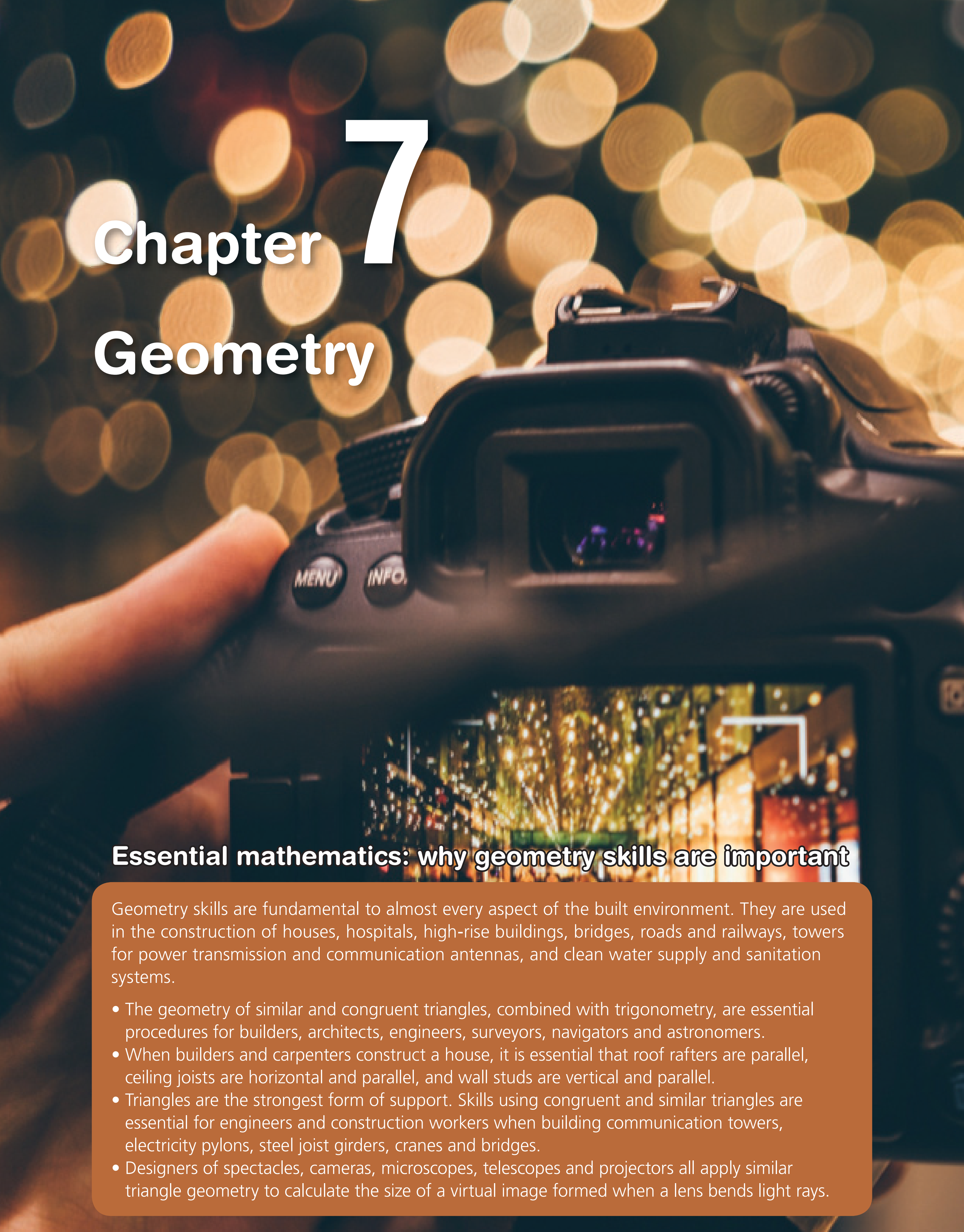

Chapter 7

Geometry

Essential mathematics: why geometry skills are important

Geometry skills are fundamental to almost every aspect of the built environment. They are used in the construction of houses, hospitals, high-rise buildings, bridges, roads and railways, towers for power transmission and communication antennas, and clean water supply and sanitation systems.

- The geometry of similar and congruent triangles, combined with trigonometry, are essential procedures for builders, architects, engineers, surveyors, navigators and astronomers.
- When builders and carpenters construct a house, it is essential that roof rafters are parallel, ceiling joists are horizontal and parallel, and wall studs are vertical and parallel.
- Triangles are the strongest form of support. Skills using congruent and similar triangles are essential for engineers and construction workers when building communication towers, electricity pylons, steel joist girders, cranes and bridges.
- Designers of spectacles, cameras, microscopes, telescopes and projectors all apply similar triangle geometry to calculate the size of a virtual image formed when a lens bends light rays.

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In this chapter

Victorian Curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Formulate proofs involving congruent triangles and angle properties (VCMMG344)

Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (VCMMG345)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Write the missing word. Choose from: *right*, *reflex*, *straight*, *complementary*, *acute*, *revolution*, *obtuse*, *supplementary*.

- **a** ________ angles are between 0° and 90°.
- **b** A ________ angle is 90°.
- **c** An ________ angle is between 90° and 180°.
- **d** A 180° angle is called a ________ angle.
- **e** A ________ angle is between 180° and 360°.
- **f** A ________ is 360°.
- **g** ________ angles sum to 90°.
- **h** ________ angles sum to 180°.

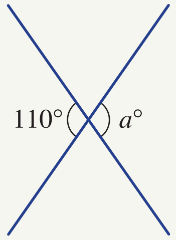
2 Match the type of triangle (**A**–**F**) with the given properties (**a**–**f**).

- **a** all sides of different length
- **b** two sides of the same length
- **c** one right angle
- **d** one obtuse angle
- **e** three sides of equal length
- **f** all angles acute

- **A** obtuse angled
- **B** isosceles
- **C** equilateral
- **D** acute angled
- **E** scalene
- **F** right angled

3 Find the values of the pronumerals.

a



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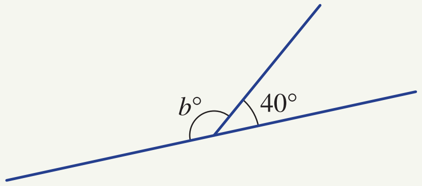

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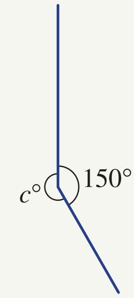

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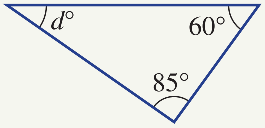

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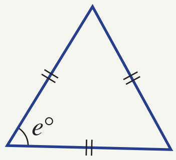

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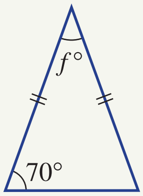
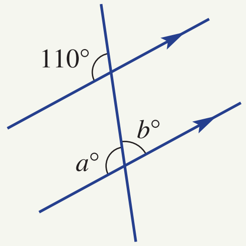


4 Find the value of the pronumerals in these sets of parallel lines.

a



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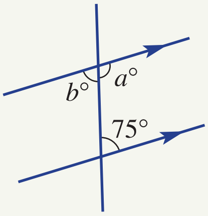

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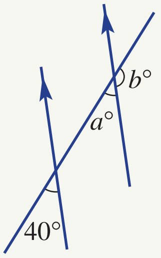

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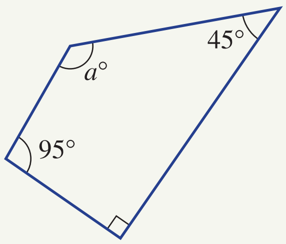
5 How many special quadrilaterals have these properties?

a All sides equal and all angles 90°
b Two pairs of parallel sides
c Two pairs of parallel sides and all angles 90°
d Two pairs of parallel sides and all sides equal
e One pair of parallel sides
f Two pairs of equal-length sides and no sides parallel

6 Use the angle sum formula, $S = 180° \times (n - 2)$, to find the angle sum of these polygons and the value of the pronumeral.

a



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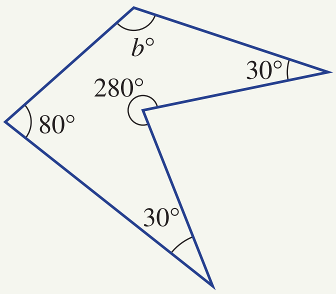

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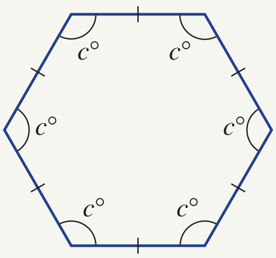
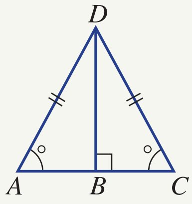


7 Select the option from **A**–**E** that represents:

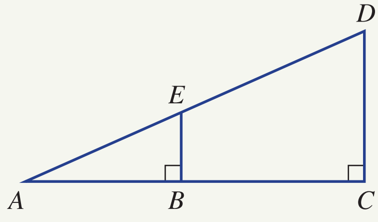
a the tests for congruence of triangles
b the tests for similarity of triangles

A SSS, SA, AA, RHS
B S, SAS, A, RAS
C SSS, SAS, AAA, RHS
D SSS, SAS, AAS, RHS
E SSS, AAA, RS

8 Complete this sentence for the given diagram. $\triangle AB\square$ is congruent to $\triangle\square BD$.



9 Complete this sentence for the given diagram.
$\triangle A\square D$ is similar to $\triangle AB\square$.



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7A Parallel lines CONSOLIDATING

Learning intentions

- To know the special pairs of angles formed when a transversal cuts two other lines
- To know the relationship between pairs of angles formed when a pair of parallel lines are cut by a transversal
- To be able to calculate unknown angles associated with parallel lines

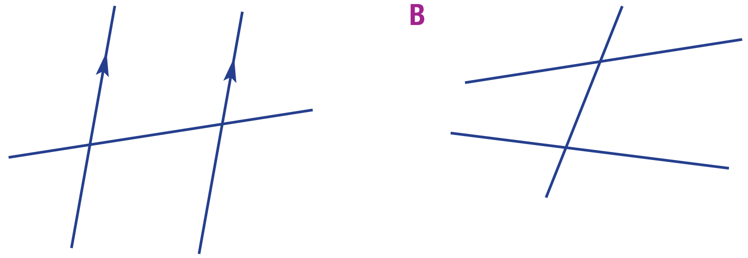
Key vocabulary: parallel, transversal, corresponding, alternate, cointerior, vertically opposite, supplementary

Parallel lines are everywhere – in buildings, in nature and on clothing patterns. Steel or concrete uprights at road intersections are an example.

Parallel lines are always the same distance apart and never meet. In diagrams, arrows are used to show that lines are parallel.

Lesson starter: 2, 4 or 8 different angles

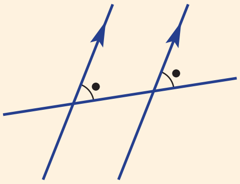
Here are two pairs of lines crossed by a transversal. One pair is parallel and the other is not.



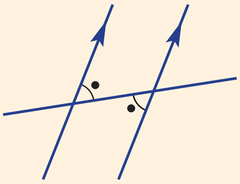
- How many angles of different size are in set A?
- How many angles of different size are in set B?
- If only one angle is known in set A, can you determine all other angles? Give reasons.

Key ideas

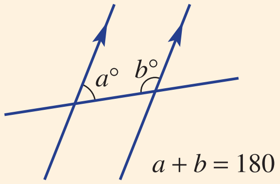
- A **transversal** is a line cutting two or more other lines.
- For parallel lines:
  - **Corresponding** angles are equal.



  - **Alternate** angles are equal.



  - **Cointerior** angles are **supplementary** (sum to 180°).



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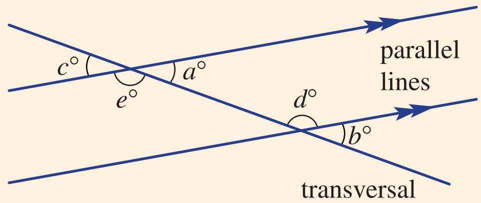
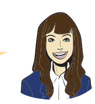


$a = b$	Corresponding angles
$a = c$	**Vertically opposite** angles
$d = e$	Alternate angles
$a + e = 180$	Supplementary angles
$a + d = 180$	Cointerior angles

Exercise 7A

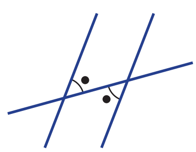
Understanding — 1, 2 — 2



Hint: Choose from:
180°, *equal, supplementary*

1 Write the missing word or number.
- **a** Supplementary angles add to __________.
- **b** Vertically opposite angles are __________.
- **c** When two lines are parallel and are crossed by a transversal, then:
  - **i** Corresponding angles are __________.
  - **ii** Alternate angles are __________.
  - **iii** Cointerior angles are __________.

2 For the given diagrams, decide whether the given pair of marked angles are corresponding, alternate, cointerior or vertically opposite.

a


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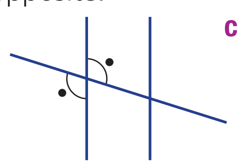

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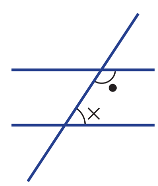

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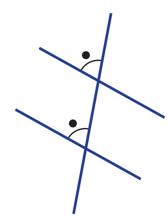
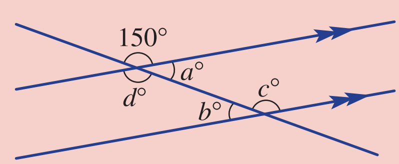


Fluency — 3–5 — 3, 4, 5(½)

Example 1 Finding angles in parallel lines

Find the values of the pronumerals in this diagram.
Write down the reason in each case.



Solution	Explanation
$a = 180 - 150 = 30$ The angles marked as $a°$ and $150°$ are supplementary.	Two angles on a straight line sum to $180°$.
$b = 30$ The angles marked as $b°$ and $a°$ are alternate.	Alternate angles are equal in parallel lines.

Continued on next page

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$c = 150$
The angles marked as $c°$ and $150°$ are corresponding.
OR, the angles marked as $c°$ and $a°$ are cointerior.

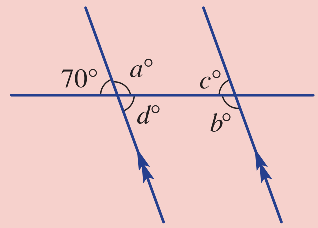
Corresponding angles are equal in parallel lines.
Cointerior angles are supplementary in parallel lines.

$d = 150$
The angles marked as $d°$ and $150°$ are vertically opposite.

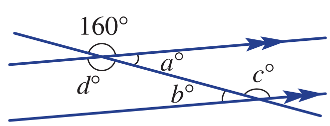
Vertically opposite angles are equal.

Now you try

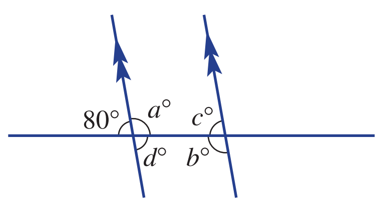
Find the values of the pronumerals in this diagram.
Write down the reason in each case.



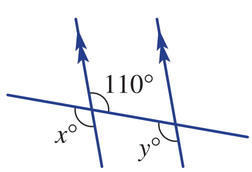
3 Find the values of the pronumerals in this diagram. Write down the reason in each case.



4 Find the values of the pronumerals in this diagram. Write down the reason in each case.



5 Find the value of x and y in these diagrams.

a


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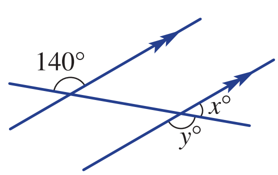

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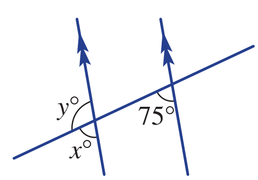

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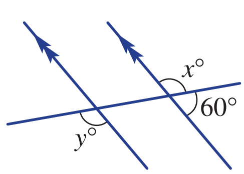

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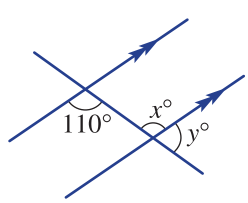

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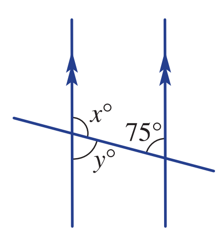

Hint: When lines are parallel:
- Corresponding angles are equal.
- Alternate angles are equal.
- Cointerior angles add to $180°$.

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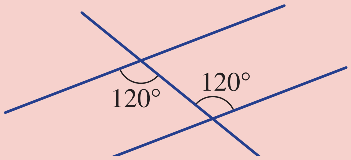
Problem-solving and reasoning

6, 7–8(½), 9 | 6–8(½), 9, 10

Example 2 Proving that two lines are parallel

Decide, with reasons, whether the given pairs of lines are parallel.

a



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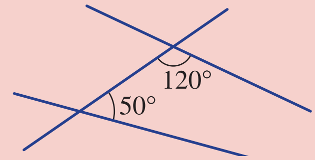
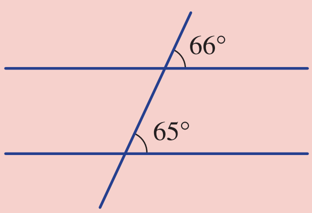


Solution	Explanation
a Yes – alternate angles are equal.	If alternate angles are equal, then lines are parallel.
b No – cointerior angles are not supplementary.	If lines are parallel, then cointerior angles should add to 180°, but $120^\circ + 50^\circ = 170^\circ$.

Now you try

Decide, with reasons, whether the given pairs of lines are parallel.

a



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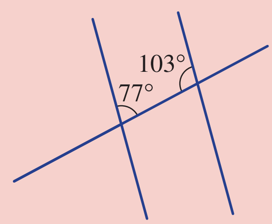
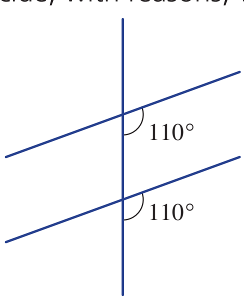


6 Decide, with reasons, whether the given pairs of lines are parallel.

a



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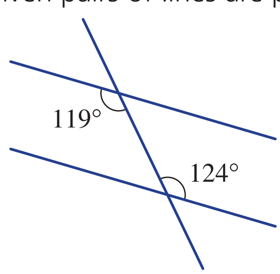

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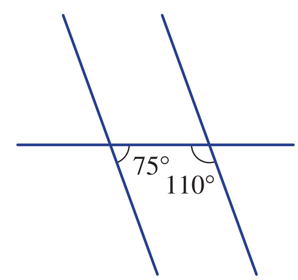

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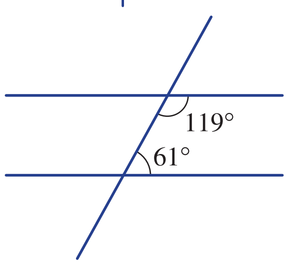

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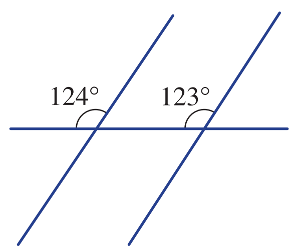

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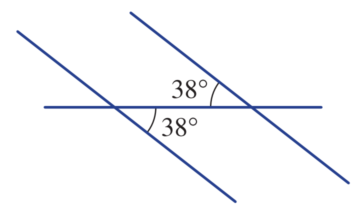
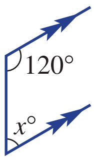


7 These diagrams have a pair of parallel lines. Find the unknown angle, x.

a



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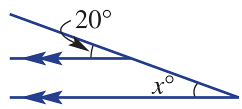

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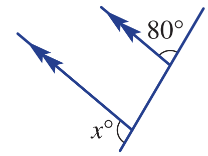

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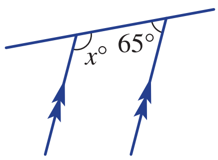

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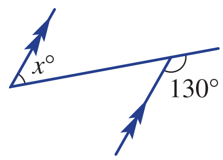

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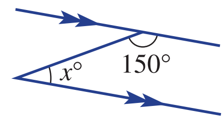

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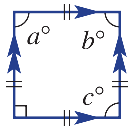
7A

8 These common shapes consist of parallel lines. One or more internal angles are given. Find the values of the pronumerals.

a


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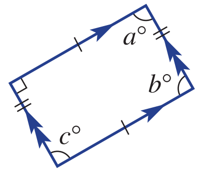

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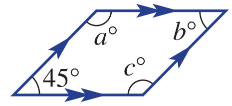

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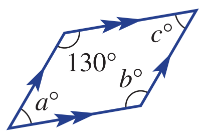

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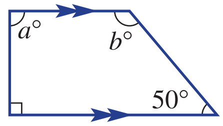

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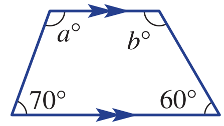
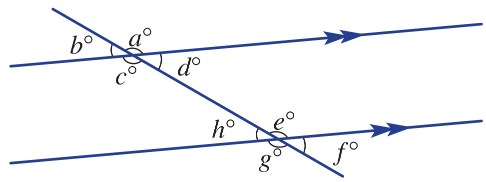
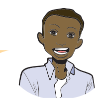


9 For this diagram, list all pairs of angles that are:

a corresponding
b alternate
c cointerior
d vertically opposite

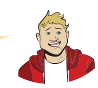


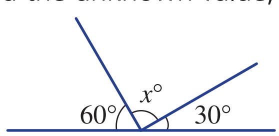
Hint: One example for part a is (a, e).



