Chapter 4

Probability

Essential mathematics: why understanding probability is important

Probability indicates the likely occurrence or risk of a particular event. Awareness of risk helps people to make informed decisions. Being able to analyse data and interpret probabilities are important skills and essential for success in many businesses.

Vast amounts of data can be hard to make sense of, so techniques are used to discover the various proportions (i.e. probabilities) within results. For example, Venn diagrams and two-way tables can show the numbers of people:

- aged under 30 and who watch sport or comedy or both
- aged under 25 who have car accidents from drink driving or speeding or both
- who buy utes or are trade workers or both
- who smoke or die from cancer or both.

Using probabilities, informed management decisions can be made about a range of things: TV scheduling; money spent on advertising; shops selecting what stock to buy; and insurance premiums based on age, accident probabilities and even whether the person is a smoker.

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In this chapter

Victorian Curriculum

STATISTICS AND PROBABILITY

Chance

Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (VCMSP347)

Use the language of 'if ... then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language (VCMSP348)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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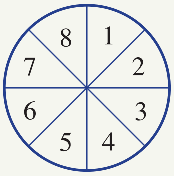
Warm-up quiz

1 A letter is selected from the word PROBABILITY.
  a How many letters are there in total?
  b Find the chance (i.e. probability) of selecting:
    i the letter R **ii** the letter B **iii** a vowel
    iv not a vowel **v** a T or an I **vi** neither a B nor a P

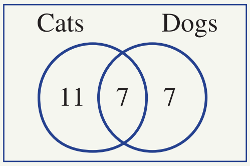
2 A spinning wheel has eight equal sectors numbered 1 to 8. On one spin of the wheel, find the following probabilities.



  a Pr(5) **b** Pr(even)
  c Pr(not even) **d** Pr(multiple of 3)
  e Pr(factor of 12) **f** Pr(odd or a factor of 12)
  g Pr(both odd and a factor of 12)

3 Arrange from lowest to highest: $\frac{1}{2}$, 0.4, 1 in 5, 39%, $\frac{3}{4}$, 1, 0, $\frac{9}{10}$, 0.62, 71%.

4 This Venn diagram shows the number of people in a group of 25 who own cats and/or dogs.



  a State the number of people who own:
    i a dog **ii** a cat or a dog (including both)
    iii only a cat
  b If a person is selected at random from this group, find the probability that they will own:
    i a cat **ii** a cat and a dog **iii** only a dog

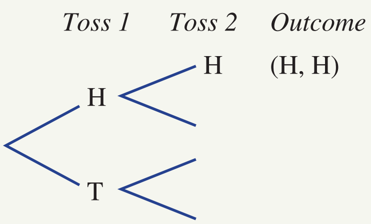
5 Drew shoots from the free-throw line on a basketball court. After 80 shots he counts 35 successful throws.
  a Estimate the probability, in simplified form, that his next throw will be successful.
  b Estimate the probability, in simplified form, that his next throw will not be successful.

6 Two 4-sided dice are rolled and the sum of the two numbers obtained is noted. Copy and complete this grid to help answer the following.

Roll 2 \ Roll 1	1	2	3	4
1				
2				
3				
4				

  a What is the total number of outcomes?
  b Find the probability that the total sum is:
    i 2
    ii 4
    iii less than 5
    iv less than or equal to 5
    v at most 6
    vi no more than 3

7 Two coins are tossed. Copy and complete this tree diagram to help answer the following.



  a State the total number of outcomes.
  b Find the probability of obtaining:
    i 2 heads
    ii no heads
    iii 1 tail
    iv at least 1 tail
    v 1 of each, a head and a tail
    vi at most 2 heads

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4A Review of probability CONSOLIDATING

Learning intentions

- To understand the idea of chance and how to describe it numerically
- To know that the level of chance is based on a numerical value
- To be able to find the probability of an event for equally likely outcomes
- To be able to calculate an experimental probability

Key vocabulary: theoretical probability, experimental probability, trial, sample space, outcome, event, chance, long run proportion

Probability is an area of mathematics concerned with the likelihood of particular random events. In some situations, such as rolling a die, we can determine theoretical probabilities because we know the total number of outcomes and the number of favourable outcomes. In other cases we can use statistics and experimental results to describe the chance that an event will occur. The chance that a particular soccer team will win its next match, for example, could be estimated using various results from previous games.

Lesson starter: Name the event

For each number below, describe an event that has that exact or approximate probability.
If you think it is exact, give a reason.

$\frac{1}{2}$ $\quad$ 25% $\quad$ 0.2 $\quad$ 0.00001 $\quad$ $\frac{99}{100}$

Key ideas

- Definitions:
  - **Probability** is the likelihood of an event happening.
  - A **trial** is a single undertaking of an experiment, such as a single roll of a die.
  - The **sample space** is the list of outcomes from an experiment, such as $\{1, 2, 3, 4, 5, 6\}$ from rolling a 6-sided die.
  - An **outcome** is a possible result of an experiment, such as a 6 on the roll of a die.
  - An **event** is the list of favourable outcomes, such as a 5 or 6 from the roll of a die.
  - Equally likely outcomes are possible results that have the same chance of occurring.
- In the study of probability, a numerical value based on a scale from 0 to 1 is used to describe levels of **chance** – the likelihood of an event happening.

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
impossible	unlikely				even chance			likely		certain

- A probability can be written as a decimal, fraction or percentage; e.g. 0.125, $\frac{1}{8}$ or 12.5%.

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- The **theoretical probability** of an event in which outcomes are equally likely is calculated as follows:

$$\text{Pr(Event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- **Experimental probability** is calculated in the same way as theoretical probability but uses the results of an experiment.

$$\text{Pr(Event)} = \frac{\text{number of favourable outcomes}}{\text{total number of trials}}$$

  - The **long run proportion** is the experimental probability for a sufficiently large number of trials.

Exercise 4A

Understanding	1–3	3

1 Complete the following by filling in the blanks.

a A list of all the possible outcomes from an experiment is called the ________________.

b The numerical values used to describe levels of chance are between ________ and ________.

c An event with a probability of 0.8 would be described as ________ to occur.

d Obtaining a tail from the toss of a coin is called an ________ of the experiment.

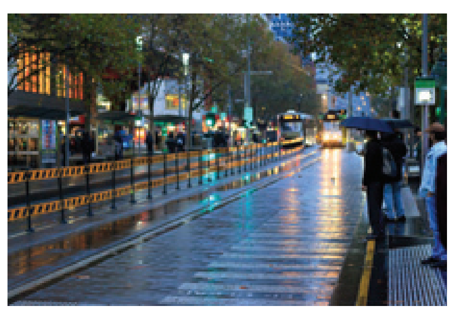
2 Order these events (**A–D**) from least likely to most likely.

A The chance that it will rain every day for the next 10 days.

B The chance that a member of class is ill on the next school day.

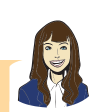
C The chance that school is cancelled next year.

D The chance that the Sun comes up tomorrow.

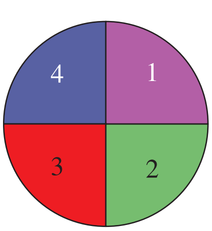


3 For the following spinners, find the probability that the outcome will be a 4.

Hint: Since each section is equal, $\text{Pr}(4) = \frac{\text{number of 4s}}{\text{total number of sections}}$



a



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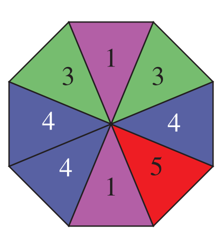

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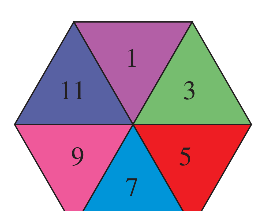

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Fluency | 4–6 | 4, 6, 7

Example 1 Calculating simple theoretical probabilities

A letter is chosen randomly from the word TELEVISION.

a How many letters are there in the word TELEVISION?

b Find the probability that the letter is:

- **i** a V
- **ii** an E
- **iii** not an E
- **iv** an E or a V

Solution	Explanation
a 10	The sample space includes 10 letters.
b i $\Pr(V) = \frac{1}{10} (= 0.1)$	$\Pr(V) = \frac{\text{number of Vs}}{\text{total number of letters}}$
ii $\Pr(E) = \frac{2}{10}$	There are 2 Es in the word TELEVISION.
$= \frac{1}{5} (= 0.2)$	Simplify the fraction.
iii $\Pr(\text{not an E}) = \frac{8}{10}$	If there are 2 Es in the word TELEVISION with 10 letters, then there must be 8 letters that are not E.
$= \frac{4}{5} (= 0.8)$	
iv $\Pr(\text{an E or a V}) = \frac{3}{10} (= 0.3)$	The number of letters that are either E or V is 3.

Now you try

A letter is chosen randomly from the word DINNER.

a How many letters are there in the word DINNER?

b Find the probability that the letter is:

- **i** a D
- **ii** an N
- **iii** not an N
- **iv** a D or an N

4 A letter is chosen randomly from the word TEACHER.

a How many letters are there in the word TEACHER?

b Find the probability that the letter is:

- **i** an R
- **ii** an E
- **iii** not an E
- **iv** an R or an E

5 A letter is chosen randomly from the word EXPERIMENT. Find the probability that the letter is:

a an E

b a vowel

c not a vowel

d an X or a vowel

Hint: The vowels are A, E, I, O and U.

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4A

Example 2 Calculating simple experimental probabilities

An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	11	40	36	13

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads
  ii 2 heads
  iii fewer than 2 heads
  iv at least 1 head

Solution

a 36

Explanation

From the table, you can see that 2 heads has a frequency of 36.

b $11 + 40 = 51$

Fewer than 2 means obtaining 0 heads or 1 head.

c **i** $\text{Pr(0 heads)} = \frac{11}{100}$
$= 0.11$

$\text{Pr(0 heads)} = \frac{\text{number of times 0 heads is observed}}{\text{total number of trials}}$

ii $\text{Pr(2 heads)} = \frac{36}{100}$
$= 0.36$

$\text{Pr(2 heads)} = \frac{\text{number of times 2 heads is observed}}{\text{total number of trials}}$

iii Pr(fewer than 2 heads)
$= \frac{11 + 40}{100}$
$= \frac{51}{100} = 0.51$

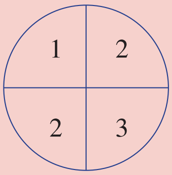
Fewer than 2 heads means to observe 0 or 1 head.

iv Pr(at least 1 head)
$= \frac{40 + 36 + 13}{100}$
$= \frac{89}{100} = 0.89$

At least 1 head means that 1, 2 or 3 heads can be observed.

Now you try

An experiment involves spinning the spinner shown 3 times and counting the number of 2s. Here are the results after running the experiment 100 times.



Number of 2s	0	1	2	3
Frequency	15	34	42	9

a How many times did one 2 occur?
b How many times did more than one 2 occur?
c Find the experimental probability of obtaining:
  i no 2s
  ii three 2s
  iii fewer than two 2s
  iv at least one 2

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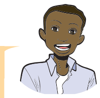
6 An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.



Hint: The total number of outcomes is 100.

Number of heads	0	1	2	3
Frequency	9	38	43	10

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
- **i** 0 heads
- **ii** 2 heads
- **iii** fewer than 2 heads
- **iv** at least 1 head

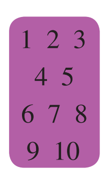
7 An experiment involves rolling two dice and counting the number of 6s. Here are the results after running the experiment 100 times.

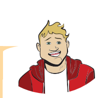
Number of 6s	0	1	2
Frequency	62	35	3

Find the experimental probability of obtaining:
a no 6s
b two 6s
c fewer than two 6s
d at least one 6

Problem-solving and reasoning | 8, 9 | 9–11

8 A 10-sided die, numbered 1 to 10, is rolled once. Find these probabilities.





Hint: Prime numbers less than 10 are 2, 3, 5 and 7.

a Pr(8)
b Pr(odd)
c Pr(even)
d Pr(less than 6)
e Pr(prime)
f Pr(3 or 8)
g Pr(8, 9 or 10)

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4A

9 Amelia is a prizewinner in a competition and will be randomly awarded a single prize chosen from a collection of 50 prizes. The type and number of prizes to be handed out are listed below.

Hint: Remember that the total number of prizes is 50.

Prize	Car	Holiday	iPad	Blu-ray player
Number	1	4	15	30

Find the probability that Amelia will be awarded the following.

a a car
b an iPad
c a prize that is not a car

10 Many of the 50 cars inspected at an assembly plant contain faults. The results of the inspection are as follows.

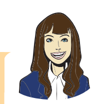
Number of faults	0	1	2	3	4
Number of cars	30	12	4	3	1

Find the experimental probability that a car selected from the assembly plant will have:

a 1 fault
b 4 faults
c fewer than 2 faults
d 1 or more faults
e 3 or 4 faults
f at least 2 faults

11 A bag contains red and yellow counters. A counter is drawn from the bag and then replaced. This happens 100 times and 41 of the counters drawn were red.

Hint: $\frac{41}{100}$ were red.

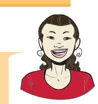


a How many counters drawn were yellow?
b If there were 10 counters in the bag, how many do you expect were red? Give a reason.
c If there were 20 counters in the bag, how many do you expect were red? Give a reason.

