sales.

Deseasonalising a time series

6 The following table shows the number of students enrolled in a 3-month computer systems training course along with some seasonal indices that have been calculated from the previous year's enrolment figures. Complete the table by calculating the seasonal index for spring and the deseasonalised student numbers for each course.

	Summer	Autumn	Winter	Spring
Number of students	56	125	126	96
Deseasonalised numbers				
Seasonal index	0.5	1.0	1.3	

7 The number of waiters employed by a restaurant chain in each quarter of 1 year, along with some seasonal indices that have been calculated from the previous years' data, are given in the following table.

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	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Number of waiters	198	145	86	168
Seasonal index	1.30		0.58	1.10

a What is the seasonal index for the second quarter?

b The seasonal index for quarter 1 is 1.30. Explain what this means in terms of the average quarterly number of waiters.

c Deseasonalise the data.

Calculating seasonal indices

Example 17

8 The table below records quarterly sales (in \$'000s) for a shop.

Quarter 1	Quarter 2	Quarter 3	Quarter 4
60	56	75	78

Use the data to determine the seasonal indices for the four quarters. Give your results rounded to two decimal places.

9 The table below records the monthly visitors (in '000s) to a museum over one year.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
12	13	14	17	18	15	9	10	8	11	15	20

Use the data to determine the seasonal indices for the 12 months. Give your results rounded to two decimal places.

Example 18

10 The table below records the monthly sales (in \$'000s) for a shop over a two year period.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
22	19	25	23	20	18	20	15	14	11	23	30
21	20	23	25	22	17	19	17	16	11	25	31

Use the data to determine the seasonal indices for the 12 months. Give your results rounded to two decimal places.

Example 19

11 The daily number of cars carried on a car ferry service each day over a two-week period, together with the daily seasonal indices, are shown in the table below:

Week	Mon	Tues	Wed	Thur	Fri	Sat	Sun
1	124	110	45	67	230	134	330
2	120	108	57	74	215	150	345
Seasonal index	0.8	0.7	0.3	0.5	1.5	1.0	2.2

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a Use the seasonal indices to deseasonalise the data, rounding the answers to the nearest whole number.

b Use your calculator to construct a time series plot the the deseasonalised data.

12 The numbers of retail job vacancies advertised on an online job board each quarter in each of three consecutive years are shown in the following table.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	212	194	196	227
2	220	197	196	239
3	231	205	203	245

a Construct a time series plot of the data.

b Use the data to calculate seasonal indices, rounded to two decimal places.

c Use the seasonal indices to construct a table of the deseasonalised data.

d Construct a time series plot of the deseasonalised data.

Exam 1 style questions

Use the following information to answer Questions 13 to 15.

The table below shows the number of customers each month at a restaurant and the long term seasonal indices for the number of customers at the restaurant each month of the year. The number of customers for August is missing.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal index	1.36	1.19	1.05	1.01	0.93	0.82	0.75	0.68	0.87	0.9	1.18	1.26
Number of customers	934	836	736	716	649	554	541		598	626	826	873

13 To correct the number of customers in May for seasonality, the actual number of customers should be:

A increased by 93.0% **B** decreased by 93.0% **C** decreased by 7.0%
D decreased 7.5% **E** increased by 7.5%

14 To correct the number of customers in November for seasonality, the actual number of customers should be:

A increased by 18.0% **B** decreased by 84.7% **C** increased by 15.3%
D decreased 15.3% **E** decreased by 18.0%

15 If the deseasonalised number of customers for August is 700, the actual number of customers in that month is closest to:

A 1029 **B** 768 **C** 570 **D** 607 **E** 476

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16 The table below records the monthly average electricity cost (in dollars) for a home.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
223	190	253	236	201	189	203	153	143	111	235	307

The seasonal index for August is closest to:

A 1.00 **B** 0.75 **C** 1.25 **D** 0.80 **E** 0.87

17 The table below shows the room occupancy rate for a chain of hotels over the summer, autumn, winter and spring quarters for the years 2020–2022.