10 Find the unknown value, x, in each of these cases.

Hint: Angles on a straight line add to $180°$.



a


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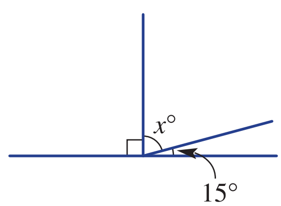

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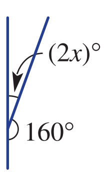

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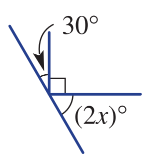

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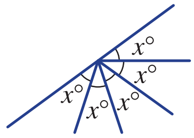

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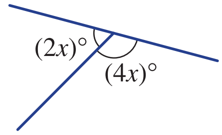
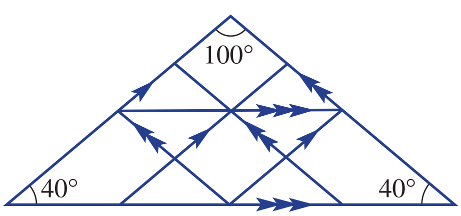


The roof truss | — | 11

11 This diagram is of a roof truss with three groups of parallel supports.
How many of the angles are:

a $100°$ in size?
b $40°$ in size?
c $140°$ in size?



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7B Triangles CONSOLIDATING

Learning intentions

- To know the angle sum of a triangle
- To know the properties of special types of triangles
- To know the exterior angle theorem
- To be able to calculate unknown angles inside a triangle
- To be able to calculate unknown angles using the exterior angle theorem

Key vocabulary: triangle, acute, right, obtuse, scalene, isosceles, equilateral, exterior angle theorem

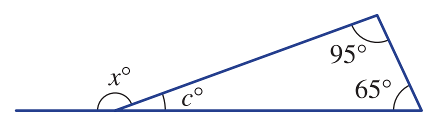
The triangle is at the foundation of geometry, and its properties are used to work with more complex geometry.

One of the best known and most useful properties of triangles is the internal angle sum (180°). You can check this by measuring and adding up the three internal angles of any triangle.

Lesson starter: Explore the exterior angle

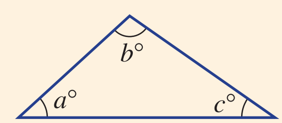
Consider this triangle with exterior angle x°.

- Use the angle sum of a triangle to find the value of c.
- Now find the value of x.
- What do you notice about x° and the two given angles? Is this true for other triangles? Give examples and reasons.

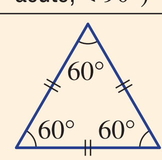
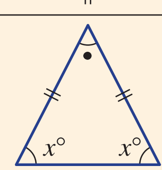
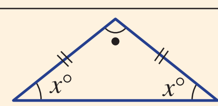
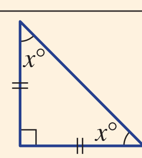
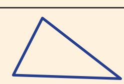
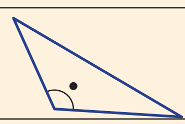
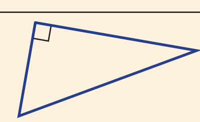


Key ideas

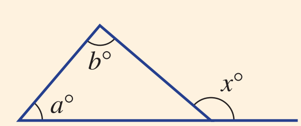
- The sum of all three internal angles of a triangle is 180°.
- Triangles can be classified by their side lengths or their internal angles.



$a + b + c = 180$

		Classified by internal angles		
		Acute-angled triangles (all angles acute, $< 90^\circ$)	Obtuse-angled triangles (one angle obtuse, $> 90^\circ$)	Right-angled triangles (one right angle, 90°)
Classified by side lengths	Equilateral triangles (three equal side lengths)		Not possible	Not possible
	Isosceles triangles (two equal side lengths)			
	Scalene triangles (no equal side lengths)			

- The **exterior angle theorem**:
  The exterior angle is equal to the sum of the two opposite interior angles.



$x = a + b$

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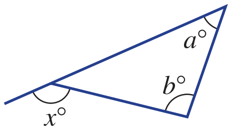
Exercise 7B

Understanding | 1–3 | 3

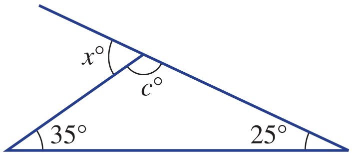
1 State the missing word or angle size.
  a The angle sum of a triangle is ________.
  b An ________ triangle has two equal side lengths.
  c An equilateral triangle's interior angles are all ________.
  d An ________ -angled triangle has one obtuse angle.
  e A ________ triangle has all sides of different length.
  f An acute-angled triangle has all angles less than ________.

2 Choose the correct expression for this exterior angle.



  A $a = x + b$ B $b = x + a$
  C $x = a + b$ D $a + b = 180$
  E $2a + b = 2x$

3 The two given interior angles for this triangle are $25°$ and $35°$.

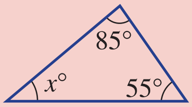


  a Use the angle sum $(180°)$ to find the value of c.
  b Hence, find the value of x.
  c What do you notice about the value of x and the two given interior angles?

Fluency | 4–6(½) | 4–6(½)

Example 3 Using the angle sum

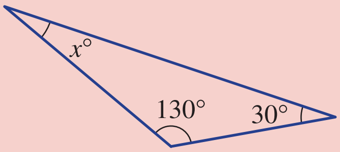
Find the value of the unknown angle, x, in this triangle.



Solution	Explanation
$x + 85 + 55 = 180$	The sum of the three internal angles in a triangle is $180°$.
$x + 140 = 180$	Simplify before solving for x.
$x = 40$	Solve for x by subtracting 140 from both sides of the 'equals' sign.
∴ The unknown angle is $40°$.	

Now you try

Find the value of the unknown angle, x, in this triangle.



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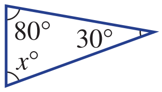
4 Find the value of the unknown angle, x, in these triangles.

a


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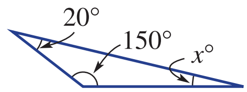
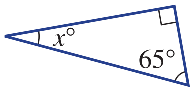


c


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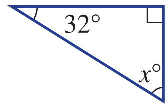

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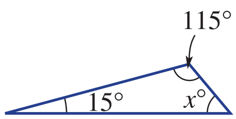

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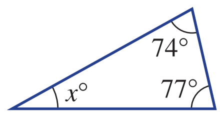
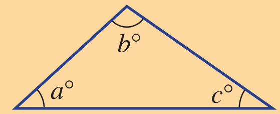
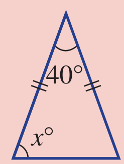


Hint:


$a + b + c = 180$

Example 4 Working with an isosceles triangle

Find the value of x in this isosceles triangle.

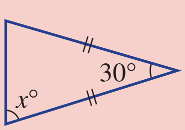


Solution	Explanation
$x + x + 40 = 180$	The triangle is isosceles and therefore the two base angles are equal.
$2x + 40 = 180$	Collect like terms.
$2x = 140$	Subtract 40 from both sides.
$x = 70$	Divide both sides by 2.

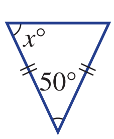
∴ The unknown angle is 70°.

Now you try

Find the value of x in this isosceles triangle.



5 Find the value of the unknown angle, x, in these triangles.

a


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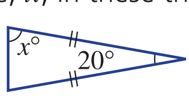

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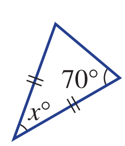

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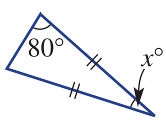

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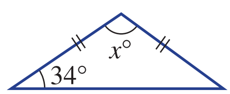

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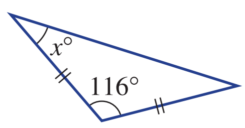

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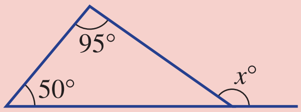
7B

Example 5 Using the exterior angle theorem

Use the exterior angle theorem to find the value of x.

a



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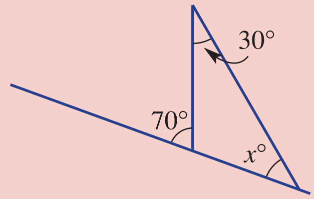
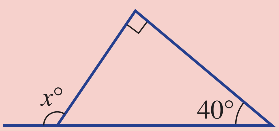


Solution	Explanation
a $x = 95 + 50$ $= 145$	The exterior angle $x°$ is the sum of the two opposite interior angles.
b $x + 30 = 70$ $x = 40$	The two opposite interior angles are $x°$ and $30°$, and $70°$ is the exterior angle.

Now you try

Use the exterior angle theorem to find the value of x.

a



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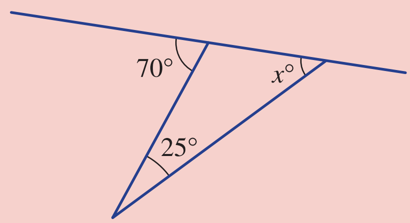
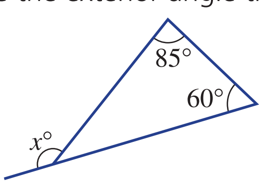


6 Use the exterior angle theorem to find the value of x.

a



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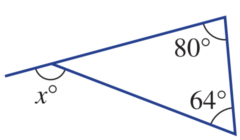
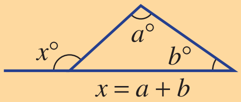


Hint:



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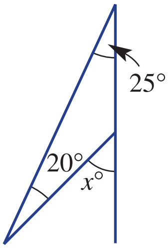

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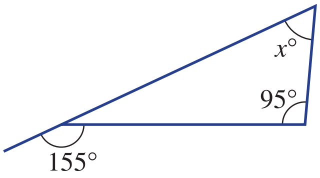

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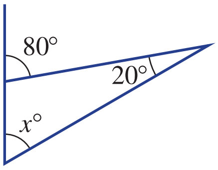

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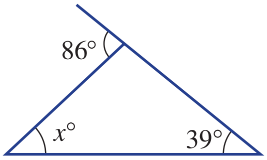

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Problem-solving and reasoning

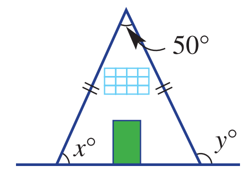
	7, 8	7, 9, 10

7 Decide whether the following are possible. If so, make a drawing.

- **a** acute scalene triangle
- **b** acute isosceles triangle
- **c** obtuse equilateral triangle
- **d** acute equilateral triangle
- **e** obtuse isosceles triangle
- **f** obtuse scalene triangle
- **g** right equilateral triangle
- **h** right isosceles triangle
- **i** right scalene triangle

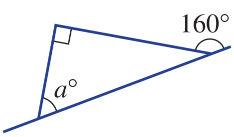
8 An architect draws the cross-section of a new ski lodge, which includes a very steep roof, as shown. The angle at the top is $50°$. Find:

- **a** the acute angle the roof makes with the floor $(x°)$
- **b** the obtuse angle the roof makes with the floor $(y°)$



9 Use your knowledge of parallel lines and triangles to find out the value of the pronumerals in these diagrams.

a



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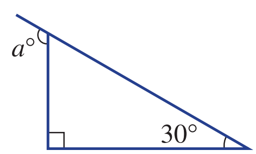

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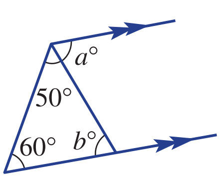

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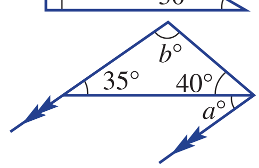

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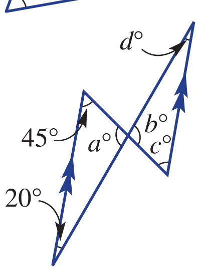

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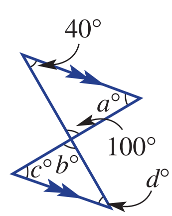
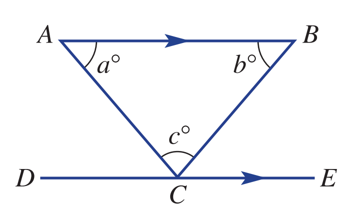


10 For this diagram, AB is parallel to DE.

- **a** What is the size of $\angle ACD$? Use a pronumeral and give a reason.
- **b** What is the size of $\angle BCE$? Use a pronumeral and give a reason.
- **c** Since $\angle DCE = 180°$, what does this tell us about a, b and c?



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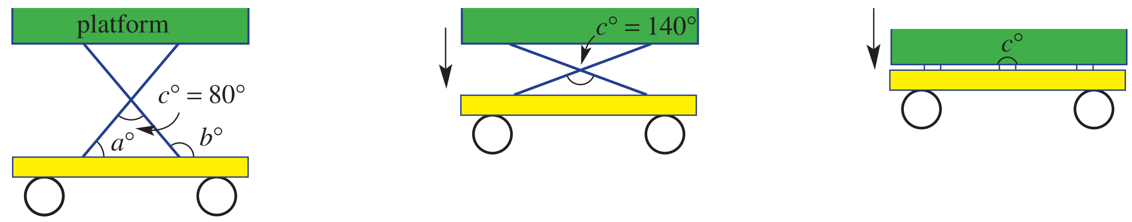
7B

The hydraulic platform

—	11

11 A hydraulic platform includes a movable 'X' shape support system, as shown. When the platform is at its highest point, the angle at the centre (c°) of the 'X' is 80°, as shown.



a Find the following when the platform is at its highest position.
  i the acute angle the 'X' makes with the platform (a°)
  ii the obtuse angle the 'X' makes with the platform (b°)

b The platform now moves down so that the angle at the centre (c°) of the 'X' changes from 80° to 140°. With this platform position, find the values of:
  i the acute angle the 'X' makes with the platform (a°)
  ii the obtuse angle the 'X' makes with the platform (b°)

c The platform now moves down to the base so that the angle at the centre (c°) of the 'X' is now 180°. Find:
  i the acute angle the 'X' makes with the platform (a°)
  ii the obtuse angle the 'X' makes with the platform (b°)

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7C Quadrilaterals

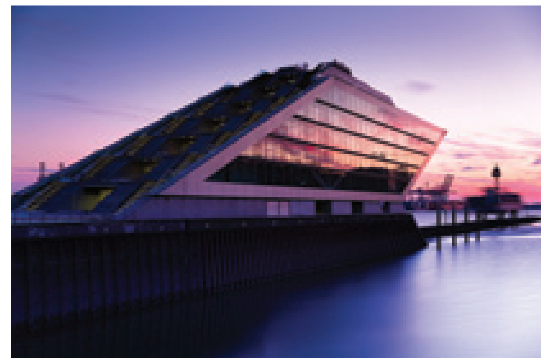
Learning intentions

- To know the properties of special quadrilaterals
- To know the angle sum of a quadrilateral
- To be able to calculate unknown angles inside a quadrilateral

Key vocabulary: quadrilateral, parallelogram, square, rectangle. rhombus, trapezium, kite, diagonal, parallel

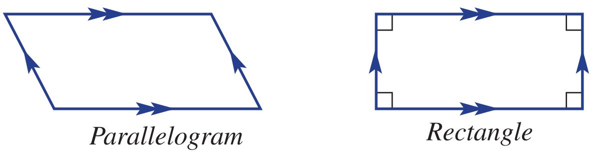
Quadrilaterals are shapes that have four straight sides with a special angle sum of 360°. There are six special quadrilaterals, each with their own special set of properties.

When you look around any old or modern building, you will see examples of these shapes.



Lesson starter: Why is a rectangle a parallelogram?

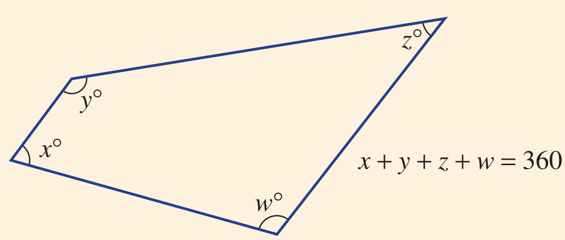
By definition, a parallelogram is a quadrilateral with two pairs of parallel sides.



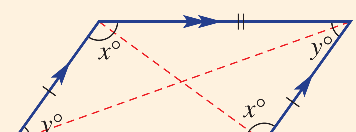
- Using this definition, do you think that a rectangle is also a parallelogram? Why?
- What properties does a rectangle have that a general parallelogram does not?
- What other special shapes are parallelograms? What are their properties?

Key ideas

- The sum of the interior angles of any quadrilateral is 360°.



- **Parallelogram** Two pairs of parallel lines
  Two pairs of equal length sides
  Opposite angles equal
  Diagonals are not equal in length



  - Parallelograms are quadrilaterals with two pairs of parallel sides. These include the square, rectangle and rhombus.

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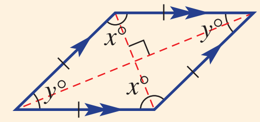
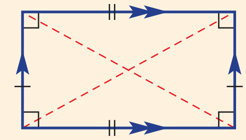
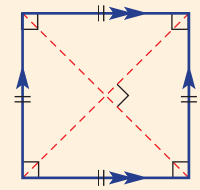
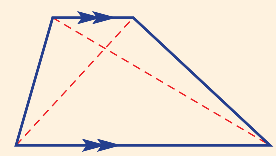
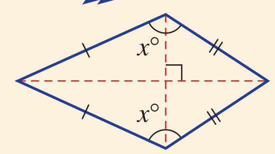
Parallelogram	**Properties**	**Drawing**
Rhombus	Two pairs of parallel lines All sides of equal length Opposite angles equal Diagonals intersect at right angles	
Rectangle	Two pairs of parallel lines Two pairs of equal-length sides All angles 90° Diagonals equal in length	
Square	Two pairs of parallel lines All sides of equal length All angles 90° Diagonals equal and intersect at right angles.	

Other quadrilaterals	**Properties**	**Drawing**
Trapezium	At least one pair of parallel sides	
Kite	Two pairs of equal-length adjacent sides One pair of equal angles Diagonals intersect at right angles	

Exercise 7C

Understanding | 1–3 | 2, 3

1 State the missing word or angle.

a The angle sum of a quadrilateral is ________ .

b The two special quadrilaterals which are not parallelograms are the ________ and the ________.

2 Which three special quadrilaterals are parallelograms?

Hint: Refer to the Key ideas for help.

3 List all the quadrilaterals that have the following properties.

a two pairs of parallel sides

b two pairs of equal-length sides

c equal opposite angles

d one pair of parallel sides

e one pair of equal angles

f all angles 90°

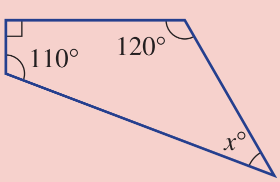
g equal-length diagonals

h diagonals intersecting at right angles

Fluency | 4–5(½) | 4–5(½)

Example 6 Using the angle sum of a quadrilateral

Find the unknown angle in this quadrilateral.



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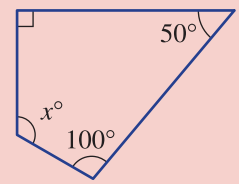
Solution	**Explanation**
$x + 110 + 120 + 90 = 360$	The sum of internal angles is 360°.
$x + 320 = 360$	Simplify.
$x = 40$	Subtract 320 from both sides.
∴ The unknown angle is 40°.	

