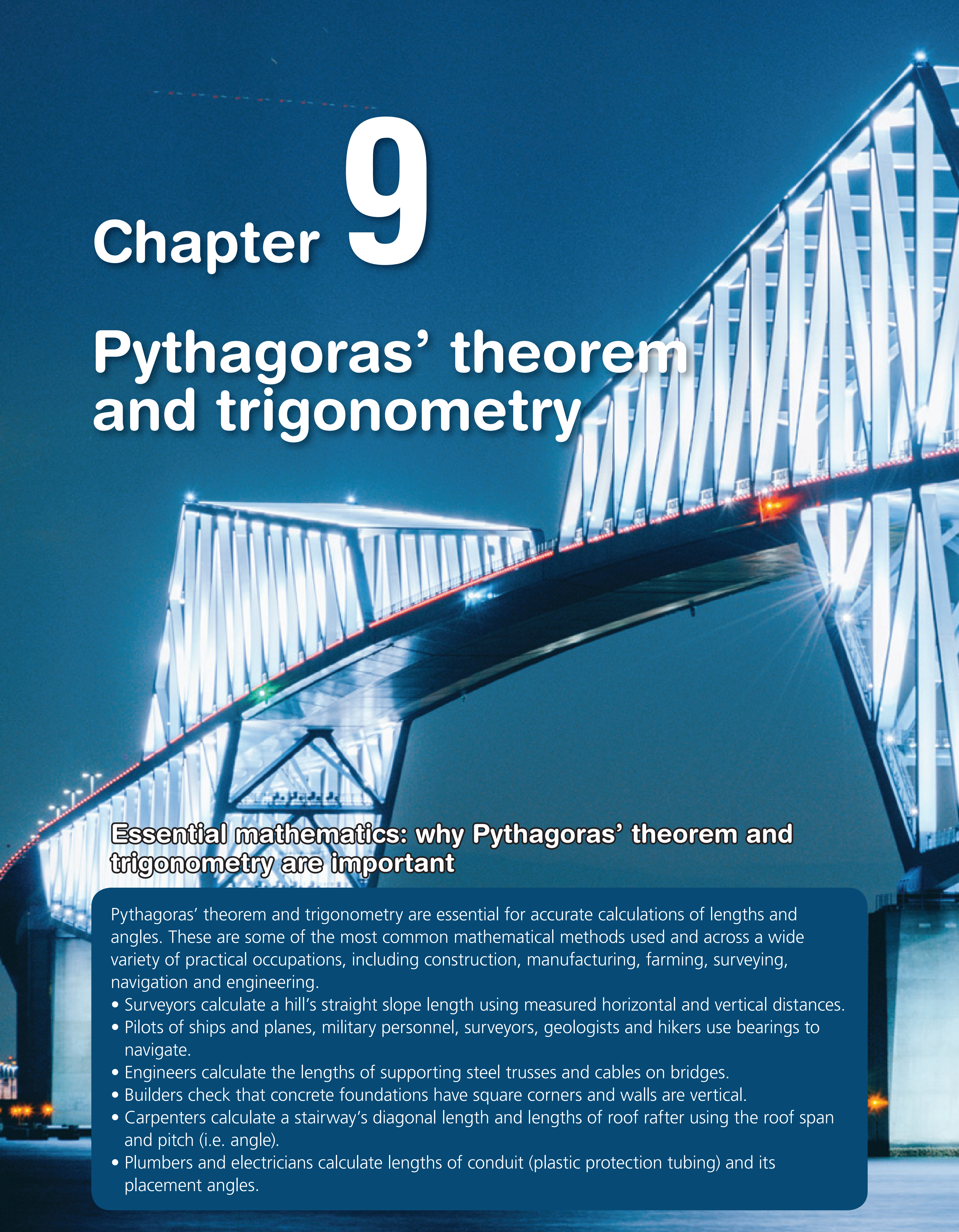

Chapter 9

Pythagoras’ theorem and trigonometry

Essential mathematics: why Pythagoras’ theorem and trigonometry are important

Pythagoras’ theorem and trigonometry are essential for accurate calculations of lengths and angles. These are some of the most common mathematical methods used and across a wide variety of practical occupations, including construction, manufacturing, farming, surveying, navigation and engineering.

- Surveyors calculate a hill’s straight slope length using measured horizontal and vertical distances.
- Pilots of ships and planes, military personnel, surveyors, geologists and hikers use bearings to navigate.
- Engineers calculate the lengths of supporting steel trusses and cables on bridges.
- Builders check that concrete foundations have square corners and walls are vertical.
- Carpenters calculate a stairway’s diagonal length and lengths of roof rafter using the roof span and pitch (i.e. angle).
- Plumbers and electricians calculate lengths of conduit (plastic protection tubing) and its placement angles.

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In this chapter

9A Reviewing Pythagoras' theorem (Consolidating)
9B Finding the length of a shorter side
9C Applications of Pythagoras' theorem ⭐
9D Trigonometric ratios (Consolidating)
9E Finding side lengths
9F Solving for the denominator ⭐
9G Finding angles
9H Angles of elevation and depression
9I Direction and bearings ⭐

Victorian Curriculum

MEASUREMENT AND GEOMETRY

Pythagoras and trigonometry

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (VCMMG346)

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Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Round the following decimals, correct to two decimal places.

a 15.84312 **b** 164.8731 **c** 0.86602 **d** 0.57735
e 0.173648 **f** 0.7071 **g** 12.99038 **h** 14.301

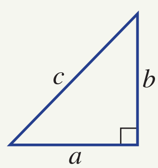
2 Find the value of each of the following.

a 5^2 **b** 6.8^2 **c** 19^2 **d** $9^2 + 12^2$ **e** $3.1^2 + 5.8^2$ **f** $41^2 - 40^2$

3 Write the following as a decimal, correct to one decimal place.

a $\sqrt{8}$ **b** $\sqrt{7}$ **c** $\sqrt{15}$ **d** $\sqrt{10}$
e $\sqrt{12.9}$ **f** $\sqrt{8.915}$ **g** $\sqrt{3.8}$ **h** $\sqrt{200}$

4 Write down the letter or letters matching the hypotenuse (i.e. the side opposite the right angle) on the following triangles.

a


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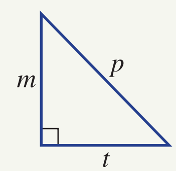
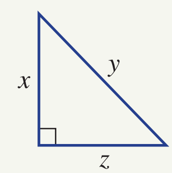
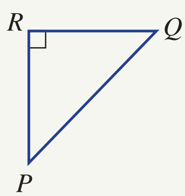


c


d


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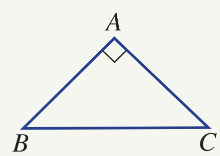

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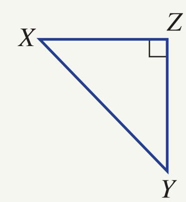


5 Solve for x.

a $3x = 9$ **b** $4x = 16$ **c** $\frac{x}{5} = 7$ **d** $\frac{2x}{3} = 6$

6 Solve for m.

a $7m = 25.55$ **b** $9m = 10.8$ **c** $\frac{m}{1.3} = 4$ **d** $\frac{m}{5.4} = 1.06$

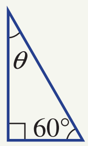
7 Solve each of the following equations, correct to one decimal place.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 17$ **c** $\frac{32}{x} = 15$ **d** $\frac{3.8}{x} = 9.2$ **e** $\frac{15}{x} = 6.2$ **f** $\frac{29.3}{x} = 3.2$

8 If x is a positive integer, solve:

a $x^2 = 16$ **b** $x^2 = 400$ **c** $x^2 = 5^2 + 12^2$ **d** $x^2 + 3^2 = 5^2$

9 Find the size of the angle θ in the following diagrams.

a


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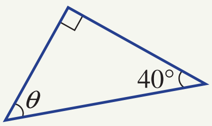

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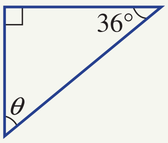

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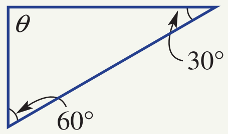
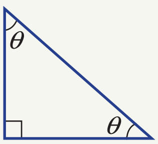


e


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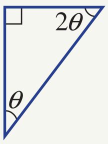
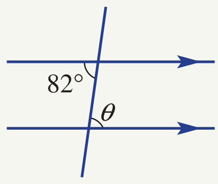
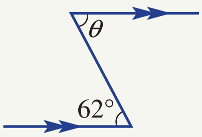
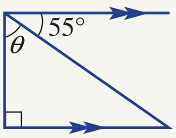


g


h


i


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9A Reviewing Pythagoras' theorem

CONSOLIDATING

Learning intentions

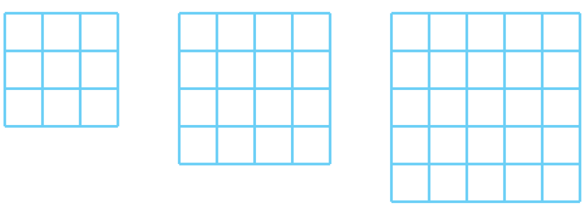
- To know that Pythagoras' theorem connects the three side lengths of a right-angled triangle
- To be able to find the length of the hypotenuse of a right-angled triangle given the other two sides.
- To be able to apply Pythagoras' theorem in finding the length of the hypotenuse in a simple application

Key vocabulary: Pythagoras' theorem, right angle, hypotenuse, square

The ancient Egyptians knew of the relationship between the numbers 3, 4 and 5 and how they could be used to form a right-angled triangle.

Greek philosopher and mathematician Pythagoras expanded on this idea and the theorem we use today is named after him.

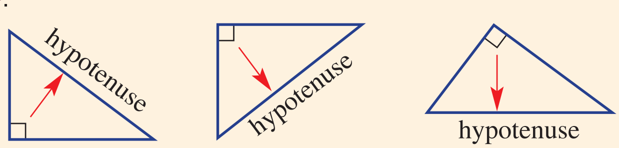
Lesson starter: Three, four and five



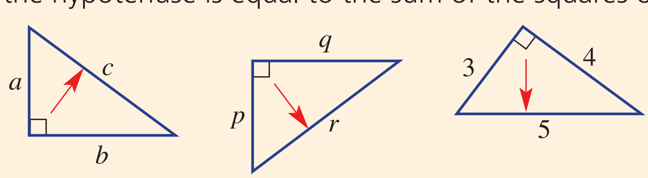
- On square grid paper, construct three squares as shown above.
- Cut them out and place the middle-sized square on top of the largest square. Then cut the smallest square into 9 smaller squares and also place them onto the largest square to finish covering it.
- What does this show about the numbers 3, 4 and 5?

Key ideas

- A right-angled triangle has its longest side opposite the right angle. This side is called the **hypotenuse**.
  For example:



- **Pythagoras' theorem** states:
  The square of the hypotenuse is equal to the sum of the squares on the other two sides.
  For example:



$c^2 = a^2 + b^2$ $\quad$ $r^2 = p^2 + q^2$ $\quad$ $5^2 = 3^2 + 4^2$

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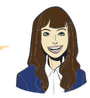
9A

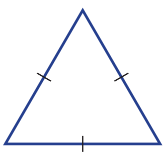
Exercise 9A

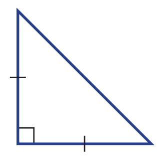
Understanding | 1–4 | 3, 4

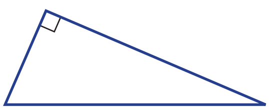
1 Which of the following triangles have a side known as the hypotenuse?

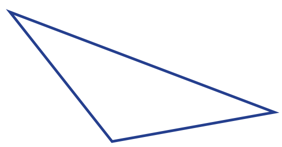


Hint: Only right-angled triangles have a hypotenuse.

a 

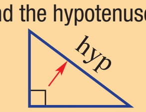
b 

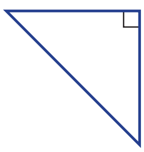
c 

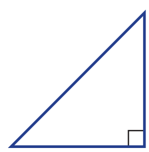
d 

2 Copy these triangles into your workbook and label the hypotenuse.

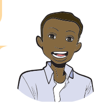
Hint: Draw an arrow across from the right angle to find the hypotenuse (hyp).



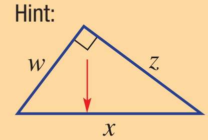
a 

b 

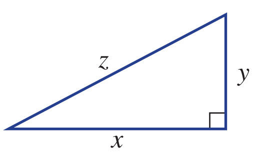
3 Write the relationship between the sides of these triangles.

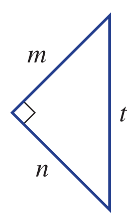


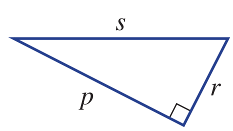
Hint:



$x^2 = w^2 + z^2$

a 

b 

c 

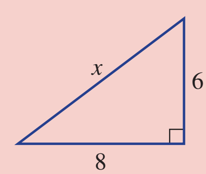
4 Find the value of $a^2 + b^2$ when:

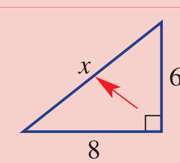
a $a = 3$ and $b = 4$ **b** $a = 3$ and $b = 5$ **c** $a = 3$ and $b = 6$

Fluency | 5–6(½) | 5–7(½)

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse (x) of the triangle shown.



Solution	Explanation
$x^2 = 6^2 + 8^2$ $= 36 + 64$ $= 100$	Write the relationship for the given triangle using Pythagoras' theorem. 
$x = \sqrt{100}$ $= 10$	Take the square root to find x.

Continued on next page

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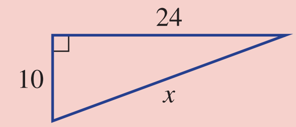
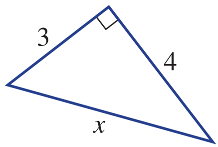
Now you try

Find the length of the hypotenuse (x) of the triangle shown.

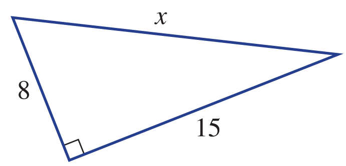


5 Find the length of the hypotenuse in these right-angled triangles.

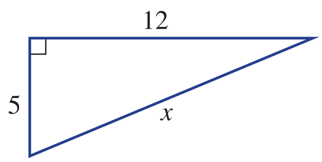
a



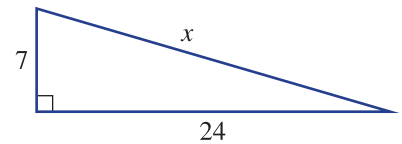
b



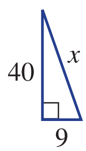
c



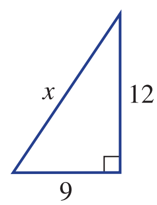
d



e

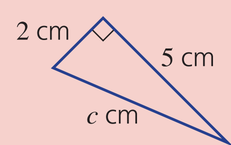


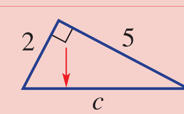
f



Example 2 Finding the length of the hypotenuse as a decimal

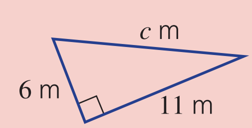
Find the length of the hypotenuse in this triangle, correct to one decimal place.



Solution	Explanation
$c^2 = 5^2 + 2^2$	Write the relationship for this triangle, where c is the length of the hypotenuse. 
$= 25 + 4$	
$= 29$	
$c = \sqrt{29}$	Simplify.
$c = 5.38516\ldots$	Take the square root to find c.
$c = 5.4$ (to 1 d.p.)	Round 5.3(8)516… to one decimal place by rounding up.

Now you try

Find the length of the hypotenuse in this triangle, correct to one decimal place.



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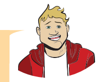
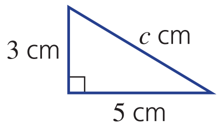
9A

6 Find the length of the hypotenuse in these triangles, correct to one decimal place.



Hint: If $c^2 = 34$, then $c = \sqrt{34}$. Use a calculator to find the decimal.

a


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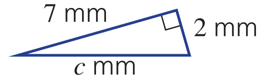

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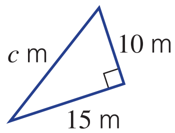
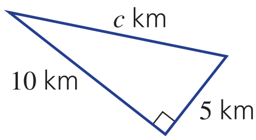


d


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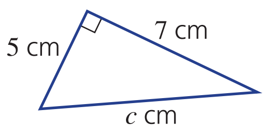

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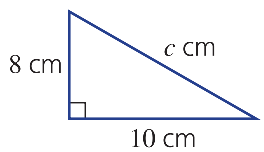
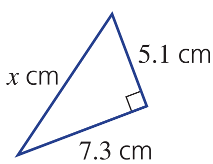


7 Find the value of the hypotenuse in these triangles, correct to two decimal places.

a


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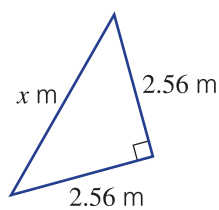
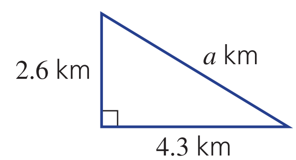
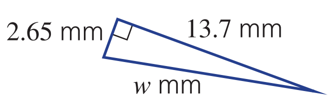
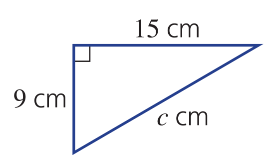


c


d


e


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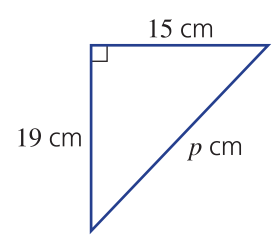
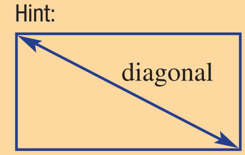
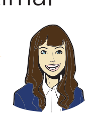


Problem-solving and reasoning | 8–11 | 11–14

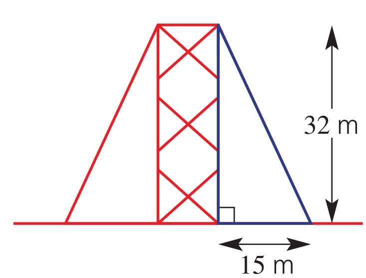
8 A LCD plasma TV is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to one decimal place.

Hint:


9 A 32 m tower is supported by cables from the top to a position on the ground 15 m from the base of the tower. Determine the length of each cable needed to support the tower, correct to one decimal place.



Hint: Set up and solve using Pythagoras' theorem.



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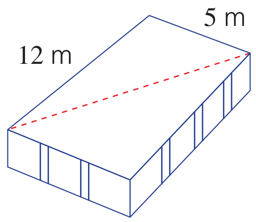
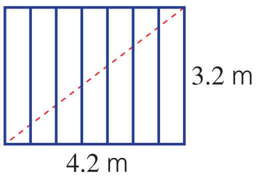
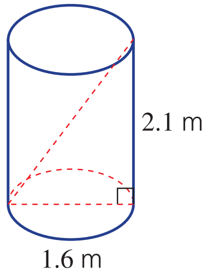
10 Boris the builder uses Pythagoras' theorem to check the corners of his concrete slab. What will be the length of the diagonal when the angle is 90°?



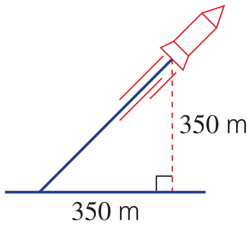
11 Find the length of the diagonal steel brace needed to support a gate of length 4.2 m and width 3.2 m, correct to two decimal places.



12 Find the length of the longest rod that will fit in a cylindrical container of height 2.1 m and diameter 1.6 m, correct to two decimal places.



13 A rocket blasts off and after a few seconds it is 350 m above the ground. At this time it has covered a horizontal distance of 350 m. How far has the rocket travelled, correct to two decimal places?



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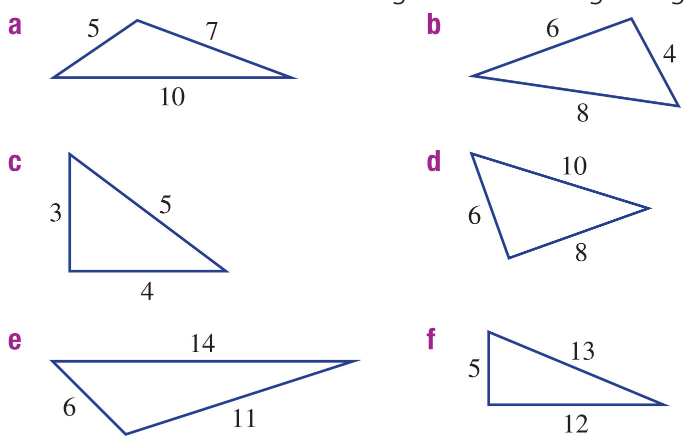
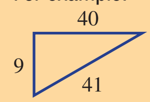
9A

14 Determine whether these triangles contain a right angle.



Hint: If Pythagoras' theorem works, then the triangle has a right angle.
For example:



$41^2 = 1681$
$40^2 + 9^2 = 1681$
$\therefore 41^2 = 40^2 + 9^2$ and the triangle has a right angle, opposite the 41.

