Chapter 3

Algebra and indices

Essential mathematics: why skills with algebra and index laws are important

Applying algebraic formulas and procedures are essential skills across the trades and professions, and are especially important for correctly entering and managing formulas in Excel spreadsheets.

- Algebraic formulas are widely used, including by welders (metal shrinkage: $S = \frac{A}{5T} + 0.05d$); nurses (child's dose $C = \frac{AD}{A + 12}$); auto mechanics (piston force: $F = \frac{\pi PD^2}{4}$); vets (a horse's weight in kg: $W = \frac{G^2L}{11\,880}$); and financial analysts (investment value: $A = P\left(1 + \frac{r}{100}\right)^n$).
- Technicians in many fields use scientific notation: boiler technicians (hospital sterilisation steam heat energy, kJ/day); lab technicians (red blood cells/L); and air-conditioner technicians (heat transfer through walls in kJ/s).
- Computer programmers first write an algebraic solution to the problem they want a computer to solve; these steps are then coded to form an algorithm. The internet and apps are powered by algebra in the form of algorithms.
- Electricians and electronics engineers require algebra in fields including designing and building microcircuits in robots, autopilots and medical equipment.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor (VCMNA329)

Simplify algebraic products and quotients using index laws (VCMNA330)

Apply the four operations to simple algebraic fractions with numerical denominators (VCMNA331)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 Write algebraic expressions for the following.

a 3 lots of x
b one more than a
c 5 less than $2m$
d 4 times the sum of x and y

2 Find the value of the following when $x = 4$ and $y = 7$.

a $5x$
b $2y + 3$
c $xy - 5$
d $2(x + y)$

3 Decide whether the following pairs of terms are like terms.

a $6x$ and 8
b $3a$ and $7a$
c $4xy$ and $2yx$
d $3x^2$ and $10x$

4 Simplify:

a $3m + 5m$
b $8ab - 3ab$
c $4x + 3y + 2x + 5y$
d $2 \times 4 \times x$
e $5 \times a \times 3 \times b$
f $6y \div 2$

5 Expand:

a $2(x + 5)$
b $3(y - 2)$
c $4(2x - 3)$
d $x(3x + 1)$

6 Find the HCF (highest common factor) of these pairs of terms.

a 8, 12
b 18, 30
c $7a$, $14a$
d $2xy$ and $8xz$
e $5x$ and $8x^2$
f $7x^2y$ and $21xy^2$

7 Simplify:

a $\frac{3}{8} + \frac{2}{5}$
b $\frac{6}{7} - \frac{1}{3}$
c $\frac{5}{9} \times \frac{6}{25}$
d $\frac{2}{3} \div \frac{4}{9}$

8 Write each of the following in index form (e.g. $5 \times 5 \times 5 = 5^3$).

a $7 \times 7 \times 7 \times 7$
b $m \times m \times m$
c $x \times x \times y \times y \times y$
d $3a \times 3a \times 3a \times 3a \times 3a$

9 Evaluate:

a 7^2
b 3^3
c 2^4
d 4^3

10 Write the following as 3 raised to a single power.

a $3^4 \times 3^5$
b $3^7 \div 3^5$
c $(3^2)^5$
d 1
e $\frac{1}{3^2}$

11 Complete the following.

a $3.8 \times 10 =$ ______________
b $2.31 \times 1000 =$ ______________
c $17.2 \div 100 =$ ______________
d $0.18 \div 100 =$ ______________
e $3827 \div$ ______________ $= 3.827$
f $6.49 \times$ ______________ $= 64\,900$

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3A Algebraic expressions CONSOLIDATING

Learning intentions

- To know the names of the parts of an algebraic expression
- To be able to form algebraic expressions from simple word problems
- To be able to evaluate expressions by substituting given values

Key vocabulary: expression, pronumeral, variable, term, constant term, coefficient, substitute, evaluate

Algebra involves the use of pronumerals (also called variables), which are letters that represent numbers. Numbers and pronumerals connected by multiplication or division form *terms*, and *expressions* are one or more terms connected by addition or subtraction.

If a ticket to an art gallery costs \$12, then the cost for y visitors is the expression $12 \times y = 12y$. By substituting values for y we can find the costs for different numbers of visitors. For example, if there are five visitors, then $y = 5$ and $12y = 12 \times 5 = 60$. So total cost = \$60.

Lesson starter: Expressions at the gallery

Ben, Alea and Victoria are visiting the art gallery. The three of them combined have \$$c$ between them. Drinks cost \$$d$ and Ben has bought x postcards in the gift shop.

Write expressions for the following.

- The cost of two drinks
- The amount of money each person has if the money is shared equally
- The number of postcards Alea and Victoria bought if Alea bought three more than Ben and Victoria bought five less than twice the number Ben bought

Key ideas

- A **pronumeral** (or **variable**) is a letter used to represent an unknown number.
- Algebraic **expressions** are made up of one or more terms connected by addition or subtraction; e.g. $3a + 7b$, $\frac{x}{2} + 3y$, $3x - 4$.
  - A **term** is a group of numbers and pronumerals connected by multiplication and division; e.g. $2x$, $\frac{y}{4}$, $5x^2$.
  - A **constant term** is a number with no attached pronumerals; e.g. 7, −3.
  - The **coefficient** is the number multiplied by the pronumerals in the term;
    e.g. 3 is the coefficient of y in $2x + 3y$.
    −4 is the coefficient of x in $5 - 4x$.
    1 is the coefficient of x^2 in $2x + x^2$.

  This expression has 3 terms: $3x$, $2y$ and 4.

  $$3x - 2y + 4$$

  3 is the coefficient of x | −2 is the coefficient of y | 4 is the constant term

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- Operations in algebraic expressions:
  - The operations for addition and subtraction are written with '+' and '−'.
  - Multiplication is written without the sign; e.g. $3 \times y = 3y$.
  - Division is written as a fraction; e.g. $y \div 4 = \frac{y}{4}$ or $\frac{1}{4}y$.
- To find the value of an expression (or to **evaluate**), **substitute** a value for each pronumeral. The order of operations (BODMAS) is followed. For example, if $x = 2$ and $y = 3$:

$$\begin{aligned} 4xy - y^2 &= 4 \times 2 \times 3 - 3^2 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

Exercise 3A

Understanding | 1–3 | 1

1 Fill in the missing word(s) in the sentences, using the words *expression, term, constant term* or *coefficient*.

a An algebraic ____________ is made up of one or more terms connected by addition and subtraction.

b A term without a pronumeral part is a ____________.

c A number multiplied by the pronumerals in a term is a ____________.

d Numbers and pronumerals connected by multiplication and division form a ____________.

2 Express in simplified mathematical form

a x plus 3 **b** $5 \times y$ **c** $a \div 5$ **d** $2 \times x \times y$

3 Substitute the value 3 for the pronumeral x in the following and evaluate.

a $x + 4$ **b** $5x$ **c** $8 - x$ **d** x^2 **e** $\frac{18}{x}$

Fluency | 4, 5, 6(½) | 4, 5–6(½)

Example 1 Naming parts of an expression

Consider the expression $\frac{xy}{2} - 4x + 3y^2 - 2$. Determine:

a the number of terms **b** the constant term

c the coefficient of:

i y^2 **ii** x

Solution	Explanation
a 4	There are four terms with different combinations of pronumerals and numbers, separated by + or −.
b −2	The term with no pronumerals is −2. The negative is included.
c i 3	The number multiplied by y^2 in $3y^2$ is 3.
ii −4	The number multiplied by x in $-4x$ is −4. The negative sign belongs to the term that follows.

Continued on next page

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Now you try

Consider the expression $4y - \frac{x}{3} - 2x^2 + 1$. Determine:

a the number of terms
b the constant term
c the coefficient of:

i y
ii x^2

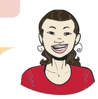
4 For these algebraic expressions, determine:

i the number of terms
ii the constant term
iii the coefficient of y

a $4xy + 5y + 8$

b $2xy + \frac{1}{2}y^2 - 3y + 2$

c $2x^2 - 4 + y$



Hint: The coefficient is the number multiplied by the pronumerals in each term. The constant term has no pronumerals.

Example 2 Writing algebraic expressions

Write algebraic expressions for the following.

a three more than x
b 4 less than 5 times y
c the sum of c and d is divided by 3
d the product of a and the square of b

Solution	Explanation
a $x + 3$	More than means add (+).
b $5y - 4$	Times means multiply ($5 \times y = 5y$) and less than means subtract (−).
c $\frac{c+d}{3}$	Sum c and d first (+), then divide by 3 (÷). Division is written as a fraction.
d ab^2	'Product' means 'multiply'. The square of b is b^2 (i.e. $b \times b$). $a \times b^2 = ab^2$

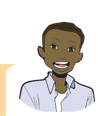
Now you try

Write algebraic expressions for the following.

a five more than y
b 7 less than 3 times x
c the sum of a and b is divided by 5
d the product of x and the square of y

5 Write an expression for the following.

a two more than x
b four less than y
c the sum of ab and y
d three less than 2 lots of x
e the product of x and 5
f twice m
g three times the value of r
h half of x
i three-quarters of m
j the quotient of x and y
k the sum of a and b is divided by 4
l the product of the square of x and y



Hint: Quotient means ÷. Product means ×
$\frac{1}{3}y = \frac{y}{3}$.

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3A

Example 3 Substituting values

Find the value of these expressions when $x = 2$, $y = 3$ and $z = -5$.

a $xy + 3y$ **b** $y^2 - \frac{8}{x}$ **c** $2x - yz$

Solution	Explanation
a $xy + 3y = 2 \times 3 + 3 \times 3$ $= 6 + 9$ $= 15$	Substitute for each pronumeral: $x = 2$ and $y = 3$. Recall that $xy = x \times y$ and $3y = 3 \times y$. Simplify, following order of operations, by multiplying first.
b $y^2 - \frac{8}{x} = 3^2 - \frac{8}{2}$ $= 9 - 4$ $= 5$	Substitute $y = 3$ and $x = 2$. $3^2 = 3 \times 3$ and $\frac{8}{2} = 8 \div 2$. Do subtraction last.
c $2x - yz = 2 \times 2 - 3 \times (-5)$ $= 4 - (-15)$ $= 4 + 15$ $= 19$	Substitute for each pronumeral. $3 \times (-5) = -15$ To subtract a negative number, add its opposite.

Now you try

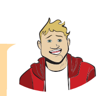
Find the value of these expressions when $x = 6$, $y = -2$ and $z = 4$.

a $xz + 2x$ **b** $x^2 + \frac{z}{2}$ **c** $3z - xy$

6 Find the value of these expressions when $a = 4$, $b = -2$ and $c = 3$.

- **a** ac
- **b** $2a - 5$
- **c** $3a - c$
- **d** $a^2 - 2c$
- **e** $ac + b$
- **f** $3b + a$
- **g** $ab + c^2$
- **h** $\frac{a}{2} - b$
- **i** $\frac{ac}{b}$
- **j** $2a - b$
- **k** $a + bc$
- **l** $\frac{6bc}{a}$

Hint:
$12 + (-2) = 12 - 2$
$2 - (-2) = 2 + 2$



Problem-solving and reasoning 7–9 8–11

7 Write an expression for the following.

- **a** The cost of 5 pencils at x cents each
- **b** The cost of y apples at 35 cents each
- **c** One person's share when \$500 is divided among n people
- **d** The cost of a pizza (\$11) equally shared between m people
- **e** Parvinda's age in x years' time if he is 11 years old now

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8 A taxi in Sydney has a pick-up charge (i.e. flagfall) of \$3.40 and charges \$2 per km.

a Write an expression for the taxi fare for a trip of d kilometres.

b Use your expression in part **a** to find the cost of a trip that is:

i 10 km

ii 22 km

Hint: The taxi fare has initial cost + cost per km × number of km.

9 **a** Ye thinks of a number, which we will call x.
Now write an expression for each of the following stages.

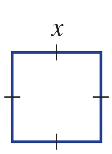
i He doubles the number.

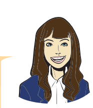
ii He decreases the result by 3.

iii He multiplies the result by 3.

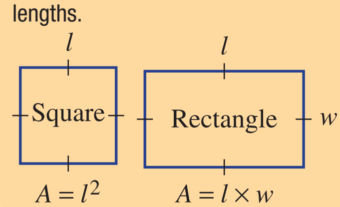
b If $x = 5$, use your answer to part **a iii** to find the final number.

10 A square with side length x is changed to a rectangle by increasing the length by 1 and decreasing the width by 1.





Hint: Perimeter is the sum of the side lengths.



a Write an expression for the new length and width of the rectangle.

b Is there any change in the perimeter of the shape?

c **i** Write an expression for the area of the rectangle.

ii Use trial and error to determine whether the area of the rectangle is more or less than the original square. By how much?

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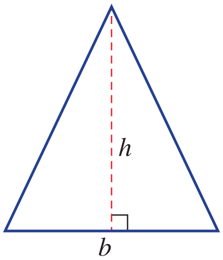
3A

11 The area of a triangle is given by $A = \frac{1}{2}bh$.



a If $b = 6$ and $h = 7$, what is the area?

b If the area is 9, what are the possible whole number values for b if h is also a whole number?

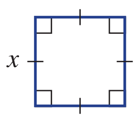
Area and perimeter — 12

12 For the shapes shown, write an expression for:

i the perimeter

ii the area

a



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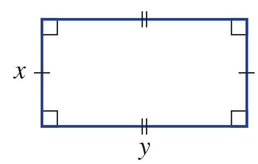

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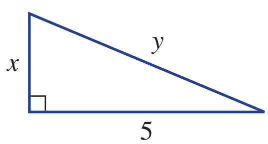
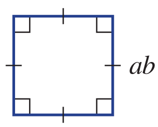
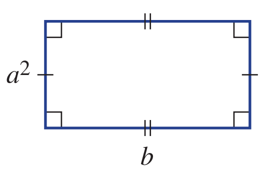


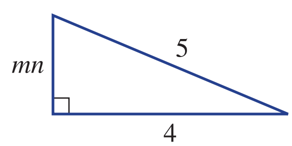
d

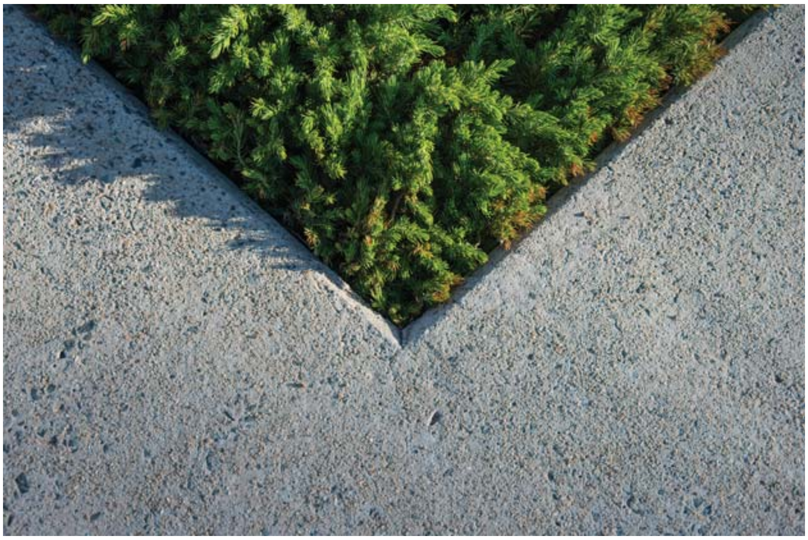


e



f





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3B Simplifying algebraic expressions

Learning intentions

- To be able to identify like terms
- To know that only like terms can be combined under addition and subtraction
- To be able to simplify algebraic expressions using the four operations: +, −, × and ÷

Key vocabulary: like terms, pronumeral

Many areas of finance and industry involve complex algebraic expressions. Often these expressions can be made simpler by applying the rules of addition, subtraction, multiplication and division.

Just as we would write $3 + 3 + 3 + 3$ as 4×3, we write $x + x + x + x$ as $4 \times x$ or $4x$. Similarly, $3x + 2x = 5x$ and $3x - 2x = 1x$ ($1x$ is written as x).

We also know that $2 \times 3 = 3 \times 2$ and $(2 \times 3) \times 4 = 2 \times 3 \times 4 = 3 \times 4 \times 2$ etc., so $2 \times x \times 4 = 2 \times 4 \times x = 8x$. By writing a division as a fraction we can also cancel common factors. For example, $9x \div 3 = \frac{9x}{3} = 3x$.

Lesson starter: Equivalent expressions

Split these expressions into two groups that are equivalent by simplifying them first.

$3x + 6x$	$17x - 5x$	$x + 7x + x$	$4x + 3 + 5x - 3$
$2 \times 6x$	$\frac{24xy}{2y}$	$3x \times 3$	$3x - 2y + 9x + 2y$
$8x + 6x - 2x$	$18x \div 2$	$\frac{9x^2}{x}$	$6x - (-6x)$

Key ideas

- **Like terms** have the exact same pronumeral factors, including powers; e.g. $3x$ and $7x$, and $4x^2y$ and $-3x^2y$.
  - Since $x \times y = y \times x$, $3xy$ and $2yx$ are like terms.
- Addition and subtraction apply to like terms only.
  For example, $5x + 7x = 12x$
  $7ab - 6ab = 1ab = ab$
  $3x + 2y$ cannot be simplified
- Multiplication and division apply to all terms.
  - In multiplication, deal with numerals and pronumerals separately:
    $2 \times 8a = 2 \times 8 \times a = 16a$
    $6x \times 3y = 6 \times 3 \times x \times y = 18xy$
- When dividing, write as a fraction and cancel common factors:

$$\frac{\cancel{8}^{4}x}{\cancel{2}^{1}} = 4x$$

$$6x^2 \div (3x) = \frac{6x^2}{3x} = \frac{\cancel{6}^{2} \times \cancel{x}^{1} \times x}{\cancel{3}^{1} \times \cancel{x}^{1}} = 2x$$

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Exercise 3B

Understanding | 1–4 | 4

1 Are the following sets of terms like terms? Answer yes (Y) or no (N).

a $3x, 2x, -5x$

b $2ax, 3xa, -ax$

c $2ax^2, 2ax, 62a^2x$

d $\frac{3}{4}x^2, 2x^2, \frac{x^2}{3}$

2 Simplify the following.

a $8g + 2g$

b $3f + 2f$

c $12e - 4e$

d $3h - 3h$

e $5x + x$

f $14st + 3st$

Hint: Add or subtract the numerals in like terms.

3 Simplify the following.

a $3 \times 2x$

b $4 \times 3a$

c $2 \times 5m$

d $-3 \times 6y$

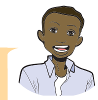
4 Simplify these fractions by cancelling.

a $\frac{4}{8}$

b $\frac{12}{3}$

c $\frac{14}{21}$

d $\frac{35}{15}$



Hint: Choose the highest common factor to cancel.

Fluency | 5–8(½) | 5–8(½)

Example 4 Identifying like terms

Write down the like terms in the following lists.

a $3x, 6a, 2ax, 3a, 5xa$

b $-2ax, 3x^2a, 3a, -5x^2a, 3x$

Solution	Explanation
a $6a$ and $3a$	Both terms contain a.
$5xa$ and $2ax$	Both terms contain ax; $x \times a = a \times x$.
b $3x^2a$ and $-5x^2a$	Both terms contain x^2a.