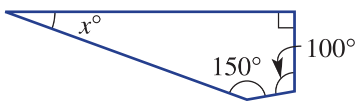
Now you try

Find the unknown angle in this quadrilateral.

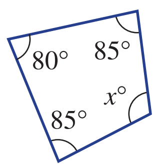


4 Find the unknown angles in these quadrilaterals.

a



b



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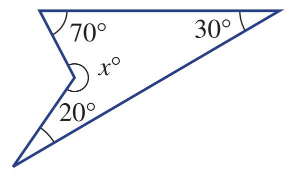
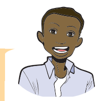


Hint: The angle sum of a quadrilateral is 360°.



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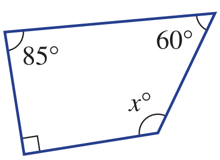

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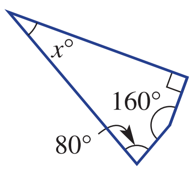

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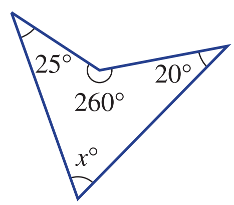
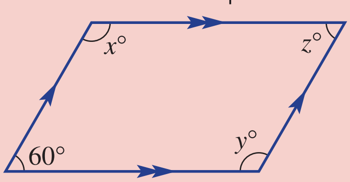


Example 7 Finding angles in special quadrilaterals

Find the value of the pronumerals in these special quadrilaterals.

a



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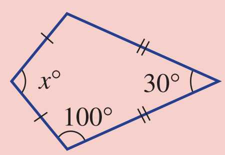

Solution	**Explanation**
a $x + 60 = 180$	$x°$ and 60° are cointerior angles and sum to 180°.
$x = 120$	Subtract 60 from both sides.
$\therefore y = 120$	$y°$ is opposite and equal to $x°$.
$\therefore z = 60$	$z°$ is opposite and equal to 60°.
b $x + 100 + 100 + 30 = 360$	A kite has a pair of equal opposite angles, so there are two 100° angles.
$x + 230 = 360$	The total sum is still 360°.
$x = 130$	

Continued on next page

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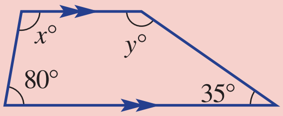
Now you try

Find the value of the pronumerals in these special quadrilaterals.

a



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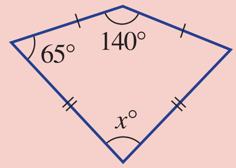
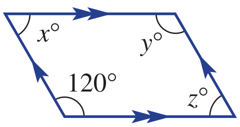


5 Find the value of the pronumerals in these special quadrilaterals.

a



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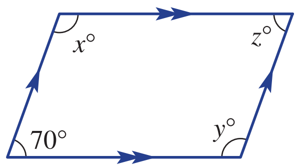

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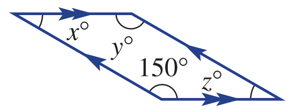
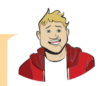
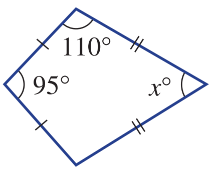




Hint: Refer to the properties of special quadrilaterals for help.

d



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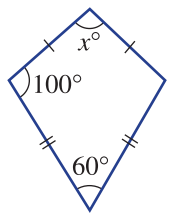

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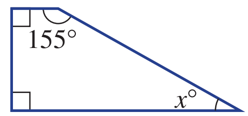

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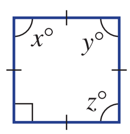

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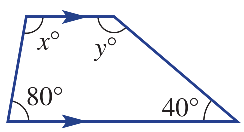
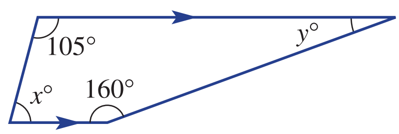


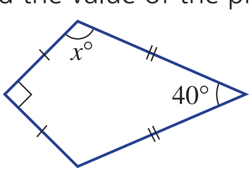
i



Problem-solving and reasoning | 6, 7 | 6–9

6 Find the value of the pronumerals in these shapes.

a



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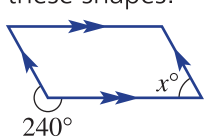

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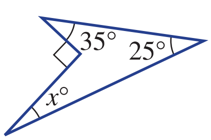

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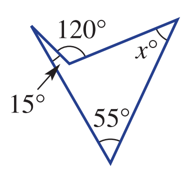

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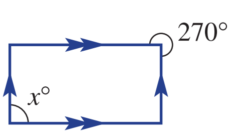

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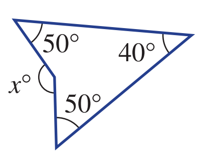
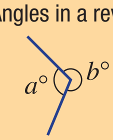


Hint:
Angles in a revolution add to 360°.



$a + b = 360$

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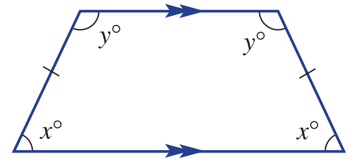
7 This shape is called an isosceles trapezium.



a Why do you think it is called an isosceles trapezium?

b **i** When $x = 60$, find y.

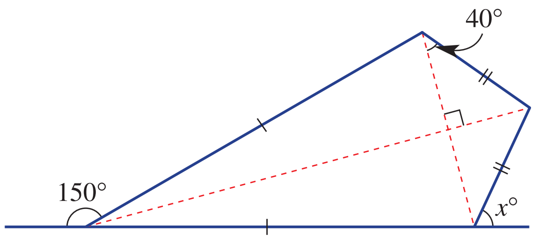
ii When $y = 140$, find x.

c List the properties of an isosceles trapezium.

8 A modern hotel is in the shape of a kite.
Some angles are given in the diagram.

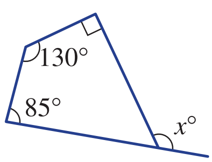
a Draw a copy of only the kite shape, including the diagonals.

b Find the angle that the right-hand wall makes with the ground ($x°$).



9 These quadrilaterals also include exterior angles. Find the value of x.

a



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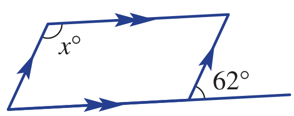

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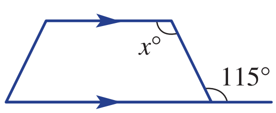

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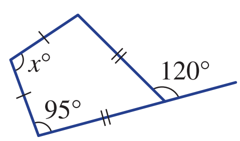
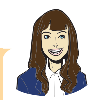
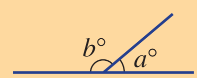




Hint:

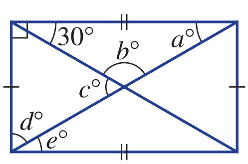


$a + b = 180$

Quadrilaterals and triangles — 10

10 The following shapes combine quadrilaterals with triangles. Find the value of the pronumerals.

a



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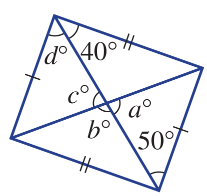

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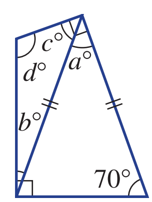

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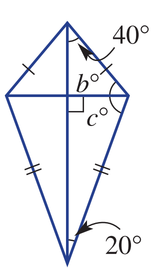

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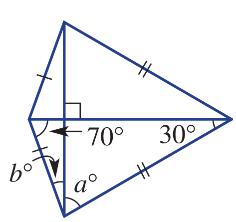

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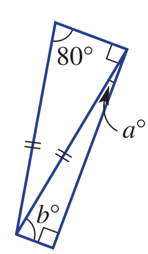

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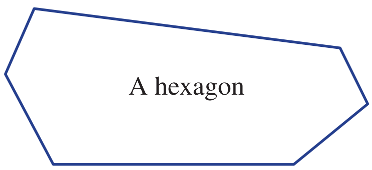
7D Polygons ★

Learning intentions

- To know the rule for the angle sum of a polygon
- To be able to calculate unknown angles inside a polygon
- To be able to calculate the interior angle of regular polygons

Key vocabulary: polygon, regular polygon

A closed shape with all straight sides is called a polygon. Like triangles and quadrilaterals (which are both polygons), they all have a special angle sum.



Lesson starter: Remember the names

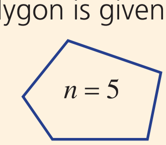
From previous years you should remember some of the names for polygons. See if you can remember them by completing this table.

Number of sides	Name
3	
4	
5	
6	
7	Heptagon

Number of sides	Name
8	
9	
10	
11	
12	

Key ideas

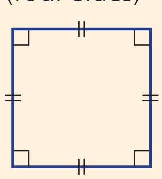
- A **polygon** is a shape with straight sides.
  - They are named by their number of sides.
- The sum of internal angles (S) of a polygon is given by this rule:
  $S = 180° \times (n - 2)$
  where n is the number of sides



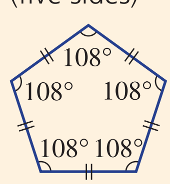
$S = 180° \times (n - 2)$
$S = 180° \times (5 - 2)$
$= 180° \times 3$
$= 540°$

- **A regular polygon** has sides of equal length and equal angles.

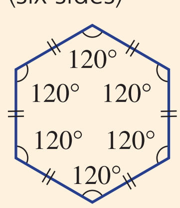
regular quadrilateral (square) (four sides)



regular pentagon (five sides)



regular hexagon (six sides)



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Exercise 7D

Understanding 1–3 2(½), 3

1 How many sides do these shapes have?
- **a** quadrilateral **b** octagon **c** decagon **d** heptagon
- **e** nonagon **f** hexagon **g** pentagon **h** dodecagon

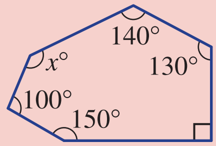
2 Use the angle sum rule, $S = 180° \times (n - 2)$, to find the angle sum of these polygons.
- **a** pentagon $(n = 5)$ **b** hexagon $(n = 6)$ **c** heptagon $(n = 7)$
- **d** octagon $(n = 8)$ **e** nonagon $(n = 9)$ **f** decagon $(n = 10)$

3 What is always true about a polygon that is regular?

Fluency 4(½), 5 4(½), 6

Example 8 Finding and using the angle sum of a polygon

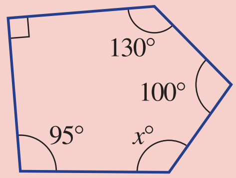
For this polygon, find the angle sum and then the value of x.



Solution	Explanation
$S = 180° \times (n - 2)$ $= 180° \times (6 - 2)$	Use the angle sum rule first, with $n = 6$ since there are 6 sides.
$= 720°$	Find the angle sum.
$x + 100 + 150 + 90 + 130 + 140 = 720$	Use the total angle sum to find the value of x.
$x + 610 = 720$ $x = 110$	Solve for the value of x.

Now you try

For this polygon, find the angle sum and then the value of x.



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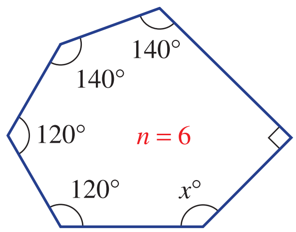
7D

4 For these polygons, find the angle sum and then find the value of x.

Hint: First use
$S = 180° \times (n - 2)$

a



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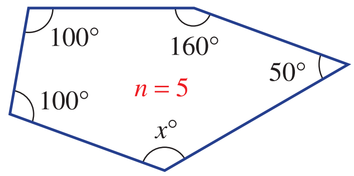

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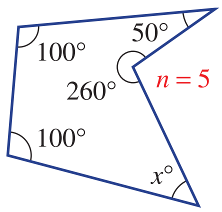

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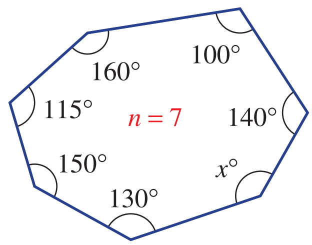

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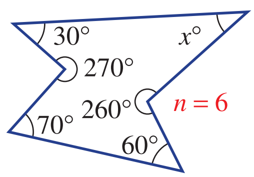

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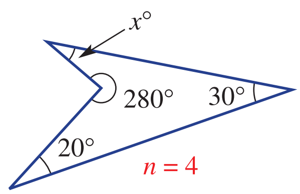
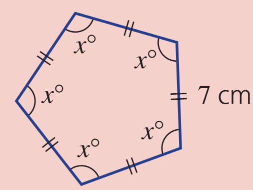


Example 9 Working with regular polygons

Shown here is a regular pentagon with a straight edge side length of 7 cm.

a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $x°$.

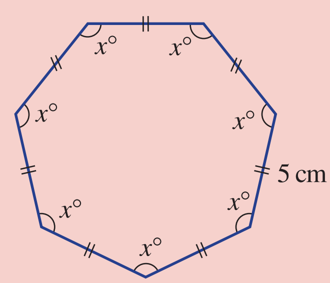


Solution	Explanation
a 35 cm	There are five sides at 7 cm each.
b $S = 180° \times (n - 2)$ $= 180° \times (5 - 2)$ $= 180° \times 3$ $= 540°$	Write the general rule for the sum of internal angles for a polygon. $n = 5$ since there are five sides. Simplify and evaluate.
c $540° \div 5 = 108°$ $x° = 108°$	There are five equally sized angles since it is a regular pentagon.

Now you try

Shown here is a regular heptagon with a straight edge side length of 5 cm.

a Find the perimeter of the heptagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $x°$, correct two decimal places.



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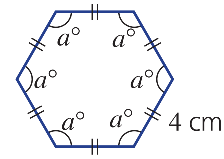
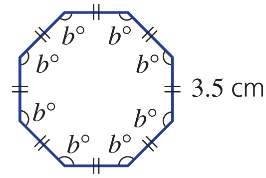
5 Shown here is a regular hexagon with straight-edge side length 4 cm.
a Find the perimeter of the hexagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $a°$.



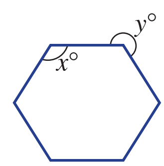
6 Shown here is a regular octagon with straight-edge side length 3.5 cm.
a Find the perimeter of the octagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $b°$.



Problem-solving and reasoning

	7, 8, 10	9–12

7 The cross-section of a pencil is a regular hexagon.
a Find the interior angle ($x°$).
b Find the outside angle ($y°$).

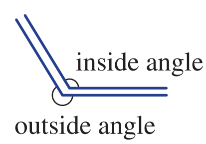


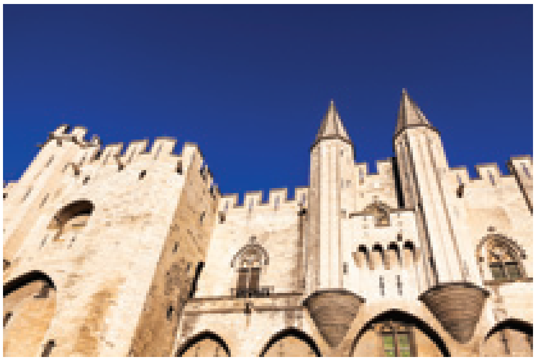
8 Find the total internal angle sum for a polygon with:
a 11 sides
b 20 sides

Hint: Remember:
$S = 180° \times (n - 2)$

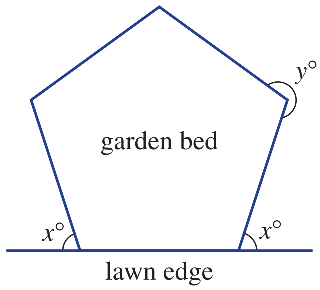
9 Find the size of a single internal angle for a regular polygon with:
a 10 sides
b 25 sides

10 A castle turret is in the shape of a regular hexagon.
At each of the six corners, find:
a the inside angle
b the outside angle





11 A garden bed is to be designed in the shape of a regular pentagon and sits adjacent to a lawn edge, as shown.
a Find the angle the lawn edge makes with the garden bed ($x°$).
b Find the outside angle for each corner ($y°$).



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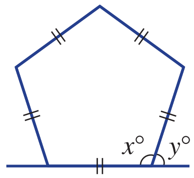
7D

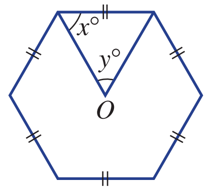
12 For these diagrams, find the values of the unknowns. The shapes are regular.

Hint: In parts **b** and **c**, the point at O is the centre.

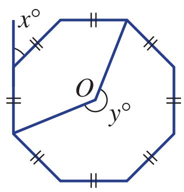
a



b



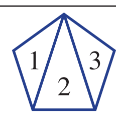
c



Develop the angle sum rule — 13

13 **a** Copy and complete this table.

Hint: For the diagram, choose one vertex and draw lines to all other vertices.

Polygon	Number of sides	Diagram	Number of triangles	Total angle sum *(S)*	Regular polygon internal angle *(A)*
Triangle					
Quadrilateral					
Pentagon	5		3	$3 \times 180 = 540$	$540 \div 5 = 108$
Hexagon					
…					
n-gon	n				

b Complete these sentences by writing the rule.

i For a polygon with n sides, the total angle sum, S, is given by $S =$ __________.

ii For a regular polygon with n sides, a single internal angle, A, is given by $A =$ __________.

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7E Congruent triangles

Learning intentions

- To understand the four tests for congruence of triangles
- To be able to recognise a pair of congruent triangles using one of the four tests
- To be able to prove that two triangles are congruent

Key vocabulary: congruent, corresponding, hypotenuse

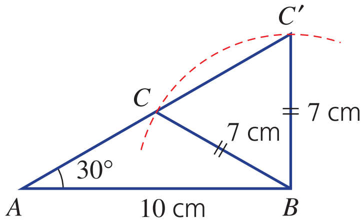
In solving problems or in the building of structures, for example, it is important to know whether or not two expressions or objects are identical. The mathematical word used to describe identical objects is *congruence*.

For congruent triangles there are four important tests that can be used to prove congruence.

Lesson starter: Why are AAA and SSA not tests for congruence?

AAA and SSA are not tests for the congruence of triangles.

- For AAA, can you draw two different triangles using the same three angles? Why does this mean that AAA is not a test for congruence?
- Look at this diagram, showing triangle ABC and triangle ABC'. Both triangles have a $30°$ angle and two sides of length 10 cm and 7 cm. Explain how this diagram shows that SSA is not a test for congruence of triangles.

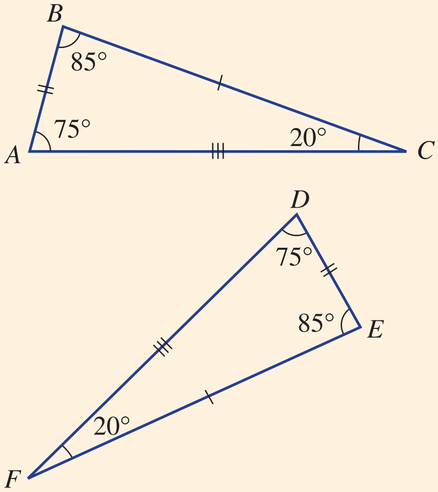


Key ideas

- Two triangles are said to be **congruent** if they are exactly the same 'size' and 'shape'.

  Corresponding sides and angles will be of the same size, as shown in the triangles below.

- If triangle ABC is congruent to triangle DEF, we write $\triangle ABC \equiv \triangle DEF$.
  - This is called a congruence statement.
  - Letters are usually written in matching order.
  - If two triangles are not congruent, we write: $\triangle ABC \not\equiv \triangle DEF$.



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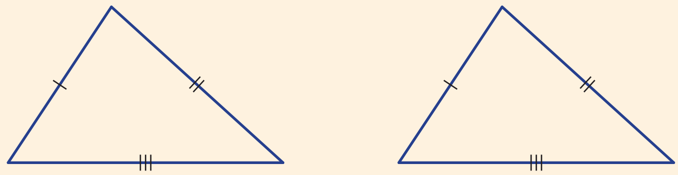
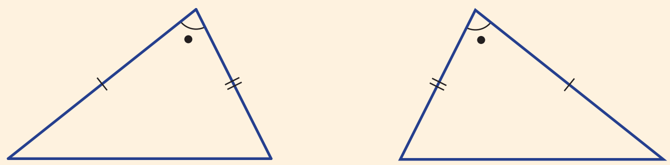
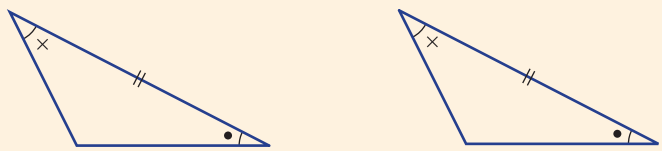
- Two triangles can be tested for congruence by considering the following necessary conditions.
  1. Three pairs of corresponding sides are equal (**SSS**).



