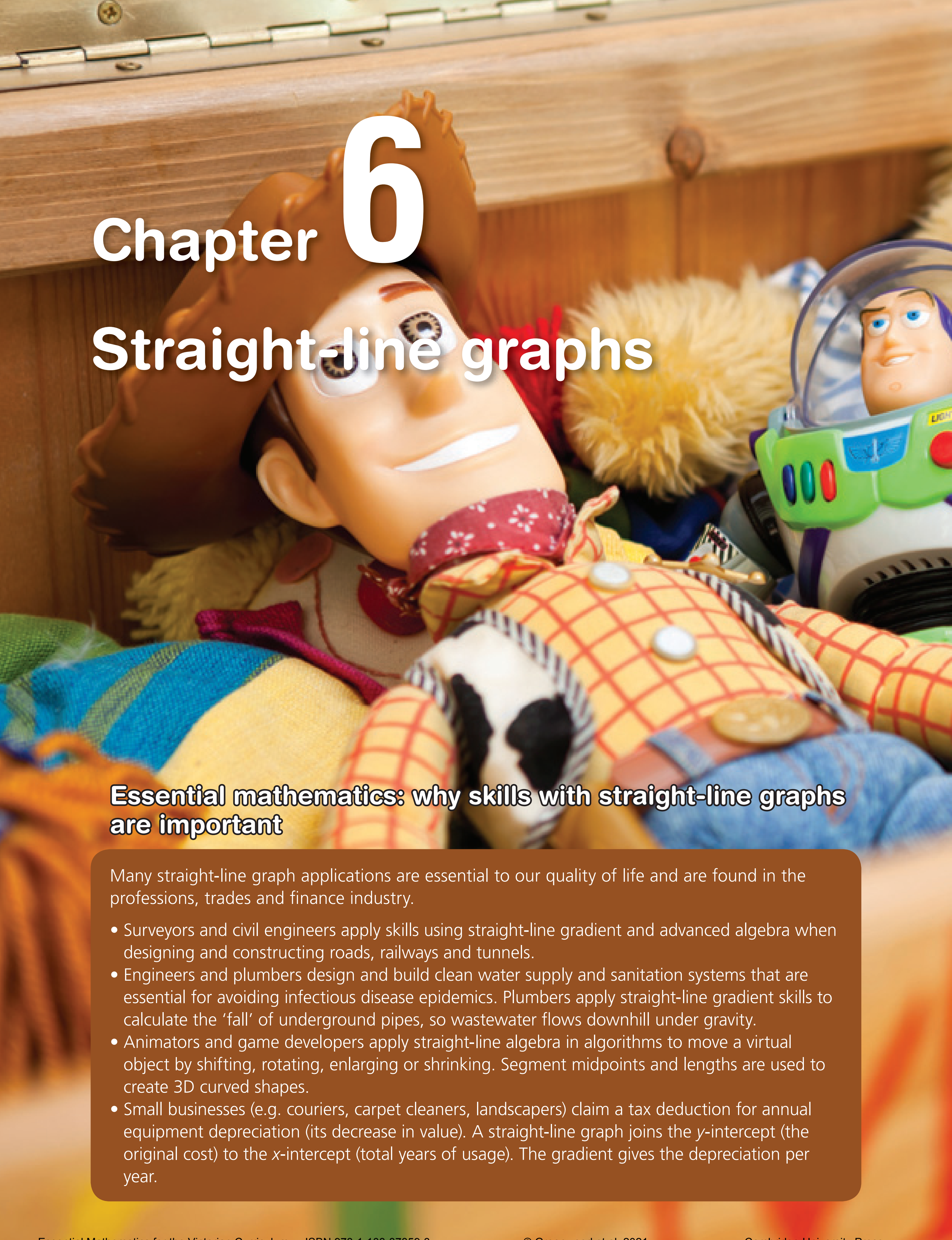

Chapter 6 Straight-line graphs

Essential mathematics: why skills with straight-line graphs are important

Many straight-line graph applications are essential to our quality of life and are found in the professions, trades and finance industry.

- Surveyors and civil engineers apply skills using straight-line gradient and advanced algebra when designing and constructing roads, railways and tunnels.
- Engineers and plumbers design and build clean water supply and sanitation systems that are essential for avoiding infectious disease epidemics. Plumbers apply straight-line gradient skills to calculate the 'fall' of underground pipes, so wastewater flows downhill under gravity.
- Animators and game developers apply straight-line algebra in algorithms to move a virtual object by shifting, rotating, enlarging or shrinking. Segment midpoints and lengths are used to create 3D curved shapes.
- Small businesses (e.g. couriers, carpet cleaners, landscapers) claim a tax deduction for annual equipment depreciation (its decrease in value). A straight-line graph joins the y-intercept (the original cost) to the x-intercept (total years of usage). The gradient gives the depreciation per year.

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In this chapter

Victorian Curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas (VCMNA335)

Solve problems involving parallel and perpendicular lines (VCMNA338)

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

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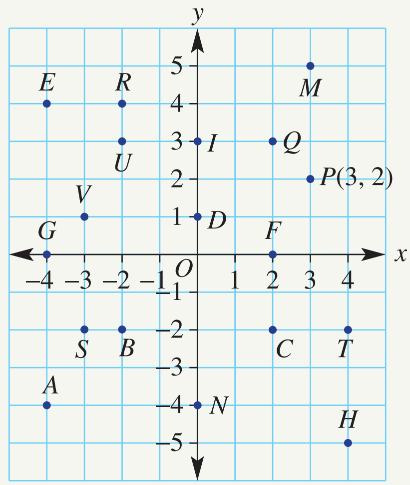
Warm-up quiz



1 The coordinates of P on this graph are $(3, 2)$.
Write down the coordinates of:

a M **b** T **c** A
d V **e** C **f** F

2 Name the point with coordinates:

A $(-4, 0)$ **B** $(0, 1)$ **C** $(-2, -2)$
D $(-3, -2)$ **E** $(0, -4)$ **F** $(2, 3)$

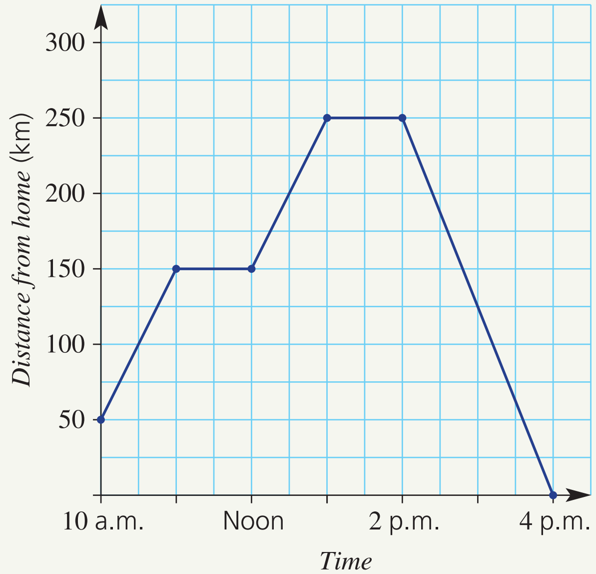
3 Draw up a four-quadrant number plane and plot the following points.
What shape do they form? (Choose from: *hexagon*, *rectangle*, *square*, *pentagon*, *equilateral triangle*, *isosceles triangle*.)

a $(0, 0), (0, 5), (5, 5), (5, 0)$
b $(-3, -1), (-3, 1), (4, 0)$
c $(-2, 3), (-4, 1), (-2, -3), (2, -3), (4, 1), (2, 3)$

4 Find the mean (i.e. average) of the following pairs.

a 10 and 12 **b** 15 and 23 **c** 6 and 14 **d** 3 and 4
e −6 and 6 **f** −3 and 1 **g** 0 and 7 **h** −8 and −10

5 **a** For how many minutes did the Heart family stop on their trip, according to their journey shown in this travel graph?



b How far had the Heart family travelled by 1 p.m., after starting at 10 a.m.?
c What was their speed in the first hour of travel?

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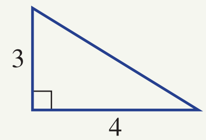
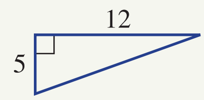
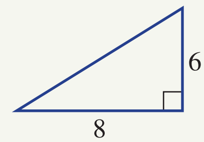
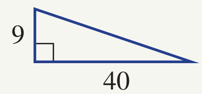
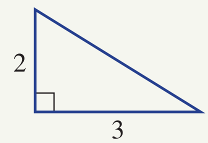
6 Find the length of the hypotenuse in each right-angled triangle. Use $a^2 + b^2 = c^2$. Round your answers to two decimal places in parts **e** and **f**.

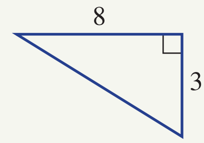
a


b


c


d


e


f


7 Copy and complete the table of values for each rule given.

a $y = x + 3$

x	0	1	2	3
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

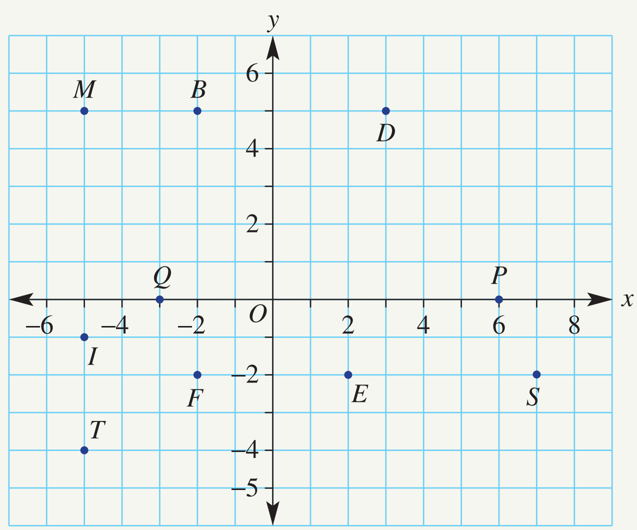
x	0	1	2
y			

d $y = 4 - x$

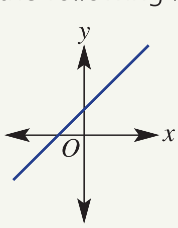
x	−2	−1	0
y			

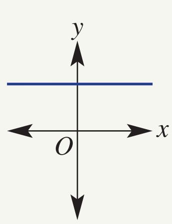
8 Use the graph to find the following distances.

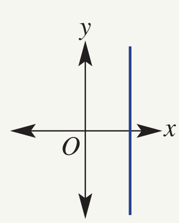
a OP
b QP
c MB
d FS
e BD
f TM

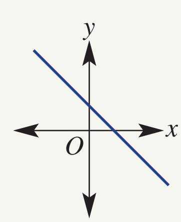


9 Select a word (i.e. *positive*, *negative*, *zero or undefined*) to describe the gradient (slope) of the following lines.

a


b


c


d


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6A Interpretation of straight-line graphs CONSOLIDATING

Learning intentions

- To understand that two variables with a linear relationship can be represented with a straight-line graph
- To be able to interpret information from a graph using the given variables
- To be able to read off values from a graph both within the graph and using an extended graph

Key vocabulary: variable, linear relationship, interpolation, extrapolation

When two variables are related, we can use mathematical rules to describe the relationship. The simplest kind of relationship forms a straight-line graph and the rule is called a linear equation.

Information can be easily read from within a linear graph – this is called interpolation. A straight line also can be extended to determine information outside of the original data – this is called extrapolation.

For example, if a swimming pool is filled at 1000 L per hour, the relationship between volume and time is linear because the volume is increasing at the constant rate of 1000 L/h.

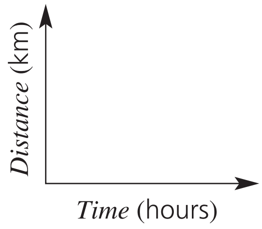
$$\text{Volume} = 1000 \times \text{number of hours}$$

This rule is a linear equation and the graph of volume versus time will be a straight line.

Lesson starter: Graphing a straight line

Jozef is an athlete who trains by running 24 km in 2 hours at a constant rate. Draw a straight-line graph to show this linear relation.

- Draw axes with time (up to 2 hours) on the horizontal axis and distance (up to 24 km) on the vertical axis.



- How far has Jozef run at zero hours? Mark this point on your graph.
- Mark the point on the graph that shows the end of Jozef's run.
- Join these two points with a straight line.
- Mark the point on the graph that shows Jozef's position after half an hour. How far had he run?
- Mark the point on the graph that shows Jozef's position after 18 km. For how long had Jozef been running?
- Name the variables shown on the graph.
- Discuss some advantages of showing information on a graph.

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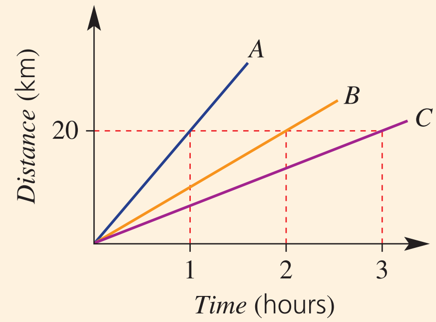
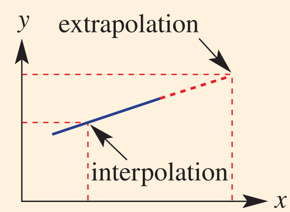
Key ideas

- A **variable** is an unknown that can take on many different values.
- When two variables have a **linear relationship** they can be represented as a straight-line graph.
  Information about one of the variables based on information about the other variable is easily determined by reading from the graph:
  - A: 20 km in 1 hour
  - B: 20 km in 2 hours
  - C: 20 km in 3 hours



- Information can be found from:
  - reading within a graph (**interpolation**) or
  - reading off an extended graph (**extrapolation**)

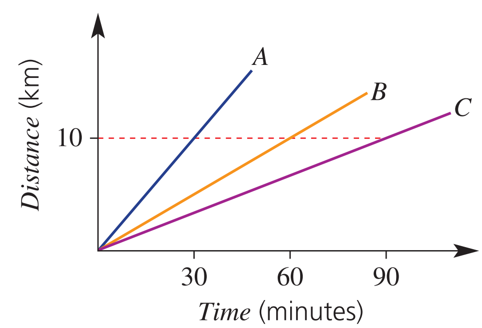


Exercise 6A

Understanding 1, 2 | 2

1 For the graph shown, determine how long it takes each cyclist to travel 10 km.

a cyclist A
b cyclist B
c cyclist C



2 Choose the correct word: *interpolation* or *extrapolation*, to complete the following.

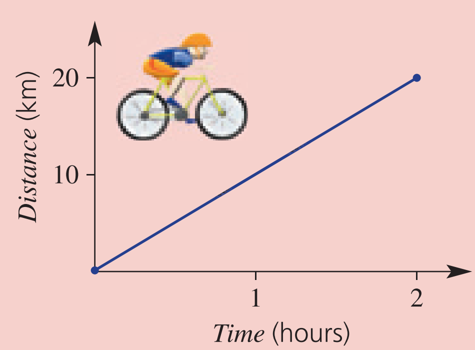
a ______________ is reading information from within a graph
b ______________ is reading information from an extended graph

Fluency 3–5 | 3–5

Example 1 Reading information from a graph

This graph shows the journey of a cyclist from one place (A) to another (B).

a How far did the cyclist travel?
b How long did it take the cyclist to complete the journey?
c If the cyclist were to ride from A to B and then halfway back to A, how far would the journey be?



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6A

Solution	**Explanation**
a 20 km	Draw an imaginary line from the end point (B) to the vertical axis; i.e. 20 km.
b 2 hours	Draw an imaginary line from the end point (B) to the horizontal axis; i.e. 2 hours.
c $20 + 10 = 30$ km	Cyclist would ride 20 km out and 10 km back.

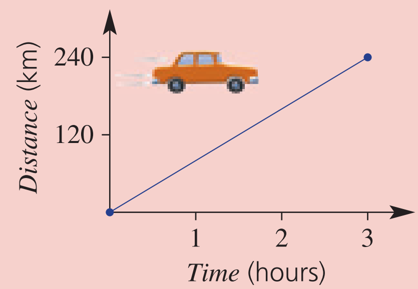
Now you try

This graph shows a car journey from one place (A) to another (B).