Now you try

Write down the like terms in the following lists.

a $4a, 3b, 5ab, 2a, 2ba$

b $-x^2y, 3x^2, 2xy, 4x, 4x^2y$

5 Write down the like terms in the following lists.

a $3ac, 2a, 5x, -2ac$

b $4pq, 3qp, 2p^2, -4p^2q$

c $7x^2y, -3xy^2, 2xy^2, 4yx^2$

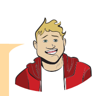
d $2r^2, 3rx, -r^2, 4r^2x$

e $-2ab, 5bx, 4ba, 7xa$

f $3p^2q, -4pq^2, \frac{1}{2}pq, 4qp^2$

g $\frac{1}{3}lm, 2l^2m, \frac{lm}{4}, 2lm^2$

h $x^2y, yx^2, -xy, yx$



Hint: Like terms have the same pronumeral factors.
$x \times y = y \times x$, so $3xy$ and $5yx$ are like terms.

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Example 5 Collecting like terms

Simplify the following.

a $4a + 5a + 3$ b $3x + 2y + 5x - 3y$ c $5xy + 2xy^2 - 2xy + xy^2$

Solution	Explanation
a $4a + 5a + 3 = 9a + 3$	Collect like terms ($4a$ and $5a$) and add coefficients.
b $3x + 2y + 5x - 3y = 3x + 5x + 2y - 3y$ $= 8x - y$	Collect like terms in x $(3 + 5 = 8)$ and y $(2 - 3 = -1)$. Note: $-1y$ is written as $-y$.
c $5xy + 2xy^2 - 2xy + xy^2$ $= 5xy - 2xy + 2xy^2 + xy^2$ $= 3xy + 3xy^2$	Collect like terms. In xy, the negative belongs to $2xy$. In xy^2, recall that xy^2 is $1xy^2$.

Now you try

Simplify the following.

a $7x + 3x + 2$ b $2a + 4b + 3a - 2b$ c $4mn + 3m^2n - mn + 2m^2n$

6 Simplify the following by collecting like terms.

a $4t + 3t + 10$ b $5g - g + 1$ c $3x - 5 + 4x$
d $4m + 2 - 3m$ e $2x + 3y + x$ f $3x + 4y - x + 2y$
g $8a + 4b - 3a - 6b$ h $2m - 3n - 5m + n$ i $3de + 3de^2 + 2de + 4de^2$
j $6kl - 4k^2l - 6k^2l - 3kl$ k $3x^2y + 2xy^2 - xy^2 + 4x^2y$ l $4fg - 5g^2f + 4fg^2 - fg$

Example 6 Multiplying algebraic terms

Simplify the following.

a $2a \times 7d$ b $-3m \times 8mn$

Solution	Explanation
a $2a \times 7d = 2 \times 7 \times a \times d$ $= 14ad$	Multiply coefficients and collect the pronumerals: $2 \times a \times 7 \times d = 2 \times 7 \times a \times d$. Multiplication can be done in any order.
b $-3m \times 8mn = -3 \times 8 \times m \times m \times n$ $= -24m^2n$	Multiply coefficients $(-3 \times 8 = -24)$ and pronumerals. Recall: $m \times m$ can be written as m^2.

Now you try

Simplify the following.

a $4x \times 5w$ b $-2a \times 6ac$

7 Simplify the following.

a $3r \times 2s$ b $2h \times 3u$ c $4w \times 4h$
d $2r^2 \times 3s$ e $-2e \times 4s$ f $5h \times (-2v)$
g $-3c \times (-4m^2)$ h $-7f \times (-5l)$ i $2x \times 4xy$
j $3ab \times 8a$ k $xy \times 3y$ l $-2a \times 8ab$
m $-3m^2n \times 4n$ n $-5xy^2 \times (-4x)$ o $5ab \times 4ab$

Hint: Multiply the numerals and collect the pronumerals.
$a \times b = ab$

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3B

Example 7 Dividing algebraic terms

Simplify the following.

a $\frac{18x}{6}$ **b** $12a^2b \div (8ab)$

Solution

a $\frac{\cancel{18}^{3}x}{\cancel{6}^{1}} = 3x$

b $12a^2b \div (8ab) = \frac{12a^2b}{8ab}$

$= \frac{^{3}\cancel{12} \times a \times \cancel{a}_1 \times \cancel{b}_1}{_{2}\cancel{8} \times \cancel{a}_1 \times \cancel{b}_1}$

$= \frac{3a}{2}$

Explanation

a Cancel highest common factor of numerals; i.e. 6.

b Write division as a fraction.

Cancel the highest common factor of 12 and 8 and cancel an a and b.

Now you try

Simplify the following.

a $\frac{20x}{4}$ **b** $9s^2t \div (15st)$

8 Simplify by cancelling common factors.

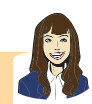
a $\frac{6a}{2}$ **b** $\frac{7x}{14}$ **c** $3a \div 9$

d $2ab \div 8$ **e** $\frac{4ab}{2a}$ **f** $\frac{15xy}{5y}$

g $4xy \div (8x)$ **h** $28ab \div (35b)$ **i** $\frac{8x^2}{20x}$

j $\frac{12xy^2}{18y}$ **k** $30a^2b \div (10a)$ **l** $12mn^2 \div (36mn)$

Hint: Write each division as a fraction first where necessary.



Problem-solving and reasoning 9, 10 9–12

9 A rectangle's length is three times its width, x. Write a simplified expression for:

a the rectangle's perimeter

b the rectangle's area

Hint: Draw a rectangle and label the width x and the length $3 \times x = 3x$.

10 Fill in the missing term to make the following true.

a $8x + 4 - \square = 3x + 4$

b $3x + 2y - \square + 4y = 3x - 2y$

c $3b \times \square = 12ab$

d $4xy \times (\square) = -24x^2y$

e $12xy \div (\square) = 6y$

f $\square \div (15ab) = \frac{2a}{3}$

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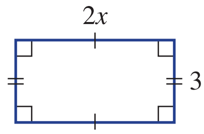
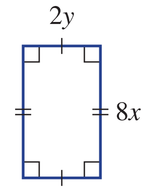
11 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes.

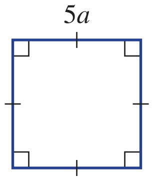
a

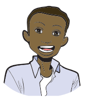


b

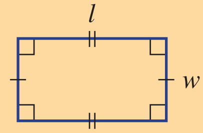


c

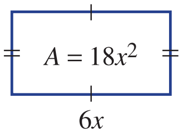




Hint: Perimeter is the sum of all the sides.
Area $= l \times w$



12 A rectangular garden bed has length given by $6x$ and area $18x^2$.
What is the width of the garden bed?



Hint: The opposite of $\times$ is $\div$.

Order of operations — 13

13 Simplify the following expressions, using order of operations.

a $4 \times 3x \div 2$

b $2 + 4a \times 2 + 5a \div a$

c $5a \times 2b \div a - 6b$

d $8x^2 \div (4x) + 3 \times 3x$

e $2x \times (4x + 5x) \div 6$

f $5xy - 4x^2y \div (2x) + 3x \times 4y$

g $(5x - x) \times (16xy \div (8y))$

h $9x^2y \div (3y) + 4x \times (-8x)$

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3C Expanding algebraic expressions

Learning intentions

- To understand the distributive law for expanding brackets
- To be able to expand expressions involving brackets

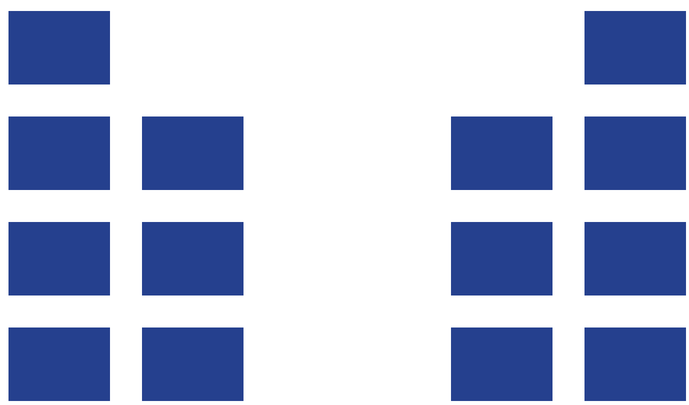
Key vocabulary: distributive law, expand

When an expression is multiplied by a term, each term in the expression must be multiplied by the term. Brackets are used to show this. For example, to double $4 + 3$ we write $2 \times (4 + 3)$, and each term within the brackets (both 4 and 3) must be doubled. The expanded version of this expression is $2 \times 4 + 2 \times 3$.

Similarly, to double the expression $x + 1$, we write $2(x + 1) = 2 \times x + 2 \times 1$. This expansion of brackets uses the distributive law.

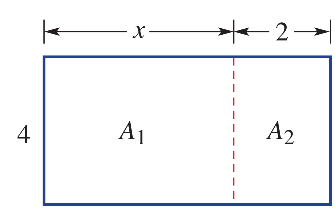
In this diagram, 7 blue blocks are doubled in groups of 4 and 3.

$2 \times (4 + 3) = 2 \times 4 + 2 \times 3$



Lesson starter: Rectangle brackets

Consider the diagram shown.



- Write an expression for the rectangle area A_1.
- Write an expression for the rectangle area A_2.
- Add your results for A_1 and A_2 to give the area of the rectangle.
- Write an expression for the total length of the rectangle.
- Using the total length, write an expression for the area of the rectangle.
- Combine your results to complete this statement: $4(x + 2) = \square + \square$.

Key ideas

- The **distributive law** is used to **expand** and remove brackets:
  - The terms inside the brackets are multiplied by the term outside the brackets.

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

For example, $2(x + 4) = 2 \times x + 2 \times 4$
$= 2x + 8$

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Exercise 3C

Understanding | 1, 2 | 2

1 The distributive law says that each term inside the ____________ is multiplied by the term ____________ the brackets.

2 Complete the following.

a $3(x+4) = 3 \times \square + 3 \times \square$
$= 3x + \square$

b $2(x-5) = 2 \times \square + \square \times (-5)$
$= \square - 10$

c $2(4x+3) = 2 \times \square + \square \times 3$
$= \square + 6$

d $x(x-3) = x \times \square + \square \times \square$
$= \square - \square$

Fluency | 3–5(½) | 3–5(½)

Example 8 Expanding expressions with brackets

Expand the following.

a $2(x+5)$ **b** $3(2x-3)$ **c** $3y(2x+4y)$

Solution	Explanation
a $2(x+5) = 2 \times x + 2 \times 5$ $= 2x + 10$	Multiply each term inside the brackets by 2.
b $3(2x-3) = 3 \times 2x + 3 \times (-3)$ $= 6x - 9$	Multiply $2x$ and -3 by 3. $3 \times 2x = 3 \times 2 \times x = 6x$.
c $3y(2x+4y) = 3y \times 2x + 3y \times 4y$ $= 6xy + 12y^2$	Multiply $2x$ and $4y$ by $3y$. $3y \times 2x = 3 \times 2 \times x \times y$ and $3y \times 4y = 3 \times 4 \times y \times y$. Recall: $y \times y$ is written as y^2.

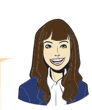
Now you try

Expand the following.

a $3(x+4)$ **b** $5(3x-2)$ **c** $4a(2a+5b)$

3 Expand the following.

a $2(x+4)$ **b** $3(x+7)$ **c** $4(y-3)$
d $5(y-2)$ **e** $2(3x+2)$ **f** $4(2x+5)$
g $3(3a-4)$ **h** $7(2y-5)$ **i** $5(2a+b)$
j $3(4a-3b)$ **k** $2x(x+5)$ **l** $3x(x-4)$
m $2a(3a+2b)$ **n** $2y(3x-4y)$ **o** $3b(2a-5b)$



Hint: Use the distributive law:

$a(b+c) = a \times b + a \times c$
$= ab + ac$

$a(b-c) = a \times b + a \times (-c)$
$= ab - ac$

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3C

Example 9 Expanding expressions with a negative out the front

Expand the following.

a $-3(x-4)$

b $-2x(3x-2y)$

Solution	Explanation
a $-3(x-4) = -3 \times x + (-3) \times (-4)$ $= -3x + 12$	Multiply each term inside the brackets by -3. $-3 \times (-4) = +12$ If there is a negative sign outside the bracket, the sign of each term inside the brackets is changed when expanded.
b $-2x(3x-2y) = -2x \times 3x + (-2x) \times (-2y)$ $= -6x^2 + 4xy$	$-2x \times 3x = -2 \times 3 \times x \times x$ and $-2x \times (-2y)$ $= -2 \times (-2) \times x \times y$

Now you try

Expand the following.

a $-4(x-5)$

b $-3y(2x-4y)$

4 Expand the following.

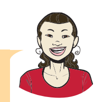
a $-2(x+3)$ **b** $-5(m+2)$ **c** $-3(w+4)$

d $-4(x-3)$ **e** $-2(m-7)$ **f** $-7(w-5)$

g $-(x+y)$ **h** $-(x-y)$ **i** $-2x(3x+4)$

j $-3x(2x+5)$ **k** $-4x(2x-2)$ **l** $-3y(2y-9)$

m $-2x(3x-5y)$ **n** $-3x(3x+2y)$ **o** $-6y(2x+3y)$



Hint: A negative out the front will change the sign of each term in the brackets when expanded.
$-2(x-3) = -2x + 6$

Example 10 Simplifying expressions by removing brackets

Expand and simplify the following.

a $8 + 3(2x-3)$

b $3(2x+2) - 4(x+4)$

Solution	Explanation
a $8 + 3(2x-3) = 8 + 6x - 9$ $= 6x - 1$	Expand the brackets first: $3 \times 2x + 3 \times (-3) = 6x - 9$. Collect like terms: $8 - 9 = -1$.
b $3(2x+2) - 4(x+4) = 6x + 6 - 4x - 16$ $= 2x - 10$	Expand the brackets first. Note that $-4(x+4) = -4 \times x + (-4) \times 4 = -4x - 16$. Collect like terms: $6x - 4x = 2x$ and $6 - 16 = -10$.

Now you try

Expand and simplify the following.

a $5 + 2(4a-3)$

b $5(y+3) - 2(2y+5)$

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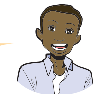
5 Expand and simplify the following.



Hint: Expand first, then collect like terms.

a $2 + 5(x + 3)$ **b** $3 + 7(x + 2)$
c $5 + 2(x - 3)$ **d** $7 - 2(x + 3)$
e $21 - 5(x + 4)$ **f** $4 + 3(2x - 1)$
g $3 + 2(3x + 4)$ **h** $8 - 2(2x - 3)$ **i** $12 - 3(2x - 5)$
j $3(x + 2) + 4(x + 3)$ **k** $2(p + 2) + 5(p - 3)$ **l** $4(x - 3) + 2(3x + 4)$
m $3(2s + 3) - 2(s + 2)$ **n** $4(3f + 2) - 2(6f + 2)$ **o** $3(2x - 5) - 2(2x - 4)$

Problem-solving and reasoning | 6, 7 | 6–9

6 Fill in the missing term/number to make each statement true.

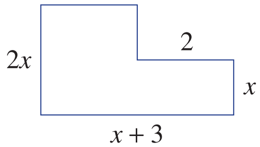
a $\square(x + 4) = 2x + 8$ **b** $\square(2x - 3) = 8x - 12$ **c** $\square(2x + 3) = 6x^2 + 9x$
d $4(\square + 5) = 12x + 20$ **e** $4y(\square - \square) = 4y^2 - 4y$ **f** $-2x(\square + \square) = -4x^2 - 6xy$

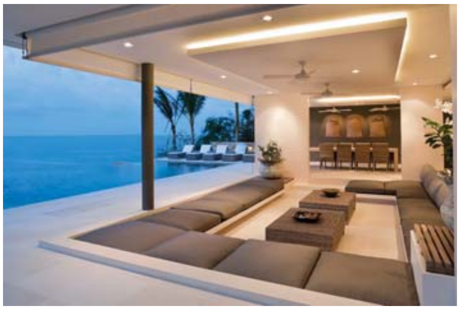
7 Four rectangular rooms in a house have floor side lengths listed below. Find an expression for the area of each floor in expanded form.

Hint: Area of a rectangle = length × width

a 2 and $x - 5$ **b** x and $x + 3$
c $2x$ and $x + 4$ **d** $3x$ and $2x - 1$

8 The deck on a house is constructed in the shape shown. Find the area of the deck in expanded form. (All lines meet at 90°.)





Hint: To find 10% of an amount, multiply by $\frac{10}{100} = 0.1$.

9 Virat earns $\$x$ but does not have to pay tax on the first $18 200.

a Write an expression for the amount of money Virat is taxed on.

b Virat is taxed 10% of his earnings in part **a**. Write an expanded expression for how much tax he pays.

Expanding binomial products | — | 10

10 A rectangle has dimensions $(x + 2)$ by $(x + 3)$, as shown. The area can be found by summing the individual areas:

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

	x	3
x	x^2	$3x$
2	$2x$	6

This can also be done using the distributive law:

$$(x + 2)(x + 3) = x(x + 3) + 2(x + 3)$$
$$= x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

Expand and simplify these binomial products using this method.

a $(x + 4)(x + 3)$ **b** $(x + 3)(x + 1)$ **c** $(x + 2)(x + 5)$
d $(x + 2)(x - 4)$ **e** $(x + 5)(x - 2)$ **f** $(x + 4)(2x + 3)$
g $(2x + 3)(x - 2)$ **h** $(x - 3)(x + 4)$ **i** $(4x - 2)(x + 5)$

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3D Factorising algebraic expressions

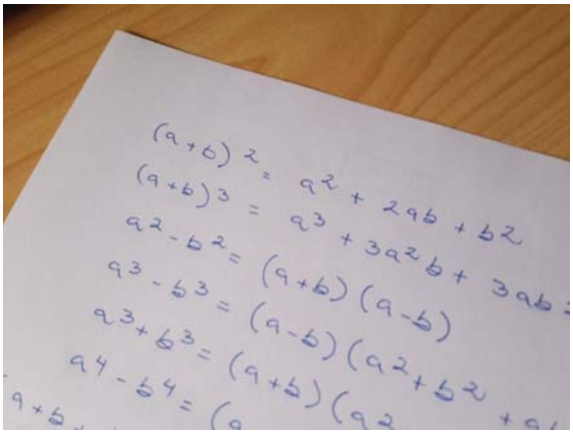
Learning intentions

- To be able to identify the highest common factor of terms
- To know the form of a factorised expression
- To understand that factorising and expanding are reverse processes
- To be able to factorise algebraic expressions involving a common factor

Key vocabulary: highest common factor, factorise, term

Factorising is an important step in solving many types of equations and in simplifying algebraic expressions.

Just as 15 can be expanded and written as 3×5, we can factorise to write an algebraic expression as the product of its factors. Factorising is therefore the opposite of expanding.



Lesson starter: Products of factors

- Expand the product $6(2x + 4)$.
- Write as many products as you can (using whole numbers) that give the same result as $6(2x + 4)$ when expanded.
- Which of your products has the highest number in front of the brackets? What is this number?
- How does this number relate to the two terms in the expanded form?
- Write a product of factors that expand to $18x + 24$, using the highest common factor.

