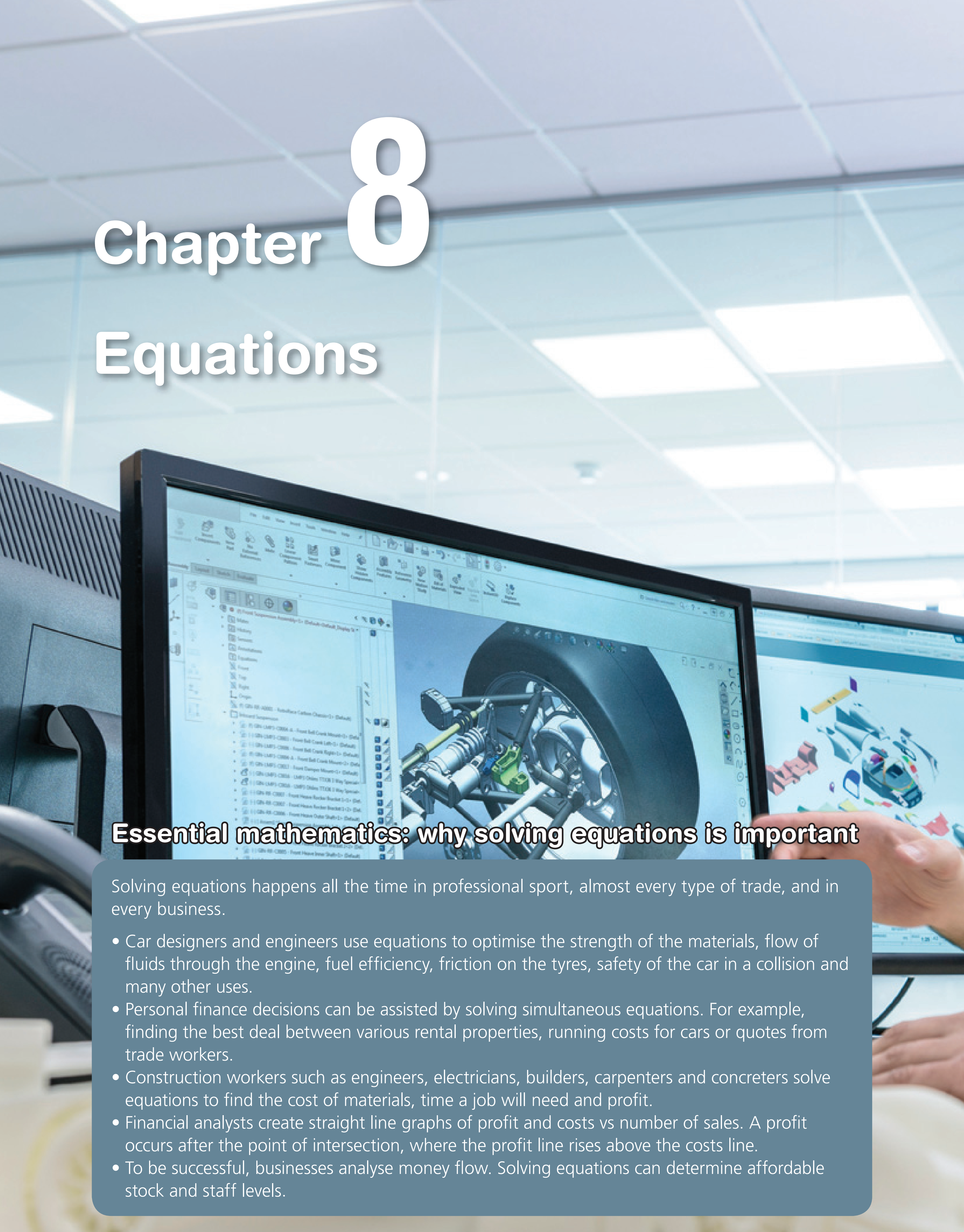

Chapter 8

Equations

Essential mathematics: why solving equations is important

Solving equations happens all the time in professional sport, almost every type of trade, and in every business.

- Car designers and engineers use equations to optimise the strength of the materials, flow of fluids through the engine, fuel efficiency, friction on the tyres, safety of the car in a collision and many other uses.
- Personal finance decisions can be assisted by solving simultaneous equations. For example, finding the best deal between various rental properties, running costs for cars or quotes from trade workers.
- Construction workers such as engineers, electricians, builders, carpenters and concreters solve equations to find the cost of materials, time a job will need and profit.
- Financial analysts create straight line graphs of profit and costs vs number of sales. A profit occurs after the point of intersection, where the profit line rises above the costs line.
- To be successful, businesses analyse money flow. Solving equations can determine affordable stock and staff levels.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Substitute values into formulas to determine an unknown and re-arrange formulas to solve for a particular term (VCMNA333)

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (VCMNA335)

Solve linear inequalities and graph their solutions on a number line (VCMNA336)

Solve simultaneous linear equations, using algebraic and graphical techniques including using digital technology (VCMNA337)

Solve linear equations involving simple algebraic fractions (VCMNA340)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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Warm-up quiz

1 When $a = 6$ and $b = -3$, evaluate the following.

a $a - b$ **b** ab **c** b^2 **d** $3(a + 2b)$

2 When $m = 4$, $n = 7$ and $p = -2$, evaluate the following.

a $4m + p$ **b** $p(4 - n)$ **c** $\frac{8m}{p}$ **d** $2m^2$

3 Simplify the following.

a $a + 2a$ **b** $4m - m$ **c** $6p + 2p$ **d** $7m - 7m$
e $2m - 7m$ **f** $8x + y - x$ **g** $8p + 4p - 3p$ **h** $7m - 4m + 3m$

4 Simplify the following.

a $5x \times 3$ **b** $4p \times 4$ **c** $8x \times 4y$
d $6a \times (-5)$ **e** $a \times b$ **f** $6x \div 6$
g $m \div m$ **h** $6a \div 3$ **i** $\frac{15a}{5a}$

5 Complete the following.

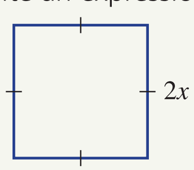
a $x + 5 - \square = x$ **b** $w - 3 + \square = w$ **c** $p - 5 + \square = p$
d $z + 1 - \square = z$ **e** $w \times 4 \div \square = w$ **f** $a \div 2 \times \square = a$
g $m - 3 + \square = m$ **h** $2m \div \square = m$ **i** $\frac{x}{4} \times \square = x$
j $\frac{m}{3} \times \square = m$ **k** $6a \div \square = a$ **l** $10x \div \square = x$

6 Write an expression for each of the following.

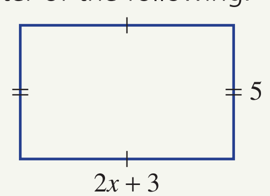
a the sum of x and 3 **b** six more than n
c double w **d** half of x
e six more than double x **f** seven less than x
g three more than x and then doubled **h** one more than triple x

7 Write an expression for the perimeter of the following.

a



b



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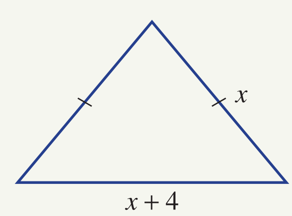

8 State true (T) or false (F) for whether the following are equations.

a $x + 3$ **b** $3x - 6 = 9$ **c** $x^2 - 8$
d $2x$ **e** $3a = 12$ **f** $x^2 = 100$
g $1 = x - 3$ **h** $m - m$ **i** $2p = 0$

9 Solve the following simple equations.

a $m + 3 = 10$ **b** $y - 7 = 12$ **c** $3x = 15$ **d** $\frac{b}{4} = 3$

10 Answer true (T) or false (F) to the following.

a $5 > 3$ **b** $-2 < 7$ **c** $4 \le 2$

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8A Solving linear equations CONSOLIDATING

Learning intentions

- To know what a solution to an equation is
- To be able to solve a simple linear equation
- To be able to verify a solution to an equation

Key vocabulary: equation, linear equation, solve, variable, pronumeral, backtracking, verify, substitute, solution

A cricket batsman will put on socks, then cricket shoes and, finally, pads in that order. When the game is over, these items are removed in reverse order: first the pads, then the shoes and finally the socks. Nobody takes their socks off before their shoes. A similar reversal occurs when solving equations.

We can undo the operations around x by doing the opposite operation in the reverse order to how they have been applied to x. To keep each equation balanced, we always apply the same operation to both sides of an equation.

For example:

Applying operations to $x = 7$

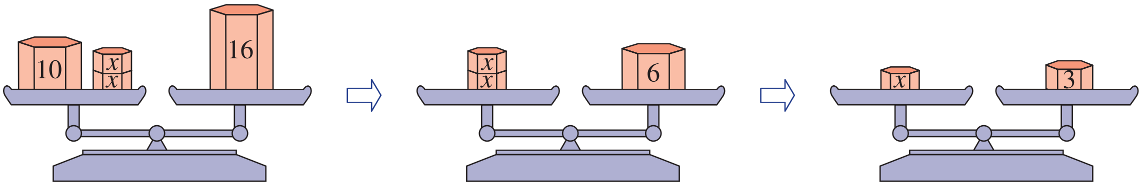
$x = 7$
$\times 2$ $\quad$ $\times 2$
$2x = 14$
$+12$ $\quad$ $+12$
$2x + 12 = 26$

Undoing the operations around x

$2x + 12 = 26$
-12 $\quad$ -12
$2x = 14$
$\div 2$ $\quad$ $\div 2$
$x = 7$

Lesson starter: Keeping it balanced

Three weighing scales are each balanced with various weights on the left and right pans.



Balanced scales 1 $\quad$ Balanced scales 2 $\quad$ Balanced scales 3

- What weight has been removed from each side of scales 1 to get to scales 2?
- What has been done to both the left and right sides of scales 2 to get to scales 3?
- What equations are represented in each of the balanced scales shown above?
- What methods can you recall for solving equations?

Key ideas

- An **equation** is a mathematical statement that includes an equals sign. The equation will be true only for certain value(s) of the pronumeral(s) that make the left-hand side equal to the right-hand side.
  For example: $\frac{5x}{6} = -2$, $3p + 2t = 6$ are equations; $6x - 13$ is not an equation.
- A **linear equation** contains a variable (e.g. x) to the power of 1 and no other powers.
  For example: $3x - 5 = 7$, $4(m - 3) = m + 6$ are linear equations; $x^2 = 49$ is not linear.

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8A

- To **solve** an equation, undo the operations built around x by doing the opposite operation in the reverse order.
  - Always perform the same operation to both sides of an equation so it remains balanced.
    For example:
    For $5x + 2 = 17$, we observe operations that have been applied to x:

$$x \xrightarrow{\times 5} 5x \xrightarrow{+2} 5x + 2$$

    So we solve the equation by 'undoing' them in reverse order on both sides of the equation:

$$5x + 2 \xrightarrow{-2} 5x \xrightarrow{\div 5} x \text{ and } 17 \xrightarrow{-2} 15 \xrightarrow{\div 5} 3$$

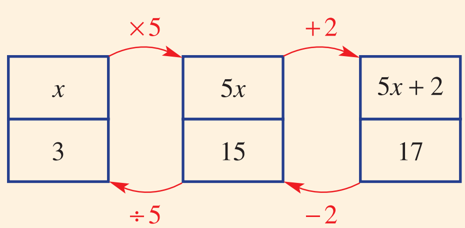
    This gives the solution:

$$\begin{aligned} 5x + 2 &= 17 \quad (-2 \text{ both sides}) \\ 5x &= 15 \quad (\div 5 \text{ both sides}) \\ x &= 3 \end{aligned}$$

  - Alternatively, a solution need not show the operations applied to each side. These can be done mentally. For example:

$$\begin{aligned} 5x + 2 &= 17 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

- A flow chart can be used to solve equations. First, the equation is built up following the order of operations applied to x and then the solution for x is found by undoing these operations in the reverse order.
  For example, here is a flow chart solution to $5x + 2 = 17$.



  Solution $x = 3$.
  - **Backtracking** is the process of undoing the operations applied to x.

- To **verify** an answer means to check that the solution is correct by substituting the answer to see if it makes the equation true.
  e.g. Verify that $x = 3$ is a solution to $5x + 2 = 17$.

$$\begin{aligned} \text{LHS} &= 5x + 2 \qquad \text{RHS} = 17 \\ &= 5(3) + 2 \\ &= 17 \qquad \therefore\ x = 3 \text{ is a solution.} \end{aligned}$$

Exercise 8A

Understanding	1–3	3

1 State the missing word or number.
  a An equation is a statement that contains an ______ sign.
  b A linear equation contains a variable to the power of ______.

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2 Consider the equation $2x + 3 = 7$.

a Complete this table by evaluating $2x + 3$ for the given values of x.

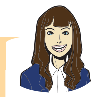
x	0	1	2	3
$2x + 3$				

b By looking at your table of values, which value of x is the solution to $2x + 3 = 7$?

3 Decide whether $x = 2$ is a solution to these equations.

a $x + 3 = 5$ **b** $2x = 7$ **c** $x - 1 = 4$
d $2x - 1 = 10$ **e** $3x + 2 = 8$ **f** $2 - x = 0$

Hint: Substitute $x = 2$ to see whether LHS = RHS.



Fluency

4–9(½) 6–9(½)

Example 1 Solving one-step equations

Solve:

a $x + 7 = 12$ **b** $x - 9 = 3$ **c** $3x = 12$ **d** $\frac{x}{4} = 20$

Solution	Explanation
a $x + 7 = 12$	Write the equation. The opposite of $+7$ is -7.
$x = 12 - 7$	Subtract 7 from both sides.
$x = 5$	Simplify.
Verify: LHS $= 5 + 7 = 12$ RHS $= 12$	Check that your answer is correct.
b $x - 9 = 3$	Write the equation. The opposite of -9 is $+9$.
$x = 3 + 9$	Add 9 to both sides.
$x = 12$	Simplify.
Verify: LHS $= 12 - 9 = 3$ RHS $= 3$	Check that your answer is correct.
c $3x = 12$	Write the equation. The opposite of $\times 3$ is $\div 3$.
$x = \frac{12}{3}$	Divide both sides by 3.
$x = 4$	Simplify.
Verify: LHS $= 3 \times 4 = 12$ RHS $= 12$	Check that your answer is correct.
d $\frac{x}{4} = 20$	Write the equation. The opposite of $\div 4$ is $\times 4$.
$x = 20 \times 4$	Multiply both sides by 4.
$x = 80$	Simplify.
Verify: LHS $= \frac{80}{4} = 20$ RHS $= 20$	Check that your answer is correct.

Now you try

Solve:

a $x + 5 = 21$ **b** $x - 6 = 12$ **c** $4x = 36$ **d** $\frac{x}{3} = -4$

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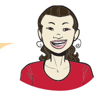
4 Solve the following.

a $t+5=8$ **b** $m+4=10$ **c** $8+x=14$
d $m+8=40$ **e** $a+1=-5$ **f** $16=m+1$
g $x-3=3$ **h** $x-7=2$ **i** $x-8=9$
j $x-3=0$ **k** $x-2=-8$ **l** $x-5=7$

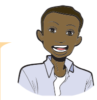
Hint:
$8+x=14$ is the same as $x+8=14$.
$16=m+1$ is the same as $m+1=16$.



5 Solve the following.

a $8p=24$ **b** $5c=30$ **c** $27=3d$
d $15p=15$ **e** $6m=-42$ **f** $-10=20p$
g $\frac{x}{5}=10$ **h** $\frac{m}{3}=7$ **i** $\frac{a}{6}=-2$
j $\frac{z}{7}=0$ **k** $\frac{w}{3}=\frac{1}{2}$ **l** $\frac{m}{2}=\frac{1}{4}$

Hint: $27=3d$ is the same as $3d=27$.

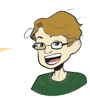


Hint: $3\times\frac{1}{2}=\frac{3}{1}\times\frac{1}{2}=\frac{3}{2}$

6 Solve the following equations.

a $x+9=12$ **b** $x+3=12$ **c** $x+15=4$
d $x-7=3$ **e** $x-2=12$ **f** $x-5=5$
g $3x=9$ **h** $4x=16$ **i** $2x=100$
j $\frac{x}{5}=4$ **k** $\frac{x}{3}=7$ **l** $\frac{x}{7}=1$

Hint: Carry out the 'opposite' operation to solve for x.