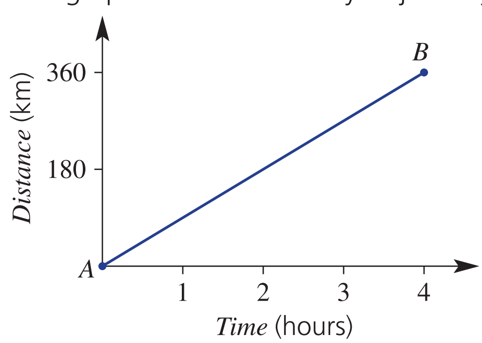
a How far did the car travel?

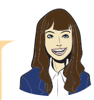
b How long did it take to complete the journey?

c If the car were to be driven from A to B, then halfway back to A, how far would the journey be?



3 This graph shows a motorcycle journey from one place (A) to another (B).



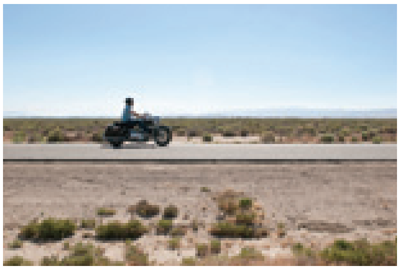


Hint: To find the total time taken to go from A to B, look on the time scale that is aligned with the right end point.

a How far did the motorcycle travel?

b How long did it take to complete the journey?

c If the motorcycle were to be driven from A to B, then halfway back to A, how far would the journey be?



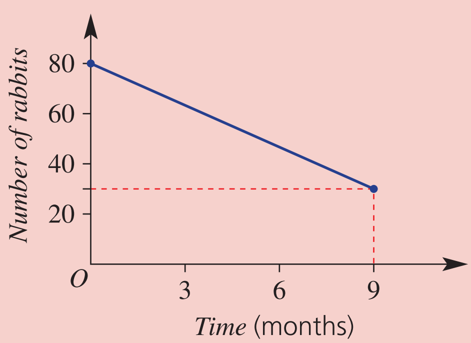
Example 2 Interpreting information from a graph

The number of rabbits in a colony has decreased according to this graph.

a How many rabbits were there in the colony to begin with?

b How many rabbits were there after 9 months?

c How many rabbits disappeared from the colony during the 9-month period?



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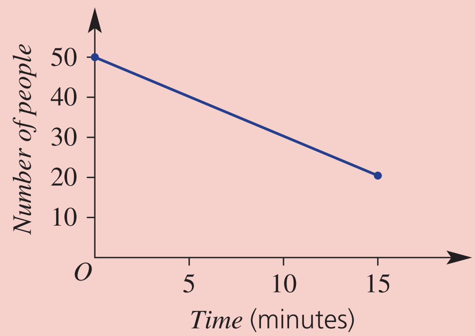
Solution	Explanation
a 80 rabbits	At $t = 0$ there were 80 rabbits.
b 30 rabbits	Read the number of rabbits from the graph at $t = 9$.
c $80 - 30 = 50$ rabbits	There were 80 rabbits at the start and 30 after 9 months.

Now you try

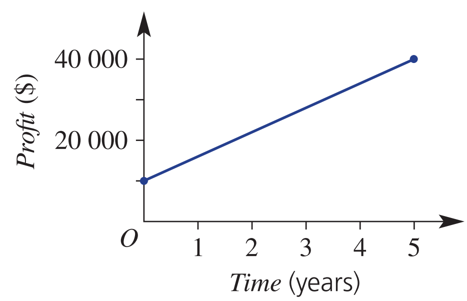
The number of people in a queue has decreased according to this graph.

a How many people were in the queue to begin with?
b How many people were in the queue after 15 minutes?
c How many people left the queue during the 15 minutes shown?



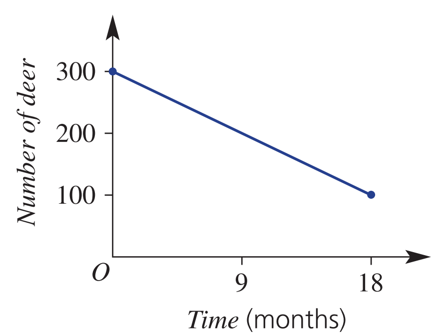
4 This graph shows a company's profit result over a 5-year period.

a What is the company's profit at:
  i the beginning of the 5-year period?
  ii the end of the 5-year period?
b Has the profit increased or decreased over the 5-year period?
c How much has the profit increased over the 5 years?

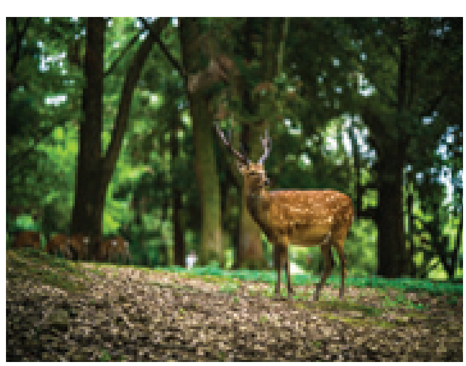


5 The number of deer in a particular forest has decreased over recent months according to the graph shown.

a How many deer were there to begin with?
b How many deer were there after 18 months?
c How many deer disappeared from the colony during the 18-month period?



Hint: 'To begin with' means time $= 0$.



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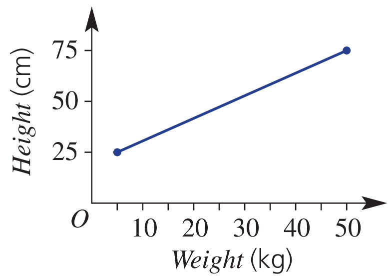
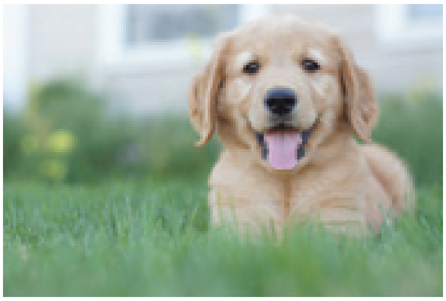
6A

Problem-solving and reasoning 6–9 7, 9, 10

6 A height versus weight graph for a golden retriever dog breed is shown.





a From the smallest to the largest dog, use the graph to find the total increase in:
 i height ii weight

b Fill in the missing numbers.
 i The largest weight is ___ times the smallest weight.
 ii The largest height is ___ times the smallest height.

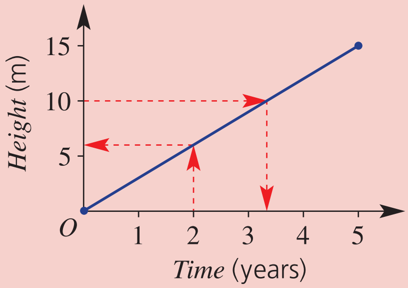
Example 3 Reading within a graph (interpolation)

This graph shows the growth of a tree over 5 years.

a How many metres has the tree grown over the 5 years?

b Use the graph to find how tall the tree is after 2 years.

c Use the graph to find how long it took for the tree to grow to 10 metres.



Solution	Explanation
a 15 metres	The end point of the graph is at 15 metres.
b 6 metres	Draw a dotted line at 2 years and read the height.
c 3.3 years	Draw a dotted line at 10 metres and read the time.

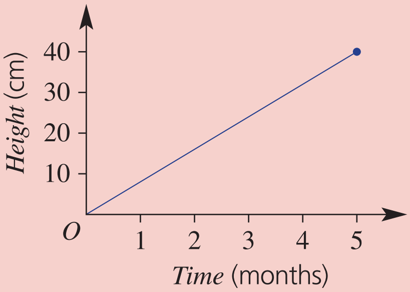
Now you try

This graph shows the growth of a seedling over 5 months.

a How many cm has the seed grown over the 5 months?

b Use the graph to find the height of the seedling after 2 months.

c Use the graph to find how long it took for the seedling to grow to 30 cm.



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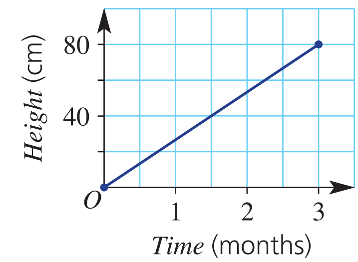
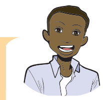
7 This graph shows the height of a tomato plant over 3 months.

a How many centimetres has the plant grown over 3 months?

b Use the graph to find how tall the tomato plant is after $1\frac{1}{2}$ months.

c Use the graph to find how long it took for the plant to grow to 60 centimetres.





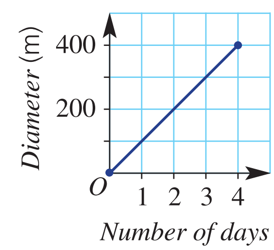
Hint: Start at 60 cm on the height axis, then go across to the straight line and down to the time axis. Read off the time.

8 The diameter of an oil slick increased every day after an oil tanker hit some rocks. Use the graph to answer the following.

a How wide is the oil slick after 4 days?

b How wide is the oil slick after 2.5 days?

c How many days does it take for the oil slick to reach a diameter of 350 m?



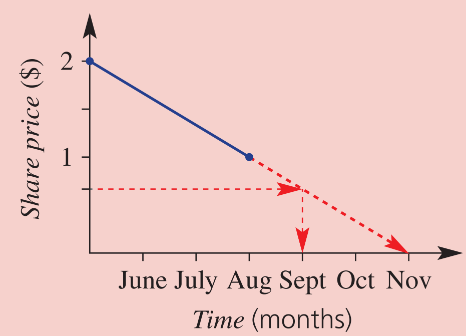
Example 4 Reading off an extended graph (extrapolation)

Due to poor performance, the value of a company's share price is falling.

a By the end of August, how much has the share price fallen?

b At the end of November what would you estimate the share price to be?

c Near the end of which month would you estimate the share price to be 70 cents?



Solution	Explanation
a Price has dropped by $1.	By August the price has changed from $2 to $1.
b $0	Use a ruler to extend your graph (shown by the dotted line) and read the share price for November.
c September	Move across from 70 cents to the extended line and read the month.

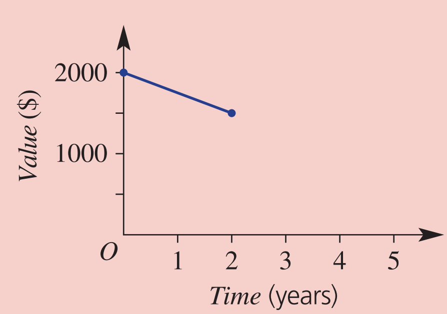
Now you try

The value of a computer is decreasing over time.

a By the end of 2 years, how much has the value fallen?

b At the end of 4 years, what would the estimated value be?

c Near the end of which year would you estimate the value to be $1200?



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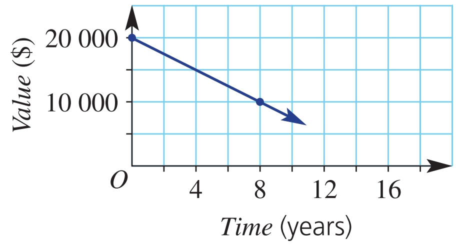
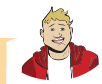
6A

9 The value of a car decreases with time.

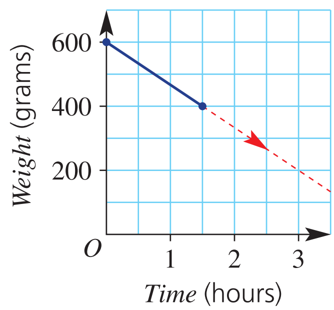




Hint: Use your ruler on the line to 'extend' it.

a By the end of 8 years, how much has the car's value fallen?
b At the end of 16 years, what would you estimate the car's value to be?
c Near the end of which year would you estimate the car's value to be \$5000?

10 The weight of a wet sponge is reduced after it is left in the sun to dry.

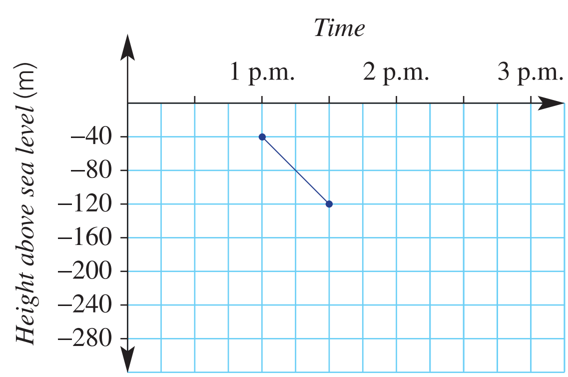


a The weight of the sponge has been reduced by how many grams over the first 1.5 hours?
b What would you estimate the weight of the sponge to be after 3 hours?
c How many hours would it take for the sponge to weigh 300 g?

Submarine depth | — | 11

11 A submarine goes to depths below sea level, as shown in this graph.

a How long does it take for the submarine to drop from 40 m to 120 m below sea level?
b At what time of day is the submarine at:
i −40 m? **ii** −80 m?
iii −60 m? **iv** −120 m?
c What is the submarine's depth at:
i 1:30 p.m.?
ii 1:15 p.m.?



d Extend the graph to find the submarine's depth at:
i 12:45 p.m. **ii** 1:45 p.m. **iii** 2:30 p.m.
e Use your extended graph to estimate the time when the submarine is at:
i 0 m **ii** −200 m **iii** −320 m

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6B Distance–time graphs

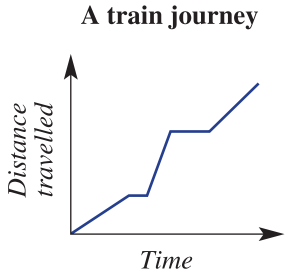
Learning intentions

- To understand that a distance–time graph can be made up of line segments joined together
- To be able to interpret the features of a distance–time graph
- To be able to sketch a distance–time graph with movement at a constant rate

Key vocabulary: line segment, stationary, horizontal axis, vertical axis

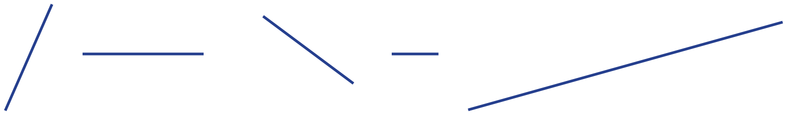
Some of the graphs considered in the previous section were distance–time graphs, which show the distance on the vertical axis and the time on the horizontal axis. Many important features of a journey can be displayed on such graphs. Each section of a journey that is at a constant rate of movement can be graphed with a straight-line segment. Several different line segments can make up a total journey.

For example, a train journey could be graphed with a series of sloping line segments that show travel between stations and flat line segments that show when the train is stopped at a station.



Lesson starter: An imaginary journey

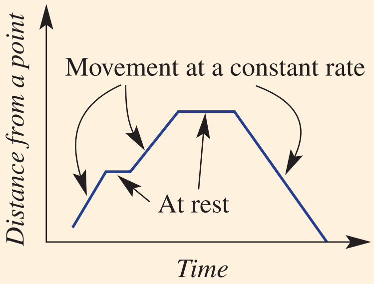
Here are five line segments.



- Use five similar line segments, arranged in any order you choose, and draw a distance–time graph. Each segment must be joined to the one next to it.
- Write a summary of the journey shown by your distance–time graph.
- Swap graphs with a classmate and explain the journey that you think your classmate's graph is showing.

Key ideas

- Graphs of *distance* versus *time* usually consist of **line segments**.
- Each segment shows whether the object is moving or at rest (**stationary**).
- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.



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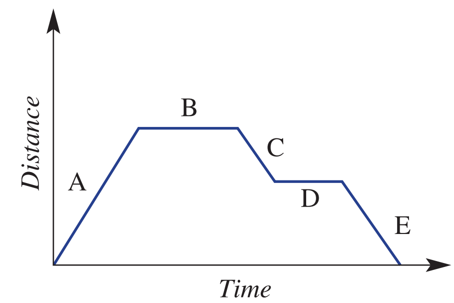
6B

Exercise 6B

Understanding | 1, 2 | 2

1 For the distance versus time graph shown, for each line segment A – E, describe the movement of the object as either moving at a constant rate or at rest.