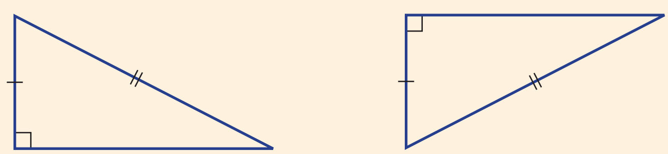
  2. Two pairs of corresponding sides and the angle between them are equal (**SAS**).



  3. Two angles and any pair of corresponding sides are equal (**AAS**).



  4. A right angle, the hypotenuse and one other pair of corresponding sides are equal (**RHS**).

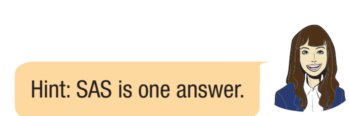


Exercise 7E

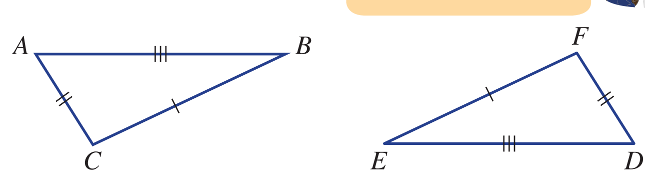
Understanding | 1–3 | 3

1 True or false?
- **a** SSA is a test for the congruence of triangles.
- **b** AAA is a test for the congruence of triangles.
- **c** Two congruent triangles are the same shape and size.
- **d** If $\triangle ABC \equiv \triangle DEF$, then triangle ABC is congruent to triangle DEF.

2 Write the four tests for congruence, using their abbreviated names.



3 Here is a pair of congruent triangles.
- **a** Which point on $\triangle DEF$ corresponds to point B on $\triangle ABC$?
- **b** Which side on $\triangle ABC$ corresponds to side DF on $\triangle DEF$?
- **c** Which angle on $\triangle DEF$ corresponds to $\angle BAC$ on $\triangle ABC$?



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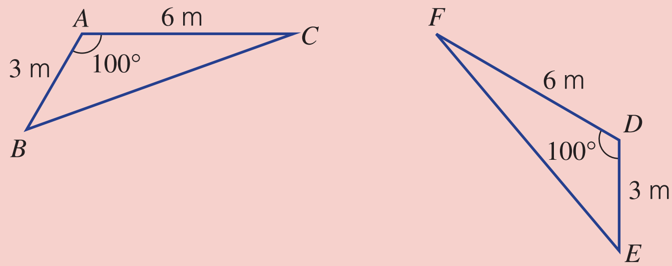
Fluency

4, 5(½) | 4–5(½)

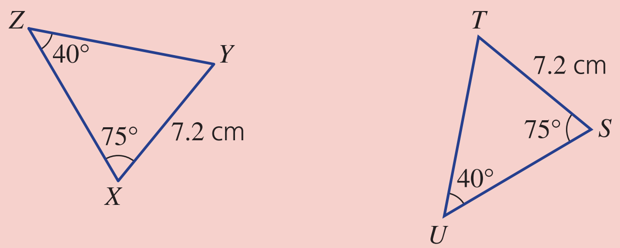
Example 10 Choosing a test for congruence

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



b

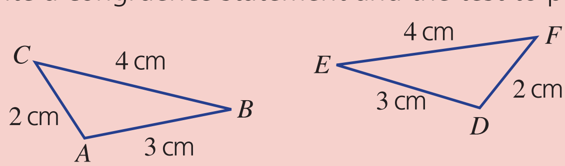


Solution	Explanation
a $\triangle ABC \equiv \triangle DEF$ (SAS)	Write letters in corresponding (matching) order. Two pairs of sides are equal as well as the angle between.
b $\triangle XYZ \equiv \triangle STU$ (AAS)	X matches S, Y matches T, and Z matches U. Two angles and one pair of matching sides are equal.

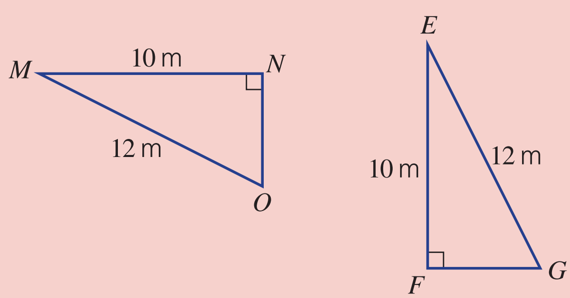
Now you try

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



b



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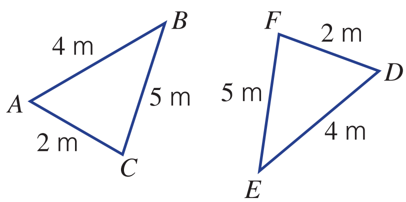
7E

4 Write a congruence statement and the test to prove congruence for these pairs of triangles.

Hint: $\triangle ABC \equiv \triangle DEF$ is a congruence statement.
Choose one of the tests SSS, SAS, AAS or RHS.

a



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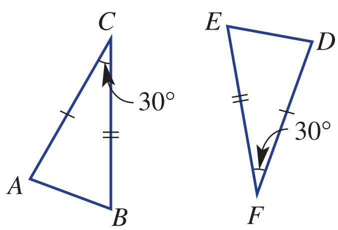

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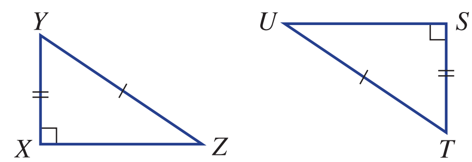

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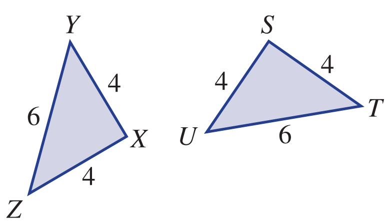

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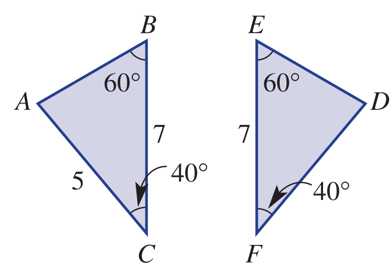

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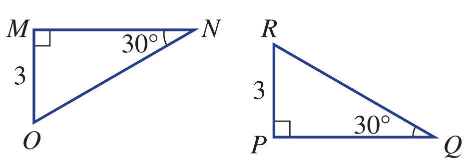
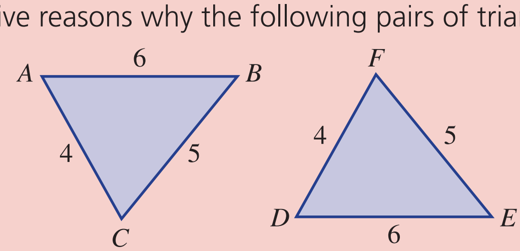


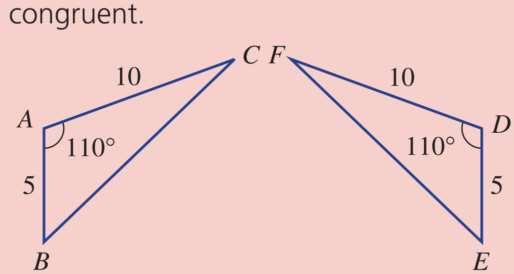
Example 11 Proving that a pair of triangles are congruent

Give reasons why the following pairs of triangles are congruent.

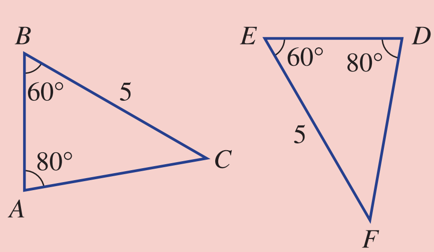
a



b



c



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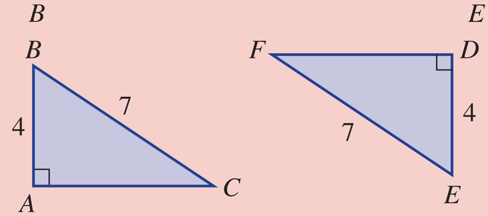

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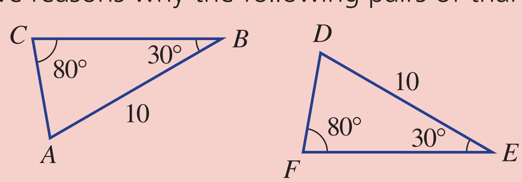
Solution		Explanation
a $AB = DE$	(S)	First choose all the corresponding side lengths.
$AC = DF$	(S)	Corresponding side lengths will have the same length.
$BC = EF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(SSS)	Write the congruence statement and the abbreviated reason.
b $AB = DE$	(S)	Note that two corresponding side lengths are equal and the included angles are equal.
$\angle BAC = \angle EDF$	(A)	
$AC = DF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(SAS)	Write the congruence statement and the abbreviated reason.
c $\angle ABC = \angle DEF$	(A)	Two of the angles are equal and one of the pairs of corresponding sides are equal. Write the congruence statement and the abbreviated reason.
$\angle BAC = \angle EDF$	(A)	
$BC = EF$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(AAS)	
d $\angle BAC = \angle EDF = 90^\circ$	(R)	Note that both triangles are right angled, the hypotenuse of each triangle is of the same length and another pair of corresponding sides is of the same length.
$BC = EF$	(H)	
$AB = DE$	(S)	
$\therefore \triangle ABC \equiv \triangle DEF$	(RHS)	Write the congruence statement and the abbreviated reason.

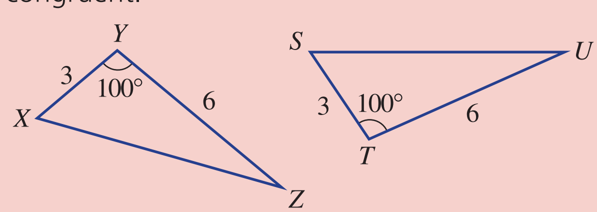
Now you try

Give reasons why the following pairs of triangles are congruent.

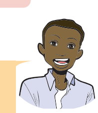
a



b

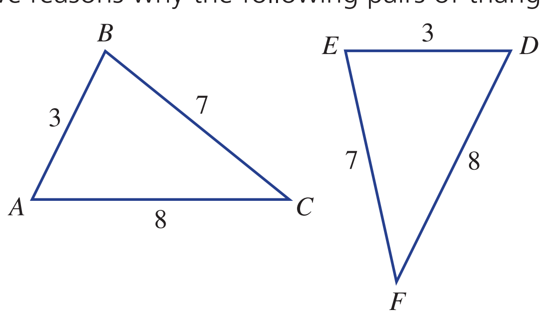


5 Give reasons why the following pairs of triangles are congruent.

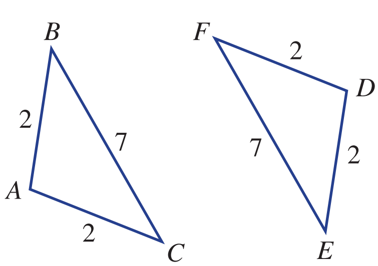


Hint: List reasons as in the examples to establish SSS, SAS, AAS or RHS.

a



b



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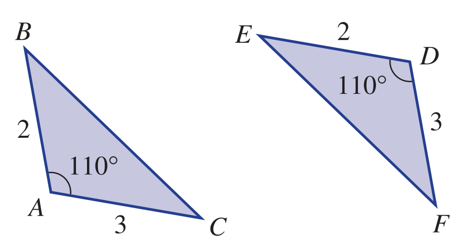

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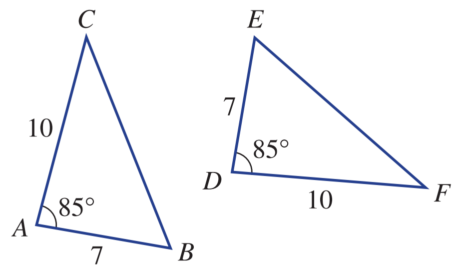
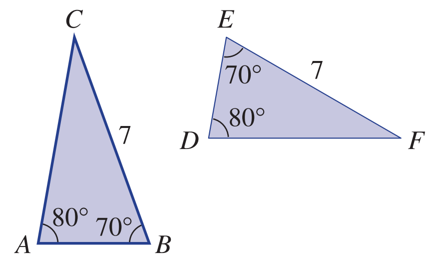
d



e



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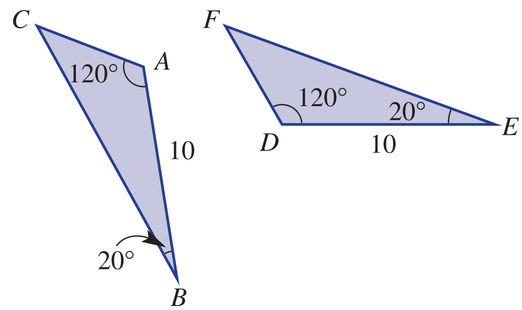

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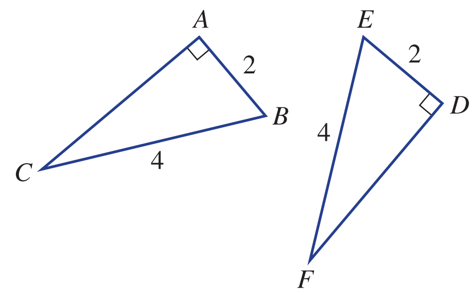
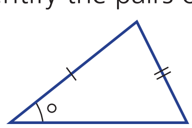


Problem-solving and reasoning

6, 7 | 6, 7, 8(½)

6 Identify the pairs of congruent triangles from those below.

a



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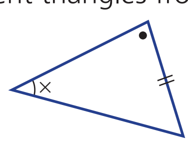

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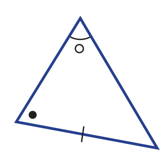
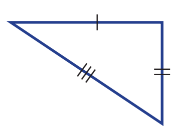


Hint: Sides with the same markings and angles with the same mark are equal.

d



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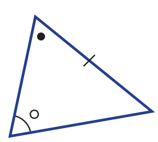

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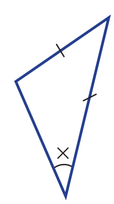

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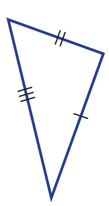

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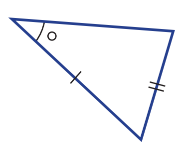
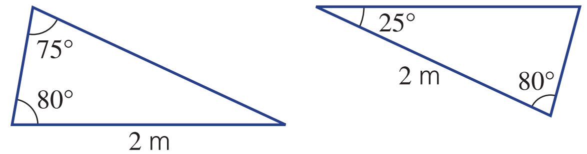


7 Two triangular windows have the given dimensions.

a Find the missing angle in each triangle.

b Are the two triangles congruent? Give a reason.



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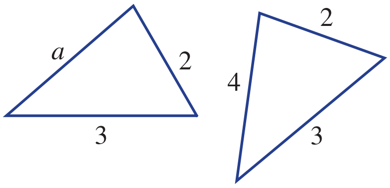
8 For the pairs of congruent triangles, find the values of the pronumerals.

Hint: Given that these triangles are congruent, corresponding sides are equal, as are corresponding angles.

a



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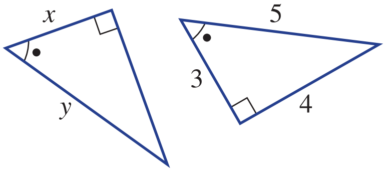

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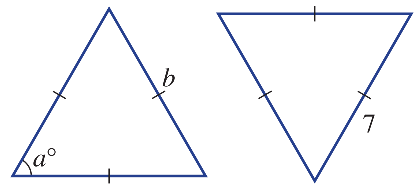

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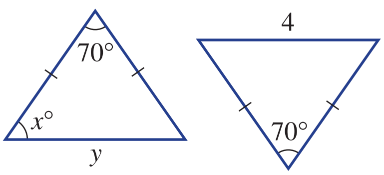

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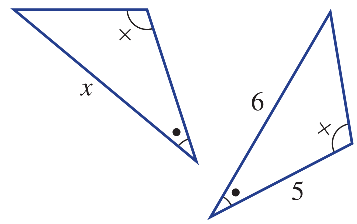

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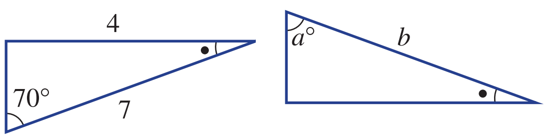

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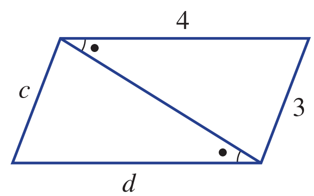

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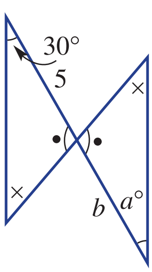
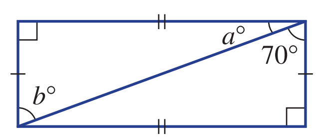


i



Lawn landscaping — 9

9 A new garden design includes two triangular lawn areas, as shown.

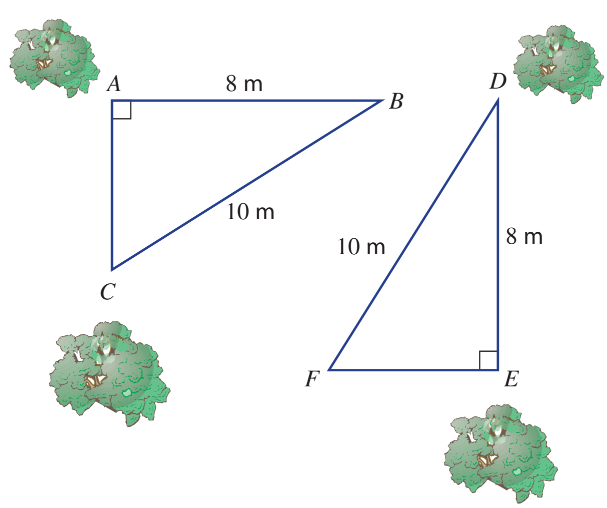
a Give reasons why the two triangular lawn areas are congruent.

b If the length of AC is 6 m, find the length of EF.

c If the angle $ABC = 37°$, find the angles:

i $\angle EDF$

ii $\angle DFE$



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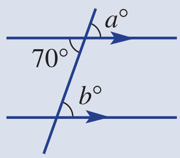
Progress quiz

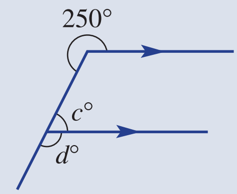
7A

1 Find the values of the pronumerals in these diagrams with parallel lines, giving reasons.

a



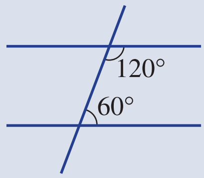
b



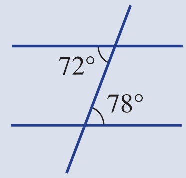
7A

2 State, with reasons, if the following pairs of lines are parallel.

a



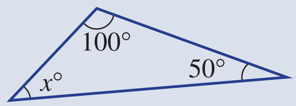
b



7B/C

3 Find the values of the pronumerals in these triangles and quadrilaterals.

a



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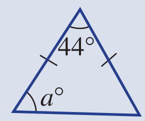

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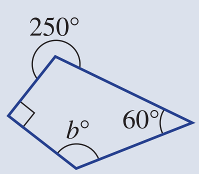
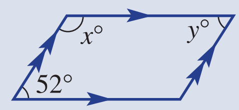
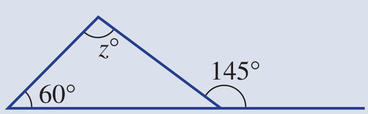


d



7B

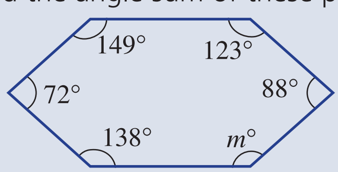
4 Use the exterior angle theorem to find the value of z.