Example 2 Solving two-step equations

Solve $4x+5=17$.

Solution	Explanation
$4x+5=17$	Write the equation.
$4x=12$	Subtract 5 from both sides first.
$x=\frac{12}{4}$	Divide both sides by 4.
$x=3$	Simplify.
Verify: LHS $=4(3)+5$ $=17$ RHS $=17$	Check your answer.

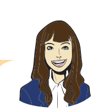
Now you try

Solve $5x-1=19$.

7 Solve the following equations.

a $2x+5=7$ **b** $3x+2=11$
c $4x-3=9$ **d** $6x+13=1$
e $8x+16=8$ **f** $10x+92=2$
g $3x-4=8$ **h** $2x-7=9$
i $5x-4=36$ **j** $2x-6=-10$
k $7x-3=-24$ **l** $6x-3=27$

Hint: First choose to add or subtract a number from both sides and then divide by the coefficient of x.



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Example 3 Solving two-step equations involving simple fractions

Solve $\frac{x}{5} - 3 = 4$.

Solution	Explanation
$\frac{x}{5} - 3 = 4$	Write the equation.
$\frac{x}{5} = 7$	Add 3 to both sides.
$x = 35$	Multiply both sides by 5.
Verify: LHS $= \frac{35}{5} - 3$ $= 4$ RHS $= 4$	Check that your answer is correct.

Now you try

Solve $\frac{x}{7} + 2 = 6$.

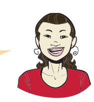
8 Solve the following equations.

a $\frac{x}{3} + 2 = 5$ **b** $\frac{x}{6} + 3 = 3$ **c** $\frac{x}{7} + 4 = 12$

d $\frac{x}{4} - 3 = 2$ **e** $\frac{x}{5} - 4 = 3$ **f** $\frac{x}{10} - 2 = 7$

g $\frac{x}{8} - 2 = -6$ **h** $\frac{x}{4} - 3 = -8$ **i** $\frac{x}{2} - 1 = -10$

Hint: When solving equations, the order of steps is important. For $\frac{x}{3} - 5$, undo the -5 first, then undo the $\div 3$.



Example 4 Solving more two-step equations

Solve $\frac{x+4}{2} = 6$.

Solution	Explanation
$\frac{x+4}{2} = 6$	Write the equation.
$x + 4 = 12$	In $\frac{x+4}{2}$ we first add 4 and then divide by 2. So to undo we first multiply both sides by 2.
$x = 8$	Subtract 4 from both sides.
Verify: LHS $= \frac{8+4}{2}$ $= 6$ RHS $= 6$	Check that your answer is correct.

Now you try

Solve $\frac{x-3}{4} = 1$.

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8A

9 Solve the following equations.

a $\frac{m+1}{2} = 3$ **b** $\frac{a-1}{3} = 2$

c $\frac{x+5}{2} = 3$ **d** $\frac{x+5}{3} = 2$

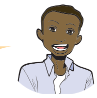
e $\frac{n-4}{5} = 1$ **f** $\frac{m-6}{2} = 8$

g $\frac{w+4}{3} = -1$ **h** $\frac{m+3}{5} = 2$

i $\frac{w-6}{3} = 7$ **j** $\frac{a+7}{4} = 2$

k $\frac{a-3}{8} = -5$ **l** $\frac{m+5}{8} = 0$

Hint: When solving equations, the order of steps is important. For $\frac{x+7}{3}$, undo the $\div 3$ first, then undo the $+7$. Never cancel a number joined by $+$ or $-$ to an x. In $\frac{x+8}{4}$, you cannot cancel the 4 into the 8.



Problem-solving and reasoning 10, 11 11–13

Example 5 Writing equations from word problems

For each of the following statements, write an equation and solve for the pronumeral.

a When 7 is subtracted from x, the result is 12.
b When x is divided by 5 and then 6 is added, the result is 10.
c When 4 is subtracted from x and that answer is divided by 2, the result is 9.

Solution	Explanation
a $x - 7 = 12$ $x = 19$	Subtract 7 from x means to start with x and then subtract 7. 'The result' means '='.
b $\frac{x}{5} + 6 = 10$ $\frac{x}{5} = 4$ $x = 20$	Divide x by 5, then add 6 and make it equal to 10. Solve the equation by subtracting 6 from both sides first.
c $\frac{x-4}{2} = 9$ $x - 4 = 18$ $x = 22$	Subtracting 4 from x gives $x - 4$, and divide that answer by 2. Undo $\div 2$ by multiplying both sides by 2, then add 4 to both sides.

Now you try

For each of the following statements, write an equation and solve for the pronumeral.

a When 3 is added to x, the result is 9.
b When x is divided by 3 then 7 is subtracted, the result is 0.
c When 6 is subtracted from x and that answer is divided by 3, the result is 10.

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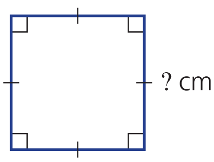
10 For each of the following statements, write an equation and solve for the pronumeral.

- **a** When 4 is added to x, the result is 6.
- **b** When x is added to 12, the result is 8.
- **c** When 5 is subtracted from x, the result is 5.
- **d** When x is divided by 3 and then 2 is added, the result is 8.
- **e** Twice the value of x is added to 3 and the result is 9.
- **f** $(x - 3)$ is divided by 5 and the result is 6.
- **g** 3 times x plus 4 is equal to 16.

Hint: 5 subtracted from x is $x - 5$.

11 Write an equation and solve it for each of these questions.

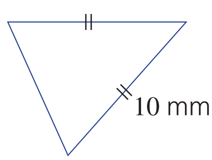
- **a** The perimeter of a square is 52 cm. Determine the length of the side.



$P = 52$ cm

Hint: Draw a diagram and choose a pronumeral to represent the unknown side. Then write an equation and solve it.

- **b** The perimeter of an isosceles triangle is 32 mm. If the equal sides are both 10 mm, determine the length of the other side.

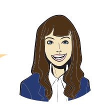


$P = 32$ mm

12 Convert the following into equations, then solve them for the unknown number.

- **a** n is multiplied by 2, then 5 is added. The result is 11.
- **b** Four times a certain number is added to 9 and the result is 29. What is the number?
- **c** Half of a number less 2 equals 12. What is the number?
- **d** A number plus 6 has been divided by 4. The result is 12. What is the number?
- **e** 12 is subtracted from a certain number and the result is divided by 5. If the answer is 14, what is the number?

Hint: Choose a pronumeral to represent the unknown number, then write an equation using the pronumeral. $\frac{1}{2}$ of x can be written as $\frac{x}{2}$.



13 Write an equation and solve it for each of these questions.

- **a** The sum of two consecutive whole numbers is 23. What are the numbers?
- **b** If I add 5 to twice a number, the result is 17. What is the number?
- **c** Three less than five times a number is 12. What is the number?
- **d** One person is 19 years older than another person. Their age sum is 69. What are their ages?
- **e** Andrew threw the shot-put 3 m more than twice the distance Barry threw it. If Andrew threw the shot-put 19 m, how far did Barry throw it?

Hint: Consecutive whole numbers are one number apart; e.g. 3, 4, 5, 6 etc. The next consecutive number after x is $x + 1$.

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8A

Modelling with equations — 14, 15

14 A service technician charges $\$40$ up front and $\$60$ for each hour she works.

a Find a linear equation for the total charge, $\$C$, of any job for h hours worked.

b What will a 4-hour job cost?

c If the technician works on a job for 3 days and averages 6 hours per day, what will be the overall cost?

d If a customer is charged $\$400$, how long did the job take?

15 A petrol tank holds 71 litres of fuel. It originally contained 5 litres. If a petrol pump fills it at 6 litres per minute, find:

a a linear equation for the amount of fuel (V litres) in the tank at time t minutes

b how long it will take to fill the tank to 23 litres

c how long it will take to fill the tank

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8B Solving more difficult linear equations ★

Learning intentions

- To be able to expand brackets and collect like terms when solving a linear equation
- To be able to collect pronumerals to one side in order to solve a linear equation
- To be able to solve a simple word problem by setting up and solving a linear equation

Key vocabulary: expand, like terms, product, equivalent

More complex linear equations may have variables on both sides of the equation and/or brackets. Examples are $6x = 2x - 8$ or $5(x + 3) = 12x + 4$.

Brackets can be removed by expanding. Equations with variables on both sides can be solved by collecting variables to one side, using addition or subtraction of a term.

More complex linear equations of this type are used when constructing buildings and in science and engineering.

Lesson starter: Steps in the wrong order

The steps to solve $8(x + 2) = 2(3x + 12)$ are listed here in the incorrect order.

$$8(x + 2) = 2(3x + 12)$$
$$x = 4$$
$$2x + 16 = 24$$
$$8x + 16 = 6x + 24$$
$$2x = 8$$

- Arrange them in the correct order, working from the problem to the solution.
- By considering all the steps in the correct order, write what has happened in each step.

Key ideas

- When solving complicated linear equations:
  1. First, **expand** any brackets.
     In this example, multiply the 3 into the first bracket and the −2 into the second bracket.
     $$3(2x - 1) - 2(x - 2) = 22$$
     $$6x - 3 - 2x + 4 = 22$$
  2. Collect any **like terms** on the LHS and any like terms on the RHS.
     Collecting like terms on each side of this example:
     $$5x - 4 - 3x - 9 = x - 5 + 2x + 10$$
     $$2x - 13 = 3x + 5$$
     $5x - 3x = 2x$, $-4 - 9 = -13$, $x + 2x = 3x$ and $-5 + 10 = 5$
  3. If an equation has variables on both sides, collect to one side by adding or subtracting one of the terms.
     For example, when solving the equation $12x + 7 = 5x + 19$, first subtract $5x$ from both sides:
     LHS: $12x + 7 - 5x = 7x + 7$, RHS: $5x + 19 - 5x = 19$:
     $$12x + 7 = 5x + 19 \quad (-5x \text{ both sides})$$
     $$7x + 7 = 19$$
  4. Start to perform the opposite operation to both sides of the equation.
  5. Repeat Step **4** until the equation is solved.
  6. Verify that the answer is correct.

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8B

- To solve a word problem using algebra:
  - Read the problem and find out what the question is asking for.
  - Define a pronumeral and write a statement such as: 'Let x be the number of …'. The pronumeral is often what you have been asked to find in the question.
  - Write an equation using your defined pronumeral.
  - Solve the equation.
  - Answer the question in words.

Exercise 8B

Understanding 1–3 2, 3

1 Choose from the words *collect*, *expand* and *one* to complete the following when solving linear equations.

a First ______ any brackets.
b ______ any like terms.
c If variables are on both sides, collect to ______ side.

2 When $-2(x-1)$ is expanded, the result is:

A $-2x-2$ **B** $-2x+1$ **C** $-2x+2$
D $2x+2$ **E** $2x+1$

3 When $2x$ is subtracted from both sides, $5x+1=2x-3$ becomes:

A $3x-1=3$ **B** $7x+1=-3$ **C** $7x+1=3$
D $3x+1=3$ **E** $3x+1=-3$

Fluency 4–9(½) 4–9(½)

Example 6 Solving equations with brackets

Solve $4(x-1)=16$.

Solution	Explanation
$4(x-1)=16$	
$4x-4=16$	Expand the brackets: $4 \times x$ and $4 \times (-1)$.
$4x=20$	Add 4 to both sides.
$x=5$	Divide both sides by 4.

Now you try

Solve $3(x+1)=15$.

4 Solve each of the following equations by first expanding the brackets.

a $3(x+2)=9$ **b** $4(x-1)=16$
c $3(x+5)=12$ **d** $4(a-2)=12$
e $5(a+1)=10$ **f** $2(x-10)=10$
g $6(m-3)=6$ **h** $3(d+4)=15$
i $7(a-8)=14$ **j** $10(a+2)=20$
k $5(3+x)=15$ **l** $2(a-3)=0$

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Example 7 Solving equations with two sets of brackets

Solve $3(2x + 4) + 2(3x - 2) = 20$.

Solution	Explanation
$3(2x + 4) + 2(3x - 2) = 20$	Use the distributive law to expand each set of brackets.
$6x + 12 + 6x - 4 = 20$	
$12x + 8 = 20$	Collect like terms on the LHS.
$12x = 12$	Subtract 8 from both sides.
$x = 1$	Divide both sides by 12.

Now you try

Solve $2(3x - 1) - 3(x - 4) = 16$.

5 Solve the following equations.

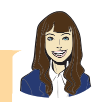
a $3(2x + 3) + 2(x + 4) = 25$ **b** $2(2x + 3) + 4(3x + 1) = 42$
c $2(2x + 3) + 3(4x - 1) = 51$ **d** $3(2x - 2) + 5(x + 4) = 36$
e $4(2x - 3) + 2(x - 4) = 10$ **f** $2(3x - 1) + 3(2x - 3) = 13$
g $2(x - 4) + 3(x - 1) = -21$ **h** $4(2x - 1) + 2(2x - 3) = -22$

Hint: Expand each pair of brackets and collect like terms before solving.

6 Solve the following equations.

a $3(2x + 4) - 4(x + 2) = 6$ **b** $2(5x + 4) - 3(2x + 1) = 9$
c $2(3x - 2) - 3(x + 1) = -7$ **d** $2(x + 1) - 3(x - 2) = 8$
e $8(x - 1) - 2(3x - 2) = 2$ **f** $5(2x - 3) - 2(3x - 1) = -9$
g $5(2x + 1) - 3(x - 3) = 35$ **h** $4(2x - 3) - 2(3x - 1) = -14$

Hint:
$-4(x + 2) = -4x - 8$
$-4(x - 2) = -4x + 8$



Example 8 Solving equations with variables on both sides

Solve $7x + 9 = 2x - 11$ for x.

Solution	Explanation
$7x + 9 = 2x - 11$	
$5x + 9 = -11$	Subtract $2x$ from both sides.
$5x = -20$	Subtract 9 from both sides.
$x = -4$	Divide both sides by 5.

Now you try

Solve $10x + 3 = 8x - 1$ for x.

7 Find the value of x in the following.

a $7x = 2x + 10$ **b** $10x = 9x + 12$
c $8x = 4x - 12$ **d** $6x = 2x + 80$
e $2x = 12 - x$ **f** $2x = 8 + x$
g $3x + 4 = x + 12$ **h** $4x + 9 = x - 3$
i $2x - 9 = x - 10$ **j** $6x - 10 = 12 + 4x$
k $9x = 10 - x$ **l** $1 - x = x + 3$

Hint: Remove the term containing x on the RHS. For parts **e**, **k** and **l**, you will need to add x to both sides.

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8B

Example 9 Solving equations with fractions

Solve $\frac{2x+3}{4} = 2$ for x.

Solution	Explanation
$\frac{2x+3}{4} = 2$	
$2x + 3 = 8$	Multiply both sides by 4.
$2x = 5$	Subtract 3 from both sides.
$x = 2.5$	Divide both sides by 2.

Now you try

Solve $\frac{4x-3}{2} = 4$.

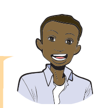
8 Solve the following equations.

a $\frac{x+2}{3} = 5$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{x-1}{3} = 4$

d $\frac{x-5}{3} = 2$ **e** $\frac{2x+1}{7} = 3$ **f** $\frac{2x+2}{3} = 4$

g $\frac{5x-3}{3} = 9$ **h** $\frac{3x-6}{2} = 9$ **i** $\frac{5x-2}{4} = -3$

Hint: First multiply by the denominator.



Example 10 Solving equations with more difficult fractions

Solve $\frac{3x}{2} - 4 = 2$ for x.

Solution	Explanation
$\frac{3x}{2} - 4 = 2$	
$\frac{3x}{2} = 6$	Add 4 to both sides.
$3x = 12$	Multiply both sides by 2.
$x = 4$	Divide both sides by 3.

Now you try

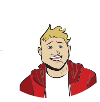
Solve $\frac{5x}{3} + 1 = 6$ for x.