Key ideas

- **Factorising** involves writing an expression as a product.
- Factorisation is the opposite process of expansion.
- To factorise an expression, take out the **highest common factor (HCF)** of each of the terms. The highest common factor is the largest number, pronumeral or product of these that divides into each term.
  - Divide each term by the HCF and leave the expression in the brackets.
  - A factorised expression can be checked by expanding to get the original expression.
  - If the HCF has been removed, the terms in the brackets should have no common factors; e.g. $2(x + 3)$ is fully factorised, but $2(4x + 6)$ is not because 2 can still be divided into both 4 and 6 within the brackets.

    For example: $3x + 12 = 3(x + 4)$ HCF: 3

    $2x^2 + 8x = 2x(x + 4)$ HCF: $2x$

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Exercise 3D

Understanding | 1–3 | 3

1 Write down the highest common factor (HCF) of these pair of numbers.
  a 10 and 16
  b 9 and 27
  c 14 and 35
  d 36 and 48

2 State true (T) or false (F) if the first expression is the factorised form of the second expression. Confirm by expanding.
  a $3(x+2)$, $3x+6$
  b $-2(x-4)$, $-2x-8$

3 Consider the expression $4x^2+8x$.
  a Which of the following factorised forms uses the HCF?
    A $2(2x^2+4x)$ B $4(x^2+8x)$ C $4x(x+2)$ D $2x(2x+4)$
  b What can be said about the terms inside the brackets once the HCF is removed, which is not the case for the other forms?

Fluency | 4–7(½) | 4–7(½)

Example 11 Finding the HCF

Determine the HCF of the following.
a $8a$ and 20 b $3x$ and $6x$ c $10a^2$ and $15ab$

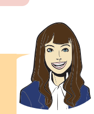
Solution	Explanation
a HCF of $8a$ and 20 is 4.	Compare numerals and pronumerals separately. The highest common factor (HCF) of 8 and 20 is 4. a is not a common factor.
b HCF of $3x$ and $6x$ is $3x$.	HCF of 3 and 6 is 3. x is also a common factor.
c HCF of $10a^2$ and $15ab$ is $5a$.	HCF of 10 and 15 is 5. HCF of a^2 and ab is a.

Now you try

Determine the HCF of the following.
a $10x$ and 25 b $7x$ and $14x$ c $9yz$ and $15y^2$

4 Determine the HCF of the following.
  a $6x$ and 12
  b 10 and $15y$
  c $8a$ and $12b$
  d $9x$ and $18y$
  e $5a$ and $20a$
  f $10m$ and $22m$
  g $14x$ and $21x$
  h $8a$ and $40ab$
  i $3a^2$ and $9ab$
  j $4x^2$ and $10x$
  k $16y$ and $24xy$
  l $15x^2y$ and $25xy$

Hint: Find the HCF of the numeral and variable factors.



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3D

Example 12 Factorising simple expressions

Factorise the following.

a $4x + 20$

b $6a - 15b$

Solution	Explanation
a $4x + 20 = 4(x + 5)$	HCF of $4x$ and 20 is 4. Place 4 in front of the brackets and divide each term by 4. Expand to check: $4(x + 5) = 4x + 20$.
b $6a - 15b = 3(2a - 5b)$	HCF of $6a$ and $15b$ is 3. Place 3 in front of the brackets and divide each term by 3.

Now you try

Factorise the following.

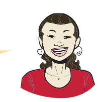
a $3x + 15$

b $12m - 18n$

5 Factorise the following.

a $3x + 9$
b $4x - 8$
c $10y - 20$
d $6a + 30$
e $5x + 5y$
f $12a + 4b$
g $18m - 27n$
h $36x - 48y$
i $8x + 44y$
j $24a - 18b$
k $121m + 55n$
l $14k - 63l$

Hint: Check your answer by expanding.
$3(x + 3) = 3x + 9$



Example 13 Factorising expressions with pronumeral common factors

Factorise the following.

a $8y + 12xy$

b $4x^2 - 10x$

Solution	Explanation
a $8y + 12xy = 4y(2 + 3x)$	HCF of 8 and 12 is 4, HCF of y and xy is y. Place $4y$ in front of the brackets and divide each term by $4y$. Check that $4y(2 + 3x) = 8y + 12xy$.
b $4x^2 - 10x = 2x(2x - 5)$	HCF of $4x^2$ and $10x$ is $2x$. Place $2x$ in front of the brackets and divide each term by $2x$. Recall: $x^2 = x \times x$.

Now you try

Factorise the following.

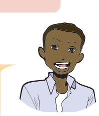
a $9a + 24ab$

b $15x^2 - 35x$

6 Factorise the following.

a $14x + 21xy$
b $6ab - 15b$
c $32y - 40xy$
d $5x^2 - 5x$
e $x^2 + 7x$
f $2a^2 + 8a$
g $12a^2 + 42ab$
h $9y^2 - 63y$
i $6x^2 + 14x$
j $9x^2 - 6x$
k $16y^2 + 40y$
l $10m - 40m^2$

Hint: Place the HCF in front of the brackets and divide each term by the HCF: $14x + 21xy = 7x(\_\_\_ + \_\_\_)$



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Example 14 Factorising expressions by removing a common negative

Factorise $-10x^2 - 18x$.

Solution	Explanation
$-10x^2 - 18x = -2x(5x + 9)$	The HCF of $-10x^2$ and $-18x$ is $-2x$, including the common negative. Place $-2x$ in front of the brackets and divide each term by $-2x$. Dividing by a negative changes the sign of each term.

Now you try

Factorise $-8y^2 - 36y$.

7 Factorise the following, including the common negative.

a $-2x - 6$ **b** $-4a - 8$ **c** $-3x - 6y$
d $-7a - 14ab$ **e** $-x - 10xy$ **f** $-3b - 12ab$
g $-x^2 - 7x$ **h** $-4x^2 - 12x$ **i** $-2y^2 - 10y$
j $-8x^2 - 14x$ **k** $-12x^2 - 8x$ **l** $-15a^2 - 5a$

Hint: Dividing by a negative changes the sign of the term.

Problem-solving and reasoning

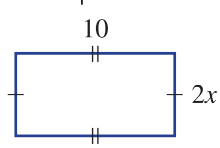
	8, 9	8(½), 9–11

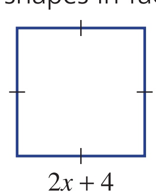
8 Factorise these mixed expressions.

a $7a^2b + ab$ **b** $4a^2b + 20a^2$ **c** $xy - xy^2$
d $x^2y + 4x^2y^2$ **e** $6mn + 18mn^2$ **f** $5x^2y + 10xy^2$
g $-y^2 - 8yz$ **h** $-3a^2b - 6ab$ **i** $-ab^2 - a^2b$

Hint: Be sure to find the highest common factor first.

9 Give the perimeter of these shapes in factorised form.

a


b


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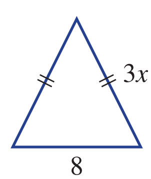
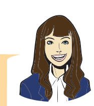


Hint: Find the perimeter first, then factorise.

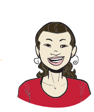


10 A square sandpit has perimeter $(4x + 12)$ metres. What is the side length of the square?

11 Common factors from expressions involving more than two terms can be removed in a similar way. Factorise these by taking out the HCF.

a $2x + 4y + 6z$ **b** $3x^2 + 12x + 6$ **c** $4x^2 + 8xy + 12$
d $6x^2 + 3xy - 9x$ **e** $10xy - 5xz + 5x$ **f** $4y^2 - 18y + 14xy$

Hint: $4a + 6b + 10c = 2(2a + 3b + 5c)$



Taking out a binomial factor

	—	12

12 A common factor may be a binomial term, such as $(x + 1)$.
For example, $3(x + 1) + x(x + 1)$ has HCF $= (x + 1)$, so $3(x + 1) + x(x + 1) = (x + 1)(3 + x)$, where $(3 + x)$ is what remains when $3(x + 1)$ and $x(x + 1)$ are divided by $(x + 1)$.
Use the method above to factorise the following.

a $4(x + 2) + x(x + 2)$ **b** $x(x + 3) + 2(x + 3)$ **c** $x(x + 4) - 7(x + 4)$
d $x(2x + 1) - 3(2x + 1)$ **e** $2x(y - 3) + 4(y - 3)$ **f** $2x(x - 1) - 3(x - 1)$

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3E Multiplying and dividing algebraic fractions ★

Learning intentions

- To know that expressions must be factorised before common factors can be cancelled
- To be able to simplify algebraic fractions by cancelling common factors
- To be able to multiply and divide algebraic fractions

Key vocabulary: algebraic fraction, common factor, factorise, numerator, denominator, reciprocal

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors to simplify the calculation and dividing by multiplying by the reciprocal of a fraction.

The process of cancelling requires cancelling of factors, for example:

$$\frac{8}{12} = \frac{2 \times \cancel{4}^{1}}{3 \times \cancel{4}_{1}} = \frac{2}{3}$$

For algebraic fractions, you need to factorise the expressions to identify and cancel common factors.

Lesson starter: Expressions as products of their factors

Factorise these expressions to write them as a product of their factors. Fill in the blanks and simplify.

$$\frac{2x+4}{2} = \frac{\square(\square)}{2} = \square \qquad \frac{6x+9}{3} = \frac{\square(\square)}{3} = \square$$

$$\frac{x^2+2x}{x} = \frac{\square(\square)}{x} = \square \qquad \frac{4x+4}{4} = \frac{\square(\square)}{4} = \square$$

Describe the errors made in these factorisations.

$$\frac{{}^{1}\cancel{3}x+2}{\cancel{3}^{1}} = x+2 \qquad \frac{x^2+\cancel{3x}^{1}}{\cancel{3x}^{1}} = x^2+1 \qquad \frac{6\cancel{x}^{1}+6}{{}^{1}\cancel{x}+1} = \frac{12}{1} = 12$$

Key ideas

- An **algebraic fraction** is a fraction containing pronumerals as well as numbers.
- Simplify algebraic fractions by cancelling common factors in factorised form.

  For example, $\frac{4x+6}{2} = \frac{\cancel{2}_{1}(2x+3)}{\cancel{2}_{1}}$
  $= 2x+3$
- To multiply algebraic fractions:
  - Factorise expressions if possible.
  - Cancel common factors.
  - Multiply numerators and denominators together.

  $$\frac{\cancel{(x+1)}^{1}}{\cancel{10}_{2}} \times \frac{{}^{1}\cancel{5}x}{4\cancel{(x+1)}_{1}} = \frac{x}{8}$$
- To divide algebraic fractions:
  - Multiply by the **reciprocal** of the fraction following the division sign (e.g. the reciprocal of 6 is $\frac{1}{6}$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$).
  - Follow the rules for multiplication.

  $$\frac{2}{(x-2)} \div \frac{8}{3(x-2)}$$
  $$= \frac{\cancel{2}_{1}}{\cancel{(x-2)}_{1}} \times \frac{3\cancel{(x-2)}_{1}}{\cancel{8}_{4}}$$
  $$= \frac{3}{4}$$

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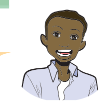
Exercise 3E

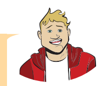
Understanding | 1–3 | 2, 3



Hint: Be sure to cancel the *highest* common factor.

1 Write these fractions in simplest form by cancelling common factors.

a $\frac{14}{21}$ **b** $\frac{9}{12}$ **c** $\frac{8x}{20}$ **d** $\frac{4x}{10}$



Hint: The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

2 Write the reciprocal of these fractions.

a $\frac{3}{2}$ **b** $\frac{5x}{3}$ **c** 7 **d** $\frac{x+3}{4}$

3 Follow the rules for multiplication and division to simplify these numeric fractions. Cancel common factors before multiplying.

a $\frac{15}{21} \times \frac{14}{25}$ **b** $\frac{4}{27} \div \frac{16}{9}$

Fluency | 4–7(½) | 4–7(½)

Example 15 Cancelling common factors

Simplify by cancelling common factors.

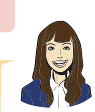
a $\frac{8xy}{12x}$ **b** $\frac{3(x+2)}{6(x+2)}$

Solution	Explanation
a $\frac{8xy}{12x} = \frac{{}^{2}\cancel{8} \times \cancel{x}^{1} \times y}{{}^{3}\cancel{12} \times \cancel{x}^{1}}$ $= \frac{2y}{3}$	Cancel the highest common factor of 8 and 12 (i.e. 4) and cancel the x.
b $\frac{3(x+2)}{6(x+2)} = \frac{{}^{1}\cancel{3} \times \cancel{(x+2)}^{1}}{{}^{2}\cancel{6} \times \cancel{(x+2)}^{1}}$ $= \frac{1}{2}$	Cancel the highest common factors: 3 and $(x+2)$.

Now you try

Simplify by cancelling common factors.

a $\frac{18ab}{8b}$ **b** $\frac{5(x-1)}{15(x-1)}$



Hint: Cancel the HCF of the numerals and pronumerals.

4 Simplify by cancelling common factors.

a $\frac{6xy}{12x}$ **b** $\frac{12ab}{30b}$ **c** $\frac{8x^2}{40x}$ **d** $\frac{25x^2}{5x}$

e $\frac{3(x+1)}{3}$ **f** $\frac{7(x-5)}{7}$ **g** $\frac{4(x+1)}{8}$ **h** $\frac{5(x-2)}{x-2}$

i $\frac{4(x-3)}{x-3}$ **j** $\frac{6(x+2)}{12(x+2)}$ **k** $\frac{9(x+3)}{3(x+3)}$ **l** $\frac{15(x-4)}{10(x-4)}$

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Example 16 Simplifying by factorising

Simplify these fractions by factorising first.

a $\frac{9x-12}{3}$ **b** $\frac{4x+8}{x+2}$

Solution	Explanation
a $\frac{9x-12}{3} = \frac{{}^{1}\cancel{3}(3x-4)}{\cancel{3}_1}$ $= 3x - 4$	Factorise the expression in the numerator, which has HCF = 3. Then cancel the common factor of 3.
b $\frac{4x+8}{x+2} = \frac{4\cancel{(x+2)}_1}{\cancel{x+2}_1}$ $= 4$	4 is the HCF in the numerator. After factorising, $(x+2)$ can be seen as a common factor and can be cancelled.

Now you try

Simplify these fractions by factorising first.

a $\frac{16x-8}{8}$ **b** $\frac{3x-6}{x-2}$

5 Simplify these fractions by factorising first.

a $\frac{4x+8}{4}$ **b** $\frac{6a-30}{6}$ **c** $\frac{8y-12}{4}$

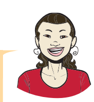
d $\frac{14b-21}{7}$ **e** $\frac{3x+9}{x+3}$ **f** $\frac{4x-20}{x-5}$

g $\frac{6x+9}{2x+3}$ **h** $\frac{12x-4}{3x-1}$ **i** $\frac{x^2+2x}{x}$

j $\frac{x^2-5x}{x}$ **k** $\frac{2x^2+6x}{2x}$ **l** $\frac{x^2+4x}{x+4}$

m $\frac{x^2-7x}{x-7}$ **n** $\frac{2x^2-4x}{x-2}$ **o** $\frac{3x^2+6x}{x+2}$

Hint: Cancel after you have factorised the numerator.



Example 17 Multiplying algebraic fractions

Simplify these products.

a $\frac{12}{5x} \times \frac{10x}{9}$ **b** $\frac{3(x-1)}{10} \times \frac{15}{x-1}$

Solution	Explanation
a $\frac{\cancel{12}_4}{\cancel{5x}_1} \times \frac{\cancel{10x}^2}{\cancel{9}_3}$ $= \frac{8}{3}\left(= 2\frac{2}{3}\right)$	Cancel common factors between numerators and denominators: $5x$ and 3. Then multiply the numerators and the denominators.
b $\frac{3\cancel{(x-1)}^1}{\cancel{10}^2} \times \frac{\cancel{15}_3}{\cancel{x-1}_1}$ $= \frac{9}{2}\left(= 4\frac{1}{2}\right)$	Cancel the common factors, which are $(x-1)$ and 5. Multiply numerators and denominators.

Continued on next page

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Now you try

Simplify these products.

a $\frac{20}{3x} \times \frac{6x}{25}$

b $\frac{4(x+2)}{9} \times \frac{12}{x+2}$

6 Simplify these products.

a $\frac{3}{x} \times \frac{2x}{9}$

b $\frac{4x}{5} \times \frac{15}{8x}$

c $\frac{9a}{14} \times \frac{7}{6a}$

d $\frac{2x^2}{5} \times \frac{25}{6x}$

e $\frac{4y^2}{7} \times \frac{21}{8y}$

f $\frac{x+1}{6} \times \frac{5}{x+1}$

g $\frac{x+3}{9} \times \frac{4}{x+3}$

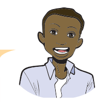
h $\frac{4(y-7)}{2} \times \frac{5}{y-7}$

i $\frac{10}{a+6} \times \frac{3(a+6)}{4}$

j $\frac{4(x-2)}{7} \times \frac{14}{5(x-2)}$

k $\frac{3(x+2)}{2x} \times \frac{8}{9(x+2)}$

l $\frac{4(2x+1)}{3x} \times \frac{9x}{2x+1}$



Hint: Cancel any common factors between numerators and denominators before multiplying.

Example 18 Dividing algebraic fractions

Simplify the following.

a $\frac{3x^2}{8} \div \frac{9x}{4}$

b $\frac{2(x-2)}{3} \div \frac{x-2}{6}$

Solution	Explanation
a $\frac{3x^2}{8} \div \frac{9x}{4} = \frac{\cancel{3}^1\cancel{x^2}^1}{\cancel{8}_2} \times \frac{\cancel{4}_1}{{}_3\cancel{9}\cancel{x}^1}$	Multiply by the reciprocal of the second fraction. The reciprocal of $\frac{9x}{4}$ is $\frac{4}{9x}$.
	Cancel common factors: $3x$ and 4.
	Note: $\frac{3x^2}{9x} = \frac{\cancel{3}^1 \times x \times \cancel{x}^1}{{}_3\cancel{9} \times \cancel{x}^1}$
$= \frac{x}{6}$	Multiply the numerators and the denominators.
b $\frac{2(x-2)}{3} \div \frac{x-2}{6} = \frac{2\cancel{(x-2)}^1}{\cancel{3}^1} \times \frac{\cancel{6}^2}{\cancel{x-2}^1}$	The reciprocal of $\frac{x-2}{6}$ is $\frac{6}{x-2}$.
$= 4$	Cancel the common factors $(x-2)$ and 3, and multiply. Recall: $\frac{4}{1} = 4$.