Season	2020	2021	2022
summer quarter	72.0	71.5	72.0
autumn quarter	72.4	71.9	72.7
winter quarter	68.4	67.7	68.3
spring quarter	72.3	73.0	73.5
Quarterly average	71.3	71.0	71.6

The seasonal index for winter is closest to:

A 0.960 **B** 0.957 **C** 1.046 **D** 0.969 **E** 1.003

5E Fitting a trend line and forecasting

Learning intentions

- To be able to use the method of least squares to fit a trend line to a time series.
- To be able to use the trend line to make predictions.
- To be able to use seasonal indices to add seasonality to predicted values as appropriate.

Fitting a trend line

If we identify a linear trend in the time series plot, we can use the least squares method to fit a line to the data to model that trend.

The following example demonstrates fitting a trend line to times series data which shows no seasonal component.

Example 20 **Fitting a trend line**

The table below shows the number of female students in Victoria enrolled in at least one subject in the Mathematics learning area at year 12 over the period 2010–18. Fit a trend line to the data, and interpret the slope.

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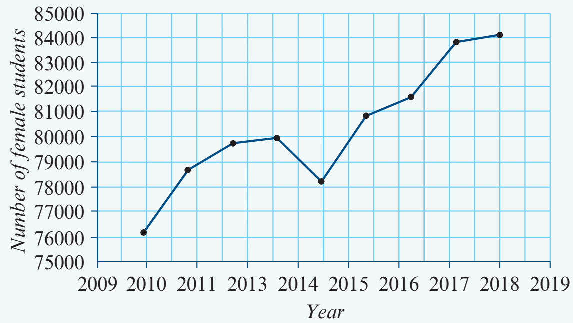
Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Number	76260	78707	79797	79952	78237	80858	81587	83820	84069

Explanation

1 Construct a time series plot of the data to confirm that the trend is approximately linear.

Solution



2 Fit a least squares line to the data, giving the coefficients rounded to six significant figures.

Number of female students = −1 633 580 + 851.017×*year*

3 Write down the slope rounded to the nearest whole number and interpret.

Slope = 851

Over the period 2010–2018 on average the number of female students in Victoria enrolled in at least one subject in the Mathematics learning area at year 12 increased by 851 per year.

Forecasting

Using a trend line fitted to a time series plot to make predictions about future values is known as **trend line forecasting**.

Example 21 Forecasting

How many female students in Victoria do we predict being enrolled in at least one subject in the Mathematics learning area at year 12 in 2026 if the same increasing trend continues? Give your answer rounded to the nearest whole number.

Explanation

Substitute 2026 in the equation determined using least squares regression, and round to the nearest whole number.

Solution

number of female students

= −1 633 580 + 851.017 × *year*

= −1 633 580 + 851.017 × 2026

= 90 580 to the nearest whole number.

Note: As with any prediction involving extrapolation, the results obtained when predicting well beyond the range of the data should be treated with caution.

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Forecasting taking seasonality into account

When time series data is seasonal, it is usual to deseasonalise the data before fitting the trend line.

Example 22 **Fitting a trend line (seasonality)**

The deseasonalised quarterly sales data from Mikki's shop are shown below.

Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Sales	893	943	955	858	1005	1026	1043	1040	1261	1151	1115	1267

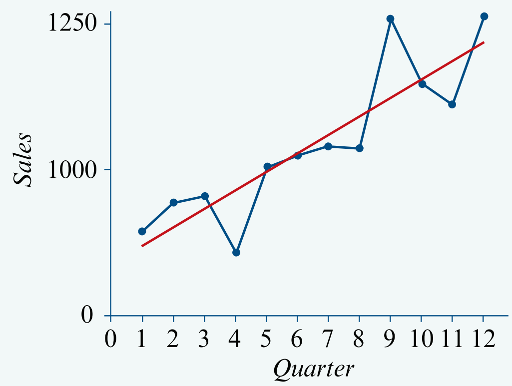
Fit a trend line and interpret the slope.