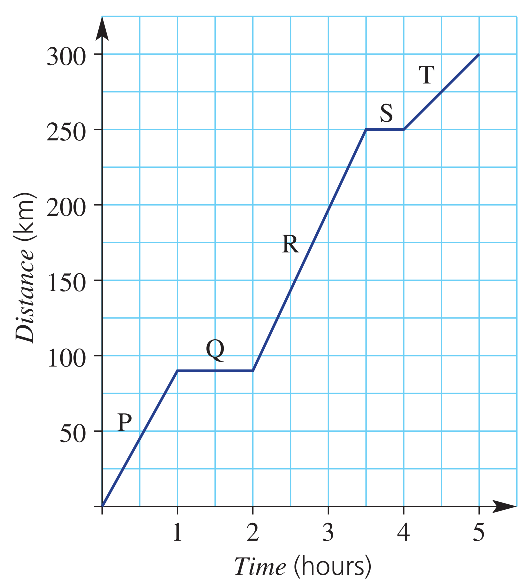


2 The Martino family makes a 300-km car journey, which takes 5 hours. The distance–time graph of this journey is shown. For each description below, choose the line segment of the graph that matches it.

Hint: A flat line segment shows that the car has stopped.

- **a** A half-hour rest break is taken after travelling 250 km.
- **b** In the first hour, the car travels 90 km.
- **c** The car is at rest for 1 hour, 90 km from the start.
- **d** The car takes 1.5 hours to travel from 90 km to 250 km.
- **e** The distance from 250 km to 350 km takes 1 hour.
- **f** The distance travelled stays constant at 250 km for half an hour.

Distance–time graph of car journey

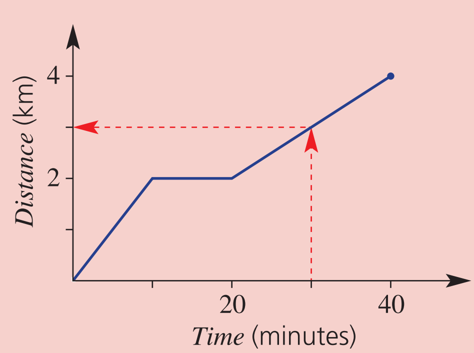


Fluency | 3–5 | 3, 5, 6

Example 5 Interpreting a distance–time graph

This distance–time graph shows a car's journey from home, to school and then to the local shopping centre.

- **a** What is the total distance travelled?
- **b** How long is the car resting while at the school?
- **c** What is the total distance travelled after 30 minutes?



Continued on next page

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Solution	Explanation
a 4 km	Read the distance from the end point of the graph.
b 10 minutes	The rest starts at 10 minutes and finishes at 20 minutes.
c 3 km	Draw a line from 30 minutes and read off the distance.

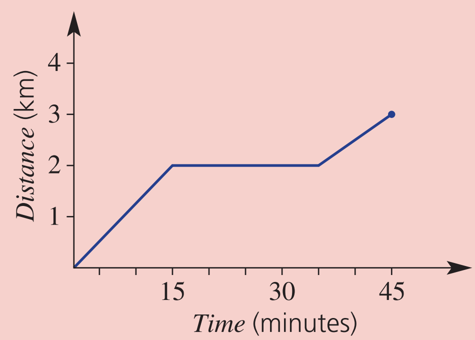
Now you try

The distance–time graph shows a person's walk from their house to the cafe for a rest and then to the post office.

a What is the total distance travelled?

b How long is the person resting at the cafe?

c What is the total distance travelled after 40 minutes?

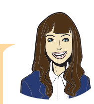


3 A bicycle journey is shown on this distance–time graph.

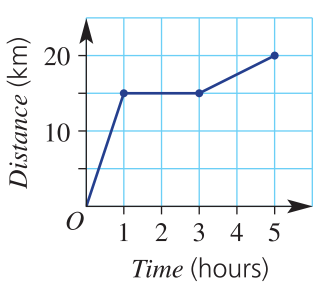
a What is the total distance travelled?

b How long is the cyclist at rest?

c How far has the cyclist travelled after 4 hours?



Hint: From the end of the line, go across to the distance scale. This will show the total distance travelled.



Hint: A flat line segment shows that the weight is not changing, so no water is being poured in at that time.

4 The weight of a water container increases while water is poured into it from a tap.

a What is the total weight of the container after:

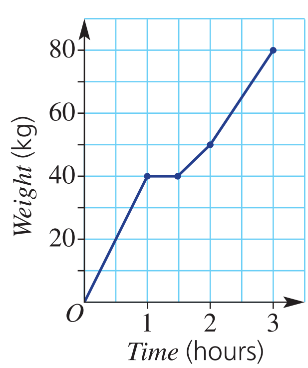
i 1 hour?

ii 2 hours?

iii 3 hours?

b During the 3 hours, how long is the container not actually being filled with water?

c During which hour is the container filling the fastest?



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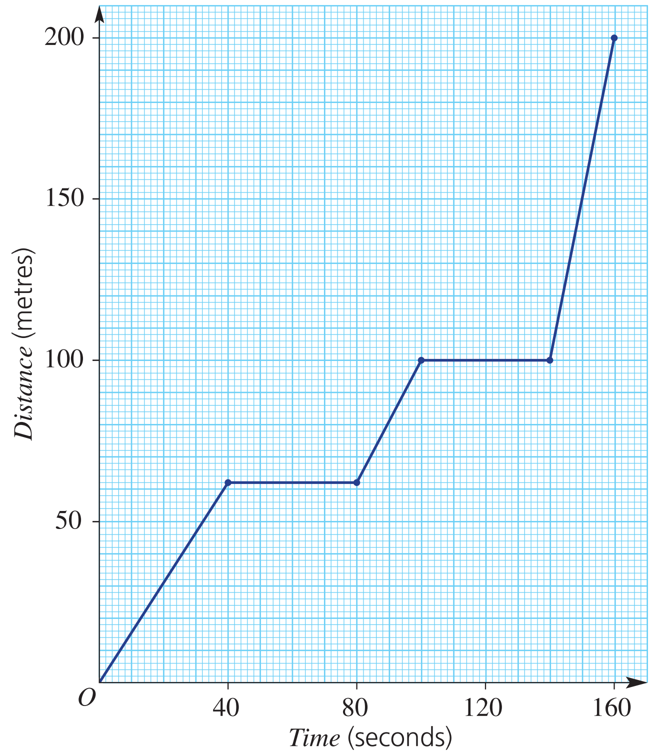
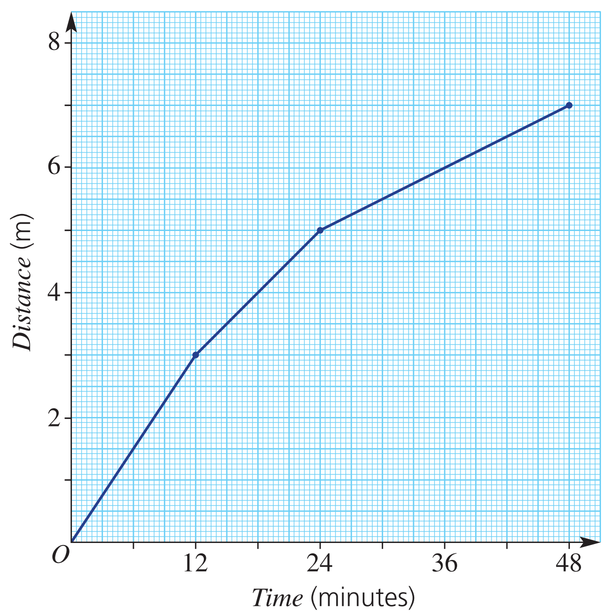
6B

5 This graph shows a shopper's short walk in a shopping mall.

  a What is the total distance the shopper travelled?
  b How long was the shopper not walking?
  c What was the total distance the shopper travelled by the following times?
    i 20 seconds
    ii 80 seconds
    iii 2.5 minutes

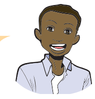


6 A snail makes its way across a footpath, garden bed and lawn according to this graph.



  a How far does the snail travel on:
    i the footpath?
    ii the garden bed?
    iii the lawn?
  b On which surface does the snail spend the most time?
  c Use your graph to find how far the snail travelled after:
    i 6 minutes
    ii 18 minutes
    iii 42 minutes

Hint: The line segment that has the largest horizontal change is the surface that the snail spent most time on.



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Problem-solving and reasoning

7–9 | 9–11

7 This graph shows the distance of a train from the city station over a period of time.

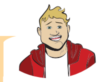
a What is the farthest distance the train travelled from the station?

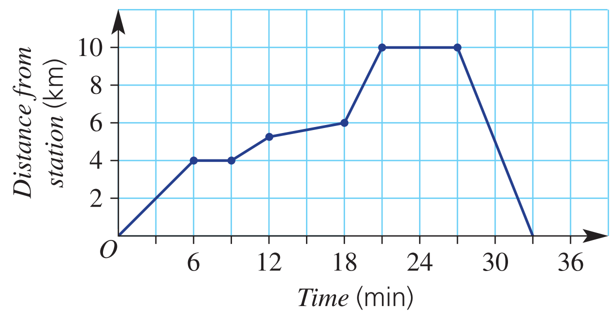
b What is the total distance travelled?

c After how many minutes does the train begin to return to the station?

d What is the total number of minutes the train was at rest?

Hint: Remember to include the return trip in the total distance travelled.



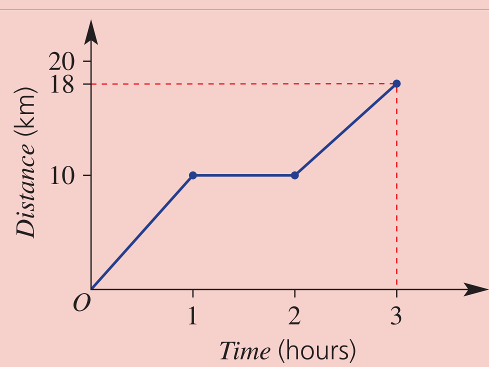


Example 6 Sketching a distance–time graph

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 18 km in 3 hours
- 10 km covered in the first hour
- a 1-hour long rest after the first hour

Solution



Explanation

Draw axes with time on the horizontal (up to 3 hours) and distance on the vertical (up to 18 km).

Start at time zero.

Draw the first hour with 10 km covered.

Draw the rest stop, which lasts for 1 hour.

Draw the remainder of the journey, so that 18 km is completed after 3 hours.

Now you try

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 20 km in 2 hours
- 14 km covered in the first hour
- a half-hour rest stop after the first hour

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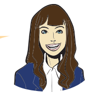
6B

8 Sketch a distance–time graph displaying all of the following information.
- total distance covered is 100 km in 2 hours
- 50 km covered in the first hour
- a half-hour rest stop after the first hour

Hint: Draw axes with time on the horizontal (up to 2 hours) and distance on the vertical (up to 100 km).

9 Sketch a graph to illustrate a journey described by the following.
- total distance covered is 15 metres in 40 seconds
- 10 metres covered in the first 10 seconds
- a 25-second rest after the first 10 seconds



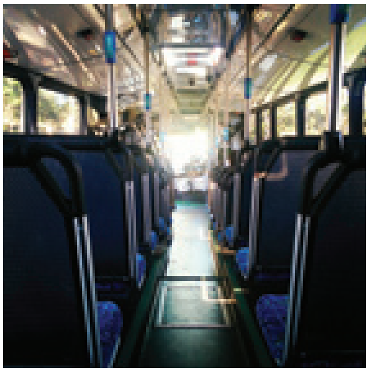
Hint: Always use a ruler to draw line segments.

10 A bus travels 5 km in 6 minutes, stops for 2 minutes, then travels 10 km in 8 minutes, stops for another 2 minutes and then completes the journey by travelling 5 km in 4 minutes.

a What is the total distance travelled?

b What is the total time taken?

c Sketch a distance–time graph for the journey.



Hint: Find the total time taken to determine the scale for the horizontal axis. Find the total distance travelled to determine the scale for the vertical axis.

11 A 1-day, 20 km bush hike included the following features.
- a 3-hour hike to waterfalls (10 km distance)
- a half-hour rest at the falls
- a 2-hour hike to the mountain peak (5 km distance)
- a $1\frac{1}{2}$-hour hike to the camp site

Sketch a distance–time graph for the journey.

Pigeon flight | — | 12

12 The distance travelled by a pigeon is described by these points.
- a half-hour flight, covering a distance of 18 km
- a 15-minute rest
- a 15-minute flight, covering 12 km
- a half-hour rest
- turning and flying 10 km back towards 'home' over the next half hour
- a rest for a quarter of an hour
- reaching 'home' after another 45-minute flight

a Sketch a graph illustrating the points above using *Distance* on the vertical axis.

b What is the fastest speed (in km/h) that the pigeon flew? $\left(\text{Speed} = \dfrac{\text{distance}}{\text{time}}\right)$

c Determine the pigeon's average speed, in km/h. $\left(\text{Average speed} = \dfrac{\text{total distance}}{\text{total flying time}}\right)$

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6C Plotting straight lines CONSOLIDATING

Learning intentions

- To review the Cartesian plane and how to position and describe points using coordinates
- To understand that a straight-line graph is formed from a linear rule
- To be able to plot a graph of a rule using a table of values
- To be able to use a straight line graph to determine the values of variables

Key vocabulary: Cartesian or number plane, x-coordinate, y-coordinate, point of intersection, origin, linear

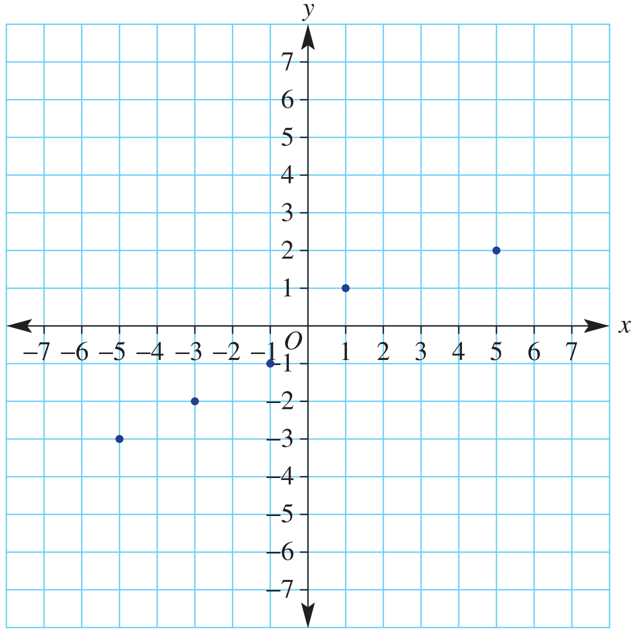
On a number plane, a pair of coordinates gives the exact position of a point. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin (O). It provides a reference point for all other points on the plane.

A rule that relates two variables can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight-line graphs are described as being linear.

Architects apply their knowledge of two-dimensional straight lines and geometric shapes to form interesting three-dimensional surfaces. Computers use line equations to produce visual models.

Lesson starter: What's the error?

- Which point is not in line with the rest of the points on this graph? What should its coordinates be so it is in line? List two other points that would be in line with the points on this graph.



- This table shows coordinates for the rule $y = 4x + 3$. Which y value has been calculated incorrectly in the table? What would be the correct y value?

x	0	1	2	3	4
y	3	7	11	12	19

- Which two points in the list below will not be in the same line as the other points?
  $(-2, 4), (-1, 2), (0, 0), (1, -2), (2, 4), (3, 6)$
  What would be the correct coordinates for these two points, using the given x values?
- Points that follow a linear rule will always be in a straight line. Discuss some ways of checking whether the coordinates of a point have been calculated incorrectly.

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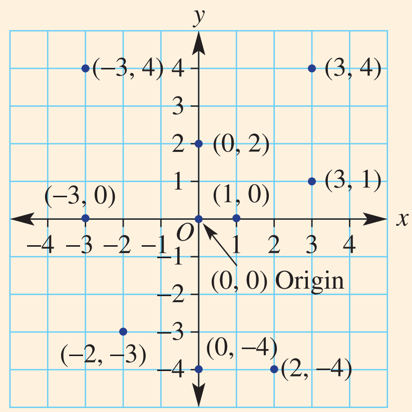
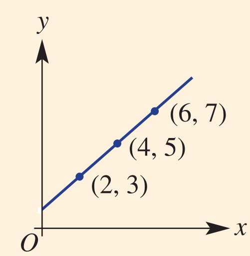
Key ideas

- A **number plane** or **Cartesian plane** includes a vertical **y-axis** and a horizontal **x-axis** intersecting at right angles.