9 Solve the following equations.

a $\frac{x}{3} + 1 = 5$ **b** $\frac{x}{3} + 1 = 7$ **c** $\frac{x}{4} - 5 = 10$

d $\frac{3x}{4} - 2 = 5$ **e** $\frac{2x}{5} - 3 = -1$ **f** $\frac{3x}{2} - 5 = -14$

Hint: First add or subtract a number from both sides.



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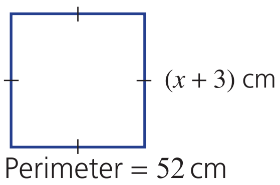
Problem-solving and reasoning

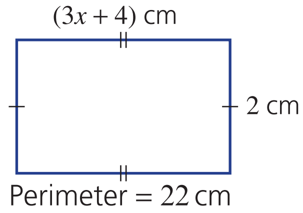
10–13 | 11–14

10 For each of these questions, write an equation and solve it for x.

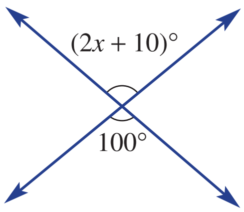
a



b



c



Hint: Vertically opposite angles are equal.

11 Solve the following equations using trial and error (guess, check and refine). Substitute your chosen values of x until you have found a value that makes the equation true.

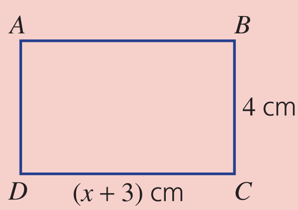
a $\frac{x+22}{3} = 4x$

b $5(3 - x) = 2(x + 7.5)$

c $\frac{2x-1}{4} = 2 - x$

Example 11 Solving a word problem

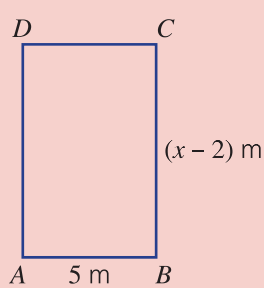
Find the value of x if the area of rectangle $ABCD$ shown is 24 cm^2.



Solution	Explanation
$A = l \times w$	Write an equation for area.
$24 = (x + 3) \times 4$	Substitute: $l = (x + 3)$, $w = 4$, $A = 24$.
$24 = 4x + 12$	Expand the brackets: $(x + 3) \times 4 = 4(x + 3)$.
$12 = 4x$	Subtract 12 from both sides.
$3 = x$	Divide both sides by 4.
$x = 3$	Write the answer.

Now you try

Find the value of x if the area of rectangle $ABCD$ shown is 40 m^2.



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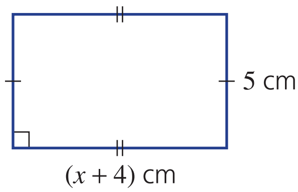
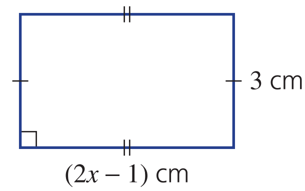
8B

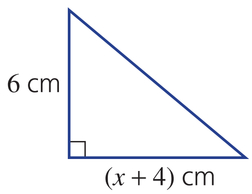
12 **a** Find the value of x if the area is 35 cm^2.



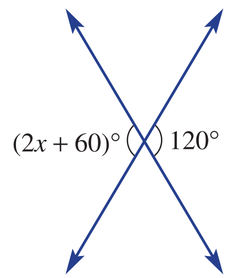
b Find the value of x if the area is 27 cm^2.



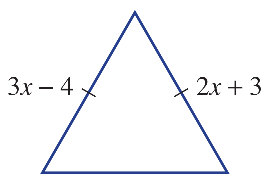
c Find the value of x if the area is 42 cm^2.

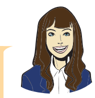


d Vertically opposite angles are equal. Find the value of x.

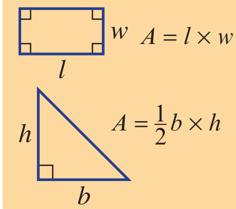


e Find the value of x.





Hint: Form the area equation first.



13 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 5 and 1 more than a number is 40.

b The product of 5 and 6 less than a number is -15.

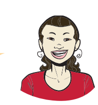
c When 6 less than 3 lots of a number is doubled, the result is 18.

d When 8 more than 2 lots of a number is tripled, the result is 36.

e 10 more than 4 lots of a number is equivalent to 6 lots of the number.

f 5 more than 4 times a number is equivalent to 1 less than 5 times the number.

g 6 more than a doubled number is equivalent to 5 less than 3 lots of the number.



Hint:
- ‘Product’ means ‘to multiply’
- The product of 5 and 1 more than a number means $5(x + 1)$.
- ‘6 less than 3 lots of a number is doubled’ will require brackets.
- ‘Tripled’ means three times a number.
- ‘Equivalent’ means ‘equal to’.

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14 Valentina and Harrison are planning to hire a car for their wedding day. 'Vehicles For You' have the following deal: \$850 hiring fee plus a charge of \$156 per hour.

a Write an equation for the cost ($\$C$) of hiring a car for h hours.

b If Valentina and Harrison have budgeted for the car to cost a maximum of \$2000, find the maximum number of full hours they can hire the car.

c If the car picks up the bride at 1:15 p.m., at what time must the event finish if the cost is to remain within budget?

More than one fraction — 15

15 Consider:

$$\frac{4x-2}{3}=\frac{3x-1}{2}$$

$$\frac{{}^{2}\cancel{6}(4x-2)}{\cancel{3}_{1}}=\frac{{}^{3}\cancel{6}(3x-1)}{\cancel{2}_{1}}$$ (Multiply both sides by 6 (LCM of 2 and 3) to get rid of the fractions.)

$2(4x-2)=3(3x-1)$ (Simplify.)

$8x-4=9x-3$ (Expand both sides.)

$-4=x-3$ (Subtract $8x$ from both sides.)

$-1=x$ (Add 3 to both sides.)

$\therefore x=-1$

Solve the following equations using the method shown above.

a $\frac{x+2}{3}=\frac{x+1}{2}$ **b** $\frac{x+1}{2}=\frac{x}{3}$ **c** $\frac{3x+4}{4}=\frac{x+6}{3}$

d $\frac{5x+2}{3}=\frac{3x+4}{2}$ **e** $\frac{2x+1}{7}=\frac{3x-5}{4}$ **f** $\frac{5x-1}{3}=\frac{x-4}{4}$

Using technology 8B: Solving linear equations

This activity is available on the companion website as a printable PDF.

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8C Using formulas

Learning intentions

- To understand that a relationship between variables can be described using formulas
- To be able to substitute into a formula and evaluate
- To be able to solve an equation after substitution into a formula

Key vocabulary: subject, formula, variable, substitute, evaluate

A formula (or rule) is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns.

You will already be familiar with many formulas. For example, $C = 2\pi r$ is the formula for finding the circumference, C, of a circle when given its radius, r.

$F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

$s = \frac{d}{t}$ is the formula for finding the speed, s, when given the distance, d, and time, t.

C, F and s are said to be the subjects of the formulas given above.

Lesson starter: Jumbled solution

Problem: The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.

Xavier was asked to find a, given that $A = 126$, $b = 10$ and $h = 14$, and to write the explanation beside each step of the solution.

Xavier's solution and explanation are below. His solution is correct but he has jumbled up the steps in the explanation. Copy Xavier's solution and write the correct instruction(s) beside each step.

Solution	Jumbled explanation
$A = \frac{h}{2}(a + b)$	Subtract 70 from both sides.
$126 = \frac{14}{2}(a + 10)$	Divide both sides by 7.
$126 = 7(a + 10)$	Substitute the given values.
$126 = 7a + 70$	Copy the formula.
$56 = 7a$	Simplify $\frac{14}{2}$.
$a = 8$	Expand the brackets.

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Key ideas

- A **formula** is an equation that relates two or more variables.
- The **subject** of a formula is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be rearranged to make another variable the subject. $C = 2\pi r$ can be rearranged to give $r = \frac{C}{2\pi}$.
- Note that $\sqrt{a^2} = a$ when $a \geq 0$ and $\sqrt{a^2 + b^2} \neq a + b$.

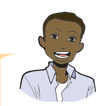
Exercise 8C

Understanding	1, 2	2

1 State the letter that is the subject in these formulas.

a $I = \frac{Prt}{100}$ **b** $F = ma$ **c** $V = \frac{4}{3}\pi r^3$

d $A = \pi r^2$ **e** $c = \sqrt{a^2 + b^2}$ **f** $P = 2x + 2y$



Hint: The subject of a formula is the letter on its own, on the left-hand side.

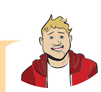
2 Substitute the given values into each of the following formulas to find the value of each subject. Round the answer to one decimal place where appropriate.

a $m = \frac{F}{a}$, when $F = 180$ and $a = 3$

b $A = lw$, when $l = 6$ and $w = 8$

c $A = \frac{1}{2}(a + b)h$, when $a = 6$, $b = 12$ and $h = 4$

d $v^2 = u^2 + 2as$, when $u = 6$, $a = 12$ and $s = 7$ **e** $m = \sqrt{\frac{x}{y}}$, when $x = 56$ and $y = 4$



Hint: Copy each formula, substitute the given values and then calculate the answer.

Fluency	3–8(½)	4–5(½), 7–9(½)

Example 12 Substituting values and solving equations

If $v = u + at$, find t when $v = 16$, $u = 4$ and $a = 3$.

Solution	Explanation
$v = u + at$	Substitute each value into the formula. $v = 16, u = 4, a = 3$
$16 = 4 + 3t$	An equation now exists. Solve this equation for t.
$12 = 3t$	Subtract 4 from both sides.
$4 = t$	Divide both sides by 3.
$t = 4$	Answer with the pronumeral on the left-hand side.

Now you try

If $A = \frac{1}{2}xy$, find y when $A = 12$ and $x = 4$.

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8C

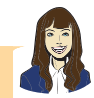
3 If $v = u + at$, find t when:
a $v = 16$, $u = 8$ and $a = 2$
b $v = 20$, $u = 8$ and $a = 3$
c $v = 100$, $u = 10$ and $a = 9$
d $v = 84$, $u = 4$ and $a = 10$

Hint: First copy the formula. Then substitute the given values. Then solve the equation.

4 If $P = 2(l + 2b)$, find b when:
a $P = 60$ and $l = 10$
b $P = 48$ and $l = 6$
c $P = 96$ and $l = 14$
d $P = 12.4$ and $l = 3.6$

5 If $V = lwh$, find h when:
a $V = 100$, $l = 5$ and $w = 4$
b $V = 144$, $l = 3$ and $w = 4$
c $V = 108$, $l = 3$ and $w = 12$
d $V = 280$, $l = 8$ and $w = 5$

6 If $A = \frac{1}{2}bh$, find b when:
a $A = 90$ and $h = 12$
b $A = 72$ and $h = 9$
c $A = 108$ and $h = 18$
d $A = 96$ and $h = 6$

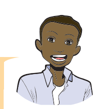


Hint: For $90 = \frac{1}{2} \times b \times 12$,
$\frac{1}{2} \times b \times 12 = \frac{1}{2} \times 12 \times b$
$= 6b$
So, $90 = 6b$
Solve for b.

7 If $A = \frac{h}{2}(a + b)$, find h when:
a $A = 20$, $a = 4$ and $b = 1$
b $A = 48$, $a = 5$ and $b = 7$
c $A = 108$, $a = 9$ and $b = 9$
d $A = 196$, $a = 9$ and $b = 5$

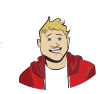
Hint: When solving the equation first undo the division by 2 by multiplying both sides by 2.

8 $E = mc^2$. Find m when:
a $E = 100$ and $c = 5$
b $E = 4000$ and $c = 10$
c $E = 72$ and $c = 1$
d $E = 144$ and $c = 6$



Hint: Square the c value before solving the equation.

9 If $V = \pi r^2 h$, find h (to one decimal place) when:
a $V = 160$ and $r = 3$
b $V = 400$ and $r = 5$
c $V = 1460$ and $r = 9$
d $V = 314$ and $r = 2.5$



Hint: For $160 = 9\pi h$, divide both sides by 9π to find h:
$h = \frac{160}{9\pi}$
Then evaluate on a calculator.

Problem-solving and reasoning

10–12 | 10, 12–14

10 The formula $F = \frac{9C}{5} + 32$ is used to convert temperature from degrees Celsius (°C) (which is used in Australia) to degrees Fahrenheit (°F) (which is used in the USA).
a When it is 30°C in Sydney, what is the temperature in Fahrenheit?
b How many degrees Celsius is 30° Fahrenheit? Answer to one decimal place.
c Water boils at 100°C. What is this temperature in degrees Fahrenheit?
d What is 0°F in degrees Celsius? Answer to one decimal place.

Hint: When finding C, you will have an equation to solve.

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11 The cost, in dollars, of a taxi is $C = 3 + 1.45d$, where d is the distance travelled, in kilometres.

a What is the cost of a 20 km trip?

b How many kilometres can be travelled for \$90?

12 $I = \frac{Prt}{100}$ calculates interest on an investment. Find:

a P when $I = 60$, $r = 8$ and $t = 1$

b t when $I = 125$, $r = 5$ and $P = 800$

c r when $I = 337.50$, $P = 1500$ and $t = 3$

13 The number of tablets a nurse must give a patient is found using the formula:

$$\text{Tablets} = \frac{\text{strength required}}{\text{tablet strength}}$$

a 750 milligrams of a drug must be administered to a patient. How many 500 milligram tablets should the nurse give the patient?

b If the nurse administers 2.5 of these tablets to another patient, how much of the drug did the patient take?

14 A drip is a way of pumping a liquid drug into a patient's blood. The flow rate of the pump, in millilitres per hour, is calculated using the formula: $\text{Rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$.

a A patient needs 300 mL of the drug administered over 4 hours. Calculate the rate, in mL/h, which needs to be delivered by the pump.

b A patient was administered 100 mL of the drug at a rate of 300 mL/h. How long was the pump running?

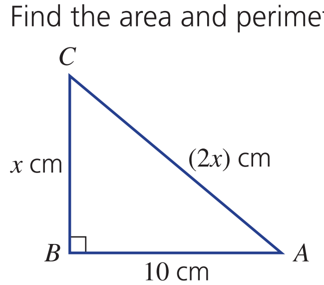
Calculation challenges — 15–17

15 A tax agent charges \$680 for an 8-hour day. The agent uses the formula $F = \frac{680x}{8}$ to calculate a fee to a client, in dollars.

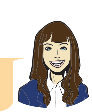
a What does the x represent?

b If the fee charged to a client is \$637.50, how many hours, to one decimal place, did the agent spend working on the client's behalf?

16 Find the area and perimeter of triangle ABC, shown. Round to two decimal places.



Hint: Use Pythagoras' theorem to find x.



17 Iqra is 10 years older than Urek. In 3 years' time, she will be twice as old as Urek. How old are they now?

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8D Linear inequalities

Learning intentions

- To know the four inequality symbols and what they mean
- To be able to illustrate an inequality using a number line
- To know when to reverse an inequality sign

Key vocabulary: inequality, inequality sign, linear inequality

There are many situations in which a solution to the problem is best described using one of the symbols <, ≤, > or ≥. For example, a medical company will publish the lowest and highest amounts for a safe dose of a particular medicine; e.g. 20 mg/day ≤ dose ≤ 55 mg/day, meaning that the dose should be between 20 and 55 mg/day.

An inequality is a mathematical statement that uses an 'is less than' (<), an 'is less than or equal to' (≤), an 'is greater than' (>) or an 'is greater than or equal to' (≥) symbol. Inequalities may result in an infinite number of solutions. These can be illustrated using a number line.

You can solve inequalities in a similar way to solving equations.

Lesson starter: What does it mean for *x*?

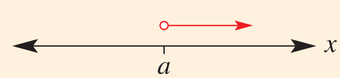
The following inequalities provide some information about the number x.

a $x < 6$ **b** $x \geq 4$ **c** $-5 \geq x$ **d** $-2 < x$

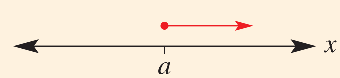
- Can you describe the possible values for x that satisfy each inequality?
- Test some values to check.
- How would you write the solution for x? Illustrate each on a number line.