Cards probability — 12

12 A card is chosen from a standard deck of 52 playing cards that includes 4 aces, 4 kings, 4 queens and 4 jacks. Find the following probabilities.

Hint: There are 4 suits in a deck of cards: hearts, diamonds, spades and clubs.



a Pr(heart)
b Pr(king)
c Pr(king of hearts)
d Pr(heart or club)
e Pr(king or jack)
f Pr(heart or king)
g Pr(not a king)
h Pr(neither a heart nor a king)

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4B Venn diagrams

Learning intentions

- To understand how a Venn diagram is used to show the distribution of the sample space among events
- To know the notation and regions of a Venn diagram that represent the union, intersection and complement
- To be able to use a Venn diagram to display the distribution of two sets
- To be able to use a Venn diagram to calculate probabilities of events

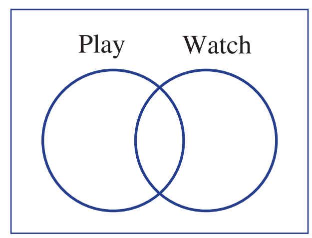
Key vocabulary: Venn diagram, union, intersection, complement, mutually exclusive

Sometimes we need to work with situations where there are overlapping events. A TV station, for example, might be collecting statistics regarding whether or not a person watches cricket and/or tennis or neither over a certain period of time. The estimated probability that a person will watch cricket *or* tennis will therefore depend on how many people responded *yes* to watching both cricket *and* tennis. Venn diagrams are a useful tool when dealing with such events.

Lesson starter: How many like both?

Of 20 students in a class, 12 people like to play tennis and 15 people like to watch tennis. Two people like neither playing nor watching tennis. Some like both playing and watching tennis.

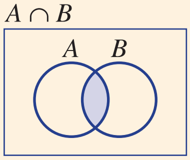
- Is it possible to represent this information in a Venn diagram?
- How many students like to play and watch tennis?
- How many students like to watch tennis only?
- From the group of 20 students, what would be the probability of selecting a person that likes watching tennis only?



Key ideas

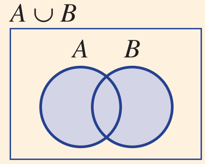
- A **Venn diagram** illustrates how all elements in the sample space are distributed among the events.
  - All elements that belong to both A *and* B make up the **intersection**: $A \cap B$.

    $A \cap B$

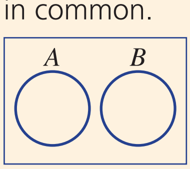
    

  - All elements that belong to either events A *or* B make up the **union**: $A \cup B$.

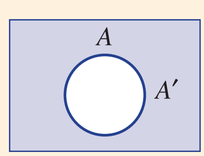
    $A \cup B$

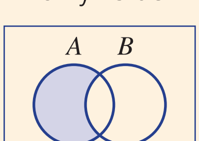
  - Two sets A and B are **mutually exclusive** if they have no elements in common.

  - For an event A, the **complement** of A is A' (or 'not A').
    $\Pr(A') = 1 - \Pr(A)$

  - 'A only' is defined as all the elements in A but not in any other set.

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Exercise 4B

Understanding 1–3 3

1 Match the words in the left column with the description in the right column for two sets.

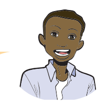
a union — **A** no elements in both sets

b intersection — **B** elements not in the set

c complement — **C** elements in both sets

d mutually exclusive — **D** elements in either set

2 Decide whether the events *A* and *B* are mutually exclusive.



Hint: Mutually exclusive events have nothing in common.

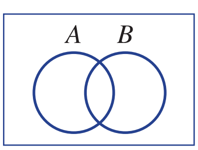
a $A = \{1, 3, 5, 7\}$
$B = \{5, 8, 11, 14\}$

b $A = \{-3, -2, \ldots, 4\}$
$B = \{-11, -10, \ldots, -4\}$

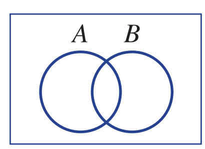
c $A = \{\text{prime numbers}\}$
$B = \{\text{even numbers}\}$

3 Copy these Venn diagrams and shade the region described by each of the following.

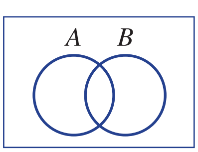
a *A*



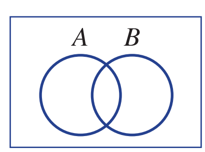
b $A \cap B$ (i.e. *A* and *B*)



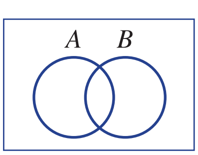
c $A \cup B$ (i.e. *A* or *B*)



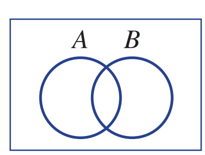
d *B* only



e A' (not *A*)



f neither *A* nor *B*



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Fluency 4–6 5–7

Example 3 Listing sets

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 3, 4, 5, 6\}$ $B = \{1, 3, 7, 8\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets.

i $A \cap B$ (i.e. A and B) **ii** $A \cup B$ (i.e. A or B)

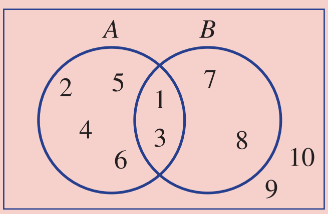
c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why or why not?

Solution / **Explanation**

a



The elements 1 and 3 are common to both sets A and B. The elements 9 and 10 belong to neither set A nor set B.

b **i** $A \cap B = \{1, 3\}$ — $A \cap B$ is the intersection of sets A and B.

ii $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ — $A \cup B$ contains elements in either A or B.

c **i** $\Pr(A) = \frac{6}{10} = \frac{3}{5}$ — There are 6 numbers in A.

ii $\Pr(A \cap B) = \frac{2}{10} = \frac{1}{5}$ — $A \cap B$ contains 2 numbers.

iii $\Pr(A \cup B) = \frac{8}{10} = \frac{4}{5}$ — $A \cup B$ contains 8 numbers.

d The sets A and B are not mutually exclusive since there are numbers inside $A \cap B$. — The set $A \cap B$ contains at least 1 number.

Now you try

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$A = \{2, 4, 6, 8, 10\}$ $B = \{2, 3, 5, 7\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets.

i $A \cap B$ (i.e. A and B) **ii** $A \cup B$ (i.e. A or B)

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** $A \cap B$ **iii** $A \cup B$

d Are the events A and B mutually exclusive? Why or why not?

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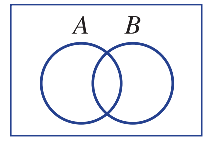
4B

4 Consider the given events A and B, which involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 4, 5, 7, 8, 10\}$ $B = \{2, 3, 5, 6, 8, 9\}$

Hint: $A \cap B$ means A and B. $A \cup B$ means A or B.



a Represent events A and B in a Venn diagram.
b List the following sets.
  i $A \cap B$ ii $A \cup B$
c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.
  i A ii $A \cap B$ iii $A \cup B$
d Are the events A and B mutually exclusive? Why or why not?

5 The elements of the events A and B described below are numbers taken from the first 10 prime numbers.

$A = \{2, 5, 7, 11, 13\}$ $B = \{2, 3, 13, 17, 19, 23, 29\}$

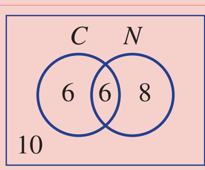
a Represent events A and B in a Venn diagram.
b List the elements belonging to the following.
  i A and B ii A or B
c If a number from the first 10 prime numbers is selected, find the probability that these events occur.
  i A ii B iii $A \cap B$ iv $A \cup B$

Example 4 Using Venn diagrams

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.
a Illustrate this information in a Venn diagram.
b State the number of students who enjoy:
  i netball only ii neither cricket nor netball
c Find the probability that a person chosen randomly from the class will enjoy:
  i netball ii netball only iii both cricket and netball

Solution | **Explanation**

a



First place the 6 in the intersection (i.e. 6 enjoy cricket and netball), then determine the other values according to the given information.

The total must be 30, with 12 in the cricket circle and 14 in netball.

b i 8 — Includes students in N but not in C.
  ii 10 — These are the students outside both C and N.

c i $\Pr(N) = \frac{14}{30} = \frac{7}{15}$ — 14 of the 30 students enjoy netball.

  ii $\Pr(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$ — 8 of the 30 students enjoy netball but not cricket.

  iii $\Pr(C \cap N) = \frac{6}{30} = \frac{1}{5}$ — 6 students like both cricket and netball.

Continued on next page

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Now you try

From a survey of 20 families, 7 enjoy camping (C), 10 enjoy beach holidays (B) and 2 enjoy both camping and beach holidays.

a Illustrate this information in a Venn diagram.
b State the number of families who enjoy:
  i camping only
  ii neither camping nor beach holidays
c Find the probability that a randomly chosen family from the survey enjoys:
  i camping
  ii camping only
  iii both camping and beach holidays

6 From a group of 50 adults, 35 enjoy reading fiction (F), 20 enjoy reading non-fiction (N) and 10 enjoy reading both fiction and non-fiction.

Hint: First enter the '10' in the intersection, then balance all the other regions.

a Illustrate the information in a Venn diagram.
b State the number of people who enjoy:
  i fiction only
  ii neither fiction nor non-fiction
c Find the probability that a person chosen randomly from the group will enjoy reading:
  i non-fiction
  ii non-fiction only
  iii both fiction and non-fiction

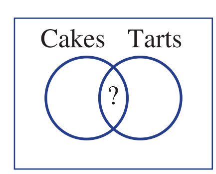
7 At a show, 45 children have the choice of riding on the Ferris wheel (F) and/or the Big Dipper (B). 35 of the children wish to ride on the Ferris wheel, 15 children want to ride on the Big Dipper and 10 children want to ride on both.

a Illustrate the information in a Venn diagram.
b State the number of children who want to:
  i ride on the Ferris wheel only
  ii ride on neither the Ferris wheel nor the Big Dipper
c For a child chosen at random from the group, find the probability that they will want to ride on:
  i the Ferris wheel
  ii both the Ferris wheel and the Big Dipper
  iii the Ferris wheel or the Big Dipper
  iv not the Ferris wheel
  v neither the Ferris wheel nor the Big Dipper

Problem-solving and reasoning

8, 9	8–10

8 In a group of 12 chefs, all enjoy baking cakes and/or tarts. In fact, 7 enjoy baking cakes and 8 enjoy baking tarts. Find out how many chefs from the group enjoy baking both cakes and tarts.



9 In a group of 32 car enthusiasts, all collect either vintage cars or modern sports cars. 18 collect vintage cars and 19 collect modern sports cars. How many from the group collect both vintage cars and modern sports cars?

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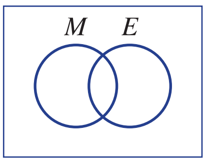
4B

10 Mario and Elisa are choosing a colour to paint the interior walls of their house. They have six colours to choose from: white (w), cream (c), navy (n), sky blue (s), maroon (m) and violet (v).

Mario would be happy with white or cream and Elisa would be happy with cream, navy or sky blue. As they can't decide, a colour is chosen at random for them.

Let M be the event that Mario will be happy with the colour and let E be the event that Elisa will be happy with the colour.

a Represent the events M and E in a Venn diagram.

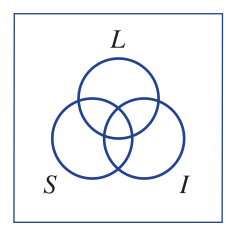


b Find the probability that the following events occur.

- **i** Mario will be happy with the colour choice; i.e. find Pr(M).
- **ii** Mario will not be happy with the colour choice.
- **iii** Both Mario and Elisa will be happy with the colour choice.
- **iv** Mario or Elisa will be happy with the colour choice.
- **v** Neither Mario nor Elisa will be happy with the colour choice.

Courier companies — 11

11 Of 15 chosen courier companies, 9 offer a local service (L), 7 offer an interstate service (S) and 6 offer an international service (I). Two companies offer all three services, 3 offer both local and interstate services, 5 offer only local services and 1 offers only an international service.



a Draw a Venn diagram displaying the given information.

b Find the number of chosen courier companies that offer neither local, interstate nor international services.

c If a courier is chosen at random from the 15 initially examined, find the following probabilities.

- **i** Pr(L)
- **ii** Pr(L only)
- **iii** Pr(L or S)
- **iv** Pr(L and S only)

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4C Two-way tables

Learning intentions

- To know that a two-way table is an alternate way of representing the information in a Venn diagram
- To understand how the rows and columns of a two-way table work
- To be able to fill out a two-way table either from a problem or from a Venn diagram
- To be able to use a two-way table to find associated probabilities

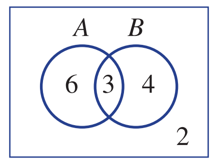
Key vocabulary: two-way table, Venn diagram

Like a Venn diagram, two-way tables are useful tools for the organisation of overlapping events. The totals at the end of each column and row help to find the unknown numbers required to solve various problems.

Lesson starter: Comparing Venn diagrams with two-way tables

Here is a Venn diagram and an incomplete two-way table.