- A point on a number plane has coordinates (x, y).
  - The **x-coordinate** is listed first, followed by the **y-coordinate**.
- The point $(0, 0)$ is called the **origin** (O).
- $(x, y) = \left(\text{horizontal units from origin}, \text{vertical units from origin}\right)$
- A rule is an equation connecting two or more variables.
- A straight-line graph will result from a rule that is **linear**.
- For two variables, a linear rule is often written with y as the subject. For example, $y = 2x - 3$ or $y = -x + 7$.
- To graph a linear relationship using a rule:
  - Construct a table of values finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

x	2	4	6
y	3	5	7

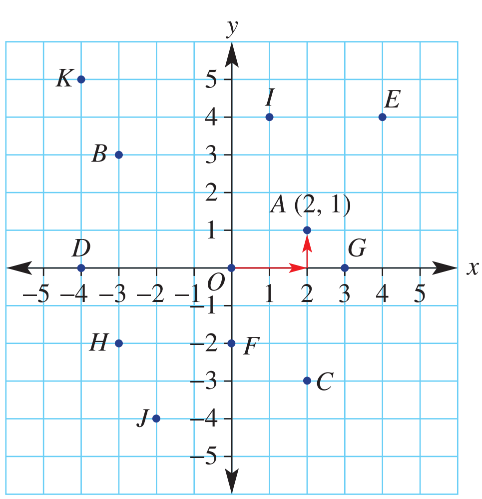


- The **point of intersection** of two lines is the point where the lines cross over each other.

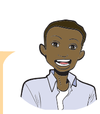
Exercise 6C

Understanding 1–3 3

1 a List the coordinates of each point (A–K) plotted on the number plane.
  b Which points are on the x-axis?
  c Which points are on the y-axis?
  d What are the coordinates of the point called the 'origin'?



Hint: $(x, y) = \left(\text{right or left}, \text{up or down}\right)$
The 'origin' is the point where the x-axis and y-axis meet.



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2 Write the coordinates for each point listed in this table.

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

Hint: Coordinates are written as (x, y).

x	−2
y	1

} $(-2, 1)$

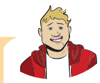
3 Ethan is finding the coordinates of some points that are on the line $y = -2x + 4$. Copy and complete these calculations, stating the coordinates for each point.

a $x = -3, y = -2 \times (-3) + 4 = 6 + 4 = (-3, \quad)$

b $x = -1, y = -2 \times (-1) + 4 = \quad = (\quad , \quad)$

c $x = 0, y = -2 \times 0 + 4 = \quad = (\quad , \quad)$

d $x = 2, y = -2 \times 2 + 4 = \quad = (\quad , \quad)$



Hint: In the order of operations, first do any multiplication, then do addition or subtraction from left to right. When multiplying, same signs make a positive and different signs make a negative.

Fluency 4, 5(½) 4–5(½), 6

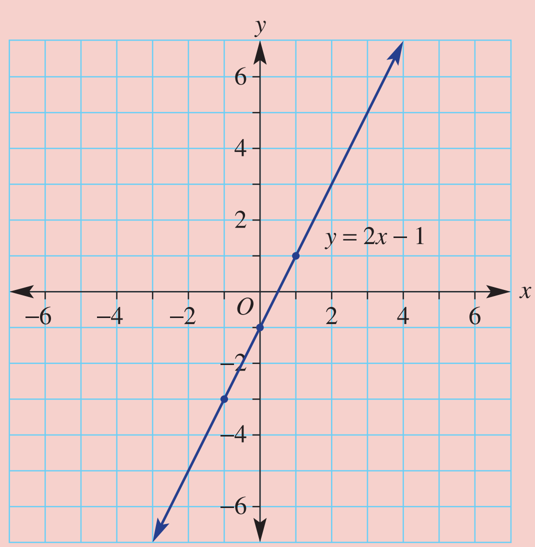
Example 7 Plotting a graph from a rule

Plot the graph of $y = 2x - 1$ by first completing the table of values.

x	−1	0	1
y			

Solution

x	−1	0	1
y	−3	−1	1



Explanation

Substitute each value into the equation:

$x = -1, y = 2 \times (-1) - 1 = -3 \quad (-1, -3)$

$x = 0, y = 2 \times 0 - 1 = -1 \quad (0, -1)$

$x = 1, y = 2 \times 1 - 1 = 1 \quad (1, 1)$

Plot the points and draw the line with a ruler.

When labelling axes, put the numbers on the grid lines, not in the spaces.

Now you try

Plot the graph of $y = 3x + 2$ by first completing the table of values.

x	−1	0	1
y			

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6C

4 Complete the following tables, then plot the graph of each one on a separate number plane.

a $y = 2x$

x	−1	0	1
y			

b $y = x + 4$

x	0	1	2
y			

Hint: When multiplying, same signs make a positive; e.g. $-2 \times (-1) = 2$

c $y = 2x - 3$

x	0	1	2
y			

d $y = -2x$

x	−1	0	1
y			

e $y = x - 4$

x	1	2	3
y			

f $y = 6 - x$

x	0	1	2
y			

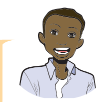
5 Complete the following tables, then plot the graph of each pair on the same axes.

a i $y = x + 2$

x	0	2	4
y			

ii $y = -x + 2$

x	0	2	4
y			



Hint: For each part, draw line i and line ii on the same axes.

b i $y = x - 4$

x	0	1	2
y			

ii $y = 4 - x$

x	0	1	2
y			

c i $y = 2 + 3x$

x	−3	0	3
y			

ii $y = 3x - 4$

x	−3	0	3
y			

6 By plotting the graphs of each of the following pairs of lines on the same axes, find the coordinates of the point of intersection. Use a table of values, with x from −2 to 2.

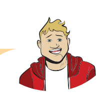
a $y = 2x$ and $y = x$

b $y = x + 3$ and $y = 2x + 2$

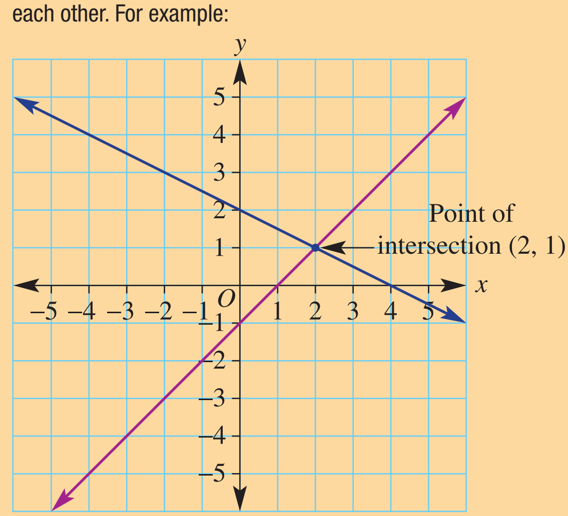
c $y = 2 - x$ and $y = 2x + 5$

d $y = 2 - x$ and $y = x + 2$

e $y = 2x - 3$ and $y = x - 4$



Hint: The point of intersection of two lines is where they cross each other. For example:



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Problem-solving and reasoning

7, 9 | 8, 10

Example 8 Interpreting a graph when given a table of values

Jasmine is organising a school dance. The venue is chosen and the costs are shown in the table.

Number of students (S)	0	100	200	300
Total cost in dollars (C)	500	1000	1500	2000

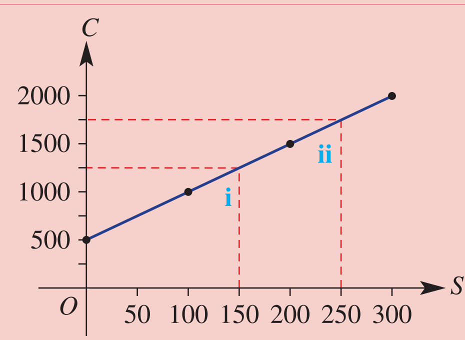
a Plot a graph of the total cost against the number of students.

b Use the graph to determine:

- **i** the total cost for 150 students
- **ii** how many students could attend the dance if Jasmine has $1750 to spend

Solution

a



Explanation

Construct a set of axes using S between 0 and 300 and C between 0 and 2000.

Number of students is placed on the horizontal axis.

Plot each point using the information in the table.

b **i** The total cost for 150 students is $1250.

Draw a vertical dotted line at $S = 150$ to meet the graph, then draw another dotted line horizontally to the C-axis.

ii 250 students could attend the dance if the budget is $1750.

Draw a horizontal dotted line at $C = 1750$ to meet the graph, then draw a dotted line vertically to the S-axis.

Now you try

Hayley is organising a party for her child. The venue is chosen and the costs are shown in the table.

Number of children (n)	0	10	20	30
Total cost in dollars (C)	200	400	600	800

a Plot a graph of the total cost against the number of children.

b Use the graph to determine:

- **i** the total cost for 25 children
- **ii** how many children could be invited if Hayley has $500 to spend

7 A furniture removalist charges per hour. His rates are shown in the table below.

No. of hours (n)	0	1	2	3	4	5
Cost (C)	200	240	280	320	360	400

a Plot a graph of cost against hours.

b Use the graph to determine the:

- **i** total cost for 2.5 hours of work
- **ii** number of hours the removalist will work for $380

Hint: Place 'Number of hours' on the horizontal axis.

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6C

8 Olive oil is sold in bulk for $8 per litre.

No. of litres (*L*)	1	2	3	4	5
Cost (*C*)	8	16	24	32	40

a Plot a graph of cost against number of litres.
b Use the graph to determine the:
 i total cost for 3.5 litres of oil
 ii number of litres of oil you can buy for $20

Example 9 Constructing a table and graph for interpretation

An electrician charges $50 for a service call and $60 an hour for labour.

a Complete the table of values.

No. of hours (*n*)	1	2	3	4	5
Cost (*C*)					

b Plot a graph of cost against number of hours.
c Use the graph to determine:
 i the cost for 4.5 hours of work
 ii how long the electrician will work for $260

Solution

a

No. of hours (*n*)	1	2	3	4	5
Cost (*C*)	110	170	230	290	350

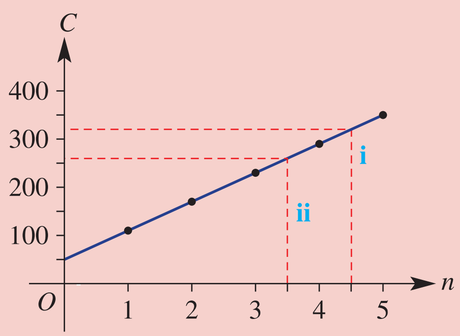
Explanation

Cost for 1 hour = $50 + $60 = $110

Cost for 2 hours = $50 + 2 × $60 = $170

Cost for 3 hours = $50 + 3 × $60 = $230 etc.

b



Plot the points from the table using C on the vertical axis and n on the horizontal axis. Join all the points to form the straight line.

c i The cost is $320.

Draw a vertical dotted line at $n = 4.5$ to meet the graph, then draw a line horizontally to the C-axis.

ii The electrician will work for 3.5 hours.

Draw a horizontal dotted line at $C = 260$ to meet the graph, then draw vertically to the n-axis.

Continued on next page

ISBN 978-1-108-87859-3

Now you try

A balloon artist charges $120 to attend a child's party and $60 per hour in attendance.

a Complete the table of values.

No. of hours (n)	1	2	3	4
Cost (C)				

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 2.5 hours of hire

ii how long the balloon artist will work for $330

9 A car rental firm charges $200 plus $1 for each kilometre travelled.

a Complete the table of values below.

No. of km (k)	100	200	300	400	500
Cost (C)					

b Plot a graph of cost against kilometres.

c Use the graph to determine:

i the cost if you travel 250 km

ii how many kilometres you can travel on a budget of $650

10 Matthew delivers pizza for a fast-food outlet. He is paid $20 a shift plus $3 per delivery.

a Complete the table of values below.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of Matthew's pay against number of deliveries.

c Use the graph to determine the:

i amount of pay for 12 deliveries

ii number of deliveries made if Matthew is paid $74

Which mechanic? — 11

11 Two mechanics charge different rates for their labour. Yuri charges $75 for a service call plus $50 per hour. Sherry charges $90 for a service call plus $40 per hour.

a Create a table for each mechanic for up to 5 hours of work.

b Plot a graph for the total charge against the number of hours worked for Yuri and Sherry, on the same axes.

c Use the graph to determine the:

i cost of hiring Yuri for 3.5 hours

ii cost of hiring Sherry for 1.5 hours

iii number of hours of work if Yuri charges $100

iv number of hours of work if Sherry charges $260

v number of hours of work if the cost from Yuri and Sherry is the same

d Write a sentence describing who is cheaper for different hours of work.

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6D Midpoint and length of a line segment

Learning intentions

- To understand what is meant by the midpoint and length of a line segment
- To be able to determine the midpoint of a line segment from a graph and using coordinates
- To understand how the length of a line segment can be found using Pythagoras' theorem
- To be able to find the length of a line segment from a graph and using coordinates

Key vocabulary: line segment, length, midpoint, coordinates, Pythagoras' theorem, square root, average, hypotenuse

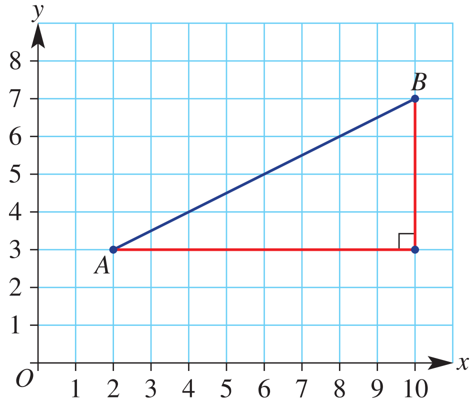
A line segment has a definite length and also has a point in the middle of the segment called the midpoint. Both the midpoint and length can be found by using the coordinates of the end points.

Builders use mathematical calculations to determine the length, midpoint and angle of inclination of wooden beams when constructing the timber frame of a house.

Lesson starter: Finding a method

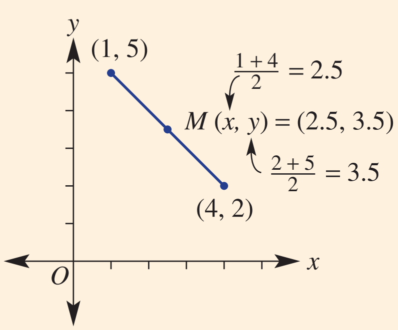
Below right is a graph of the line segment AB. A right-angled triangle has been drawn so that AB is the hypotenuse.

- How many units long are the horizontal and vertical sides of this right-angled triangle?
- Discuss and explain a method for finding the length of the line segment AB.
- What is the x value of the middle point of the horizontal side of the right-angled triangle?
- What is the y value of the middle point of the vertical side of the right-angled triangle?
- What are the coordinates of the point in the middle of the line segment AB?
- Discuss and explain a method for finding the midpoint of a line segment.



Key ideas

- A **line segment** is a part of a line and has two end points.
- The **midpoint** (M) of a line segment is the halfway point between the two end points.
  - We find the mean of the two x values at the end points and the mean of the two y values at the end points.
  - $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
  - When finding the mean, add the values in the numerator before dividing by 2.



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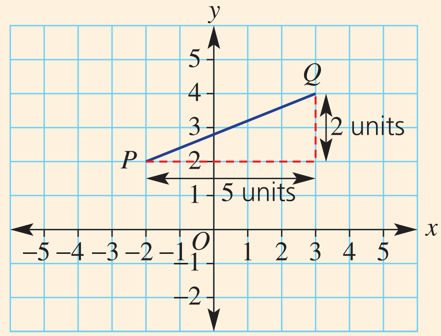
- The length of a line segment is found using **Pythagoras' theorem**.
  To find the length of the line segment PQ, follow these steps.
  - Draw a right-angled triangle with the line segment PQ as the hypotenuse (i.e. longest side).
  - Count the grid squares to find the length of each smaller side.
  - Apply Pythagoras' theorem:
    $$\begin{aligned} PQ^2 &= 5^2 + 2^2 \\ &= 25 + 4 \\ &= 29 \text{ (take the square root)} \\ PQ &= \sqrt{29} \text{ units} \end{aligned}$$
  - $\sqrt{29}$ is the length of line segment PQ, in square root form.



Exercise 6D

Understanding | 1–3 | 3

1 Complete the following.

a The ____________ of a line segment is the halfway point between the two end points.

b The ____________ of a line segment is found using Pythagoras' theorem.