Explanation

1 Using a calculator plot the time series.

2 Fit a least squares line (with quarter the EV and sales as the RV).

Solution



3 Write down the equation of the least squares line with the intercept and slope rounded to 4 significant figures.

Sales = 838.0 + 32.07 × quarter

4 Interpret the slope in terms of the variables involved.

Over the 3-year period, on average sales at Mikki's shop increased by 32.07 sales per quarter.

Making predictions with deseasonalised data

When using deseasonalised data to fit a trend line, you must remember that the result of any prediction is a deseasonalised value. To be meaningful, this result must then be reseasonalised by multiplying by the appropriate seasonal index.

Example 23 **Forecasting (seasonality)**

What sales do we predict for Mikki's shop in the winter of year 4? (Because many items have to be ordered well in advance, retailers often need to make such decisions.)

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Explanation	Solution
1 Substitute the appropriate value for the time period in the equation for the trend line. Since summer year 1 is quarter 1, then winter year 4 is quarter 15.	Deseasonalised sales $= 838.0 + 32.07 \times \text{quarter}$ $= 838.0 + 32.07 \times 15$ $= 1319.05$
2 To obtain the actual predicted sales figure reseasonalise the predicted value by multiplying this value by the seasonal index for winter, 1.30.	Actual sales prediction for winter of year 4 $= 1319.05 \times 1.30$ $= 1714.765$ $= 1715$ (to the nearest whole number)

Exercise 5E

Fitting a least squares line to a time series plot (no seasonality)

Example 21
Example 20

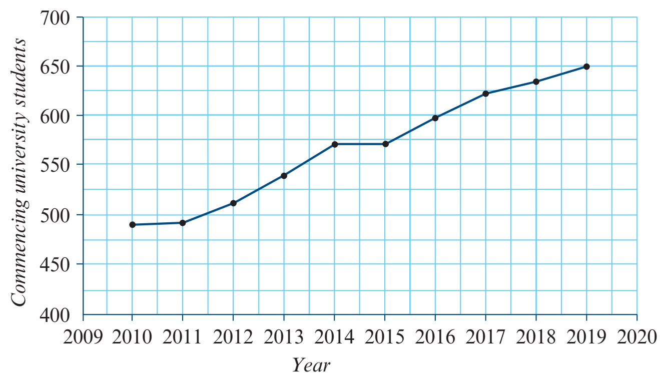
1 The data show the number of commencing university students (in thousands) in Australia for the period 2010–2019.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Number	488	490	510	538	569	569	595	619	632	645

The time series plot of the data is shown below.

a Comment on the plot.

b Fit a least squares regression trend line to the data, giving the values of the coefficients to 5 significant figures, and interpret the slope.



c Use this equation to predict the number of students expected to commence university in Australia in 2030 to the nearest 1000 students.

2 The table below shows the percentage of total retail sales that were made in department stores over an 11-year period:

Year	1	2	3	4	5	6	7	8	9	10	11
Sales (%)	12.3	12.0	11.7	11.5	11.0	10.5	10.6	10.7	10.4	10.0	9.4

a Using your CAS calculator, construct a time series plot.

b Comment on the time series plot in terms of trend.

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c Fit a trend line to the time series plot, find its equation and interpret the slope. Give your answer rounded to 3 significant figures.

d Use the trend line to forecast the percentage of retail sales which will be made by department stores in year 15.

3 The median ages of mothers in Australia over the years 2010–2020 are shown below.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Age	30.7	30.7	30.7	30.8	30.8	30.9	31.1	31.2	31.3	31.4	31.5

a Fit a least squares regression trend line to the data, and interpret the slope. (Give the values of the coefficients rounded to 3 significant figures.)

b Use the trend line to forecast the average ages of mothers having their first child in Australia in 2030. Explain why this prediction is not likely to be reliable.

4 The average weekly earnings (in dollars) in Australia during the period 2014–2021 are given in the following table.

Year	2014	2015	2016	2017	2018	2019	2020	2021
Earnings	1454.10	1483.10	1516.00	1543.20	1585.30	1634.80	1713.90	1737.10

a Fit a least squares regression trend line to the data, and interpret the slope. Give the values of the coefficients rounded to four significant figures.

b Use this trend relationship to forecast average weekly earnings in 2030. Explain why this prediction is not likely to be reliable.