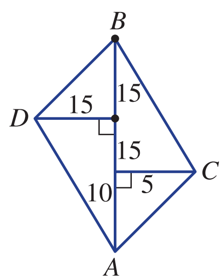
An offset survey

	—	15, 16

15 An offset survey measures distances perpendicular to the baseline offset. A notebook entry is made showing these distances, and then perimeters and areas are calculated.

		B		
		40		
D	15	25		
		10	5	C
		0		
		A		

Notebook entry



Field diagram
(Not to scale)

a Using the diagrams above, find these lengths, correct to one decimal place.
i AC **ii** BC **iii** DB **iv** AD

b Find the perimeter of the field $ACBD$, correct to the nearest metre.

c Find the area of the field.

16 At right is a notebook entry. Draw the field diagram and find the perimeter of the field, to one decimal place.

		B		
		60		
		40	10	E
D	25	30		
C	15	10		
		0		
		A		

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9B Finding the length of a shorter side

Learning intentions

- To be able to find the length of a shorter side of a right-angled triangle given the other two sides
- To be able to find the length of a shorter side of a right-angled triangle in a simple application

Key vocabulary: hypotenuse

Using Pythagoras' theorem, we can determine the length of the shorter sides of a right-angled triangle. The angled support beams on a rollercoaster ride, for example, create right-angled triangles with the ground. The vertical and horizontal distances are the shorter sides of the triangle.

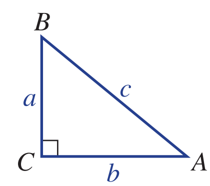
Lesson starter: Choosing the correct numbers

For the triangle ABC, Pythagoras' theorem is written $c^2 = a^2 + b^2$.

Choose the 3 numbers from each group that work for $c^2 = a^2 + b^2$.

Group 1: 6, 7, 8, 9, 10 Group 2: 15, 16, 20, 25

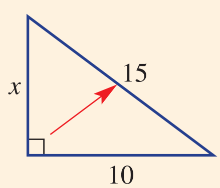
Group 3: 9, 10, 12, 15 Group 4: 9, 20, 21, 40, 41



Key ideas

- We can use Pythagoras' theorem to determine the length of one of the shorter sides if we know the length of the hypotenuse and the other side.

For example:



$15^2 = x^2 + 10^2$ becomes $x^2 = 15^2 - 10^2$.

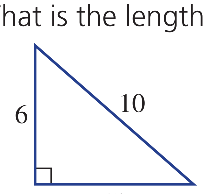
So $x^2 = 125$ and $x = \sqrt{125}$.

Exercise 9B

Understanding 1–3 3

1 What is the length of the hypotenuse in each of these triangles?

a



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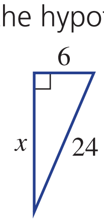

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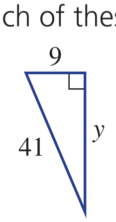

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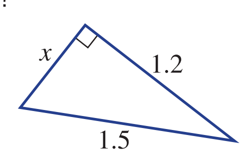
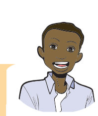


2 Copy and complete:

a When $10^2 = 6^2 + w^2$, then $w^2 = 10^2 - \square$.

b When $13^2 = 5^2 + x^2$, then $x^2 = 13^2 - \square$.

c When $30^2 = p^2 + 18^2$, then $p^2 = \square - 18^2$.



Hint: Follow a step as if you were solving an equation.

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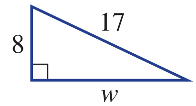
3 Substitute the numbers and pronumerals into Pythagoras' theorem $c^2 = a^2 + b^2$, for each of these triangles. Do not solve for the unknown.

a



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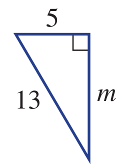

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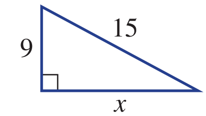
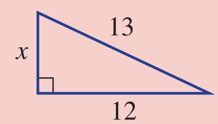


Fluency

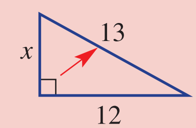
Fluency	4–5(½)	4–6(½)

Example 3 Calculating a shorter side

Determine the value of x in the triangle shown, using Pythagoras' theorem.

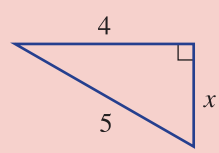


Solution	Explanation
$13^2 = x^2 + 12^2$	Write the relationship for this triangle using Pythagoras' theorem, with 13 as the hypotenuse.
$x^2 = 13^2 - 12^2$ $= 169 - 144$	Rewrite the rule with the x^2 on the left-hand side.
$= 25$	Simplify.
$x = \sqrt{25}$ $\therefore\ x = 5$	Find the square root to find x.



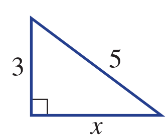
Now you try

Determine the value of x in the triangle shown, using Pythagoras' theorem.



4 Determine the value of x in these triangles, using Pythagoras' theorem.

a



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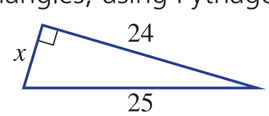
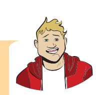


Hint: In $c^2 = a^2 + b^2$, c is always the hypotenuse.



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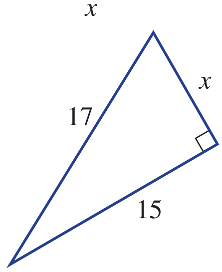

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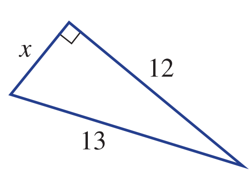

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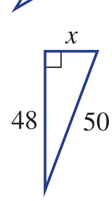

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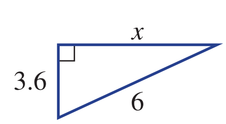

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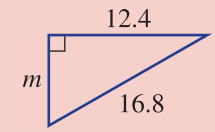
Example 4 Finding a shorter side length as a decimal value

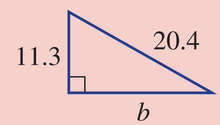
Determine the value of m in the triangle, correct to one decimal place.



Solution	Explanation
$16.8^2 = m^2 + 12.4^2$	Write the relationship for this triangle.
$m^2 = 16.8^2 - 12.4^2$	Make m^2 the subject.
$= 128.48$	Simplify, using your calculator.
$m = \sqrt{128.48}$	Take the square root of both sides to find m.
$= 11.3349\ldots$	
$m = 11.3$ (to 1 d.p.)	Round your answer to one decimal place.

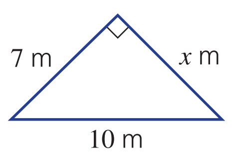
Now you try

Determine the value of b in the triangle, correct to one decimal place.



5 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to one decimal place.

a



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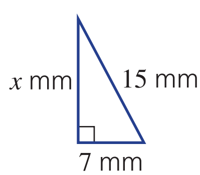

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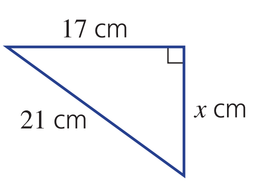

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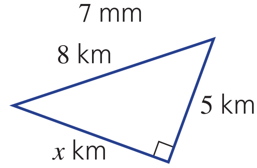

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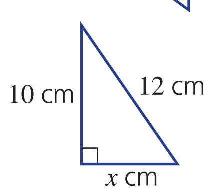

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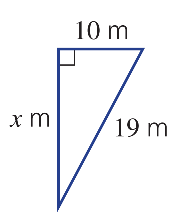
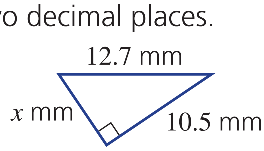


Hint: To round to one decimal place, look at the second decimal place. If it is 5 or more, round up. If it is 4 or less, round down. For example, 7.1 (4) 14… rounds to 7.1.

6 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to two decimal places.

a



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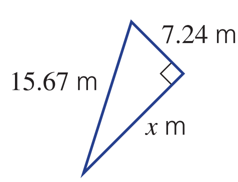

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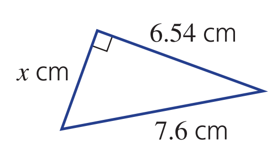

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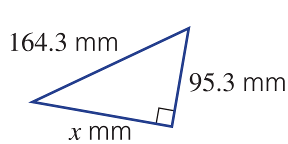

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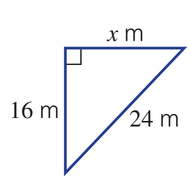

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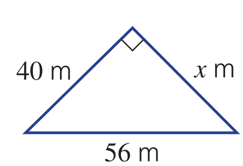

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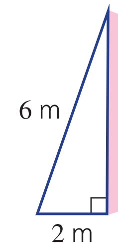
9B

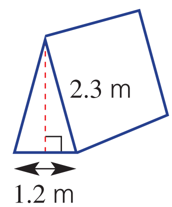
Problem-solving and reasoning

7–9 | 9–11

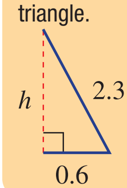
7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to two decimal places.



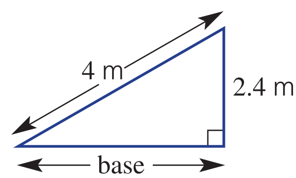
8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to one decimal place.



Hint: Identify the right-angled triangle.



9 A city council wants to build a skateboard ramp measuring 4 m long and 2.4 m high. How long should the base of the ramp be, correct to one decimal place?



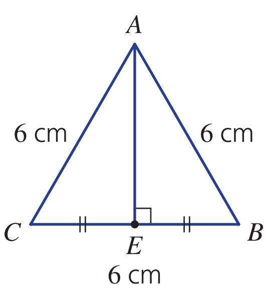
10 Triangle ABC is equilateral. AE is an axis of symmetry.

a Find the length of:

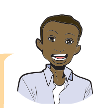
i EB

ii AE, to one decimal place

b Find the area of triangle ABC, to one decimal place.

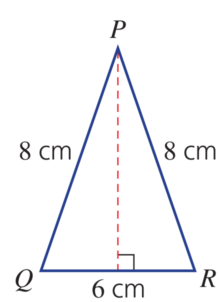


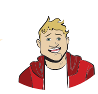
Hint: An equilateral triangle has 3 equal sides.



Hint: Remember: $A = \frac{1}{2}bh$ is the area of a triangle.

11 What is the height of this isosceles triangle, to one decimal place?





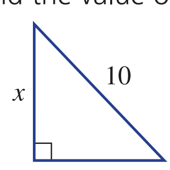
Hint: Pythagoras' theorem applies only to right-angled triangles.

More than one pronumeral

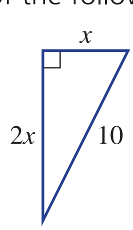
— | 12

12 Find the value of x in each of the following. Answer to one decimal place.

a



b



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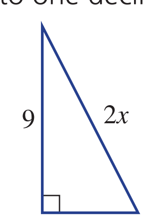

Hint: Remember to square the entire side. The square of $2x$ is $(2x)^2$ or $4x^2$.

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9C Applications of Pythagoras’ theorem

Learning intentions

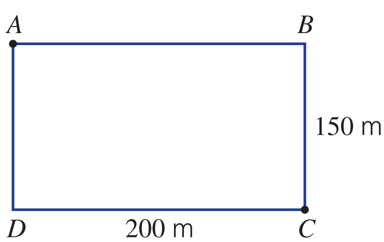
- To be able to identify right-angled triangles in simple applications
- To be able to apply Pythagoras’ theorem in a real situation to find an unknown length

Key vocabulary: identify, pronumeral

Pythagoras’ theorem has many applications, some of which you may have noticed already in this chapter. Some areas where Pythagoras’ theorem is useful include drafting, building and navigation.

Lesson starter: Finding the shortest path

A rectangular field is 200 m by 150 m. Marco wants to walk from the corner of the field marked A to the corner of the field marked C. How many metres are saved by walking along the diagonal AC rather than walking along AB and then BC?



Key ideas

- When applying Pythagoras’ theorem follow these steps.
  - Identify and draw the right-angled triangle or triangles you need in order to solve the problem.
  - Label the triangle and place a pronumeral (letter) on the side length that is unknown.
  - Use Pythagoras’ theorem to find the value of the pronumeral.
  - Answer the question. (Written questions should have written answers.)

Exercise 9C

Understanding	1–3	3

Hint: Each one involves a right-angled triangle.

1 Draw a diagram for each of the following questions. You don’t need to answer the question.

  a A 2.4 m ladder is placed 1 m from the foot of a building. How far up the building will the ladder reach?

  b The diagonal of a rectangle with length 18 cm is 24 cm. How wide is the rectangle?

  c Sebastian walks 5 km north, then 3 km west. How far is he from his starting point?

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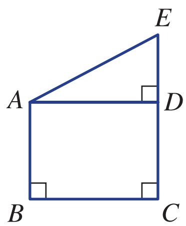
9C

2 Name the right-angled triangles in each of the following diagrams; e.g. $\triangle ABC$.

a



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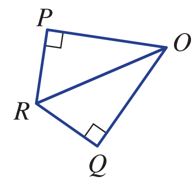

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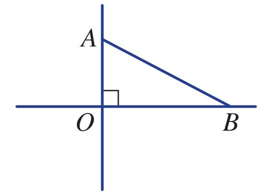
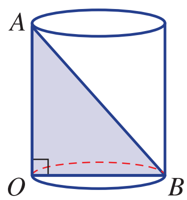


3 Name the hypotenuse in each of the shaded right-angled triangles found within these three-dimensional shapes; e.g. FG.

a



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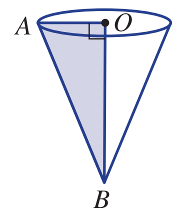

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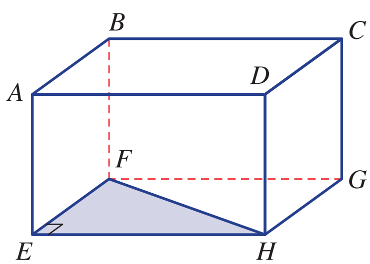

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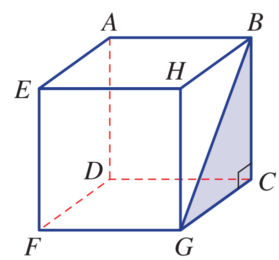
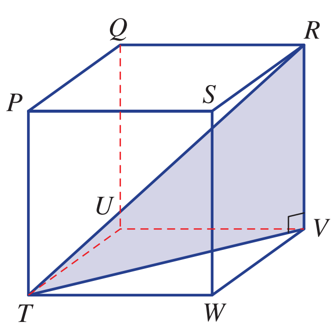


e



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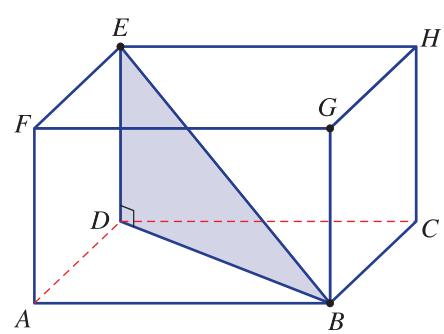
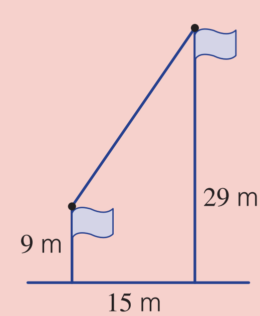


Fluency | 4–7 | 4, 5, 7, 8

Example 5 Applying Pythagoras' theorem

Two flag poles are 15 m apart and a rope links the tops of both poles. Find the length of the rope if one flag pole is 9 m tall and the other is 29 m tall.



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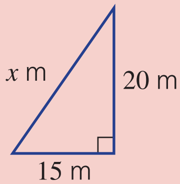
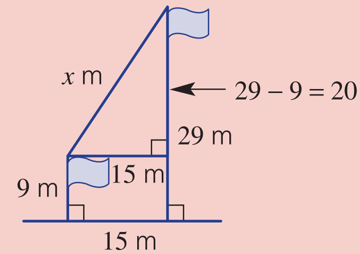
Solution	**Explanation**
Let x metres be the length of rope.	Locate and draw the right-angled triangle, showing all measurements. Introduce a pronumeral for the missing side.





$$x^2 = 15^2 + 20^2$$
$$= 225 + 400$$
$$= 625$$
$$x = \sqrt{625}$$
$$= 25$$

Write the relationship, using Pythagoras' theorem.

Simplify.

Take the square root to find x.

The rope is 25 m long. — Answer the question.

Now you try

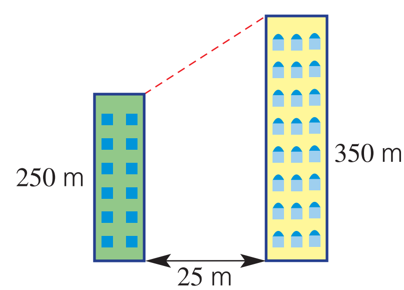
Two vertical poles are 2 m and 5 m high and are 4 m apart. Find the distance between the top of the two poles.

4 Two skyscrapers are 25 m apart and a cable runs from the top of one building to the top of the other. One building is 350 m tall and the other is 250 m.

a Determine the difference in the heights of the buildings.

b Draw an appropriate right-angled triangle you could use to find the length of the cable.

c Find the length of the cable, correct to two decimal places.

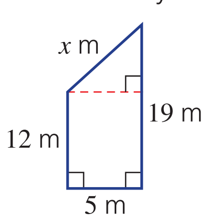


(Not to scale)

5 Find the value of x in each of the following, correct to one decimal place where necessary.

Hint: Label the two known lengths of each triangle first.

a



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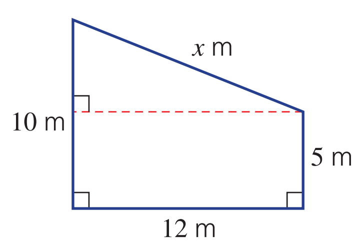

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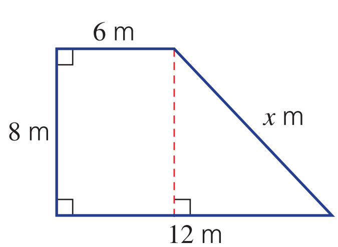

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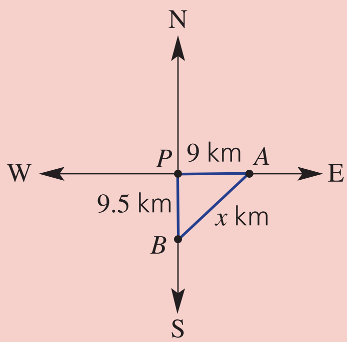
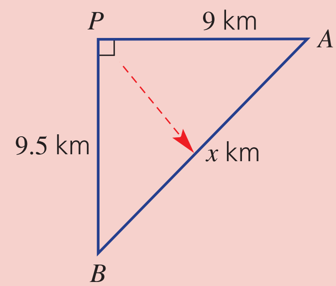
9C

Example 6 Using direction with Pythagoras' theorem

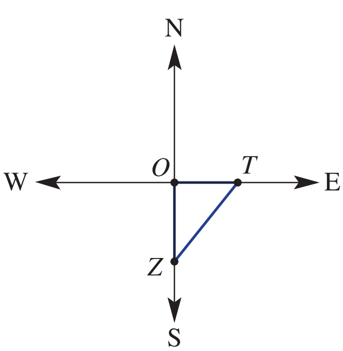
Two hikers leave their camp (P) at the same time. One walks due east for 9 km; the other walks due south for 9.5 km. How far apart are the two hikers at this point? (Answer to one decimal place.)

Solution	Explanation
	Draw a diagram. Consider $\triangle PAB$. 
$\therefore x^2 = 9^2 + 9.5^2$	Write Pythagoras' theorem and evaluate.
$x^2 = 171.25$	Square root to find x.
$x = \sqrt{171.25}$	
$= 13.086$	
$= 13.1$ (to 1 d.p.)	Round to one decimal place.
$\therefore$ The hikers are 13.1 km apart.	Answer the question in words.

Now you try

From a floating oil rig, one ship travels due north for 20 km and another travels due west for 30 km. How far apart are the two ships at this point? (Answer to one decimal place.)