2 Complete the working to find the midpoint of the line segment with the following end point coordinates.

a $(1, 4)$ and $(3, 8)$

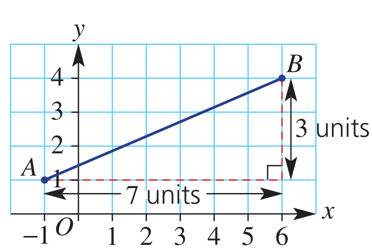
$$M = \left(\frac{1+\square}{2}, \frac{4+\square}{2}\right)$$
$$M = (\square, \square)$$

b $(-1, 3)$ and $(5, -3)$

$$M = \left(\frac{\square+\square}{2}, \frac{\square+\square}{2}\right)$$
$$M = (\square, 0)$$

3 For the line segment shown, complete the Pythagoras' theorem working below to find the length.

$$\begin{aligned} AB^2 &= \square^2 + \square^2 \\ &= \square + \square \\ &= 58 \\ AB &= \sqrt{58} \end{aligned}$$



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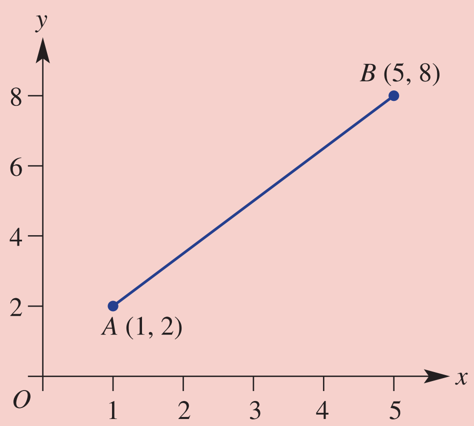
6D

Fluency 4–8(½) 4–9(½)

Example 10 Finding the midpoint of a line segment from a graph

Find the midpoint of the interval between $A(1, 2)$ and $B(5, 8)$.



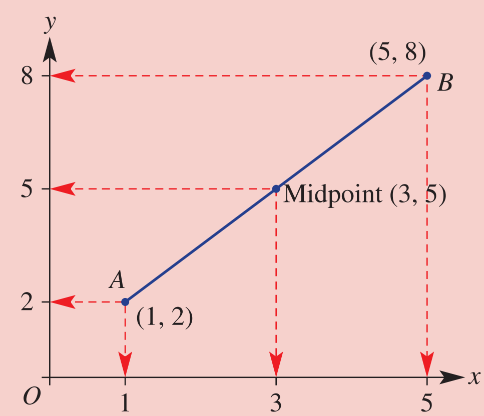
Solution

$$\text{Average of } x \text{ values} = \frac{1+5}{2} = \frac{6}{2} = 3$$

$$\text{Average of } y \text{ values} = \frac{2+8}{2} = \frac{10}{2} = 5$$

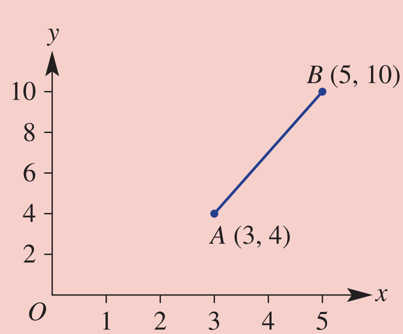
Midpoint is $(3, 5)$.

Explanation



Now you try

Find the midpoint of the interval between $A(3, 4)$ and $B(5, 10)$.



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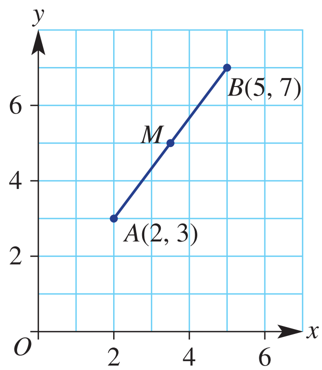
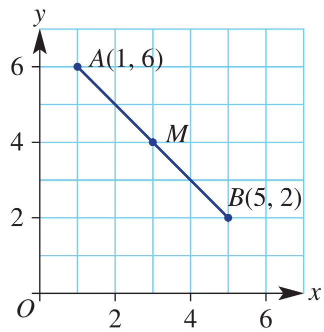
4 Find the midpoint, M, of each of the following intervals.

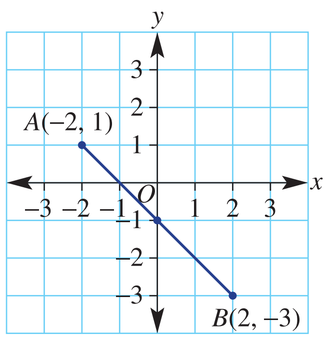
a



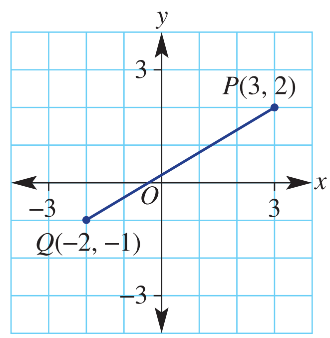
b



c



d



Hint: In finding the average, add the numerator values before dividing by 2.

Example 11 Finding the midpoint of a line segment when given the coordinates of the end points

Find the midpoint of the line segment joining $P(-3, 1)$ and $Q(5, -4)$.

Solution	**Explanation**
$x = \frac{-3 + 5}{2}$	Average the x-coordinates.
$= \frac{2}{2}$	Calculate the numerator before dividing by 2; i.e. $-3 + 5 = 2$.
$= 1$	
$y = \frac{1 + (-4)}{2}$	Average the y-coordinates.
$= \frac{-3}{2}$	Calculate the numerator before dividing by 2; i.e. $1 + (-4) = 1 - 4 = -3$.
$= -1.5$	
Midpoint is $(1, -1.5)$.	Write the coordinates of the midpoint.

Now you try

Find the midpoint of the line segment joining $P(3, 1)$ and $Q(6, -5)$.

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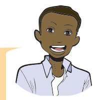
6D

5 Find the midpoint of the line segment joining the following points.

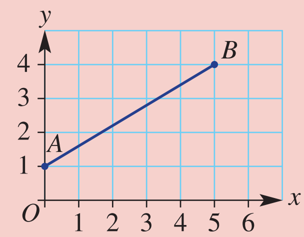
a $(1, 4)$ and $(3, 6)$
b $(3, 7)$ and $(5, 9)$
c $(0, 4)$ and $(6, 6)$
d $(2, 4)$ and $(3, 5)$
e $(7, 2)$ and $(5, 3)$
f $(1, 6)$ and $(4, 2)$
g $(0, 0)$ and $(-2, -4)$
h $(-2, -3)$ and $(-4, -5)$
i $(-3, -1)$ and $(-5, -5)$
j $(-3, -4)$ and $(5, 6)$
k $(0, -8)$ and $(-6, 0)$
l $(3, -4)$ and $(-3, 4)$



Hint: Check that your answer appears to be halfway between the end points.

Example 12 Finding the length of a line segment from a graph

Find the length of the line segment between $A(0, 1)$ and $B(5, 4)$.



Solution

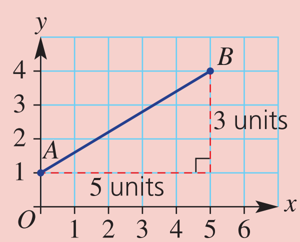
$AB^2 = 5^2 + 3^2$

$AB^2 = 25 + 9$

$AB^2 = 34$

$AB = \sqrt{34}$

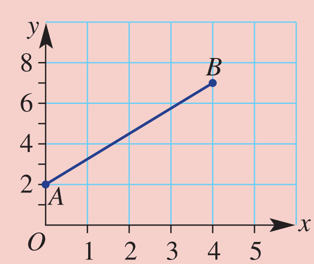
Explanation



Create a right-angled triangle and use Pythagoras' theorem.
For $AB^2 = 34$, take the square root of both sides to find AB.
$\sqrt{34}$ is the exact answer.

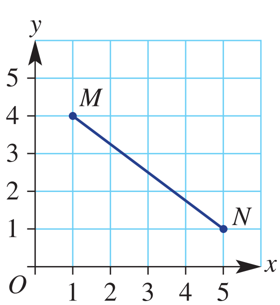
Now you try

Find the length of the line segment between $A(0, 2)$ and $B(4, 7)$.

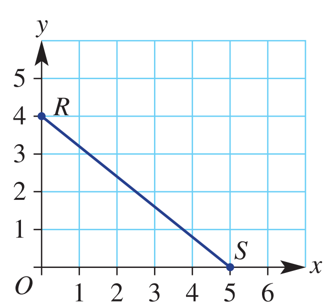


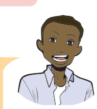
6 Find the length of each of the following line segments.

a

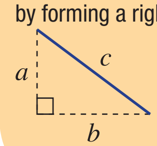


b





Hint: Use Pythagoras' theorem ($c^2 = a^2 + b^2$) by forming a right-angled triangle:



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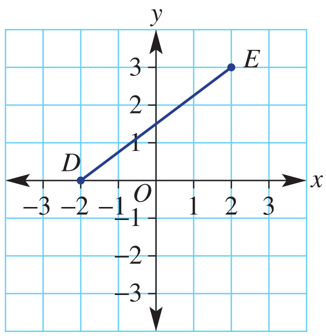
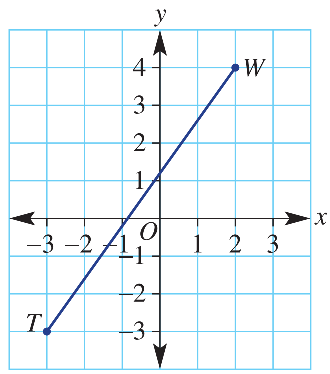
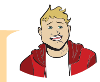
c



d

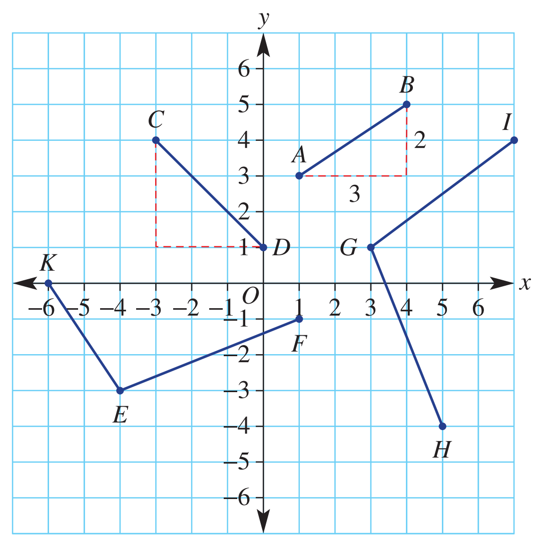


Hint: Write the answer in square root form if it is not a known square root.

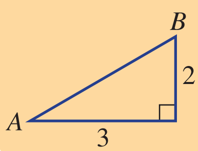


7 Find the length of each line segment on the following number plane. Leave your answers in square root form.

a AB **b** CD
c EF **d** GH
e KE **f** GI

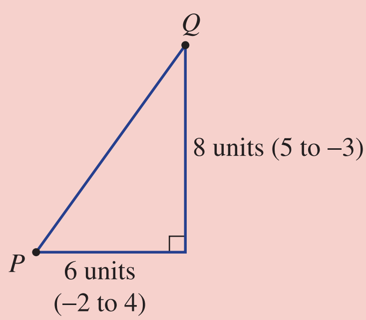
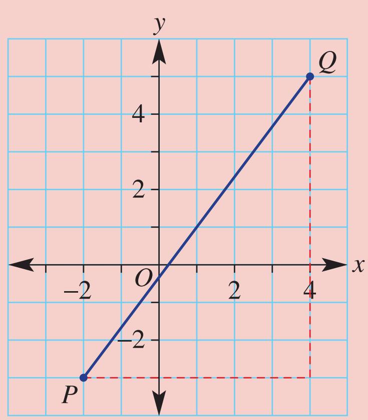


Hint: First sketch a right-angled triangle for each line segment, labelling the known sides. For example:



Example 13 Finding the length of a line segment when given the coordinates of the end points

Find the distance between the points P and Q if P is at $(-2, -3)$ and Q is at $(4, 5)$.

Solution	Explanation
	Use Pythagoras' theorem to find PQ, the hypotenuse. 
$PQ^2 = 6^2 + 8^2$ $PQ^2 = 36 + 64$ $PQ^2 = 100$ $PQ = \sqrt{100}$ $PQ = 10$ units	If you know the value of the square root, write its value.

Now you try

Find the distance between the points P and Q if P is at $(-1, -4)$ and Q is at $(3, 2)$.

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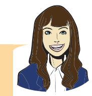
8 Plot each of the following pairs of points and find the distance between them, correct to one decimal place where necessary.

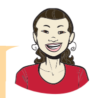
a $(2, 3)$ and $(5, 7)$
b $(0, 1)$ and $(6, 9)$
c $(0, 0)$ and $(-5, 10)$
d $(-4, -1)$ and $(0, -5)$
e $(-3, 0)$ and $(0, 4)$
f $(0, -1)$ and $(2, -4)$



Hint: First rule up your axes with x from -5 to 10 and y from -5 to 10.

9 Find the exact length between these pairs of points.

a $(1, 3)$ and $(2, 2)$
b $(4, 1)$ and $(7, 3)$
c $(-3, -1)$ and $(0, 4)$
d $(-2, -3)$ and $(3, 5)$
e $(-1, 0)$ and $(-6, 1)$
f $(1, -3)$ and $(4, -2)$

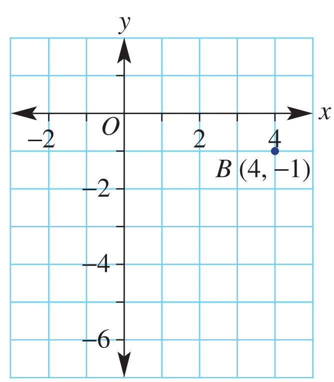


Hint: Exact length means leave the $\sqrt{}$ sign in the answers.

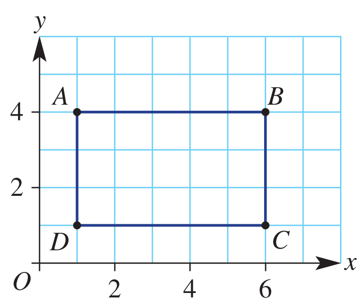
Problem-solving and reasoning

10, 11 | 10–12

10 Copy the diagram below. Mark the point $B(4, -1)$, as shown, then mark the point $M(1, -3)$. Find the coordinates of A if M is the midpoint of the interval AB.



11 Copy the diagram of rectangle $ABCD$.



a What are the coordinates of each vertex?
b Find the midpoint of the diagonal AC.
c Find the midpoint of the diagonal BD.
d What does this tell us about the diagonals of a rectangle?

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12 **a** Draw up a four-quadrant number plane like the one shown.

b Plot the points $A(-4, 0)$, $B(0, 3)$ and $C(0, -3)$ and form the triangle ABC.

c What is the length of:

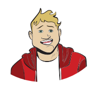
i AB? **ii** AC?

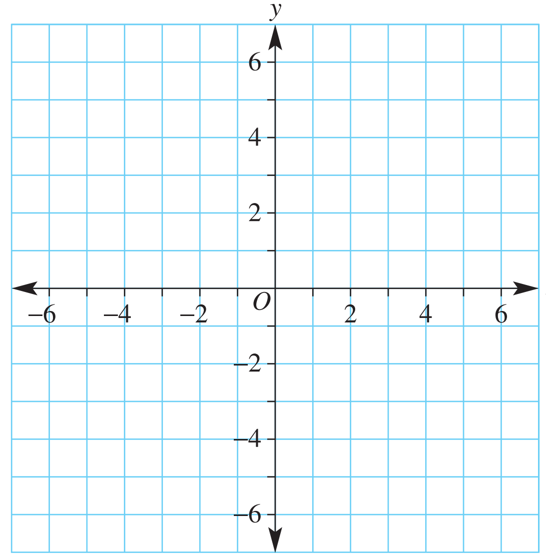
d What type of triangle is ABC?

e Calculate its perimeter and area.

f Write down the coordinates of D such that $ABDC$ is a rhombus.

Hint: A rhombus has all sides of equal length.





Features of a circle

	—	13

13 The diameter of a circle is shown on this graph.

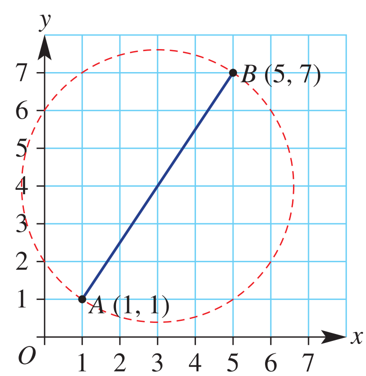
a What are the coordinates of X, the centre of the circle? Mark this point on your graph.

b What is the length of the radius XA?

c Find the distance from X to the point $(5, 1)$. How can we tell that $(5, 1)$ lies on the circle?

d Use $C = 2\pi r$ to find the circumference of the circle shown. Round your answer to one decimal place.

e Calculate the area of this circle using $A = \pi r^2$, correct to one decimal place.