Key ideas

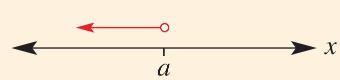
- The four **inequality signs** are <, ≤, > and ≥.
  - $x > a$ means x is greater than a.



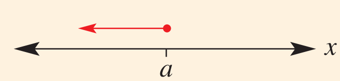
  - $x \geq a$ means x is greater than or equal to a.



  - $x < a$ means x is less than a.



  - $x \leq a$ means x is less than or equal to a.



- On the number line, a closed circle (•) indicates that the number is included. An open circle (◦) indicates that the number is not included.
- Solving **linear inequalities** follows the same rules as solving linear equations, except:
  - We reverse an inequality sign if we multiply or divide by a negative number. For example, $-5 < -3$ and $5 > 3$, and if $-2x < 4$ then $x > -2$.
  - We reverse the inequality sign if the sides are switched. For example, if $2 \geq x$, then $x \leq 2$.

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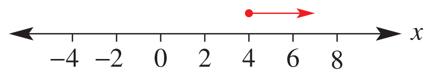
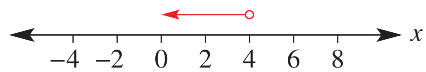
Exercise 8D

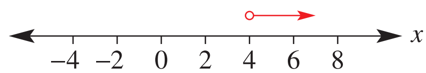
Understanding 1, 2(½) 2(½)

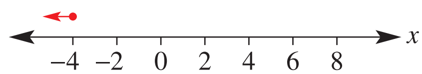
1 Match each inequality given with the correct number line.

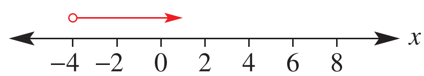
Hint: Look back at the Key ideas. The direction of the arrowhead is the same as the direction of the inequality sign.

a $x > 4$ **A** 

b $x < 4$ **B** 

c $x \geq 4$ **C** 

d $x > -4$ **D** 

e $x \leq -4$ **E** 

2 Match each inequality with the correct description.

a $x < 2$ **A** x is greater than 2

b $x \geq 2$ **B** x is less than or equal to 2

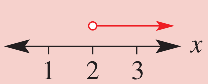
c $x \leq 2$ **C** x is less than 2

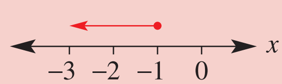
d $x > 2$ **D** x is greater than or equal to 2

Fluency 3–5(½), 6, 7 4–5(½), 7, 8

Example 13 Writing inequalities from number lines

Write each number line as an inequality.

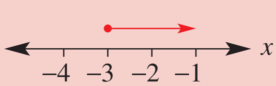
a 

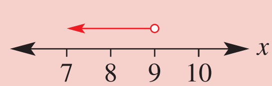
b 

Solution	Explanation
a $x > 2$	An open circle means 2 is not included.
b $x \leq -1$	A closed circle means -1 is included.

Now you try

Write each number line as an inequality.

a 

b 

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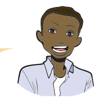
8D

3 Write each graph as an inequality.



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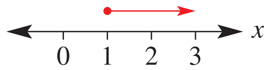

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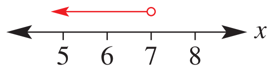

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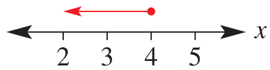

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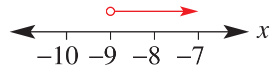

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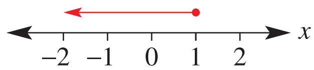

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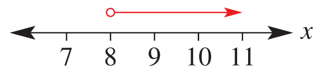

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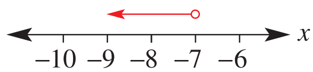

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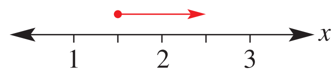

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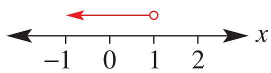

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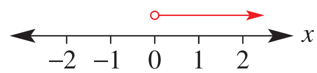
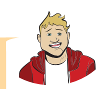
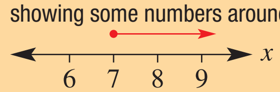


4 Show each of the following on separate number lines.

a $x \geq 7$	**b** $x > 1$	**c** $x < 1$
d $x \leq 1$	**e** $x \geq -1$	**f** $a \geq 0$
g $p \geq -2$	**h** $a > -15$	**i** $h < 5$

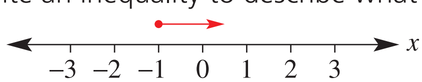


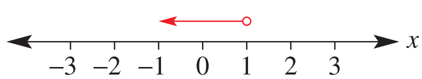
Hint: For $x \geq 7$, draw a number line showing some numbers around 7.

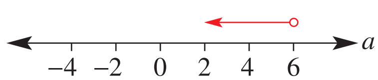


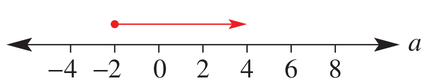
Use a closed circle (•) for ≥ and ≤.
Use an open circle (∘) for > and <.

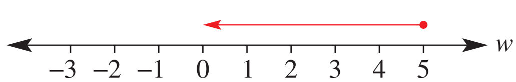
5 Write an inequality to describe what is shown on each of the following number lines.

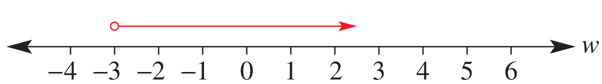
a


b


c


d


e


f


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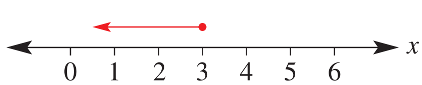

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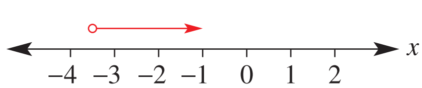

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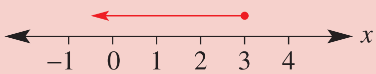
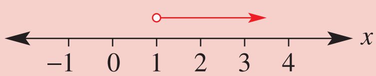
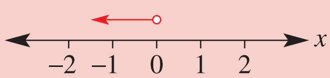
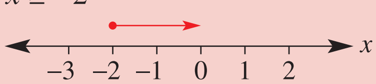
Example 14 Writing and graphing inequalities

Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 3　　**b** x is greater than 1

c x is less than 0　　**d** x is greater than or equal to -2

Solution	Explanation
a $x \leq 3$ 	Less than or equal to, ≤, closed circle
b $x > 1$ 	Greater than, >, open circle
c $x < 0$ 	Less than, <, open circle
d $x \geq -2$ 	Greater than or equal to, ≥, closed circle

Now you try

Write each of the following as an inequality and then show each solution on a number line.

a x is greater than -4　　**b** x is less than or equal to 6

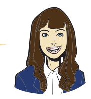
6 Write each of the following as an inequality and then show each solution on a number line.

a x is less than or equal to 6　　**b** x is greater than 4

c x is less than 2　　**d** x is greater than or equal to 5

7 Write each of the following as an inequality, using the pronumeral n.

Hint: 'At least 50' means '50 or more'.



a The number of people who visit the Sydney Opera House each year is more than 100 000.

b The number of lollies in a bag should be at least 50.

c A factory worker must pack more than three boxes a minute.

d More than 100 penguins take part in the nightly parade on Phillip Island.

e The weight of a suitcase is 30 kg or less.

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8D

8 Write each of the following statements as an inequality and determine which of the numbers below make each inequality true.

$-6, -2, -\frac{1}{2}, 0, 2, 5, 7, 10, 15, 24$

Hint: Write the inequality, then list the given numbers that make it true.

a x is less than zero
b x is greater than 10
c x is greater than or equal to 10
d x is less than or equal to zero
e x is greater than or equal to -1
f x is less than 10

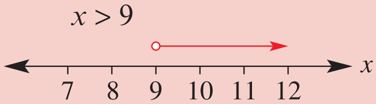
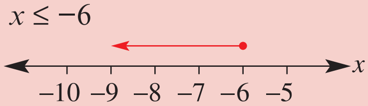
Problem-solving and reasoning

9–12(½) | 10–13(½)

Example 15 Solving and graphing inequalities

Solve the following and show your solution on a number line.

a $2x - 1 > 17$
b $\frac{x}{3} \le -2$

Solution	Explanation
a $2x - 1 > 17$	Add 1 to both sides.
$2x > 18$	Divide both sides by 2.
$x > 9$	
	> uses an open circle.
b $\frac{x}{3} \le -2$	Multiply both sides by 3.
$x \le -6$	
	≤ uses a closed circle.

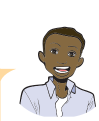
Now you try

Solve the following and show your solution on a number line.

a $3x + 2 \le 11$
b $\frac{x-1}{2} > -3$

9 Solve each of the following inequalities and show your solution on a number line.

a $2x > 10$
b $x + 2 < 7$
c $3x > 15$
d $\frac{x}{2} \ge 8$
e $x - 3 > 4$
f $x - 3 < 4$
g $p + 8 \le 0$
h $3a > 0$
i $x - 7 < 0$
j $2x \le 14$
k $5m > -15$
l $d - 3 > 2.4$
m $\frac{x}{7} \le 0.1$
n $\frac{1}{2}x \le 6$
o $5 + x > 9$



Hint: Keep the inequality sign the same when:
- adding or subtracting a number from both sides
- multiplying or dividing both sides by a positive number.

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10 Solve the following.

a $2 + 4a \leq 10$ **b** $5 + 2y > 11$ **c** $3p - 1 > 14$

d $3x - 2 \geq 10$ **e** $3x - 2 < 1$ **f** $5 + 2w \geq 8$

g $5x + 5 < 10$ **h** $5x - 5 \geq 0$ **i** $10p - 2 < 8$

11 Give the solution set for each of the following.

a $\frac{x+2}{4} \leq 1$ **b** $\frac{a-3}{2} \leq -1$

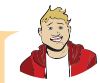
c $\frac{x}{4} - 1 \geq 6$ **d** $\frac{x}{3} + 7 > 2$

e $5 + \frac{x}{2} < 7$ **f** $\frac{x+2}{4} < 8$

g $\frac{2x-7}{3} > 4$ **h** $\frac{2x+1}{5} < 0$

i $\frac{3x}{2} + 1 \geq -3$ **j** $5x - 4 > 2 - x$

k $4(2x + 1) \geq 16$ **l** $3x + 7 < x - 2$



Hint: For $\frac{x+2}{4} \leq 1$, first multiply both sides by 4.
For $\frac{x}{4} - 1 \geq 6$, first add 1 to both sides.

12 For each of the following, write an inequality and solve it to find the possible values of x.

a When a number, x, is multiplied by 3, the result is less than 9.

b When a number, x, is multiplied by 3 and the result divided by 4, it creates an answer less than 6.

c When a number, x, is doubled and then 15 is added, the result is greater than 20.

d Thuong is x years old and Gary is 4 years older. The sum of their ages is less than 24.

e Kaitlyn has x rides on the Ferris wheel at \$4 a ride and spends \$7 on food. The total amount she spends is less than or equal to \$27.

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Example 16 Solving inequalities when the pronumeral has a negative coefficient

Solve $4 - x \geq 6$.

Solution	Explanation
$4 - x \geq 6$	Subtract 4 from both sides.
$-x \geq 2$	Divide both sides by -1.
$x \leq -2$	When we divide both sides by a *negative* number, the inequality sign is reversed.
Alternative solution:	
$4 - x \geq 6$	Add the x to both sides so that it is positive.
$4 \geq 6 + x$	Subtract 6 from both sides.
$-2 \geq x$	Switch the sides to have the x on the left-hand side.
$x \leq -2$	Reverse the inequality sign. Note that the inequality sign still 'points' to the x.

Now you try

Solve $5 - 2x < 17$.

13 Choose an *appropriate strategy* to solve the following.

a $5 - x < 6$
b $7 - x \geq 10$
c $-p \leq 7$
d $9 - a < -10$
e $-w \geq 6$
f $-3 - 2p < 9$
g $5 - 2x < 7$
h $-2 - 7a \geq 4$

Hint: Remember to reverse the inequality sign when multiplying or dividing by a negative number.
e.g.
$\times(-1)$ $-x < 7$ → $x > -7$ $\times(-1)$
$\div(-2)$ $-2x \geq 20$ → $x \leq -10$ $\div(-2)$

Investigating inequalities — 14

14 a Let us start with the numbers 4 and 6 and the true relationship $4 < 6$. Copy and complete the following table.

				True or false?
4 and 6	**4**	**6**	**4 < 6**	**True**
Add 3	$4 + 3$	$6 + 3$	$7 < 9$	True
Subtract 3	$4 - 3$		$1 < 3$	True
Multiply by 2				
Divide by 2				
Multiply by -2				False $(-8 > -12)$
Divide by -2	$4 \div (-2)$	$6 \div (-2)$		

b Copy and complete the following.
When solving an inequality, you can add or _______ a number from both sides and the inequality remains true. You can multiply or _______ by a _______ number and the _______ also remains true. However, if you _______ or _______ by a negative _______ the inequality sign must be reversed for the inequality to remain _______.

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8E Solving simultaneous equations graphically

Learning intentions

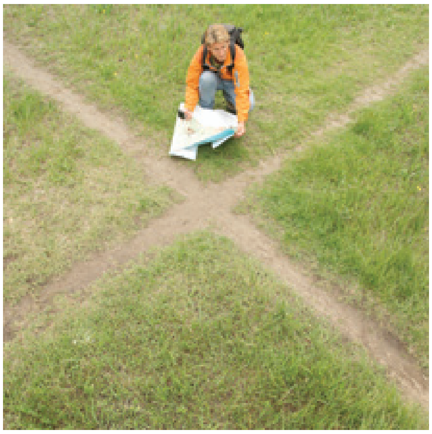
- To understand that an intersection point represents the solution to simultaneous linear equations
- To be able to locate an intersection point graphically
- To be able to interpret an intersection point as the solution to a real problem involving simultaneous equations

Key vocabulary: intersection point, coordinates, parallel, gradient, simultaneous

When we approach an intersection while driving, we near the shared position of two or more roads.

Like two roads, two straight lines in the same plane will always intersect unless they are parallel.

If we try to find the point of intersection, we are said to be solving the equations simultaneously.



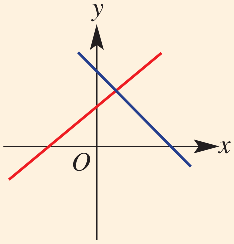
Lesson starter: Which job has better pay?

You start working as a delivery person for the Hasty Tasty Pizza Company. You're paid \$25 per shift and \$4 per pizza delivery. A second pizza company, More-2-Munch Pizzas, offers you a job at \$15 per shift and \$5 per pizza delivery.

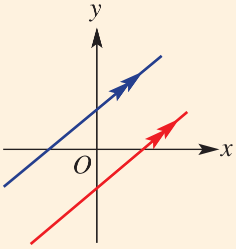
- How much does each company pay for delivery of 7 pizzas in one shift? How much does each company pay for delivery of 12 pizzas in one shift?
- For each pizza company, draw up a table to show the money you could earn for delivery of up to 15 pizzas delivered in one shift.
- On the same sheet, draw a graph of the information in your tables for each pizza company. Draw the graph for each pizza company on the same set of axes.
- What does the point of intersection show us?
- Write a sentence describing which job pays better for different numbers of pizzas delivered.
- Write down one advantage of using a graph to compare these two wages.

Key ideas

- At a point of **intersection**, two lines will have the same coordinates.
- The point of intersection represents the solution of two linear simultaneous equations.
- To find the point of intersection, sketch each straight line and read off the coordinates of where the lines meet.
- When two lines are **parallel**, they have the same gradient and there is no point of intersection.



1 point of intersection



0 points of intersection

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8E

Exercise 8E

Understanding | 1–3 | 2, 3

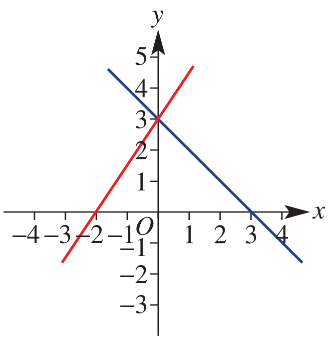
1 State the missing number.

a If two lines are parallel, then there are _______ points of intersection.

b If two lines are not parallel, then there is _______ point of intersection.