Fitting a least-squares line to a time series with seasonality

Example 23

5 The table below shows the deseasonalised quarterly washing-machine sales of a company over 3 years.

	Year 1				Year 2				Year 3			
Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Deseasonalised sales	53	51	54	55	64	64	61	63	67	69	68	66

a Use least squares regression to fit a trend line to the data.

Example 22

b Use this trend equation for washing-machine sales, with the seasonal indices below, to forecast the sales of washing machines in the fourth quarter of year 4.

Quarter	1	2	3	4
Seasonal index	0.90	0.81	1.11	1.18

6 The quarterly seasonal indices for the sales of boogie boards in a surf shop are as follows.

Seasonal index	1.13	0.47	0.62	1.77

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The actual sales of the boogie boards over a 2-year period are given in the table.

Year	*Quarter 1*	*Quarter 2*	*Quarter 3*	*Quarter 4*
1	138	60	73	230
2	283	115	163	417

a Use the seasonal indices to calculate the deseasonalised sales figures for this period to the nearest whole number.

b Use a CAS calculator to plot the actual sales figures and the deseasonalised sales figures for this period and comment on the plot.

c Fit a trend line to the deseasonalised sales data. Write the slope and intercept rounded to three significant figures.

d Use the relationship calculated in **c**, together with the seasonal indices, to forecast the sales for the first quarter of year 4 (you will need to reseasonalise here).

Exam 1 style questions

7 The number of visitors to an adventure park is seasonal. A least squares regression line has been fitted to the data, and the equation is:

$$\textit{deseasonalised number of visitors} = 38345 + 286.5 \times \textit{quarter}$$

where *quarter* number one is January – March 2022.

The quarterly seasonal indices for visitors to the adventure park are shown in the table below.

Quarter	Jan-Mar	Apr-Jun	Jul-Sept	Oct-Dec
Seasonal index	1.17	0.91	0.78	1.14

The predicted number of actual visitors for the April-June quarter in 2025 is closest to:

A 42070 **B** 42356 **C** 38544 **D** 46545 **E** 37501

8 An electrical goods retailer knows that the sales of air conditioners are seasonal. A least squares regression line has been fitted to the data collected by the retailer in 2021 and 2022, and the equation is:

$$\textit{deseasonalised number of air conditioners} = 197 + 1.2 \times \textit{month}$$

where *month* number one is January 2021.

The monthy seasonal indices for air conditioner sales are shown in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal index	1.36	1.19	1.05	1.01	0.93	0.82	0.75	0.68	0.87	0.90	1.18	1.26

The predicted number of actual sales for November 2025 is closest to:

A 268 **B** 227 **C** 299 **D** 333 **E** 316

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Key ideas and chapter summary

Assign-ment

Time series data — **Time series data** are a special case of bivariate data, where the explanatory variable is the time at which the values of the response variable were recorded.

Time series plot — A **time series plot** is a bivariate plot where the values of the response variable are plotted in time order. Points in a time series plot are joined by line segments.

Features to look for in a time series plot

- Trend
- Seasonality
- Possible outliers
- Cycles
- Structural change
- Irregular (random) fluctuations

Trend — A general increase or decrease over a significant period of time in a times series plot is called a **trend**.

Cycles — **Cycles** are present when there is a periodic movement in a time series. The period is the time it takes for one complete up and down movement in the time series plot. This term is generally reserved for periodic movements with a period greater than one year.

Seasonality — **Seasonality** is present when there is a periodic movement in a time series that has a calendar related period – for example, a year, a month, a week.

Structural change — **Structural change** is present when there is a sudden change in the established pattern of a time series plot.

Outliers — **Outliers** are present when there are individual values that stand out from the general body of data.

Irregular (random) fluctuations — **Irregular (random) fluctuations** are always present in any real-world time series plot. They include all of the variations in a time series that we cannot reasonably attribute to systematic changes like trend, cycles, seasonality, structural change or the presence of outliers.