- First, can you complete the two-way table?
- Describe what each box in the two-way table means.
- Was it possible to find all the missing numbers in the two-way table without referring to the Venn diagram?

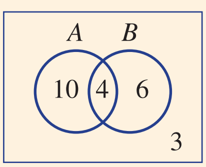


	A	A'	
B		4	
B'			8
	9		15

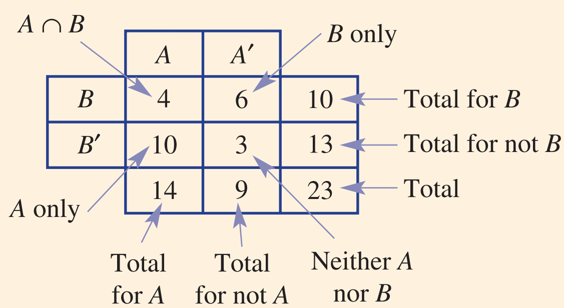
Key ideas

- **Two-way tables** use rows and columns to describe the number of elements in different regions of overlapping events. Each row and column sums to the total at the end.

Venn diagram



Two-way table



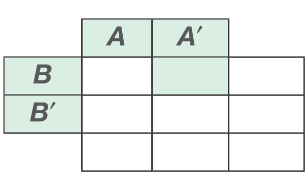
Exercise 4C

Understanding 1, 2 2

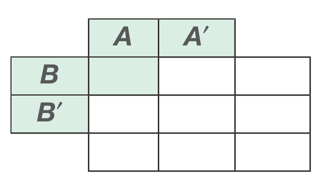
1 Match the shaded two-way tables (**A–D**) with each description (**a–d**).

a $A \cap B$ **b** B only **c** A **d** $A \cup B$

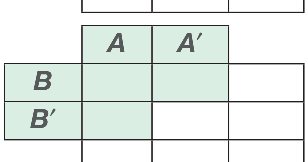
A



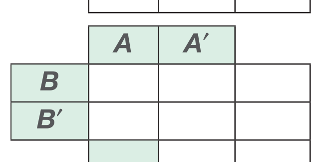
B



C



D



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4C

2 Look at this two-way table.

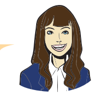
	A	A'	
B	4	3	7
B'	6	1	7
	10	4	14

a State the number of elements in these events.

i A and B **ii** A only
iii B only **iv** neither A nor B
v A **vi** B
vii A' **viii** B′

b $A \cup B$ (i.e. A or B) includes $A \cap B$, A only and B only.
Find the total number of elements in $A \cup B$.

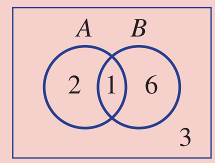
Hint: *A* only is at the intersection of column *A* and row *B*′.



Fluency 3, 4 3–5

Example 5 Using two-way tables

The Venn diagram shows the distribution of elements in two sets, A and B.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements for these regions.

i A and B **ii** B only **iii** A only
iv neither A nor B **v** A **vi** not B
vii A or B

c Find:

i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

Solution

a

	A	A'	
B	1	6	7
B'	2	3	5
	3	9	12

Explanation

	A	A'	
B	$A \cap B$	B only	Total the row
B'	A only	Neither A nor B	Total the row
	Total the column	Total the column	Overall total

b **i** 1 — In both A and B
ii 6 — In B but not A
iii 2 — In A but not B
iv 3 — In neither A nor B
v 3 — Total of A
vi 5 — Total not in B
vii $2 + 1 + 6 = 9$ — In A only or B only or both (3 regions)

c **i** $\Pr(A \cap B) = \frac{1}{12}$

ii $\Pr(A') = \frac{9}{12} = \frac{3}{4}$

iii $\Pr(A \text{ only}) = \frac{2}{12} = \frac{1}{6}$

When calculating probabilities, you will need to divide the number of elements in each set by the number of elements in the sample space, which is 12.

Continued on next page

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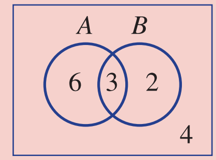
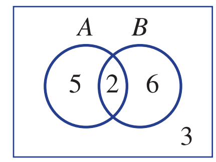
Now you try

The Venn diagram shows the distribution of elements in two sets, *A* and *B*.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements for these regions.
  i *A* and *B* **ii** *B* only **iii** *A* only
  iv neither *A* nor *B* **v** *A* **vi** not *B*
  vii *A* or *B*
c Find:
  i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

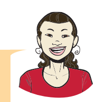
3 The Venn diagram shows the distribution of elements in two sets, *A* and *B*.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements in these regions.
  i *A* and *B* **ii** *B* only **iii** *A* only **iv** neither *A* nor *B*
  v *A* **vi** not *B* **vii** *A* or *B*
c Find:
  i $\Pr(A \cap B)$ **ii** $\Pr(A')$ **iii** $\Pr(A \text{ only})$

4 From a total of 10 people, 5 like apples (*A*), 6 like bananas (*B*) and 4 like both apples and bananas.
a Draw a Venn diagram for the 10 people.
b Draw a two-way table for the 10 people.
c Find the number of people who like:
  i only bananas **ii** apples
  iii apples and bananas **iv** apples or bananas
d Find:
  i $\Pr(B)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A \text{ only})$ **iv** $\Pr(B')$ **v** $\Pr(A \cup B)$

Hint: Once you have your Venn diagram, you can transfer to the two-way table.



5 Of 12 people interviewed at a train station, 7 like staying in hotels, 8 like staying in apartments and 4 like staying in hotels and apartments.
a Draw a two-way table for the 12 people.
b Find the number of people who like:
  i only hotels
  ii neither hotels nor apartments
c Find the probability that one of the people interviewed likes:
  i hotels or apartments
  ii only apartments

Problem-solving and reasoning

6–8 7–10

6 Complete the following two-way tables.

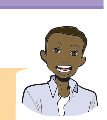
a

	A	*A′*	
B		3	6
B′			
		4	11

b

	A	*A′*	
B	2	7	
B′			3
	4		

Hint: All the rows and columns should add up correctly.



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4C

7 In a class of 24 students, 13 like Mathematics, 9 like English and 3 like both.

a Find the probability that a randomly selected student from the class likes both Mathematics and English.

b Find the probability that a randomly selected student from the class likes neither Mathematics nor English.

8 Two sets, A and B, are mutually exclusive.

a Find $\Pr(A \cap B)$.

b Now complete this two-way table.

	A	A'	
B		6	
B'			12
	10		18

9 Of 32 cars at a show, 18 cars have four-wheel drive, 21 are sports cars and 27 have four-wheel drive or are sports cars.

a Find the probability that a randomly selected car at the show is both four-wheel drive and a sports car.

b Find the probability that a randomly selected car at the show is neither four-wheel drive nor a sports car.

10 A card is selected from a deck of 52 playing cards. Find the probability that the card is:

a a heart or a king

b a club or a queen

c a black card or an ace

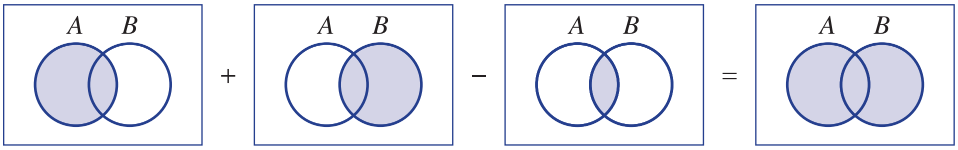
d a red card or a jack

Hint: Make sure you don't count some cards twice; e.g. the king of hearts in part a.

The addition rule — 11

11 For some of the problems above you will have noticed the following, which is called the addition rule.

$$A \quad + \quad B \quad - \quad A \cap B \quad = \quad A \cup B$$



Use the addition rule to find $A \cup B$ in these problems.

a Of 20 people at a sports day, 12 people like archery (A), 14 like basketball (B) and 8 like both archery and basketball ($A \cap B$). How many from the group like archery or basketball?

b Of 100 households, 84 have wide-screen TVs, 32 have high-definition TVs and 41 have both. How many of the households have wide-screen or high-definition TVs?

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4D Conditional probability

Learning intentions

- To understand the notion of conditional probability
- To know the notation of conditional probability and how to calculate it
- To be able to calculate simple conditional probabilities from a Venn diagram or two-way table

Key vocabulary: conditional probability

The mathematics associated with the probability that an event occurs, given that another event has already occurred, is called conditional probability.

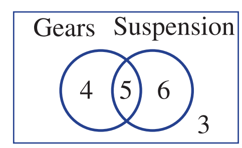
Consider, for example, a group of primary school students who own bicycles. Some of the bicycles have gears, some have suspension and some have both gears and suspension. Consider these two questions.

- What is the probability that a randomly selected bicycle has gears?
- What is the probability that a randomly selected bicycle has gears, given that it has suspension?

The second question is conditional, in that we already know that the bicycle has suspension.

Lesson starter: Gears and suspension

Suppose that, in a group of 18 bicycles, 9 have gears, 11 have suspension and 5 have both gears and suspension. Discuss the solution to the following question by considering the points below.

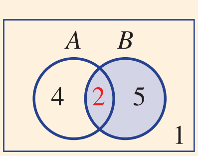


What is the probability that a randomly selected bicycle will have gears, given that it has suspension?

- First look at the information set out in a Venn diagram.
- How many of the bicycles that have suspension also have gears?
- Out of the 11 that have suspension, what is the probability that a bike will have gears?
- What would be the answer to the question in reverse; i.e. what is the probability that a bicycle will have suspension, given that it has gears?

Key ideas

- **Conditional probability** is the probability of an event occurring given that another event has already occurred.
- The probability of event A occurring given that event B has occurred is denoted by $\Pr(A|B)$, which reads 'the probability of A given B'.
- $\Pr(A \text{ given } B) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } B}$ for equally likely outcomes



$\Pr(A|B) = \dfrac{2}{7}$

	A	**A′**	
B	2	5	7
B′	4	1	5
	6	6	12

$\Pr(A|B) = \dfrac{2}{7}$

- $\Pr(B \text{ given } A) = \dfrac{\text{number of elements in } A \cap B}{\text{number of elements in } A}$ for equally likely outcomes

  For the diagrams above, $\Pr(B|A) = \dfrac{2}{6} = \dfrac{1}{3}$.

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Exercise 4D

Understanding 1–3 3

1 Complete the following by filling in the blanks

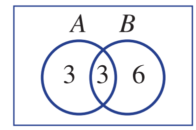
a The probability of A given B is denoted by ________

b Pr(*A* given *B*) = $\dfrac{\text{number of elements in \_\_\_\_\_\_\_\_}}{\text{number of elements in \_\_\_\_\_\_\_\_}}$

2 Consider this Venn diagram.

a What fraction of the elements in *A* are also in *B*? (This finds $\Pr(B|A)$.)

b What fraction of the elements in *B* are also in *A*? (This finds $\Pr(A|B)$.)



3 Use this two-way table to answer these questions.

a What fraction of the elements in *A* are also in *B*? (This finds $\Pr(B|A)$.)

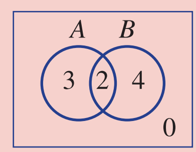
b What fraction of the elements in *B* are also in *A*? (This finds $\Pr(A|B)$.)

	A	**A′**	
B	7	5	12
B′	3	1	4
	10	6	16

Fluency 4, 5 4–6

Example 6 Finding conditional probabilities using a Venn diagram

Consider this Venn diagram, displaying the number of elements belonging to the events *A* and *B*.



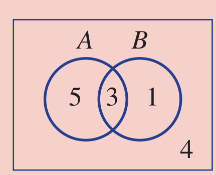
Find the following probabilities.

a $\Pr(A)$ **b** $\Pr(A \cap B)$ **c** $\Pr(A|B)$ **d** $\Pr(B|A)$

Solution	Explanation
a $\Pr(A) = \dfrac{5}{9}$	There are 5 elements in *A* and 9 in total.
b $\Pr(A \cap B) = \dfrac{2}{9}$	There are 2 elements common to *A* and *B*.
c $\Pr(A|B) = \dfrac{2}{6} = \dfrac{1}{3}$	2 of the 6 elements in *B* are in *A*.
d $\Pr(B|A) = \dfrac{2}{5}$	2 of the 5 elements in *A* are in *B*.

Now you try

Consider this Venn diagram, displaying the number of elements belonging to the events *A* and *B*.



Find the following probabilities.

a $\Pr(A)$ **b** $\Pr(A \cap B)$ **c** $\Pr(A|B)$ **d** $\Pr(B|A)$

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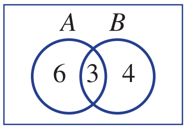
4 The following Venn diagrams display information about the number of elements associated with the events A and B. For each Venn diagram, find:

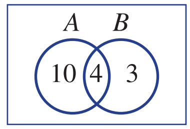
i $\Pr(A)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A|B)$ **iv** $\Pr(B|A)$

Hint:
- $A \cap B$ means both A and B.
- $\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$
- $\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

a



b



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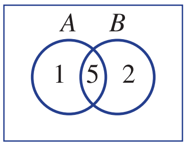

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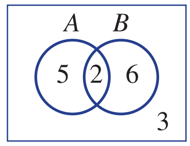

Example 7 Finding conditional probabilities using a two-way table

From a group of 15 hockey players at a game of hockey, 13 played on the field, 7 sat on the bench and 5 both played and sat on the bench.
A hockey player is chosen at random from the team.
Let A be the event 'the person played on the field' and B be the event 'the person sat on the bench'.

a Represent the information in a two-way table.
b Find the probability that the person only sat on the bench.
c Find the probability that the person sat on the bench, given that they played on the field.
d Find the probability that the person played on the field, given that they sat on the bench.