6 Tranh (T) walks 4.5 km east while Zara (Z) walks 5.2 km south. How far from Tranh is Zara? Answer to one decimal place.



7 Find the distance between Sui and Kevin if:

a Sui walks 6 km north from camp O and Kevin walks 8 km west from camp O.

b Sui walks 40 km east from point A and Kevin walks 9 km south from point A.

c Kevin walks 15 km north-west from O and Sui walks 8 km south-west also from O.

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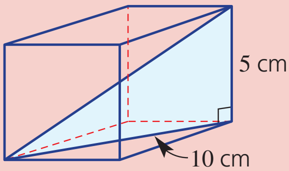
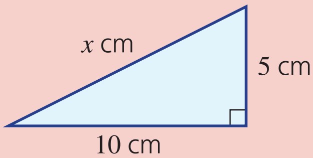
Example 7 Using Pythagoras' theorem in 3D

Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



Solution



$x^2 = 5^2 + 10^2$
$= 25 + 100$
$x = \sqrt{125}$
$= 11.18$ cm (to 2 d.p.)

∴ The distance between the opposite corners is 11.18 cm.

Explanation

Draw the triangle you need and mark the lengths.

Write the relationship for this triangle.

Simplify.

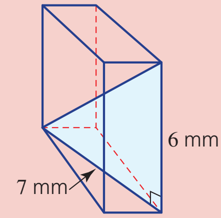
Take the square root to find x.

Round your answer to two decimal places.

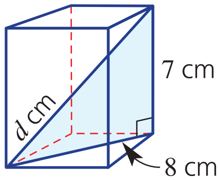
Write the answer.

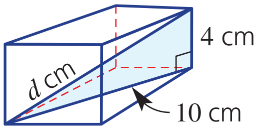
Now you try

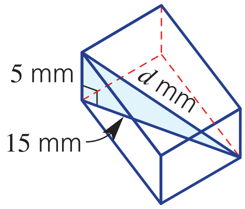
Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.

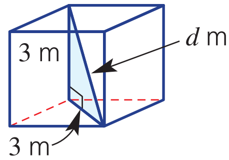


8 Find the distance of d from one corner to the opposite corner in the following rectangular prisms, correct to one decimal place.

a 

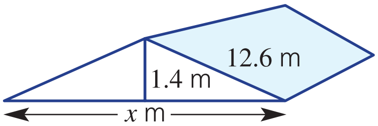
b 

c 

d 

Problem-solving and reasoning | 9–11 | 11–14

9 The height of a roof is 1.4 m. If the length of a gable (the diagonal) is to be 12.6 m, determine the length of the horizontal beam needed to support the roof, correct to two decimal places.



Hint: Find the base length of the right-angled triangle first.

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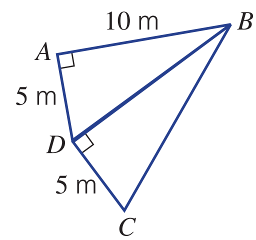
9C

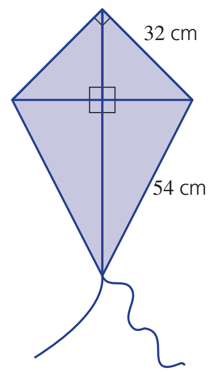
10 For the diagram shown, find the lengths of:

a BD, correct to two decimal places

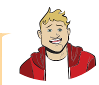
b BC, correct to one decimal place



11 A kite is constructed with six pieces of wooden dowel and covered in fabric. The four pieces around the edge have two 32 cm rods and two 54 cm rods. If the top of the kite is right angled, find the length of the horizontal and vertical rods, correct to two decimal places.

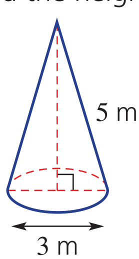


Hint: Find the length of the horizontal rod first. What type of triangle is the top of the kite? Find the length of the vertical rod using two calculations.

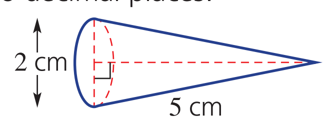


12 Find the height of the following cones, correct to two decimal places.

a

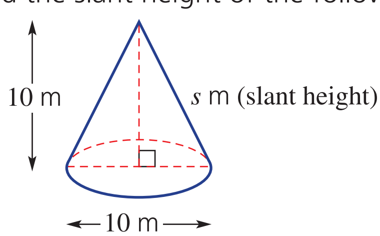


b



13 Find the slant height of the following, correct to one decimal place.

a



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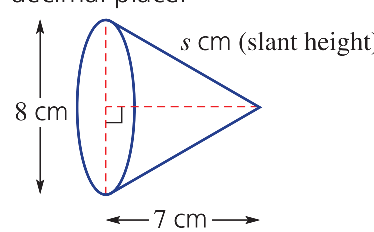
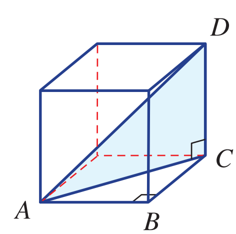


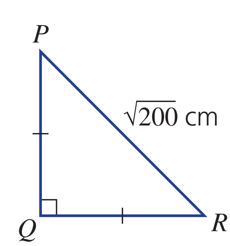
14 This cube has 1 cm sides. Find, correct to two decimal places, the lengths of:

a AC b AD



How much do you know? — 15

15 Write down everything you know about $\triangle PQR$, including the things that you can calculate.



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9D Trigonometric ratios CONSOLIDATING

Learning intentions

- To know the three trigonometric ratios for a right-angled triangle
- To be able to write down the ratio for sine, cosine and tangent for a triangle with given side lengths

Key vocabulary: trigonometry, sine, cosine, tangent, hypotenuse, opposite, adjacent, angle of reference

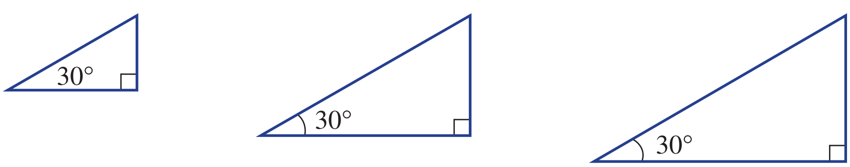
Trigonometry deals with the relationship between the sides and angles of triangles.

In this section we look at the relationship between right-angled triangles and the three trigonometric ratios: sine (sin), cosine (cos) and tangent (tan).

Using your calculator and knowing how to label the sides of right-angled triangles, you can use trigonometry to find missing sides and angles.

Lesson starter: Thirty degrees

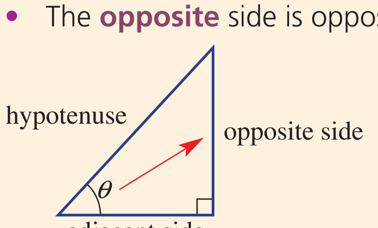
- Draw three different right-angled triangles that each have a 30° angle.

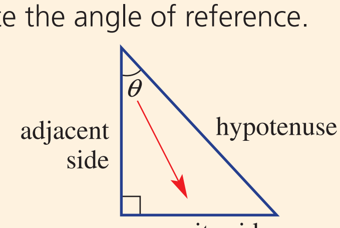


- Measure each side of each triangle and add these measurements to your diagrams.
- The hypotenuse, as we know, is opposite the right angle. The side opposite the 30° is called the opposite side. For each of your three triangles, write down the ratio of the opposite side divided by the hypotenuse. What do you notice?
- Type '$\sin 30^\circ$' into your calculator. What do you notice?

Key ideas

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The **angle of reference** is the angle in a right-angled triangle that is used to determine the opposite side and the adjacent side.
  - The **hypotenuse** is always opposite the right angle.
  - The **adjacent** side is next to the angle of reference.
  - The **opposite** side is opposite the angle of reference.





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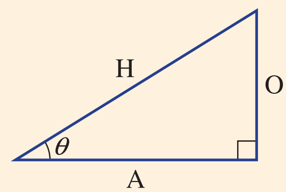
9D

- For a right-angled triangle with a given angle θ (theta), the three trigonometric ratios of **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine of angle θ: $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ: $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ: $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$
- When working with right-angled triangles, label each side of the triangle O (opposite), A (adjacent) and H (hypotenuse).

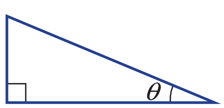


- The three trigonometric ratios are:
  $\sin\theta = \dfrac{O}{H}$ $\cos\theta = \dfrac{A}{H}$ $\tan\theta = \dfrac{O}{A}$
  We can remember this as **SOH CAH TOA**.

Exercise 9D

Understanding 1–4 4

1 By referring to the angles marked, copy each triangle and label the sides opposite, adjacent and hypotenuse, using O, A and H.

a


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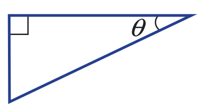

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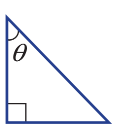

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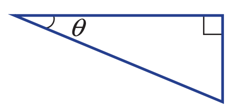

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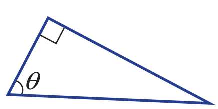

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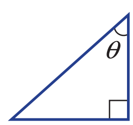
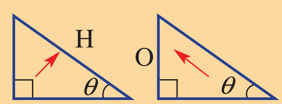
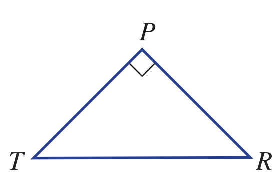


Hint: Arrows help you find the hypotenuse and the opposite side:

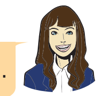


2 Referring to triangle PTR, name the:

a side opposite the angle at T
b side adjacent to the angle at T
c side opposite the angle at R
d side adjacent to the angle at R
e hypotenuse
f angle opposite the side PR



Hint: 'Adjacent' means 'next to'.



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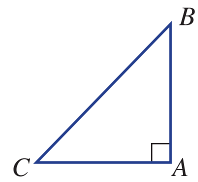
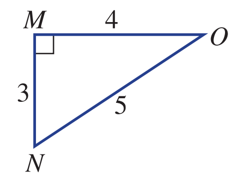
3 Referring to triangle ABC, name the:
  a hypotenuse
  b side opposite the angle at B
  c side opposite the angle at C
  d side adjacent to the angle at B



4 In triangle MNO, write the ratio (i.e. fraction) of:
  a $\dfrac{\text{the side oposite angle } O}{\text{hypotenuse}}$
  b $\dfrac{\text{the side opposite angle } N}{\text{hypotenuse}}$
  c $\dfrac{\text{the side adjacent angle } O}{\text{hypotenuse}}$

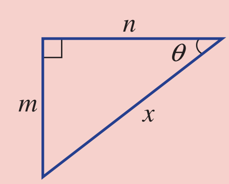


Fluency | 5–6(½) | 5–6(½)

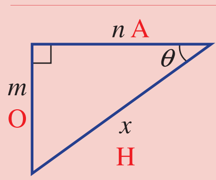
Example 8 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ b $\cos\theta$ c $\tan\theta$



Solution



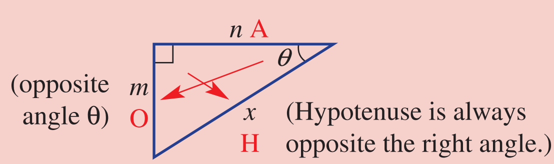
a $\sin\theta = \dfrac{m}{x}$

b $\cos\theta = \dfrac{n}{x}$

c $\tan\theta = \dfrac{m}{n}$

Explanation

Use arrows to label the sides correctly.



SOH CAH TOA

$\sin\theta = \dfrac{\text{O}}{\text{H}} = \dfrac{m}{x}$

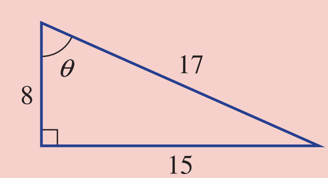
$\cos\theta = \dfrac{\text{A}}{\text{H}} = \dfrac{n}{x}$

$\tan\theta = \dfrac{\text{O}}{\text{A}} = \dfrac{m}{n}$

Now you try

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ b $\cos\theta$ c $\tan\theta$



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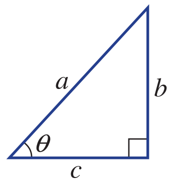
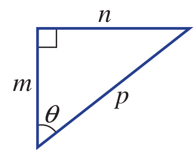
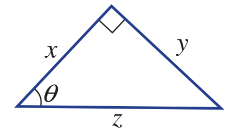
9D

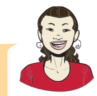
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

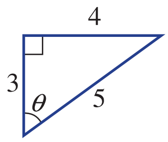
a


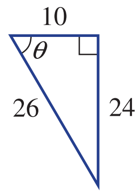
b


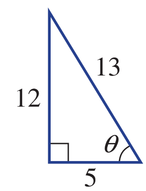
c




Hint: Use SOH CAH TOA after labelling the sides as O, A and H.

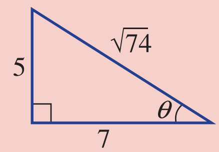
d


e


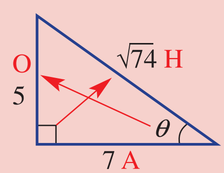
f


Example 9 Writing a trigonometric ratio

Write down the ratio of $\cos\theta$ for this triangle.



Solution	**Explanation**
$\cos\theta = \frac{A}{H}$	Label the sides of the triangle.
$\cos\theta = \frac{7}{\sqrt{74}}$	

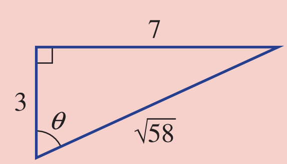


SOH CAH TOA tells us $\cos\theta$ is $\frac{\text{adjacent}}{\text{hypotenuse}}$.

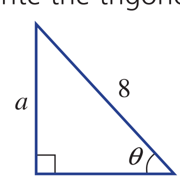
Substitute the values for the adjacent (A) and hypotenuse (H).

Now you try

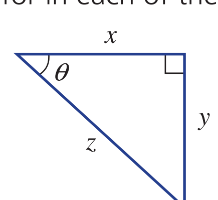
Write down the ratio of $\sin\theta$ for this triangle.



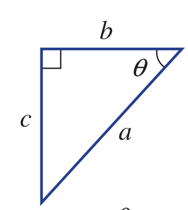
6 Write the trigonometric ratio asked for in each of the following.

a


$\tan\theta =$

b


$\sin\theta =$

c


$\cos\theta =$

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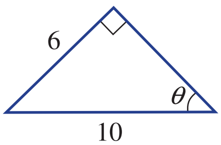
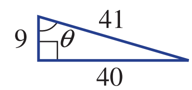
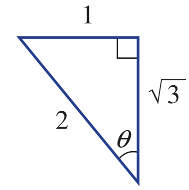
d 

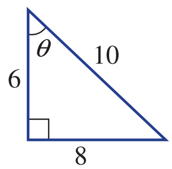
sin $\theta =$

e 

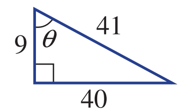
sin $\theta =$

f 

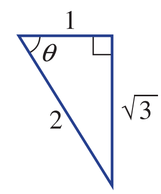
tan $\theta =$

g 

tan $\theta =$

h 

cos $\theta =$

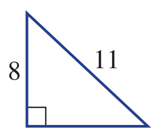
i 

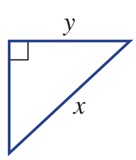
tan $\theta =$

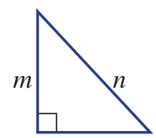
Problem-solving and reasoning

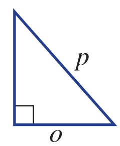
7–9 | 8-11

7 Copy each of these triangles and mark the angle θ that will enable you to write a ratio for $\sin\theta$.

a 

b 

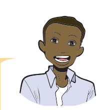
c 

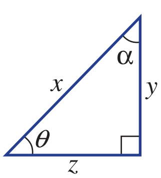
d 

8 For the triangle shown, write a ratio for:

a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$
d $\cos\alpha$ **e** $\tan\theta$ **f** $\tan\alpha$

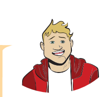
Hint: θ and α are letters of the Greek alphabet that are used to mark angles.

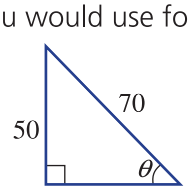


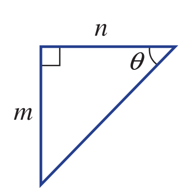


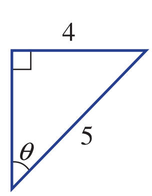
9 For each of the triangles below, decide which trigonometric ratio (sin, cos or tan) you would use for the given information.

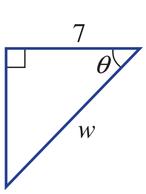
Hint: First decide which two sides are involved.

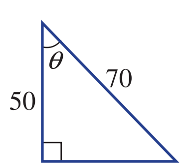


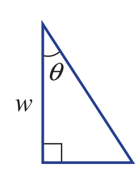
a 

b 

c 

d 

e 

f 

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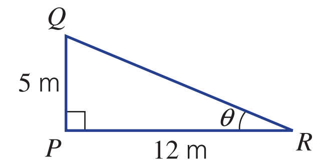
9D

10 Consider the triangle PQR.



a Use Pythagoras' theorem to find the length of QR.

b Write down the ratio of $\sin\theta$.

11 For a given right-angled triangle, $\sin\theta = \frac{1}{2}$.

a Draw a right-angled triangle and show this information.

b What is the length of the third side? Use Pythagoras' theorem.

c Find the value of:

i $\cos\theta$ ii $\tan\theta$

Relationship between sine and cosine — 12

12 Use your calculator to complete the table, answering to three decimal places where necessary.

a For what angle is $\sin\theta = \cos\theta$?

b Copy and complete:

i $\sin 5° = \cos$ ____°

ii $\sin 10° = \cos$ ____°

iii $\sin 60° = \cos$ ____°

iv $\sin 90° = \cos$ ____°

c Write down a relationship, in words, between sin and cos.

d Why do you think it's called cosine?

Angle (θ)	sin θ	cos θ
0°		
5°		
10°		
15°		
20°		
25°		
30°		
35°		
40°		
45°		
50°		
55°		
60°		
65°		
70°		
75°		
80°		
85°		
90°		

Hint: For most calculators, you enter the values in the same order as they are written. That is, $\sin 30° \rightarrow$ [sin] $30 = 0.5$.

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9E Finding side lengths

Learning intentions

- To be able to identify which trigonometric ratio can be used to set up an equation
- To be able to find a missing length on a right-angled triangle given an angle and another side

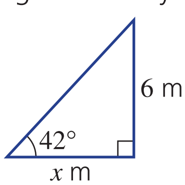
Key vocabulary: sine, cosine, tangent, hypotenuse, opposite, adjacent

In any right-angled triangle, given one of the acute angles and a side length, you can find the length of the other two sides. This can help builders find special lengths in right-angled triangles if they know an angle and the length of another side.

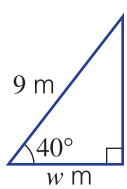
Lesson starter: Is it sin, cos or tan?

Of the six triangles below, only two provide enough information to directly use the sine ratio. Which two triangles are they?

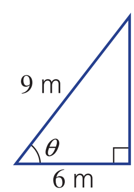
A



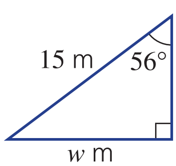
B



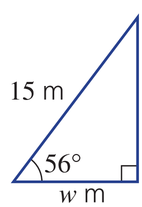
C



D



E



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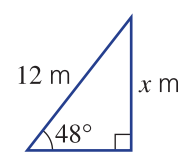
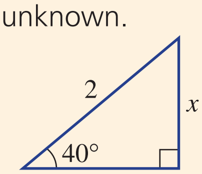


Key ideas

- To find a missing side when given a right-angled triangle with one acute angle and one of the sides:
  - Label the triangle using O (opposite), A (adjacent) and H (hypotenuse).
  - Use SOH CAH TOA to decide on the correct trigonometric ratio.
  - Write down the equation, using your chosen ratio.
  - Solve the equation, using your calculator, to find the unknown.

Write $\rightarrow \dfrac{x}{2} = \sin 40°$

Solve $\rightarrow x = 2 \times \sin 40°$



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9E

Exercise 9E

Understanding — 1–2(½), 3 — 2(½), 3

1 Use a calculator to find the value of each of the following, correct to four decimal places.

a $\sin 10°$ **b** $\cos 10°$ **c** $\tan 10°$
d $\tan 30°$ **e** $\cos 40°$ **f** $\sin 70°$
g $\cos 80°$ **h** $\tan 40°$ **i** $\sin 80°$

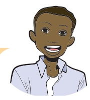
Hint: Locate the sin, cos and tan buttons on your calculator.