Now you try

Simplify the following.

a $\frac{7x^2}{10} \div \frac{14x}{5}$

b $\frac{3(x+1)}{4} \div \frac{x+1}{12}$

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3E

7 Simplify the following.

Hint: To divide, multiply by the reciprocal of the fraction following the division sign. $\frac{x}{5} \div \frac{x}{15} = \frac{x}{5} \times \frac{15}{x}$

a $\frac{x}{5} \div \frac{x}{15}$ **b** $\frac{3x}{10} \div \frac{x}{20}$

c $\frac{4a^2}{9} \div \frac{a}{18}$ **d** $\frac{3x^2}{10} \div \frac{6x}{5}$

e $\frac{4a}{9} \div \frac{5a^2}{6}$ **f** $\frac{2x}{7} \div \frac{x^2}{14}$

g $\frac{x+4}{2} \div \frac{x+4}{6}$ **h** $\frac{5(x-2)}{8} \div \frac{x-2}{4}$ **i** $\frac{3(x+4)}{10} \div \frac{6(x+4)}{15}$

j $\frac{2}{5(2x-1)} \div \frac{10}{2x-1}$ **k** $\frac{2(x-3)}{x-4} \div \frac{x-3}{5(x-4)}$ **l** $\frac{3(x+1)}{14(x-1)} \div \frac{6(x+1)}{35(x-1)}$

Problem-solving and reasoning 8, 9(½) 8–10(½)

8 Find the error in the simplifying of these fractions and correct it.

Hint: Remember that common factors can be easily identified when expressions are in factorised form.

a $\frac{3x+\cancel{6}_2}{\cancel{3}_1} = 3x+2$ **b** $\frac{x^2+2\cancel{x}_1}{\cancel{x}_1} = x^2+2$

c $\frac{4x}{5} \div \frac{10x}{3} = \frac{4x}{\cancel{5}_1} \times \frac{{}^2\cancel{10}x}{3} = \frac{8x^2}{3}$ **d** $\frac{\cancel{x}_1+4}{\cancel{15}_5} \times \frac{\cancel{3}_1}{\cancel{x}_1} = \frac{4}{5}$

9 Simplify these algebraic fractions by factorising expressions first.

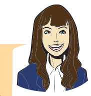
a $\frac{7a+14a^2}{21a}$ **b** $\frac{4x+8}{5x+10}$ **c** $\frac{x^2+3x}{4x+12}$

d $\frac{2m+4}{15} \times \frac{3}{m+2}$ **e** $\frac{5-x}{12} \times \frac{14}{15-3x}$ **f** $\frac{x^2+2x}{4} \times \frac{8}{3x+6}$

g $\frac{2x-1}{10} \div \frac{4x-2}{25}$ **h** $\frac{2x+4}{6x} \div \frac{3x+6}{x^2}$ **i** $\frac{2x^2-4x}{3x-6} \div \frac{6x}{x+5}$

10 By removing a negative factor, further simplifying is sometimes possible.

For example, $\frac{-2x-4}{x+2} = \frac{-2\cancel{(x+2)}_1}{\cancel{x+2}_1} = -2$.



Hint: Taking out a negative factor changes the sign of each term inside the brackets.

Use this idea to simplify the following.

a $\frac{-3x-9}{x+3}$ **b** $\frac{-4x-10}{2x+5}$ **c** $\frac{-x^2-4x}{x+4}$

d $\frac{-3x^2-6x}{-9x}$ **e** $\frac{-2x+12}{-2}$ **f** $\frac{-10x+15}{-5}$

Cancelling of powers — 11

11 Just as $\frac{x^{\cancel{2}1}}{\cancel{x}^1} = x$, $\frac{(x+1)^{\cancel{2}1}}{\cancel{x+1}_1} = x+1$. Use this idea to simplify these algebraic fractions.

Some will need factorising first.

a $\frac{(x+1)^2}{8} \times \frac{4}{x+1}$ **b** $\frac{(x+1)^2}{7x} \times \frac{14x}{3(x+1)}$ **c** $\frac{9}{x-2} \div \frac{18}{(x-2)^2}$

d $\frac{(x+2)^2}{10} \times \frac{5}{4x+8}$ **e** $\frac{(x-3)^2}{9x} \times \frac{3x}{4x-12}$ **f** $\frac{15}{8x+4} \div \frac{6}{(2x+1)^2}$

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3F Adding and subtracting algebraic fractions

Learning intentions

- To know that the steps for adding and subtracting algebraic fractions are the same as for numerical fractions
- To be able to find the lowest common denominator of fractions
- To be able to add and subtract algebraic fractions

Key vocabulary: lowest common denominator, equivalent fraction, algebraic fraction, numerator, denominator

As with multiplying and dividing, the steps for adding and subtracting numerical fractions can be applied to algebraic fractions. A lowest common denominator is required before the fractions can be combined.

Lesson starter: Steps for adding fractions

- Write out the list of steps you would give to someone to show them how to add $\frac{3}{5}$ and $\frac{2}{7}$.
- Follow your steps to add the fractions $\frac{3x}{5}$ and $\frac{2x}{7}$.
- What is different when these steps are applied to $\frac{x+2}{5}$ and $\frac{x}{7}$?

Key ideas

- To add or subtract **algebraic fractions**:
  - Determine the **lowest common denominator (LCD)** – the smallest common multiple of the denominators.
    For example, the LCD of 3 and 5 is 15 and the LCD of 4 and 12 is 12.
  - Write each fraction as an equivalent fraction by multiplying the denominator(s) to equal the LCD. When denominators are multiplied, numerators should also be multiplied.
    For example, $\frac{x}{3}+\frac{2x}{5}$ (LCD of 3 and 5 = 15.)
    $$=\frac{x_{(\times 5)}}{3_{(\times 5)}}+\frac{2x_{(\times 3)}}{5_{(\times 3)}}$$
    $$=\frac{5x}{15}+\frac{6x}{15}$$
    and $\frac{2x}{4}-\frac{x}{12}$ (LCD of 4 and 12 = 12.)
    $$=\frac{2x_{(\times 3)}}{4_{(\times 3)}}-\frac{x}{12}$$
    $$=\frac{6x}{12}-\frac{x}{12}$$
  - Add or subtract the numerators.
    For example, $\frac{5x}{15}+\frac{6x}{15}=\frac{11x}{15}$ and $\frac{6x}{12}-\frac{x}{12}=\frac{5x}{12}$
- To express $\frac{x+1}{3}$ with a denominator of 12, both the numerator and denominator must be multiplied by 4 with brackets required to multiply the numerator:
  $$\frac{(x+1)_{(\times 4)}}{3_{(\times 4)}}=\frac{4x+4}{12}$$

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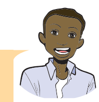
Exercise 3F

Understanding 1–3 3

Hint: The LCD is not always the two denominators multiplied together; e.g. $3 \times 6 = 18$ but the LCD of 3 and 6 is 6.



Hint: For equivalent fractions, whatever the denominator is multiplied by, the numerator must be multiplied by the same amount.

1 Write down the lowest common denominator (LCD) for these pairs of fractions.

a $\frac{2x}{5}, \frac{x}{4}$ **b** $\frac{x}{3}, \frac{x}{12}$ **c** $\frac{3x}{10}, \frac{2x}{15}$

2 Complete these equivalent fractions by giving the missing term.

a $\frac{x}{4} = \frac{\square}{12}$ **b** $\frac{2x}{5} = \frac{\square}{15}$ **c** $\frac{x-1}{4} = \frac{\square(x-1)}{20}$

3 Complete the following by filling in the boxes.

a $\frac{x}{4} + \frac{x}{5} = \frac{\square}{20} + \frac{\square}{20}$
$= \frac{\square}{20}$

b $\frac{2x}{5} - \frac{x}{10} = \frac{\square}{10} - \frac{\square}{10}$
$= \frac{\square}{10}$

Fluency 4–6(½) 4–6(½)

Example 19 Adding and subtracting algebraic fractions

Simplify the following.

a $\frac{x}{2} + \frac{x}{3}$ **b** $\frac{4x}{5} - \frac{x}{2}$ **c** $\frac{x}{2} - \frac{5}{6}$

Solution	Explanation
a $\frac{x_{(\times 3)}}{2_{(\times 3)}} + \frac{x_{(\times 2)}}{3_{(\times 2)}} = \frac{3x}{6} + \frac{2x}{6}$ $= \frac{5x}{6}$	The LCD of 2 and 3 is 6. Express each fraction with a denominator of 6 and add numerators.
b $\frac{4x_{(\times 2)}}{5_{(\times 2)}} - \frac{x_{(\times 5)}}{2_{(\times 5)}} = \frac{8x}{10} - \frac{5x}{10}$ $= \frac{3x}{10}$	The LCD of 5 and 2 is 10. Express each fraction with a denominator of 10 and subtract $5x$ from $8x$.
c $\frac{x_{(\times 3)}}{2_{(\times 3)}} - \frac{5}{6} = \frac{3x}{6} - \frac{5}{6}$ $= \frac{3x-5}{6}$	The LCD of 2 and 6 is 6. Multiply the numerator and denominator of $\frac{x}{2}$ by 3 to express with a denominator of 6. Write as a single fraction; $3x - 5$ cannot be simplified.

Now you try

Simplify the following.

a $\frac{x}{4} + \frac{x}{5}$ **b** $\frac{3x}{2} - \frac{x}{7}$ **c** $\frac{x}{3} + \frac{7}{9}$

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4 Simplify the following.

a $\frac{x}{3}+\frac{x}{4}$ **b** $\frac{x}{5}+\frac{x}{2}$ **c** $\frac{x}{3}-\frac{x}{9}$

d $\frac{x}{5}-\frac{x}{7}$ **e** $\frac{2x}{3}+\frac{x}{5}$ **f** $\frac{3x}{4}+\frac{5x}{12}$

g $\frac{5x}{6}-\frac{4x}{9}$ **h** $\frac{7x}{10}-\frac{3x}{8}$ **i** $\frac{x}{7}-\frac{x}{2}$

j $\frac{x}{10}-\frac{2x}{5}$ **k** $\frac{5x}{6}-\frac{13x}{15}$ **l** $\frac{3x}{10}-\frac{3x}{2}$

Hint: Express each fraction with a common denominator using the LCD, then add or subtract numerators.

5 Simplify the following.

a $\frac{x}{2}+\frac{3}{4}$ **b** $\frac{x}{5}+\frac{2}{3}$ **c** $\frac{2x}{15}+\frac{7}{20}$

d $\frac{x}{4}-\frac{2}{5}$ **e** $\frac{2x}{3}-\frac{5}{9}$ **f** $\frac{5}{6}-\frac{x}{4}$

Example 20 Adding and subtracting with binomial numerators

Simplify the following algebraic expressions.

a $\frac{x+2}{4}-\frac{x}{6}$ **b** $\frac{x+3}{3}+\frac{x-4}{7}$

Solution	Explanation
a $\frac{(x+2)(\times 3)}{4(\times 3)}-\frac{x(\times 2)}{6(\times 2)}=\frac{3(x+2)}{12}-\frac{2x}{12}$ $=\frac{3x+6-2x}{12}$ $=\frac{x+6}{12}$	The LCD of 4 and 6 is 12. Express each fraction with a denominator of 12. When multiplying $(x+2)$ by 3, brackets are required. Expand the brackets and collect the terms: $3x+6-2x=3x-2x+6$
b $\frac{(x+3)(\times 7)}{3(\times 7)}+\frac{(x-4)(\times 3)}{7(\times 3)}=\frac{7(x+3)}{21}+\frac{3(x-4)}{21}$ $=\frac{7x+21+3x-12}{21}$ $=\frac{10x+9}{21}$	The LCD of 3 and 7 is 21. Express each fraction with a denominator of 21. Expand each pair of brackets first and sum by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\frac{x-3}{10}-\frac{x}{15}$ **b** $\frac{x+2}{4}+\frac{x-3}{5}$

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3F

6 Simplify these algebraic expressions.

a $\frac{x+2}{3}+\frac{x}{2}$ **b** $\frac{x+4}{5}+\frac{2x}{3}$

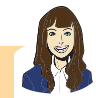
c $\frac{x-2}{4}+\frac{3x}{8}$ **d** $\frac{x+4}{3}-\frac{x}{6}$

e $\frac{x+2}{2}-\frac{2x}{5}$ **f** $\frac{6x+7}{12}-\frac{3x}{8}$

g $\frac{x+3}{5}+\frac{x+2}{4}$ **h** $\frac{2x+3}{7}+\frac{x+1}{2}$

i $\frac{x+8}{6}+\frac{x-3}{4}$ **j** $\frac{2x+5}{3}+\frac{x-2}{4}$

k $\frac{x-3}{5}+\frac{x+4}{10}$ **l** $\frac{2x+1}{8}+\frac{x-2}{3}$



Hint: LCD of 2 and 3 is 6:

$\frac{x+2}{3}+\frac{x}{2}=\frac{\square(x+2)}{6}+\frac{\square x}{6}$

Problem-solving and reasoning

	7, 8	7–9(½)

7 Find the error in each of the following and then correct it.

a $\frac{2x}{3}+\frac{3x}{4}=\frac{5x}{12}$ **b** $\frac{3x}{5}-\frac{x}{2}=\frac{2x}{3}$

c $\frac{x+2}{5}+\frac{x+4}{3}=\frac{3x+2+5x+4}{15}$
$=\frac{8x+6}{15}$

d $\frac{x+4}{2}+\frac{x-3}{6}=\frac{3x+12+x+3}{6}$
$=\frac{4x+15}{6}$

8 Recall that the expansion of $-5(x-2)$ is $-5x+10$,
so $6(x+1)-5(x-2)=6x+6-5x+10=x+16$.
Use this method to simplify these subtractions.

a $\frac{x+1}{5}-\frac{x-2}{6}$ **b** $\frac{x+2}{3}-\frac{x-4}{5}$ **c** $\frac{x-3}{4}-\frac{x+2}{5}$ **d** $\frac{x+8}{2}-\frac{x+7}{4}$

9 The LCD of the fractions $\frac{4}{x}+\frac{2}{3}$ is $3\times x=3x$.

Use this to find the LCD and simplify these fractions.

a $\frac{4}{x}+\frac{2}{3}$ **b** $\frac{3}{4}+\frac{2}{x}$ **c** $\frac{2}{5}+\frac{3}{x}$

d $\frac{3}{7}-\frac{2}{x}$ **e** $\frac{1}{5}-\frac{4}{x}$ **f** $\frac{3}{x}-\frac{5}{8}$

Hint:

$\frac{4}{x}+\frac{2}{3}=\frac{\square}{3x}+\frac{\square}{3x}$

$=\frac{\square}{3x}$

Pronumerals in the denominator

	—	10

10 As seen in Question **9**, pronumerals may form part of the LCD.
The fractions $\frac{5}{2x}$ and $\frac{3}{4}$ would have a LCD of $4x$, whereas the fractions $\frac{3}{x}$ and $\frac{5}{x^2}$ would have a LCD of x^2.
By first finding the LCD, simplify these algebraic fractions.

a $\frac{3}{4}+\frac{5}{2x}$ **b** $\frac{1}{6}+\frac{5}{2x}$ **c** $\frac{3}{10}-\frac{1}{4x}$ **d** $\frac{3}{x}+\frac{5}{x^2}$

e $\frac{4}{x}+\frac{1}{x^2}$ **f** $\frac{3}{x^2}-\frac{5}{x}$ **g** $\frac{3}{2x}+\frac{2}{x^2}$ **h** $\frac{4}{x}+\frac{7}{3x^2}$

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Progress quiz

3A **1** For the expression $2x + \frac{y}{2} - 3x^2 + 5$, determine:

- **a** the number of terms
- **b** the constant term
- **c** the coefficient of:
  - **i** x^2
  - **ii** y

3A **2** Find the value of the following expressions if $a = 2$, $b = -5$ and $c = 8$.

a $ab + 2c$ **b** $b^2 - ac$ **c** $\frac{c}{a} - 2b$

3B **3** Simplify the following by collecting like terms.

- **a** $4x - 3 + 2x$
- **b** $7x - 3y - 2x + 8y$
- **c** $3x^2y + 5xy^2 - x^2y + 2xy^2$

3B **4** Simplify the following.

a $3r \times 4rs$ **b** $\frac{2x}{6}$ **c** $15mn^2 \div (6mn)$

3C **5** Expand the following.

- **a** $3(2x + 3)$
- **b** $4x(5x - 2)$
- **c** $-6(2x - 3)$

3C **6** Expand and simplify the following.

- **a** $4 + 2(4x - 5)$
- **b** $4(2x + 3) - 5(x - 4)$

3D **7** Factorise the following by first identifying the highest common factor. (Include any common negatives.)

a $6m + 12$ **b** $15a - 20ab$
c $4xy + x$ **d** $6x^2 - 10x$
e $-8x - 20$ **f** $-3y^2 - 6y$

3E **8** Simplify by cancelling common factors. You will need to factorise first in parts **b** and **c**.

a $\frac{8(x-1)}{4(x-1)}$ **b** $\frac{15x - 35}{5}$ **c** $\frac{2x^2 + 6x}{x + 3}$

3E ★ **9** Simplify the following algebraic fractions.

a $\frac{8x}{7} \times \frac{21}{16x}$ **b** $\frac{9x}{4(x-1)} \div \frac{12}{x-1}$

3F ★ **10** Simplify the following algebraic fractions.

a $\frac{3x}{5} + \frac{x}{4}$ **b** $\frac{2x}{3} - \frac{4}{9}$

3F ★ **11** Simplify $\frac{x+1}{4} + \frac{x-2}{6}$.

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3G Index notation and index laws 1 and 2

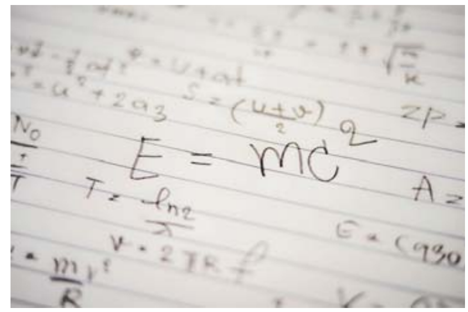
Learning intentions

- To understand index notation and know that it is a shorthand way of writing multiplication of the same base
- To know the index laws for multiplication and division involving a common base
- To be able to apply the index laws for multiplication and division
- To be able to use a calculator to evaluate a power

Key vocabulary: index form, exponent, power, base, index

When 9 is written as 3^2, we are using index form. The 2 in 3^2 is called the index, exponent or power, and the 3 is called the base.

When the same factor is multiplied repeatedly, index form provides an efficient way to represent this. For example, $5 \times 5 \times 5 \times 5$ is written as 5^4 instead. For terms with the same base, calculations can be carried out in index form using the index laws.



Lesson starter: Simplifying in index form

- Express the following calculations in expanded form and then simplify the result in index form.
  a $5^3 \times 5^4$ **b** $6^2 \times 6^3$
- Describe how you could write these in index form without first expanding them.
- Express the following in expanded form and then cancel common factors to write the result as a single term in index form.
  a $\dfrac{4^5}{4^2}$ **b** $\dfrac{7^6}{7^3}$
- Describe how you could write these in index form without first expanding them.

Key ideas

- For a number in **index form**; e.g. a^m, we say 'this is a to the power of m', where a is the **base** and m is the **index**, **exponent** or **power**.
  - The exponent, index or power tells us how many times to multiply the base number by itself. For example:

index form		expanded form		basic numeral
5^3	=	$5 \times 5 \times 5$	=	125

  - Note that $a = a^1$
  - The power button (^) on a calculator allows you to evaluate 5^4 using 5(^)4.
- The first two index laws deal with multiplication and division of numbers expressed in index form.
  - Index law 1: $a^m \times a^n = a^{m+n}$; e.g. $5^3 \times 5^4 = 5^{3+4} = 5^7$.
  - Index law 2: $a^m \div a^n = a^{m-n}$
    or $\dfrac{a^m}{a^n} = a^{m-n}$; e.g. $7^5 \div 7^2 = 7^{5-2} = 7^3$.