Solution | **Explanation**

a

	A	**A′**	
B	5	2	7
B′	8	0	8
	13	2	15

$A \cap B$ has 5 elements, A has a total of 13 and B a total of 7. There are 15 players in total.

b $\Pr(\text{bench only}) = \dfrac{2}{15}$

Two people sat on the bench and did not play on the field.

c $\Pr(B|A) = \dfrac{5}{13}$

$\Pr(B|A) = \dfrac{\text{number in } A \cap B}{\text{number in } A}$

d $\Pr(A|B) = \dfrac{5}{7}$

$\Pr(A|B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

Now you try

From a class of 20 students, 12 own a cat, 10 own a dog and 5 owned both a cat and a dog.
A student is chosen at random from the class.
Let A be the event 'the student owns a cat' and B be the event 'the student owns a dog'.

a Represent the information in a two-way table.
b Find the probability that the student only owns a dog.
c Find the probability that the student owns a dog, given that they own a cat.
d Find the probability that the student owns a cat, given that they own a dog.

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4D

5 Of a group of 20 English cricket fans at a match, 13 purchased a pie, 15 drank beer and 9 purchased a pie *and* drank beer.
Let A be the event 'the fan purchases a pie'.
Let B be the event 'the fan drank beer'.

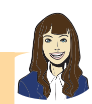
a Copy and complete this two-way table.

	A	A'	
B	9		
B'			
			20

b Find the probability that a fan only purchased a pie (and did not drink beer).
c Find the probability that a fan purchased a pie, given that they drank beer.
d Find the probability that a fan drank beer, given that they purchased a pie.

6 The following two-way tables show information about the number of elements in the events A and B.
For each two-way table, find:

i $\Pr(A)$
ii $\Pr(A \cap B)$
iii $\Pr(A | B)$
iv $\Pr(B | A)$



Hint: First decide on the total that gives the denominator of your fraction.

a

	A	A'	
B	2	8	10
B'	5	3	8
	7	11	18

b

	A	A'	
B	1	4	5
B'	3	1	4
	4	5	9

c

	A	A'	
B	7	3	10
B'	1	6	7
	8	9	17

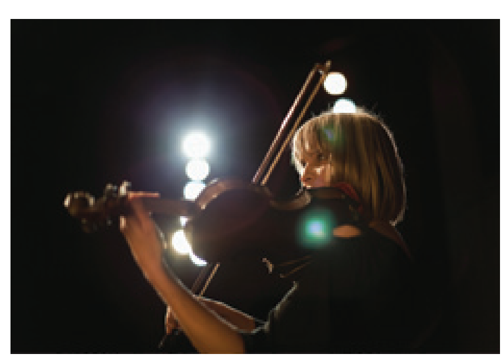
d

	A	A'	
B	4	2	6
B'	8	2	10
	12	4	16

Problem-solving and reasoning 7 7–9

7 Of 15 musicians surveyed to find out whether they play the violin or the piano, 5 play the violin, 8 play the piano and 2 play both instruments.

a Represent the information in a Venn diagram.
b How many of the musicians surveyed do not play either the violin or the piano?
c Find the probability that one of the 15 musicians surveyed plays piano, given that they play the violin.
d Find the probability that one of the 15 musicians surveyed plays the violin, given that they play piano.



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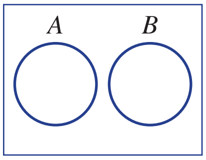
8 A card is drawn from a deck of 52 playing cards. Find the probability that:
   a the card is a king given that it is a heart
   b the card is a jack given that it is a red card

Hint: 13 of the cards are hearts. There are 4 kings, including one king of hearts.

9 Two events, A and B, are mutually exclusive. What can be said about the probability of A given B (i.e. $\Pr(A|B)$) or the probability of B given A (i.e. $\Pr(B|A)$)? Give a reason.



Cruise control and airbags — 10

10 On a car production line, 30 cars are due to be completed by the end of the day. Fifteen of the cars have cruise control and 20 have airbags, and 6 have both cruise control and airbags.
   a Represent the information provided in a Venn diagram or two-way table.
   b Find the probability that a car chosen at random will have:
      i cruise control only ii airbags only
   c Given that the car chosen has cruise control, find the probability that the car will have airbags.
   d Given that the car chosen has airbags, find the probability that the car will have cruise control.

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4E Using tables for two-step experiments

Learning intentions

- To know how to list the sample space of a two-step experiment in a table
- To understand the difference between experiments carried out with replacement and without replacement
- To be able to construct tables for two-step experiments with and without replacement and find associated probabilities

Key vocabulary: with replacement, without replacement, two-step experiments, sample space

Some experiments contain more than one step and are called multi-stage experiments. Examples include rolling a die twice, selecting a number of chocolates from a box or choosing people at random to fill positions on a committee. Tables can be used to list all the outcomes from two-step experiments. The number of outcomes depend on whether or not the experiment is conducted with or without replacement.

Lesson starter: Two prizes, three people

Two special prizes are to be awarded in some way to Bill, May and Li for their efforts in helping at the school fete. This table shows how the prizes might be awarded.

		2nd prize		
		Bill	May	Li
	Bill	(B, B)	(B, M)	(B, L)
1st prize	May	(M, B)		
	Li			

- Complete the table to show how the two prizes can be awarded.
- Does the table show that the same person can be awarded both prizes?
- What is the probability that Bill and Li are both awarded a prize?
- How would the table change if the same person could not be awarded both prizes?
- How do the words ‘with replacement’ and ‘without replacement’ relate to the situation above? Discuss.

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Key ideas

- A **two-step experiment** involves two stages of an experiment eg. tossing a coin twice.
- Tables are used to list the sample space for two-step experiments.
- If replacement is allowed, then outcomes from each selection can be repeated, and such experiments are called **with replacement**.
- If selections are made **without replacement**, then outcomes from each selection cannot be repeated.
  For example, two selections are made from the digits $\{1, 2, 3\}$.

With replacement

		1st		
		1	2	3
2nd	1	(1, 1)	(2, 1)	(3, 1)
	2	(1, 2)	(2, 2)	(3, 2)
	3	(1, 3)	(2, 3)	(3, 3)

9 outcomes

Without replacement

		1st		
		1	2	3
2nd	1	×	(2, 1)	(3, 1)
	2	(1, 2)	×	(3, 2)
	3	(1, 3)	(2, 3)	×

6 outcomes

Exercise 4E

Understanding 1–3 1, 3

1 Choose either *with replacement* or *without replacement* to complete the following.

a Two chocolates are selected from a box and eaten. This is an example of ________________.

b Two cards are selected from a pack one after the other and their suit recorded. Each card is returned to the pack after its suit is recorded. This is an example of ________________.

2 Two letters are chosen from the word DOG.

a Complete a table listing the sample space if selections are made:

i with replacement

		1st		
		D	O	G
2nd	D	(D, D)	(O, D)	
	O			
	G			

ii without replacement

		1st		
		D	O	G
2nd	D	×	(O, D)	
	O		×	
	G			×

b State the total number of outcomes if selection is made:

i with replacement **ii** without replacement

3 Two digits are selected from the set $\{2, 3, 4\}$ to form a two-digit number. Find the number of two-digit numbers that can be formed if the digits are selected:

a with replacement

	2	3	4
2	22	32	
3			
4			

b without replacement

	2	3	4
2	×	32	
3		×	
4			×

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4E

Fluency 4–6 4–6

Example 8 Constructing a table with replacement

A fair 6-sided die is rolled twice.

a List all the outcomes, using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome $(1, 5)$.

d Find:

i Pr(double) **ii** Pr(sum of at least 10) **iii** Pr(sum not equal to 7)

Solution

a

		Roll 2					
		1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Roll 1	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Explanation

Be sure to place the number from roll 1 in the first position for each outcome.

b 36 outcomes

There is a total of $6 \times 6 = 36$ outcomes.

c $\text{Pr}(1, 5) = \frac{1}{36}$

Only one outcome is $(1, 5)$.

d **i** $\text{Pr(double)} = \frac{6}{36}$

$= \frac{1}{6}$

Six outcomes have the same number repeated.

ii $\text{Pr(sum of at least 10)} = \frac{6}{36} = \frac{1}{6}$

Six outcomes have a sum of either 10, 11 or 12.

iii $\text{Pr(sum not equal to 7)} = 1 - \frac{6}{36}$

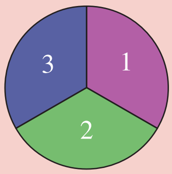
$= \frac{30}{36}$

$= \frac{5}{6}$

This is the complement of having a sum of 7.
Six outcomes have a sum of 7.
$\text{Pr(not } A) = 1 - \text{Pr}(A)$

Now you try

The spinner shown is spun twice.



a List all the outcomes, using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome $(3, 2)$.

d Find:

i Pr(same 2 numbers) **ii** Pr(product is more than 3) **iii** Pr(product is not equal to 9)

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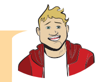
4 A fair 4-sided die is rolled twice.

a List all the outcomes, using a table.

		1st			
		1	2	3	4
2nd	1	(1, 1)	(2, 1)		
	2				
	3				
	4				



Hint: Questions **4** and **5** are making selections 'with replacement' because outcomes can be repeated.

b State the total number of possible outcomes.

c Find the probability of obtaining the outcome $(2, 4)$.

d Find the probability of:

i a double **ii** a sum of at least 5 **iii** a sum not equal to 4

5 Two coins are tossed, each landing with a head (H) or tail (T).

a List all the outcomes, using a table.

		1st	
		H	T
2nd	H	(H, H)	(T, H)
	T		

b State the total number of possible outcomes.

c Find the probability of obtaining the outcome (H, T).

d Find the probability of obtaining:

i exactly one tail **ii** at least one tail

e If the two coins are tossed 1000 times, how many times would you expect to get two tails?

Example 9 Constructing a table without replacement

Two letters are chosen from the word KICK, without replacement.

a Construct a table to list the sample space.

b Find the probability of:

i obtaining the outcome (K, C) **ii** selecting two letters that are K **iii** selecting a K and a C

Solution

a

		1st			
		K	I	C	K
2nd	K	×	(I, K)	(C, K)	(K, K)
	I	(K, I)	×	(C, I)	(K, I)
	C	(K, C)	(I, C)	×	(K, C)
	K	(K, K)	(I, K)	(C, K)	×

Explanation

Selection is without replacement, so the same letter (from the same position) cannot be chosen twice.

b **i** $\Pr(\text{K, C}) = \frac{2}{12}$

$= \frac{1}{6}$

Two of the 12 outcomes are (K, C).

ii $\Pr(\text{K, K}) = \frac{2}{12}$

$= \frac{1}{6}$

Two of the outcomes are K and K, which use different Ks from the word KICK.

iii $\Pr(\text{K and C}) = \frac{4}{12}$

$= \frac{1}{3}$

Four outcomes contain a K and a C.

Continued on next page

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4E

Now you try

Two letters are chosen from the word TREE, without replacement.

a Construct a table to list the sample space.

b Find the probability of:

i obtaining the outcome (T, E) ii selecting two letters that are E iii selecting an E and a T

6 Two letters are chosen from the word SETS, without replacement.

a Complete this table to list the sample space.

b Find the probability of:

i obtaining the outcome (E, S)

ii selecting one T

iii selecting two letters that are S

iv selecting an S and a T

v selecting an S or a T

		1st			
		S	E	T	S
2nd	S	×	(E, S)	(T, S)	(S, S)
	E		×		
	T			×	
	S				×

Problem-solving and reasoning 7, 8 8–10

7 A letter is chosen from the word LEVEL without replacement and then a second letter is chosen from the same word.

Hint: Remember that this is 'without replacement'.

a Draw a table displaying the sample space for the pair of letters chosen.

b State the total number of outcomes possible.

c State the number of outcomes that contain exactly one of the following letters.

i V ii L iii E

d Find the probability that the outcome will contain exactly one of the following letters.

i V ii L iii E

e Find the probability that the two letters chosen will be the same.

8 In a quiz, Min guessed that the probability of rolling a sum of 10 or more from 2 six-sided dice is 10%. Complete the following to decide whether or not this guess is correct.

a Copy and complete the table representing all the outcomes for possible totals that can be obtained.

b State the total number of outcomes.

c Find the number of the outcomes that represent a sum of:

i 3 ii 7 iii less than 7

d Find the probability that the following sums are obtained.

i 7

ii less than 5

iii greater than 2

iv at least 11

e Find the probability that the sum is at least 10, and decide whether or not Min's guess is correct.