2 State the point of intersection (x, y) for the following lines, if there is one.

a



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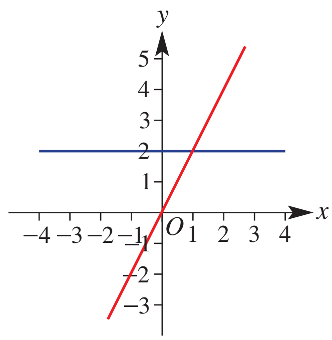

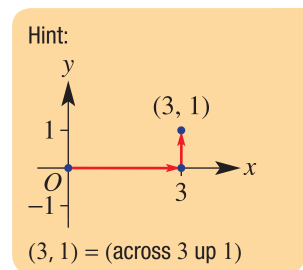

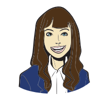

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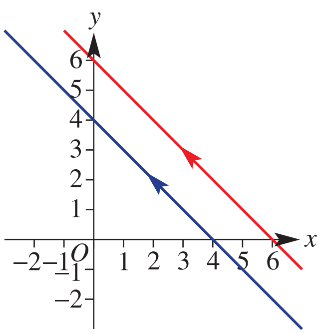

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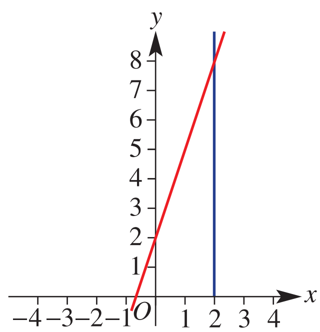

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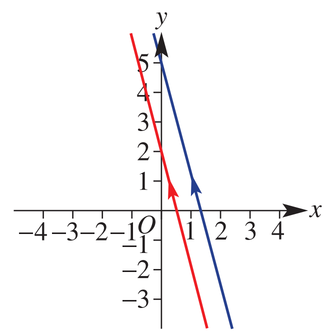

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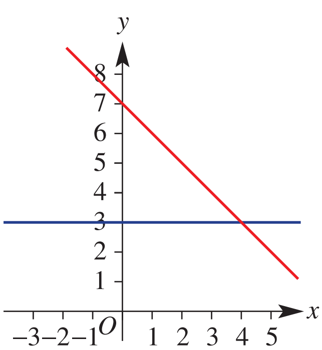

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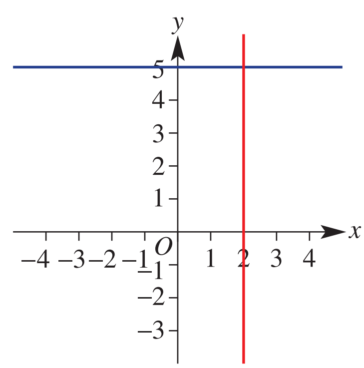

3 Use the method of trial and error (guess, check and refine) to find the point of intersection for these pairs of linear equations. Remember, your chosen point must satisfy both equations, i.e. substitute your x and y values into the equations to see if both are true.

a $y = 2x - 1$ and $y = 5 - x$

b $y = x - 3$ and $2x + y = -6$

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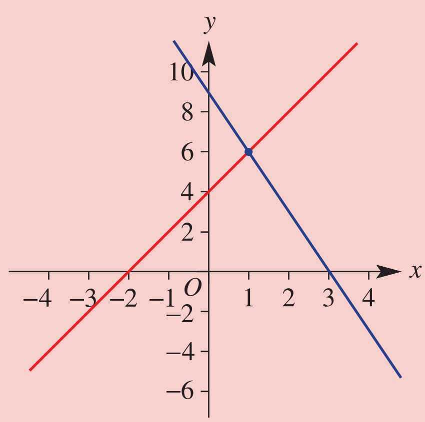
Fluency

Fluency	4(½), 5, 6	4(½), 5–7

Example 17 Finding the point of intersection by graphing

Find the point of intersection (x, y) of $y = 2x + 4$ and $3x + y = 9$ by sketching accurate graphs on the same axes.

Solution	**Explanation**
$y = 2x + 4$	First, find the x- and y-intercepts of each graph.
y-intercept at $x = 0$: $y = 2(0) + 4 = 4$	Substitute $x = 0$.
x-intercept at $y = 0$: $0 = 2x + 4$	Substitute $y = 0$.
$2x = -4$	Subtract 4 from both sides.
$x = -2$	Divide both sides by 2.
$3x + y = 9$	
y-intercept at $x = 0$: $3(0) + y = 9$	Substitute $x = 0$.
$y = 9$	Simplify.
x-intercept at $y = 0$: $3x + (0) = 9$	Substitute $y = 0$.
$3x = 9$	Simplify.
$x = 3$	Divide both sides by 3.



Sketch the graphs using the x- and y-intercepts.
For $y = 2x + 4$, the x-intercept $= -2$ and the y-intercept $= 4$.
For $3x + y = 9$, the x-intercept $= 3$ and the y-intercept $= 9$.

The point of intersection is $(1, 6)$. — Read off the intersection point, listing x followed by y.

Now you try

Find the point of intersection (x, y) of $y = x - 2$ and $2x - y = 3$ by sketching accurate graphs on the same axes.

4 Find the point of intersection (x, y) of each pair of equations by plotting an accurate graph.

a $y = x + 1$ and $3x + 2y = 12$
b $y = 3x + 2$ and $2x + y = 12$
c $y = 2x + 9$ and $3x + 2y = 18$
d $y = x + 11$ and $4x + 3y = 12$

Hint:
y-intercept: $x = 0$
x-intercept: $y = 0$

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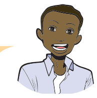
5 Find the point of intersection of each pair of equations by plotting an accurate graph.
  a $y = 3$ and $x = 2$
  b $y = -2$ and $x = 3$



Hint: $y = 3$ cuts the y-axis at 3 and is horizontal.
$x = 2$ cuts the x-axis at 2 and is vertical.

6 Find the point of intersection of each pair of equations by plotting an accurate graph.
  a $y = 3x$ and $y = 2x + 3$
  b $y = -3x$ and $y = 2x - 5$

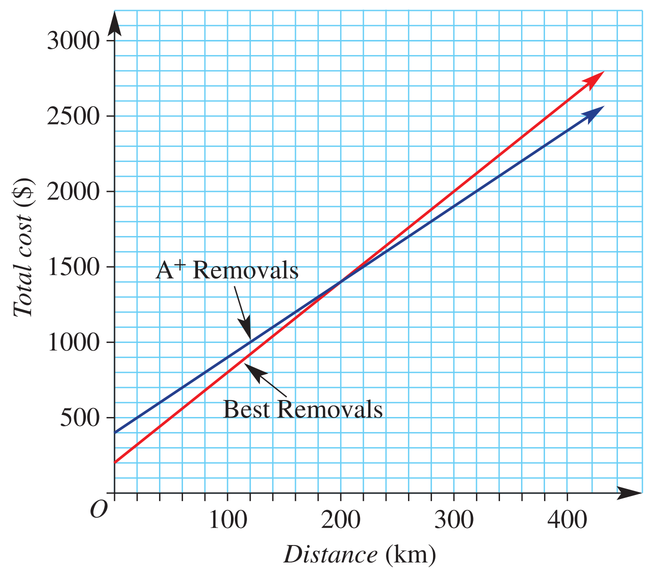
7 Find the point of intersection of each pair of equations by plotting an accurate graph.
  a $y = 2x - 6$ and $y = 3x - 7$
  b $y = -2x + 3$ and $y = 3x - 2$

Problem-solving and reasoning

	8, 9	9–11

8 This graph represents the cost of hiring two different removalist companies to move a person's belongings for various distances.
  a Determine the number of kilometres for which the total cost of the removalists is the same.

Hint: The cost is the same at the point of intersection.



  b What is the price when the total cost is equal?
  c If a person wanted to move 100 km, which company would be cheaper and by how much?
  d If a person wanted to move 400 km, which company would be cheaper and by how much?

9 The pay structures for baking companies A and B are given by the following.
Company A: $20 per hour
Company B: $45 plus $15 per hour
  a Complete two tables, showing the pay by each company for up to 12 hours.
  b Draw a graph of the pay by each company (on the vertical axis) versus time, in hours (on the horizontal axis). Draw the graphs for both companies on the same set of axes.
  c State the number of hours worked for which the earnings are the same for the two companies.
  d State the amount earned when the earnings are the same for the two companies.

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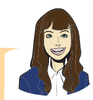
10 **a** Graph these three lines on the same coordinate axes by plotting the axis intercepts for each: $y = 3, y = x + 1, y = 1 - x$.
b Write the coordinates of the points of intersection.
c Find the length of each line segment formed between the intersection points.
d What type of triangle is formed by these line segments?

Hint: Use Pythagoras' theorem to find the length of a line segment.

11 The value of two cars is depreciating (i.e. decreasing) at a constant rate according to the information in this table.

Car	Initial value	Annual depreciation
Luxury sports coupe	$70 000	$5000
Family sedan	$50 000	$3000



Hint: Annual depreciation means how much the car's value goes down by each year.

a Complete two tables showing the value of each car every second year from zero to 12 years.
b Draw a graph of the value of each car (on the vertical axis) versus time, in years (on the horizontal axis). Draw both graphs on the same set of axes.
c From the graph, determine the time taken for the cars to have the same value.
d State the value of the cars when they have the same value.

Multiple intersections — 12, 13

Use a calculator to complete these questions.

12 On the same axes, plot the graphs of $y = 2x, y = 2x + 1, y = 2x + 2$ and $y = 2x + 3$.
a Are there any points of intersection?
b Suggest a reason for your answer to part **a**.
c Plot the graph of $y = 3x + 6$.
d Determine the points of intersection of the graphs already drawn and $y = 3x + 6$.

13 On the same axes, plot $y = x - 1, y = 2x - 1, y = 3x - 1$ and $y = 4x - 1$.
a Are there any points of intersection?
b Suggest a reason for your answer to part **a**.
c Plot the graph of $y = 2x + 1$.
d Determine the points of intersection of the graphs already drawn and $y = 2x + 1$.

Using technology 8E: Finding intersections
This activity is available on the companion website as a printable PDF.

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Progress quiz

8A **1** Solve the following one-step equations.

a $x + 12 = 18$ **b** $y - 9 = 8$ **c** $3m = 21$ **d** $\frac{x}{5} = -2$

8A/B **2** Solve the following equations.

a $2x - 3 = 15$ **b** $1 - 2y = 9$ **c** $\frac{x}{3} + 4 = 7$

d $\frac{x-2}{4} = 1$ **e** $\frac{2y+2}{5} = 4$ **f** $2 - \frac{x}{5} = 3$

8B **3** Solve these equations with brackets.

a $3(x - 1) = 6$ **b** $5(2a + 3) = -45$

c $3(y + 4) + 2(2y - 3) = 27$ **d** $4(2x - 1) - 3(x - 2) = 22$

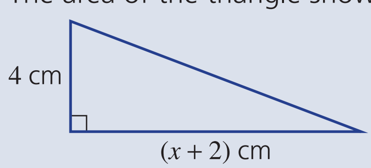
8A/B **4** Write equations for the following and solve for the pronumeral.

a When x is doubled and then 3 is added, the result is 25.

b When 3 is subtracted from x and the answer is divided by 2, the result is 6.

c When 2 is subtracted from x and the answer is multiplied by 3, the result is 12.

d The area of the triangle shown is 16 cm^2.



8B **5** Solve these equations with variables on both sides.

a $8x = 2x - 12$ **b** $9x + 4 = 4x + 14$ **c** $3x + 4 = 20 - x$

8C **6** Find the value of the unknown in these formulas by substituting the given values.

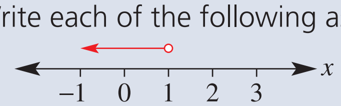
a For $A = \frac{1}{2}bh$, find b when $A = 28$ and $h = 14$.

b For $s = ut + \frac{1}{2}at^2$, find u when $a = 2$, $t = 3$ and $s = 24$.

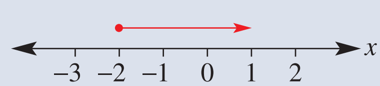
c For $S = 2\pi rh$, find h (to one decimal place) when $S = 30$ and $r = 2$.

8D **7** Write each of the following as inequalities.

a



b



8D **8** Solve the following inequalities and show the solution on a number line.

a $3x + 1 < 10$ **b** $\frac{x+1}{3} \geq 2$ **c** $3 - 2x < 13$

8E **9** Find the point of intersection of $x + y = 4$ and $y = 2x + 1$ by sketching accurate graphs on the same axes.

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8F Solving simultaneous equations using substitution ★

Learning intentions

- To understand that a solution to a pair of simultaneous equations can be found algebraically
- To be able to use the method of substitution to find a solution to a pair of simultaneous equations
- To be able to apply the method of substitution to solve simultaneous equations in a real context

Key vocabulary: substitute, subject, define

A pair of simultaneous equations is formed when there are two unknown quantities (i.e. variables) and two pieces of information relating these quantities. The solution to these simultaneous equations gives the variable values that make both equations true.

In the previous section, the solution was found from the point of intersection of two line graphs. In this section, you will learn how to find the solution using the algebraic method of *substitution*.

An example of two variables is the cost of a wedding reception and the number of invited guests. Two simultaneous equations could be made from two different catering companies. The solution will be the number of guests that make the costs equal for the two companies. Using equations helps to accurately compare the two deals.

Lesson starter: Equations and solutions

Albert is 11 years older than Jenny and the sum of their ages is 69. What are the ages of Albert and Jenny? Here are the steps to solve this problem but they are in the wrong order. Decide on the correct order.

A $x + (x + 11) = 69$
$2x + 11 = 69$
$2x = 58$
$x = 29$

B Let x = Jenny's age
Let y = Albert's age

C Jenny is 29 years old.
Albert is 40 years old.

D $x + y = 69$
$y = x + 11$

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8F

Key ideas

- The algebraic method of **substitution** is generally used when at least one of the linear equations has x or y as the subject;

  e.g. $y = 3x + 4$ and $3x + y = 2$ or $y = -2x + 6$ and $y = -x - 1$ or $x = 2$ and $2x - y = 5$

- The method involves:
  - substituting one equation into the other
  - solving for the remaining variable
  - substituting to find the value of the second variable

- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two unknowns using pronumerals
  - Write down two equations using your pronumerals
  - Solve the equations using the method of substitution
  - Answer the original question in words

Exercise 8F

Understanding | 1, 2 | 2

1 Write the missing words to complete each statement. Choose from *intersection*, *substituted*, *simultaneous* and *substitution*.

a __________ equations involve two equations and two variables.

b When two equations have been graphed, the x and y values that make both equations true are the coordinates of the point of __________.

c If x (or y) is replaced with a number, then we have __________ that number for x.

d If x (or y) is replaced with an algebraic expression, then we have __________ that expression for x (or y).

e When we algebraically substitute one equation into another, this is called solving simultaneous equations by the method of __________.

Hint: In part a, replace y in $2x + y = 6$ with $x + 1$.

2 Choose the correct option.

a When $y = x - 1$ is substituted into $2x + y = 6$, the result is:

A $2x + (x - 1) = 6$
B $2x - 1 = 6$
C $x - 1 = 6$
D $2x - x + 1 = 6$
E $3x = 6$

b When $y = 2x + 3$ is substituted into $x - 3y = 1$, the result is:

A $x + 3(2x + 3) = 1$
B $3(2x + 3) = 1$
C $x - 3(2x + 3) = 1$
D $x - (2x + 3) = 1$
E $2x - 3 = 1$

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Fluency

3–4(½), 6 | 3–4(½), 5–7

Example 18 Using the substitution method to solve simultaneous equations

Determine the point of intersection of $y = 5x$ and $y = 2x + 6$.

Solution	Explanation
$y = 5x$ [1] $y = 2x + 6$ [2]	Label the two equations.
Substitute [1] into [2]:	Explain how you are substituting the equations.
$5x = 2x + 6$	Replace y in the second equation with $5x$.
$3x = 6$	Subtract $2x$ from both sides.
$x = 2$	Divide both sides by 3.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$y = 5(2)$	Replace x with the number 2.
$y = 10$	Simplify.
The point of intersection is $(2, 10)$.	Write the solution.
Check: $10 = 2(2) + 6$	Substitute your solution into the other equation to check.