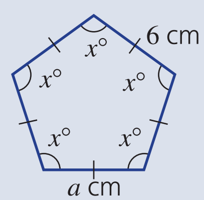
7D

5 Find the angle sum of these polygons and then find the value of each pronumeral.

a



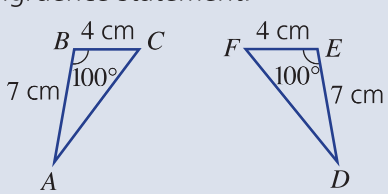
b



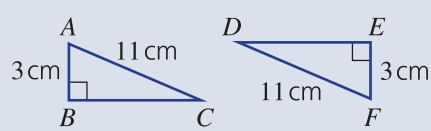
7E

6 Give reasons why the following pairs of triangles are congruent and write a congruence statement.

a



b



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7F Similar triangles

Learning intentions

- To know what it means for triangles to be similar
- To understand the four tests for similarity of triangles
- To be able to recognise a pair of similar triangles using one of the four tests
- To be able to prove that two triangles are similar
- To be able to calculate and use the scale factor to find an unknown length

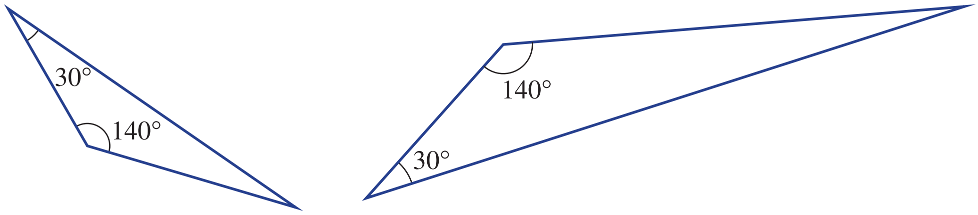
Key vocabulary: similar, scale factor, ratio, corresponding, hypotenuse

When two objects are similar, they are the same shape but of different size. For example, a computer image reproduced on a large screen with the same aspect ratio (e.g. 16:9) will show all aspects of the image in the same way except in size.

The computer image and screen image are said to be similar figures.

Lesson starter: Is AA the same as AAA?

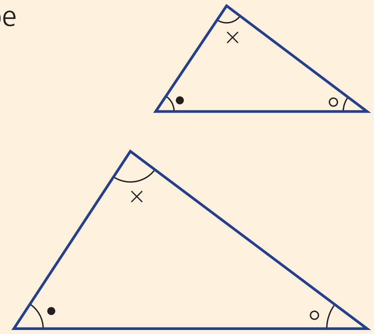
Look at these two triangles.



- What is the missing angle in each triangle?
- Do you think the triangles are similar? Why?
- Is the AA test equivalent to the AAA test?

Key ideas

- Two triangles are said to be **similar** if they are the same shape but different in size. Corresponding angles will be equal and corresponding side lengths will be in the same ratio.
- If $\triangle ABC$ is similar to $\triangle DEF$, then we write $\triangle ABC \parallel\!\mid \triangle DEF$ or $\triangle ABC \sim \triangle DEF$.



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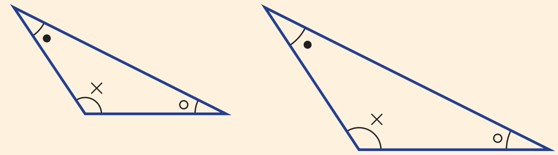
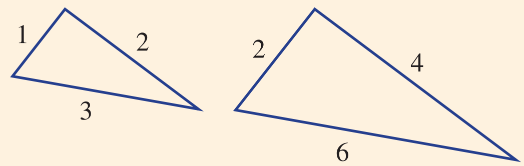
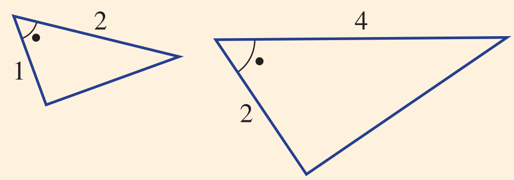
- Two triangles can be tested for similarity by considering the following necessary conditions.
  1. All three pairs of corresponding angles are equal (**AAA**). (Remember that if two pairs of corresponding angles are equal then the third pair of corresponding angles is also equal.)



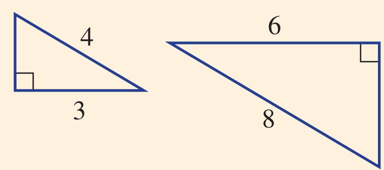
  2. All three pairs of corresponding sides are in the same ratio (**SSS**).
     $\frac{6}{3} = \frac{4}{2} = \frac{2}{1} = 2$



  3. Two pairs of corresponding sides are in the same ratio and the included corresponding angles between these sides are equal (**SAS**).
     $\frac{4}{2} = \frac{2}{1} = 2$



  4. The hypotenuses and a pair of corresponding sides in a right-angled triangle are in the same ratio (**RHS**).
     $\frac{8}{4} = \frac{6}{3} = 2$

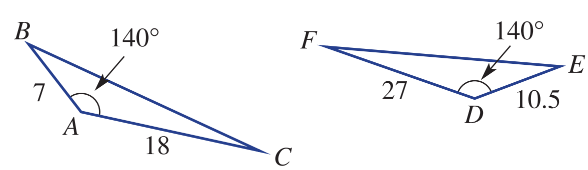


- The **scale factor** is calculated using a pair of corresponding sides.
  In the three examples above, the scale factor is 2.

Exercise 7F

Understanding — 1–3 — 3

1. Which of the following is not a test for the similarity of triangles?
   SSS, SAS, RHS, SSA, AAA
2. Why is the AA test the same as the AAA test for similar triangles?
3. Consider this pair of triangles.



   a. Work out $\frac{DE}{AB}$.
   b. Work out $\frac{DF}{AC}$. What do you notice?
   c. What is the scale factor?
   d. Which of SSS, SAS, AAA or RHS would be used to explain their similarity?

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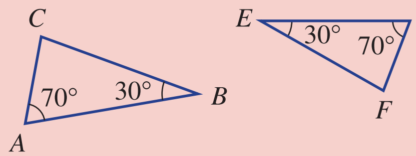
Fluency

4(½), 5 | 4(½), 5

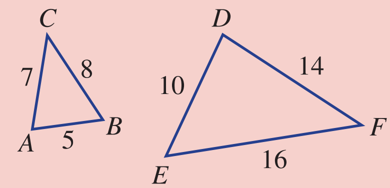
Example 12 Proving similar triangles

Decide whether the pairs of triangles are similar, giving reasons.

a



b



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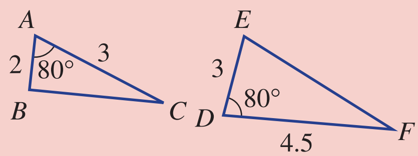
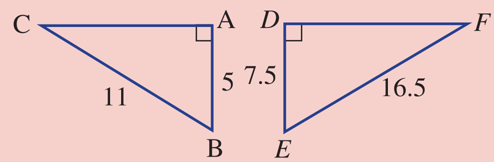


d



Solution	Explanation

a

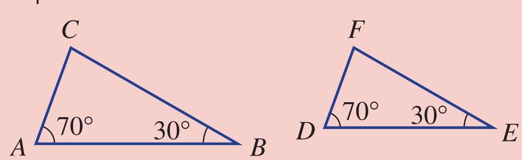
$\angle BAC = \angle EDF$ (A)

$\angle ABC = \angle DEF$ (A)

$\angle ACB = \angle DFE$ (A)

$\therefore \triangle ABC \,|||\, \triangle DEF$ (AAA)

Two corresponding angles are equal and therefore the third corresponding angle is also equal.



b

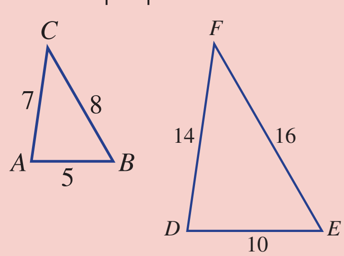
$\dfrac{DE}{AB} = \dfrac{10}{5} = 2$ (S)

$\dfrac{EF}{BC} = \dfrac{16}{8} = 2$ (S)

$\dfrac{DF}{AC} = \dfrac{14}{7} = 2$ (S)

$\therefore \triangle ABC \,|||\, \triangle DEF$ (SSS)

All three corresponding sides are in the same ratio or proportion.



c

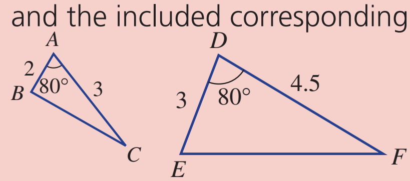
$\dfrac{DE}{AB} = \dfrac{3}{2} = 1.5$ (S)

$\angle BAC = \angle EDF$ (A)

$\dfrac{DF}{AC} = \dfrac{4.5}{3} = 1.5$ (S)

$\therefore \triangle ABC \,|||\, \triangle DEF$ (SAS)

Two corresponding sides are in the same ratio and the included corresponding angles are equal.



d

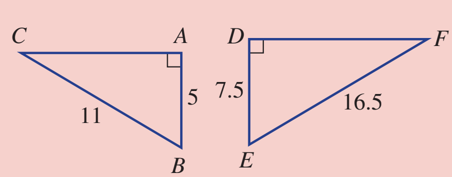
$\angle BAC = \angle EDF = 90°$ (R)

$\dfrac{EF}{BC} = \dfrac{16.5}{11} = 1.5$ (H)

$\dfrac{DE}{AB} = \dfrac{7.5}{5} = 1.5$ (S)

$\therefore \triangle ABC \,|||\, \triangle DEF$ (RHS)

They are right-angled triangles with the hypotenuses and one other pair of corresponding sides in the same ratio.



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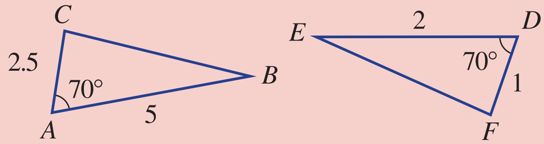
Now you try

Decide whether the pairs of triangles are similar, giving reasons.

a



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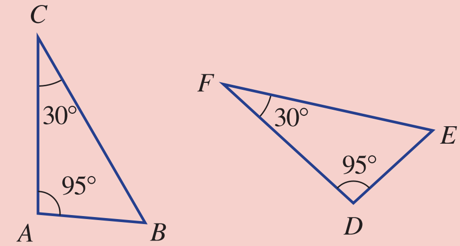
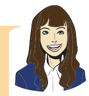
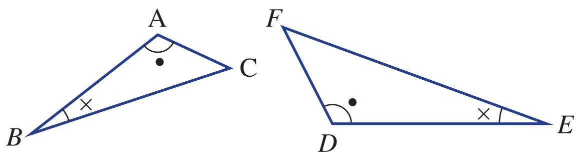


4 Decide whether the pairs of triangles are similar, giving reasons.



Hint: List all the equal angles and corresponding pairs of sides, as in Example 12.

a



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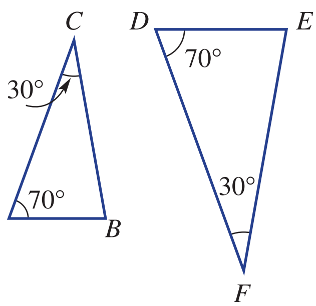

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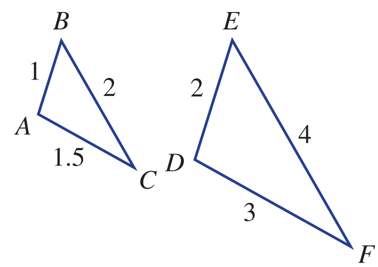

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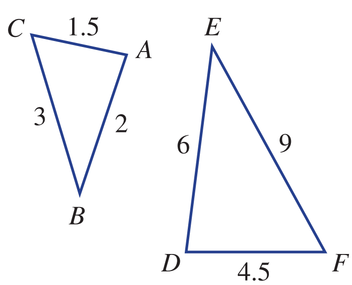

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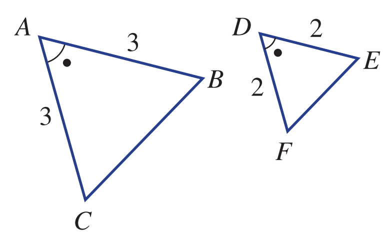

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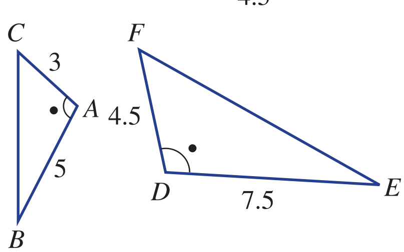

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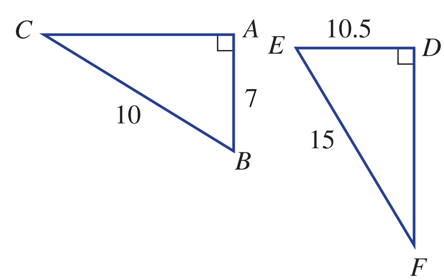

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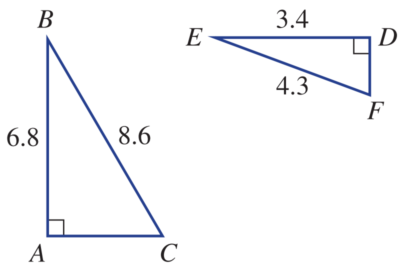

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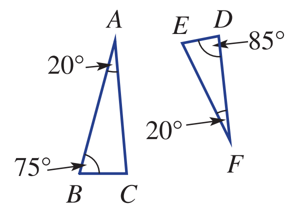

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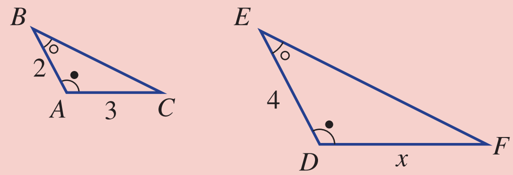
Example 13 Using similar triangles to find an unknown length

If the given pair of triangles are known to be similar, find the value of x.



Solution

Scale factor $= \dfrac{DE}{AB} = \dfrac{4}{2} = 2$

$x = 3 \times 2$

$= 6$

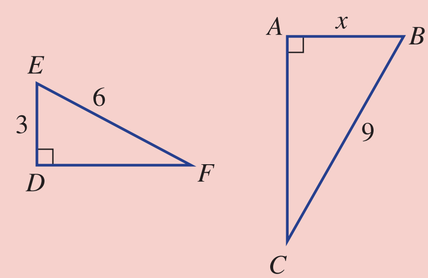
Explanation

First find the scale factor using a pair of corresponding sides. Divide the larger number by the smaller number.

Multiply the corresponding length on the smaller triangle using the scale factor.

Now you try

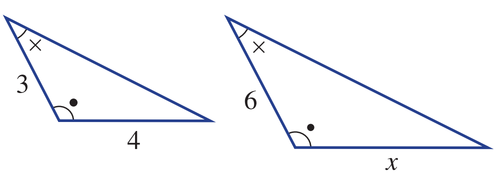
If the given pair of triangles are known to be similar, find the value of x.



Hint: For parts **c** and **d**, use division to find x.

5 If the given pair of triangles are known to be similar, find the value of x.

a



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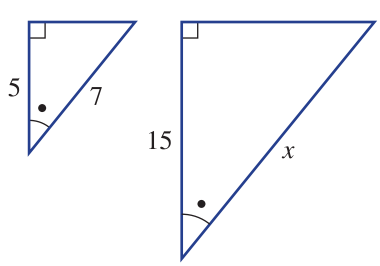
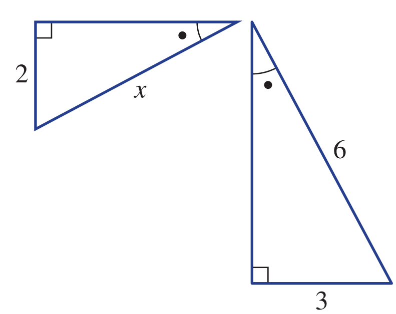
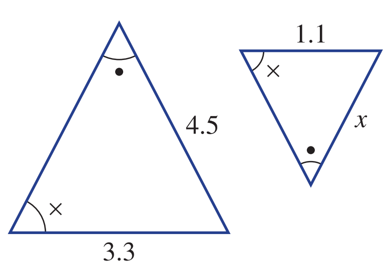


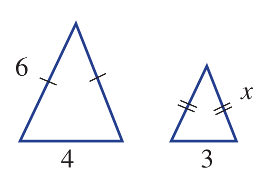
c



d



e



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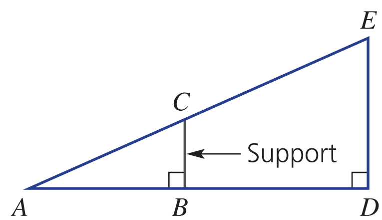
7F

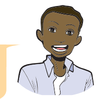
Problem-solving and reasoning — 6 — 6, 7

6 A ski ramp has a vertical support, as shown.

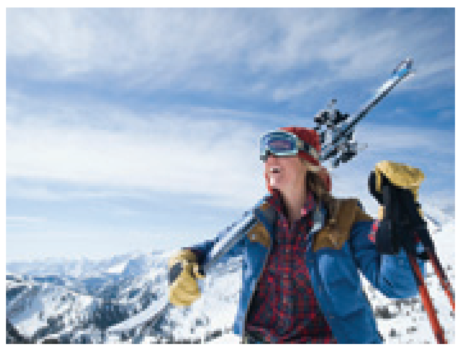
 a List the two triangles that are similar.
 b Why are the two triangles similar?



Hint: List triangles like this: $\triangle STU$.

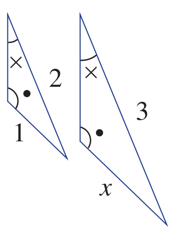


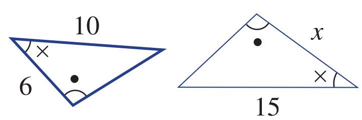
 c If $AB = 4$ m and $AD = 10$ m, find the scale factor.
 d If $BC = 1.5$ m, find the height of the ramp, DE, using the scale factor from part **c**.

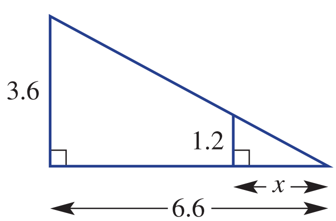


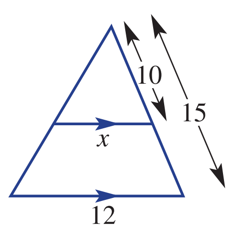
7 State why the pairs of triangles are similar (give the abbreviated reason) and determine the value of x in each case.

Hint: They all have the same reason.

 a


 b


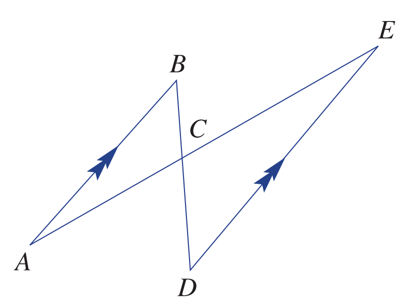
 c


 d


Triangles in parallel lines — — — 8

8 In the given diagram, AB is parallel to DE.

 a List the three pairs of angles that are equal and give a reason.
 b If $AB = 8$ cm and $DE = 12$ cm, find:
  i DC if $BC = 4$ cm
  ii AC if $CE = 9$ cm



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7G Applying similar triangles

Learning intentions

- To be able to identify a pair of similar triangles in a given context
- To be able to calculate and use the scale factor to find an unknown length in a real situation

Key vocabulary: similar, scale factor

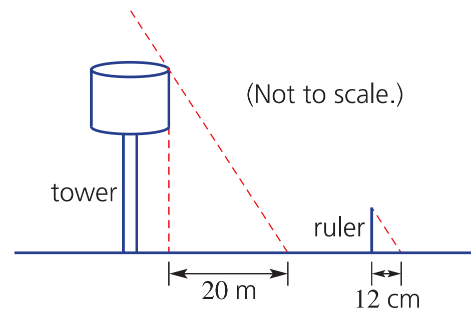
Once it is established that two triangles for a particular situation are similar, the ratio or scale factor between side lengths can be used to find unknown side lengths.

Similar triangles have many applications in the real world. One application is finding an inaccessible distance, like the height of a tall object or the distance across a deep ravine.