2 Evaluate each of the following, correct to two decimal places.

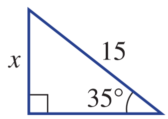
a $12 \tan 10°$ **b** $12 \sin 25°$ **c** $18 \tan 60°$
d $56 \sin 56°$ **e** $8 \tan 45°$ **f** $20 \sin 70°$

Hint: On your calculator, enter $12 \tan 10°$ as $12 \times \tan 10$.

3 Decide which of the three trigonometric ratios (sin, cos or tan) is suitable for these triangles. Do not solve.



Hint: Remember to label the triangle and think SOH CAH TOA. Consider which two sides are involved.

a


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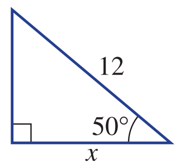
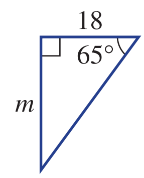
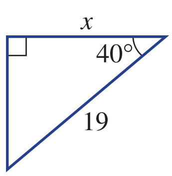
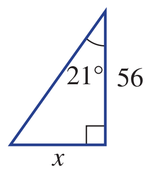
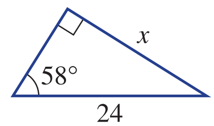


c


d


e


f


Fluency — 4–8(½) — 4–9(½)

Example 10 Solving a trigonometric equation

Find the value of x, correct to two decimal places, for $\cos 30° = \frac{x}{12}$.

Solution	Explanation
$\cos 30° = \frac{x}{12}$	Multiply both sides by 12 to get x on its own.
$x = 12 \times \cos 30°$	$12 \times \cos 30° = \frac{x}{\cancel{12}_1} \times \cancel{12}_1$
$= 10.39230\ldots$	Use your calculator.
$= 10.39$ (to 2 d.p.)	Round as indicated.

Now you try

Find the value of x, correct to two decimal places, for $\sin 40° = \frac{x}{6}$.

4 Find the value of x in these equations, correct to two decimal places.

a $\sin 20° = \frac{x}{4}$ **b** $\cos 43° = \frac{x}{7}$ **c** $\tan 85° = \frac{x}{8}$
d $\tan 30° = \frac{x}{24}$ **e** $\sin 50° = \frac{x}{12}$ **f** $\cos 40° = \frac{x}{12}$

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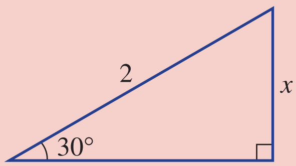
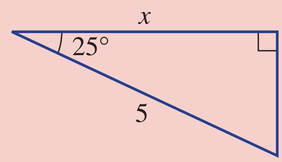
Example 11 Finding a missing side, using SOH CAH TOA

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

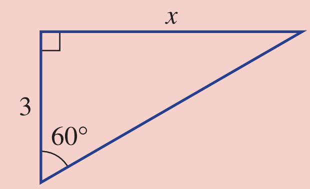
a



b



c



Solution | **Explanation**

a

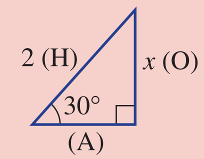
$$\sin \theta = \frac{\text{O}}{\text{H}}$$

$$\sin 30° = \frac{x}{2}$$

$$2 \times \sin 30° = x$$

$$\therefore\ x = 1$$

Label the triangle and decide on your trigonometric ratio SOH CAH TOA.



Write the ratio.
Substitute values and solve the equation, using your calculator.

b

$$\cos \theta = \frac{\text{A}}{\text{H}}$$

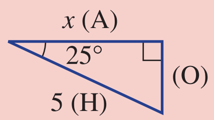
$$\cos 25° = \frac{x}{5}$$

$$5 \times \cos 25° = x$$

$$x = 4.5315\ldots$$

$$\therefore\ x = 4.53 \text{ (to 2 d.p.)}$$

Label the triangle.
SOH CAH TOA



Write the ratio.
Substitute values and solve the equation, using your calculator.

Round to two decimal places.

c

$$\tan \theta = \frac{\text{O}}{\text{A}}$$

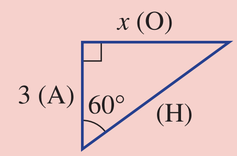
$$\tan 60° = \frac{x}{3}$$

$$3 \times \tan 60° = x$$

$$x = 5.1961\ldots$$

$$\therefore\ x = 5.20 \text{ (to 2 d.p.)}$$

Label the triangle.
SOH CAH TOA



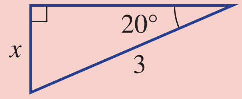
Write the ratio.
Substitute values and solve the equation, using your calculator.

Round to two decimal places.

Now you try

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

a



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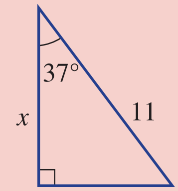

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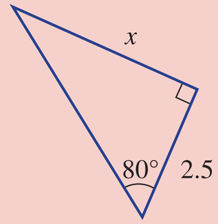

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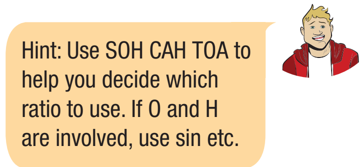
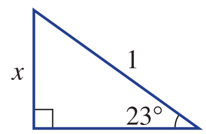
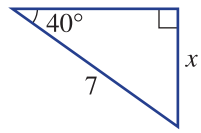
9E

5 Triangles with one unknown side are given below.

- **i** Copy each one and label the three sides opposite (O), adjacent (A) and hypotenuse (H).
- **ii** Decide on a trigonometric ratio.
- **iii** Find the value of each pronumeral, correct to two decimal places.



a


b


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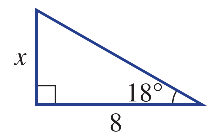
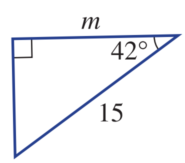


d


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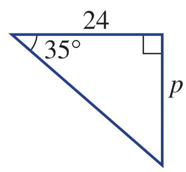

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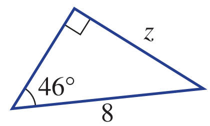
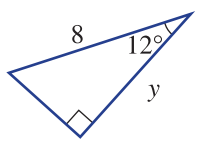
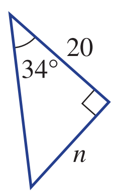
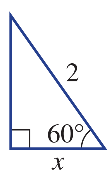
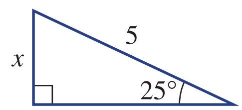


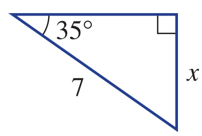
g


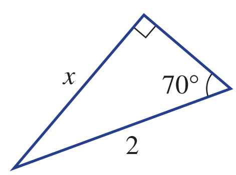
h


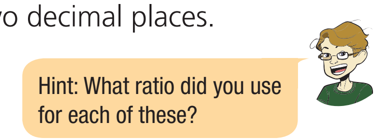
i


6 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

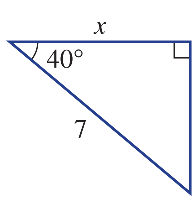
a


b


c




7 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

a


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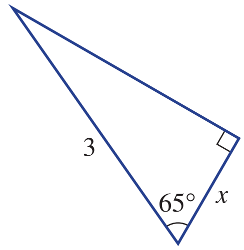

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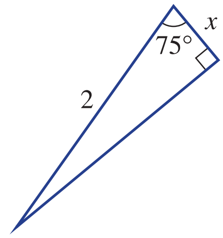
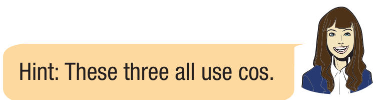
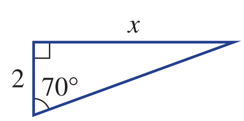




8 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

a


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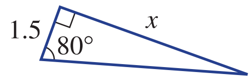

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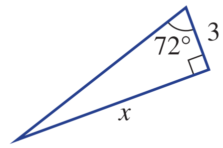

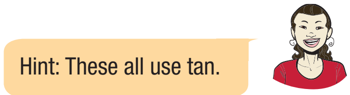

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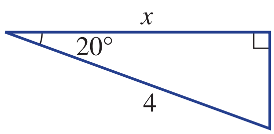
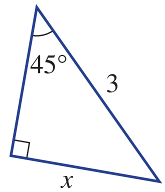
9 Decide whether to use sin, cos or tan, then find the value of x in these triangles. Round to two decimal places.

a



b



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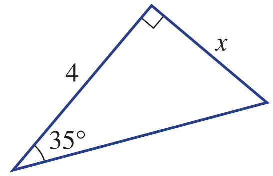
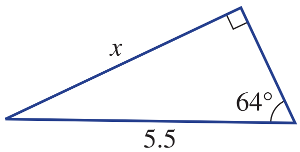
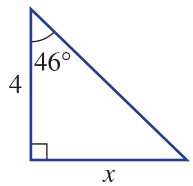


d



e



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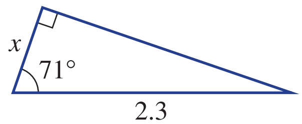
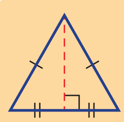


Problem-solving and reasoning

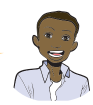
	10, 11	10, 12

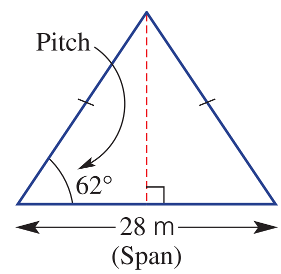
10 **a** Find the height of this isosceles triangle, which is similar to a roof truss, to two decimal places.

b If the span doubles to 56 m, what is the height of the roof, to two decimal places?

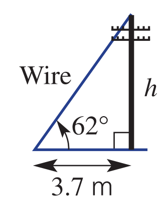


Hint: In an isosceles triangle, the perpendicular cuts the base in half.



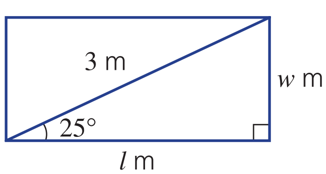


11 The stay wire of a power pole joins the top to the ground. It makes an angle of 62° with the ground. It is fixed to the ground 3.7 m from the bottom of the pole. How high is the pole, correct to two decimal places?

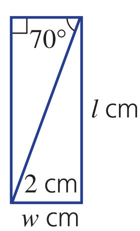


12 Find the length and width of these rectangles, to two decimal places.

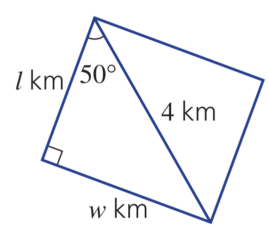
a

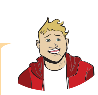


b



c





Hint: Use the hypotenuse in each calculation.

Accuracy and errors

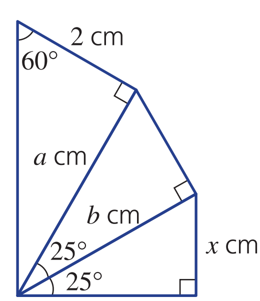
	—	13

13 Our aim is to find the value of x, correct to two decimal places, by first finding the value of a and b.

a Find the value of a, then b and then x, using one decimal place for a and b.

b Repeat this process, finding a and b, correct to three decimal places each, before finding x.

c Does it make any difference to your final answer for x if you round off the values of a and b during calculations?



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9F Solving for the denominator ★

Learning intentions

- To be able to identify which trigonometric ratio can be used to set up an equation
- To be able to find a missing length on a right-angled triangle if the variable sits in the denominator of the equation

Key vocabulary: denominator, hypotenuse, opposite, adjacent

So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator and these problems can be solved with an extra step in your mathematical working.

Lesson starter: Solving equations with x in the denominator

Consider the equations $\frac{x}{3} = 4$ and $\frac{3}{x} = 4$.

- Do the equations have the same solution?
- What steps are used to solve the equations?
- Now solve $\frac{4}{x} = \sin 30°$ and $\frac{2}{x} = \cos 40°$.

Key ideas

- When the unknown value is in the **denominator**, you need to do two algebraic steps to find the unknown. For example,

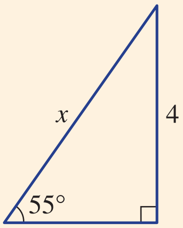
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55° = \frac{4}{x}$$

$$x \times \sin 55° = 4$$

$$x = \frac{4}{\sin 55°}$$

$$= 4.88 \text{ (to two decimal places)}$$



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Exercise 9F

Understanding 1–3 2, 3

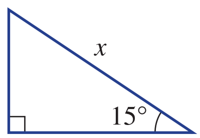
1 Find the value, correct to two decimal places, of:

a $\frac{10}{\tan 30^\circ}$ **b** $\frac{12}{\sin 60^\circ}$ **c** $\frac{15}{\tan 8^\circ}$

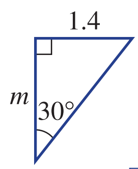
d $\frac{12.4}{\tan 32^\circ}$ **e** $\frac{15.2}{\sin 38^\circ}$ **f** $\frac{9}{\cos 47^\circ}$

Hint: For part **a**, enter $10 \div \tan 30$ into your calculator.

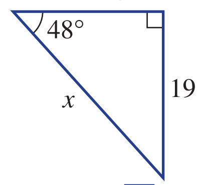
2 For each of these triangles, complete the required trigonometric ratio.

a


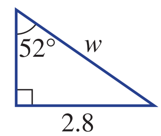
$\cos 15^\circ = \frac{\square}{\square}$

b


$\tan 30^\circ = \frac{\square}{\square}$

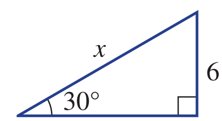
c


$\sin 48^\circ = \frac{\square}{\square}$

d


$\sin 52^\circ = \frac{\square}{\square}$

3 Which one of the following is the correct working to solve for x in this triangle?



A
$$\cos 30^\circ = \frac{6}{x}$$
$$x \cos 30^\circ = 6$$
$$x = \frac{6}{\cos 30^\circ}$$
$$= 6.93$$

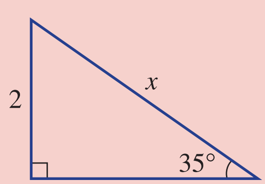
B
$$\sin 30^\circ = \frac{6}{x}$$
$$x = 6 \times \sin 30^\circ$$
$$= 3$$

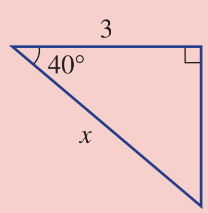
C
$$\sin 30^\circ = \frac{6}{x}$$
$$x \sin 30^\circ = 6$$
$$x = \frac{6}{\sin 30^\circ}$$
$$= 12$$

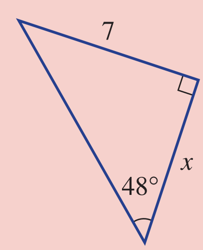
Fluency 4–6 4–6, 7(½)

Example 12 Finding the value of the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

a


b


c


Solution	Explanation
a $\sin 35^\circ = \frac{2}{x}$	Use $\sin\theta = \frac{\text{O}}{\text{H}}$, as we can use the opposite (2) and hypotenuse (x).
$x \times \sin 35^\circ = 2$	Multiply both sides by x.
$x = \frac{2}{\sin 35^\circ}$	Divide both sides by $\sin 35^\circ$ to get x on its own. Recall that $\sin 35^\circ$ is just a number.
$\therefore\ x = 3.49$ (to 2 d.p.)	Evaluate and round your answer.

Continued on next page

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9F

b
$$\cos 40^\circ = \frac{3}{x}$$
$$x \times \cos 40^\circ = 3$$
$$x = \frac{3}{\cos 40^\circ}$$
$$\therefore\ x = 3.92 \text{ (to 2 d.p.)}$$

Use $\cos\theta = \frac{A}{H}$, as we can use the adjacent (3) and hypotenuse (x).
Multiply both sides by x.
Divide both sides by $\cos\ 40^\circ$ to get x on its own.

Evaluate and round your answer.

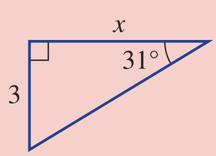
c
$$\tan 48^\circ = \frac{7}{x}$$
$$x \times \tan 48^\circ = 7$$
$$x = \frac{7}{\tan 48^\circ}$$
$$\therefore\ x = 6.30 \text{ (to 2 d.p.)}$$

Use $\tan\theta = \frac{O}{A}$, as we can use the adjacent (x) and opposite (7).
Multiply both sides by x.
Divide both sides by $\tan 48^\circ$ to get x on its own.

Evaluate and round your answer.

Now you try

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

a


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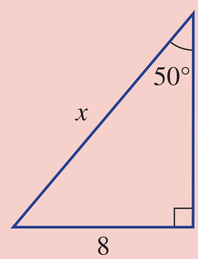

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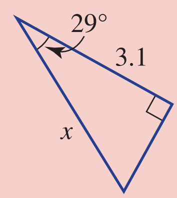
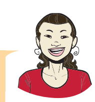
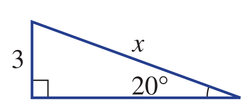


4 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.



Hint: In $\sin 20^\circ = \frac{3}{x}$, multiply both sides by x first.

a


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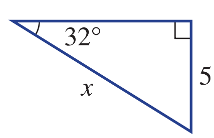

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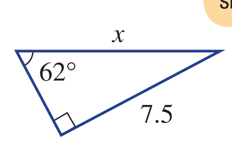
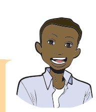
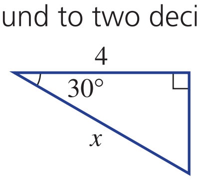


5 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.



Hint: $\cos\theta = \frac{A}{H}$

a


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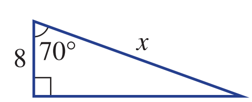

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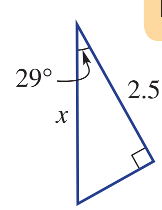

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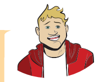
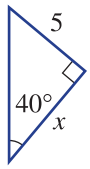
6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

Hint: $\tan\theta = \frac{O}{A}$



a



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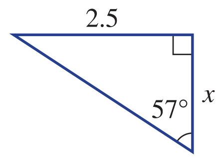

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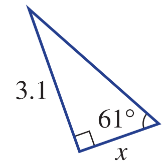
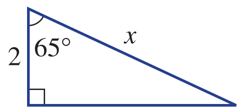


7 By first deciding whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$, find the value of x in these triangles. Round to two decimal places.

Hint: SOH CAH TOA

a



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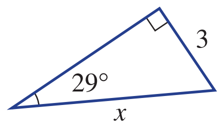

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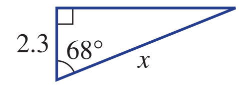

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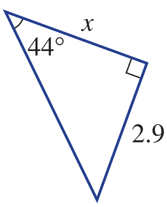

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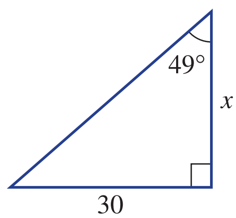

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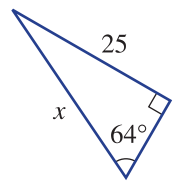
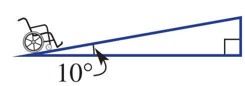


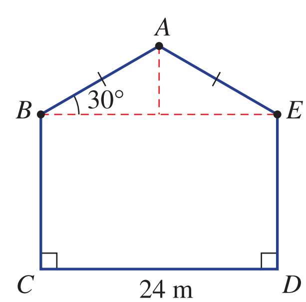
Problem-solving and reasoning

8–10	9–12

8 The recommended angle of a wheelchair ramp to the horizontal is approximately 10°. How long is the ramp if the horizontal distance is 2.5 metres? Round your answer to two decimal places.

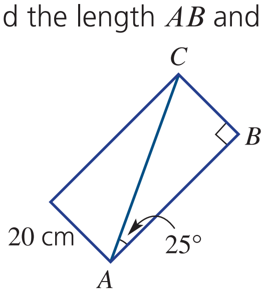


9 The roof of this barn has a pitch of 30°, as shown. Find the length of roof section AB, to one decimal place.

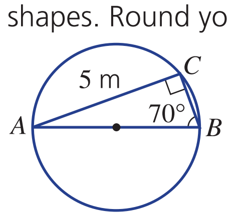


10 Find the length AB and BC in these shapes. Round your answers to two decimal places.

a



b



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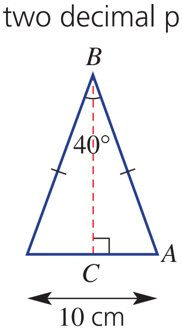

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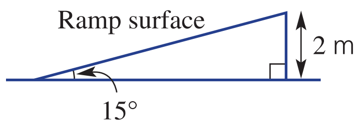
9F

11 The ramp shown has an incline angle of 15° and a height of 2 m. Find, correct to three decimal places:

a the base length of the ramp

b the length of the ramp surface

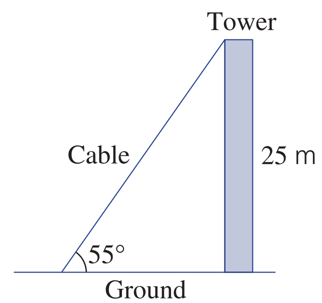
Hint: The 'incline' is the angle to the horizontal.