Now you try

Determine the point of intersection of $y = 7x$ and $y = 2x + 5$.

3 Determine the point of intersection for the following pairs of lines.

a $y = 5x$
$y = 3x + 4$

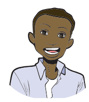
b $y = 3x$
$y = 2x - 5$

c $y = 2x$
$y = 4x + 8$

d $y = 4x$
$y = -3x + 7$

e $y = x$
$y = -5x + 12$

f $y = 6x$
$y = -2x + 16$



Hint:
$y = 5x$
$y = 3x + 4$
$5x = 3x + 4$

Example 19 Solving simultaneous equations with the substitution method

Solve the simultaneous equations $y = x + 3$ and $2x + 3y = 19$ using the substitution method; i.e. find the point of intersection.

Solution	Explanation
$y = x + 3$ [1] $2x + 3y = 19$ [2]	Label the two equations.

Continued on next page

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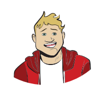
Substitute [1] into [2]:	Explain how you are substituting the equations.
$2x + 3(x + 3) = 19$	Replace y in the second equation with $(x + 3)$.
$2x + 3x + 9 = 19$	Expand the brackets.
$5x + 9 = 19$	Simplify.
$5x = 10$	Subtract 9 from both sides.
$x = 2$	Divide both sides by 5.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation (2).
$y = 2 + 3$	Replace x with the number 2.
$y = 5$	Simplify.
The point of intersection is (2, 5).	Write the solution.
Check: $2(2) + 3(5) = 19$	Substitute your solution into the other equation to check.

Now you try

Solve the simultaneous equations $y = x - 1$ and $3x + 2y = -12$ using the substitution method; i.e. find the point of intersection.

4 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

- **a** $y = x + 3$ and $2x + 3y = 19$
- **b** $y = x + 2$ and $3x + y = 6$
- **c** $y = x - 1$ and $3x + 2y = 8$
- **d** $y = x - 1$ and $3x + 5y = 27$
- **e** $y = x + 2$ and $2x + 3y = -19$
- **f** $y = x + 5$ and $5x - y = -1$
- **g** $y = x - 3$ and $5x - 2y = 18$
- **h** $y = x - 4$ and $3x - y = 2$



Hint: In part **a**, replace y in the second equation with $(x + 3)$. It is important to use brackets.

$y = x + 3$

$2x + 3y = 19$

Remember that

$3(x + 3) = 3x + 9$

5 Solve the following pairs of simultaneous equations; i.e. find the point of intersection.

a $y = 2$	**b** $y = -1$	**c** $y = 4$
$y = 2x + 4$	$y = 2x - 7$	$2x + 3y = 20$

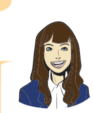
Hint: Replace y in the second equation with 2.

$y = 2$

$y = 2x + 4$

6 Determine the point of intersection for the following.

a $x = 2$	**b** $x = -3$	**c** $x = 7$
$3x + 2y = 14$	$y = -2x - 4$	$4x - 3y = 31$



Hint: Replace x in the second equation with 2.

$x = 2$

$3x + 2y = 14$

Remember that $3x$ means $3 \times x$.

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7 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 2x + 3$
$11x - 5y = -14$

b $y = 3x - 2$
$7x - 2y = 8$

c $y = 3x - 5$
$3x + 5y = 11$

d $y = 4x + 1$
$2x - 3y = -23$

Hint: Be careful with signs when expanding brackets.
$-5 \times (+3) = -15$

$11x - 5(2x + 3)$
$= 11x - 10x - 15$

When multiplying numbers with different signs, the answer is negative.

Problem-solving and reasoning

Problem-solving and reasoning	8, 9	8, 10, 11

Example 20 Solving word problems with simultaneous equations (substitution)

Jade is 5 years older than Marian. If their combined age is 33, find their ages.

Solution	Explanation
Let j be Jade's age and m be Marian's age.	Define two pronumerals using words.
$j = m + 5$ [1]	The first piece of information is that Jade is 5 years older than Marian.
$j + m = 33$ [2]	The second piece of information is that their combined age is 33.
$(m + 5) + m = 33$	Substitute $m + 5$ for j in the second equation.
$2m + 5 = 33$	Collect any like terms, so $m + m = 2m$.
$2m = 28$	Subtract 5 from both sides.
$m = 14$	Divide both sides by 2.
$j = m + 5$ [1] $j = 14 + 5$ $j = 19$	Use the first equation, $j = m + 5$, to find j.
Jade is 19 years old and Marian is 14 years old.	Answer the original question in words.

Now you try

A rectangle's length is 3 cm more than its width. If its perimeter is 32 cm, determine its dimensions.

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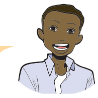
8F

8 Kye is 5 years older than Viviana. If their combined age is 81, determine their ages.

Hint: First define a pronumeral for Kye's age and another pronumeral for Viviana's age. Then write two equations before solving.



9 The length of a rectangle is three times the width. If the perimeter of the rectangle is 48 cm, determine its dimensions.

Hint: Draw a diagram to help form the perimeter equation.

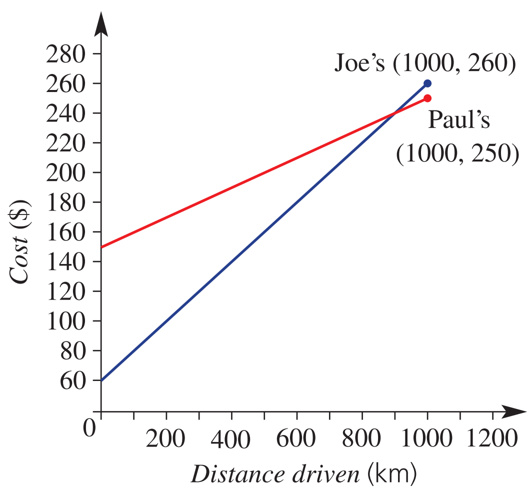
10 A vanilla thick shake is \$2 more than a fruity twirl. If three vanilla thick shakes and five fruity twirls cost \$30, determine their individual prices.

Hint: If a fruity twirl costs $\$x$, then 5 will cost $\$5x$.

11 Carlos is 3 more than twice Ella's age. If the sum of their ages is 54 years, determine their ages.

Rentals — 12

12 The given graph represents the rental cost of a new car from two car rental firms called Paul's Motor Mart and Joe's Car Rental.



a Determine:
  - **i** the initial rental cost from each company
  - **ii** the cost per kilometre when renting from each company
  - **iii** the linear equations for the total cost from each company
  - **iv** the number of kilometres at which the total cost is the same for both rental firms, using the method of substitution.

b Describe when you would use Joe's or Paul's rental firm.

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8G Solving simultaneous equations using elimination ★

Learning intentions

- To be able to use the method of elimination to find a solution to a pair of simultaneous equations
- To be able to apply the method of elimination to solve simultaneous equations in a real context

Key vocabulary: elimination, simultaneous, multiple

A second method for solving simultaneous equations, called elimination, can sometimes be more efficient, depending on how the equations are structured in the first place.

When setting up equations for real situations, we should define the unknown quantities using pronumerals. When solving simultaneous linear equations there should be only two unknown quantities for two equations formed from the given information.

For example, two related variables are the cost of owning a car and the number of kilometres driven. For two different cars, two equations could be made relating these variables. The simultaneous solution gives the number of kilometres that makes the total running costs of each car equal. Solving simultaneous equations provides information for an accurate comparison of costs between two vehicles.

Lesson starter: Eliminating a variable

One step in the elimination method involves adding or subtracting two equations in order to eliminate one of the variables. When adding, we write [1] + [2]; when subtracting, we write [1] − [2].

- A student has either added or subtracted pairs of equations, but has many incorrect answers.
- Determine which answers are incorrect and write the correct answer for these. (Note: Do not solve the equations for x or y.)

A
$5x + 3y = 34$ [1]
$7x - 3y = 26$ [2]
[1] + [2] gives:
$12x + 0 = 60$

B
$3x + 2y = 18$ [1]
$2x - 2y = 2$ [2]
[1] + [2] gives:
$5x - 4y = 20$

C
$3x - 3y = 9$ [1]
$2x - 3y = 4$ [2]
[1] − [2] gives:
$5x + 0 = 5$

D
$2x - 2y = 8$ [1]
$4x - 2y = 24$ [2]
[1] − [2] gives:
$2x - 4y = 16$

E
$4x + 3y = 16$ [1]
$-4x + 2y = 3$ [2]
[1] + [2] gives:
$0 + y = 19$

F
$3x + 2y = 25$ [1]
$2x + 2y = 18$ [2]
[1] − [2] gives:
$x + 0 = 43$

G
$5x + 3y = 31$ [1]
$5x - 3y = 19$ [2]
[1] + [2] gives:
$0 + 0 = 12$

H
$x + 3y = 15$ [1]
$x + 2y = 12$ [2]
[1] − [2] gives:
$2x + y = 3$

I
$7x - y = 5$ [1]
$3x - y = -2$ [2]
[1] − [2] gives:
$4x = 3$

Key ideas

- **Elimination** is generally used to solve simultaneous equations when both equations are in the form $ax + by = d$.
  e.g. $2x - y = 6$ and $3x + y = 10$ or $-5x + y = -2$ and $6x + 3y = 5$

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8G

- Adding or subtracting multiples of these two equations allows one of the variables to be eliminated.
  - Add $x - y = 10$ and $3x + y = 34$ to eliminate y.
  - Subtract $5x + 2y = 7$ and $5x + y = 6$ to eliminate the x.
  - Form a matching pair by multiplying by a chosen factor.
    For example, $2x - y = 3 \quad \Rightarrow \times 2 \Rightarrow \quad 4x - 2y = 6$
    $5x + 2y = 12 \quad \Rightarrow \quad 5x + 2y = 12$
- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two variables using letters.
  - Write down two equations using your variables.
  - Solve the equations using the method of elimination.
  - Answer the original question in words.

Exercise 8G

Understanding — 1–3 — 2, 3

1 What operation (i.e. + or −) will make these equations true?
a $2x$ _____ $2x = 0$ **b** $-3y$ _____ $3y = 0$ **c** $4x$ _____ $(-4x) = 0$

2 Multiply both sides of the equation $3x - 2y = -1$ by the following numbers. Write the new equations.
a 2 **b** 3 **c** 4

3 Choose the correct option.
a When $2x + y = 3$ is added to $5x - y = 11$ the result is:
A $7x = 11$ **B** $7x = 14$ **C** $3x = 14$ **D** $3x = 1$ **E** $7x = 8$
b When $x + y = 5$ is subtracted from $3x + y = 7$ the result is:
A $2x = 7$ **B** $4x = 2$ **C** $4x = 12$ **D** $x = 2$ **E** $2x = 2$

Fluency — 4, 5, 6–9(½) — 6–11(½)

Example 21 Eliminating a variable by addition of equations then solving

Add equation [1] to equation [2] and then solve for x and y.

$x + 2y = 10$ [1]
$x - 2y = 2$ [2]

Solution	Explanation
$x + 2y = 10$ [1] $x - 2y = 2$ [2]	Copy equations with the labels [1] and [2].
[1] + [2] gives:	Write the instruction to add: [1] + [2].
$2x + 0 = 12$	Add the x column: $x + x = 2x$.
$2x = 12$	Add the y column: $2y + (-2y) = 0$.
$x = 6$	Add the RHS: $10 + 2 = 12$ and then divide both sides by 2.

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Substitute $x = 6$ into [1]:	In equation [1], replace x with 6. Equation [2] could also have been used.
$6 + 2y = 10$	
$2y = 4$	Subtract 6 from both sides.
$y = 2$	Divide both sides by 2.
Solution is (6, 2).	Write the solution as an ordered pair.
Check: [2] $6 - 2 \times 2 = 2$, true	Check that the solution satisfies equation [2].

Now you try

Add equation [1] to equation [2] then solve for x and y.

$3x + y = 11$ [1]

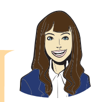
$x - y = 5$ [2]

4 Copy each pair of equations, add equation [1] to [2], then solve for x and y.

a $x + y = 7$ [1]
$x - y = 5$ [2]
[1] + [2]

b $x + 2y = 11$ [1]
$x - 2y = -5$ [2]
[1] + [2]

c $3x + 2y = 20$ [1]
$-3x + y = 1$ [2]
[1] + [2]



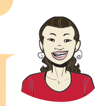
Hint:
Adding equations:
[1] + [2]
$x + y = 7$ [1]
$x - y = 5$ [2]
$2x + 0 = 12$
Remember that
$+y + (-y) = +y - y = 0$

5 Copy each pair of equations, subtract equation [2] from equation [1] and then solve for x and y, showing all steps.

a $2x + y = 16$ [1]
$x + y = 9$ [2]
[1] − [2]

b $3x + 5y = 49$ [1]
$3x + 2y = 25$ [2]
[1] − [2]

c $5x - 4y = 16$ [1]
$2x - 4y = 4$ [2]
[1] − [2]



Hint:
Subtracting equations:
[1] − [2]
$5x - 2y = 16$ [1]
$2x - 2y = 4$ [2]
$3x + 0 = 12$
Remember that
$-2y - (-2y)$
$= -2y + 2y = 0$

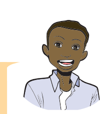
6 Determine the point of intersection of the following lines, using the elimination method.

a $x + y = 7$ and $5x - y = 5$

b $x + y = 5$ and $3x - y = 3$

c $x - y = 2$ and $2x + y = 10$

d $x - y = 0$ and $4x + y = 10$



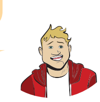
Hint: Label the two equations, one under the other, and decide whether to eliminate x or y; i.e. eliminate whichever variable has the same number in each equation.
Remember that $+y + (-y) = 0$.
The point of intersection is the same as the simultaneous solution of the equations.

7 Solve the following pairs of simultaneous equations, using the elimination method. You will need to subtract the equations to eliminate one of the variables.

a $3x + 4y = 7$
$2x + 4y = 6$

b $4x + 3y = 11$
$x + 3y = 5$

c $2x + 3y = 1$
$2x + 5y = -1$



Hint: Always label the equations and write the instruction; e.g. [1] − [2] or [2] − [1].

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8G

Example 22 Forming a matching pair

Determine the point of intersection of the lines $x + y = 6$ and $3x + 2y = 14$, using the elimination method.

Solution	Explanation
$x + y = 6$ [1] $3x + 2y = 14$ [2]	Label the two equations and decide where to form a matching pair.
$[1] \times 2: \quad 2x + 2y = 12$ [3] $[2]: \quad 3x + 2y = 14$ [2]	
$[2] - [3]:$ $x = 2$	Subtract the two equations because $2y - 2y = 0$, $3x - 2x = x$ and $14 - 12 = 2$
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$2 + y = 6$	Replace x with the number 2.
$y = 4$	Subtract 2 from both sides.
Point of intersection is $(2, 4)$.	Write the solution as an ordered pair.
Check: $3(2) + 2(4) = 14$	Check that the solution satisfies the other equation.

Now you try

Determine the point of intersection of the lines $x + y = 4$ and $5x + 2y = 11$, using the elimination method.

8 Solve these simultaneous equations by first forming a matching pair.

a $x - 3y = 1$
$2x + y = 9$

b $4x + 2y = 10$
$x + 3y = 10$

c $3x + 4y = 19$
$x - 3y = 2$

Hint: Multiply one equation by a number to form a matching pair.

Example 23 Forming a matching pair by multiplying both equations

Solve the simultaneous equations $3x + 2y = 6$ and $5x + 3y = 11$, using the elimination method.

Solution	Explanation
$3x + 2y = 6$ [1] $5x + 3y = 11$ [2]	Label the two equations and decide whether to eliminate x or y.
$[1] \times 3: \quad 9x + 6y = 18$ [3] $[2] \times 2: \quad 10x + 6y = 22$ [4]	Multiplying the first equation by 3 and the second by 2 results in a matching pair $6y$ in each equation.
$[4] - [3]: \quad x = 4$	Subtract the equations since $6y - 6y = 0$.