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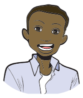
Exercise 3G

Understanding

	1–4	3, 4

1 Fill in the missing words, using *index, power, multiply, expanded* and *base*.
   a In 3^5, 3 is the ____________ and 5 is the ____________.
   b 4^6 is read as 4 to the ____________ of 6.
   c 7^4 is the ____________ form of $7 \times 7 \times 7 \times 7$.
   d The power tells you how many times to ____________ the base number by itself.
   e $6 \times 6 \times 6$ is the ____________ form of 6^3.



Hint: 4 times
$5^{④} = 5 \times 5 \times 5 \times 5$
index form — expanded form

2 Write the following in expanded form.
   a 8^3 b 7^5
   c x^6 d $(ab)^4$

3 Complete the following to write each as a single term in index form.
   a $7^3 \times 7^4 = 7 \times 7 \times 7 \times \square$
   $= 7^{\square}$
   b $\dfrac{5^6}{5^2} = \dfrac{5 \times 5 \times \square}{5 \times 5}$
   $= 5^{\square}$

4 Choose from the words *add* or *subtract* to fill in the missing words.
   a Index law 1 says that when two terms with the same base are multiplied, _______ the powers.
   b Index law 2 says that when two terms with the same base are divided, _______ the powers.

Fluency

	5–7(½)	5–7(½)

Example 21 Writing in index form

Write each of the following in index form.
a $5 \times 5 \times 5$ b $4 \times x \times x \times x \times 4 \times x$ c $a \times b \times b \times a \times b \times b$

Solution	Explanation
a $5 \times 5 \times 5 = 5^3$	The factor 5 is repeated 3 times.
b $4 \times x \times x \times x \times 4 \times x = 4 \times 4 \times x \times x \times x$	Group the factors of 4 and the factors of x together.
$= 4^2x^3$	The factor x is repeated 3 times; 4 is repeated twice.
c $a \times b \times b \times a \times b \times b$	Group the like pronumerals.
$= a \times a \times b \times b \times b \times b$ $= a^2b^4$	The factor a is repeated twice and the factor b is repeated 4 times.

Now you try

Write each of the following in index form.
a $3 \times 3 \times 3 \times 3$ b $7 \times y \times 7 \times y \times y$ c $m \times m \times n \times m \times n$

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3G

5 Write the following in index form.

a $9 \times 9 \times 9 \times 9$

b $3 \times 3 \times 3 \times 3 \times 3 \times 3$

c $15 \times 15 \times 15$

d $5 \times x \times x \times x \times 5$

e $4 \times a \times 4 \times a \times 4 \times a \times a$

f $b \times 7 \times b \times b \times b$

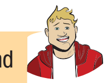
g $x \times y \times x \times x \times y$

h $a \times b \times a \times b \times b \times b$

i $3 \times x \times 3 \times y \times x \times 3 \times y \times y$

j $4 \times x \times z \times 4 \times z \times x$

Hint: Group different bases together and write each base in index form.



Example 22 Using index law 1: $a^m \times a^n = a^{m+n}$

Simplify the following, using the first index law.

a $x^7 \times x^4$ **b** $a^2b^2 \times ab^3$ **c** $3x^2y^3 \times 4x^3y^4$

Solution	Explanation
a $x^7 \times x^4 = x^{7+4}$ $= x^{11}$	Use law 1, $a^m \times a^n = a^{m+n}$, to add the indices.
b $a^2b^2 \times ab^3 = a^{2+1}b^{2+3}$ $= a^3b^5$	Add the indices of base a and base b. Recall that $a = a^1$.
c $3x^2y^3 \times 4x^3y^4 = (3 \times 4)x^{2+3}y^{3+4}$ $= 12x^5y^7$	Multiply the coefficients and add indices of the common bases x and y.

Now you try

Simplify the following, using the first index law.

a $x^5 \times x^3$ **b** $x^2y \times x^3y^4$ **c** $5ab^2 \times 2a^3b^4$

6 Simplify the following, using the first index law.

a $x^3 \times x^4$ **b** $p^5 \times p^2$

c $t^7 \times t^2$ **d** $d^4 \times d$

e $g \times g^3$ **f** $f^2 \times f$

g $2p^2 \times p^3$ **h** $3c^4 \times c^4$

i $2s^4 \times 3s^7$ **j** $a^2b^3 \times a^3b^5$

k $d^7f^3 \times d^2f^2$ **l** $v^3z^5 \times v^2z^3$

m $3a^2b \times 5ab^5$ **n** $2x^2y \times 3xy^2$

o $3e^7r^2 \times 6e^2r$ **p** $-4p^3c^2 \times 2pc$

q $-2r^2s^3 \times 5r^5s^5$ **r** $-3d^4f^2 \times (-2f^2d^2)$

Hint: Index law 1: $a^m \times a^n = a^{m+n}$
Group common bases and add indices when multiplying.

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Example 23 Using index law 2: $a^m \div a^n = a^{m-n}$

Simplify the following, using the second index law.

a $p^5 \div p^3$ b $12m^8 \div (6m^3)$ c $\dfrac{4x^2y^4}{8xy^2}$

Solution	Explanation
a $p^5 \div p^3 = p^{5-3}$ $= p^2$	Use law 2, $a^m \div a^n = a^{m-n}$, to subtract the indices.
b $12m^8 \div (6m^3) = \dfrac{{}^{2}\cancel{12}m^8}{{}^{1}\cancel{6}m^3}$ $= 2m^{8-3}$ $= 2m^5$	Write in fraction form. Cancel the highest common factor of 12 and 6. Use law 2 to subtract indices.
c $\dfrac{4x^2y^4}{8xy^2} = \dfrac{{}^{1}\cancel{4} \times x^2 \times y^4}{{}^{2}\cancel{8} \times x \times y^2}$ $= \dfrac{x^{2-1}y^{4-2}}{2}$ $= \dfrac{xy^2}{2}$ $\left(\text{or } \dfrac{1}{2}xy^2\right)$	Cancel the common factors of the numerals and subtract the indices of base x and base y.

Now you try

Simplify the following, using the second index law.

a $b^7 \div b^2$ b $20a^6 \div (8a^2)$ c $\dfrac{6a^3b^4}{9ab^3}$

7 Simplify the following, using the second index law.

a $a^4 \div a^2$ b $d^7 \div d^6$ c $r^3 \div r$

d $\dfrac{c^{10}}{c^6}$ e $\dfrac{l^4}{l^3}$ f $\dfrac{b^5}{b^2}$

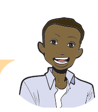
g $\dfrac{4d^4}{d^2}$ h $\dfrac{f^2}{2f^2}$ i $\dfrac{9n^4}{3n}$

j $6p^4 \div (3p^2)$ k $24m^7 \div (16m^3)$ l $10d^3 \div (30d)$

m $\dfrac{8t^4r^3}{2tr^2}$ n $\dfrac{5h^6d^4}{3d^3h^2}$ o $\dfrac{2p^2q^3}{p^2q}$

p $\dfrac{4x^2y^3}{8xy}$ q $\dfrac{3r^5s^2}{9r^3s}$ r $6c^4d^6 \div (15c^3d)$

s $2a^4y^2 \div (4ay)$ t $13m^4n^6 \div (26m^4n^5)$ u $18x^4y^3 \div (-3x^2y)$



Hint: Index law 2:
$a^m \div a^n = a^{m-n}$
or
$\dfrac{a^m}{a^n} = a^{m-n}$
When dividing, subtract indices of common bases.

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3G

Problem-solving and reasoning | 8 | 8(½), 9

Example 24 Combining laws 1 and 2

Simplify $\frac{2a^3b \times 3a^2b^3}{12a^4b^2}$ using index laws 1 and 2.

Solution	Explanation
$\frac{2a^3b \times 3a^2b^3}{12a^4b^2} = \frac{(2 \times 3)a^{3+2}b^{1+3}}{12a^4b^2}$	Simplify numerator first by multiplying coefficients and using law 1 to add indices of a and b.
$= \frac{\cancel{6}^{1}a^5b^4}{{}_{2}\cancel{12}a^4b^2}$	Cancel common factor of numerals and use law 2 to subtract indices of common bases.
$= \frac{a^{5-4}b^{4-2}}{2}$	
$= \frac{ab^2}{2}$	

Now you try

Simplify $\frac{4ab^2 \times 3a^5b^4}{18a^3b^5}$ using index laws 1 and 2.

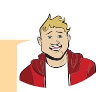
8 Simplify the following, using index laws 1 and 2.

a $\frac{x^2y^3 \times x^2y^4}{x^3y^5}$ **b** $\frac{m^2w \times m^3w^2}{m^4w^3}$

c $\frac{r^4s^7 \times r^4s^7}{r^6s^{10}}$ **d** $\frac{16a^8b \times 4ab^7}{32a^7b^6}$

e $\frac{9x^2y^3 \times 6x^7y^7}{12xy^6}$ **f** $\frac{4e^2w^2 \times 12e^2w^3}{12e^4w}$

Hint: Simplify the numerator first using index law 1, then apply index law 2.



9 When Stuart uses a calculator to raise -2 to the power 4 he gets -16, when in fact the answer is actually 16. What has he done wrong?

Index laws and calculations | — | 10, 11

10 Consider the following use of negative numbers.

a Evaluate:

i $(-3)^2$ **ii** -3^2

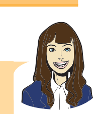
b What is the difference between your two answers in part **a**?

c Evaluate:

i $(-2)^3$ **ii** -2^3

d What do you notice about your answers in part **c**? Explain.

Hint:
$(-3)^2 = -3 \times (-3)$
$(-2)^3 = -2 \times (-2) \times (-2)$
Consider order of operations for -3^2 and -2^3.



11 Use index law 2 to evaluate these expressions without the use of a calculator.

a $\frac{13^3}{13^2}$ **b** $\frac{18^7}{18^6}$ **c** $\frac{9^8}{9^6}$ **d** $\frac{3^{10}}{3^7}$ **e** $\frac{4^8}{4^5}$ **f** $\frac{2^{12}}{2^8}$

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3H Index laws 3–5 and the zero power

Learning intentions

- To know how to apply indices to terms in brackets
- To know the rule for the zero power
- To be able to simplify expressions involving indices and brackets
- To be able to simplify using the zero power

Key vocabulary: index/indices, base

Using index laws 1 and 2, we can work out four other index laws to simplify expressions, especially those using brackets.

For example, $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= 4^{2+2+2} = 4^6$ (Add indices using law 1.)

Therefore, $(4^2)^3 = 4^{2\times 3} = 4^6$.

We also have a result for the zero power. Consider $5^3 \div 5^3$, which clearly equals 1.

Using index law 2, we can see that $5^3 \div 5^3 = 5^{3-3} = 5^0$

Therefore, $5^0 = 1$, leading to the zero power rule: $a^0 = 1$, $(a \neq 0)$.

Lesson starter: Indices with brackets

Brackets are used to show that the power outside the brackets applies to each factor inside the brackets.

Consider $(2x)^3 = 2x \times 2x \times 2x$.

- Write this in index form without using brackets.
- Can you suggest the index form of $(3y)^4$ without brackets?

Consider $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$.

- Write the numerator and denominator of this expanded form in index form.
- Can you suggest the index form of $\left(\frac{x}{4}\right)^5$ without brackets?

Write a rule for removing the brackets of the following.

- $(ab)^m$
- $\left(\frac{a}{b}\right)^m$

Key ideas

- Index law 3: $(a^m)^n = a^{m\times n}$
  Remove brackets and multiply indices. For example, $(x^3)^4 = x^{3\times 4} = x^{12}$.
- Index law 4: $(a \times b)^m = a^m \times b^m$
  Apply the index to each factor in the brackets. For example, $(3x)^4 = 3^4x^4$.
- Index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  Apply the index to the numerator and denominator. For example, $\left(\frac{y}{3}\right)^5 = \frac{y^5}{3^5}$.
- The zero power: a^0
  Any number (except zero) to the power of zero is 1. For example, $5^0 = 1$, $y^0 = 1$, $4y^0 = 4 \times 1 = 4$.

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Exercise 3H

Understanding | 1, 2 | 2

1 Complete the following index laws.

a Any number (except 0) to the power of zero is equal to ______.

b Index law 3 states $(a^m)^n =$ ______.

c Index law 4: $(a \times b)^m =$ ______ $\times$ ______.

d Index law 5: $\left(\frac{a}{b}\right)^m = \frac{\square}{\square}$.

2 Copy and complete the following.

a $(2a)^3 = 2a \times \square \times \square$
$= 2 \times \square \times \square \times a \times \square \times \square$
$= 2^{\square} a^{\square}$

b $\left(\frac{4}{7}\right)^4 = \frac{4}{7} \times \square \times \square \times \square$
$= \frac{4 \times \square \times \square \times \square}{7 \times \square \times \square \times \square}$
$= \frac{4^{\square}}{7^{\square}}$

Fluency | 3–6(½) | 3–6(½)

Example 25 Using the zero power: $a^0 = 1$

Evaluate the following by using the zero power.

a $4^0 + 2^0$ b $3a^0$ c $(-3)^0 + 6x^0$

Solution	Explanation
a $4^0 + 2^0 = 1 + 1$ $= 2$	Zero power: $a^0 = 1$, any number to the power zero (except zero) is 1.
b $3a^0 = 3 \times a^0$ $= 3 \times 1$ $= 3$	The zero power applies only to a, so $a^0 = 1$.
c $(-3)^0 + 6x^0 = 1 + 6 \times 1$ $= 7$	Any number to the power of zero is 1. $(-3)^0 = 1, 6x^0 = 6 \times x^0 = 6 \times 1$

Now you try

Evaluate the following by using the zero power.

a $5^0 - 3^0$ b $4y^0$ c $(-2)^0 + 3b^0$

3 Evaluate the following, using the zero power.

a 4^0 b 5^0 c x^0 d a^0

e $3e^0$ f $4y^0$ g $3^0 + 6^0$ h $10 - 10x^0$

i $3d^0 - 2$ j $(-4)^0 + 2x^0$ k $\frac{2}{m^0}$ l $5a^0 + 4b^0$

Hint: Any number (except zero) to the power of 0 is 1.
$4a^0 = 4 \times a^0 = 4 \times 1$

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Example 26 Using index law 3: $(a^m)^n = a^{m \times n}$

Simplify the following by using the third index law.

a $(x^5)^7$ **b** $3(f^4)^3$

Solution	Explanation
a $(x^5)^7 = x^{5\times 7}$ $= x^{35}$	Apply index law 3: $(a^m)^n = a^{m\times n}$ to multiply indices.
b $3(f^4)^3 = 3f^{4\times 3}$ $= 3f^{12}$	Apply index law 3 to the value inside the bracket only.

Now you try

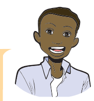
Simplify the following by using the third index law.

a $(y^3)^6$ **b** $5(m^2)^4$

4 Simplify the following by using the third index law.

a $(b^3)^4$ **b** $(f^5)^4$ **c** $(k^3)^7$

d $3(x^2)^3$ **e** $5(c^9)^2$ **f** $4(s^6)^3$



Hint:
Index law 3:
$(a^m)^n = a^{m\times n}$

Example 27 Using index laws 3, 4 and 5: $(a^m)^n = a^{m\times n}$, $(a \times b)^m = a^m \times b^m$ and $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplify the following by using the third, fourth and fifth index laws.

a $(2s)^4$ **b** $(x^2y^3)^5$ **c** $\left(\frac{x}{4}\right)^3$

Solution	Explanation
a $(2s)^4 = 2^4 \times s^4$ $= 16s^4$	Apply index law 4: $(a \times b)^m = a^m \times b^m$. Evaluate $2^4 = 2 \times 2 \times 2 \times 2$.
b $(x^2y^3)^5 = (x^2)^5 \times (y^3)^5$ $= x^{10}y^{15}$	Using index law 4, apply the index 5 to each factor in the brackets. Using index law 3, multiply indices: $(x^2)^5 = x^{2\times 5}$, $(y^3)^5 = y^{3\times 5}$
c $\left(\frac{x}{4}\right)^3 = \frac{x^3}{4^3}$ $= \frac{x^3}{64}$	Apply index law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. Evaluate $4^3 = 4 \times 4 \times 4$.

Now you try

Simplify the following by using the third, fourth and fifth index laws.

a $(3m)^3$ **b** $(a^3b^2)^4$ **c** $\left(\frac{y}{3}\right)^4$

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3H

5 Simplify the following by using the third, fourth and fifth index laws.

a $(3x)^2$ b $(4m)^3$ c $(5y)^3$

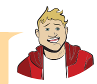
d $(2x^3)^4$ e $(x^2y)^5$ f $(3a^3)^3$

g $(x^4y^2)^6$ h $(a^2b)^3$ i $(m^3n^3)^4$

j $\left(\frac{x}{5}\right)^2$ k $\left(\frac{y}{3}\right)^4$ l $\left(\frac{m}{2}\right)^4$

m $\left(\frac{x^2}{y}\right)^3$ n $\left(\frac{x^3}{y^2}\right)^4$ o $\left(\frac{x}{y^5}\right)^3$

Hint:
Index law 4 says $(3 \times x)^2 = 3^2 \times x^2$
Index law 5 says $\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2}$



Example 28 Combining index laws

Simplify the following by using index laws.

a $\frac{3x^2y \times 2x^3y^2}{10xy^3}$ b $\left(\frac{2x^2}{y}\right)^4$ c $(2x^2)^3 + (3x)^0$

Solution	Explanation
a $\frac{3x^2y \times 2x^3y^2}{10xy^3}$	Simplify the numerator by multiplying coefficients and adding indices, using index law 1.
$= \frac{6x^5y^3}{10xy^3}$	Cancel the common factor of 6 and 10 and apply index law 2 to subtract indices of common bases.
$= \frac{3x^4y^0}{5}$	The zero power says $y^0 = 1$.
$= \frac{3x^4}{5}$	
b $\left(\frac{2x^2}{y}\right)^4 = \frac{(2x^2)^4}{y^4}$	Apply index law 5 to apply the index to the numerator and denominator.
$= \frac{2^4 \times (x^2)^4}{y^4}$	Apply laws 3 and 4 to multiply indices.
$= \frac{16x^8}{y^4}$	
c $(2x^2)^3 + (3x)^0 = 2^3 \times x^6 + 3^0 \times x^0$	Using index law 4, apply the power to each factor inside the brackets:
$= 8x^6 + 1 \times 1$	$(x^2)^3 = x^{2\times3} = x^6$
$= 8x^6 + 1$	Any number to the power of zero is 1.