12 For this communications tower, find, correct to one decimal place:

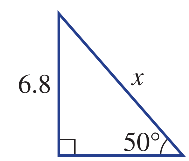
a the length of the cable

b the distance from the base of the tower to the point where the cable is attached to the ground



Inverting the fraction — 13

Shown below is another way of solving trigonometric equations with x in the denominator.

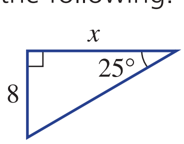


Find x, to two decimal places.

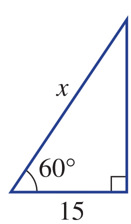
$\sin 50^\circ = \dfrac{6.8}{x}$	First, write the correct ratio.
$\dfrac{1}{\sin 50^\circ} = \dfrac{x}{6.8}$	Invert both fractions so x is in the numerator.
$x = \dfrac{1}{\sin 50^\circ} \times 6.8$	$\dfrac{1}{\sin 50^\circ} \times 6.8 = \dfrac{x}{6.8} \times 6.8$
$x = \dfrac{6.8}{\sin 50^\circ}$	Multiply both sides by 6.8 to get x on its own.
$x = 8.876769\ldots$	Use your calculator.
$x = 8.88$ (to 2 d.p.)	Round your answer as required.

13 Use the method shown above to find the value of x, to two decimal places where necessary, in each of the following.

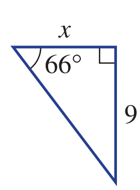
a



b



c



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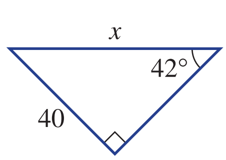
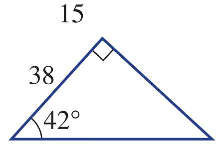


e



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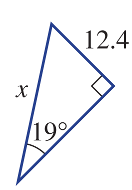

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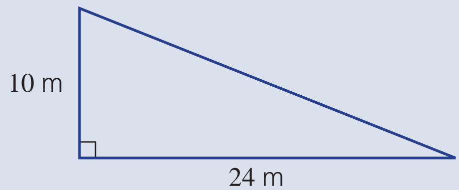
Progress quiz

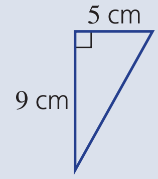
9A

1 Find the length of the hypotenuse in the following right-angled triangles. Round to one decimal place in part **b**.

a



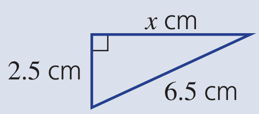
b



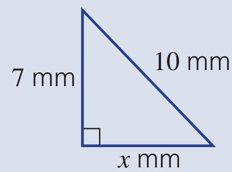
9B

2 Find the value of the pronumeral in the following triangles. Round to one decimal place in part **b**.

a

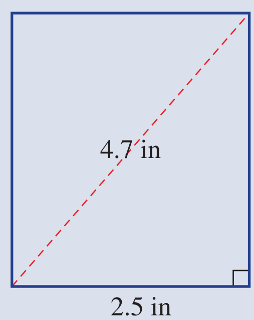


b



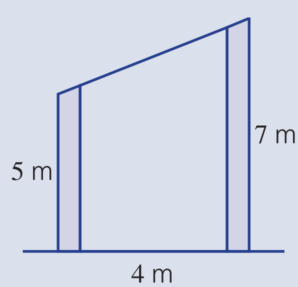
9B

3 A phone has a screen size of 4.7 inches (i.e. this is the distance from corner to corner). If the screen has a width of 2.5 inches, what is the screen height, correct to one decimal place?



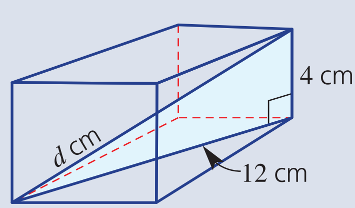
9C

4 A straight wire connects the tops of two telegraph poles. The poles are 7 m and 5 m high and are located 4 m apart. What is the length of the wire, correct to two decimal places?



9C

5 Find the distance (d cm) from one corner to the opposite corner of this rectangular prism, correct to one decimal place.



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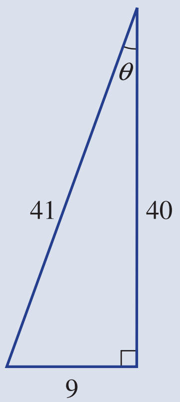
9D

6 For the triangle shown, write down the ratio for:

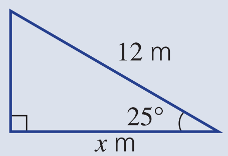
a $\cos \theta$
b $\sin \theta$
c $\tan \theta$



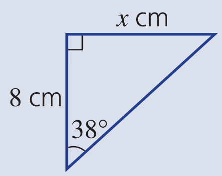
9E

7 Find the value of x in these triangles. Round your answers to one decimal place.

a



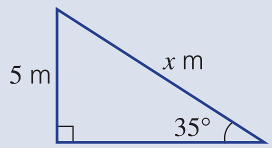
b



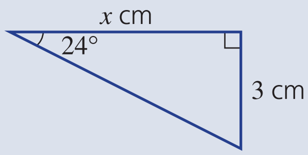
9F

8 Find the value of x in these triangles. Round your answers to one decimal place.

a



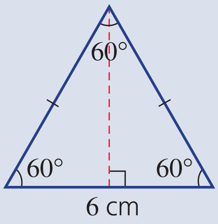
b



9B/F

9 Use each of the following methods to find the area of the equilateral triangle shown. Give your answer to one decimal place.
(Hint: First you will need to find the height of the triangle.)

a Pythagoras' theorem
b Trigonometry



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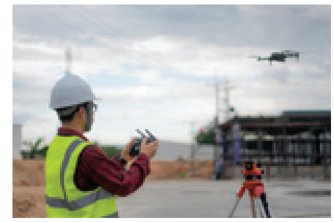
9G Finding angles

Learning intentions

- To know that angles can be found using inverse trigonometric functions
- To be able to find an angle in a right-angled triangle given any two side lengths

Key vocabulary: inverse trigonometric function or ratio

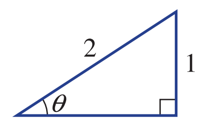
Given two side lengths of a right-angled triangle, you can find either of the acute angles. There are many different situations where you might be given two side lengths of a right-angled triangle and be asked to find the associated angles. Angles associated with flight for example, can be calculated using trigonometry.



Lesson starter: Knowing the angle

Imagine a triangle that produces $\sin\theta = 0.5$.

- Use your calculator and trial and error to find a value of θ for which $\sin\theta = 0.5$.
- Repeat for $\tan\theta = 1$ and $\cos\theta = \frac{\sqrt{3}}{2}$.
- Do you know of a quicker method, rather than using trial and error?



Key ideas

- To find an angle, you use **inverse trigonometric ratios** on your calculator.

  If $\sin\theta = x$, then $\theta = \sin^{-1}(x)$; $\sin^{-1}$ is inverse sin.
  If $\cos\theta = y$, then $\theta = \cos^{-1}(y)$; $\cos^{-1}$ is inverse cos.
  If $\tan\theta = z$, then $\theta = \tan^{-1}(z)$; $\tan^{-1}$ is inverse tan.

- Look for the three calculator buttons/functions:

  $\boxed{\sin^{-1}}$ $\boxed{\cos^{-1}}$ $\boxed{\tan^{-1}}$

Exercise 9G

Understanding	1–3	3

1 Use your calculator to find $\sin 30°$ and $\sin^{-1}(0.5)$. What do you notice?

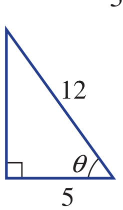
2 Use your calculator to evaluate the following, correct to the nearest whole degree.

a $\sin^{-1}(0.71)$ **b** $\cos^{-1}(0.866)$ **c** $\tan^{-1}(1.6)$

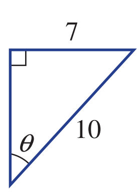
Hint: Many calculators use $\boxed{\text{shift}}$ to access $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$.

3 Write down the trigonometric ratio for these triangles, e.g. $\sin\theta = \frac{2}{3}$.

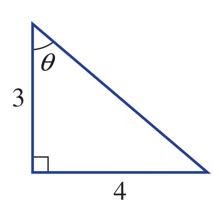
a



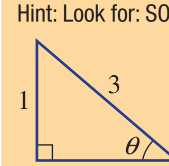
b



c



Hint: Look for: SOH CAH TOA



$\sin\theta = \frac{1}{3}$

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9G

Fluency

4–5(½) 4–6(½)

Example 13 Finding an angle

Find the angle θ, correct to the nearest degree where necessary, in each of the following.

a $\sin\theta = \frac{2}{3}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1.7$

Solution	Explanation
a $\sin\theta = \frac{2}{3}$ $\theta = \sin^{-1}\left(\frac{2}{3}\right)$ $\theta = 42°$ (to nearest degree)	Look for the $\sin^{-1}$ button on your calculator. Round as required.
b $\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta = 60°$	Look for the $\cos^{-1}$ button on your calculator.
c $\tan\theta = 1.7$ $\theta = \tan^{-1}(1.7)$ $\theta = 60°$ (to nearest degree)	Look for the $\tan^{-1}$ button on your calculator. Round to the nearest degree.

Now you try

Find the angle θ, correct to the nearest degree where necessary, in each of the following.

a $\cos\theta = 0.7$ **b** $\tan\theta = 2.5$ **c** $\sin\theta = \frac{4}{5}$

4 Find the angle θ, to the nearest degree, for the following.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{3}{5}$ **c** $\sin\theta = \frac{7}{8}$

d $\tan\theta = 1$ **e** $\tan\theta = \frac{7}{8}$ **f** $\sin\theta = \frac{8}{10}$

g $\cos\theta = \frac{2}{3}$ **h** $\sin\theta = \frac{1}{10}$ **i** $\cos\theta = \frac{4}{5}$

j $\tan\theta = 6$ **k** $\cos\theta = \frac{3}{10}$ **l** $\tan\theta = \sqrt{3}$

m $\sin\theta = \frac{4}{6}$ **n** $\cos\theta = \frac{4}{6}$ **o** $\tan\theta = \frac{4}{6}$

Hint: Remember: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on the calculator.

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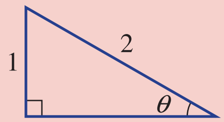
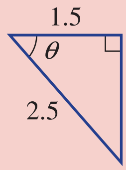
Example 14 Using SOH CAH TOA to find angles

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

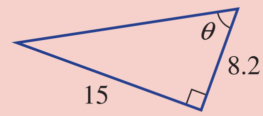
a



b



c

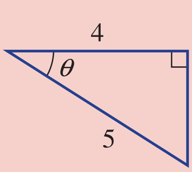


Solution	Explanation
a $\sin\theta = \frac{\text{O}}{\text{H}}$	Use $\sin\theta$ since we know the opposite and the hypotenuse.
$\sin\theta = \frac{1}{2}$	Substitute O = 1 and H = 2.
$\theta = \sin^{-1}\left(\frac{1}{2}\right)$	Use your calculator to find $\sin^{-1}\left(\frac{1}{2}\right)$.
$= 30^\circ$	
b $\cos\theta = \frac{\text{A}}{\text{H}}$	Use $\cos\theta$ since we know the adjacent and the hypotenuse.
$\cos\theta = \frac{1.5}{2.5}$	Substitute A = 1.5 and H = 2.5.
$\theta = \cos^{-1}\left(\frac{1.5}{2.5}\right)$	Use your calculator to find $\cos^{-1}\left(\frac{1.5}{2.5}\right)$.
$= 53.13^\circ$ (to 2 d.p.)	Round to two decimal places.
c $\tan\theta = \frac{\text{O}}{\text{A}}$	Use $\tan\theta$ since we know the opposite and the adjacent.
$\tan\theta = \frac{15}{8.2}$	Substitute O = 15 and A = 8.2.
$\theta = \tan^{-1}\left(\frac{15}{8.2}\right)$	Use your calculator to find $\tan^{-1}\left(\frac{15}{8.2}\right)$.
$= 61.34^\circ$ (to 2 d.p.)	Round to two decimal places.

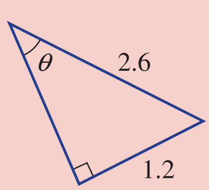
Now you try

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

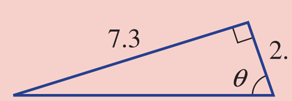
a



b



c



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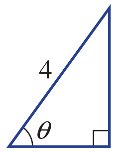
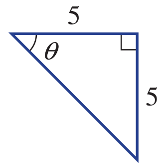
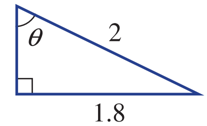
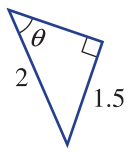
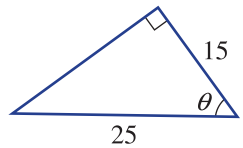
5 Use one of sin, cos or tan to find θ in these triangles, rounding to two decimal places where necessary.

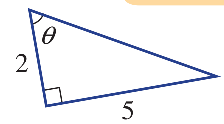
a


b


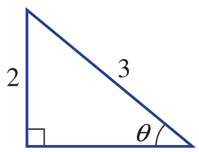
c


d


e


f


6 Find the angle θ, correct to the nearest degree, in these triangles. You will need to decide whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$.

a


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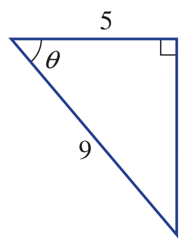

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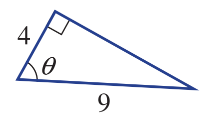

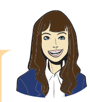

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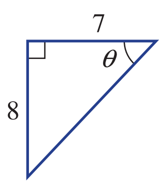

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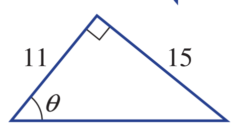

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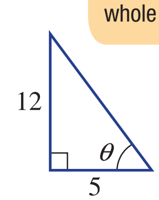
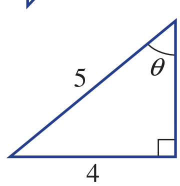
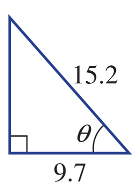
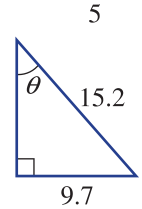


g


h


i


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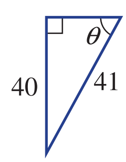

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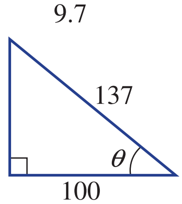

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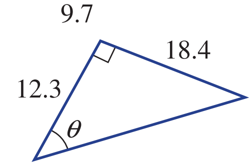
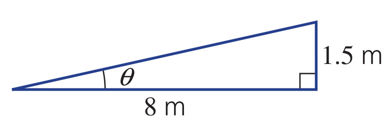


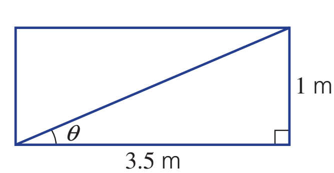
Problem-solving and reasoning

7, 8	8–10

7 A ramp is 8 m long horizontally and 1.5 m high. Find the angle the ramp makes with the ground, correct to two decimal places.



8 A rectangular piece of wood 1 m wide and 3.5 m long is to be cut across the diagonal. Find the angle the cut makes with the long side (correct to two decimal places).



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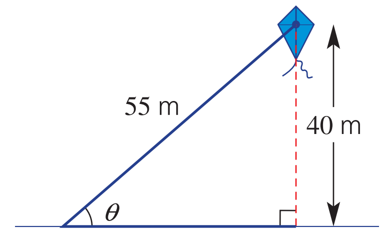
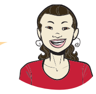
9 At what angle to the ground is a kite (shown) with height 40 m and string length 55 m? Round to two decimal places.

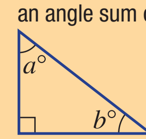




10 Find the two acute angles in a right-angled triangle with the given side lengths, correct to one decimal place.

a hypotenuse 5 cm, other side 3 cm
b hypotenuse 7 m, other side 4 m
c hypotenuse 0.5 mm, other side 0.3 mm
d the two shorter side lengths are 3 cm and 6 cm
e the two shorter side lengths are 10 m and 4 m

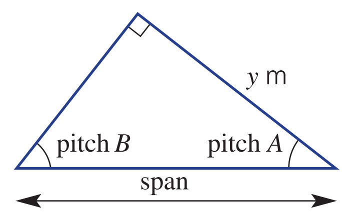
Hint:
- Draw a picture.
- Use SOH CAH TOA.
- Find one acute angle using trigonometry.
- Remember: all triangles have an angle sum of $180°$.



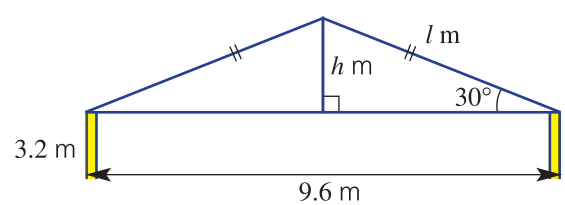
$a + b + 90 = 180$

Building and construction — 11–13

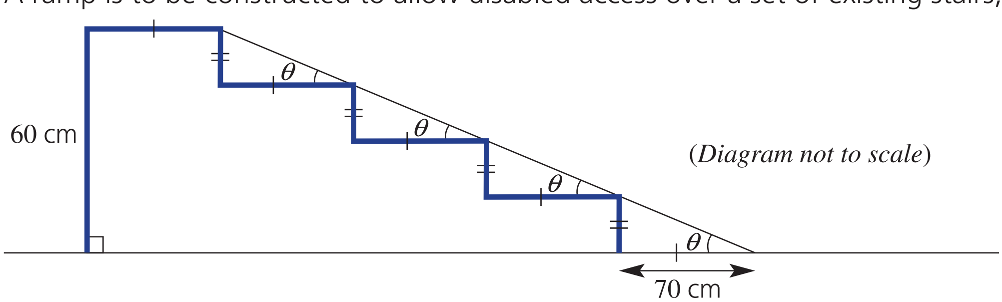
11 A roof is pitched so that the angle at its peak is $90°$. If each roof truss spans 10.5 m and distance y is 7.2 m, find the pitch angles A and B, to the nearest whole number.



12 **a** Find the length of the slats (l metres) needed along each hypotenuse for this roof cross-section, correct to two decimal places.
b Find the height of the highest point of the roof above ground level, correct to two decimal places.



13 A ramp is to be constructed to allow disabled access over a set of existing stairs, as shown.



a What angle does the ramp make with the ground, to the nearest degree?
b Government regulations state that the ramp cannot be more than $13°$ to the horizontal. Does this ramp meet these requirements?
c How long is the ramp? Round your answer to one decimal place.

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9H Angles of elevation and depression

Learning intentions

- To know how angles of elevation and depression can be identified in a real context
- To be able to use trigonometry to solve problems involving angles of elevation and depression

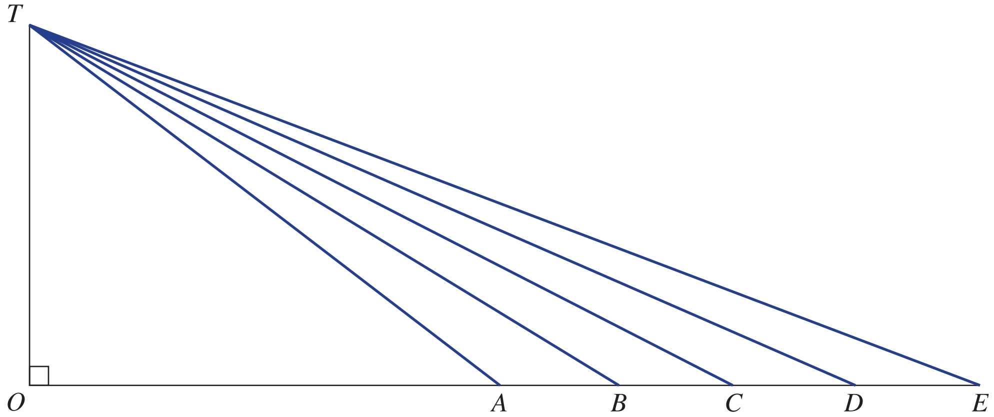
Key vocabulary: elevation, depression, horizontal

Many applications of trigonometry involve angles of elevation and angles of depression. These angles are measured up or down from a horizontal level. Whenever you view something above or below the horizontal, you form an angle of elevation or depression.