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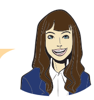
Substitute $x = 4$ into [1]:	Alternatively, substitute into equation [2].
$3(4) + 2y = 6$	Replace x with the number 4.
$2y = -6$	Subtract 12 from both sides, since $3 \times 4 = 12$.
$y = -3$	Divide both sides by 2.
Solution is $(4, -3)$.	Write the solution as an ordered pair.
Check: $5(4) + 3(-3) = 11$	Check the solution with the other equation.

Now you try

Solve the simultaneous equations $5x + 3y = 1$ and $2x + 5y = 8$, using the elimination method.

9 Solve the following pairs of simultaneous equations, using the elimination method.

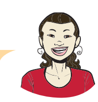
- **a** $3x + 2y = 6$ and $5x + 3y = 11$
- **b** $3x + 2y = 5$ and $2x + 3y = 5$
- **c** $2x + y = 4$ and $5x + 2y = 10$
- **d** $2x + 5y = 7$ and $x + 3y = 4$



Hint: When multiplying an equation by a number, multiply every term on the LHS and RHS by that number. Always write the instruction for multiplying; e.g. $[2] \times 4$.

10 Solve the following pairs of simultaneous equations, using the elimination method.

- **a** $3x + 5y = 8$
  $x - 2y = -1$
- **b** $2x + y = 10$
  $3x - 2y = 8$
- **c** $4x - 3y = 0$
  $3x + 4y = 25$



Hint: Choose to eliminate x or y. The coefficients need to be the same size (with + or −); e.g. $-4x$ and $4x$ or $-5y$ and $5y$. Choose to add or subtract the equations to eliminate one variable.

11 Solve the following pairs of simultaneous equations.

- **a** $5x + 3y = 18$ and $3y - x = 0$
- **b** $3x - y = 13$ and $x + y = -9$
- **c** $2x + 7y = -25$ and $5x + 7y = -31$
- **d** $2x + 6y = 6$ and $3x - 2y = -2$
- **e** $4x - 5y = -14$ and $7x + y = -5$
- **f** $7x - 3y = 41$ and $3x - y = 17$

Problem-solving and reasoning — 12–15 — 12, 13, 16–18

Example 24 Solving word problems with simultaneous equations (elimination)

Kathy is older than Blake. The sum of their ages is 17 years and the difference is 5 years. Find Kathy and Blake's ages.

Solution	Explanation
Let k be Kathy's age and b be Blake's age.	Define two variables.
$k + b = 17$ [1]	The first piece of information is 'the sum of their ages is 17'.
$k - b = 5$ [2]	The second is 'the difference is 5 and Kathy is older than Blake'.

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Solution	Explanation
[1] + [2]: $2k = 22$ $k = 11$	Add the two equations to eliminate b or, alternatively, subtract to eliminate k.
Substitute $k = 11$ into [1]: $11 + b = 17$	Alternatively, substitute into [2].
$b = 6$	Subtract 11 from both sides.
Kathy is 11 years old and Blake is 6.	Answer the original question in words.

Now you try

The sum of two numbers is 97 and their difference is 13. Find the two numbers.

12 Ayden is older than Tamara. The sum of their ages is 56 years and the difference is 16 years. Use simultaneous equations to find Ayden and Tamara's ages.

Example 25 Problem solving with simultaneous equations

Reese purchases three daffodils and five petunias from the local nursery and the cost is $25.
Giuliana buys four daffodils and three petunias and the cost is $26.
Determine the cost of each type of flower.

Solution	Explanation
Let $\$d$ be the cost of a daffodil and $\$p$ be the cost of a petunia.	Define your variables.
$3d + 5p = 25$ [1]	Three daffodils and five petunias from the local nursery cost $25.
$4d + 3p = 26$ [2]	Four daffodils and three petunias cost $26.
[1] × 4: $12d + 20p = 100$ [3] [2] × 3: $12d + 9p = 78$ [4]	Multiply [1] by 4 and [2] by 3 to obtain a matching pair ($12d$ and $12d$).
[3] − [4]: $11p = 22$	Subtract the equations to eliminate d.
$p = 2$	Divide both sides by 2.
Substitute $p = 2$ into [1]:	Alternatively, substitute into [2].
$3d + 5(2) = 25$	Replace p with the number 2.
$3d + 10 = 25$	Simplify.
$3d = 15$	Subtract 10 from both sides.
$d = 5$	Divide both sides by 3.
Check: $4(5) + 3(2) = 26$	Check your solutions by substituting into the second equation.
Daffodils cost $5 and petunias cost $2 each.	Answer the question in words.

Now you try

Jess purchases 4 buckets of chips and 3 drinks for $23.50, while Nigel purchases 3 buckets of chips and 4 drinks for $22. Find the price of a bucket of chips and a drink.

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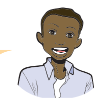
13 A market stall sells two fruit packs:
Pack 1: 10 apples and 5 mangoes ($12.50)
Pack 2: 15 apples and 4 mangoes ($13.50)

a Define two pronumerals and set up a pair of linear equations to eventually find the cost of each fruit.

b Solve the two simultaneous equations to determine the individual prices of each piece of fruit.

c Determine the cost of one apple and five mangoes.

14 Tickets to a basketball game cost $3 for children and $7 for adults. If 5000 people attended the game and the total takings at the door was $25 000, determine the number of children and adults who attended the game.



Hint: What you are being asked to find is often how you define your variables.

15 A Maths test contains multiple-choice questions worth 2 marks each and short-answer questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.

Hint: Total marks is 50. Number of questions is 22.

a Define two pronumerals to represent the number of each question type.

b Set up two linear equations.

c Solve the two equations simultaneously to determine the number of multiple-choice questions.

16 Let x and y be two numbers that satisfy the following statements. Set up two linear equations according to the information and solve them simultaneously to determine the numbers in each case.

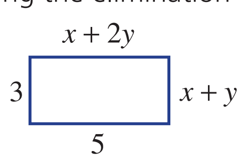
a Their sum is 16 but their difference is 2.

b Their sum is 30 but their difference is 10.

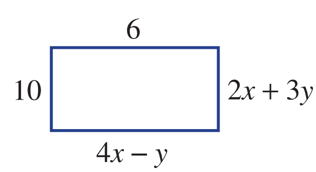
c Twice the larger number plus the smaller is 12 and their sum is 7.

17 Find the value of x and y in the following rectangles. You will need to write two equations and solve using the elimination method.

a



b



Hint: Opposite sides of rectangles are equal.

18 Gordon is currently 31 years older than his daughter. In 30 years' time he will be twice his daughter's age. Using g for Gordon's current age and d for Gordon's daughter's current age, complete the following.

a Write down expressions for:

i Gordon's age in 30 years' time

ii Gordon's daughter's age in 30 years' time

b Write down two linear equations, using the information at the start.

c Solve the equations to find the current ages of Gordon and his daughter.

Using technology — 19

19 Use technology to solve these simultaneous equations.

a $3x + 2y = 6$ and $5x + 3y = 11$

b $3x + 2y = 5$ and $2x + 3y = 5$

c $4x - 3y = 0$ and $3x + 4y = 25$

d $2x + 3y = 10$ and $3x - 4y = -2$

e $-2y - 4x = 0$ and $3y + 2x = -2$

f $-7x + 3y = 22$ and $3x - 6y = -11$

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Maths@Work: Nurse

Nursing is a career that is both challenging and rewarding. It requires a person to be caring and empathetic. Good communication skills, as well as an understanding of mathematics and science, are also important.

Nurses need to be competent in many mathematical areas, including fractions, ratios, converting units of measurement and equations. They must be able to calculate medical dosages, substitute into equations and also know how to program the correct flow rate for intravenous (IV) drips.

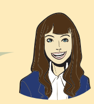
Complete these questions that are typical of a nurse's job administering medication.

1 Use these formulas to answer each of the following questions.

$$\text{Volume required} = \frac{\text{strength needed}}{\text{strength in stock}} \times \text{volume of stock solution}$$

$$\text{Number of tablets} = \frac{\text{strength required}}{\text{strength per tablet in stock}}$$

Hint: For 100 mg in 2 mL, need 75 mg in ? mL.



a What volume, in mL, of Pethidine should be given if the patient is prescribed 75 mg and the existing stock contains 100 mg in 2 mL?

b Calculate the volume, in mL, of insulin that is required for a patient who has been prescribed 60 units of the drug, if the stock is 100 units/1 mL.

c Pethidine 50 mg has been ordered to alleviate a patient's pain. The stock strength is 75 mg/1.5 mL. How much Pethidine should be given?

d How many tablets does a nurse need to give for a prescription of 500 mg of amoxicillin per day, if the stock available in the ward is 250 mg per capsule?

e How many tablets are needed for a dosage of 125 mg, if the stock available is labelled 25 mg per tablet?

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2 Paediatrics is a branch of medicine dealing with young children. Different formulas are used to calculate the doses suitable for children. Apply the rules given below to complete the following calculations and state each answer to the nearest mg.

Clarke's body weight rule:

$$\text{Child's dose} = \frac{\text{weight of child (kg)}}{\text{average adult weight (70 kg)}} \times \text{adult dose}$$

Clarke's body surface area rule:

$$\text{Child's dose} = \frac{\text{surface area of child (m}^2\text{)}}{\text{average adult surface area (1.7 m}^2\text{)}} \times \text{adult dose}$$

Fried's rule (used for infants under 1 year old):

$$\text{Child's dose} = \frac{\text{age in months}}{150} \times \text{adult dose}$$

Young's rule (used for children aged 2 to 12 years):

$$\text{Child's dose} = \frac{\text{age in years}}{\text{age} + 12} \times \text{adult dose}$$

a Use Young's rule to calculate the Amoxil dose needed for a 10-year-old boy, if the adult dose of the drug Amoxil is 250 mg.

b Use Clarke's body weight rule to find a child's dose for the drug Ampicillin, given the child's weight is 15 kg and an adult's dose is 500 mg.

c Use Fried's rule to calculate the dose required for an 8-month-old baby girl for the drug amoxicillan, given that an adult's dose is 500 mg.

d Use Clarke's body surface area formula to find the dose of penicillin, in mg, required for a child whose surface area is 8000 cm^2, given that the adult dose is 1 gram.

Hint: Recall 1 m^2 = 10 000 cm^2

3 Drugs that are given with an intravenous (IV) drip use a different set of equations to calculate the time needed or the drop rate per minute.

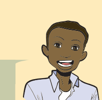
$$\text{Time (in minutes)} = \frac{\text{volume (mL)}}{\text{flow rate (drops/min)}} \times \text{drip factor}$$

$$\text{Flow rate (drops/minute)} = \frac{\text{volume (mL)}}{\text{time (mins)}} \times \text{drip factor}$$

Use the equations above to answer these questions about IV drug dosage. The drip factor is in drops/mL. State all answers rounded to one decimal place.

a Find the flow rate, in drops per minute, when a $\frac{1}{2}$ litre bag of saline solution is run over 4 hours with the IV machine set at a drip factor of 20 drops per mL.

Hint: Recall 1 L = 1000 mL



b An IV drip of saline solution started at 4:15 p.m. Tuesday. The machine has 700 mL to run and is set at 40 drops/min with a drip factor of 18 drops per mL. At what time will the IV be finished?

c How long will it take 180 mL of zero negative blood to flow through an IV at 36 drops/minute when the blood supply machine is set at a drip factor of 15 drops/mL?

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Using technology

4 Set up this Excel worksheet to calculate the ending times of intravenous drips for various patients. You will need to copy the given data and enter formulas into the shaded cells.

	A	B	C	D	E	F	G
1	Intravenous drip administration						
2	IV bag	IV machine settings		Times			
3	Total volume in mL	Flow rate in drops per minute	Drip factor drops per mL	Starting time	Time in minutes for IV	Time in hours and minutes for IV	Ending time
4	700	40	18	4:15 PM			
5	500	36	16	10:00 AM			
6	800	30	24	1:25 PM			
7	1500	40	20	6:00 PM			
8	2000	45	22	2:00 AM			

Hint:
- Format starting and ending times as *Number Category: Custom, Type: h:mm AM/PM.*
- To calculate time in hours and minutes, divide the time in minutes by the number of minutes in 24 hours, and format cells as *Number Category: Custom and Type: h:mm.*

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1 The answers to these equations will form a magic square, where each row, column and diagonal will add to the same number. Draw a 4 by 4 square for your answers and check that they do make a magic square.

$x - 3 = 6$	$x + 15 = 10$	$\frac{x}{2} = -2$	$5x = 30$
$3x + 7 = 1$	$\frac{x}{4} - 8 = -7$	$\frac{x + 7}{2} = 5$	$3(x + 4) = x + 14$
$\frac{x}{2} - 5 = -4$	$4x - 9 = -9$	$x + 7 = 4x + 10$	$2(3x - 12) - 5 = 1$
$\frac{9 - 3x}{3} = 6$	$-2(3 - x) = x + 1$	$x - 16 = -x$	$5x + 30 - 3x = -3x$

2 Write an equation and solve it to help you find each unknown number in these puzzles.

a Three-quarters of a number plus 16 is equal to 64.

b A number is increased by 6, then that answer is doubled and the result is four more than triple the number.

c The average of a number and its triple is equal to 58.6.

d In 4 years' time, Ahmed's age will be double the age he was 7 years ago. How old is Ahmed now?

3 By applying at least two operations to x, write three different equations so that each equation has the solution $x = -2$. Verify that $x = -2$ makes each equation true.
For example, $3 \times (-2) + 10 = 4$, so one possible equation would be $3x + 10 = 4$.

4 Write two sets of simultaneous equations so that each pair has the solution $(3, -2)$.

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Puzzles and games

5 Which Australian city has its centre on the intersection of the Warrego Highway and the New England Highway?

To decode this puzzle, solve the inequalities and simultaneous equations below, and match them to a number line or graph. Place the corresponding letters above the matching numbers to find the answer.

___ ___ ___ ___ ___ ___ ___ ___ ___
6 4 4 3 4 4 5 1 2

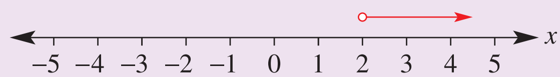
Solve these inequalities and match the solution to a number line (**1–3**).

W $2 - 3x > 8$

A $3x + 10 \geq 1$

B $x + 5 > 7$

1



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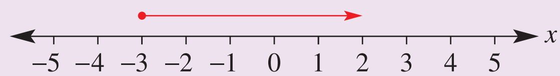

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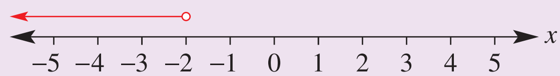


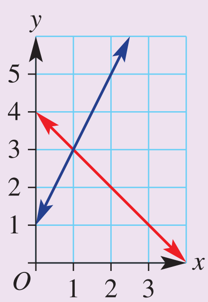
Solve these simultaneous equations and match the solution to a graph (**4–6**).

M $3x - y = 7$
$2x + y = 3$

O $y = 2x + 1$
$x + y = 4$

T $2x - y = -1$
$x - 2y = 4$

4



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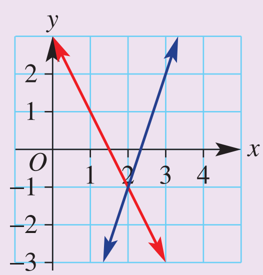

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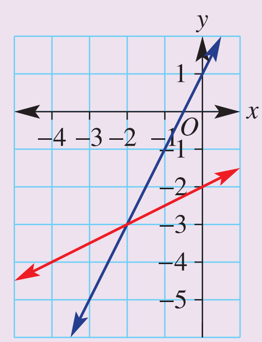

6 Jules and Enzo are participating in a long-distance bike race. Jules rides at 18 km/h and has a 2 hour head start. Enzo travels at 26 km/h.
a How long does it take for Enzo to catch up to Jules? (Use: Distance = speed × time.)
b How far did they both ride before Enzo caught up to Jules?

7 Emily travelled a distance of 138 km by jogging for 2 hours and cycling for 5 hours. She could have travelled the same distance by jogging for 4 hours and cycling for 4 hours. Find the speed at which she was jogging and the speed at which she was cycling.