Smoothing — **Smoothing** is a technique used to eliminate some of the irregular fluctuations in a time series plot so that features such as trend are more easily seen.

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Review

Moving mean smoothing In **moving mean smoothing**, each original data value is replaced by the mean of itself and a number of data values on either side. When smoothing over an even number of data points, centring is required to ensure the smoothed mean is centred on the chosen point of time.

Moving median smoothing **Moving median smoothing** is a graphical technique for smoothing a time series plot using moving medians rather than means.

Seasonal indices **Seasonal indices** are used to quantify the seasonal variation in a time series.

Deseasonalise The process of accounting for the effects of seasonality in a time series is called **deseasonalisation**.

Reseasonalise The process of a converting seasonal data back into its original form is called **reseasonalisation**.

Trend line forecasting **Trend line forecasting** uses the equation of a trend line to make predictions about the future.

Skills checklist

Check-list

Download this checklist from the Interactive Textbook, then print it and fill it out to check your skills.

5A **1** I can construct a time series plot. ☐

See Example 1, and Exercise 5A Question 1

5A **2** I can identify trend in a time series plot. ☐

See Example 2, and Exercise 5A Question 4

5A **3** I can identify cycles in a time series plot. ☐

See Example 3, and Exercise 5A Question 5

5A **4** I can identify seasonality in a time series plot. ☐

See Example 4, and Exercise 5A Question 5

5A **5** I can identify structural change in a time series plot. ☐

See Example 5, and Exercise 5A Question 6

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5A **6** I can identify outliers in a time series plot. ☐

See Example 6, and Exercise 5A Question 7

5B **7** I can smooth a time series using moving mean smoothing. ☐

See Example 7, and Exercise 5B Question 1

5B **8** I can smooth a time series using moving mean smoothing with centring. ☐

See Example 9, and Exercise 5B Question 6

5C **9** I can smooth a time series using moving median smoothing. ☐

See Example 12, and Exercise 5C Question 2

5D **10** I can interpret seasonal indices. ☐

See Example 14, and Exercise 5D Question 1

5D **11** I can use seasonal indices to deseasonalise and reseasonalise data. ☐

See Example 15, and Exercise 5D Question 2

5D **12** I can use seasonal indices to determine percentage change required to correct for seasonality. ☐

See Example 16, and Exercise 5D Question 4

5D **13** I can calculate seasonal indices from 1 year of data. ☐

See Example 17, and Exercise 5D Question 8

5D **14** I can calculate seasonal indices from several years of data. ☐

See Example 18, and Exercise 5D Question 10

5D **15** I can use seasonal indices to deseasonalise a time series. ☐

See Example 19, and Exercise 5D Question 11

5E **16** I can fit a trend line to a time series plot. ☐

See Example 21, and Exercise 5E Question 1

5E **17** I can use a trend line to forecast a future value (no seasonality). ☐

See Example 20, and Exercise 5E Question 1

5E **18** I can fit a trend line to a time series plot with seasonality. ☐

See Example 22, and Exercise 5E Question 4

5E **19** I can use a trend line to forecast a future value (with seasonality). ☐

See Example 23, and Exercise 5E Question 4

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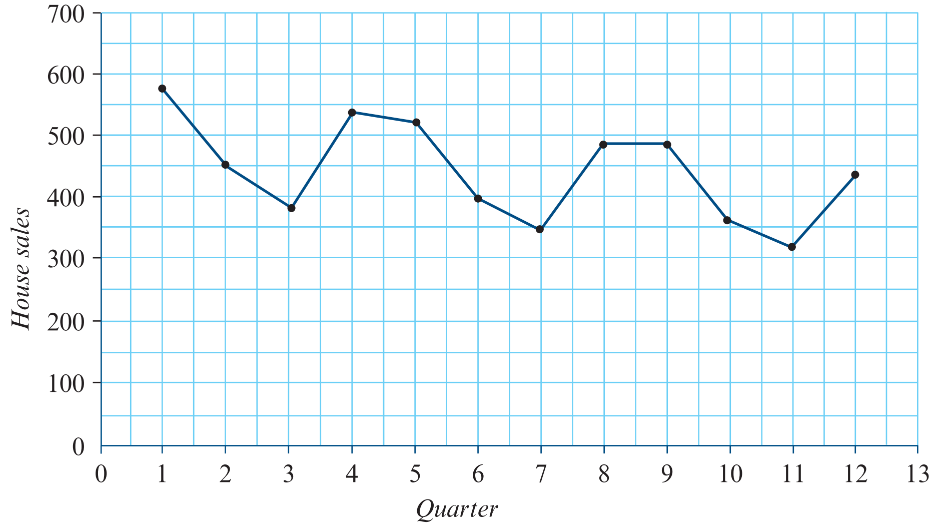
Multiple-choice questions