Now you try

Simplify the following by using index laws.

a $\frac{4a^2b^3 \times 3a^2b}{6a^3b^4}$ b $\left(\frac{3x}{y^2}\right)^3$ c $(5m^4)^2 + (5m)^0$

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6 Simplify the following by using index laws.

a $\dfrac{m^7w \times m^3w^2}{m^4w^3}$ b $\dfrac{x^3y^2 \times x^2y^7}{10x^5y^4}$

c $\dfrac{b^3c^5 \times 4b^5c^3}{3b^4c^8}$ d $\dfrac{9c^4s^2 \times 3c^3s^5}{2c^3s^7}$

Hint: First simplify the numerator, then consider the denominator.

e $\dfrac{(5r^6)^2}{3r^8}$ f $\dfrac{(2p^4)^3}{3p^7}$

g $\left(\dfrac{2s^2}{t^3}\right)^4$ h $\left(\dfrac{r^2}{5s^3}\right)^4$

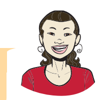
Problem-solving and reasoning

7–8(½)	7–8(½)

7 Evaluate the following without the use of a calculator.

a $\dfrac{(5^2)^2}{5^4}$ b $\dfrac{36^2}{6^4}$ c $\dfrac{27^2}{3^4}$ d $\dfrac{16^2}{4^3}$

Hint: $36^2 = (6^2)^2$



8 Simplify the following.

a $2p^2q^4 \times pq^3$ b $4(a^2b)^3 \times (3ab)^3$

c $(4r^2y)^2 \times r^2y^4 \times 3(ry^2)^3$ d $2(m^3n)^4 \div m^3$

e $\dfrac{(7s^2y)^2 \times 3sy^2}{7(sy)^2}$ f $\dfrac{3(d^4c^3)^3 \times 4dc}{(2c^2d)^3}$

g $\dfrac{4r^2t \times 3(r^2t)^3}{6r^2t^4}$ h $\dfrac{(2xy)^2 \times 2(x^2y)^3}{8xy \times x^7y^3}$

All laws together

—	9

9 Simplify the following, expressing your answer with positive indices.

a $(a^3b^2)^3 \times a^2b^4$ b $2x^2y \times (xy^4)^3$

c $2(p^2)^4 \times (3p^2q)^2$ d $\dfrac{2a^3b^2}{a^3} \times \dfrac{2a^2b^5}{b^4}$

e $\dfrac{(3rs^2)^4}{r^3s^4} \times \dfrac{(2r^2s)^2}{s^7}$ f $\dfrac{4(x^2y^4)^2}{x^2y^3} \times \dfrac{xy^4}{2s^2y}$

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3I Negative indices

Learning intentions

- To know how negative indices can be equivalently expressed using positive indices
- To be able to express negative indices in terms of positive indices
- To be able to use the index laws with negative indices

Key vocabulary: index/indices, base

We have seen how positive indices are used as a shorthand way of writing repeated multiplication of the same base but what do negative powers represent; e.g. 3^{-2} and x^{-1}. Negative powers are used in many areas of science.

Consider $\frac{3^3}{3^5}$:

Expanding and simplifying gives

$$\frac{3^3}{3^5} = \frac{\cancel{3}^1 \times \cancel{3}^1 \times \cancel{3}^1}{3 \times 3 \times \cancel{3}_1 \times \cancel{3}_1 \times \cancel{3}_1}$$

$$= \frac{1}{3 \times 3}$$

$$= \frac{1}{3^2}$$

Using index law 2, however, we get

$$\frac{3^3}{3^5} = 3^{3-5}$$

$$= 3^{-2}$$

$$\therefore \quad 3^{-2} = \frac{1}{3^2}.$$

Scientists use negative powers when describing the mass or size of very small objects.

Lesson starter: Continuing the pattern

Complete this table to consider the value of powers of 3 including negative powers.

Index form	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	27	9				$\frac{1}{9} = \frac{1}{3^2}$	

$\div 3$ $\div 3$ $\div 3$

Complete a similar table for powers of 5.

- What do you notice about the fractions in the second row compared to the numbers with negative indices in the top row in each table?
- Can you write this connection as a rule?
- What would be a way of writing 3^{-5} and 5^{-4} with a positive index?

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Key ideas

- Negative indices can be expressed as positive indices using the following rules:
  $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$
- The negative index only applies to the pronumeral or term it is a power of.
  For example, $2a^{-4}b^5 = 2 \times \frac{1}{a^4} \times b^5 = \frac{2b^5}{a^4}$
- All the index laws can be applied to negative indices.

Exercise 3I

Understanding | 1, 2 | 2

1 Complete the following, to express with positive indices.

a $a^{-m} = \square$ **b** $\frac{1}{a^{-m}} = \square$

2 Complete the following by filling in the boxes to express with positive indices.

a $x^{-3} = \frac{1}{x^{\square}}$

b $5 \times m^{-2} = 5 \times \frac{1}{\square}$
$= \frac{5}{\square}$

c $\frac{1}{a^{-4}} = a^{\square}$

Fluency | 3–5(½) | 3–5(½)

Example 29 Expressing negative indices in positive index form

Express the following with positive indices.

a x^{-2} **b** $4y^{-2}$ **c** $2a^{-3}b^2$

Solution	Explanation
a $x^{-2} = \frac{1}{x^2}$	Use $a^{-m} = \frac{1}{a^m}$.
b $4y^{-2} = 4 \times \frac{1}{y^2}$ $= \frac{4}{y^2}$	The negative index applies only to y; i.e. $y^{-2} = \frac{1}{y^2}$. $4 \times \frac{1}{y^2} = \frac{4}{1} \times \frac{1}{y^2} = \frac{4}{y^2}$
c $2a^{-3}b^2 = 2 \times \frac{1}{a^3} \times b^2$ $= \frac{2b^2}{a^3}$	$a^{-3} = \frac{1}{a^3}$, $\frac{2}{1} \times \frac{1}{a^3} \times \frac{b^2}{1} = \frac{2b^2}{a^3}$ Multiply the numerators and the denominators.

Now you try

Express the following with positive indices.

a b^{-4} **b** $3a^{-3}$ **c** $4x^{-2}y^4$

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3 Express the following with positive indices.

a y^{-3} **b** x^{-4} **c** x^{-2} **d** a^{-5}

e $3x^{-2}$ **f** $5b^{-3}$ **g** $4x^{-1}$ **h** $2m^{-9}$

i $2x^2y^{-3}$ **j** $3xy^{-4}$ **k** $3a^{-2}b^4$ **l** $5m^{-3}n^2$

Hint: Use $a^{-m} = \frac{1}{a^m}$.

Example 30 Using $\frac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices only.

a $\frac{1}{x^{-3}}$ **b** $\frac{4}{x^{-5}}$ **c** $\frac{5}{a^2b^{-4}}$

Solution

a $\frac{1}{x^{-3}} = x^3$

b $\frac{4}{x^{-5}} = 4 \times \frac{1}{x^{-5}}$
$= 4 \times x^5$
$= 4x^5$

c $\frac{5}{a^2b^{-4}} = \frac{5}{a^2} \times \frac{1}{b^{-4}}$
$= \frac{5}{a^2} \times b^4$
$= \frac{5b^4}{a^2}$

Explanation

a: Use $\frac{1}{a^{-m}} = a^m$.

b: The 4 remains unchanged.
Note: $\frac{1}{x^{-5}} = x^5$.

c: The negative index applies to b only; i.e. $\frac{1}{b^{-4}} = b^4$.
$\frac{5}{a^2} \times b^4 = \frac{5}{a^2} \times \frac{b^4}{1} = \frac{5b^4}{a^2}$

Now you try

Rewrite the following with positive indices only.

a $\frac{1}{y^{-2}}$ **b** $\frac{3}{a^{-4}}$ **c** $\frac{7}{x^{-2}y^3}$

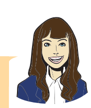
4 Rewrite the following with positive indices only.

a $\frac{1}{b^{-4}}$ **b** $\frac{1}{x^{-7}}$ **c** $\frac{1}{y^{-1}}$

d $\frac{5}{m^{-3}}$ **e** $\frac{2}{y^{-2}}$ **f** $\frac{3}{x^{-4}}$

g $\frac{5a^2}{b^{-3}}$ **h** $\frac{4}{x^2y^{-5}}$ **i** $\frac{10}{a^{-2}b^4}$

Hint: Use $\frac{1}{a^{-m}} = a^m$.



5 Rewrite the following with positive indices only.

a $\frac{4x^{-2}}{y^3}$ **b** $\frac{b^{-3}}{5a^2}$ **c** $\frac{2a^3}{b^{-2}}$

d $\frac{a^4}{3b^{-5}}$ **e** $\frac{y^{-2}}{x^{-3}}$ **f** $\frac{xy^{-3}}{x^{-2}y}$

Hint: For part **e**, $\frac{y^{-2}}{x^{-3}} = y^{-2} \times \frac{1}{x^{-3}} = \dots$

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Problem-solving and reasoning

6–7(½), 8 | 6–7(½), 8, 9(½)

Example 31 Combining index laws with negative indices

Simplify the following, using index laws. Express answers with positive indices.

a $\dfrac{x^4y^3 \times x^{-2}y^5}{x^5y^4}$ **b** $\dfrac{4(x^2y^{-1})^{-2}}{y^5}$

Solution	Explanation
a $\dfrac{x^4y^3 \times x^{-2}y^5}{x^5y^4} = \dfrac{x^{4+(-2)}y^{3+5}}{x^5y^4}$ $= \dfrac{x^2y^8}{x^5y^4}$	Use law 1 to add indices of x and y in numerator: For x: $4 + (-2) = 4 - 2 = 2$ For y: $3 + 5 = 8$
$= x^{2-5}y^{8-4}$ $= x^{-3}y^4$	Express with positive indices; i.e. $x^{-3} = \dfrac{1}{x^3}$.
$= \dfrac{y^4}{x^3}$	$x^{-3}y^4 = \dfrac{1}{x^3} \times \dfrac{y^4}{1}$
b $\dfrac{4(x^2y^{-1})^{-2}}{y^5} = \dfrac{4x^{-4}y^2}{y^5}$	Remove the brackets by applying index laws 3 and 4 to distribute the power to each pronumeral: $x^{2\times(-2)}$ and $y^{-1\times(-2)}$.
$= 4x^{-4}y^{-3}$	Apply index law 2 to subtract the powers with a base of y.
$= \dfrac{4}{x^4y^3}$	Express with positive indices: $4x^{-4}y^{-3} = 4 \times \dfrac{1}{x^4} \times \dfrac{1}{y^3}$

Now you try

Simplify the following, using index laws. Express answers with positive indices.

a $\dfrac{x^2y^{-1} \times x^2y^4}{x^6y^2}$ **b** $\dfrac{9(x^{-3}y^2)^{-2}}{x^7}$

6 Simplify the following, expressing answers using positive indices.

a $\dfrac{a^6b^2 \times a^{-2}b^3}{a^7b}$ **b** $\dfrac{x^5y^3 \times x^2y^{-1}}{x^3y^5}$

c $\dfrac{x^4y^7 \times x^{-2}y^{-5}}{x^4y^6}$ **d** $\dfrac{a^5b^{-2} \times a^{-3}b^4}{a^6b}$

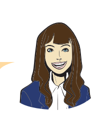
Hint: Index laws 1 and 2 apply to negative indices also.
$x^5 \times x^{-2} = x^{5+(-2)} = x^3$
$\dfrac{x^4}{x^6} = x^{4-6} = x^{-2} = \dfrac{1}{x^2}$

7 Simplify, using index laws, and express with positive indices.

a $(x^{-4})^2$ **b** $(x^3)^{-2}$ **c** $(x^{-2})^0$

d $(2y^{-2})^3$ **e** $(ay^{-3})^2$ **f** $(4x^{-3})^{-2}$

g $\dfrac{3(x^{-4}y^3)^{-2}}{4x^7}$ **h** $(a^{-3}b^2)^{-2} \times (a^{-1}b^{-2})^3$ **i** $\dfrac{(2m^{-3}n)^2}{4m^2n^{-3}}$



Hint: Remove brackets using index laws, then use $a^{-m} = \dfrac{1}{a^m}$ to express with a positive index.

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3I

8 The mass of a small insect is 3^{-6} kg. How many grams is this, correct to two decimal places?

9 Evaluate the following without the use of a calculator.

Hint: Express each one with a positive index first.

a 2^{-2} **b** 5^{-3} **c** $\frac{4}{3^{-2}}$ **d** $\frac{5}{2^{-3}}$

e -3×2^{-2} **f** $6^4 \times 6^{-6}$ **g** $\frac{2^3}{2^{-3}}$ **h** $8 \times (2^2)^{-2}$

The power of −1

	—	10

10 Consider the number $\left(\frac{3}{4}\right)^{-1}$. Using a positive index this becomes $\frac{1}{\left(\frac{3}{4}\right)} = 1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}$.

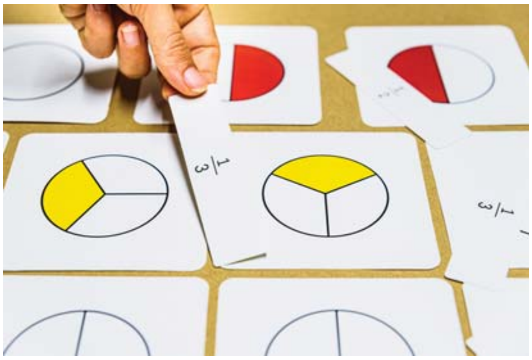
a Complete similar working to simplify the following.

i $\left(\frac{5}{3}\right)^{-1}$ **ii** $\left(\frac{1}{4}\right)^{-1}$ **iii** $\left(\frac{x}{2}\right)^{-1}$ **iv** $\left(\frac{a}{b}\right)^{-1}$

b What conclusion can you come to regarding the simplification of fractions raised to the power of −1?

c Simplify these fractions.

i $\left(\frac{3}{2}\right)^{-2}$ **ii** $\left(\frac{3}{5}\right)^{-2}$ **iii** $\left(\frac{1}{3}\right)^{-3}$ **iv** $\left(\frac{2}{3}\right)^{-4}$



3J Scientific notation

Learning intentions

- To know that scientific notation is a way of representing very large and very small numbers
- To know the form of numbers written in scientific notation
- To be able to express numbers using scientific notation and as a basic numeral
- To be able to use and interpret scientific notation on a calculator
- To know how significant figures are counted
- To be able to round to a number of significant figures

Key vocabulary: scientific notation, significant figures

Scientific notation is useful when working with very large or very small numbers. Combined with the use of significant figures, numbers can be written down with an appropriate degree of accuracy and without the need to write all the zeroes that define the position of the decimal point. The approximate distance between Earth and the Sun is 150 million kilometres or 1.5×10^8 km, when written in scientific notation. Negative indices can be used for very small numbers, such as 0.0000382 g $= 3.82 \times 10^{-5}$ g.

Lesson starter: Amazing facts large and small

Think of an object, place or living thing that is associated with a very large or small number.

- Give three examples of very large numbers.
- Give three examples of very small numbers.
- Can you remember how to write these numbers using scientific notation? List the rules you remember.

Key ideas

- **Scientific notation** is a way to express very large and very small numbers.
- A number written using scientific notation is of the form $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$ and m is an integer.
- To write numbers using scientific notation, place the decimal point after the first non-zero digit and then multiply by the power of 10 that corresponds to how many places the decimal point is moved.
  - Large numbers will use positive powers of 10.
    For example, $24\,800\,000 = 2.48 \times 10^7$
    $9\,020\,000\,000 = 9.02 \times 10^9$
  - Small numbers will use negative powers of 10.
    For example, $0.00307 = 3.07 \times 10^{-3}$
    $0.000001\,2 = 1.2 \times 10^{-6}$
- **Significant figures** are counted from left to right, starting at the first non-zero digit. Rounding occurs by considering the digit following the last significant digit; 5 or more round up, less than 5 round down. For example:
  - $47\,086\,120$ is written $47\,086\,000$ using five significant figures.
  - 2.03684 is written 2.037 using four significant figures.
  - 0.00143 is written 0.0014 using two significant figures.
  - 0.0014021 is written 0.00140 using three significant figures.
    Zeroes at the end of a number are counted for decimals (see 0.00140 above) but not whole numbers (see $47\,086\,000$ above).

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- When using scientific notation, the first significant figure sits to the left of the decimal point. For example:
  - 20 190 000 is written 2.02×10^7 using three significant figures.
- The EE or Exp keys on calculators can be used to enter numbers that use scientific notation: 2.3E−4 means 2.3×10^{-4}.

Exercise 3J

Understanding — 1–3 — 1, 3

1 State yes (Y) or no (N) as to whether the following numbers are written in scientific notation form.
a 1.27×10^3 **b** 15.2×10^{-2} **c** 0.8×10^2 **d** -4.1×10^{-3}

2 State whether these numbers would have positive or negative indices when written in scientific notation.
a 7800 **b** 0.0024 **c** 27 000 **d** 0.0009

3 Write the number 4.8721, using the following numbers of significant figures.
a three **b** four **c** two

Hint: Start counting significant digits from the 4.

Fluency — 4–5(½) — 4–5(½)

Example 32 Converting from scientific notation to a basic numeral

Write these numbers as a basic numeral.
a 5.016×10^5 **b** 3.2×10^{-7}

Solution	Explanation
a $5.016 \times 10^5 = 501\,600$	Move the decimal point 5 places to the right, inserting zeroes after the last digit. 5.01600
b $3.2 \times 10^{-7} = 0.00000032$	Move the decimal point 7 places to the left due to the −7, and insert zeroes where necessary.

Now you try

Write these numbers as a basic numeral.
a 3.27×10^4 **b** 1.2×10^{-3}

4 Write these numbers as a basic numeral.
a 3.12×10^3 **b** 5.4293×10^4 **c** 7.105×10^5
d 8.213×10^6 **e** 5.95×10^4 **f** 8.002×10^5
g 1.012×10^4 **h** 9.99×10^6 **i** 2.105×10^8
j 4.5×10^{-3} **k** 2.72×10^{-2} **l** 3.085×10^{-4}
m 7.83×10^{-3} **n** 9.2×10^{-5} **o** 2.65×10^{-1}
p 1.002×10^{-4} **q** 6.235×10^{-6} **r** 9.8×10^{-1}

Hint: For a positive index, move the decimal point right (number gets bigger). For a negative index, move the decimal point left (number gets smaller).

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Example 33 Writing numbers using scientific notation

Write these numbers in scientific notation.

a 5 700 000 **b** 0.0000006

Solution	Explanation
a $5700000 = 5.7 \times 10^6$	Place the decimal point after the first non-zero digit (5) and then multiply by 10^6, as the decimal point has been moved 6 places to the left.
b $0.0000006 = 6 \times 10^{-7}$	6 is the first non-zero digit. Multiply by 10^{-7} since the decimal point has been moved 7 places to the right.

Now you try

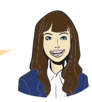
Write these numbers in scientific notation.

a 320 000 **b** 0.0002

5 Write these numbers in scientific notation.

a 43 000 **b** 712 000 **c** 901 200
d 10 010 **e** 23 900 **f** 703 000 000
g 0.00078 **h** 0.00101 **i** 0.00003
j 0.03004 **k** 0.112 **l** 0.00192

Hint: For scientific notation, place the decimal point after the first non-zero digit and multiply by the power of 10.



Example 34 Converting to scientific notation using significant figures

Write these numbers in scientific notation using three significant figures.

a 5 218 300 **b** 0.0042031

Solution	Explanation
a $5218300 = 5.22 \times 10^6$	Put the decimal point after 5 and multiply by 10^6: 5.218300 The digit following the third digit (8) is at least 5, so round the 1 up to 2.
b $0.0042031 = 4.20 \times 10^{-3}$	Put the decimal point after 4 and multiply by 10^{-3}: 0.0042031 Round down in this case, since the digit following the third digit (3) is less than 5, but retain the zero to show the value of the third significant figure.