Lesson starter: The tower and the ruler

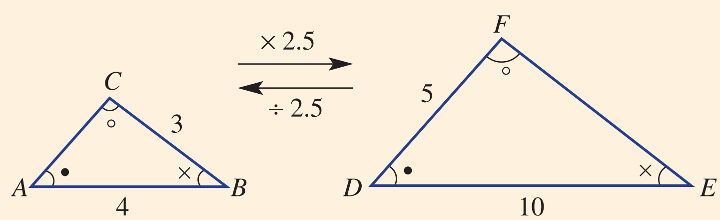
Franklin wants to know how tall a water tower is in his town. At a particular time of day he measures its shadow to be 20 m long. At the same time he stands a 30 cm ruler near the tower, which gives a 12 cm shadow.

- Explain why the two formed triangles are similar.
- What is the scale factor?
- What is the height of the tower?



Key ideas

- For two similar triangles, the ratio of the corresponding side lengths, written as a single number, is called the scale factor.



- Once the scale factor is known, it can be used to find unknown side lengths.

$$\frac{DE}{AB} = \frac{10}{4} = \frac{5}{2} = 2.5$$

$\therefore$ Scale factor is 2.5.

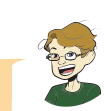
$\therefore\ EF = 3 \times 2.5 = 7.5$

$AC = 5 \div 2.5 = 2$

Exercise 7G

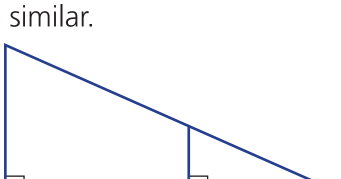
Understanding	1, 2	2

1 Give reasons why the pairs of triangles in each diagram are similar.

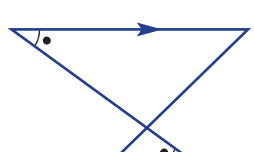


Hint: Think about all the corresponding pairs of angles. How many equal pairs are there?

a



b



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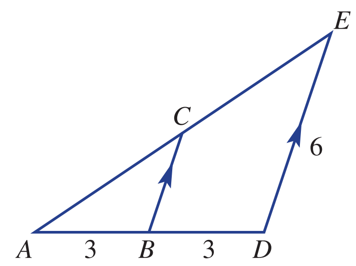
7G

2 For the pair of triangles in the given diagram:
  a Which reason would be chosen to explain their similarity: SSS, SAS, AAA or RHS?
  b What is the scale factor?
  c What is the length of BC?

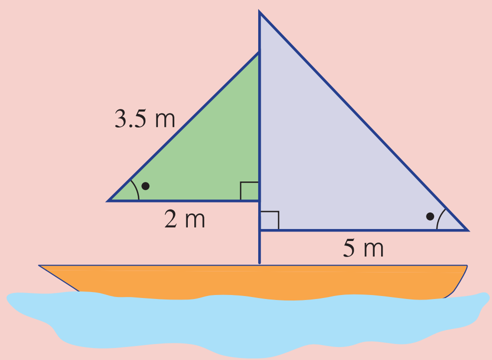


Fluency

3, 4 | 3, 5

Example 14 Applying similar triangles

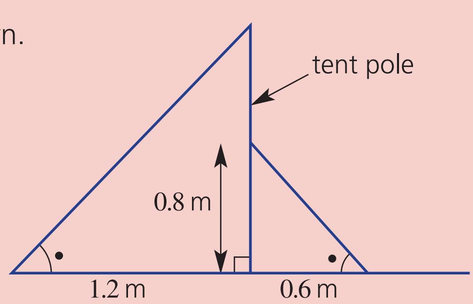
A home-made raft consists of two sails with measurements and angles as shown in this diagram.
  a Give reasons why the two sails are similar in shape.
  b Find the scale factor for the side lengths of the sails.
  c Find the length of the longest side of the large sail.



Solution	Explanation
a AAA (there are three equal pairs of angles)	Two of the three angles are clearly equal, so the third must be equal.
b Scale factor $= \frac{5}{2} = 2.5$	Choose two corresponding sides with known lengths and divide the larger by the smaller.
c Longest side $= 3.5 \times 2.5$ $= 8.75$ m	Multiply the corresponding side on the smaller triangle by the scale factor.

Now you try

A vertical tent pole is held in place with two guy ropes as shown.
  a Give reasons why the two triangles formed by the guy ropes are similar.
  b Find the scale factor for the guy ropes.
  c Find the height of the tent pole.



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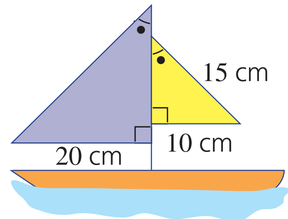
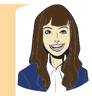
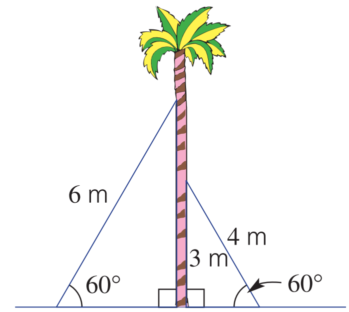
3 A toy yacht consists of two sails with measurements and angles as shown in this diagram.
  - **a** Give reasons why the two sails are similar in shape.
  - **b** Find the scale factor for the side lengths of the sails.
  - **c** Find the length of the longest side of the large sail.



Hint: You can't choose SSS, SAS or RHS because only one pair of corresponding sides is given.

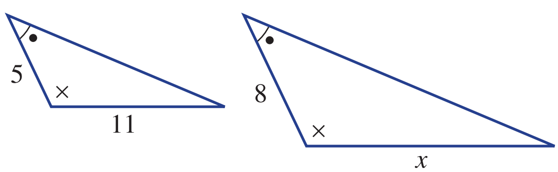


4 A tall palm tree is held in place with two cables of length 6 m and 4 m, as shown.
  - **a** Give reasons why the two triangles created by the cables are similar in shape.
  - **b** Find the scale factor for the side lengths of the cables.
  - **c** Find the height of the point above the ground where the longer cable is attached to the palm tree.

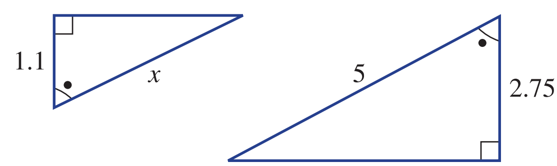


5 These pairs of triangles are known to be similar. By finding the scale factor, find the value of x.

a



b

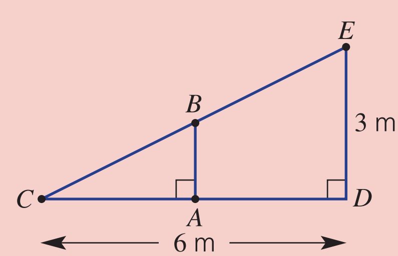


Problem-solving and reasoning | 6–8, 10 | 7–10

Example 15 Working with combined triangles

A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 6$ m and that the ramp is 3 m high.

- **a** Using the letters given, name the two triangles that are similar and give your reason.
- **b** Find the length of the stud AB.



Solution	Explanation
a $\triangle ABC$ and $\triangle DEC$ (AAA)	The angle at C is common to both triangles and they both have a right angle.
b $AC = 3$ m	Since A is in the centre of CD, then AC is half of CD.
Scale factor $= \frac{6}{3} = 2$	$CD = 6$ m and $AC = 3$ m.
$\therefore AB = 3 \div 2$ $= 1.5$ m	Divide the larger side length, DE, by the scale factor.

Continued on next page

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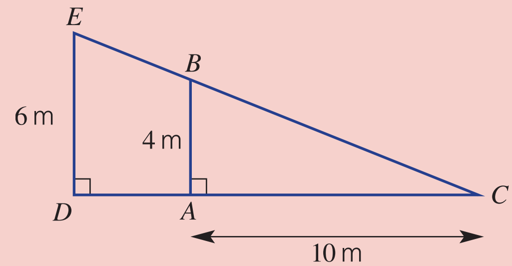
7G

Now you try

A 'lean–to' shelter is made with two vertical poles AB and DE as shown.

a Using the letters given, name the two triangles that are similar and give your reason.

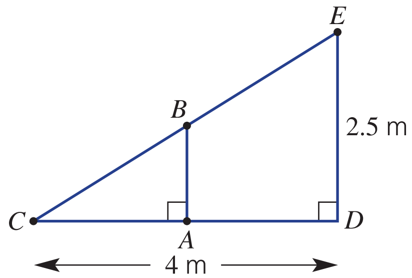
b Find the length CD.



6 A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 4$ m and that the ramp is 2.5 m high.

a Using the letters given, name the two triangles that are similar and give your reason.

b Find the length of the stud AB.



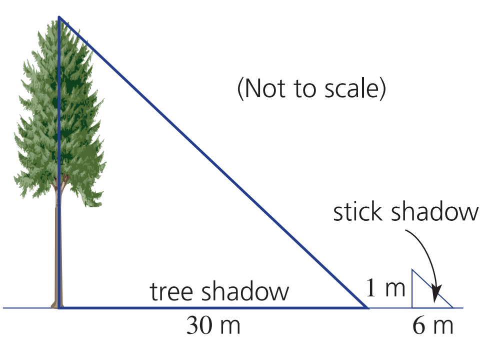
7 A 1 m vertical stick and a tree cast their shadows at a particular time in the day. The shadow lengths are shown in this diagram.

a Give reasons why the two triangles shown are similar in shape.

b Find the scale factor for the side lengths of the triangles.

c Find the height of the tree.

Hint: At the same time of day, the angle that the light makes with the ground will be the same.



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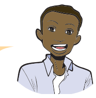
8 From a place on the river (C), a tree (E) is spotted on the opposite bank. The distances between selected trees A, B, C and D are measured as shown.

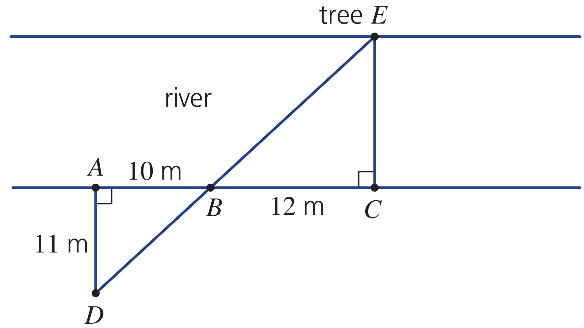
a List two similar triangles and give a reason why they are similar.

b Find the scale factor.

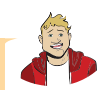
c Find the width of the river.



Hint: AB corresponds to CB and AD corresponds to CE.



9 At a particular time of day, Leon casts a shadow 1.3 m long, whereas Jackson, who is 1.75 m tall, casts a shadow 1.2 m long. Find the height of Leon, to two decimal places.



Hint: Draw a diagram to find the scale factor.

10 Try this activity with a classmate but ensure that at least one person knows their height.

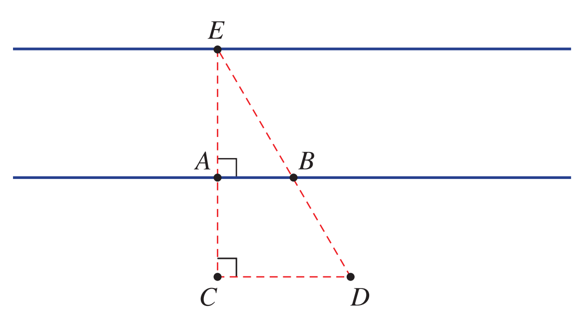
a Go out into the sun and measure the length of each person's shadow.

b Use these measurements plus the known height of one person to find the height of the other person.

Gorge challenge — 11

11 Mandy sets up a series of rocks alongside a straight section of a deep gorge. She places rocks A, B, C and D as shown. Rock E sits naturally on the other side of the gorge. Mandy then measures the following distances.

- $AB = 10$ m
- $AC = 10$ m
- $CD = 15$ m



a Explain why $\triangle ABE \,|||\, \triangle CDE$.

b What is the scale factor?

c Use trial and error to find the distance across the gorge from rocks A to E.

d Can you find the length AE by setting up an equation?

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7H Applications of similarity in measurement

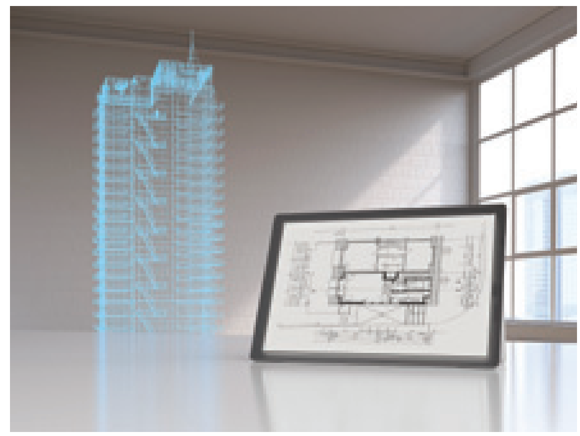
Learning intentions

- To know how the length, area and volume ratios are related in similar objects
- To be able to calculate and use the scale factor to find an unknown area or volume given similar objects

Key vocabulary: length, area, volume, ratio, scale factor

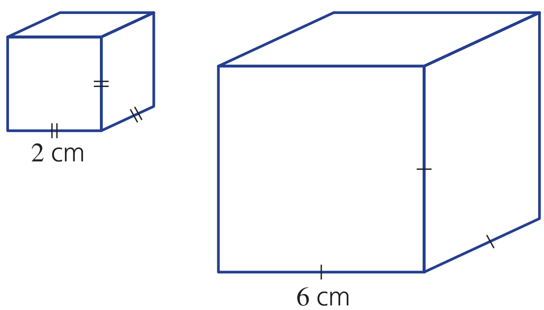
Shapes or objects that are similar have a special length, area and volume ratio relationship.

For example, if the lengths on a model of a building are one-hundredth of the actual structure, then the length ratio is $1:100$. From this, the surface area and volume ratios are $1^2:100^2$ $(1:10\ 000)$ and $1^3:100^3$ $(1:1\ 000\ 000)$, respectively. These ratios can be used to calculate the amount of material that is needed for the construction of the building.



Lesson starter: Cube analysis

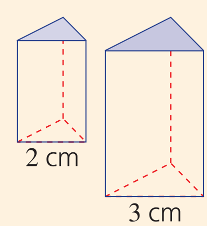
These two cubes have a 2 cm and 6 cm side length.



- What is the side length ratio when comparing the two cubes?
- What are the surface areas of the two cubes?
- What is the surface area ratio? What do you notice?
- What are the volumes of the two cubes?
- What is the volume ratio? What do you notice?

Key ideas

- When two objects are similar and have a length ratio of $a:b$, then:
  - Length ratio $= a:b$ Scale factor $= \frac{b}{a}$
  - Area ratio $= a^2:b^2$ Scale factor $= \frac{b^2}{a^2}$
  - Volume ratio $= a^3:b^3$ Scale factor $= \frac{b^3}{a^3}$
  - one dimension: length ratio $= 2^1:3^1 = 2:3$
  - two dimensions: area ratio $= 2^2:3^2 = 4:9$
  - three dimensions: volume ratio $= 2^3:3^3 = 8:27$



Exercise 7H

Understanding | 1, 2 | 2

1 The length ratio for two objects is $2:5$.

a What would be the area ratio?

b What would be the volume ratio?

Hint:
Length ratio $a:b$
Area ratio $a^2:b^2$
Volume ratio $a^3:b^3$

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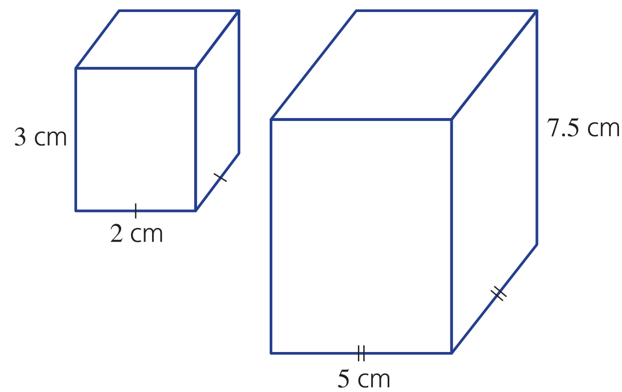
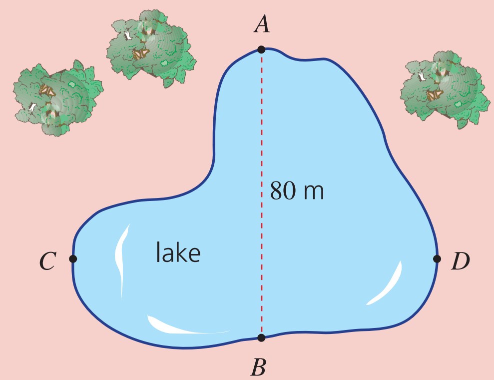
2 These two rectangular prisms are similar.
a What is the side length ratio?
b What is the surface area of each prism?
c What is the surface area ratio? What do you notice?
d What is the volume of each prism?
e What is the volume ratio? What do you notice?



Fluency | 3, 4, 7 | 4–8

Example 16 Measuring to find actual lengths

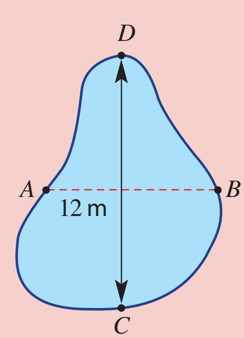
The given diagram is a simple map of a park lake.
a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)
b Find the scale factor between the map and ground distance.
c Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)
d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



Solution	Explanation
a 4 cm	Check with your ruler.
b $\frac{8000}{4} = 2000$	Using the same units, divide the real distance (80 m = 8000 cm) by the measured distance (4 cm).
c 5 cm	Check with your ruler.
d $5 \times 2000 = 10\,000$ cm $= 100$ m	Multiply the measured distance by the scale factor and convert to metres by dividing by 100.

Now you try

The given diagram is a simple map of a swimming pool.
a Use a ruler to measure the distance across the pool (AB). (Answer in cm.)
b Find the scale factor between the map and ground distance.
c Use a ruler to measure the map distance across the pool (CD). (Answer in cm.)
d Use your scale factor to find the real length of the pool (CD). (Answer in m.)



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7H

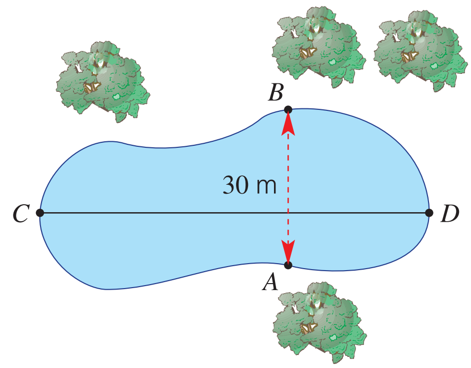
3 The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



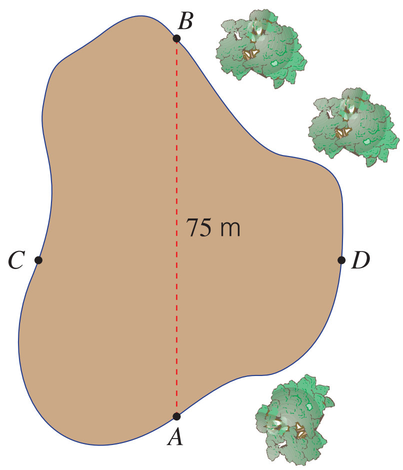
4 The given diagram is a simple map of a children's play area.

a Use a ruler to measure the distance across the children's play area (AB). (Answer in cm.)

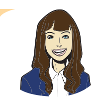
b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the children's play area (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the children's play area (CD). (Answer in m.)

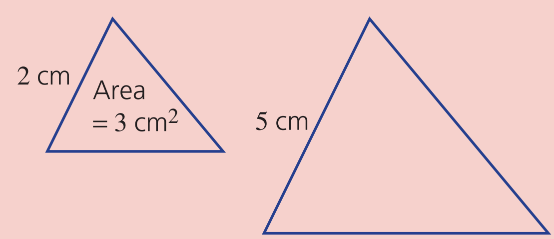


Hint: Use the measured distance AB and the actual distance AB to find the scale factor.



Example 17 Using similarity to find areas

The two given triangles are known to be similar.
Find the area of the larger triangle.



Solution	Explanation
Length ratio $= 2^1 : 5^1 = 2 : 5$	First, write the length ratio.
Area ratio $= 2^2 : 5^2 = 4 : 25$	Square each number in the length ratio to get the area ratio.
Area scale factor $= \frac{25}{4} = 6.25$	Divide the two numbers in the area ratio to get the scale factor.
$\therefore$ Area of larger triangle $= 3 \times 6.25$ $= 18.75\text{ cm}^2$	Multiply the area of the smaller triangle by the scale factor.