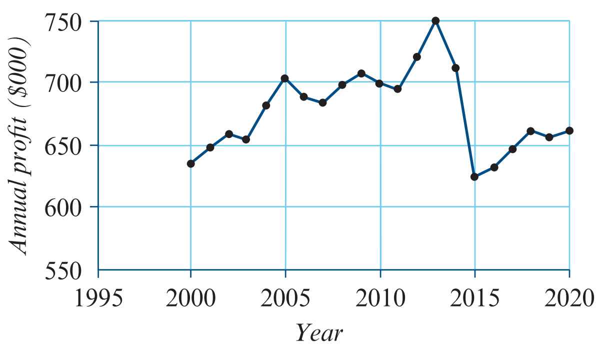
1 The time series plot below shows quarterly house sales for a real estate agency over a three year period.



The time series plot is best described as showing

A seasonality only
B irregular fluctuations only
C seasonality with irregular fluctuations
D an increasing trend seasonality and irregular fluctuations
E a decreasing trend with seasonality and irregular fluctuations

2 The time series plot below shows the annual profit (in $000) for a manufacturing company.



The time series plot is best described as having

A increasing trend
B decreasing trend

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C seasonality with irregular fluctuations
D increasing trend with an outlier
E increasing trend with a structural change

Use the following table to answer Questions 3 to 6.

Time period	1	2	3	4	5	6
Data value	2.3	3.4	4.4	2.7	5.1	3.7

3 The three-moving mean for time period 2 is closest to:

A 3.4 **B** 3.6 **C** 3.9 **D** 4.0 **E** 4.2

4 The five-moving mean for time period 3 is closest to:

A 3.4 **B** 3.6 **C** 3.9 **D** 4.1 **E** 4.2

5 The two-moving mean for time period 5 with centring is closest to:

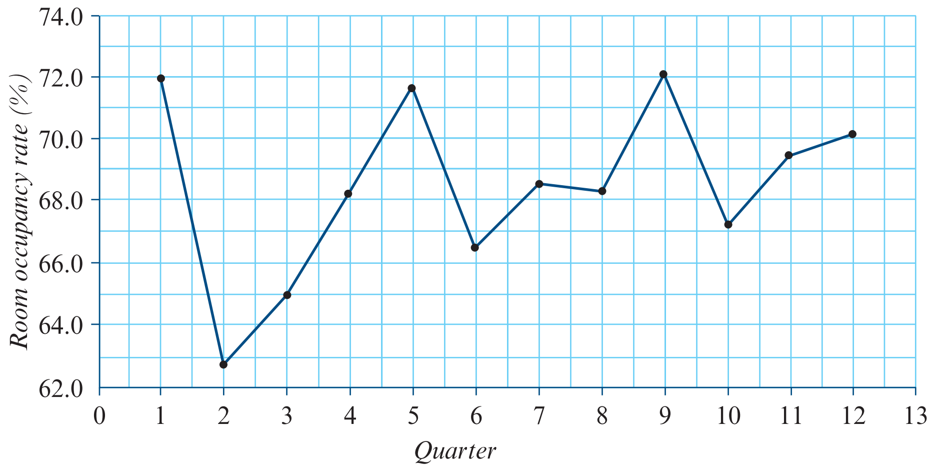
A 2.7 **B** 3.6 **C** 3.9 **D** 4.0 **E** 4.2

6 The four-moving mean for time period 4 with centring is closest to:

A 2.7 **B** 3.6 **C** 3.9 **D** 4.1 **E** 4.2

Use the following information to answer Questions 7 and 8.