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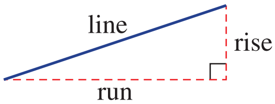
6E Exploring gradient

Learning intentions

- To know that gradient is a numerical measure of the slope of a line
- To understand that the gradient of a straight line is constant
- To be able to identify on a graph if the gradient is positive, negative, zero or undefined
- To be able to use the rise and the run between two points to calculate the gradient

Key vocabulary: gradient, slope, rise, run

The gradient of a line is a measure of its slope. It is a number that shows the steepness of a line. It is calculated by knowing how far a line rises or falls (called the *rise*) within a certain horizontal distance (called the *run*). The gradient is equal to the *rise* divided by the *run*. The letter m is used to represent gradient.

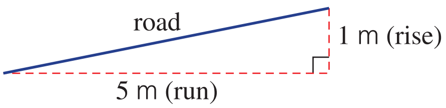


$$\text{Gradient of line} = m = \frac{\text{rise}}{\text{run}}$$

Engineers apply their knowledge of gradients when designing roads, bridges, railway lines and buildings. Some mountain railways have a gradient greater than 1, which is a slope far too steep for a normal train or even a powerful car.

For example, a train takes tourists to the Matterhorn, a mountain in Switzerland. To cope with the very steep slopes it has an extra wheel with teeth, which grips a central notched line.

Lesson starter: What's the gradient?



A road that rises by 1 m for each 5 m of horizontal distance has a gradient of 0.2 or 20%.

Trucks would find this gradient very steep.

The gradient is calculated by finding the rise divided by the run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1}{5} = 0.2 = 20\%$$

1 Find the gradient for each of these roads. Give the answer as a decimal and a percentage.

a Baldwin Street, Dunedin, New Zealand is known as the steepest street in the world. For each 2.86 m of horizontal (run) distance, the road rises by 1 m.

b Gower Street, Toowong, is Brisbane's steepest street. For each 3.2 m of horizontal (run) distance, the road rises by 1 m.

2 The Scenic Railway, Katoomba, NSW has a maximum gradient of 122% as it passes through a gorge in the cliff. What is its vertical distance (rise) for each 1 m of horizontal distance (run)?

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Use computer software (dynamic geometry) to produce a set of axes and grid.

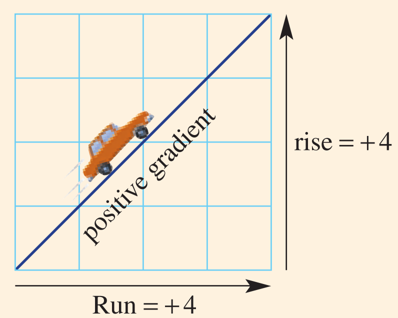
- Construct a line segment with end points on the grid. Show the coordinates of the end points.
- Calculate the rise (i.e. vertical distance between the end points) and the run (i.e. horizontal distance between the end points).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the end points and explore the effect on the gradient.
- Can you drag the end points but retain the same gradient value? Explain why this is possible.
- Can you drag the end points so that the gradient is zero or undefined? Describe how this can be achieved.

Key ideas

- **Gradient (m)** $= \frac{\text{rise}}{\text{run}}$, it describes the steepness of a **slope**.

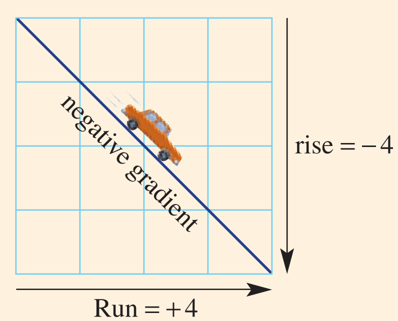
  Always move from left to right when considering the rise and the run.
  - The horizontal **'run'** always goes to the right and is always positive. The vertical **'rise'** can go up (positive) or down (negative).
  - If the line slopes up from left to right, the rise is positive and the gradient is positive.

    e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{+4}{+4} = 1$

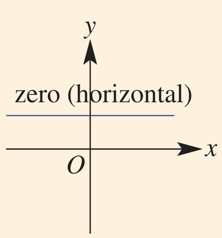


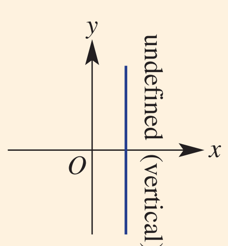
  - If the line slopes down from left to right, the rise is considered to be negative and the gradient is negative.

    e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{+4} = -1$



- The gradient can also be zero (when a line is horizontal) and undefined (when a line is vertical).





- Between two points (x_1, y_1) and (x_2, y_2), the gradient (m) is $m = \frac{y_2 - y_1 \text{ (rise)}}{x_2 - x_1 \text{ (run)}}$.

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6E

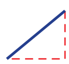
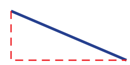
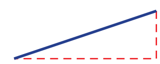
Exercise 6E

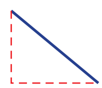
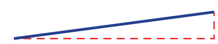
Understanding | 1, 2 | 1

1 Use the words *positive*, *negative*, *zero* or *undefined* to complete each sentence.
- **a** The gradient of a horizontal line is ______________.
- **b** The gradient of the line joining $(0, 3)$ and $(5, 0)$ is ______________.
- **c** The gradient of the line joining $(-6, 0)$ and $(1, 1)$ is ______________.
- **d** The gradient of a vertical line is ______________.

Hint: Lines going downhill from left to right have a negative gradient.

2 Decide whether each of the following lines would have a positive or negative gradient.

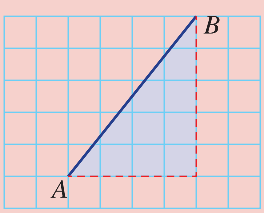
a  **b**  **c** 

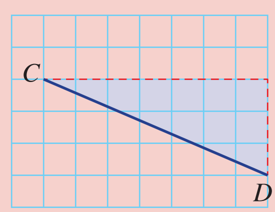
d  **e**  **f** 

Fluency | 3–5, 7 | 3, 4, 5(½), 6, 7

Example 14 Finding the gradient from a grid

Find the gradient of the following line segments, where each grid box equals 1 unit.

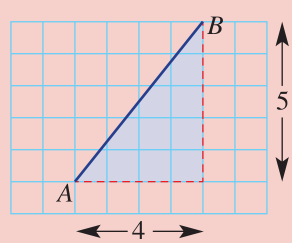
a


b


Solution | **Explanation**

a Gradient of $AB = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{4}$

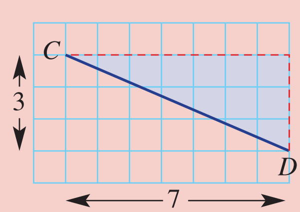


The slope is upwards, therefore the gradient is positive.
The rise is 5 and the run is 4.

b Gradient of $CD = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{7}$

$= -\frac{3}{7}$



The slope is downwards, therefore the gradient is negative.
The fall is 3, so we write rise $= -3$, and the run is 7.

Continued on next page

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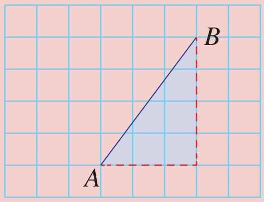
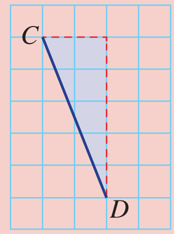
Now you try

Find the gradient of the following line segments, where each grid box equals 1 unit.

a

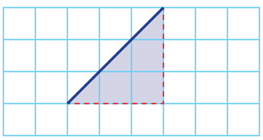


b

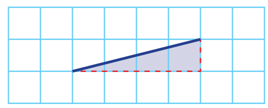


3 Find the gradient of the following line segments.

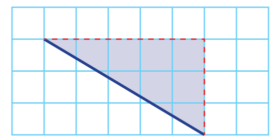
a



b

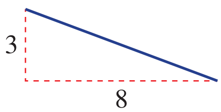


c

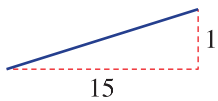


4 Find the gradient of the following.

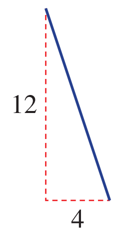
a



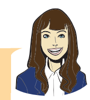
b



c



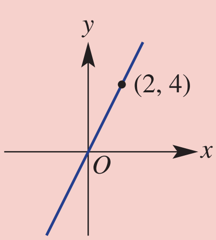
Hint: The gradient is written as a fraction or a whole number.



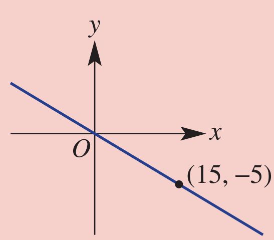
Example 15 Finding the gradient from graphs

Find the gradient of the following lines.

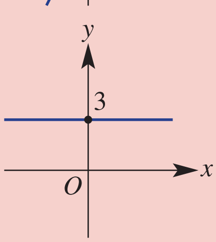
a



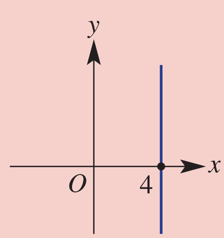
b



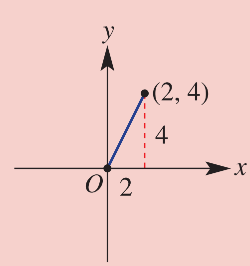
c



d



Solution	Explanation
a Gradient $= \frac{\text{rise}}{\text{run}}$	Write the rule each time.
$= \frac{4}{2}$	The rise is 4 and the run is 2 between the two points $(0, 0)$ and $(2, 4)$.
$= 2$	Simplify by cancelling.



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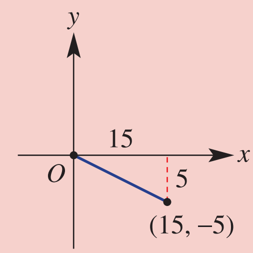
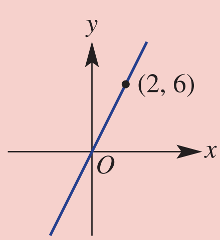
Solution	Explanation
b Gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{-5}{15}$ $= \frac{-1}{3}$ $= -\frac{1}{3}$	Note this time that, when working from left to right, there will be a slope downwards. The fall is 5 (rise = −5) and the run is 15. Simplify.
c Gradient = 0	Horizontal lines have a zero gradient.
d Gradient is undefined.	Vertical lines have an undefined gradient.



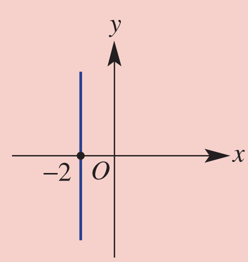
Now you try

Find the gradient of the following lines.

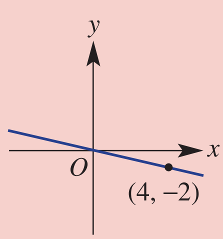
a



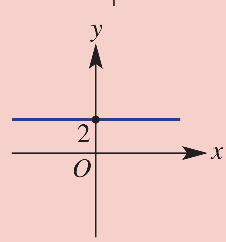
b



c

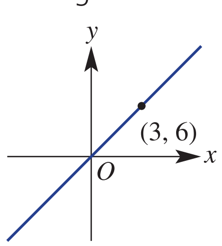


d

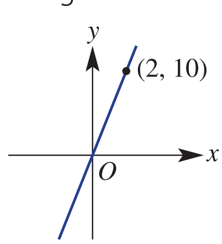


5 Find the gradient of the following lines.

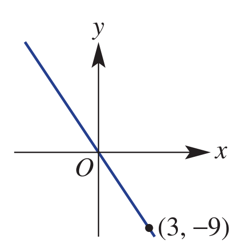
a



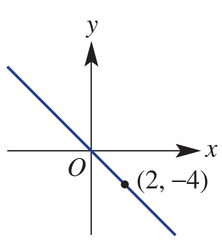
b



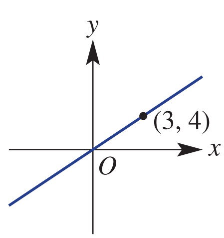
c



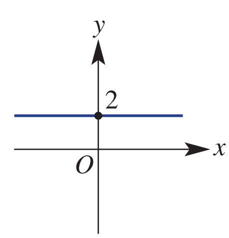
d



e



f



Hint: A horizontal line has zero rise, so its gradient is zero.

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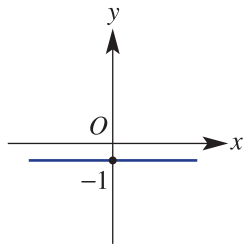
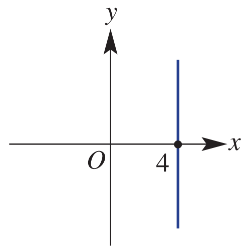
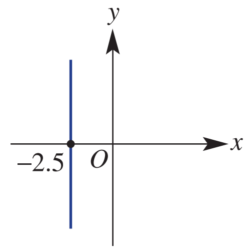
g

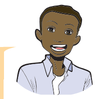


h



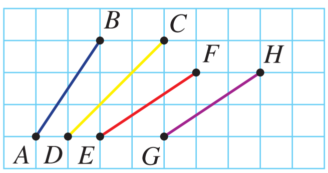
i





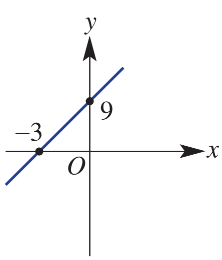
Hint: A vertical line has no 'run', so it has undefined gradient.

6 Use the grid to find the gradient of these line segments. Then order the segments from least to steepest gradient.

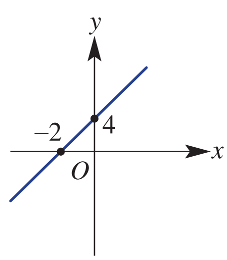


7 Determine the gradient of the following lines.

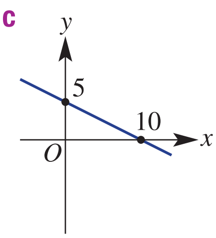
a

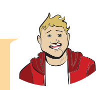


b

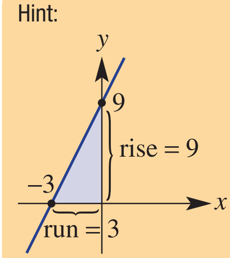


c

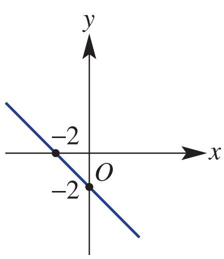




Hint:



d



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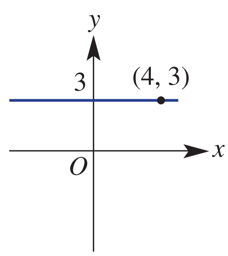
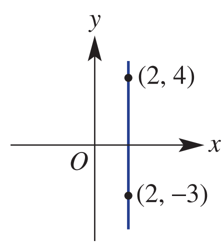


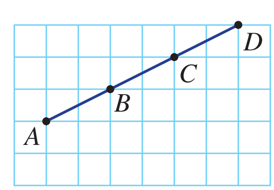
f



Problem-solving and reasoning | 8, 9 | 9–10(½), 11

8 **a** Copy and complete the table below.

Line segment	Rise	Run	Gradient
AB			
AC			
AD			
BC			
BD			
CD			



b What do you notice about the gradient between points on the same line?

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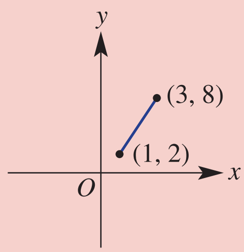
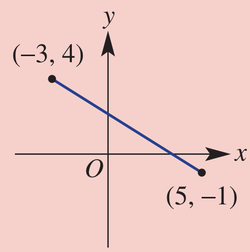
Example 16 Using a formula to calculate gradient

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b



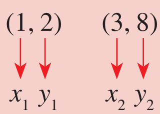
Solution

Explanation

a $m = \frac{y_2 - y_1}{x_2 - x_1}$

Write the rule.

$= \frac{8 - 2}{3 - 1}$



$= \frac{6}{2}$

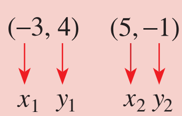
$= 3$

It does not matter which point is labelled (x_1, y_1) and which is (x_2, y_2).

b $m = \frac{y_2 - y_1}{x_2 - x_1}$

Write the rule.