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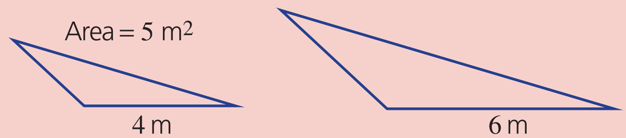
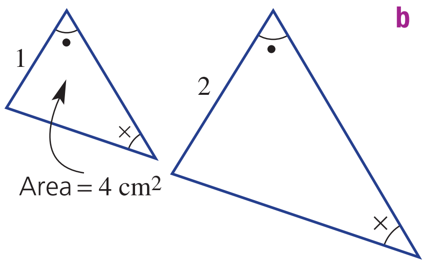
Now you try

The two given triangles are known to be similar.
Find the area of the larger triangle.

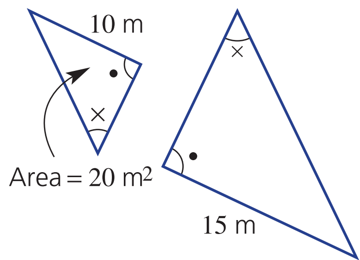


5 The two given triangles are known to be similar. Find the area of the larger triangle.

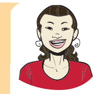
a



b

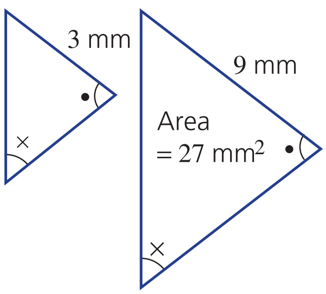


Hint:
Length ratio = $a:b$
Area ratio = $a^2:b^2$
Area scale factor = $\frac{b^2}{a^2}$

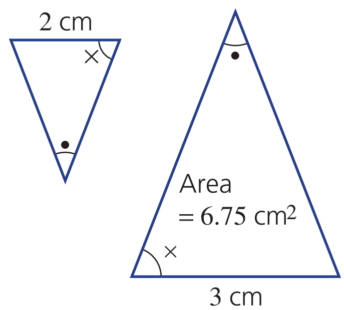


6 The two given triangles are known to be similar. Find the area of the smaller triangle.

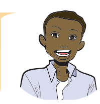
a



b

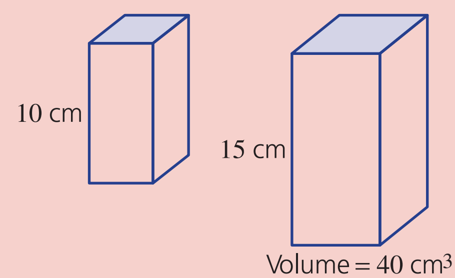


Hint: You will need to divide the larger area by the area scale factor.



Example 18 Using similarity to find volume

The two given prisms are known to be similar.
Find the volume of the smaller prism, correct to two decimal places.



Solution	Explanation
Length ratio = $10^1 : 15^1 = 2:3$	First, write the length ratio and simplify.
Volume ratio = $2^3 : 3^3 = 8:27$	Cube each number in the length ratio to get the volume ratio.
Volume scale factor = $\frac{27}{8} = 3.375$	Divide the two numbers in the volume ratio to get the scale factor.
∴ Volume of smaller prism = $40 \div 3.375$ $= 11.85\text{ cm}^3$ (to 2 d.p.)	Divide the volume of the larger prism by the scale factor and round as required.

Continued on next page

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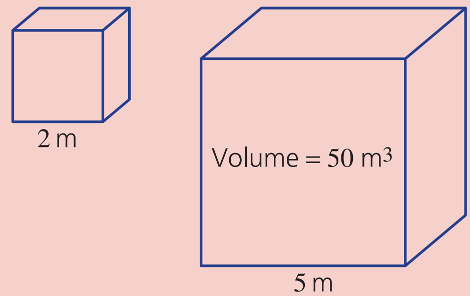
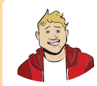
Now you try

The two given prisms are known to be similar. Find the volume of the smaller prism, correct to two decimal places.

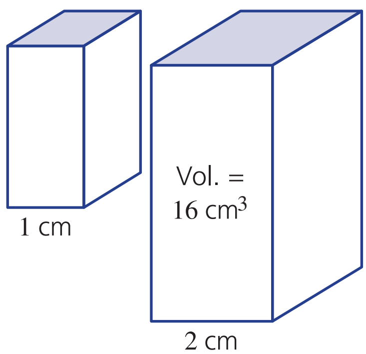


7 The two given prisms are known to be similar. Find the volume of the smaller prism (to two decimal places).

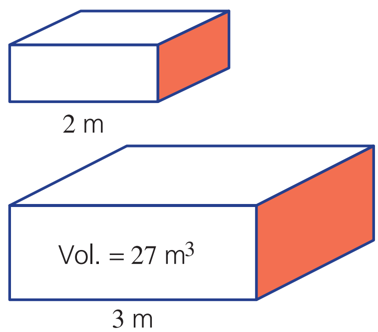
Hint: Volume scale factor $= \frac{b^3}{a^3}$ if the length ratio is $a:b$.



a

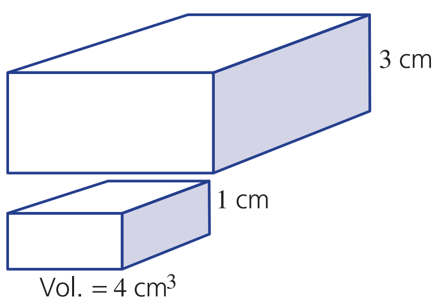


b

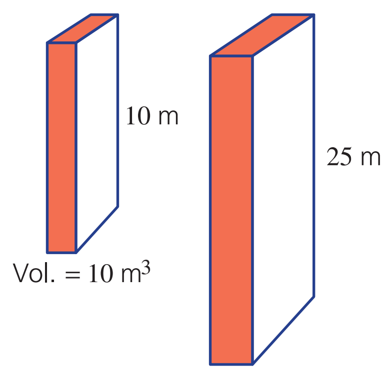


8 The two given prisms are known to be similar. Find the volume of the larger prism.

a



b



Problem-solving and reasoning 9, 10 10–12

9 The given map has a scale factor of 50 000 (i.e. ratio 1 : 50 000).

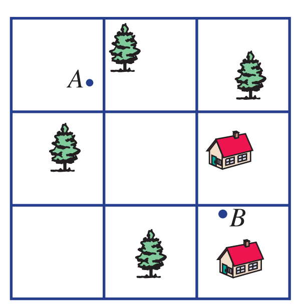
a How far on the ground, in km, is represented by these map distances?

i 2 cm **ii** 6 cm

b How far on the map, in cm, is represented by these ground distances?

i 5 km **ii** 0.5 km

c What is the actual ground distance between the two points A and B? Use your ruler to measure the distance between A and B first.



Hint:
1 m = 100 cm
1 km = 1000 m

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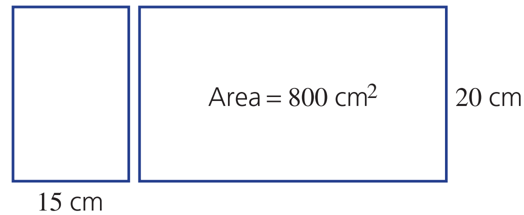
10 Two pieces of paper are similar in shape, as shown.

a What is:

i the length ratio?

ii the area ratio?

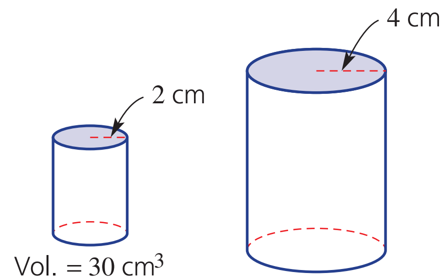
b Find the area of the smaller piece of paper.



11 Two cylinders are similar in shape, as shown.

a Find the volume ratio.

b Find the volume of the larger cylinder.



12 Two rectangular prisms are known to be similar.

a Find the following ratios.

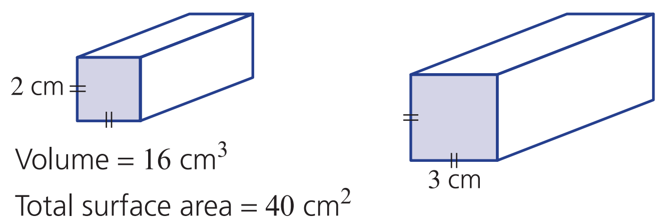
i length

ii area

iii volume

b Find the total surface area of the larger prism.

c Find the volume of the larger prism.



Skyscraper model — 13

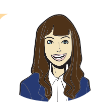
13 A scale model of a skyscraper is 1 m tall and the volume is 2 m^3. The actual height of the skyscraper is 300 m tall.

a Find the volume ratio between the model and actual skyscraper.

b Find the volume of the actual skyscraper.

c If the area of a window on the model is 1 cm^2, find the area of the actual window, in m^2.

Hint: $1\ m^2 = 100 \times 100$
$= 10\ 000\ cm^2$



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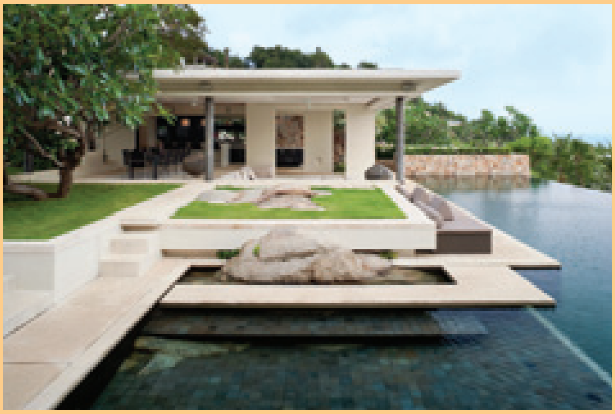
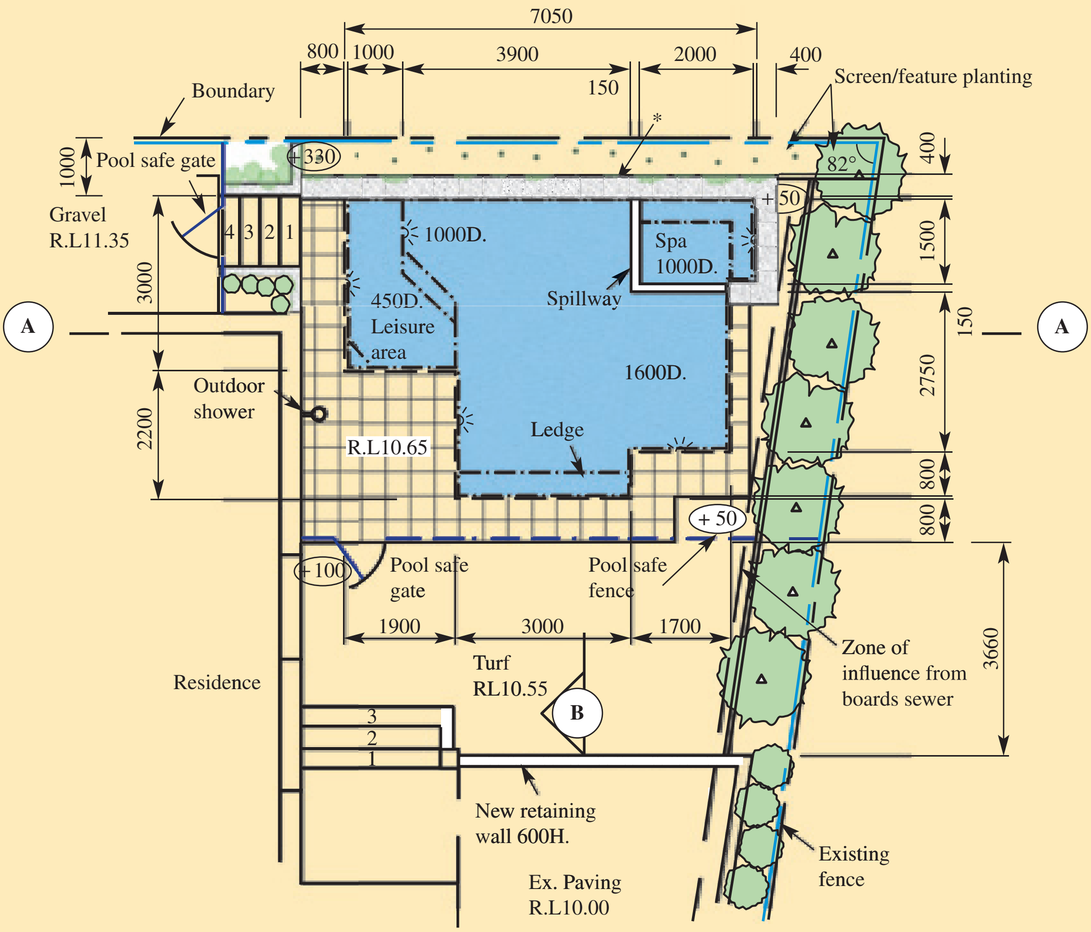
Maths@Work: Pool builder

Pool builders and designers need to understand many aspects of measurement and geometry to be able to accurately undertake the task of designing and building pools that meet the specifications of the client. They must be able to read plans, measure the angles, create similar figures and work with parallel lines, especially when they create and work from the pool plans and drawings.



Below are the plans for a pool drawn by a designer. All lengths are given in mm. Answer the questions on the next page relating to the design of this pool.



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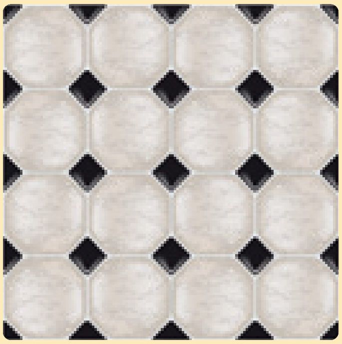
Cambridge University Press

1 Refer to the plans to answer the following questions.
- **a** What is the total length of the pool when looking across the top of the plans, in metres?
- **b** How deep is the proposed spa, in metres?
- **c** What is the height of the new retaining wall, in cm?
- **d** What is the area taken up by the internal dimensions of the spa? Include the seated area and give your answer in square metres.
- **e** What is the angle between the back fence and the right side fence?
- **f** If you walked around the outside of the pool/spa area, how many right angles would you turn? Assume you start and finish at the same point, facing in the same direction.

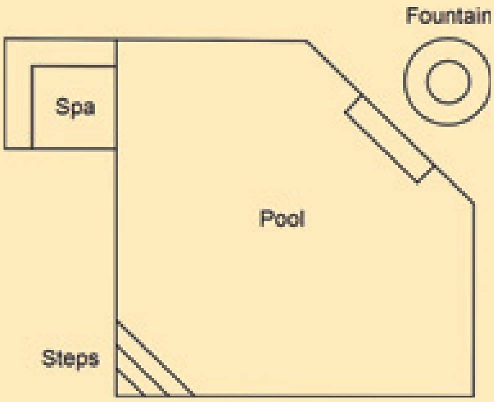
2 Consider the section of the pool called Leisure area, including the two steps which are a part of this area.
- **a** By counting the number of outside edges, decide what is the shape of the proposed leisure area.
- **b** What is the sum of the interior angles of the leisure area, using your knowledge of the angle sum of any polygon?
- **c** Use a protractor to measure the angles of the leisure area to check that the total is what you expect from part **b**.
- **d** Using your geometrical equipment, ruler and protractor, create an enlargement of the leisure area. Use the enlargement factor of 4. Your drawing will be mathematically similar to the original. Add all the labelled measurements from the original drawing.

3 Consider this pool tile, which will be used in the spa of this pool.
- **a** What are two shapes being used in its design?
- **b** Are both shapes regular?
- **c** What are the internal angles of the two shapes?
- **d** This pattern is a tessellation of two shapes. Why does there need to be two different shapes used here? Explain.
- **e** Design your own tessellation using two regular shapes, ensuring they are different from the ones used here.



Using technology

4 Use a geometry package, like GeoGebra, Desmos, Cabri or Geometer's sketchpad, to come up with your own pool design. You may wish to add length and angle measurements to add detail. Here is a simple example.



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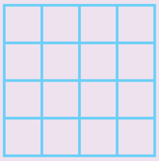
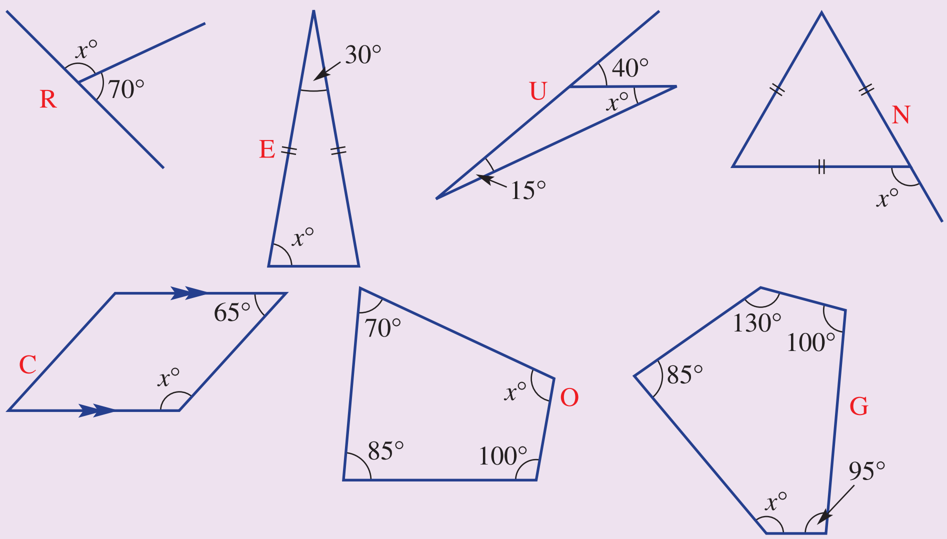
Puzzles and games

1 How many squares can you see in this diagram?

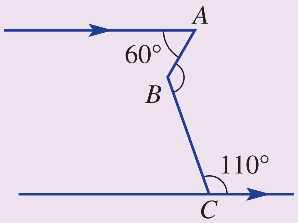


2 'I think of this when I look in the mirror.' Find the value of x in each diagram, then match the letters beside the diagrams to the answers below.



$\overline{115}$ $\overline{105}$ $\overline{120}$ $\overline{130}$ $\overline{110}$ $\overline{25}$ $\overline{75}$ $\overline{120}$ $\overline{115}$ $\overline{75}$

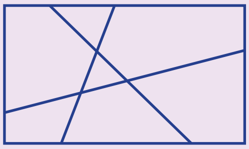
3 What is the size of the obtuse angle $\angle ABC$ in this diagram?



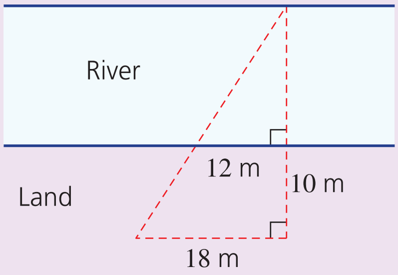
4 This rectangle is subdivided by three straight lines.

a How many regions are formed?

b What is the maximum number of regions formed if four lines are used instead of three?