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	…			
	2	3	…				
	3	4					
	4	⋮					
	5	⋮					
	6						

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9 A coin and a six-sided die are tossed. Heads on the coin is worth 2 points and a tail is worth 4 points. These points are added to the score on the die. Use a table to find the probability that the total score is:

a 4

b 6

c greater than 7

d at most 6

10 Decide whether the following situations would naturally involve selections with replacement or without replacement.

a selecting two people to play in a team

b tossing a coin twice

c rolling two dice

d choosing two chocolates to eat

Random weights — 11

11 In a gym, Justine considers choosing two weights to fit onto a leg weights machine to make the load heavier. She can choose from 2.5 kg, 5 kg, 10 kg or 20 kg, and there are plenty of each weight available. Justine's friend randomly chooses both weights, with equal probability that she will choose each weight, and places them on the machine. Justine then attempts to operate the machine without knowing which weights were chosen.

a Complete a table that displays all possible total weights that could be placed on the machine.

b State the total number of outcomes.

c How many of the outcomes deliver a total weight described by the following?

i equal to 10 kg

ii less than 20 kg

iii at least 20 kg

d Find the probability that Justine will be attempting to lift the following weights?

i 20 kg

ii 30 kg

iii no more than 10 kg

iv less than 10 kg

e If Justine is unable to lift more than 22 kg, what is the probability that she will not be able to operate the leg weights machine?

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Progress quiz

4A **1** A letter is chosen from the word ELEPHANT. Find the probability that the letter is:
a an E **b** a T or an E **c** not an E **d** a vowel

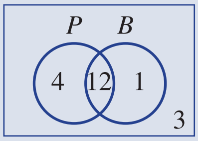
4A **2** An experiment involves rolling two dice and counting the number of even numbers. Here are the results after running the experiment 100 times.

Number of even numbers	0	1	2
Frequency	21	46	33

a How many times did more than 1 even number occur?
b Find the experimental probability of obtaining:
  i 0 even numbers
  ii fewer than 2 even numbers
  iii at least 1 even number

4B/D **3** From a group of 25 students on a school camp, 18 enjoy sailing (S), 15 enjoy bushwalking (B) and 8 enjoy both sailing and bushwalking.
a Illustrate this information in a Venn diagram.
b State the number of students who enjoy:
  i sailing only
  ii neither sailing nor bushwalking
c Find the following probabilities for a student chosen at random from the group.
  i $\Pr(B)$ **ii** $\Pr(S \text{ only})$ **iii** $\Pr(B \cap S)$
  iv $\Pr(B')$ **v** $\Pr(S|B)$

4C/D **4** The Venn diagram shown shows the distribution of 20 guests staying at a resort in Noosa. Some guests liked to swim at the hotel pool (P), others liked to swim at the beach (B) and others liked both.



a Transfer the information to a two-way table.
b Find the number of guests who like:
  i only swimming at the hotel pool
  ii swimming at either the beach or the hotel pool
c Find:
  i $\Pr(P)$ **ii** $\Pr(B \text{ only})$ **iii** $\Pr(P \cap B)$ **iv** $\Pr(P|B)$

4E **5** A fair six-sided die is rolled and a coin is tossed.
a List all the outcomes, using a table.
b State the total number of outcomes.
c Find the probability of obtaining the outcome (3, H).
d Find the probability of obtaining:
  i an even number and a tail
  ii at least a 2 and a head

	1	2	3	4	5	6
H	(1, H)	(2, H)...				
T						

4E **6** Two counters are chosen randomly from a bag containing 2 red, 1 blue and 1 green counter, without replacement.
a Construct a table to list the sample space and state the number of outcomes.
b Find the probability of:
  i obtaining the outcome (R, G)
  ii selecting 2 red counters
  iii selecting 1 blue and 1 red counter

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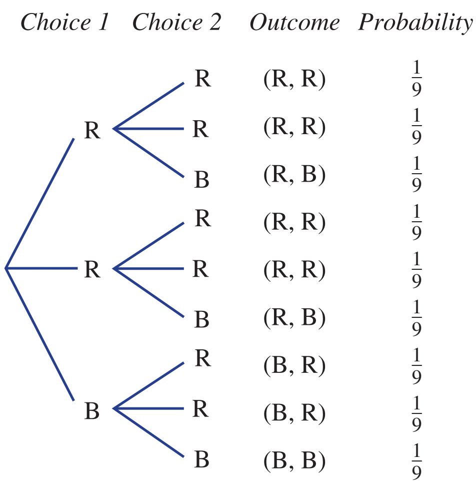
4F Using tree diagrams

Learning intentions

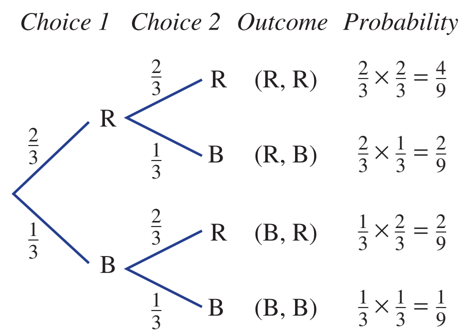
- To know how to use a tree diagram to list the sample space from experiments with two or more components
- To understand the difference between the probabilities on tree diagrams for experiments with replacement and those without replacement
- To be able to use a tree diagram to find the probability of outcomes in experiments

Key vocabulary: tree diagram, with replacement, without replacement, sample space

Tree diagrams can also be used to help list outcomes for multi-stage experiments. Suppose that a bag contains two red counters and one blue counter and that two counters are selected at random with replacement. One way to display the outcomes is with a tree diagram in which all equally likely outcomes are listed in columns, as shown below left. A more efficient way, however, is to group similar outcomes and write their corresponding probabilities on the branches, as shown below right.



$\Pr(R, B) = \frac{2}{9}$



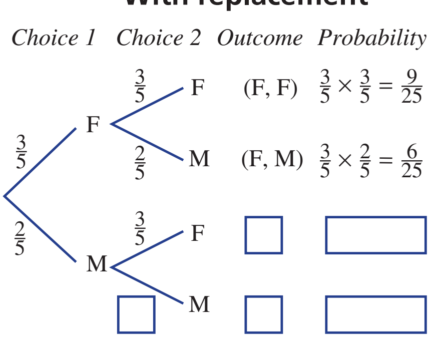
$\Pr(R, B) = \frac{2}{9}$

In the tree diagram on the right, the probability of each outcome is obtained by multiplying the branch probabilities. This also applies when selection is made without replacement.

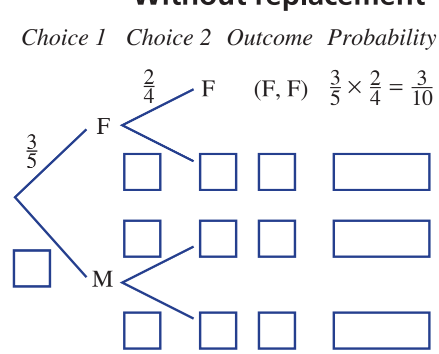
Lesson starter: Trees with and without replacement

Suppose that two selections are made from a group of 2 male and 3 female workers to complete two extra tasks.

With replacement



Without replacement



- Complete these two tree diagrams to show how these selections can be made, both with and without replacement.
- Explain where the branch probabilities come from on each branch of the tree diagrams.
- What is the total of all the probabilities on each tree diagram?

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Key ideas

- **Tree diagrams** can be used to list the sample space for experiments involving two or more components.
  - Branch probabilities are used to describe the chance of each outcome at each step.
  - The probability of each outcome for the experiment is obtained by multiplying the branch probabilities.
  - Branch probabilities will depend on whether selection is made with or without replacement. For experiments *with replacement*, probabilities do not change. For experiments *without replacement*, probabilities do change.

Exercise 4F

Understanding 1, 2 2

1 A coin is tossed three times and a head or tail is obtained each time as shown in the tree diagram.

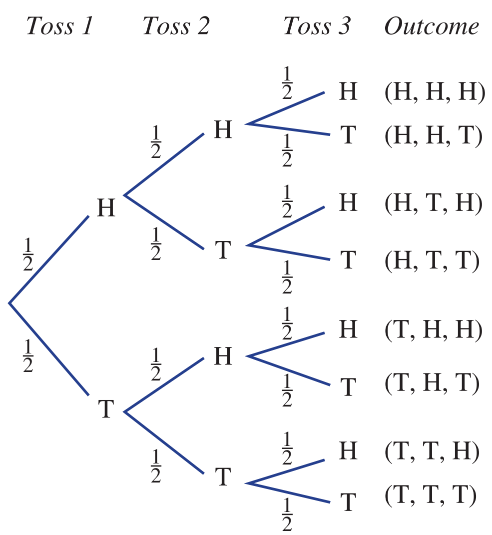
a How many outcomes are there?

b What is the probability of the outcome HHH?

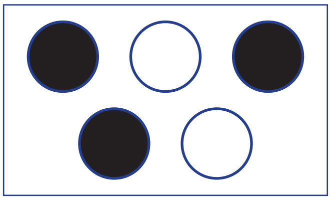
c How many outcomes obtain:

i 2 tails?

ii 2 or 3 heads?



2 A box contains 2 white (W) and 3 black (B) counters.



a A single counter is drawn at random. Find the probability that it is:

i white **ii** black

b Two counters are now drawn at random. The first one is replaced before the second one is drawn. Find the probability that the second counter is:

i white **ii** black

c Two counters are drawn and the first counter is not replaced before the second one is drawn. If the first counter is white, find the probability that the second counter is:

i white **ii** black

Hint: After one white counter is taken out, how many of each remain?

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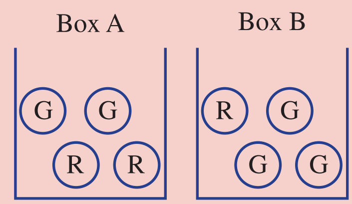
Fluency 3, 4 | 3–5

Example 10 Constructing a tree diagram for multi-stage experiments

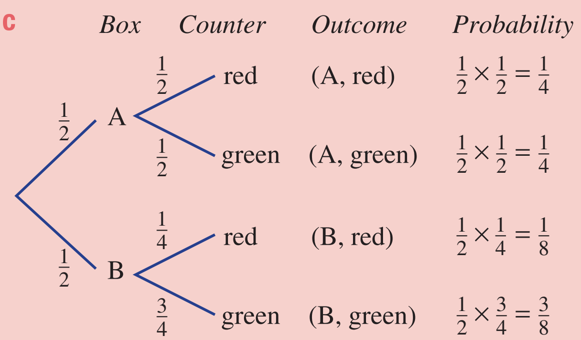
Boxes A and B contain 4 counters each. Box A contains 2 red and 2 green counters and box B contains 1 red and 3 green counters. A box is chosen at random and then a single counter is selected.

a What is the probability of selecting a red counter from box A?
b What is the probability of selecting a red counter from box B?
c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.
d What is the probability of selecting box B and a red counter?
e What is the probability of selecting a red counter?



Solution	Explanation
a Pr(red from box A) $= \frac{2}{4} = \frac{1}{2}$	2 of the 4 counters in box A are red.
b Pr(red from box B) $= \frac{1}{4}$	1 of the 4 counters in box B is red.

c



First selection is a box followed by a counter.
Multiply each of the probabilities along the branch pathways to find the probability of each outcome.

d Pr(B, red) $= \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{8}$

The probability of choosing box B is $\frac{1}{2}$ and a red counter from box B is $\frac{1}{4}$, so multiply the probabilities for these two outcomes together.

e Pr(1 red) $= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{8}$

$= \frac{3}{8}$

The outcomes (A, red) and (B, red) both contain 1 red counter, so add together the probabilities for these two outcomes.

Continued on next page

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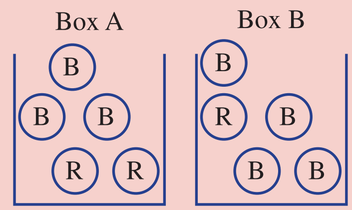
Now you try

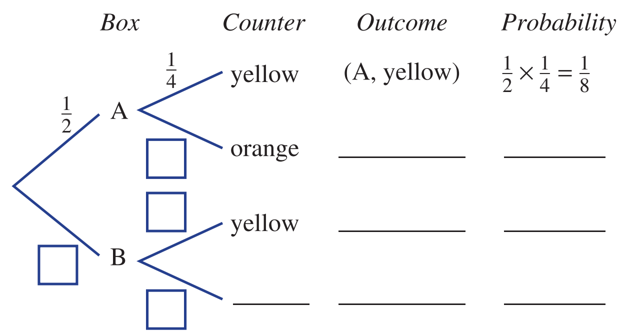
Boxes A and B contain 5 counters each. Box A contains 2 red and 3 blue counters and box B contains 1 red and 4 blue counters.



a What is the probability of selecting a red counter from box A?
b What is the probability of selecting a red counter from box B?
c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.
d What is the probability of selecting box B and a red counter?
e What is the probability of selecting a red counter?

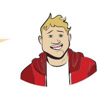
3 Boxes A and B contain 4 counters each. Box A contains 1 yellow and 3 orange counters and box B contains 3 yellow and 1 orange counter. A box is chosen at random and then a single counter is selected.

a If box A is chosen, what is the probability of selecting a yellow counter?
b If box B is chosen, what is the probability of selecting a yellow counter?
c Represent the options available by completing this tree diagram.



d What is the probability of selecting box B and a yellow counter?
e What is the probability of selecting 1 yellow counter?