Now you try

Write these numbers in scientific notation using three significant figures.

a 53 721 **b** 0.0003625

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3J

Problem-solving and reasoning 6–8 6–10

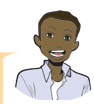
6 Write these numbers in scientific notation using three significant figures.

a 6241
b 572 644
c 30 248
d 423 578
e 10 089
f 34 971 863
g 72 477
h 356 088
i 110 438 523
j 0.002423
k 0.018754
l 0.000125
m 0.0078663
n 0.0007082
o 0.11396
p 0.000006403
q 0.00007892
r 0.000129983

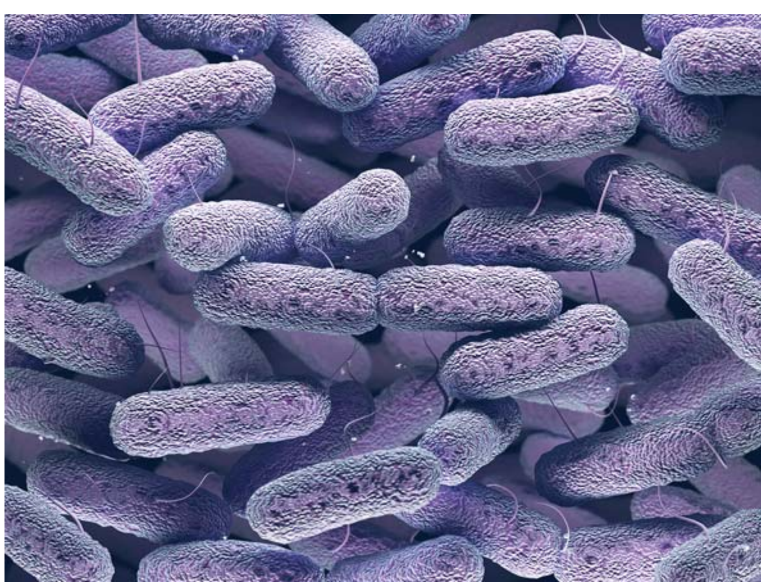
Hint: For three significant figures, count from the first non-zero digit. Look at the digit after the third digit to determine whether you should round up or down.

7 Write the following numerical facts using scientific notation.

a The area of Australia is about 7 700 000 km^2.
b The number of stones used to build the Pyramid of Khufu is about 2 500 000.
c The greatest distance of Pluto from the Sun is about 7 400 000 000 km.
d A human hair is about 0.01 cm wide.
e The mass of a neutron is about 0.000000000000000000000000001675 kg.
f The mass of a bacteria cell is about 0.00000000000095 g.



Hint: Large numbers have a positive index. Small numbers have a negative index.



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8 Explain why 38×10^7 is not written using scientific notation and then convert it to scientific notation.

9 Use a calculator to evaluate the following, giving the answers in scientific notation using three significant figures.

a $(2.31)^{-7}$ **b** $(5.04)^{-4}$

c $(2.83 \times 10^2)^{-3}$ **d** $5.1 \div (8 \times 10^2)$

e $9.3 \times 10^{-2} \times 8.6 \times 10^8$ **f** $(3.27 \times 10^4) \div (9 \times 10^{-5})$

g $\sqrt{3.23 \times 10^{-6}}$ **h** $\pi(3.3 \times 10^7)^2$

Hint: Locate the $\times 10^x$ or EE or Exp button on your calculator.

10 The speed of light is approximately 3×10^5 km/s and the average distance between Pluto and the Sun is about 5.9×10^9 km. How long does it take for light from the Sun to reach Pluto? Answer correct to the nearest minute. (Divide by 60 to convert seconds to minutes.)

Hint: Time = $\frac{\text{distance}}{\text{speed}}$

$E = mc^2$ — 11

11 $E = mc^2$ is a formula derived by Albert Einstein (1879–1955). The formula relates the energy (E joules) of an object to its mass (m kg), where c is the speed of light (approximately 3×10^8 m/s).

Use $E = mc^2$ to answer these questions using scientific notation.

a Find the energy, in joules, contained inside an object with these given masses.

i 10 kg **ii** 26 000 kg

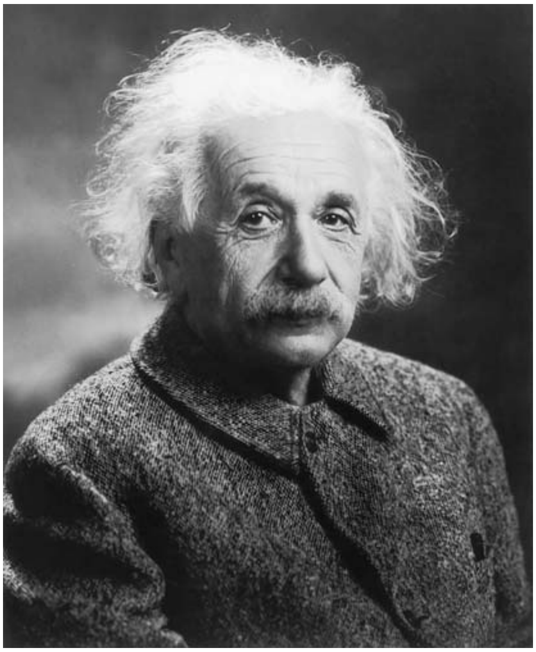
iii 0.03 kg **iv** 0.00001 kg

b Find the mass, in kilograms, of an object that contains the given amounts of energy. Give your answer using 3 significant figures.

i 1×10^{25} J **ii** 3.8×10^{16} J

iii 8.72×10^4 J **iv** 1.7×10^{-2} J

c The mass of Earth is about 6×10^{24} kg. How much energy does this convert to?



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3K Exponential growth and decay ★

Learning intentions

- To understand the concept of exponential growth and decay
- To know the rule that models exponential growth and decay
- To be able to form a rule for exponential growth or decay
- To be able to apply an exponential rule including compound interest

Key vocabulary: exponential growth, exponential decay, compound interest, principal

Exponential change occurs when a quantity is continually affected by a constant multiplying factor. The change in quantity is not the same amount each time.
If you have a continual percentage increase, it is called exponential growth. If you have a continual percentage decrease, it is called exponential decay.
Some examples include:

- compound interest at a rate of 5% per year, where the interest is calculated as 5% of the investment value each year, including the previous year's interest
- a radioactive element has a 'half-life' of 5 years, which means the element decays at a rate of 50% every 5 years.

Lesson starter: A compound rule

Imagine that you have an investment valued at $100 000 and you hope that it will return 10% p.a. (per annum).
The 10% return is to be added to the investment balance each year.

- Discuss how to calculate the investment balance in the first year.
- Discuss how to calculate the investment balance in the second year.
- Complete this table.

Year	0	1	2	3
Balance ($)	100 000	$100\,000 \times 1.1 =$ ________	$100\,000 \times 1.1 \times$ ____ $=$ ________	________ $=$ ________

- Recall how indices can be used to calculate the balance after the second year.
- Discuss how indices can be used to calculate the balance after the 10th year.
- What might be the rule connecting the investment balance ($\$A$) and the time, n years?

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Key ideas

- **Exponential growth** and **decay** is a repeated increase or decrease of a quantity by a constant percentage over time. It can be modelled by the rule $A = A_0\left(1 \pm \frac{r}{100}\right)^n$.
  - A is the amount.
  - A_0 is the initial amount (the subscript zero represents time zero).
  - r is the percentage rate of increase or decrease.
  - n is time; i.e. how many times the percentage increase/decrease is applied.
- For a growth rate of $r\%$ p.a., use $1 + \frac{r}{100}$.
  - For example, for a population increasing at 2% per year, $P = P_0(1.02)^n$.
- For a decay rate of $r\%$ p.a., use $1 - \frac{r}{100}$.
  - For a population decreasing at 3% per year, $P = P_0(0.97)^n$.
- Compound interest involves *adding* any interest earned to the balance at the end of each year or other period. The rule for the investment amount ($\$A$) is given by: $A = P\left(1 + \frac{r}{100}\right)^n$.
  - P is the initial amount or principal.
  - r is the interest rate expressed as a percentage.
  - n is the time.

Exercise 3K

Understanding 1–3 3

1 An investment of \$1000 is increasing at 5% per year.

a Find the value of the investment at the end of the first year.

b Copy and complete the rule for the value of the investment (\$V) after *n* years.

$$V = 1000(1 + \_\_\_\_\_\_)^n = 1000 \times \_\_\_\_\_^n$$

c Use your rule to calculate the value of the investment after 4 years, correct to two decimal places.

2 The mass of a 5 kg limestone rock exposed to the weather is decreasing at a rate of 2% per annum.

Hint: For decrease in part **b**, use $1 - \frac{r}{100}$.

a Find the mass of the rock at the end of the first year.

b Copy and complete the rule for the mass of the rock (M kg) after n years.
$M = 5(1 - \_\_\_\_)^n = 5 \times \_\_\_\_^n$

c Use your rule to calculate the mass of the rock after 5 years, correct to two decimal places.

3 Decide whether the following represent exponential *growth* or exponential *decay*.

a $A = 1000 \times 1.3^n$
b $A = 200 \times 1.78^n$
c $A = 350 \times 0.9^n$
d $P = 50\,000 \times 0.85^n$
e $P = P_0\left(1 + \frac{3}{100}\right)^n$
f $T = T_0\left(1 - \frac{7}{100}\right)^n$

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3K

Fluency

4–6 | 4(½),5–7

Example 35 Writing exponential rules

Form exponential rules for the following situations.

a Paloma invests her $100 000 in savings at a rate of 14% per annum.

b A city's initial population of 50 000 is decreasing by 12% per year.

Solution	Explanation
a Let A = the amount of money at any time n = the number of years the money is invested $r = 14$ $A_0 = 100\,000$ (initial amount)	Define your variables. The basic formula is $A = A_0\left(1 \pm \frac{r}{100}\right)^n$.
$A = 100\,000\left(1 + \frac{14}{100}\right)^n$ $\therefore\ A = 100\,000(1.14)^n$	Substitute $r = 14$ and $A_0 = 100\,000$ and use '+' since we have growth. $\frac{14}{100} = 0.14$.
b Let P = the population at any time n = the number of years the population decreases $P_0 = 50\,000$ (starting population) $r = 12$	Define your variables. The basic formula is $P = P_0\left(1 \pm \frac{r}{100}\right)^n$.
$P = 50\,000\left(1 - \frac{12}{100}\right)^n$ $\therefore\ P = 50\,000(0.88)^n$	Substitute $r = 12$ and $P_0 = 50\,000$ and use '−' since we have decay. $\frac{12}{100} = 0.12$ and $1 - 0.12 = 0.88$.

Now you try

Form exponential rules for the following situations.

a A town population of 3000 is increasing by 2% per year.

b A car purchased for $36 000 is losing value at 6% per year.

4 Define variables and form exponential rules for the following situations.

a $200 000 is invested at 17% per annum.

b A house initially valued at $530 000 is losing value at 5% per annum.

c The value of a car, bought for $14 200, is decreasing at 3% per annum.

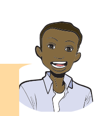
d A population, initially 172 500, is increasing at 15% per year.

e A tank with 1200 litres of water is leaking at a rate of 10% of the water in the tank every hour.

f A cell of area 0.01 cm^2 doubles its size every minute.

g An oil spill, initially covering an area of 2 square metres, is increasing at 5% per minute.

h A substance of mass 30 g is decaying at a rate of 8% per hour.



Hint: The exponential rule is of the form $A = A_0\left(1 \pm \frac{r}{100}\right)^n$

- A is the amount
- A_0 is the initial amount
- r is the percentage increase/decrease
- n is the time

Use + for growth and − for decay.

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Example 36 Applying exponential rules

House prices are rising at 9% per year and Zoe's flat is currently valued at $600 000.

a Determine a rule for the value of Zoe's flat ($$V$) in n years' time.

b What will be the value of her flat:

i next year? **ii** in 3 years' time?

c Use trial and error to find when Zoe's flat will be valued at $900 000, to one decimal place.

Solution

a Let V = value of Zoe's flat at any time

V_0 = starting value $600 000

n = number of years from now

$r = 9$

$V = V_0(1.09)^n$

$\therefore V = 600\,000(1.09)^n$

Explanation

Define your variables.

$V = V_0\left(1 \pm \frac{r}{100}\right)^n$

Use '+' since we have growth.

b i When $n = 1$, $V = 600\,000(1.09)^1$

$= 654\,000$

Zoe's flat would be valued at $654 000 next year.

Substitute $n = 1$ for next year.

ii When $n = 3$, $V = 600\,000(1.09)^3$

$= 777\,017.40$

In 3 years' time Zoe's flat will be valued at about $777 017.

For 3 years, substitute $n = 3$.

c

n	4	5	4.6	4.8	4.7
V	846 949	923 174	891 894	907 399	899 613

Zoe's flat will be valued at $900 000 in about 4.7 years' time.

Try a value of n in the rule. If V is too low, increase your n value; if V is too high, decrease your n value. Continue this process until you get close to 900 000.

Now you try

An investment is increasing at 4% per year and is currently valued at $20 000.

a Determine a rule for the value of the investment ($$V$) in n years' time.

b What will be the value of the investment

i next year? **ii** in 5 years' time?

c Use trial and error to find when the investment will be valued at $25 000, to one decimal place.

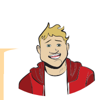
5 The value of a house purchased for $500 000 is expected to grow by 10% per year. Let $$A$ be the value of the house after n years.

a Copy and complete the rule connecting A and n. $A = 500\,000 \times \_\_\_\_\_^n$

b Use your rule to find the expected value of the house after the following number of years. Round your answers to the nearest cent.

i 3 years **ii** 10 years **iii** 20 years

Hint: An increase of 10% is $1 + \frac{10}{100}$.



c Use trial and error to estimate when the house will be worth $1 million. Round your answer to one decimal place.

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3K

6 A share portfolio initially worth $\$300\,000$ is reduced by 15% p.a. over a number of years. Let $\$A$ be the share portfolio value after n years.

Hint: A decrease of 15% is $1 - \frac{15}{100}$.

a Copy and complete the rule connecting A and n.
$A =$ ________ $\times 0.85^n$

b Use your rule to find the value of the shares after the following number of years. Round your answers to the nearest cent.
i 2 years **ii** 7 years **iii** 12 years

c Use trial and error to estimate when the share portfolio will be valued at $\$180\,000$. Round your answer to one decimal place.

7 A water tank containing $15\,000$ L has a small hole that reduces the amount of water by 6% per hour.

a Determine a rule for the volume of water (V litres) left after n hours.

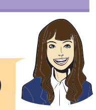
b Calculate (to the nearest litre) the amount of water left in the tank after:
i 3 hours **ii** 7 hours

c How much water is left after 2 days? Round your answer to two decimal places.

d Using trial and error, determine when the tank holds less than 500 L of water, to one decimal place.

Problem-solving and reasoning — 8, 9 | 8, 10

8 A certain type of bacteria grows according to the equation $N = 3000(2.6)^n$, where N is the number of cells present after n hours.



Hint: 'At the start' is $n = 0$ and $a^0 = 1$.

a How many bacteria are there at the start?

b Determine the number of cells (round to the whole number) present after:
i 0 hours **ii** 2 hours **iii** 4.6 hours

c If $5\,000\,000$ bacteria are needed to make a drop of serum, determine by trial and error how long you will have to wait to make a drop (to the nearest minute).

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9 A car tyre has 10 mm of tread when new. It is considered unroadworthy when there is only 3 mm left. The rubber wears at 12.5% every 10 000 km.

a Write an equation relating the depth of tread (D) for every 10 000 km travelled.

b If a tyre lasts 80 000 km ($n = 8$) before becoming unroadworthy, it is considered to be a 'good' tyre. Is this a good tyre?

c Using trial and error, determine when the tyre becomes unroadworthy ($D = 3$), to the nearest 10 000 km.

Hint: Use $D = D_0\left(1 - \frac{12.5}{100}\right)^n$ where n is $\frac{\text{number of km}}{10\,000}$ and D_0 is initial tread. In part **b**, is D greater than 3 when $n = 8$?

10 A cup of coffee has an initial temperature of 90°C.

a If the temperature reduces by 8% every minute, determine a rule for the temperature of the coffee (T) after n minutes.

b What is the temperature of the coffee (to one decimal place) after:

i 2 minutes? **ii** 90 seconds?

c Using trial and error, when is the coffee suitable to drink if it is best consumed at a temperature of 68.8°C? Give your answer to the nearest second.

Hint: The rule is of the form: $T = T_0\left(1 - \frac{r}{100}\right)^n$

Time periods — 11, 12

11 Interest on investments can be calculated using different time periods. Consider $1000 invested at 10% p.a. over 5 years.

- If interest is compounded annually, then $r = 10$ and $n = 5$, so $A = 1000(1.1)^5$.
- If interest is compounded monthly, then $r = \frac{10}{12}$ and $n = 5 \times 12 = 60$, so $A = 1000\left(1 + \frac{10}{1200}\right)^{60}$.

a If interest is calculated annually, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

b If interest is calculated monthly, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

12 You are given $2000 and you invest it in an account that offers 7% p.a. compound interest. What will the investment be worth, to the nearest cent, after 5 years if interest is compounded:

a annually? **b** monthly? **c** weekly? (Assume 52 weeks in the year.)

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Maths@Work: Electrical trades

Electricians must be able to work in teams and also independently. They need to be good at calculating with decimals, as well as using scientific notation. Understanding and working with electrical charges is one example where this is important.

When evaluating academic readiness for apprenticeship training in the construction trades, which include electricians, plumbers and air conditioning mechanics, scientific notation is seen as important and appears in different areas of their courses.

Complete these questions, which an apprentice electrician may face during their training.

1 The electrical charge (Q) of an object is determined by the number of electrons it has in excess to the number of protons it has.
The unit for measuring electrical charge is the coulomb (C). One coulomb (1C) is approximately 6.24 quintillion electrons (*e*).

$$1\ \text{C} = 6\,240\,000\,000\,000\,000\,000\ \text{e}$$

a Convert the following electrical charges, in coulombs, to the number of electrons for each. Use scientific notation using three significant figures.

i 1 C **ii** 2 C **iii** 3 C
iv 250 C **v** $\frac{1}{2}$ C **vi** 12 C

b The charge on one electron in coulombs is $(1 \div 6\,240\,000\,000\,000\,000\,000)$ C. Write down the value of the charge of one electron in scientific notation, using two significant figures.

c Amperes (or amps) are a measure of how much electrical charge in coulombs per second is being transmitted. We call this flow of charge electrical current. This means that an electrical current of 1 A (ampere) has 1 coulomb of charge per second, which is exactly $6.24150975 \times 10^{18}$ electrons per second flowing through a point in the wire at any given time.
Calculate the exact number of electrons per second (e/s) flowing through a wire if the current is:

i 2 A **ii** 10 A **iii** 20 A
iv $\frac{1}{2}$ A **v** 5 A

2 When working with metal it is important to know how it behaves under increases in temperature. For each degree Celsius increase in temperature of hard steel, it has a linear expansion by a factor of 0.0000132.
Write the following scientific notation answers, using four significant figures.

a Express the value 0.0000132 in scientific notation.

b If a section of hard steel measuring 12 mm thick is subject to a 2°C increase in temperature, what is its increase in length, in mm?

Hint: Use $12 \times (1 + 0.0000132)^2$, then substract 12 to find the increase.

c Give one example to illustrate the importance of this information when working with steel.

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Using technology

3 Using an Excel spreadsheet, set up a conversion table between electrical charges measured in coulombs and in electrons, as shown below.