Lesson starter: How close should you sit?

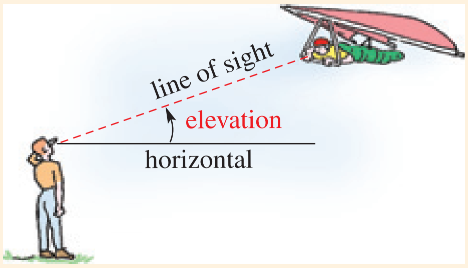
The diagram below shows an outdoor movie screen (OT). The point T is the top of the screen. The points $A–E$ are the five rows of seats in the theatre, from which a person's line of sight is taken. The line OE is the horizontal line of sight.

- Use your protractor to measure the angle of elevation from each point along the horizontal to the top of the movie screen.
- Where should you sit if you wish to have an angle of elevation between 25° and 20° and not be in the first or last row of the theatre?



Key ideas

- Looking up at an object forms an **angle of elevation** above the horizontal.



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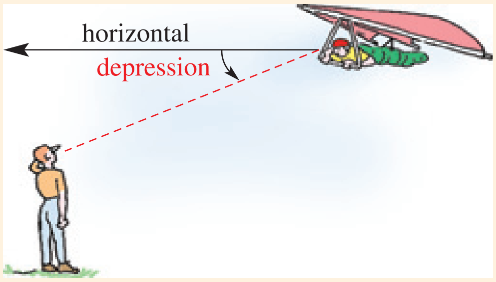
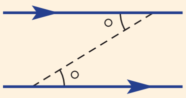
- Looking down at an object forms an **angle of depression** below the horizontal.



- The angle of elevation will equal the angle of depression in the same context.



Exercise 9H

Understanding | 1–3 | 2, 3

1 Give the missing word.

a Looking down at an object forms an angle of ________________.

b Looking up at an object forms an angle of ________________.

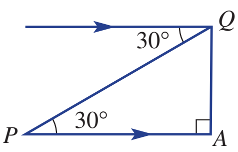
c Angles of elevation and depression are measured up or down from the ________________.

2 This diagram shows angles of elevation and depression.

a What is the angle of elevation of Q from P?

b What is the angle of depression of P from Q?

c What is the size of $\angle PQA$?



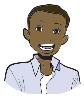
3 For each description, draw a triangle diagram that matches the information given.

a The angle of elevation to the top of a tower from a point 50 m from its base is 55°.

b The angle of depression from the top of a 200 m cliff to a boat out at sea is 22°.

c The angle of elevation of the top of a castle wall from a point on the ground 30 m from the castle wall is 33°.

Hint: Always measure angles of elevation and depression from the horizontal.



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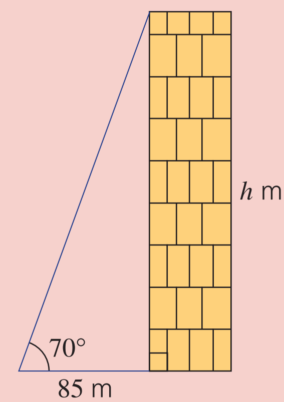
9H

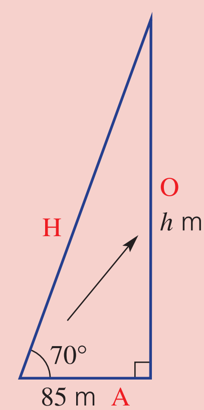
Fluency 4–6(½) 4–6(½)

Example 15 Using an angle of elevation

To find the height of a tall building, Johal stands 85 m away from its base and measures the angle of elevation at the top of the building as 70°. Find the height of the building, correct to the nearest metre.



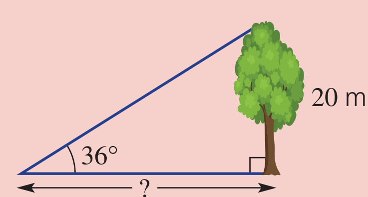
Solution	**Explanation**
$\tan\theta = \frac{\text{O}}{\text{A}}$	Label the triangle with O, A and H.
$\tan 70° = \frac{h}{85}$	Use tan since the opposite and adjacent are given.
$h = 85 \times \tan 70°$	Find h by solving the equation.
$= 233.53558\ldots$	$85 \times \tan 70° = \frac{h}{85} \times 85$
$= 234$ m (to the nearest metre)	Round to the nearest metre.
∴ The building is 234 m tall.	



Now you try

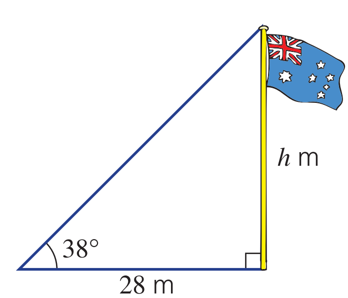
A 20 m high tree casts a shadow of unknown length.

Find the length of the shadow if the angle of elevation of the top of the tree from the end of the shadow is 36°. Round to the nearest metre.



4 Answer the following questions about angles of elevation.

a The angle of elevation to the top of a flagpole from a point 28 m from its base is 38°. How tall is the flagpole, correct to two decimal places?



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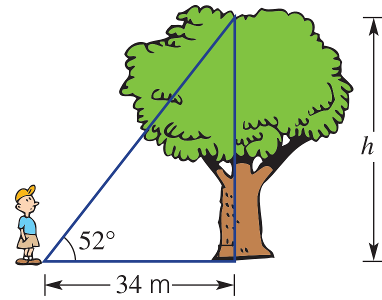
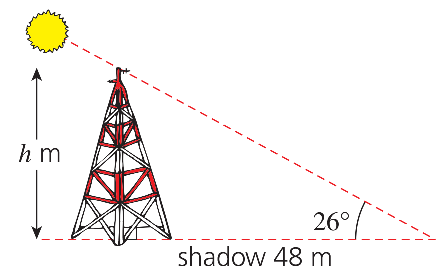
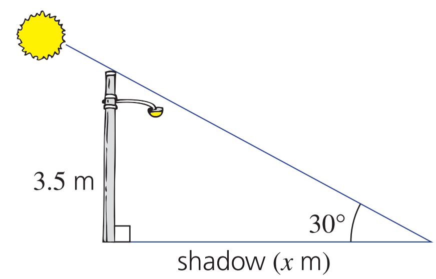
b Alvin is 34 m away from a tree and the angle of elevation to the top of the tree from the ground is 52°. What is the height of the tree, correct to one decimal place?



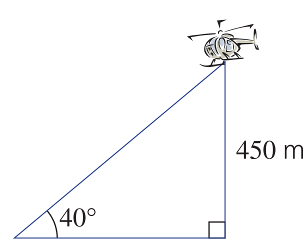
c The Sun's rays shining over a tower make an angle of elevation of 26° and cast a 48 m shadow on the ground. How tall, to two decimal places, is the tower?



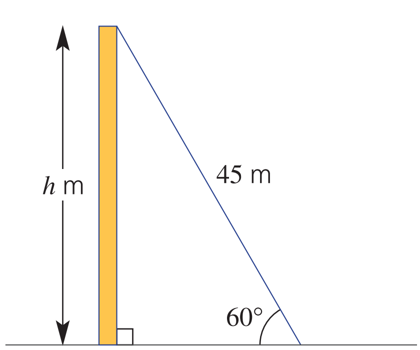
d The Sun makes an angle of elevation of 30° with a lamp post that is 3.5 m tall. How long is the shadow on the ground, correct to two decimal places?



e The altitude of a hovering helicopter is 450 m, and the angle of elevation from the helipad to the helicopter is 40°. Find the horizontal distance from the helicopter to the helipad, correct to two decimal places.



f A cable of length 45 m is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, correct to two decimal places.



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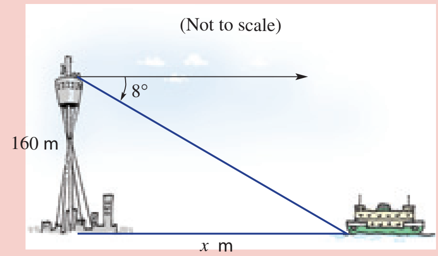
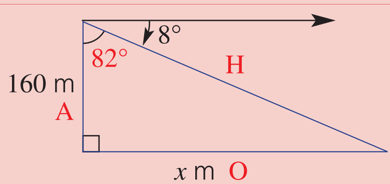
9H

Example 16 Using an angle of depression

From the observation room of Centrepoint Tower in Sydney, which has a height of 160 m, the angle of depression of a boat moored at Circular Quay is observed to be 8°. How far from the base of the tower is the boat, correct to the nearest metre?



Solution



$$\tan\theta = \frac{O}{A}$$

$$\tan 82° = \frac{x}{160}$$

$$x = 160 \times \tan 82°$$

$$= 1138.459\ldots$$

$$= 1138 \text{ (to the nearest metre)}$$

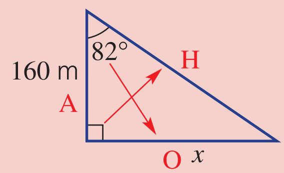
∴ The boat is about 1138 m from the base of the tower.

Explanation

Draw the triangle and find the angle inside the triangle:
$90° - 8° = 82°$

Use this angle to label the triangle.

Use tan since we have the opposite and adjacent.



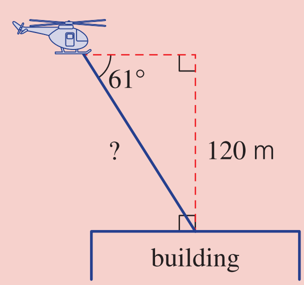
Find x by solving the equation.

Round to the nearest metre.

Now you try

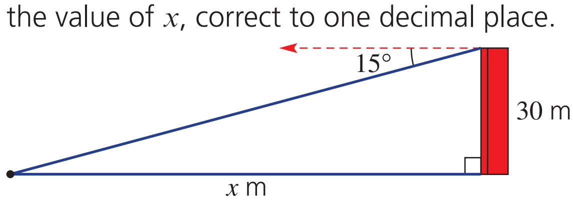
The angle of depression from a hovering helicopter to a building on the ground is 61°.

If the vertical height of the helicopter above the building is 120 m, find the direct distance from the helicopter to the building. Round to one decimal place.

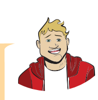


5 Answer these problems relating to angles of depression.

a The angle of depression from the top of a tower 30 m tall to a point x m from its base is 15°. Find the value of x, correct to one decimal place.



Hint: Use 15° to label an angle inside the triangle.



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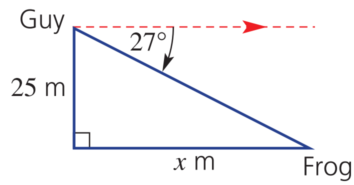
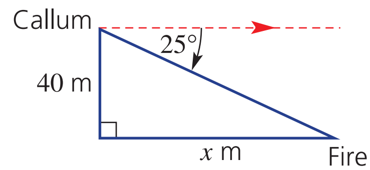
b From a bridge 25 m above a stream, Guy spots two frogs on a lilypad. He estimates the angle of depression to the frogs to be 27°. How far from the bridge are the frogs, to the nearest metre?

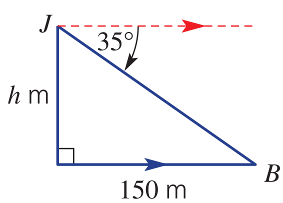


Hint: The angle of depression is the angle below the horizontal, looking down at an object.

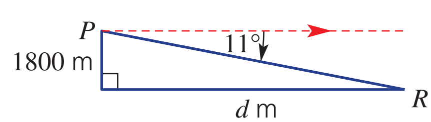
c From a lookout tower, Callum spots a bushfire at an angle of depression of 25°. If the lookout tower is 40 m tall, how far away (to the nearest metre) is the bushfire from the base of the tower?

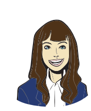


d From the top of a vertical cliff, Jung spots a boat 150 m out to sea. The angle of depression from Jung to the boat is 35°. How many metres (to the nearest whole number) above sea level is Jung?



e A plane is flying 1800 m above the ground. At the time the pilots spot the runway, the angle of depression to the edge of the runway is 11°. How far does the plane have to fly to be above the edge of the runway at its current altitude? Answer to the nearest whole number.





Hint: 'Altitude' means height.

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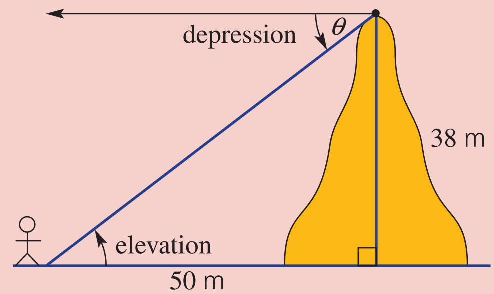
9H

Example 17 Finding angles of elevation and depression

a Find the angle of depression from the top of the hill to a point on the ground 50 m from the middle of the hill. Answer to the nearest degree.

b What is the angle of elevation from the point on the ground to the top of the 38 m hill? Answer to the nearest degree.



Solution

a $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{38}{50}$

$\theta = \tan^{-1}\left(\frac{38}{50}\right)$

$\theta = 37.2348\ldots$

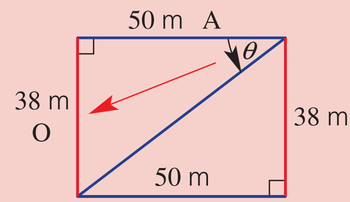
$\theta = 37°$ (to the nearest degree)

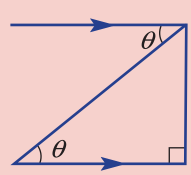
Angle of depression is 37°.

b Angle of elevation is 37°.

Explanation

Aim to find θ. Redraw the diagram as a rectangle so that θ is inside the triangle. Label the triangle, opposite and adjacent. Use tan.

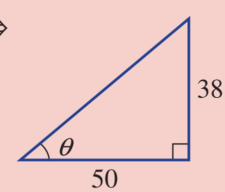




Alternate angles are equal when lines are parallel.

angle of elevation = angle of depression

Alternatively ⇒



$\Rightarrow \tan\theta = \frac{38}{50}$

Now you try

From a 20 m high lookout platform, you can see a fire that is 150 m away horizontally. Find the angle of depression from the lookout platform to the fire, correct to the nearest degree.

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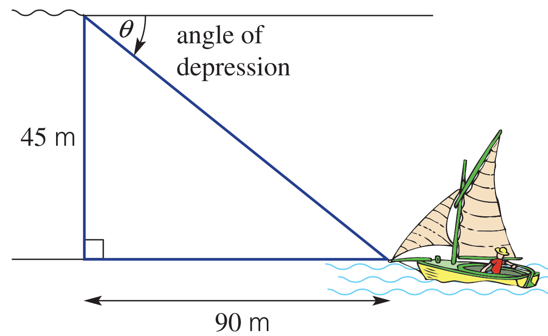
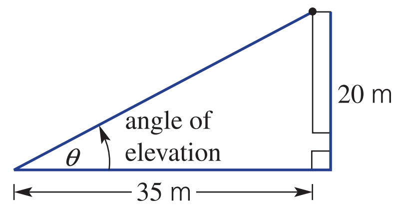
6 Answer these questions about finding angles of elevation and depression. Round all answers to one decimal place.

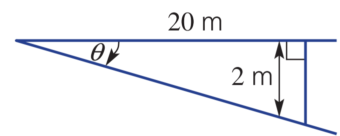
a From the top of a vertical cliff, Jacquie spots a boat 90 m out to sea. If the top of the cliff is 45 m above sea level, find the angle of depression from the top of the cliff to the boat.



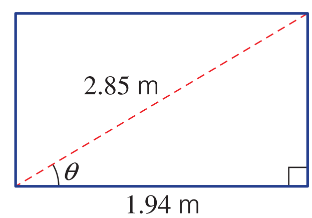
b Find the angle of elevation from a person sitting 35 m from a movie screen to the top of the screen at 20 m above the ground.



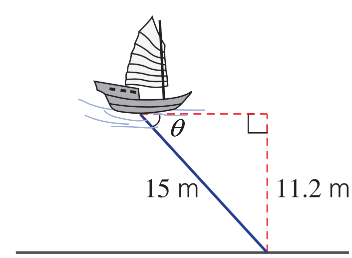
c A person sits 20 m away from a stage that is 2 m below the horizontal viewing level. Find the angle of depression of the person's viewing level to the stage.



d A diagonal cut 2.85 m long is to be made on a piece of plaster board attached to a wall, as shown. The base of the plaster board measures 1.94 m. Find the angle of elevation of the diagonal cut from the base.

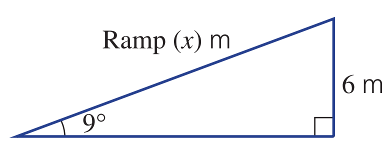


f A 15 m chain with an anchor attached, as shown, is holding a boat in a position against a current. If the water depth is 11.2 m, find the angle of depression from the boat to where the anchor is fixed to the seabed.

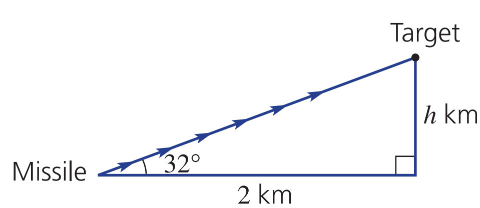


Problem-solving and reasoning | 7, 9 | 8–10

7 A ramp for wheelchairs is to be constructed into a footbridge 6 m high. The angle of elevation is to be 9°. What will be the length of the ramp, correct to two decimal places?



8 A missile is launched at an angle of elevation of 32°. If the target is 2 km away on the horizontal, how far above ground level is the target, correct to two decimal places?



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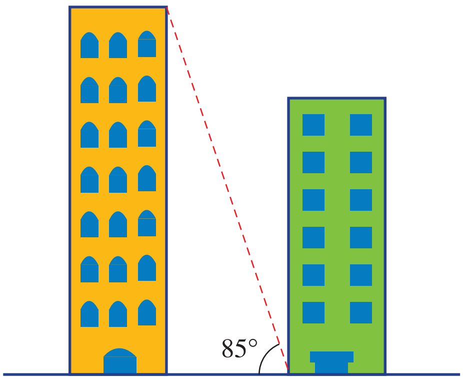
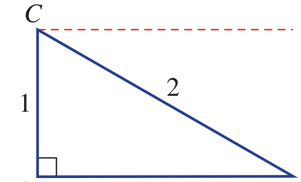
9H

9 The distance between two buildings is 24.5 m, as shown. Find the height of the taller building, correct to two decimal places, if the angle of elevation from the base of the shorter building to the top of the taller building is 85°.



10 Right-angled triangle ABC is shown:

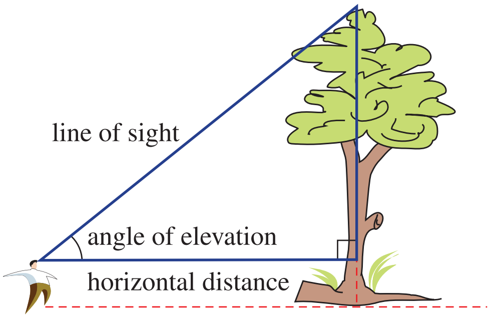
a Find the angle of elevation from B to C.
b State the angle of depression from C to B.
c Describe the relationship that exists between these two angles.
d Find the length AB, correct to one decimal place.



Practical trigonometry: measuring heights — 11

11 It is not always possible or practical to measure the height of an object directly. Here you will find the height of an object that is difficult to measure. Select a building or other structure (e.g. a statue or flagpole) for height calculation. You must be able to measure right up to the base of the structure.

a Choose a position from which you can see the top of your structure and measure the angle of elevation, θ, from your eye level. (Use an inclinometer, if your teacher has one, or simply estimate the angle using a protractor.)



b Measure the distance along the ground (d) from your location to the base of the structure.
c Calculate the height of the structure. *Remember to make an adjustment for the height of your eye level from the ground*.
d Move to another position and repeat the measurements. Calculate the height using your new measurements.
e Was there much difference between the calculated heights? Suggest reasons for any differences.