Hint: For part **e**, add the probabilities for both outcomes that have a yellow counter.



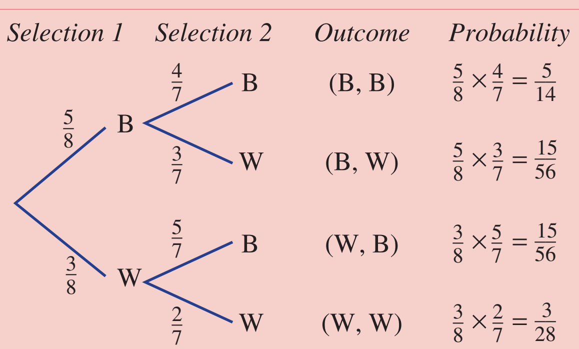
Example 11 Using a tree diagram without replacement

A bag contains 5 blue (B) and 3 white (W) marbles and two marbles are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.
b Find the probability of selecting:
  i a blue marble followed by a white marble; i.e. the outcome (B, W)
  ii 2 blue marbles
  iii exactly one blue marble
c If the experiment is repeated with replacement, find the answers to each question in part **b**.

Solution

a



Explanation

After one blue marble is selected there are 7 marbles remaining: 4 blue and 3 white.

After one white marble is selected there are 7 marbles remaining: 5 blue and 2 white.

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b **i** $\Pr(B, W) = \frac{5}{8} \times \frac{3}{7}$

$= \frac{15}{56}$

Multiply the probabilities on the (B, W) pathway.

ii $\Pr(B, B) = \frac{5}{8} \times \frac{4}{7}$

$= \frac{5}{14}$

Only 4 blue marbles remain after the first selection. Multiply the probabilities on the (B, B) pathway.

iii $\Pr(1 \text{ blue}) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$

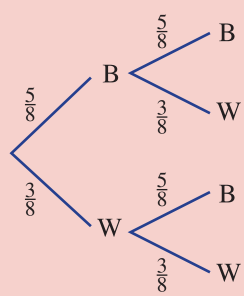
$= \frac{15}{28}$

The outcomes (B, W) and (W, B) both have 1 blue marble. Multiply probabilities to find individual probabilities, then sum for the final result.

c **i** $\Pr(B, W) = \frac{5}{8} \times \frac{3}{8}$

$= \frac{15}{64}$

When selecting objects with replacement, remember that the number of marbles in the bag remains the same for each selection.



ii $\Pr(B, B) = \frac{5}{8} \times \frac{5}{8}$

$= \frac{25}{64}$

iii $\Pr(1 \text{ blue}) = \frac{5}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8}$

$= \frac{15}{32}$

Now you try

A jar contains 4 toffees (T) and 3 mints (M) and two lollies are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.

b Find the probability of selecting:

- **i** a toffee followed by a mint: i.e. the outcome (T, M)
- **ii** 2 mints
- **iii** exactly one toffee

c If the experiment is repeated with replacement, find the answers to each question in part **b**.

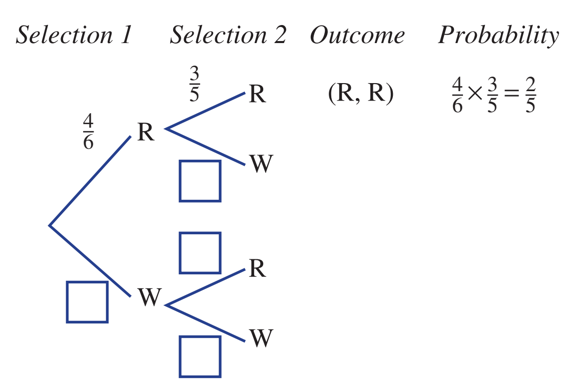
4 A bag contains 4 red (R) and 2 white (W) marbles, and two marbles are selected without replacement.

a Complete this tree diagram, showing all outcomes and probabilities.

b Find the probability of selecting:

- **i** a red marble and then a white marble (R, W)
- **ii** 2 red marbles
- **iii** exactly 1 red marble

c If the experiment is repeated with replacement, find the answers to each question in part **b**. You may need to redraw the tree diagram.



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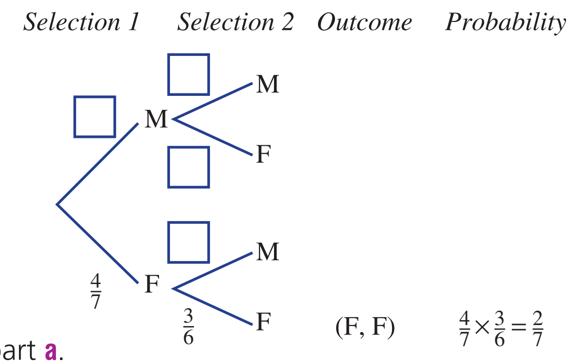
5 Two students are selected from a group of 3 males (M) and 4 females (F), without replacement.

a Complete this tree diagram to help find the probability of selecting:

- **i** 2 males
- **ii** 2 females
- **iii** 1 male and 1 female
- **iv** 2 people either both male or both female

b If the experiment is repeated with replacement, find the answers to each question in part **a**.



Problem-solving and reasoning

Problem-solving and reasoning	6, 7	6–8

6 A fair 4-sided die is rolled twice and the pair of numbers is recorded.

a Complete this tree diagram to list the outcomes.

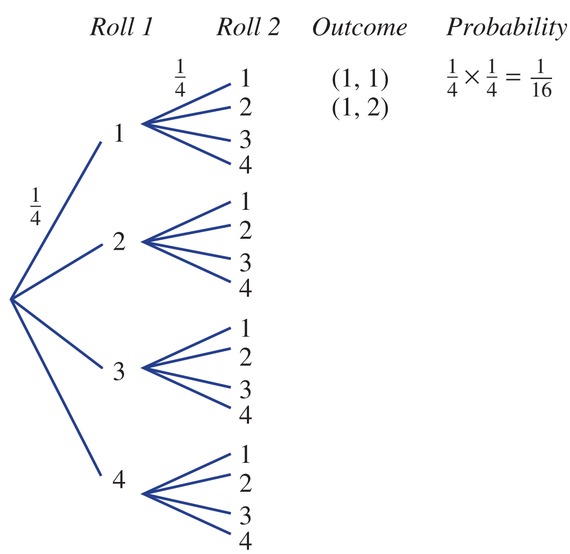
b State the total number of outcomes.

c Find the probability of obtaining:

- **i** a 4 then a 1; i.e. the outcome (4, 1)
- **ii** a double

d Find the probability of obtaining a sum described by the following:

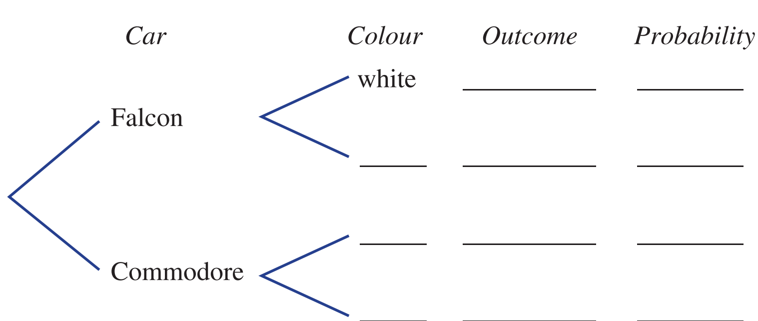
- **i** equal to 2
- **ii** equal to 5
- **iii** less than or equal to 5



7 As part of a salary package, a person can select either a Falcon or a Commodore. There are 3 white Falcons, 1 silver Falcon, 2 white Commodores and 1 red Commodore to choose from.

a Complete a tree diagram showing a random selection of a car make, then a colour.

Hint: Since car make selection is random, Pr(Falcon) $= \frac{1}{2}$.



b Find the probability that the person chooses:

- **i** a white Falcon
- **ii** a red Commodore
- **iii** a white car
- **iv** a car that is not white
- **v** a silver car or a white car
- **vi** a car that is neither a Falcon nor red

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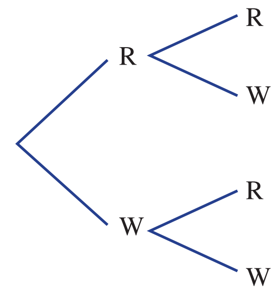
8 Two bottles of wine are randomly selected for tasting from a box containing 2 red and 2 white wines. Use a tree diagram to help answer the following.



a If the first bottle is replaced before the second is selected, find:
  i Pr(2 red)
  ii Pr(1 red)
  iii Pr(not 2 white)
  iv Pr(at least 1 white)

b If the first bottle is not replaced before the second is selected, find:
  i Pr(2 red)
  ii Pr(1 red)
  iii Pr(not 2 white)
  iv Pr(at least 1 white)

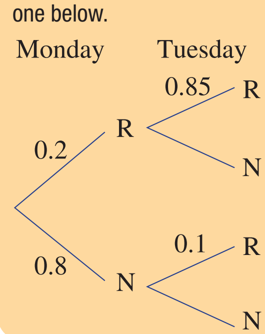
Rainy days — 9

9 Imagine that the probability of rain next Monday is 0.2. The probability of rain on a day after a rainy day is 0.85, whereas the probability of rain on a day after a non-rainy day is 0.1.

a Next Monday and Tuesday, find the probability of having:
  i 2 rainy days
  ii exactly 1 rainy day
  iii at least 1 dry day

b Next Monday, Tuesday and Wednesday, find the probability of having:
  i 3 rainy days
  ii exactly 1 dry day
  iii at most 2 rainy days

Hint: Draw a tree diagram, like the one below.



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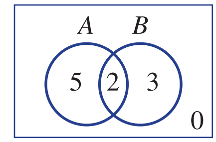
4G Independent events

Learning intentions

- To understand the concept of independent events
- To be able to determine if two events are independent using a Venn diagram or two-way table
- To know how with and without replacement affects independent events

Key vocabulary: independent events, with replacement, without replacement, conditional probability

In previous sections we have looked at problems involving conditional probability. This Venn diagram, for example, gives the following results.

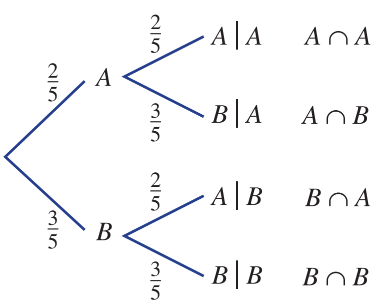


$$\Pr(A) = \frac{7}{10} \text{ and } \Pr(A \mid B) = \frac{2}{5}$$

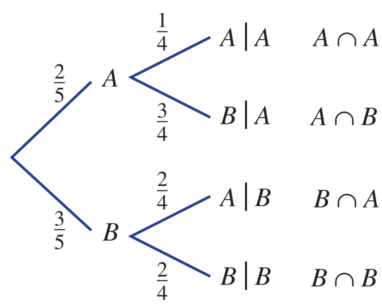
The condition B in $\Pr(A \mid B)$ has changed the probability of A. The events A and B are therefore not independent.

For multi-stage experiments we can consider events either with or without replacement. These tree diagrams, for example, show two selections of marbles from a bag of 2 aqua (A) and 3 blue (B) marbles.

With replacement



Without replacement

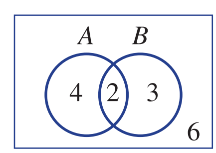


In the first tree diagram $\Pr(A \mid B) = \Pr(A)$, so the events are independent.

In the second tree diagram $\Pr(A \mid B) \neq \Pr(A)$, so the events are not independent.

Lesson starter: Is it the same to be mutually exclusive and independent?

Use the Venn diagram to consider the following questions.



- Are the events mutually exclusive? Why?
- Find $\Pr(A)$ and $\Pr(A \mid B)$. Does this mean that the events A and B are independent?

Key ideas

- Two events are **independent** if the outcome of one event does not change the probability of obtaining the other event.
  - $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
  - $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- For multi-stage experiments where selection is made with replacement, successive events are independent.
- For multi-stage experiments where selection is made without replacement, successive events are not independent.

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Exercise 4G

Understanding 1–3 | 2, 3

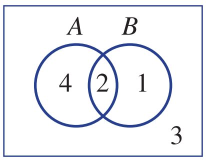
1 State if events A and B are independent for the following probabilities.

a $\Pr(A) = 0.4$, $\Pr(A|B) = 0.6$

b $\Pr(A) = 0.7$, $\Pr(A|B) = 0.7$

Hint: $\Pr(A|B) = \Pr(A)$ if A and B are independent

2 This Venn diagram shows the number of elements in events A and B.

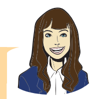


a Find:

i $\Pr(B)$ ii $\Pr(B|A)$

b Is $\Pr(B|A) = \Pr(B)$?

c Are the events A and B independent?



Hint: Recall: $\Pr(B|A) = \frac{\text{number in } A \cap B}{\text{number in } A}$

3 Complete each sentence.

a For multi-stage experiments, successive events are independent if selections are made ________ replacement.

b For multi-stage experiments, successive events are not independent if selections are made ________ replacement.

Hint: Choose from 'with' or 'without'.