Hint: The formula for cell D3 : $= (1.60217646 \times 10^{-19}) \times$ C3. Excel uses capital E to represent a power of 10.

D3   =(1.60217646*(10^-19))*C3

	A	B	C	D
1	Conversion table between charge (*Q*) in coulombs (C) and electrons (e)			
2	Charge *Q* (in coulombs, C)	Charge *Q* (in number of electrons, e)	Charge *Q* (in number of electrons, e)	Charge *Q* (in coulombs, C)
3	1	6.24150975E+18	1	1.60217646E-19
4	10		10	
5	100		100	
6	1000		1000	
7	10000		10000	
8	100000		100000	
9	1000000		1000000	

Use your spreadsheet to find the answers to the following questions and write them in scientific notation to nine significant figures.

- **i** State the charge of 100 C as the number of electrons.
- **ii** State the charge in coulombs of 1 million electrons.
- **iii** What is the increase in the number of electrons between charges of 1000 C and 1 000 000 C?

4 Using an Excel spreadsheet, set up a conversion table between electrical current in amperes (A), time in seconds and charge in units of C and e, as shown below.
Note: Charge in coulombs = amperes × time in seconds

Formulas using amperes will need $ signs since $A22$ is a fixed cell.

Hint: When referring to cell A22, type A22.

	A	B	C	D
20	Conversion table between amperes, time and electrical charge measured in coulombs and numbers of electrons			
21	Electrical current (A)	Time (*s*)	Charge *Q* (in coulombs, C)	Charge *Q* (in number of electrons, e)
22	0.5	1		
23		30		
24		60		
25		90		
26		120		
27		150		
28		180		

Use your spreadsheet to find the answers to the following questions and write them in scientific notation, to nine significant figures where possible.

a If a current of 0.50 A flows through a circuit for 90 seconds, how much charge will have passed into the circuit:
- **i** in coulombs?
- **ii** in number of electrons?

b If a current of 1.5 A flows through a circuit for 150 seconds, how much charge will have passed into the circuit:
- **i** in coulombs?
- **ii** in number of electrons?

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Puzzles and games

1 In this magic square, each row and column adds to a sum that is an algebraic expression. Complete the square to find the sum.

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$x - 2y$		$2y$

2 Write $3^{n-1} \times 3^{n-1} \times 3^{n-1}$ as a single power of 3.

3 You are offered a choice of two prizes:
- One million dollars right now, or
- You can receive 1 cent on the first day of a 30-day month, double your money every day for 30 days and receive the total amount on the 30th day.

Which prize offers the most money?

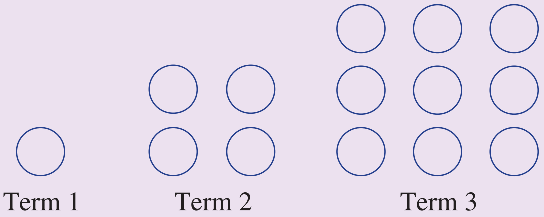
4 Simplify $\frac{25^6 \times 5^4}{125^5}$ without the use of a calculator.

5 Write $(((2^1)^2)^3)^4$ as a single power of 2.

6 How many zeroes are there in 100^{100} in expanded form?

7 Simplify $\frac{x}{2} + \frac{3x}{5} - \frac{4x}{3} + \frac{x+1}{6}$.

8 Write a rule for the number of counters in the nth term of the pattern below. Use this to find the number of counters in the 15th term.



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Chapter summary

Algebra

Expanding

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

e.g. $2(4x + 3) = 8x + 6$

$-3x(2x - y) = -6x^2 + 3xy$

Simplifying expressions

Add/subtract like terms only. Like terms have the same pronumeral factors.

e.g. $3x$ and $7x$, $2xy$ and $4yx$,
not $2x$ and x^2 or $3y$ and $4xy$.

For example,

$3x + 2y - x + 7y = 3x - x + 2y + 7y$

$= 2x + 9y$

Multiply/Divide

$3a \times 2b = 3 \times 2 \times a \times b$

$= 6ab$

$\frac{{}^{1}\cancel{7}x\cancel{y}^{1}}{{}^{2}\cancel{14}\cancel{y}_{1}} = \frac{x}{2}$

Algebraic expressions

$3xy$ $+7x$ $-4y$ $+3$

4 terms

3 is the constant term.

7 is the coefficient of x.

-4 is the coefficient of y.

Adding/subtracting algebraic fractions ★

Find the lowest common denominator and combine.

e.g. $\frac{x}{2} + \frac{2x}{3}$ LCD is 6.

$= \frac{3x}{6} + \frac{4x}{6}$

$= \frac{7x}{6}$

Factorising

This is the opposite of expanding

Factorised form		Expanded form
$2(x + 4)$	=	$2x + 8$

Look for highest common factor of terms.

e.g. $3ab + 6a$ HCF is $3a$.

$= 3a(b + 2)$

$-2x^2 - 6x$ HCF is $-2x$.

$= -2x(x + 3)$

Multiplying/dividing algebraic fractions ★

Multiply: Cancel common factors in factorised form and then multiply.

e.g. $\frac{3\cancel{(x+4)}^{1}}{\cancel{2}_{1}} \times \frac{{}^{4}\cancel{8}x}{\cancel{x+4}_{1}}$

$= 12x$

Divide: Multiply by the reciprocal of the fraction following the ÷ sign.

Reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

Index form a^m

a is the base.

m is the index/power/exponent.

Index form		Expanded form
5^4	=	$5 \times 5 \times 5 \times 5$

e.g. $3 \times a \times b \times a \times a \times b = 3a^3b^2$

Indices

Scientific notation

Used for very large and small numbers in the form $a \times 10^m$, where $1 \le a < 10$ or $-10 < a \le -1$.

$3\,870\,000 = 3.87 \times 10^6$

$0.000\,21 = 2.1 \times 10^{-4}$

Exponential growth and decay ★

For a repeated percentage increase or decrease, the rule can be

$A = A_0\left(1 \pm \frac{r}{100}\right)^n$, where

A_0 is the initial amount

r is % rate

n is the amount of time.

Use '+' for growth and '–' for decay.

Significant Figures

These are counted from left to right from the first non-zero digit

e.g. 213 270 is 2.13×10^5 in scientific notation to 3 significant figures.

0.000429 1 is 4.3×10^{-4} in scientific notation to 2 significant figures

Negative indices

$a^{-m} = \frac{1}{a^m}$

e.g. $x^{-3} = \frac{1}{x^3}$

$\frac{1}{a^{-m}} = a^m$

e.g. $\frac{4}{2^{-3}} = 4 \times 2^3 = 32$

Index laws

Law 1: $a^m \times a^n = a^{m+n}$

Law 2: $a^m \div a^n = a^{m-n}$

Law 3: $(a^m)^n = a^{m \times n}$

Law 4: $(ab)^m = a^m \times b^m$

Law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Zero power: $a^0 = 1$ when $a \neq 0$.

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

		✔
3A	**1 I can identify the parts of an algebraic expression.** e.g. For the expression $4a + 3b - 7$, state the: **a** number of terms **b** constant term **c** coefficient of b	
3A	**2 I can form an algebraic expression.** e.g. Write an algebraic expression for: **a** 3 more than 2 lots of a **b** the product of x and y, divided by 2	
3A	**3 I can evaluate an algebraic expression using substitution.** e.g. If $x = 2$, $y = 5$ and $z = -3$, evaluate: **a** $xy + 2z$ **b** $y^2 - xz$	
3B	**4 I can identify like terms.** e.g. Write down the like terms in the following list: $ax, 7b, 6x, -3b, -2xa$	
3B	**5 I can collect like terms.** e.g. Simplify: **a** $5b + 4b - 2$ **b** $4xy + 3x - 5xy + 3x$	
3B	**6 I can multiply and divide algebraic terms.** e.g. Simplify: **a** $3a \times 5ab$ **b** $9xy \div (18x)$	
3C	**7 I can expand expressions with brackets.** e.g. Expand the following. **a** $4(3x - 2)$ **b** $-2y(5x - 7y)$	
3C	**8 I can simplify expressions by removing brackets.** e.g. Expand and simplify $3(2x + 5) - 2(x + 2)$.	
3D	**9 I can determine the HCF.** e.g. Determine the HCF of the following. **a** $6x$ and $24x$ **b** $8ab$ and $20b^2$	
3D	**10 I can factorise expressions with common factors.** e.g. Factorise the following. **a** $8a + 12$ **b** $6x^2 - 10xy$ **c** $-4ab - 18a$ (including common negative)	
3E	**11 I can simplify algebraic fractions.** e.g. Simplify this fraction by factorising first: $\frac{4x - 12}{x - 3}$.	
3E	**12 I can multiply algebraic fractions.** e.g. Simplify this product: $\frac{3(x - 2)}{4x} \times \frac{10x}{x - 2}$.	

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		✔
3E	**13 I can divide algebraic fractions.** e.g. Simplify $\frac{5x^2}{9} \div \frac{10x}{3}$.	
3F	**14 I can add and subtract simple algebraic fractions.** e.g. Simplify $\frac{3x}{4} - \frac{x}{16}$.	
3F	**15 I can add and subtract with binomial numerators.** e.g. Simplify $\frac{x+2}{4} - \frac{x}{10}$.	
3G	**16 I can write in index form.** e.g. Write the following in index form. **a** $7 \times 7 \times 7 \times 7$ **b** $x \times y \times x \times x \times y$	
3G	**17 I can use index law 1.** e.g. Simplify the following using the first index law. **a** $y^3 \times y^5$ **b** $3a^3b \times 5a^2b^3$	
3G	**18 I can use index law 2.** e.g. Simplify the following using the second index law. **a** $\frac{4m^5}{2m^3}$ **b** $5x^5y^4 \div (15xy^2)$	
3G	**19 I can combine index laws 1 and 2.** e.g. Simplify $\frac{2x^5y^3 \times 3x^2y^2}{9x^3y^4}$.	
3H	**20 I can use the zero power.** e.g. Evaluate using the zero power: $5^0 + a^0$.	
3H	**21 I can use index law 3.** e.g. Simplify using the third index law: $4(x^2)^5$.	
3H	**22 I can use index laws 4 and 5.** e.g. Simplify, using index laws. **a** $(2m)^3$ **b** $\left(\frac{x^2}{3}\right)^4$	
3H	**23 I can combine index laws.** e.g. Simplify: **a** $\frac{(m^2n)^5}{4m^3n^5}$ **b** $(3a)^0 - (4a^3)^2$	
3I	**24 I can express negative indices in positive index form.** e.g. Rewrite the following with positive indices only. **a** $3y^{-2}$ **b** $\frac{4}{x^2y^{-3}}$	

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		✔
3I	**25 I can use index laws with negative indices.** e.g. Simplify, expressing with positive indices: $\frac{a^4b^3 \times a^{-2}(b^2)^{-3}}{a^6}$.	
3J	**26 I can convert from scientific notation to a basic numeral.** e.g. Write these numbers as a basic numeral. **a** 3.02×10^4 **b** 7.21×10^{-5}	
3J	**27 I can write numbers using scientific notation.** e.g. Write these numbers in scientific notation. **a** $64\,000$ **b** 0.000035	
3J	**28 I can write in scientific notation rounding to significant figures.** e.g. Write these numbers in scientific notation using three significant figures. **a** $472\,815$ **b** 0.0053821	
3K	**29 I can form exponential rules.** e.g. Write a rule for this statement: A substance of mass 450 g is decaying at a rate of 14% per day.	
3K	**30 I can apply exponential rules.** e.g. A share portfolio valued at $\$80\,000$ is expected to grow by 6% per year. Determine a rule for the value of the portfolio ($\$V$) in n years' time then use this to find the value in 4 years' time. Use trial and error to find when the portfolio will be valued at $\$115\,000$, correct to one decimal place.	

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Short-answer questions

3A **1** Consider the expression $3xy - 3b + 4x^2 + 5$.
- **a** How many terms are in the expression?
- **b** What is the constant term?
- **c** State the coefficient of:
  - **i** x^2 **ii** b

3A **2** Write an algebraic expression for the following.
- **a** 3 more than y
- **b** 5 less than the product of x and y
- **c** the sum of a and b is divided by 4

3A **3** Evaluate the following if $x = 3$, $y = 5$ and $z = -2$.
- **a** $3x + y$ **b** xyz **c** $y^2 - 5z$

3B **4** Simplify the following expressions.
- **a** $4x - 5 + 3x$ **b** $4a - 5b + 9a + 3b$ **c** $3xy + xy^2 - 2xy - 4y^2x$
- **d** $3m \times 4n$ **e** $-2xy \times 7x$ **f** $\frac{8ab}{12a}$

3C **5** Expand the following and collect like terms where necessary.
- **a** $5(2x + 4)$ **b** $-2(3x - 4y)$ **c** $3x(2x + 5y)$
- **d** $3 + 4(a + 3)$ **e** $3(y + 3) + 2(y + 2)$ **f** $5(2t + 3) - 2(t + 2)$

3D **6** Factorise the following expressions.
- **a** $16x - 40$ **b** $10x^2y + 35xy^2$
- **c** $4x^2 - 10x$ **d** $-2xy - 18x$ (include the common negative)

3F **7** Simplify the following algebraic fractions involving addition and subtraction.
- **a** $\frac{2x}{3} + \frac{4x}{15}$ **b** $\frac{3}{7} - \frac{a}{2}$ **c** $\frac{x+4}{4} + \frac{x-3}{5}$

3E **8** Simplify these algebraic fractions by first cancelling common factors in factorised form.
- **a** $\frac{5x}{12} \times \frac{9}{10x}$ **b** $\frac{x+2}{4} \times \frac{16x}{x+2}$ **c** $\frac{12x-4}{4}$ **d** $\frac{x-3}{4} \div \frac{3(x-3)}{8}$

3G **9** Simplify the following, using index laws 1 and 2.
- **a** $3x^5 \times 4x^2$ **b** $4xy^6 \times 2x^3y^2$ **c** $\frac{b^7}{b^3}$ **d** $\frac{4a^3b^5}{6ab^2}$

3H **10** Simplify the following, using the third, fourth and fifth index laws.
- **a** $(b^2)^4$ **b** $(2m^2)^3$ **c** $\left(\frac{x}{7}\right)^2$ **d** $\left(\frac{4y^2}{z^4}\right)^3$

3H **11** Simplify the following, using the zero power.
- **a** 7^0 **b** $4x^0$ **c** $5a^0 + (2y)^0$ **d** $(x^2 + 4y)^0$

3I **12** Express the following, using positive indices.
- **a** $4x^{-3}$ **b** $3r^4s^{-2}$ **c** $\frac{2x^{-3}y^4}{3}$ **d** $\frac{4}{m^{-5}}$

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Chapter review

Chapter review

3I **13** Simplify the following, using index laws. Express all answers with positive indices.

a $\dfrac{3x^2y^4 \times 5xy^7}{12x^3y^5}$ **b** $\dfrac{(5x^2y)^2 \times 4xy^2}{8(xy)^2}$

c $\dfrac{2x^3y^2 \times 5xy^2}{x^7y^4}$ **d** $\dfrac{4x^2y^5}{8(x^2)^{-3}y^8}$

3J **14** Write these numbers as a basic numeral.

a 4.25×10^3 **b** 3.7×10^7 **c** 2.1×10^{-2} **d** 7.25×10^{-5}

3J **15** Convert these numbers to scientific notation, using three significant figures.

a 123 574 **b** 39 452 178 **c** 0.0000090241 **d** 0.00045986

3K **16** Form an exponential equation for the following.

a The population of a colony of kangaroos, which starts at 20 and is increasing at a rate of 10%.

b The amount of petrol in a petrol tank fuelling a generator if it starts with 100 000 litres and uses 15% of its fuel every hour.

Multiple-choice questions

3A **1** The coefficient of x in $3xy - 4x + 7$ is:

A 4 **B** 7 **C** -4 **D** 3 **E** -1

3B **2** The simplified form of $7ab + 2b - 5ab + b$ is:

A $2ab + 2b^2$ **B** $2ab + 3b$ **C** $5ab$ **D** $2ab + b$ **E** $12ab + 3b$

3C **3** The expanded form of $2x(3x - 5)$ is:

A $6x^2 - 5$ **B** $6x - 10$ **C** $6x^2 - 10x$ **D** $5x^2 - 10x$ **E** $-4x$

3D **4** The fully factorised form of $8xy - 24y$ is:

A $4y(2x - 6y)$ **B** $8(xy - 3y)$ **C** $8y(x - 24)$ **D** $8y(x - 3)$ **E** $8x(y - 24)$

3E **5** The simplified form of $\dfrac{2(x+1)}{5x} \times \dfrac{15}{x+1}$ is:

A $\dfrac{6}{x+1}$ **B** $\dfrac{6}{x}$ **C** $\dfrac{3(x+1)}{x}$ **D** $6x$ **E** $\dfrac{3x}{x+1}$

3F **6** The sum of the algebraic fractions $\dfrac{3x}{8} + \dfrac{x}{12}$ is:

A $\dfrac{x}{5}$ **B** $\dfrac{x}{6}$ **C** $\dfrac{x}{24}$ **D** $\dfrac{11x}{24}$ **E** $\dfrac{9x}{24}$

3G **7** $3x^3y \times 2x^5y^3$ is equal to:

A $5x^{15}y^3$ **B** $6x^{15}y^3$ **C** $6x^8y^4$ **D** $5x^8y^4$ **E** $6x^8y^3$

3H **8** $(2x^4)^3$ can be written as:

A $2x^{12}$ **B** $2x^7$ **C** $6x^{12}$ **D** $8x^{12}$ **E** $8x^7$

3H **9** $5x^0 - (2x)^0$ is equal to:

A 4 **B** 0 **C** 3 **D** 2 **E** -1

3I **10** $12a^4 \div (4a^7)$ simplifies to:

A $3a^3$ **B** $8a^3$ **C** $3a^{11}$ **D** $\dfrac{8}{a^3}$ **E** $\dfrac{3}{a^3}$

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3J **11** 417 000 converted to scientific notation is:

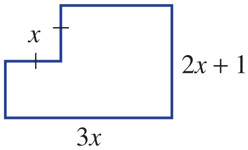
A 4.17×10^{-5} **B** 417×10^3 **C** 4.17×10^5
D 0.417×10^6 **E** 41.7×10^{-2}

3K **12** A rule for the amount of money, A, in an account after n years, if \$1200 is invested at 4% per year, is:

A $A = 1200(4)^n$ **B** $A = 1200(1.4)^n$ **C** $A = 1200(0.96)^n$
D $A = 1200(1.04)^n$ **E** $A = 1200(0.04)^n$

Extended-response questions

1 A room in a house has the shape and dimensions, in metres, shown. All lines meet at 90°.



a Find the perimeter of the room, in factorised form.

b If $x = 3$, what is the room's perimeter?

The floor of the room is to be recarpeted.

c Give the area of the floor in terms of x and in expanded form.

d If the carpet costs \$20 per square metre and $x = 3$, what is the cost of laying the carpet?

2 During the growing season, a certain type of water lily spreads by 9% per week. The water lily covers an area of $2\,m^2$ at the start of the growing season.

a Write a rule for the area, $A\,m^2$, covered by the water lily after n weeks.

b Calculate the area covered, correct to four decimal places, after:
- **i** 2 weeks
- **ii** 5 weeks

c Use trial and error to determine, to one decimal place, when there will be a coverage of $50\,m^2$.

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