The time series plot for hotel room occupancy rate (%) in a large city over a three year period is shown below.



7 The three-median smoothed value for Quarter 2 is closest to:

A 62 **B** 63 **C** 64 **D** 65 **E** 67

8 The five-median smoothed value for Quarter 3 is closest to:

A 62 **B** 64 **C** 65 **D** 68 **E** 69

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Review

9 The seasonal indices for the number of customers at a restaurant are as follows.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1.0	p	1.1	0.9	1.0	1.0	1.2	1.1	1.1	1.1	1.0	0.7

The value of p is:

A 0.5 **B** 0.7 **C** 0.8 **D** 1.0 **E** 1.2

The following information relates to Questions 10 and 11

10 The table shows the closing price (*price*) of a company's shares on the stock market over a 10 day period.

Day	1	2	3	4	5	6	7	8	9	10
Price($)	2.85	2.80	2.78	2.40	2.80	3.15	3.42	3.95	4.05	3.35

The six-mean smoothed with centring closing share *price* on Day 5 is closest to:

A $2.80 **B** $2.89 **C** $2.91 **D** $2.99 **E** $3.08

11 If five-mean smoothing was used to smooth this time series, the number of smoothed values would be:

A 5 **B** 6 **C** 7 **D** 8 **E** 9

12 Suppose that Lyn spent a total of $427 on dining out over the period from January to March, and then another $230 over the period April-May. The five-mean smoothed amount that she spent in March is closest to:

A $115 **B** $129 **C** $131 **D** $142 **E** $329

Use the following information to answer Questions 13 to 16.

The seasonal indices for the number of bathing suits sold at a surf shop are given in the table.

Quarter	Summer	Autumn	Winter	Spring
Seasonal index	1.8	0.4	0.3	1.5

13 The number of bathing suits sold one summer is 432. The deseasonalised number is closest to:

A 432 **B** 240 **C** 778 **D** 540 **E** 346

14 The deseasonalised number of bathing suits sold one winter was 380. The actual number was closest to:

A 114 **B** 133 **C** 152 **D** 380 **E** 1267

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15 The seasonal index for spring tells us that, over time, the number of bathing suits sold in spring tends to be:

A 50% less than the seasonal average
B 15% less than the seasonal average
C the same as the seasonal average
D 15% more than the seasonal average
E 50% more than the seasonal average

16 To correct for seasonality, the actual number of bathing suits sold in Autumn should be:

A reduced by 50% **B** reduced by 40% **C** increased by 40%
D increased by 150% **E** increased by 250%

17 The number of visitors to an information centre each quarter was recorded for one year. The results are tabulated below.

Quarter	Summer	Autumn	Winter	Spring
Visitors	1048	677	593	998

Using this data, the seasonal index for autumn is estimated to be closest to:

A 0.25 **B** 1.0 **C** 1.22 **D** 0.82 **E** 0.21

Use the following information to answer Questions 18 and 19.

A trend line is fitted to a time series plot displaying the percentage change in commencing international student enrolments in Australia each year compared to the previous year (*enrolments*) for the period 2012–2019.

The equation of this line is: $\%\ \textit{change in enrolments} = -3480 + 1.73 \times \textit{year}$

18 Using this trend line, the percentage change in enrolments from the previous year forecast for 2026 is:

A 24.98 **B** -11.05 **C** 1.73 **D** 12.11 **E** 24.62

19 From the slope of the trend line it can be said that:

A on average, the number of commencing international student enrolments in Australia is increasing by by 1.73 each year.
B on average, the number of commencing international student enrolments in Australia is increasing by 1.73% each year.
C on average, the number of commencing international student enrolments in Australia is decreasing by 1.73% each month.
D on average, the number of commencing international student enrolments in Australia is decreasing by 1730 each year.
E on average, the number of commencing international student enrolments in Australia is decreasing by 1.73% each year.