$= \frac{-1 - 4}{5 - (-3)}$



$= \frac{-5}{8}$

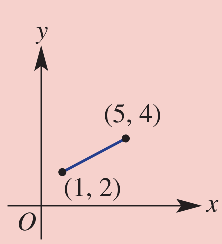
Remember that $5 - (-3) = 5 + 3 = 8$.

$= -\frac{5}{8}$

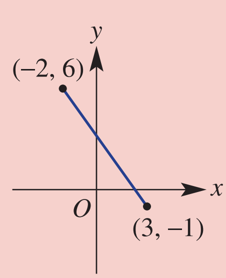
Now you try

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b



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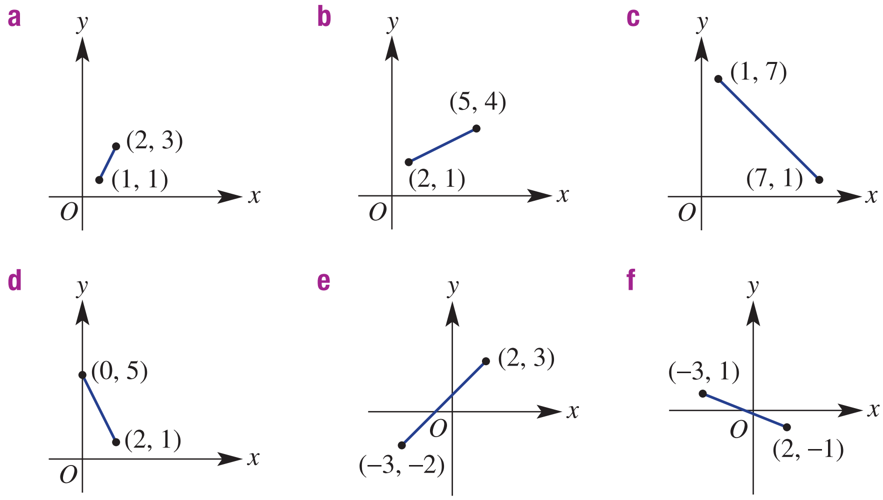
9 Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between these pairs of points.



Hint: First copy the coordinates and label them.
e.g. $(1, 1)$ $(2, 3)$ → x_1 y_1 x_2 y_2

Hint: You can choose either point to be (x_1, y_1).

10 Find the gradient between the following pairs of points.

- a $(1, 3)$ and $(5, 7)$
- b $(-1, -1)$ and $(3, 3)$
- c $(-3, 4)$ and $(2, 1)$
- d $(-6, -1)$ and $(3, -1)$
- e $(1, -4)$ and $(2, 7)$
- f $(-4, -2)$ and $(-1, -1)$

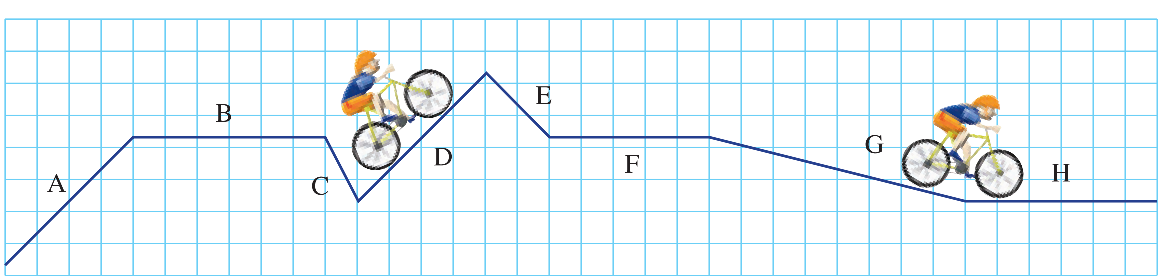
Hint: Use $m = \frac{y_2 - y_1}{x_2 - x_1}$.

11 The first section of the Cairns Skyrail travels from Caravonica terminal at 5 m above sea level to Red Peak terminal, which is 545 m above sea level. This is across a horizontal distance of approximately 1.57 km. What is the overall gradient of this section of the Skyrail? Round your answer to three decimal places.

Hint: Both distances must be in the same units.

From Bakersville to Rolland — 12

12 A transversal map for a bike ride from Bakersville to Rolland is shown.



- a Which sections, A, B, C, D, E, F, G or H, indicate travelling a positive gradient?
- b Which sections indicate travelling a negative gradient?
- c Which will be the hardest section to ride?
- d Which sections show a zero gradient?
- e Which section is the flattest of the downhill rides?

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6F Rates from graphs

Learning intentions

- To know that a rate compares two quantities
- To understand the connection between rate and the gradient of a straight line
- To be able to calculate speed (using distance ÷ time) and other rates from a graph using the gradient

Key vocabulary: rate, gradient, speed

The speed or rate at which something changes can be analysed by looking at the gradient (steepness) of a graph. Two common rates are kilometres per hour (km/h) and litres per second (L/s).

Graphs of a patient's records provide valuable information for a doctor. For example, from a graph of temperature versus time, the rate of temperature change in °C/min can be calculated. This rate provides important information to help a doctor diagnose an illness.

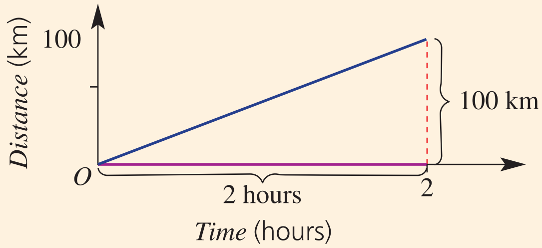
Lesson starter: What's the rate?

Calculate each of these rates.

a $60 000 for 200 tonnes of wheat.
b Lee travels 840 km in 12 hours.
c A foal grows 18 cm in height in 3 months.
d Petrol costs $96 for 60 litres.
e Before take-off, a hot-air balloon of volume 6000 m^3 is filled in 60 seconds.

Key ideas

- A **rate** compares two quantities. Many rates show how a quantity changes over *time*.



$$\text{Rate} = \frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50 \text{ km/h}$$

- Rate = change in quantity ÷ change in time
  (L, kg, ...) (seconds, hours, ...)
- The gradient of a line gives the rate.
- A common rate is **speed**.
  - Speed = change in distance ÷ change in time
    (cm, km, ...) (seconds, hours, ...)
- To determine a rate from a linear graph, calculate the gradient and include the units (y unit/x unit); e.g. km/h.

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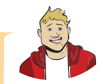
Exercise 6F

Understanding | 1, 2 | 2

1 Complete the sentences.

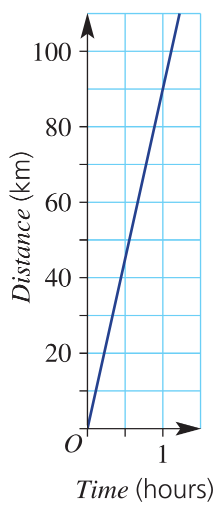
- **a** A rate is found from a linear graph by calculating the ________ of the line.
- **b** A rate compares ________ quantities.
- **c** A rate has two ________.
- **d** A speed of 60 kilometres per hour is written as 60 ________.
- **e** If the rate of filling a bath is 50 litres per minute, this is written as 50 ________.



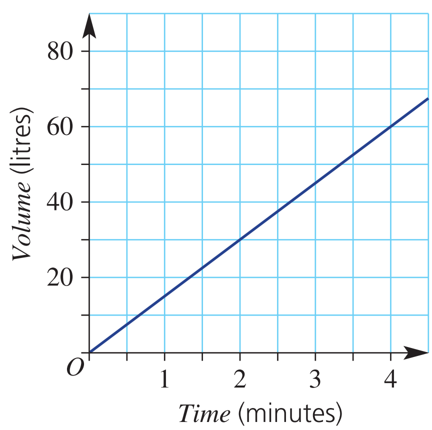
Hint: Choose from: *km/h, gradient, units, L/min, two.*

2 Write down the rate by calculating the gradient of each line graph. Include units.

a



b

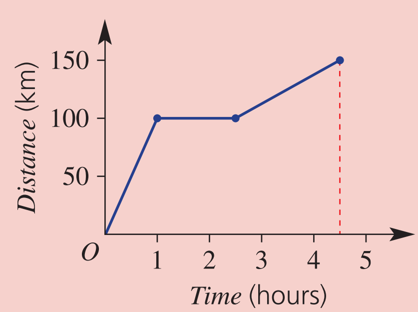


Hint: A rate = gradient with units.
Gradient = $\frac{\text{rise}}{\text{run}}$

Fluency | 3–5 | 3, 5, 6

Example 17 Calculating speed from a graph

A 4WD vehicle completes a journey, which is described by this graph.



- **a** For the first hour, find:
  - **i** the total distance travelled
  - **ii** the speed
- **b** How fast was the 4WD travelling during:
  - **i** the first hour?
  - **ii** the second section?
  - **iii** the third section?

Continued on next page

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6F

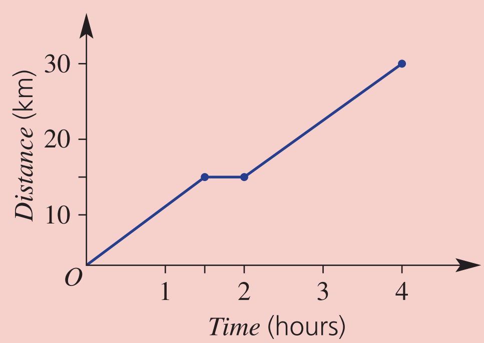
Solution	Explanation
a **i** 100 km	Read the distance at 1 hour.
ii 100 km/1 h = 100 km/h	Speed = distance ÷ time
b **i** 100 km/1 h = 100 km/h	Speed = distance ÷ time
ii 0 km/1.5 h = 0 km/h	The vehicle is at rest.
iii (150 − 100) km/(4.5 − 2.5) h = 50 km/2 h = 25 km/h	Determine the distance travelled and the amount of time, then apply the rate formula. Speed = distance ÷ time. 50 km in 2 hours is $\frac{50}{2}$ = 25 km in 1 hour.

Now you try

A marathon entrant completes a training run which is described by this graph.

a For the first hour, find:
- **i** the total distance travelled
- **ii** the speed

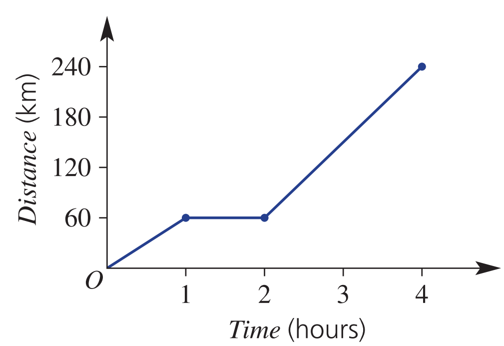
b How fast was the runner travelling during:
- **i** the first hour?
- **ii** the second section?
- **iii** the third section?



3 A car completes a journey, which is described by this graph.

a For the first hour, find:
- **i** the total distance travelled
- **ii** the speed

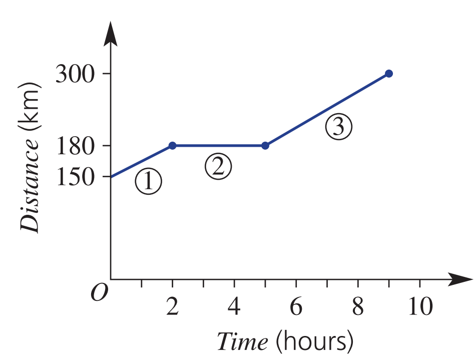
b How fast was the car travelling during:
- **i** the first hour?
- **ii** the second section?
- **iii** the third section?



4 A cyclist training for a professional race includes a rest stop between two travelling sections.

a For the first hour, find:
- **i** the total distance travelled
- **ii** the speed

b How fast was the cyclist travelling during:
- **i** the second section?
- **ii** the third section?



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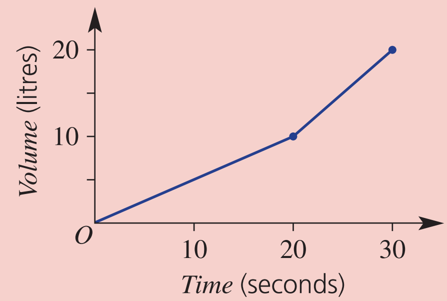
Example 18 Calculating the rate of change of volume in L/s

A container is being filled with water from a hose.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?



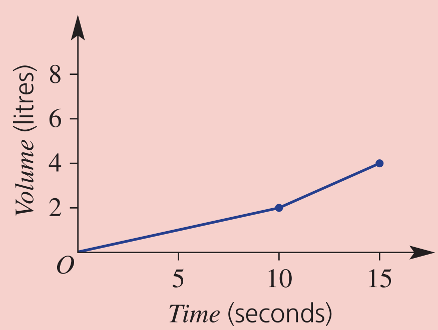
Solution	Explanation
a **i** 5 litres	Read the number of litres after 10 seconds.
ii 10 litres	Read the change in litres from 20 to 30 seconds.
b **i** 5 L/10 s = 0.5 L/s	5 litres is added in the first 10 seconds.
ii 10 L/10 s = 1 L/s	10 litres is added in the final 10 seconds.
iii 5 L/10 s = 0.5 L/s	5 litres is added between 10 and 20 seconds.

Now you try

A bucket is being filled with water from a tap.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 5 seconds?

b How fast (i.e. what rate in L/s) is the bucket being filled:
- **i** during the final 5 seconds?
- **ii** between the 5 and 10 second marks?

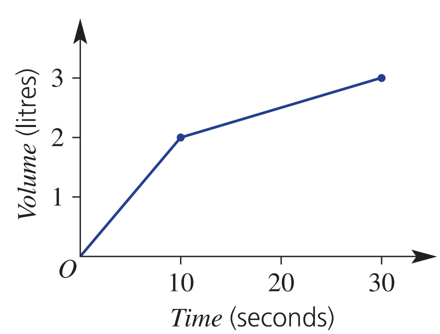


5 A large carton is being filled with milk.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

Hint:
Rate = volume ÷ time



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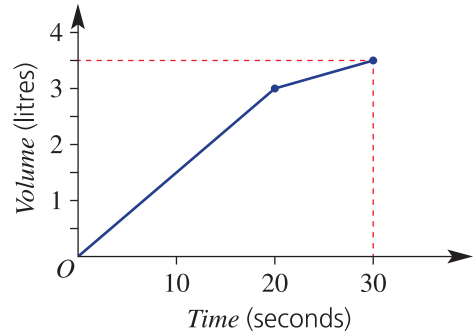
6F

6 A large bottle with a long narrow neck is being filled with water.

a How many litres are filled during:
  - **i** the first 10 seconds?
  - **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the bottle being filled:
  - **i** during the first 10 seconds?
  - **ii** during the final 10 seconds?
  - **iii** between the 10 and 20 second marks?



Problem-solving and reasoning

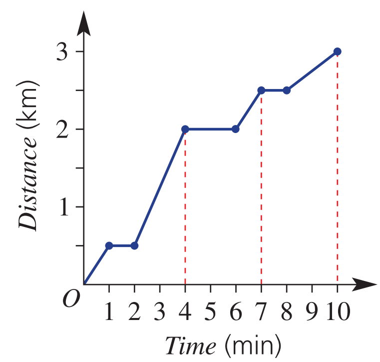
7, 8 | 8–10

7 A postal worker stops to deliver mail to each of three houses along a country lane.

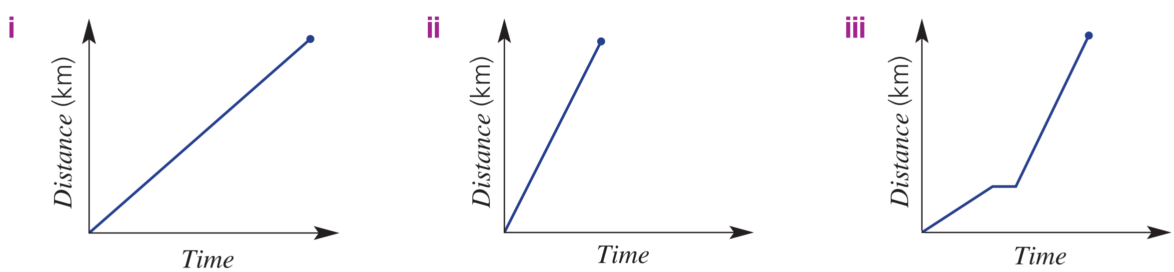
a What is the total length of the country lane?

b What is the total time the postal worker spends standing still?

c Find the speed (use km/min) of the postal worker at the following times.
  - **i** before the first house
  - **ii** between the first and the second house
  - **iii** between the second and the third house
  - **iv** after his delivery to the third house



8 Three friends, Anna, Billy and Cianne, travel 5 km from school to the library. Their journeys are displayed in the three graphs below. All three graphs are drawn to the same scale.



a If Anna walked a short distance before getting picked up by her mum, which graph represents her trip?

b If Cianne arrived at the library last, which graph best represents her journey?

c Which graph represents the fastest journey? Explain your answer.