Fluency 4 | 4, 5

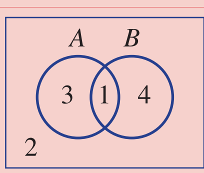
Example 12 Using Venn diagrams to consider independence

A selection of 10 mobile phone plans includes 4 with free connection and 5 with a free second battery. 1 plan has both free connection and a free second battery.
Let A be the event 'choosing a mobile phone plan with free connection'.
Let B be the event 'choosing a mobile phone plan with a free second battery'.

a Summarise the information about the 10 mobile phone plans in a Venn diagram.

b Find: i $\Pr(A)$ ii $\Pr(A|B)$

c State whether or not the events A and B are independent.

Solution	Explanation
a 	Start with the 1 element that belongs to both A and B and complete according to the given information.
b i $\Pr(A) = \frac{4}{10} = \frac{2}{5}$	4 of the 10 elements belong to A.
ii $\Pr(A\|B) = \frac{1}{5}$	1 of the 5 elements in B belongs to A.
c The events A and B are not independent.	$\Pr(A\|B) \neq \Pr(A)$

Continued on next page

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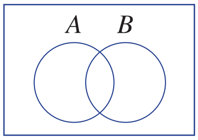
4G

Now you try

A selection of 8 hotel deals includes 3 with a free breakfast and 4 with free parking. 2 deals have both a free breakfast and free parking.
Let A be the event 'choosing a hotel deal with a free breakfast'.
Let B be the event 'choosing a hotel deal with free parking'.

a Summarise the information about the 8 hotel deals in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A \mid B)$
c State whether or not the events A and B are independent.

4 A selection of 8 offers for computer printers includes 3 with a free printer cartridge and 4 with a free box of paper. 2 have both a free printer cartridge and a free box of paper.
Let A be the event 'choosing a printer with a free printer cartridge'.
Let B be the event 'choosing a printer with a free box of paper'.



a Summarise the given information about the 8 computer printer offers in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A \mid B)$
c State whether or not the events A and B are independent.

Hint: If $\Pr(A) = \Pr(A \mid B)$, then the events A and B are independent.

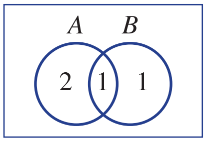
5 A selection of 6 different baby strollers includes 3 with a free rain cover and 4 with a free sunshade. 2 offer both a free rain cover and a free sunshade.
Let A be the event 'choosing a stroller with a free sunshade'.
Let B be the event 'choosing a stroller with a free rain cover'.

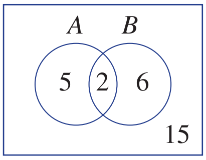
a Summarise the given information about the six baby strollers in a Venn diagram.
b Find: **i** $\Pr(A)$ **ii** $\Pr(A \mid B)$
c State whether or not the events A and B are independent.

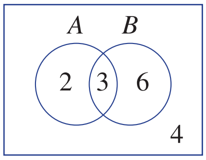
Problem-solving and reasoning 6, 7 6–9

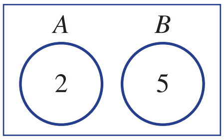
6 Events A and B are given in the Venn diagrams below.
i Find $\Pr(A)$ and $\Pr(A \mid B)$.
ii Hence, decide whether or not events A and B are independent.

Hint: Remember:
$\Pr(A \mid B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$

a


b


c


d


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7 For the events A and B with details provided in the given two-way tables, find $\Pr(A)$ and $\Pr(A \mid B)$. Decide whether or not the events A and B are independent.

a

	A	A'	
B	1	1	2
B'	3	3	6
	4	4	8

b

	A	A'	
B	1	3	4
B'	2	4	6
	3	7	10

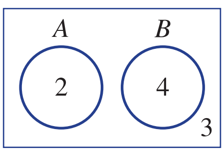
c

	A	A'	
B	3	17	20
B'	12	4	16
	15	21	36

d

	A	A'	
B	1		9
B'			
	5		45

8 Use the diagram below to help decide if this statement is true or false:
If two events, A and B, are mutually exclusive, then they are also independent.



9 A coin is tossed 5 times. Find the probability of obtaining:

a 5 heads

b at least 1 tail

c at least 1 head

Hint: Coin tosses are independent. From two coins, the probability of two heads is $\frac{1}{2} \times \frac{1}{2}$.

Tax and investment advice — 10

10 Of 17 leading accountants, 15 offer advice on tax (T) and 10 offer advice on business growth (G). Eight of the accountants offer advice on both tax and business growth. One of the 17 accountants is chosen at random.

a Use a Venn diagram or two-way table to help find:

i $\Pr(T)$ **ii** $\Pr(T \text{ only})$ **iii** $\Pr(T \mid G)$

b Are the events T and G independent?

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Maths@Work: Business analyst

Businesses employ analysts to look at how their businesses run, and how they can run more efficiently. Analysts use probabilities, relative frequencies and graphs to look at data, undertake calculations and make recommendations.

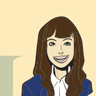
Being able to summarise and interpret data and understand likelihoods are important skills for analysts and business owners to have.

Complete these questions that a business analyst may face in their day-to-day job.

1 An online photo printing company wishes to analyse their customer orders and needs based on the products that are currently purchased from them. They looked at the results for a typical week over two areas of the business: photos and other services, such as canvases, mugs and calendars.
The results of one week's orders are shown in the table below.

	Other services	No other services	Total orders
Photos ordered	561	1179	1740
No photos ordered	260	–	260
Totals	821	1179	2000

a Percentages are often used in probability. Rewrite the table above using percentages, where 100% equals the total orders for the week.

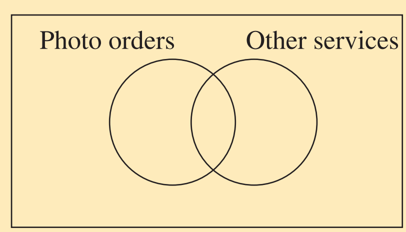


$\frac{561}{2000} \times \frac{100}{1} = 28.05\%$

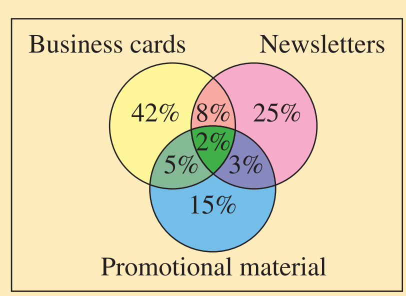
b Show this information in a Venn diagram, using percentages.

c Each customer ordering photos averages 30 prints (size 4×6). The company charges 12 cents a print. How much is the weekly income generated by photos, using the week's orders shown above?

d The company wishes to invest money in some advertising. Should this advertising be for the promotion of their photo printing service or should it be aimed at the other services they supply? Explain your answer.



2 Consider the information of the services offered by a different company wishing to look into how its business operates. They offer three types of products: business cards, newsletter printing and promotional lines, which include calendars, mugs etc. This company does not offer separate photo printing.
The breakdown of their business can be seen in the Venn diagram on the right.



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a If 1000 customers were surveyed, how many of them purchased:
 i only one of the three product lines on offer?
 ii exactly two of the product lines?
 iii at least two of the product lines?
 iv all three product lines available?

b What is the single most important product that the company offers? State the statistics for your selection and explain the significance to the company of this information.

c If you were the owners or share holders in this company, ideally how would you like to see the Venn diagram change over time to see growth within the company?

Using technology

3 'Get Set' is an online business selling sportswear. It has been very successful over the past year and its quarterly sales are shown in this table.

	A	B	C	D	E	F	G
1	Sportswear sales from Get Set online business						
2	Sport	QUARTER 1	QUARTER 2	QUARTER 3	QUARTER 4	Totals	Percentages
3	Golf	159	98	104	137		
4	Running	278	312	320	287		
5	Cycling	209	276	248	268		
6	Dance	94	77	107	68		
7	Gym	169	176	195	284		
8	Yoga	29	47	26	32		
9	Totals						
10	Percentages						

Use dollar signs (e.g. H12) in a formula that links to a fixed cell.

a Copy the data into an Excel spreadsheet and enter appropriate formulas into the shaded cells to find their values.

b Which type of sportswear is:
 i most likely to be sold?
 ii least likely to be sold?

Select the sports and sales area of the table and click on Insert and then Column.

c Insert a column graph showing the quarterly sales for each sport.

d In the fourth quarter, Get Set offered special prices for one line of sportswear as part of an advertising drive. Which sportswear do you think was on special? Was the advertising successful? Give reasons for your answer.

Hold the Ctrl button while selecting the titles Quarter 1 to Quarter 4 and also their total percentages row.

e Insert a column graph showing the total percentages for each quarter of sales.

f Businesses use graphs to show comparisons between data sets.
What is the increase in total percentage from Quarter 1 to Quarter 4? Comment on whether your graph in part **e** exaggerates this increase and, if so, explain why. Make an adjustment on the graph so that it is not misleading.

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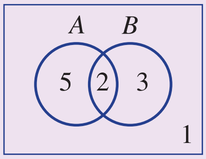
Puzzles and games

1 'I have nothing in common.' Match the answers to the letters in parts **a** and **b** to uncover the code.

$\frac{5}{14}$ 5 2 5 7 10 10 $\frac{7}{11}$

$\frac{5}{11}$ $\frac{3}{14}$ $\frac{1}{2}$ 10 5 $\frac{10}{11}$ $\frac{1}{7}$ 3 $\frac{5}{11}$

a These questions relate to the Venn diagram at right.



- T How many elements in $A \cap B$?
- L How many elements in $A \cup B$?
- V How many elements in B only?
- Y Find $\Pr(A)$.
- S Find $\Pr(A \cup B)$.
- E Find $\Pr(A \text{ only})$.

b These questions relate to the two-way table at right.

	P	P'	
Q	U	4	9
Q'	2		
		A	14

- U What number should be in place of the letter U?
- A What number should be in place of the letter A?
- M Find $\Pr(P \cap Q)$.
- C Find $\Pr(P')$.
- X Find Pr(neither P nor Q).
- I Find $\Pr(P \text{ only})$.

2 What is the chance of rolling a sum of at least 10 from rolling two 6-sided dice?

3 *Game for two people*: You will need a bag or pocket and coloured counters.

- One person places 8 counters of 3 different colours in a bag or pocket. The second person must not look!
- The second person then selects a counter from the bag. The colour is noted, then the counter is returned to the bag. This is repeated 100 times.
- Complete this table.

Colour	Tally	Frequency	Guess
Total:	100	100	

- Using the experimental results, the second person now tries to guess how many counters of each colour are in the bag.

4 Two digits are chosen without replacement from the set $\{1, 2, 3, 4\}$ to form a two-digit number. Find the probability that the two-digit number is:

a 32   **b** even   **c** less than 40   **d** at least 22

5 A coin is tossed 4 times. What is the probability that at least 1 tail is obtained?

6 Two leadership positions are to be filled from a group of 2 girls and 3 boys. What is the probability that the positions will be filled by 1 girl and 1 boy?

7 The letters of the word DOOR are jumbled randomly. What is the probability that the final arrangement will spell DOOR?

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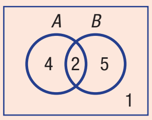
Chapter summary

Probability

Review
- Sample space is the list of all possible outcomes
- Pr(event) $= \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Venn diagram



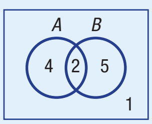
Two-way table

	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

Conditional probability ★

$\Pr(A \mid B)$ is read as the probability of A given B

$\Pr(A \mid B) = \dfrac{\text{number in } A \cap B}{\text{number in } B}$



	A	A'	
B	2	5	7
B'	4	1	5
	6	6	12

$\Pr(A \mid B) = \frac{2}{7}$

$\Pr(B \mid A) = \frac{2}{6} = \frac{1}{3}$

Independent events ★
- $\Pr(A \mid B) = \Pr(A)$ or $\Pr(B \mid A) = \Pr(B)$
- $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Tables

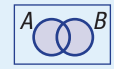
With replacement

	A	B	C
A	(A, A)	(B, A)	(C, A)
B	(A, B)	(B, B)	(C, B)
C	(A, C)	(B, C)	(C, C)

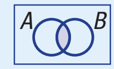
Without replacement

	A	B	C
A	×	(B, A)	(C, A)
B	(A, B)	×	(C, B)
C	(A, C)	(B, C)	×

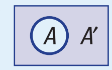
Notation
- Union $A \cup B$ (A or B)



- Intersection $A \cap B$ (A and B)



- Complement of A is A' (not A)



- A only

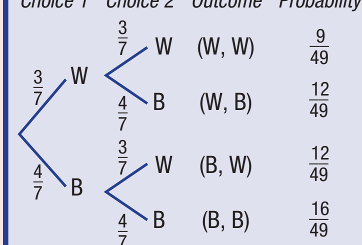
- Mutually exclusive events $\Pr(A \cap B) = 0$

Tree diagrams

3 white
4 black

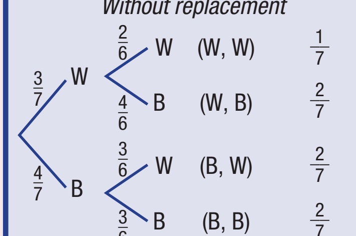
With replacement

Choice 1 *Choice 2* *Outcome* *Probability*



$\Pr(\text{W, B}) = \frac{3}{7} \times \frac{4}{7}$
$= \frac{12}{49}$

Without replacement



$\Pr(\text{1 white}) = \frac{2}{7} + \frac{2}{7}$
$= \frac{4}{7}$

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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

4A

1 I can calculate a theoretical probability.

e.g. A letter is chosen from the word ALLIGATOR. Find the probability that the letter is:

a an A
b not a G
c an A or a T

4A

2 I can find an experimental probability.

e.g. An experiment involves tossing 3 coins and counting the number of tails. The results from running the experiment 100 times are shown.