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Equations

Solving linear equations that have brackets ⭐

- Expand all brackets.
- Collect like terms on each side of the equation.
- Collect terms with a pronumeral to one side (usually the LHS).
- Solve for unknown.

e.g.

$12(x + 1) - 2(3x - 3) = 4(x + 10)$

$12x + 12 - 6x + 6 = 4x + 40$

$6x + 18 = 4x + 40$

$2x + 18 = 40$

$2x = 22$

$x = 11$

Solving linear equations

Solving involves finding the value that makes an equation true.

e.g. $2x + 5 = 9$

$2x = 4$ (subtract 5)

$x = 2$ (divide by 2)

Equations with fractions ⭐

e.g.

$\frac{3x}{4} - 2 = 7$

$\frac{3x}{4} = 9$ (first $+\ 2$ to both sides)

$3x = 36$ ($\times$ 4 both sides)

$x = 12$ ($\div$ 3 both sides)

e.g.

$\frac{2x - 5}{3} = 7$

$2x - 5 = 21$ (first $\times 3$ to both sides)

$2x = 26$ (+5 to both sides)

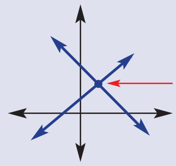
$x = 13$ ($\div$2 to both sides)

Solving word problems

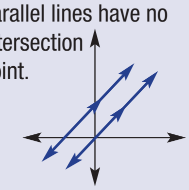
1 Define variable(s).
2 Set up equation(s).
3 Solve equation(s).
4 Check each answer and write in words.

Graphical solutions of simultaneous equations

Graph each line and read off point of intersection.



Parallel lines have no intersection point.



Formulas

Some common formulas

e.g. $A = \pi r^2$, $C = 2\pi r$

An unknown value can be found by substituting values for the other variables.

A formula can be rearranged to make a different variable the subject; i.e. the variable is out the front on its own.

e.g. $E = mc^2$, find m when $E = 320$ and $c = 4$.

$320 = m \times 4^2$ (substitute values)

$320 = 16m$

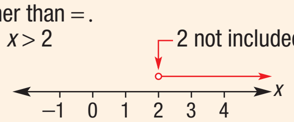
$20 = m$ (divide both sides by 16)

$m = 20$ (Write the answer with m on the left.)

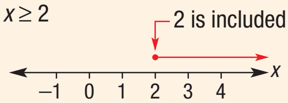
Inequalities

These are represented using $>, <, \geq, \leq$ rather than $=$.

e.g. $x > 2$



e.g. $x \geq 2$



Solving inequalities uses the same steps as solving equations, except when multiplying or dividing by a negative number. In this case, the inequality sign must be reversed.

e.g. $4 - 2x > 10$ (-4)

$-2x > 6$ ($\div -2$)

$x < -3$ (reverse sign)

Simultaneous equations ⭐

Use substitution or elimination to find the solution that satisfies two equations.

Substitution

e.g. $2x + y = 12$ [1]

$y = x + 3$ [2]

In [1] replace y with [2]:

$2x + (x + 3) = 12$

$3x + 3 = 12$

$3x = 9$

$x = 3$

Sub. $x = 3$ to find y.

In [2] $y = 3 + 3 = 6$

Solution is (3, 6).

Elimination

Ensure both equations have a matching pair.

Add two equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ [1]

$2x + 3y = 5$ [2]

[1] $\times$ 2: $2x + 4y = 4$ [3]

[3] – [2]: $y = -1$

In [1]: $x + 2(-1) = 2$

$x - 2 = 2$

$x = 4$

Solution is (4, –1).

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Chapter checklist

Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

8A **1 I can solve simple linear equations.**
e.g. Solve these linear equations.
a $2x - 1 = 7$ **b** $\frac{x+4}{2} = 12$

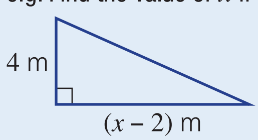
8A **2 I can set up and solve worded problems using linear equations.**
e.g. 5 less than twice a number is equal to 31. Find the number.

8B **3 I can solve equations with brackets.**
e.g. Solve $3(x - 2) = 21$.

8B **4 I can solve equations with variables on both sides.**
e.g. Solve $5x - 2 = 3x + 6$.

8B **5 I can solve equations with fractions.**
e.g. Solve $\frac{2x-1}{4} = -1$.

8B **6 I can set up and solve an equation with brackets from a real situation.**
e.g. Find the value of x if the area of this triangle is 32 m^2.



8C **7 I can substitute into a formula and solve for a variable.**
e.g. If $A = \frac{h}{2}(a + b)$ and $A = 20, h = 4$ and $b = 3$ find the value of a.

8D **8 I can illustrate an inequality on a number line.**
e.g. Represent $x > -2$ on a number line.

8D **9 I can solve a simple inequality.**
e.g. Solve $3x - 2 > 7$.

8D **10 I can solve an inequality when the pronumeral has a negative coefficient.**
e.g. Solve $6 - 3x \geq 18$.

8E **11 I can find an intersection point of two linear relations which represents the solution to a pair of simultaneous equations.**
e.g. Find the point of intersection (x, y) of these equations by plotting a graph.
$y = 4x - 9$ and $2x + y = 3$

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			✔
8F	12	**I can use the method of substitution to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $y = x - 2$ and $2x + 3y = -1$ using the substitution method; i.e. find the point of intersection.	
8F	13	**I can use the method of substitution to find a solution to a real problem involving simultaneous equations.** e.g. Jonah is 9 years older then Penny and their combined ages is 47. Find their ages.	
8G	14	**I can use the method of elimination to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $x + y = 1$ and $4x + 3y = 5$ using the elimination method.	
8G	15	**I can use the method of elimination to find a solution to a real problem involving simultaneous equations.** e.g. Jill buys 5 pens and 2 pencils from her favourite store for $\$13$, while Michael buys 4 pens and 3 pencils from the same store for $\$12.50$. Find the cost of a pen and a pencil from this store.	

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Short-answer questions

8A **1** Solve the following.

a $4a = 32$ **b** $\frac{m}{5} = -6$ **c** $x + 9 = 1$

d $x + x = 16$ **e** $9m = 0$ **f** $w - 6 = 9$

g $8m = -1.6$ **h** $\frac{w}{4} = 1$ **i** $r - 3 = 3$

8A **2** Find the solution to the following.

a $2m + 7 = 11$ **b** $3w - 6 = 18$ **c** $\frac{m}{2} + 1 = 6$

d $\frac{5w}{4} - 3 = 7$ **e** $\frac{m-6}{2} = 4$ **f** $\frac{3m+2}{6} = 1$

g $6a - 9 = 0$ **h** $4 - x = 3$ **i** $9 = x + 6$

8B **3** Solve the following by first expanding the brackets.

a $3(m + 1) = 12$ **b** $4(a - 3) = 16$ **c** $5(2 + x) = 30$

d $4(2x + 1) = 16$ **e** $2(3m - 3) = 9$ **f** $2(1 + 4x) = 9$

g $2(2x + 3) + 3(5x - 1) = 41$ **h** $3(2x + 4) - 4(x - 7) = 56$

8B **4** Find the value of p in the following.

a $7p = 5p + 8$ **b** $2p = 12 - p$ **c** $6p + 9 = 5p$

d $2p + 10 = p + 8$ **e** $3p + 1 = p - 9$ **f** $4p - 8 = p - 2$

8A **5** Write an equation for the following and then solve it.

a Six times a number equals 420. What is the number?

b Eight more than a number equals 5. What is the number?

c A number divided by 9 gives 12. What is the number?

d Seven more than a number gives 3. What is the number?

e The sum of a number and 2.3 equals 7. What is the number?

8C **6** **a** For $A = \frac{1}{2}hb$, find b when $A = 24$ and $h = 6$.

b For $V = lwh$, find w when $V = 84$, $l = 6$ and $h = 4$.

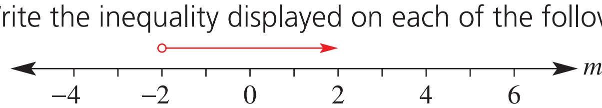
c For $A = \frac{x+y}{2}$, find x when $A = 3.2$ and $y = 4$.

d For $E = mc^2$, find m when $E = 40$ and $c = 2$.

e For $F = \frac{9}{5}C + 32$, find C when $F = 95$.

8D **7** Write the inequality displayed on each of the following number lines.

a



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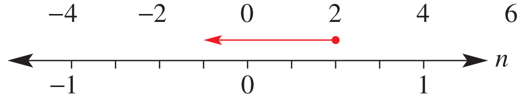

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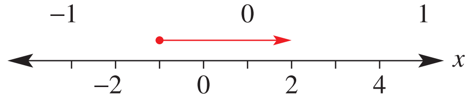

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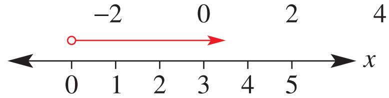
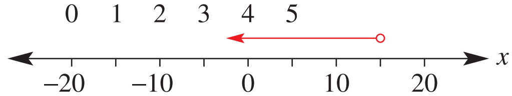


e



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8D **8** Solve the following.

a $x + 8 \geq -10$ **b** $2m < 7$ **c** $2x + 6 > 10$

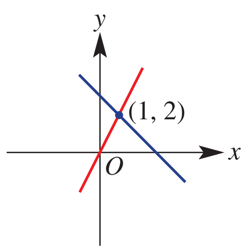
d $x - 3 < 0$ **e** $\frac{x}{4} + 1 \leq 3$ **f** $m - 6 \geq 4$

8D **9** Solve the following.

a $-6x \leq 12$ **b** $8 - x \leq 10$ **c** $-x > 0$

8E **10** Determine the point of intersection of the following lines.

a



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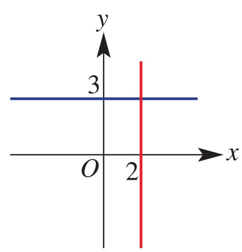

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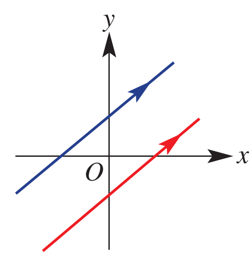


8E/F **11** Find the point of intersection (x, y) of the following by plotting an accurate graph.

a $y = 2x + 4$, $3x + y = 9$ **b** $y = 2$, $x = 3$ **c** $y = 3x$, $y = -3x$

8F **12** Solve the simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 5x - 13$, $2x + 3y = 12$ **b** $y = -1$, $y = 2x - 11$

8G **13** Determine the point of intersection of the following lines, using the elimination method.

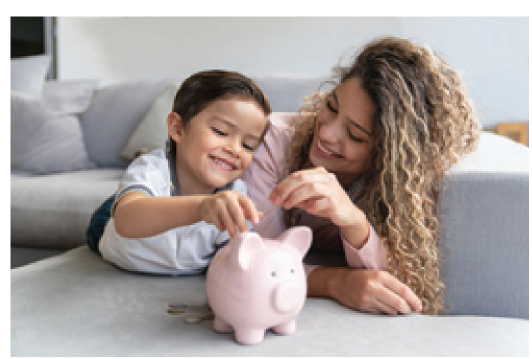
a $2x + 7y = -25$, $5x + 7y = -31$ **b** $3x + 2y = 8$, $x - 2y = 0$

8G **14** The sum of two numbers is 15 and their difference is 7. Use simultaneous equations to find the two numbers.

8G **15** A money box contains 20 cent and 50 cent coins. The amount in the money box is \$50 and there are 160 coins.

a Define two variables and set up a pair of linear equations.

b Solve the two simultaneous equations to determine the number of 20 cent and 50 cent coins.



8F **16** There are twice as many adults as children at a local grand final football match. It costs \$10 for adults and \$2 for children to attend the match. If the football club collected \$1100 at the entrance gates, how many children went to see the match?

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Multiple-choice questions

8A **1** The solution to $x + 7 = 9$ is:
A $x = 16$ **B** $x = -2$ **C** $x = 2$ **D** $x = 1$ **E** $x = -16$

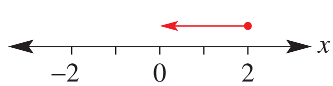
8B **2** To solve the equation $3(2x + 4) - 4(x + 2) = 6$, you would first:
A divide both sides by 12
B expand the brackets
C subtract 6 from both sides
D multiply both sides by 6
E add $4(x + 2)$ to both sides

8A **3** A number is increased by 6 and then doubled. The result is 36. This translates to:
A $6x + 2 = 36$ **B** $2x + 6 = 36$ **C** $2(x + 6) = 36$
D $2(x - 6) = 36$ **E** $x + 12 = 36$

8B **4** If $4a - 6 = 2a$, then a equals:
A -1 **B** 1 **C** 6 **D** 3 **E** -3

8D **5** $x \le 4$ is a solution to:
A $x + 1 < 3$ **B** $3x - 1 \le 11$ **C** $\frac{x}{2} - 1 \ge 0$
D $x - 1 \ge 1$ **E** $-x \le -4$

8D **6** Which number line shows $x + 4 < 6$?

A


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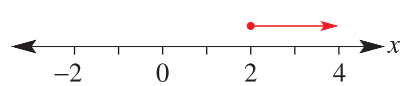

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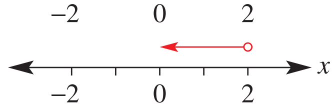

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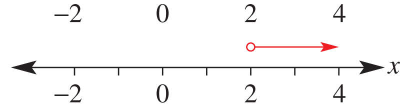

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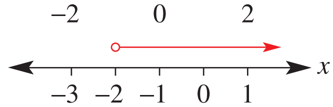

8B **7** The solution to $\frac{5x}{9} - 4 = 1$ is:
A $x = 6$ **B** $x = -9$ **C** $x = -5$ **D** $x = 9$ **E** $x = 5$

8E **8** If two lines are not parallel, the number of intersection points they will have is:
A 0 **B** 1 **C** 2 **D** 3 **E** 4

8E **9** The intersection point for the graphs of $y = 2$ and $x = 3$ is:
A $(-1, 2)$ **B** $(2, 2)$ **C** $(3, 2)$ **D** $(3, 3)$ **E** $(2, 3)$

8B **10** The solution to $3(x - 1) = 12$ is:
A $x = -1$ **B** $x = 2$ **C** $x = 0$ **D** $x = 5$ **E** $x = 4$

8F **11** $y = 3x$ and $x + y = 4$ has the solution:
A $(1, 3)$ **B** $(3, 1)$ **C** $(2, 6)$ **D** $(2, 2)$ **E** $(-1, 5)$

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8F

12 Substituting $y = x - 1$ into $x + 2y = 3$ gives:

A $x - 2x - 2 = 3$ **B** $x + 2y - 2 = 3$ **C** $x - x - 1 = 3$
D $x + 2x - 1 = 3$ **E** $x + 2(x - 1) = 3$

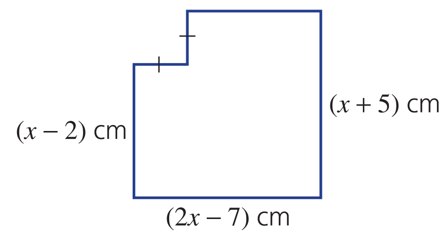
8G

13 Adding $x + y = 3$ to $x - y = 4$ gives:

A $2x - 2y = 7$ **B** $2x = 7$ **C** $x = 7$ **D** $y = 7$ **E** $2y = 7$

Extended-response questions

1 All the lines meet at 90° in this shape.



a Determine the equation of its perimeter, P.
b **i** If the perimeter is 128 cm, determine the value of x.
ii Find the actual side lengths.
c Repeat part **b** for perimeters of:
i 152 cm **ii** 224 cm

2 Two computer consultants have an up-front fee plus an hourly rate. Rhys charges \$50 plus \$70 per hour, whereas Agnes charges \$100 plus \$60 per hour.

a Using $\$C$ for the cost and t hours for the time, write a rule for the cost of hiring:
i Rhys **ii** Agnes
b If Agnes charges \$280, solve an equation to find how long she was hired for.
c By drawing a graph of C versus t for both Rhys and Agnes on the same set of axes, find the coordinates of the intersection point.
d Use the algebraic method of substitution to solve the simultaneous equations and confirm your answer to part **c**.

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