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9I Direction and bearings

Learning intentions

- To know the 8-point compass rose and how true bearings are measured
- To be able to draw a diagram including a right-angled triangle for a problem involving bearings
- To be able to find an unknown distance in a problem involving bearings

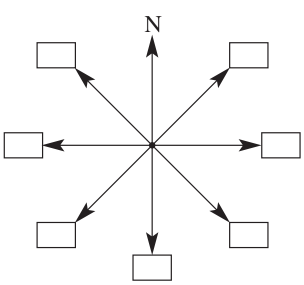
Key vocabulary: true bearing, clockwise

True bearings are used to communicate a direction and are important in navigation. Ships, planes, bushwalkers and the military all use bearings when communicating direction.

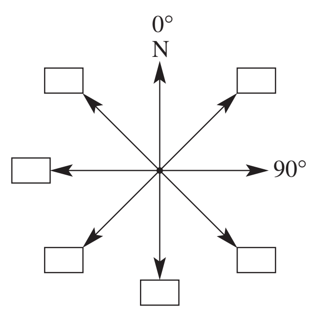
Lesson starter: Compass bearings

Work together as a class to label the 8-point compass rose using letters/words in **1** and angles in **2**.

1

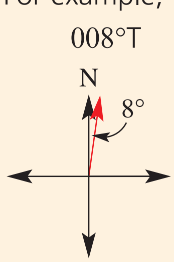


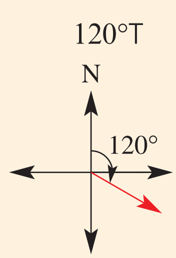
2

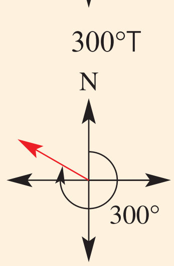


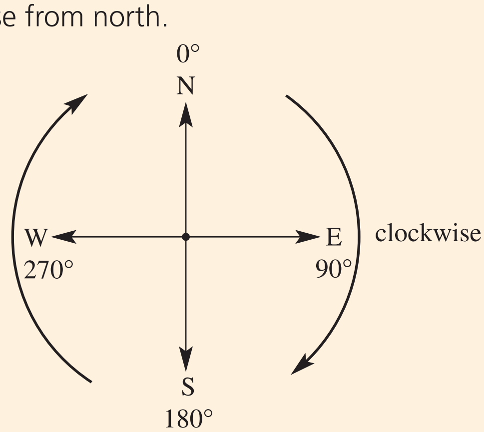
Key ideas

- A **true bearing** (°T) is an angle measured clockwise from north.
  - It is written using three digits.
    For example,









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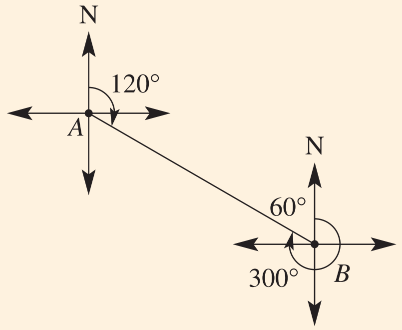
9I

- The word *from* indicates the direction from which a bearing is being taken. For example,



The bearing of B from A is 120°T.

The bearing of A from B is 300°T.

- When solving problems relating to bearings, always draw a diagram using N, S, E and W each time a bearing is used.

Exercise 9I

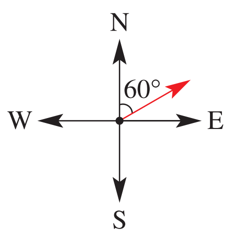
Understanding 1–4 3, 4

1 What is the opposite direction to:
  - **a** north (N)?
  - **b** east (E)?
  - **c** south (S)?
  - **d** north-east (NE)?

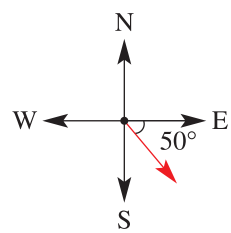
2 Match each diagram below with the correct true bearing from the list below.

i 300° **ii** 260° **iii** 225°
iv 140° **v** 060° **vi** 350°

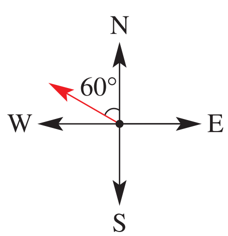
a



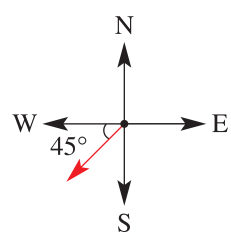
b



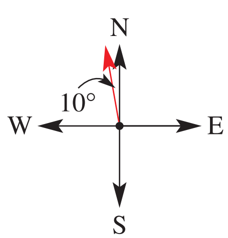
c



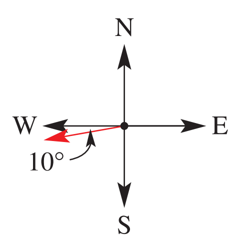
d



e



f



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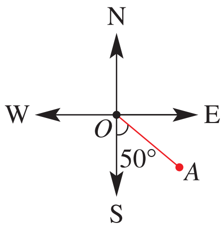
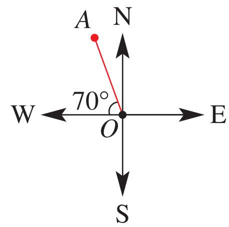
3 Write down the true bearings of A from O, as shown in these diagrams.

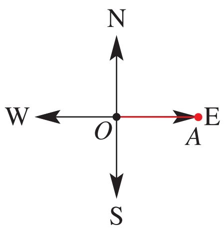
a



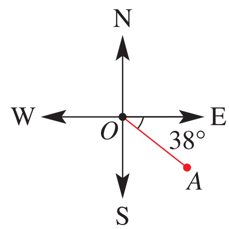
b



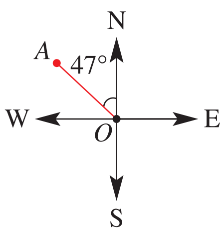
c



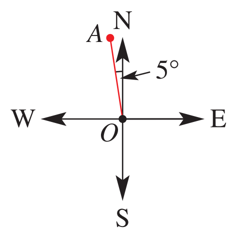
d



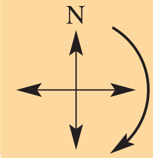
e



f



Hint: Remember to use 3 digits and to go clockwise from north.



4 Fill in the missing terms and values for the diagram shown.

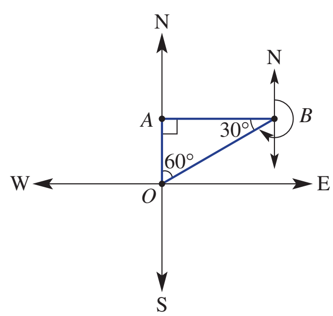
a A is due ______ of O.

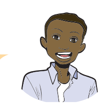
b B is due ______ of A.

c A is due ______ of B.

d The true bearing of B from O is ______.

e The true bearing of O from B is ______.





Hint: For part **e**, start from north and move clockwise to the line BO.

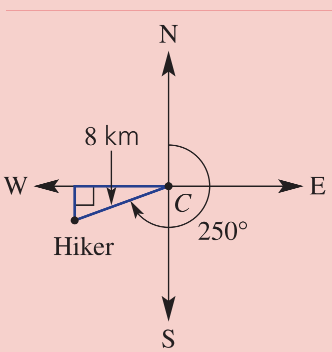
Fluency

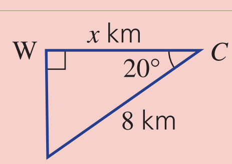
5–7	5, 6, 8

Example 18 Drawing a diagram

A hiker leaves camp (C) and walks on a bearing of 250°T for 8 km. How far west of camp (x km) is the hiker? Show all this information on a right-angled triangle. You do not need to solve for x.

Solution





Explanation

Draw the compass points first.

Start your diagram with the camp at the centre.

Mark in 250° clockwise from north, 8 km.

Draw a line from the hiker to the west line at right angles.

Redraw the triangle, showing any angles and lengths known ($270° - 250° = 20°$). Place a pronumeral on the required side.

Continued on next page

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9I

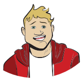
Now you try

A plane leaves the airport and heads on a bearing of $165°$ T for 100 km. How far south of the airport (x km) is the plane?

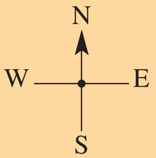
Show all this information on a right-angled triangle. You do not need to solve for x.

5 Draw a right-angled triangle for each of the situations outlined below.

- **a** Zahra runs on a true bearing of $300°$ from her home for 6 km. How far north of home is she when she stops?
- **b** Bailey walks 12.5 km from camp, C, on a bearing of $135°$T. How far south is he now from camp?
- **c** Samir walks due south 10 km, then turns and walks due east 12 km. What is his bearing from O, his starting point?

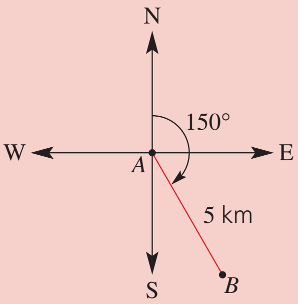


Hint: Start by making the starting point at the centre of a compass.

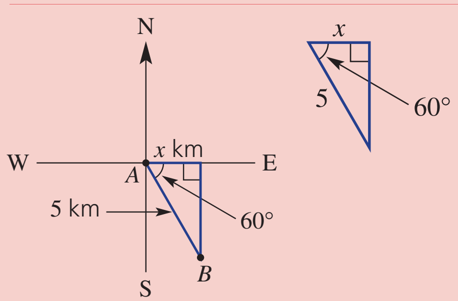


Example 19 Finding distances with bearings

A bushwalker walks 5 km on a true bearing of $150°$ from point A to point B. Find how far east point B is from point A.



Solution



Explanation

Copy the diagram and draw a line from B up to the east line.

Use the pronumeral x along the east line.

Find the angle within the triangle: $150° - 90° = 60°$

Redraw the triangle.

$\cos\theta = \frac{A}{H}$ — Since the adjacent (A) and hypotenuse (H) are given, use cos.

$\cos 60° = \frac{x}{5}$

$x = 5 \times \cos 60°$

$x = 2.5$ — Solve the equation to find x.

$\therefore$ Point B is 2.5 km east of point A. — Answer the question.

Now you try

A ship travels 200 km on a true bearing of $290°$ from point A to point B. Find how far west the point B is from point A. Round to one decimal place.

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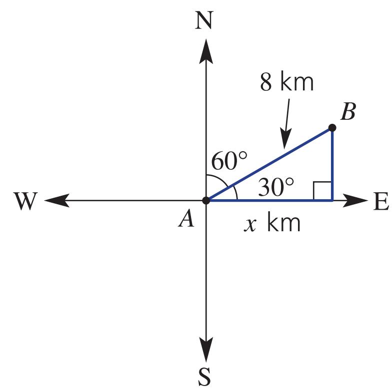
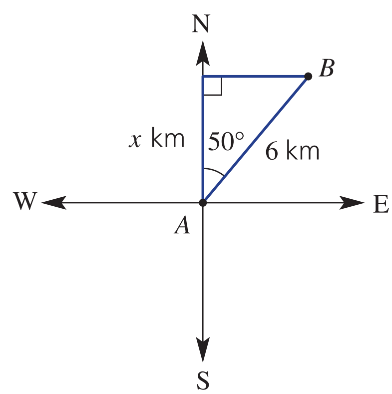
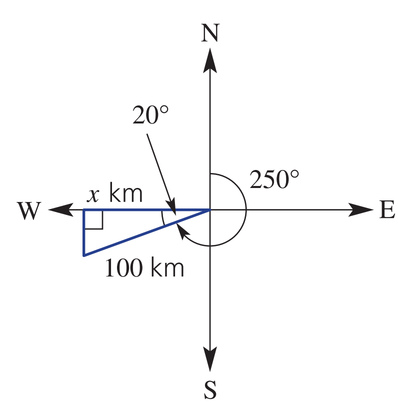
6 Rihanna walks 8 km from point A to point B on a true bearing of 060°. How far east, correct to one decimal place, is point B from point A?



7 A bushwalker walks 6 km on a true bearing of 050° from point A to point B. Find how far north point B is from point A, correct to two decimal places.



8 A speed boat travels 100 km on a true bearing of 250°. Find how far west of its starting point the speed boat is, correct to two decimal places.



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9I

Problem-solving and reasoning

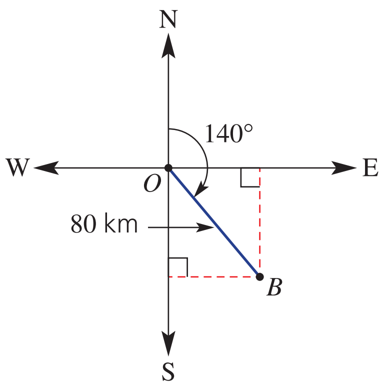
9, 10 | 9, 11

9 A fishing boat starts from point O and sails 80 km on a true bearing of 140° to point B, as shown below.

a How far east of point O is point B? (Answer to two decimal places.)

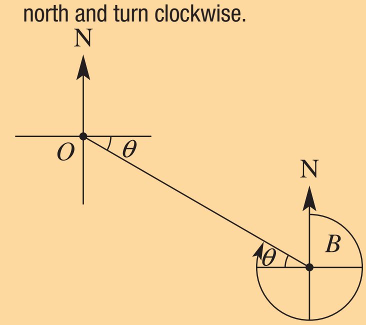
b How far south of point O is point B? (Answer to two decimal places.)

c What is the bearing of point O from point B?



Hint:
Remember what 'from' means!

Hint:
Remember: to find a bearing, face north and turn clockwise.

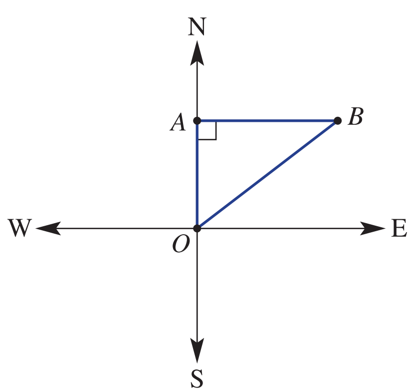


Bearings are given as a 3-digit angle.

10 A plane flies from point O 50 km due north to point A, and then turns and flies 60 km east to point B.

a Copy the diagram below and mark in the lengths 50 km and 60 km.

Hint: Pythagoras' theorem: $c^2 = a^2 + b^2$



b Use Pythagoras' theorem to find the distance of B from O, correct to two decimal places.

c Use trigonometry to find the size of angle AOB. Round to the nearest degree.

d What is the bearing of B from O?

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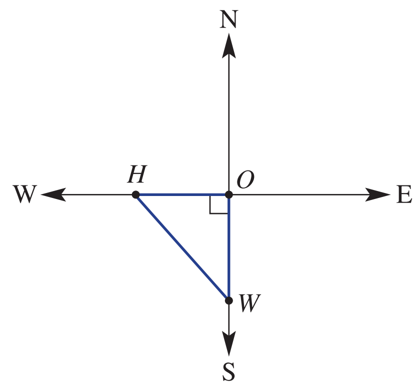
11 William and Harry both leave camp, O, at the same time. William walks south from O for 10 km. Harry walks west from O for 8 km.

a Copy and complete the diagram for this question.

b How far is Harry from William (to one decimal place)?

c Find the size of angle OWH, correct to the nearest degree.

d What is the bearing of Harry from William?



Hint: You can use Pythagoras' theorem here.

Drawing your own diagrams — 12–14

12 Huang walks on a true bearing of $210°$ for 6.5 km. How far west of his starting point is he?

13 A plane flies on a true bearing of $320°$ from an airport, A, for 150 km. At this time how far north of the airport is the plane? Answer to the nearest kilometre.

14 Point A is 10 km due east of point O, and point B is 15 km due south of point A.

a How far is it, correct to two decimal places, from point B to point O?

b What is the bearing, to the nearest degree, of point B from point O?

Hint: Remember: The word 'from' indicates where the bearing is being taken.

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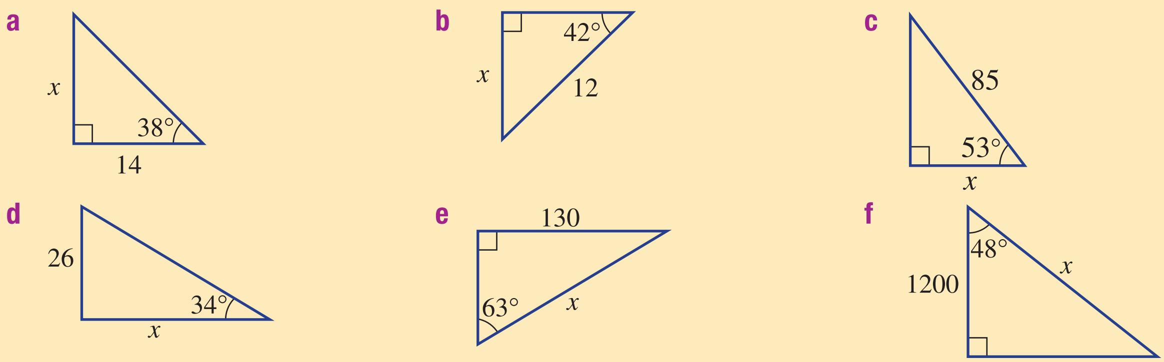
Maths@Work: Surveyor

Surveyors collect and measure data about land and the environment, including measurements for subdivisions of new housing estates, new roads and open-pit mines. They work with plans, files, charts and reports.

To become a surveyor you can either do a university course, requiring Year 12 mathematics, or apply to TAFE and complete the training there. All courses require a student to have studied mathematics.

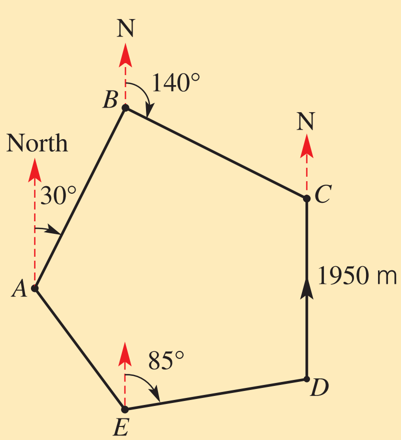
Mathematical skills needed in becoming a land surveyor include geometry and trigonometry. Complete these questions that a surveyor may face in their day-to-day job.

1 Refresh your trigonometry skills by finding the value of each pronumeral in the following triangles. All measurements are in metres, and give answers correct to two decimal places.



In surveying, there are two types of traverse surveys: the closed traverse survey and the open traverse survey.

2 The example below is of a closed traverse survey that could be around a large contained development, such as a school or airport. The diagram is not to scale.



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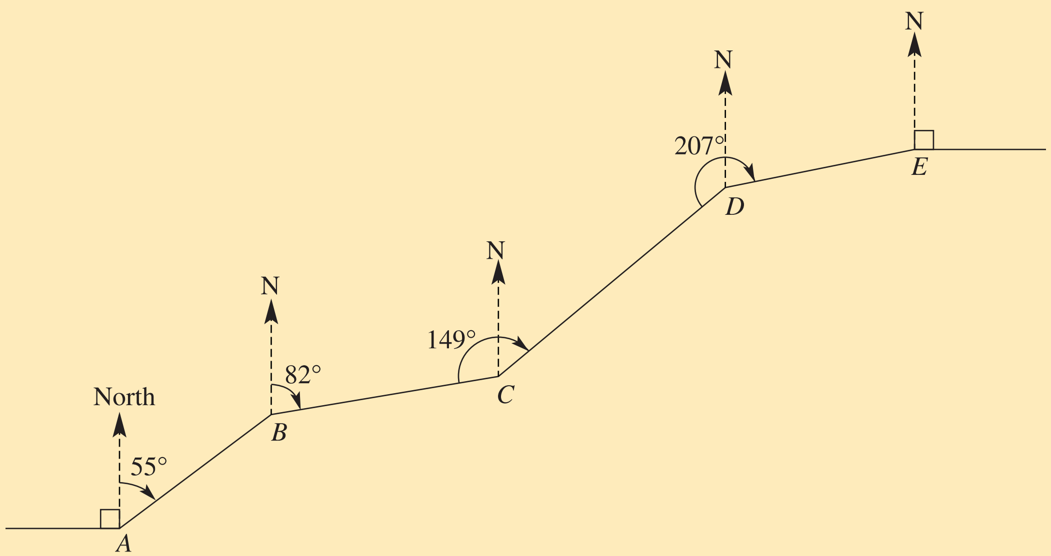
State answers to the following questions in metres, to two decimal places unless told otherwise.

a Given that marker B is 650 m east of A, calculate the distance AB.
b Given that marker C is 900 m south of B, calculate the distance BC.
c Given that marker E is 160 m south of D, calculate the distance DE.
d Marker E is 600 m east of A and 900 m south of A. Calculate the distance AE.
e Calculate the perimeter of the figure $ABCDE$, correct to the nearest metre.
f Calculate the true bearing of B from C.
g Calculate the true bearing of E from D.