5 Find the distance across the river.



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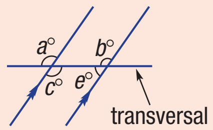
Parallel lines

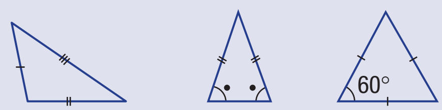


$a = b$ corresponding angles
$b = c$ alternate angles
$c + e = 180$ cointerior angles
$a = c$ vertically opposite angles

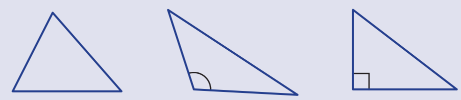
Triangles

Angle sum $= 180^\circ$

Scalene | Isosceles | Equilateral

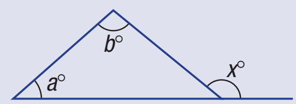


Acute | Obtuse | Right



Exterior angles

$x = a + b$



Congruent triangles

Equal in size and shape

Tests: SSS, SAS, AAS, RHS

A congruence statement:
$\triangle ABC \equiv \triangle DEF$

Geometry

Similar triangles

Equal in shape and sides are in the same ratio

Tests: SSS, SAS, AAA, RHS

A similarity statement:
$\triangle ABC \,|||\, \triangle DEF$

Quadrilaterals

Angle sum $= 360^\circ$

Special types include:
- parallelogram
- square
- rectangle
- rhombus
- kite
- trapezium

Applications of similarity

When $\triangle ABC \,|||\, \triangle DEF$, then:
- Scale factor $= \frac{DE}{AB}$ or $\frac{DF}{AC}$ or $\frac{EF}{BC}$
- Length ratio $a : b$, Scale factor $= \frac{b}{a}$ ⭐
- Area ratio $a^2 : b^2$, Scale factor $= \frac{b^2}{a^2}$ ⭐
- Volume ratio $a^3 : b^3$, Scale factor $= \frac{b^3}{a^3}$ ⭐

Polygons ⭐

Angle sum, $S = 180^\circ \times (n - 2)$

Regular polygons have:
- equal side lengths
- equal angles

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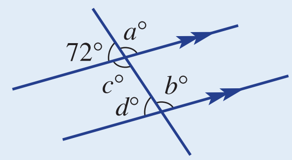
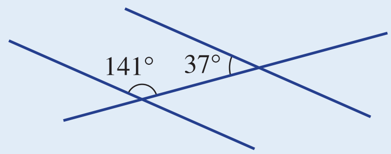
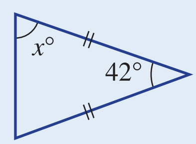
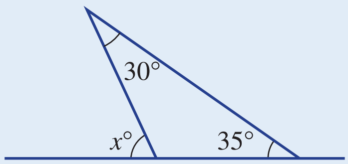
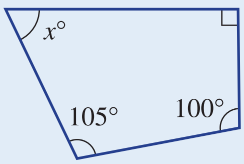
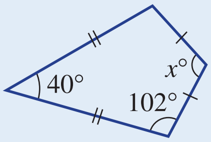
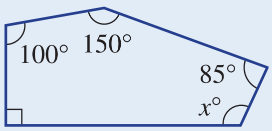
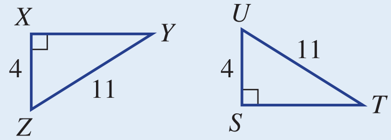
Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

Section			✔
7A	1	**I can find unknown angles in parallel lines.** e.g. Find the values of the pronumerals in this diagram and give reasons for your answers. 	
7A	2	**I can prove that two lines are parallel.** e.g. Decide, with reasons, whether the given pair of lines are parallel. 	
7B	3	**I can find unknown angles in any type of triangle.** e.g. Find the value of x in this triangle. 	
7B	4	**I can use the exterior angle theorem to find unknown angles.** e.g. Use the exterior angle theorem to find the value of x in this diagram. 	
7C	5	**I can find an unknown angle in a quadrilateral.** e.g. Find the value of x in this quadrilateral. 	
7C	6	**I can find an unknown angle in a special quadrilateral.** e.g. Find the value of x in this kite. 	
7D	7	**I can find an angle sum of a polygon and an unknown angle in a polygon.** e.g. Find the value of x in this pentagon after finding the angle sum. 	
7D	8	**I can find the internal angle in a regular polygon.** e.g. Find the size of an internal angle inside a regular hexagon.	
7E	9	**I can choose a test and write a congruence statement for a pair of congruent triangles.** e.g. Write a congruence statement and the test to prove congruence for this pair of triangles. 	

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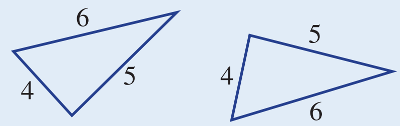
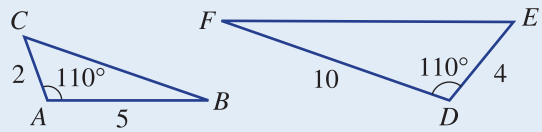
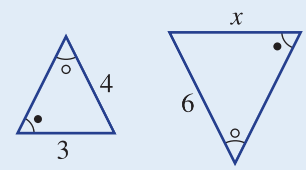
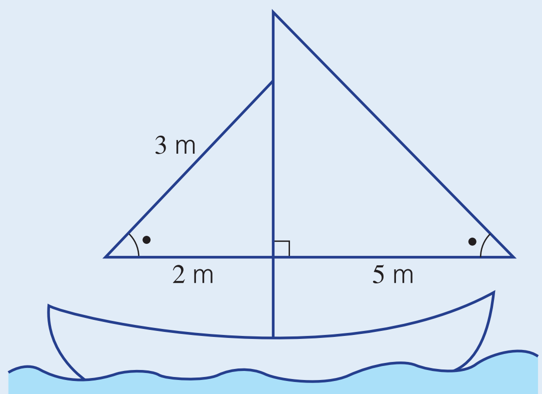
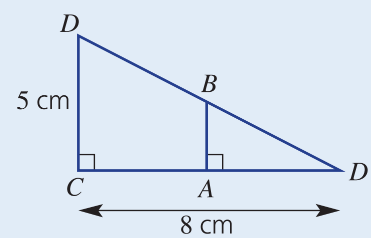
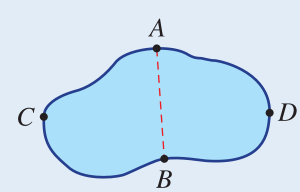
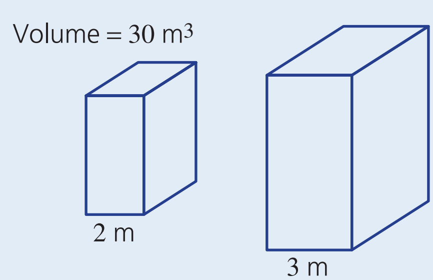
Chapter checklist

		✔
7E	**10 I can prove that a pair of triangles are congruent.** e.g. Prove that this pair of triangles are congruent, giving full reasons. 	
7F	**11 I can prove that a pair of triangles are similar.** e.g. Prove that this pair of triangles are similar, giving full reasons. 	
7F	**12 I can find a scale factor and use this to find an unknown length.** e.g. If the given pair of triangles are known to be similar, find the value of x. 	
7G	**13 I can use the scale factor for similar triangles to find an unknown length in a real context.** e.g. For the triangles formed from these yacht sails, give a reason why they are similar then use the scale factor to find the length of the hypotenuse on the larger sail. 	
7G	**14 I can use the scale factor for similar triangles to find an unknown length inside combined triangles.** e.g. If the point A is at the centre of CD in this diagram, find the length AB. 	
7H	**15 I can use measurement and the map scale factor to find a real distance on a map.** e.g. Use measurement and the map scale factor to find the distance across the pool (CD). Scale 1 : 300 	
7H	**16 I can use an area or volume ratio to find an area or volume of a similar object.** e.g. Find the volume of the larger prism if it is known that they are similar. 	

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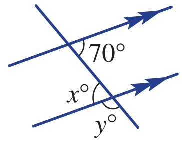
Chapter review

Short-answer questions

7A **1** Find the value of x and y in these diagrams.

a


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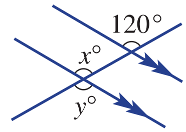

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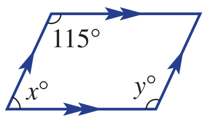

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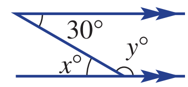

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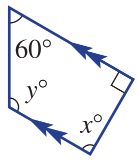

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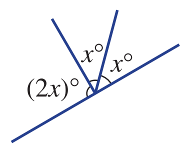
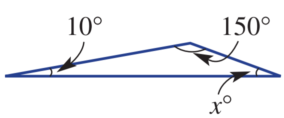


7B **2** Find the value of x in these triangles.

a


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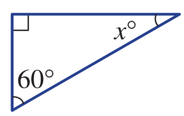

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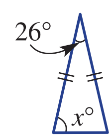

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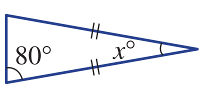

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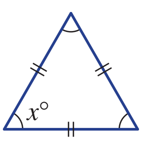

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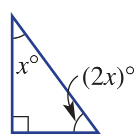

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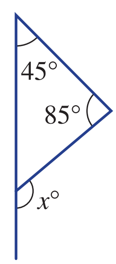

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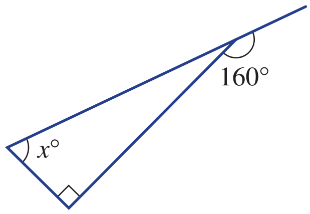

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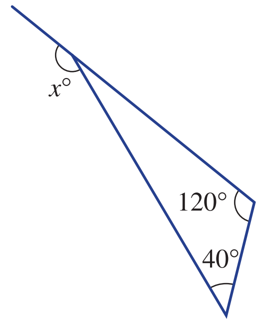

7C **3** List all the quadrilaterals that have:
- **a** two pairs of parallel lines
- **b** opposite angles that are equal
- **c** one pair of equal angles
- **d** diagonals intersecting at right angles

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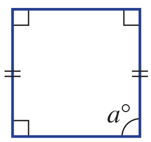
7C **4** Find the values of the pronumerals in these quadrilaterals.

a



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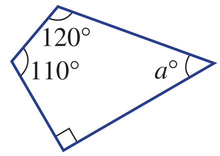

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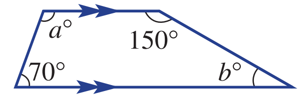
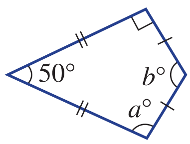
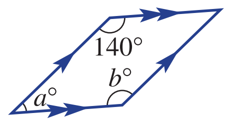


d



e



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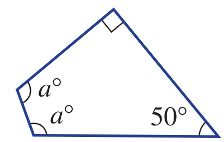
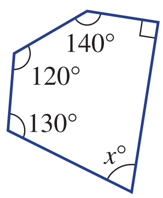


7D **5** Find the value of x by first finding the angle sum. Use $S = 180° \times (n - 2)$.

a



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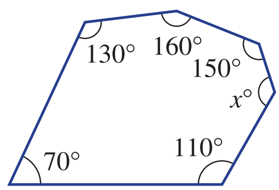

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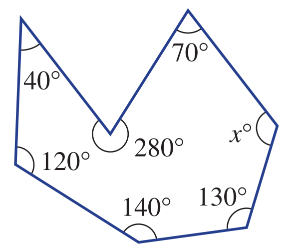
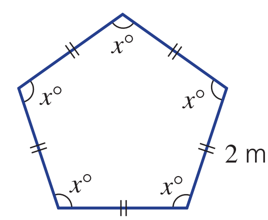


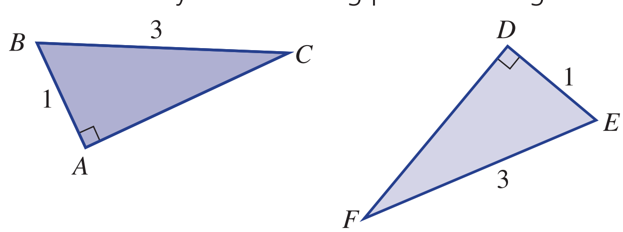
7D **6** Shown here is an example of a regular pentagon ($n = 5$) with side lengths 2 m.

a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle (x).

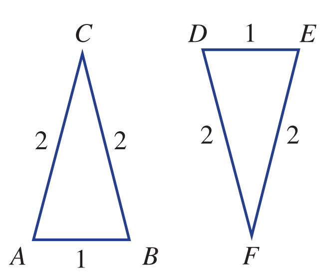


7E **7** Give reasons why the following pairs of triangles are congruent.

a



b



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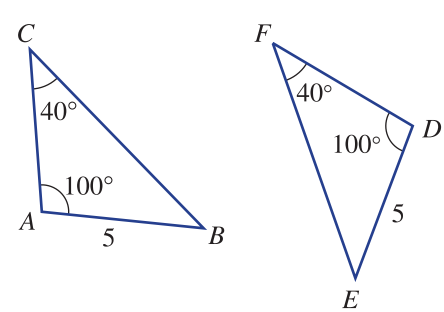

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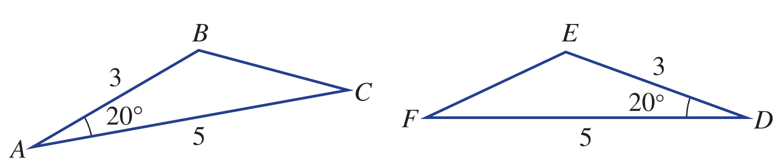

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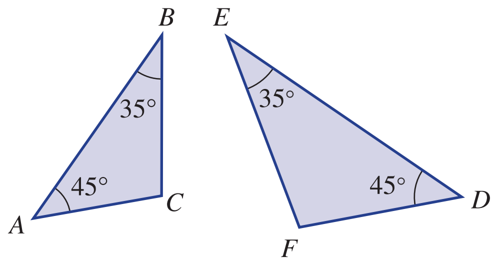
Chapter review

7F **8** Decide whether the given pairs of triangles are similar and give your reasons.

a



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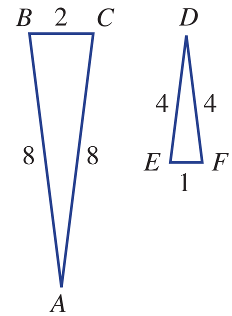

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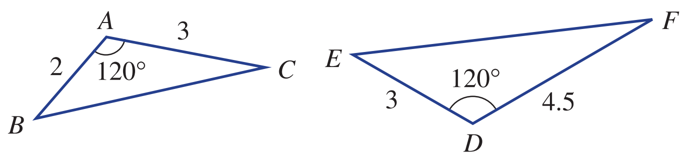
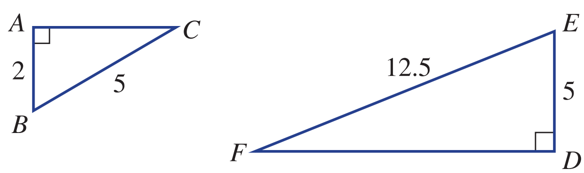
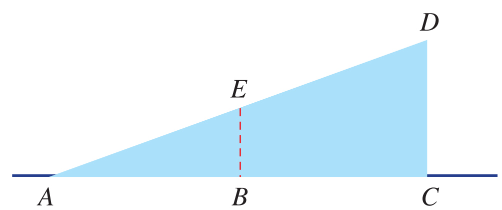


d



7G **9** A skateboard ramp is supported by two vertical struts, BE (2 m) and CD (5 m).



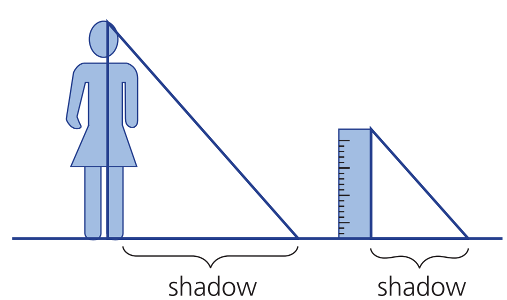
a Name two triangles that are similar, using the letters A, B, C, D and E.

b Give a reason why the triangles are similar.

c Find the scale factor from the smallest to the larger triangle.

d If the length AB is 3 m, find the horizontal length of the ramp AC.

7G **10** The shadow of Clara standing in the sun is 1.5 m long, whereas the shadow of a 30 cm ruler is 24 cm.



a Give a reason why the two created triangles are similar.

b Find the scale factor between the two triangles.

c How tall is Clara?

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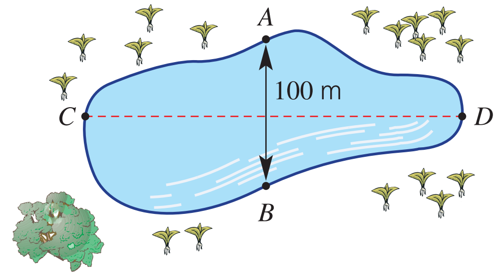
7H

11 The given diagram is a simple map of a swamp in bushland.



a Use a ruler to measure the distance across the swamp (AB). (Answer in cm.)

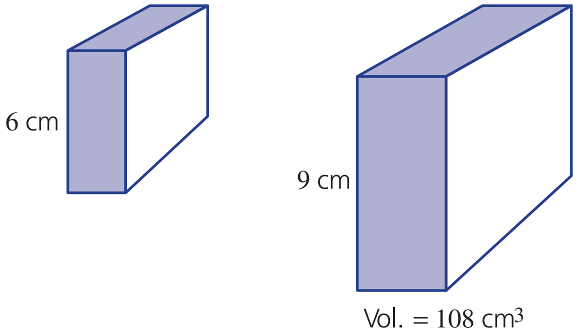
b Find the scale factor between the map and actual ground distance.

c Use a ruler to find the map distance across the swamp (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the swamp (CD). (Answer in m.)

7H

12 The two rectangular prisms shown are known to be similar.



a Find:

- **i** the length ratio
- **ii** the area ratio
- **iii** the volume ratio

b Find the volume of the smaller prism.

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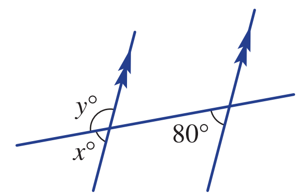
Multiple-choice questions

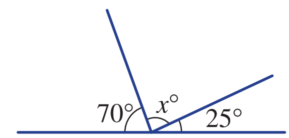
7A **1** The values of x and y in this diagram are, respectively:

A 100, 100 **B** 80, 100
C 80, 80 **D** 60, 120
E 80, 60



7A **2** The unknown value x in this diagram is:

A 85 **B** 105 **C** 75
D 80 **E** 90

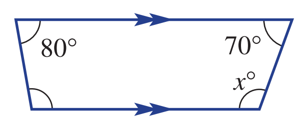


7B **3** A triangle has one angle of $60°$ and another angle of $70°$. The third angle is:

A $60°$ **B** $30°$ **C** $40°$
D $50°$ **E** $70°$

7C **4** The value of x in this quadrilateral is:

A 130 **B** 90 **C** 100
D 120 **E** 110



7D **5** The sum of the internal angles of a hexagon is:

A $180°$ **B** $900°$ **C** $360°$
D $540°$ **E** $720°$

7E **6** Which abbreviated reason is not relevant for proving congruent triangles?

A AAS **B** RHS **C** SSS
D AAA **E** SAS

7F **7** Two similar triangles have a length ratio of $2:3$. If one side on the smaller triangle is 5 cm, the length of the corresponding side on the larger triangle is:

A 3 cm **B** 7.5 cm **C** 9 cm
D 8 cm **E** 6 cm

7G **8** A stick of length 2 m and a tree of unknown height stand vertically in the sun. The shadow lengths cast by each are 1.5 m and 30 m, respectively. The height of the tree is:

A 40 m **B** 30 m **C** 15 m **D** 20 m **E** 60 m

7H **9** Two similar triangles have a length ratio of $1:3$ and the area of the large triangle is 27 cm^2. The area of the smaller triangle is:

A 12 cm^2 **B** 1 cm^2 **C** 3 cm^2 **D** 9 cm^2 **E** 27 cm^2

7H **10** Two similar prisms have a length ratio of $2:3$. The volume ratio is:

A $4:9$ **B** $8:27$ **C** $2:27$ **D** $2:9$ **E** $4:27$

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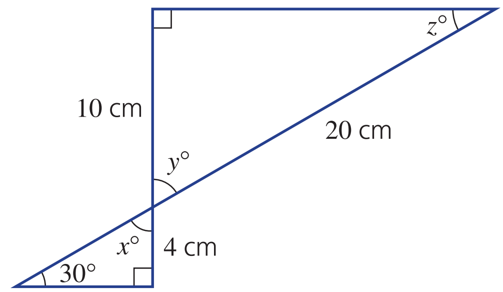
Extended-response questions

1 A company logo contains two triangles, as shown.



- **a** Write down the value of x, y and z.
- **b** Explain why the two triangles are similar.
- **c** Write down the scale factor for length.
- **d** Find the length of the longest side of the smaller triangle.
- **e** Write down the area ratio of the two triangles.
- **f** Write down the area scale factor of the two triangles.

2 A toy model of a car is 8 cm long and the actual car is 5 m long.

- **a** Write down the length ratio of the toy car to the actual car.
- **b** If the toy car is 4.5 cm wide, what is the width of the actual car?
- **c** What is the surface area ratio?
- **d** If the actual car needs 5 litres of paint, what amount of paint would be needed for the toy car?

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