Number of tails	0	1	2	3
Frequency	15	39	37	9

Find the experimental probability of obtaining:

a 1 tail
b at least 1 tail

4B

3 I can list sets from a Venn diagram.

e.g. Events A and B involve numbers taken from the first 10 positive integers:
$A = \{1, 4, 5, 7\}$ and $B = \{4, 5, 6, 7, 8\}$.
Represent the two events in a Venn diagram and hence:

a list the set $A \cup B$
b find the probability that a randomly selected number from the first 10 positive integers is in $A \cap B$
c decide if the events are mutually exclusive

4B

4 I can use a Venn diagram.

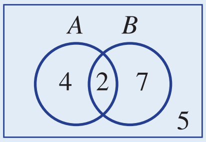
e.g. From a group of 20 people in a swimming squad, 12 train for freestyle (F), 6 train for butterfly (B) and 3 train for both freestyle and butterfly.
Illustrate this information in a Venn diagram and hence:

a state the number in the squad who train for butterfly only
b find the probability a randomly selected swimmer from the squad trains for neither freestyle nor butterfly

4C

5 I can use a two-way table.

e.g. The Venn diagram shows the distribution of elements from two events A and B.



Transfer the information into a two-way table and hence, find:

a the number of elements in A and B
b the number of elements in neither A nor B
c Pr(B') and Pr(A only)

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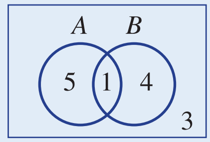
Chapter checklist

✔

4D

6 I can find conditional probability using a Venn diagram.
e.g. The Venn diagram shows the distribution of elements belonging to two events A and B.



Find the following probabilities:
a $\Pr(A \mid B)$
b $\Pr(B \mid A)$

4D

7 I can find a conditional probability in a word problem using a two-way table.
e.g. From a team of 11 cricketers in a cricket match, 9 batted in the match, 6 bowled in the match and 4 both batted and bowled. A cricketer is chosen at random from the team. Let A be the event 'the cricketer batted in the match' and B be the event 'the cricketer bowled in the match'. Represent the information in a two-way table and hence, find the probability that:
a the cricketer batted, given that they bowled
b the cricketer bowled, given that they batted

4E

8 I can construct a table with replacement.
e.g. A fair 5-sided die is rolled twice. List all the outcomes using a table and find:
a Pr(2 even numbers)
b Pr(sum of at least 7)
c Pr(sum not equal to 9)

4E

9 I can construct a table without replacement.
e.g. Two letters are chosen from the word TENT, without replacement. Construct a table to list the sample space and find the probability of:
a obtaining the outcome (T, E)
b selecting a T and an N

4F

10 I can construct a tree diagram.
e.g. Two jars contain 4 jelly beans each. Jar A has 3 black and 1 pink jelly bean and Jar B contains 2 black and 2 pink jelly beans. A jar is chosen at random followed by a single jelly bean. Represent the options available in a tree diagram that shows all outcomes and the associated probabilities. Find the probability of selecting:
a jar A and a black jelly bean
b a black jelly bean

4F

11 I can use a tree diagram without replacement.
e.g. A mixed bag of chocolates contains 2 Mars bars (M) and 4 Snickers (S). Draw a tree diagram to show the outcomes and probabilities of the selection of two chocolates without replacement. Find the probability of selecting:
a 2 Snickers
b exactly 1 Mars bar

4G

12 I can determine if events are independent.
e.g. A selection of 10 soccer club memberships found that 5 included a free scarf and 3 included a free soccer ball. One had both a free scarf and a free soccer ball.
Let A be the event 'soccer membership had a free scarf', and B be the event 'soccer membership had a free soccer ball'. Summarise the information in a Venn diagram and find $\Pr(A|B)$ and $\Pr(B|A)$ to determine whether or not events A and B are independent.

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Chapter review

Short-answer questions

4A **1** A fair 6-sided die is rolled once. Find:

a Pr(4) **b** Pr(even) **c** Pr(at least 3)

4A **2** A letter is chosen from the word INTEREST. Find the probability that the letter will be:

a I **b** E **c** a vowel
d not a vowel **e** E or T

4A **3** An engineer inspects 20 houses in a street for cracks. The results are summarised in this table.

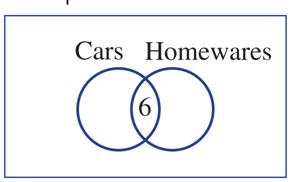
Number of cracks	0	1	2	3	4
Frequency	8	5	4	2	1

a From these results, estimate the probability that the next house inspected in the street will have the following number of cracks.
i 0 **ii** 1 **iii** 2 **iv** 3 **v** 4

b Estimate the probability that the next house will have:
i at least 1 crack
ii no more than 2 cracks

4B/C **4** Of 36 people, 18 have an interest in cars, 11 have an interest in homewares and 6 have an interest in both cars and homewares.

a Complete this Venn diagram.



b Complete this two-way table.

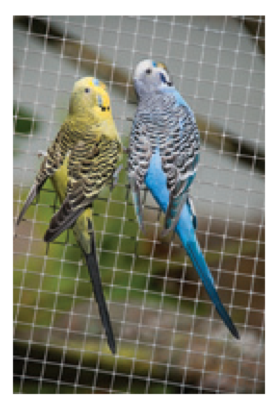
	C	C'	
H	6		
H'			

c State the number of people from the group who do not have an interest in either cars or homewares.

d If a person is chosen at random from the group, find the probability that the person will:
i have an interest in cars and homewares
ii have an interest in homewares only
iii not have any interest in cars

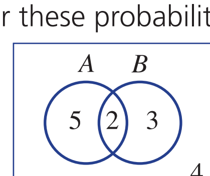
4B/C **5** All 26 birds in an aviary have clipped wings and/or a tag. In total, 18 birds have tags, 14 have clipped wings and 6 have both clipped wings and a tag.

a Find the number of birds that have only clipped wings.
b Find the probability that a bird chosen at random will have a tag only.



4D **6** For these probability diagrams, find $\Pr(A \mid B)$.

a



b

	A	A'	
B	1	4	5
B'	2	2	4
	3	6	9

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4E **7** A letter is chosen at random from the word HAPPY and a second letter is chosen from the word HEY.

a List the sample space by completing this table.

b State the total number of outcomes.

c Find the probability that the two letters chosen will be:

i H then E **ii** the same **iii** not the same

		1st				
		H	**A**	**P**	**P**	**Y**
	H	(H, H)	(A, H)	(P, H)		
2nd	**E**					
	Y					

4F **8** A fair 4-sided die is rolled twice and the total is noted.

a Complete this tree diagram to list the sample space.

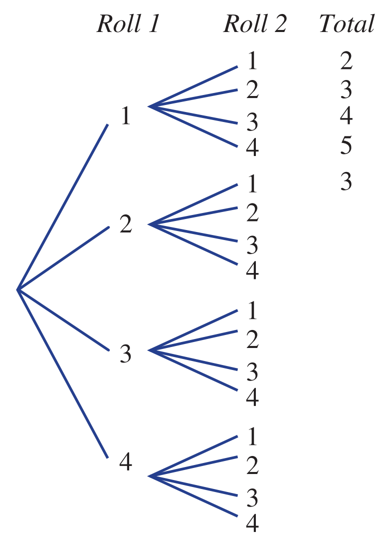
b Find these probabilities.

i Pr(2)

ii Pr(5)

iii Pr(1)

iv Pr(not 1)



4F **9** Two people are selected from a group of 2 females and 3 males, without replacement. Use a tree diagram to find the probability of selecting:

a a female on the first selection

b a male on the second selection given that a female was chosen on the first selection

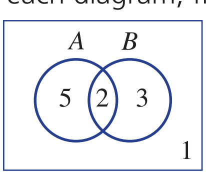
c 2 males

d 1 male

e at least 1 female

4D/G **10** For each diagram, find Pr(A) and Pr($A|B$), then decide if events A and B are independent.

a



b

	A	A'	
B	3		6
B'			
	5		10

Multiple-choice questions

4A **1** A letter is chosen from the word SUCCESS. The probability that the letter is not a C is:

A $\frac{2}{7}$ **B** $\frac{3}{5}$ **C** $\frac{5}{7}$ **D** $\frac{4}{7}$ **E** $\frac{3}{7}$

4A **2** The number of manufacturing errors spotted in a car plant on 20 randomly selected days is given by this table.

Number of errors	0	1	2	3	Total
Frequency	11	6	2	1	20

An estimate of the probability that on the next day no errors will be observed is:

A $\frac{3}{10}$ **B** $\frac{9}{20}$ **C** $\frac{11}{20}$ **D** $\frac{17}{20}$ **E** $\frac{3}{20}$

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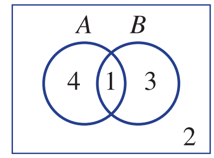
4B **3** For this Venn diagram, $\Pr(A \cup B)$ is equal to:



A $\frac{4}{5}$ **B** $\frac{1}{2}$ **C** $\frac{5}{8}$
D $\frac{1}{4}$ **E** $\frac{1}{10}$

4B **4** 15 people like apples or bananas. Of those 15 people, 10 like apples and 3 like both apples and bananas. How many from the group like only apples?

A 5 **B** 3 **C** 13 **D** 7 **E** 10

4E **5** A letter is chosen from each of the words CAN and TOO. The probability that the pair of letters will not have an O is:

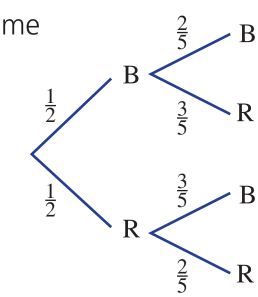
	C	A	N
T	(C, T)	(A, T)	(N, T)
O	(C, O)	(A, O)	(N, O)
O	(C, O)	(A, O)	(N, O)

A $\frac{2}{3}$ **B** $\frac{1}{2}$ **C** $\frac{1}{3}$
D $\frac{1}{9}$ **E** $\frac{5}{9}$

4B **6** The sets A and B are known to be mutually exclusive. Which of the following is therefore true?

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A \cap B) = 0$ **C** $\Pr(A) = 0$
D $\Pr(A \cap B) = 1$ **E** $\Pr(A \cup B) = 0$

4F **7** For this tree diagram, what is the probability of the outcome (B, R)?



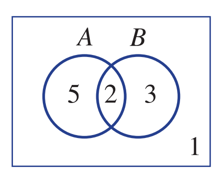
A $\frac{1}{5}$ **B** $\frac{3}{10}$ **C** $\frac{3}{7}$
D $\frac{1}{10}$ **E** $\frac{6}{11}$

4C **8** For this two-way table, $\Pr(A \cap B)$ is:

	A	A'	
B		1	3
B'			4
		4	

A $\frac{2}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{7}$
D $\frac{1}{3}$ **E** $\frac{2}{7}$

4D **9** For this Venn diagram, $\Pr(A | B)$ is:



A $\frac{5}{7}$ **B** $\frac{2}{5}$ **C** $\frac{5}{8}$
D $\frac{5}{3}$ **E** $\frac{3}{11}$

4G **10** Two events are independent when:

A $\Pr(A) = \Pr(B)$ **B** $\Pr(A') = 0$ **C** $\Pr(A \cup B) = 0$
D $\Pr(A | B) = \Pr(B)$ **E** $\Pr(A) = \Pr(A | B)$

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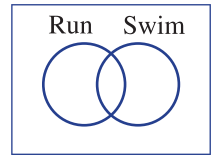
Extended-response questions

1 Of 15 people surveyed to find out if they run or swim for exercise, 6 said they run, 4 said they swim and 3 said they both run and swim.



a How many people surveyed neither run nor swim?
b One of the 15 people is selected at random. Find the probability that they:
  i run or swim
  ii only swim
c Represent the information in a two-way table.
d Find the probability that:
  i a person swims, given that they run
  ii a person runs, given that they swim

2 A bakery sells three types of bread: raisin (R) at \$2 each, sourdough (S) at \$3 each, and white (W) at \$1.50 each. Judy is in a hurry. She randomly selects 2 loaves and quickly takes them to the counter. (Assume an unlimited loaf supply.)

		1st		
		R	S	W
2nd	R			
	S			
	W			

a Complete this table, showing the possible combination of loaves that Judy could have selected.
b Find the probability that Judy selects:
  i 2 raisin loaves
  ii 2 loaves that are the same
  iii at least 1 white loaf
  iv not a sourdough loaf

Judy has only \$4 in her purse.

c How many different combinations of bread will Judy be able to afford?
d Find the probability that Judy will not be able to afford her two chosen loaves.

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