Hint: For part **a**, start by forming a right-angled triangle with AB as the hypotenuse.

3 Below is an example of an open traverse survey that could be the simplified model of a section of road. The diagram is not to scale.



a Complete this table of measurements for the open traverse survey above.
Hints: Draw a large, ruled and labelled diagram with a north–south line at each marker. Recall the rules for angles between parallel lines.

Marker	Distance to the next marker (metres)	True bearing of the next marker from the current marker	Angle between line segments at marker (clockwise)	Distance East to the next marker (in m to 2 d.p.)	Distance North to the next marker (in m to 2 d.p.)
A	98.00	055°			
B	125.36	082°			
C	157.80		149°		
D	104.42		207°		
E	–	–		–	–

b How far east is marker E from A? Give your answer in metres to two decimal places.
c How far north of marker A is E? Give your answer in metres to two decimal places.
d By considering a right-angle triangle with hypotenuse AE, what is the true bearing of E from A? Give your answer to the nearest degree.

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Using technology

4 Use FxDraw or CAD software or another digital drawing program to create a labelled diagram of a closed traverse survey for a town park. Your diagram of the park must include:

- a true north line on each marker
- true north bearings of each next marker from each marker
- the length, in metres, of each boundary segment
- internal angles between line segments (these need to be calculated)
- the perimeter of the park

Your diagram doesn't need to be to scale but it should be a close representation of the survey data.

Closed Traverse survey data for Penny Park			
Marker	True north bearing of next marker from current marker	Distance from current marker to next marker	Internal angle between line segments at marker
A	30°	394 m	
B	99°	658 m	
C	180°	500 m	
D	266°	590 m	
E	324°	360 m	

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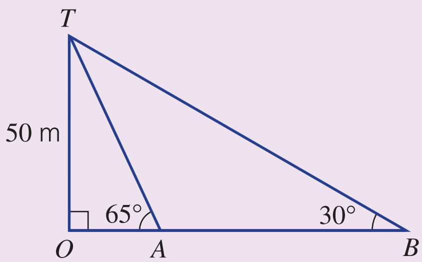
Puzzles and games

1 What is the opposite direction to:

a East? **b** NE? **c** SE? **d** 018°? **e** 300°?

2 Use two different right-angled triangles to find the distance from A to B in this diagram, correct to two decimal places.



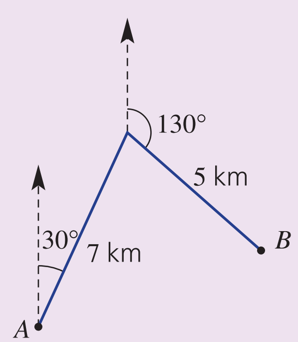
3 Make up your own saying using the letters in SOH CAH TOA as the first letter of each word.

S___ O___ H___ C___ A___ H___ T___ O___ A___

4 In the wordfind below there are 17 terms that are used in this chapter. See if you can locate all 17 terms and write a definition or draw a diagram for each of them.

S	I	D	R	T	Y	I	P	Y	T	S	O	H	T	H
I	D	E	P	R	E	S	S	I	O	N	D	Y	O	Y
N	T	E	I	N	T	E	W	P	Y	A	D	J	H	P
E	I	O	E	E	D	S	V	B	Y	T	P	U	W	O
Q	U	O	T	R	S	I	A	D	J	A	C	E	N	T
A	D	A	N	G	E	D	E	P	R	A	N	G	L	E
S	E	N	A	R	T	E	E	G	R	I	L	K	O	N
D	P	T	R	I	A	N	G	L	E	H	E	I	P	U
C	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
B	A	C	G	B	L	I	N	O	Y	A	A	W	O	E
H	D	O	H	T	E	A	N	G	L	N	T	H	S	H
U	J	S	T	R	G	A	O	K	Y	G	I	I	I	Y
J	E	I	W	F	L	N	R	M	K	E	O	D	T	P
K	N	N	T	J	T	R	N	I	O	N	P	Z	E	O
E	L	E	V	A	T	I	O	N	N	T	P	A	M	B
E	T	B	A	S	P	Y	T	H	A	G	O	R	A	S

5 Find the bearing from A to B in this diagram.



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Chapter summary

Pythagoras' theorem and trigonometry

Trigonometric ratios

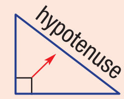
S O H → $\sin\theta = \frac{O}{H}$

C A H → $\cos\theta = \frac{A}{H}$

T O A → $\tan\theta = \frac{O}{A}$

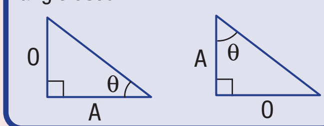
Hypotenuse

The *hypotenuse* is the longest side in a right-angled triangle. It is opposite the right angle.



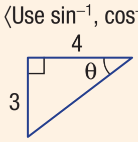
Opposite and Adjacent

In a right-angled triangle the sides marked 'opposite' and 'adjacent' are named according to the angle used.



Finding angles

(Use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$)



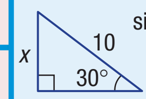
$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$= 37^\circ$

(to the nearest degree)

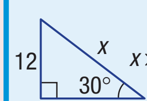
Finding lengths with trigonometry



$\sin 30^\circ = \frac{x}{10}$

$x = 10\sin 30^\circ$

$x = 5$



$\sin 30^\circ = \frac{12}{x}$

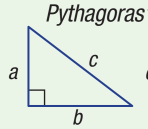
$x \times \sin 30^\circ = 12$

$x = \frac{12}{\sin 30^\circ}$

$x = 24$

Pythagoras' theorem

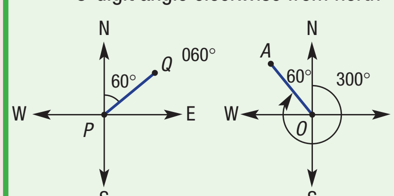
For the triangle shown, *Pythagoras' theorem* is



$c^2 = a^2 + b^2$

True bearings

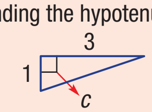
3-digit angle clockwise from north



Bearing of Q from P is 060°.

Bearing of A from O is 300°.

Finding the hypotenuse



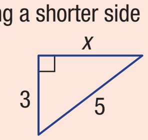
$c^2 = 1^2 + 3^2$

$c^2 = 10$

$c = \sqrt{10}$

$= 3.2$ (to 1 d.p.)

Finding a shorter side



$5^2 = x^2 + 3^2$

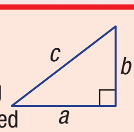
$x^2 = 5^2 - 3^2$

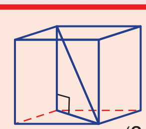
$x^2 = 16$

$x = 4$

Applications

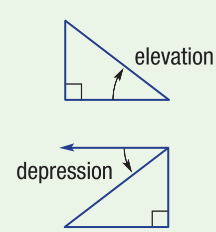
Start by identifying right-angled diagrams





(Cube)

Elevation and depression



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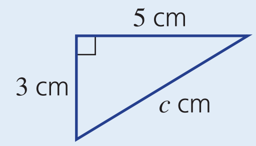
Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

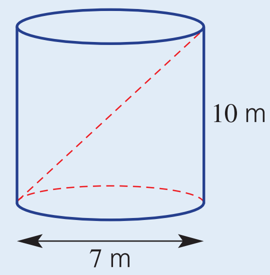
9A

1 I can find the length of the hypotenuse for a right-angled triangle given the length of the other two sides.
e.g. Find the length of the hypotenuse of this triangle, correct to one decimal place.



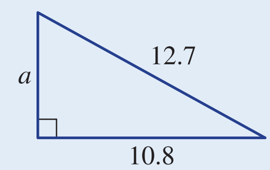
9A

2 I can apply Pythagoras' theorem and find the length of the hypotenuse in a real situation.
e.g. Find the length of the longest rod that will fit into this cylinder, correct to two decimal places.



9B

3 I can use Pythagoras' theorem to find the length of a shorter side in a right-angled triangle.
e.g. Find the value of a in this triangle, correct to one decimal place.

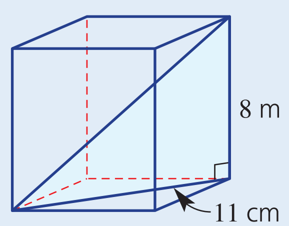


9C

4 I can apply Pythagoras' theorem to solve a range of real-life problems.
e.g. Two bushwalkers leave their camp at the same time. One walks due south for 5 km and the other walks due west for 4 km. How far apart are the bushwalkers at this point. Round to two decimal places.

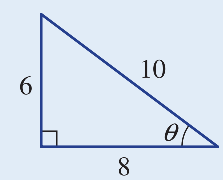
9C

5 I can apply Pythagoras' theorem to find a length in a three-dimensional solid.
e.g. Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



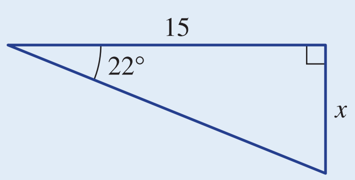
9D

6 I can write a ratio for sine, cosine and tangent of an angle in a triangle with given side lengths.
e.g. Write a ratio for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for this triangle.



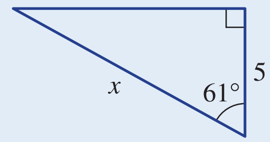
9E

7 I can find a missing length on a right-angled triangle given an angle and another side.
e.g. Find the value of x in this triangle, correct to two decimal places.



9F

8 I can find a missing length in a right-angled triangle if the pronumeral sits in the denominator of the fraction.
e.g. Find the value of x in this triangle, correct to two decimal places.



Chapter checklist

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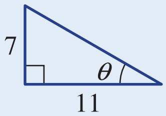
Chapter checklist

		✔
9G	**9 I can find an angle in a right-angled triangle given any two side lengths.** e.g. Find the angle θ in this triangle, correct to one decimal place. 	
9H	**10 I can solve problems in trigonometry using angles of elevation and depression.** e.g. The angle of elevation to the top of a vertical pole from a point on the ground 50 m away horizontally is $27°$. Find the height of the pole, correct to the nearest metre.	
9H	**11 I can find angles of elevation or depression in a real situation if given two other lengths from a right-angled triangle.** e.g. Find the angle of elevation of a 40 m high tower from a point which is 30 m horizontally from the base of the tower. Round to one decimal place.	
9I	**12 I can interpret a bearings problem and draw a suitable right-angled triangle.** e.g. A yacht travels 10 km on a true bearing of $210°$. How far east of the starting point is the yacht? Show all this information on a diagram using a right-angled triangle. You do not need to solve the problem.	
9I	**13 I can find an unknown distance in a problem involving bearings.** e.g. A hiker leaves base camp and walks 8 km on a true bearing of $235°$. How far west of base camp is the hiker at this point? Round to the nearest degree.	

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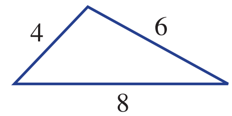
Short-answer questions

9A

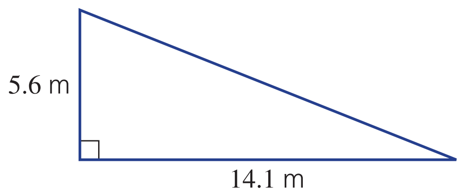
1 Determine whether the triangle shown contains a right angle.



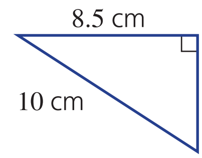
9A/B

2 Find the missing length, correct to two decimal places, in these triangles.

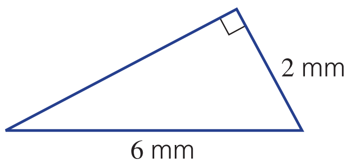
a



b



c

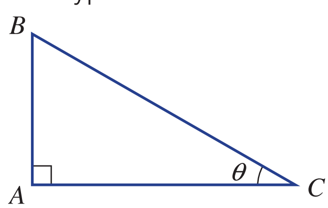


9D

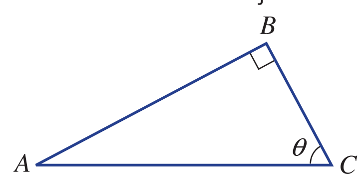
3 Which side (AB, AC or BC) of these triangles is:

i the hypotenuse? **ii** opposite to θ? **iii** adjacent to θ?

a



b



9D

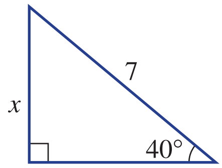
4 Use a calculator to find the value of the following, rounding to two decimal places.

a $\sin 35^\circ$ **b** $\cos 17^\circ$ **c** $\tan 83^\circ$

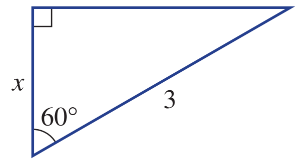
9E

5 Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

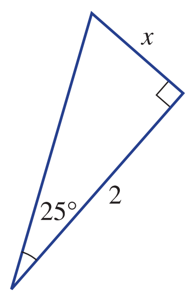
a



b



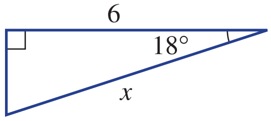
c



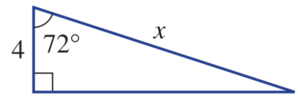
9F

6 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

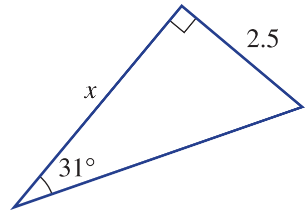
a



b



c



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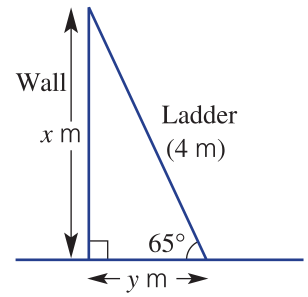
Chapter review

9E

7 A 4 m ladder leans, as shown, against a wall at an angle of 65° to the horizontal.

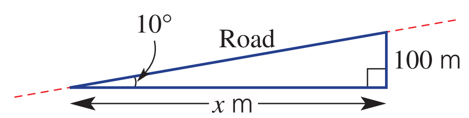
a Find how high up the wall the ladder reaches (x m), correct to two decimal places.

b Find how far the bottom of the ladder is from the wall (y m), correct to two decimal places.



9F

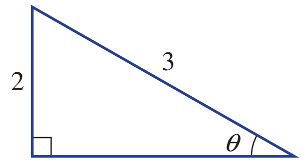
8 A section of road has a slope of 10° and gains 100 m in height. Find the horizontal length of the road (x m), correct to two decimal places.



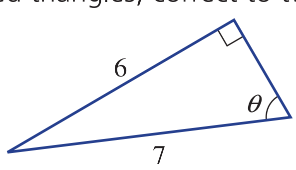
9G

9 Find θ in the following right-angled triangles, correct to two decimal places.

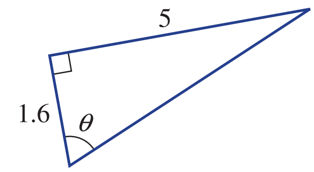
a



b



c

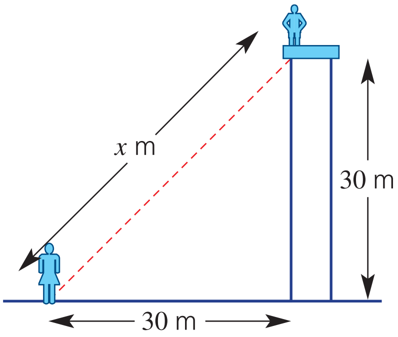


9H

10 Raymond and Sofia view each other from two different places, as shown. Raymond is on a viewing platform, whereas Sofia is 30 m from the base of the platform, on the ground. The platform is 30 m above the ground.

a Find the angle of elevation from Sofia's feet to Raymond's feet.

b Using your answer to part **a**, find the distance (x) between Sofia and Raymond, correct to one decimal place.



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9I

11 A hiker walks on a true bearing of 220° from point A for 3 km to point B.

a Find how far south the hiker has walked, correct to one decimal place.

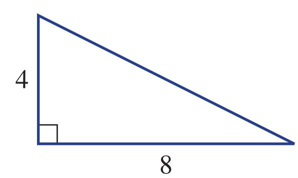
b Find how far west the hiker has walked, correct to one decimal place.

c What is the bearing of point A from B?

Multiple-choice questions

9A

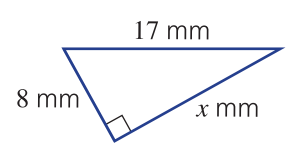
1 The length of the hypotenuse in the triangle shown is closest to:



A 10 **B** 9 **C** 4
D 100 **E** 64

9B

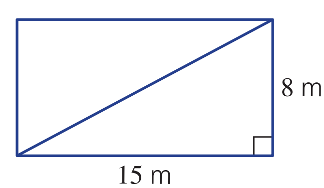
2 The length of the side marked x in the triangle shown is:



A 23 **B** 17 **C** 12
D 19 **E** 15

9C

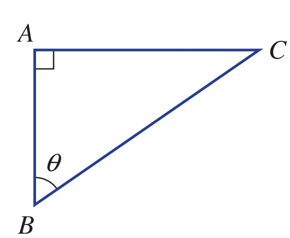
3 For the shape shown to be a rectangle, the length of the diagonal must be:



A 15 m **B** 8 m **C** 17 m
D 23 m **E** 32 m

9D

4 Which side (AB, AC or BC) is adjacent to θ in this triangle?



A AC **B** AB **C** BC
D hypotenuse **E** opposite

9D

5 The value of $\cos 21^\circ$ is closest to:

A -0.55 **B** 0.9 **C** 0.9336
D 0.93 **E** 0.934

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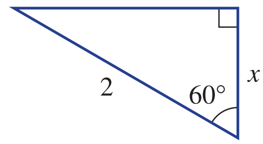
Chapter review

9E

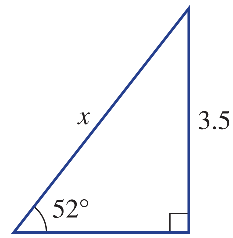
6 The value of x in this triangle is:



A $2 \div \cos 60°$ B $2 \div \sin 60°$
C $2 \times \cos 60°$ D $2 \times \sin 60°$
E $2 \times \tan 60°$

9F

7 The value of x in this triangle is closest to:



A 2.76 B 4.48 C 5.68
D 4.44 E 2.73

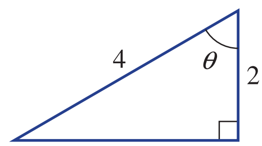
9F

8 A metal brace sits at 55° to the horizontal and reaches 4.2 m up a wall. The distance between the base of the wall and the base of the brace is closest to:

A 6.00 m B 2.41 m C 7.32 m
D 5.13 m E 2.94 m

9G

9 The angle θ in this triangle is:



A 60° B 30° C 26.57°
D 20° E none of the above

9H

10 The angle of depression from the roof of a building to a trampoline is 75°. If the roof is 12 m above the level of the trampoline, then the distance of the trampoline from the building is closest to:

A 12.42 m B 11.59 m C 3.22 m
D 44.78 m E 3.11 m

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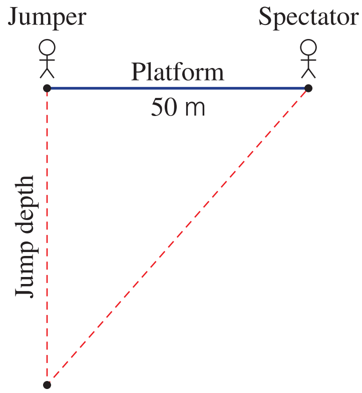
Extended-response questions

1 A spectator is viewing bungee jumping from a point 50 m to the side but level with the jumping platform.



a The first bungee jumper has a maximum fall of 70 m. Find the angle of depression from the spectator to the bungee jumper at the maximum depth, correct to two decimal places.

b The second bungee jumper's maximum angle of depression from the spectator is 69°. Find the jumper's maximum depth, correct to two decimal places.

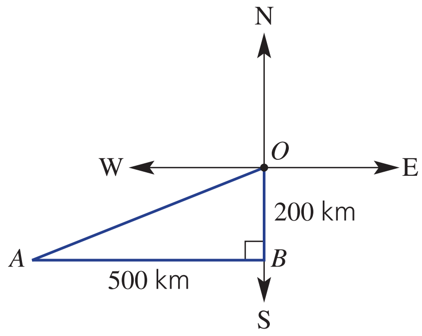
c The third jumper wants to do the 'Head Dunk' into the river below. This occurs when the spectator's angle of depression to the river is 75°. Find, correct to the nearest metre, the height of the platform above the river.

2 A military plane flies 200 km from point O to point B, then west 500 km to point A.

a How far is A from O, to the nearest kilometre?

b What is angle BOA, correct to the nearest degree?

c What is the bearing of A from O?



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