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Review

20 Suppose that the seasonal indices for the wholesale price of petrol are:

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Index	1.2	1.0	0.9	0.8	0.7	1.2	1.2

The equation of the least squares regression line that could enable us to predict the *deseasonalised price* per litre in cents from the *day number* is

$$\textit{deseasonalised price} = 189.9 + 0.23 \times \text{day number}$$

where day number 1 is Sunday March 20. The predicted actual price for Sunday April 3 is closest to:

A 161.1 cents **B** 193.1 cents **C** 193.4 cents **D** 231.7 cents **E** 232.0 cents

Written response questions

1 The table below shows the carbon dioxide emissions in Australia (in tonnes per capita) for the period 2013 to 2020.

Year	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
CO_2	18.2	17.6	17.3	17.0	16.4	15.8	15.8	15.9	15.7	15.5

a Use your calculator to construct a times series plot of the data.

b Briefly describe the general trend in the data.

c Fit a least squares line to the time series plot that will enable *emissions* to be predicted from *year*. Write down the equation for the least squares line, rounding the intercept and slope to four significant figures.

d Use the least squares equation to predict the carbon dioxide emissions in Australia in 2026. Round to three significant figures.

e Explain why the prediction you made in part d may not be reliable.

2 The table below shows the annual inflation rates in Australia and China for the period 2010–20.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Inflation Australia (%)	2.9	3.3	1.7	2.5	2.5	1.5	1.3	2.0	1.9	1.6	0.9
Inflation China (%)	3.3	5.4	2.6	2.6	2.0	1.4	2.0	1.6	2.1	2.9	2.4

These data are plotted in the time series plot below.

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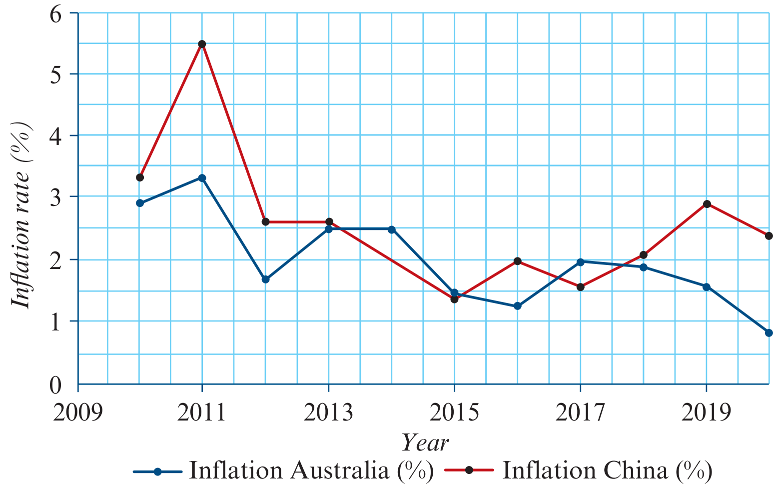

a **i** Find the equation of the least squares line which allows *inflation* to be predicted from *year* for China.

ii Draw the least squares line on the time series plot.

b **i** Find the equation of the least squares line which allows *inflation* to be predicted from *year* for Australia.

ii Draw the least squares line on the time series plot.

c Explain why the equations of the least squares lines predict that the inflation rate for China will always remain higher than the inflation rate for Australia.

d Find the two-mean centred smoothed inflation rate for Australia for 2015.

3 The table below shows the number of dolphins spotted in a bay over each of the four seasons for the years 2020-2021.

Year	Summer	Autumn	Winter	Spring
2020	97	112	480	678
2021	107	145	496	730

a Use the data in the table to find seasonal indices. Give your answers rounded to two decimal places.

b The number of dolphins spotted in each of the four seasons in 2022 is shown in the table below.

Year	Summer	Autumn	Winter	Spring
2022	78	86	350	540

Use the seasonal indices from part a to deseasonalise the data. Round your answers to the nearest whole number.

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