9 **a** Draw your own graph to show the following journey: 10 km/h for 2 hours, then rest for 1 hour, and then 20 km/h for 2 hours.

b Then use your graph to find the total distance travelled.

Hint:
Mark each segment one at a time. 10 km/h for 2 hours covers a distance of $10 \times 2 = 20$ km.

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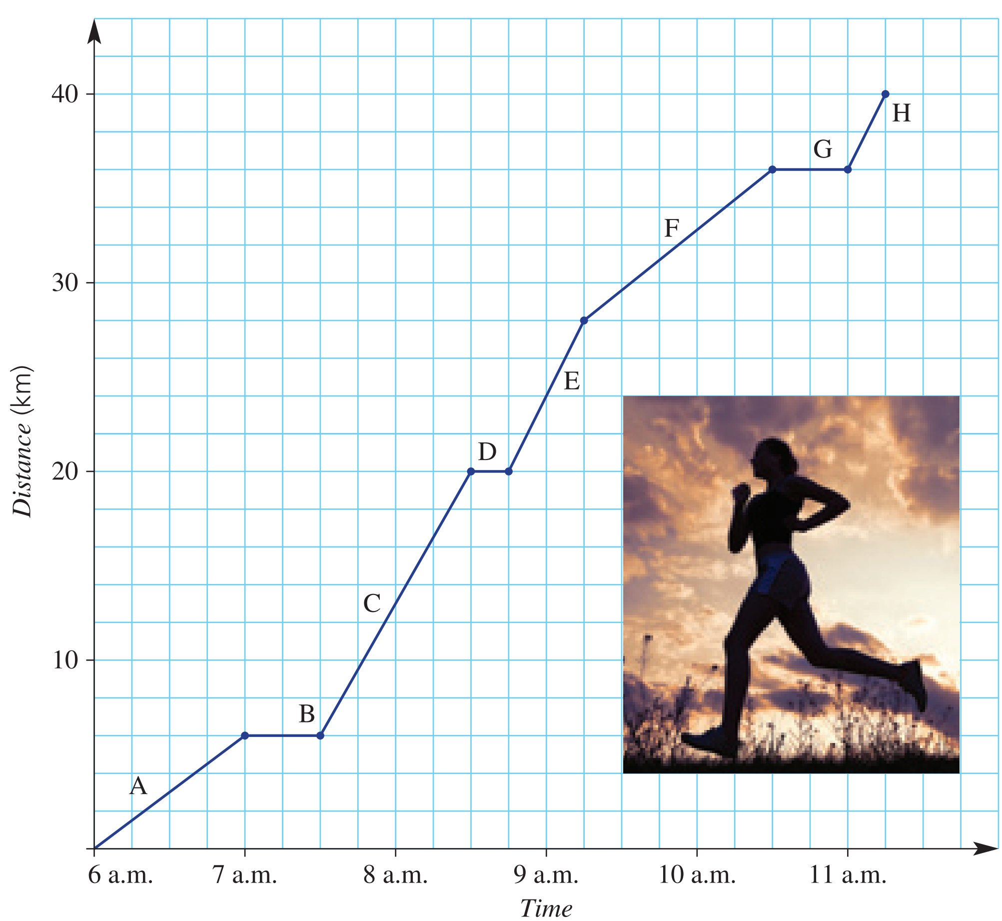
10 A lift starts on the ground floor (height 0 m) and moves to floor 3 at a rate of 3 m/s for 5 seconds. After waiting at floor 3 for 9 seconds, the lift rises 45 m to floor 9 in 9 seconds. The lift rests for 11 seconds before returning to ground level at a rate of 6 m/s. Draw a graph to help find the total time taken to complete the movements described. Use time, in seconds, on the horizontal axis.

Sienna's training — 11

11 Sienna is training for the Sydney Marathon. Her distance–time graph is shown below.

- **a** How many stops does Sienna make?
- **b** How far does she jog between:
  - **i** 6 a.m. and 7 a.m.?
  - **ii** 7:30 a.m. and 8:30 a.m.?
- **c** Which sections of the graph have a zero gradient?
- **d** Which sections of the graph have the steepest gradient?
- **e** At what speed does Sienna run in section:
  i A? **ii** C? **iii** E? **iv** F? **v** H?
- **f** In which sections is Sienna travelling at the same speed? How does the graph show this?
- **g** How long does the training session last?
- **h** What is the total distance travelled by Sienna during the training session?
- **i** What is her average speed for the entire trip, excluding rest periods?

Sienna's training



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Progress quiz

6A

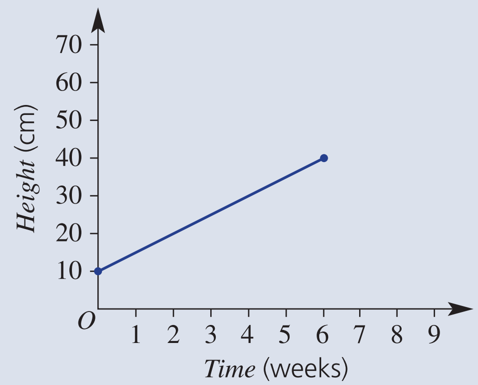
1 The graph at right shows the growth of a plant measured at weekly intervals.

a What is the initial height of the plant?

b How much does the plant grow in 6 weeks?

c Use the graph to find the height of the plant after 4 weeks.

d Extend the graph to estimate the height of the plant after 8 weeks.



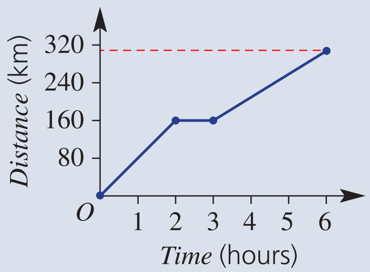
6B/F

2 The graph at right shows the journey of a car.

a What is the distance travelled in the first 2 hours?

b What is the speed in the first 2 hours?

c What is the speed in the final section after the rest break?



6B

3 Sketch a distance–time graph for a runner's training session. Display the following information in your graph:

- covers 6 km in 30 minutes
- 4 km covered in first 15 minutes
- a 5-minute rest after 15 minutes.

6C

4 Plot the graphs of the following by first completing the table of values.

a $y = 3x + 1$

x	-2	-1	0	1	2
y					

b $y = -2x + 3$

x	-2	-1	0	1	2
y					

6C

5 A sailing boat can be hired for \$60 per hour with an initial fee of \$40.

a Complete the table of values.

No. of hours (n)	1	2	3	4	5
Cost (\$$C$)					

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 2.5 hours of hire

ii how long the sailing boat was hired if the cost was \$250

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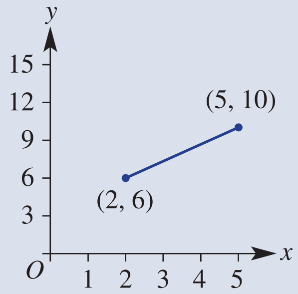
6D **6** Find the midpoint and length of the following.

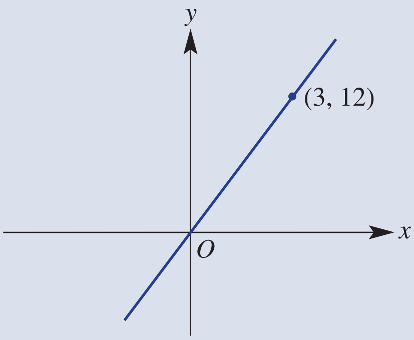
a



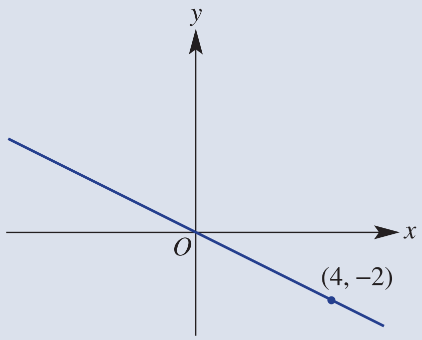
b a line segment joining $(-2, 7)$ and $(4, 12)$

6E **7** Find the gradient of the following lines.

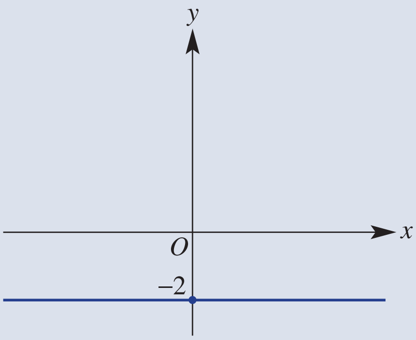
a



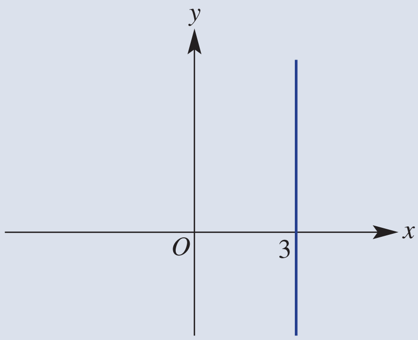
b



c



d



6E **8** Calculate the gradient of the line segment with end points $(-3, 2)$ and $(2, 6)$.

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6G $y = mx + c$ and special lines

Learning intentions

- To know that the y-intercept is the point where a graph crosses the y-axis
- To know how the gradient and y-intercept can be determined from a straight line equation
- To be able to sketch a straight-line graph using the gradient and the y-intercept
- To be able to sketch vertical and horizontal lines and lines passing through the origin from their equation
- To be able to determine if a point is on a line

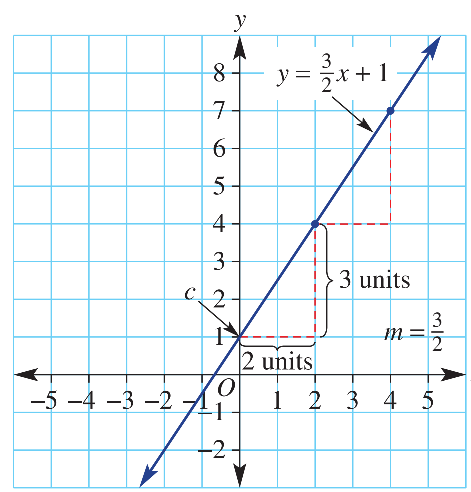
Key vocabulary: gradient–intercept form, gradient, coefficient, y-intercept, horizontal, vertical

Most straight-line graphs can be described by a linear equation $y = mx + c$.

The gradient, m, is the coefficient of x, and c is the y-intercept. This is why this rule is called the gradient–intercept form.

At right is a graph of $y = \frac{3}{2}x + 1$

the gradient $m = \frac{3}{2}$ the y-intercept $c = +1$



Mathematicians use rules and graphs to help determine how many items should be manufactured to make the maximum profit. For example, profit would be reduced by making too many of a certain style of mobile phone that soon will be outdated. A knowledge of graphs is important in business.

Lesson starter: Matching lines with equations

Below are some equations of lines and some graphs. Work with a classmate and help each other to match each equation with its correct line graph.

a $y = 2x - 3$

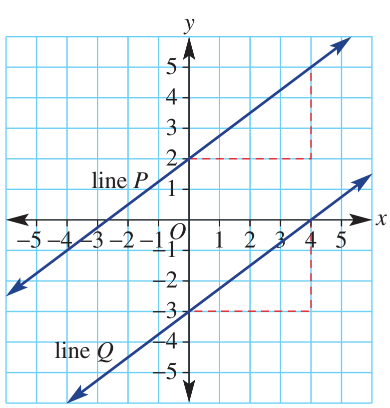
b $y = \frac{3}{4}x + 2$

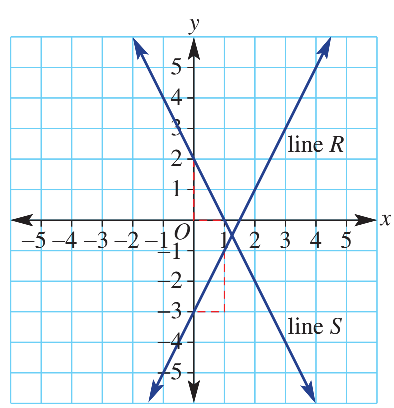
c $y = -\frac{2}{3}x + 1$

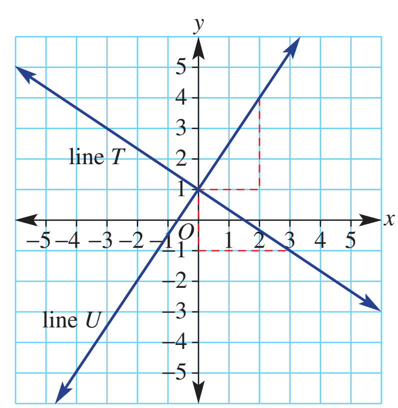
d $y = -2x + 2$

e $y = \frac{3}{4}x - 3$

f $y = \frac{3}{2}x + 1$







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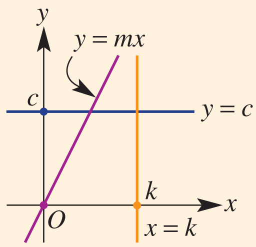
Key ideas

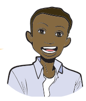
- $y = mx + c$ is called the **gradient–intercept form**, where m and c are constants. Examples are $y = 3x + 4$, $y = \frac{1}{2}x - 3$ and $y = 2$.
  - The **gradient** or slope equals m (the **coefficient** of x).
  - The ***y*-intercept** is the y value at the point where the line cuts the y-axis. In $y = mx + c$, the y-intercept is c.
- Some special lines include:
  - **horizontal** lines: $y = c$ $(m = 0)$
  - **vertical** lines: $x = k$ (m is undefined)
  - lines passing through the origin $(0, 0)$: $y = mx$ $(c = 0)$



Exercise 6G

Understanding | 1–3 | 2, 3

1 Complete the sentences.
- **a** $y = mx + c$ is called the ___________ — ___________ form of a straight line.
- **b** The symbol m stands for the ___________.
- **c** In the equation, the gradient, m, is the ___________ of x.
- **d** The symbol c stands for the ___________.
- **e** A horizontal line has a ___________ gradient.



2 Form rules of the form $y = mx + c$ for the following straight-line graphs:
- **a** gradient $= 3$, y-intercept $= -1$
- **b** gradient $= -\frac{3}{2}$, y-intercept $= 2$
- **c** gradient $= -1$, y-intercept $= 0$

3 Identify the following special lines as: *horizontal*, *vertical* or *passes though the origin*.
- **a** $y = 2$ **b** $x = -3$ **c** $y = 2x$ **d** $x = 0$ **e** $y = -x$

Fluency | 4, 5, 6(½) | 4–7(½)

Example 19 Reading the gradient and *y*-intercept from an equation

For the following equations, state:

i the gradient **ii** the y-intercept

a $y = 3x + 4$

b $y = -\frac{3}{4}x - 7$

Continued on next page

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6G

Solution	Explanation
a **i** Gradient is 3.	The coefficient of x is 3; i.e. $m = 3$.
ii y-intercept is 4.	The value of c is $+4$.
b **i** Gradient is $-\frac{3}{4}$.	The coefficient of x is $-\frac{3}{4}$; i.e. $m = -\frac{3}{4}$.
ii y-intercept is -7.	The value of c is -7; don't forget to include the – sign.

Now you try

For the following equations, state the:

i gradient **ii** y-intercept

a $y = 2x - 5$

b $y = -\frac{1}{3}x + 2$

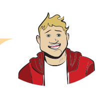
4 For each of the following equations, state the:

i gradient **ii** y-intercept

a $y = 2x + 4$ **b** $y = 6x - 7$

c $y = -\frac{2}{3}x + 7$ **d** $y = -7x - 3$

e $y = \frac{3}{5}x - 8$ **f** $y = 9x - 5$

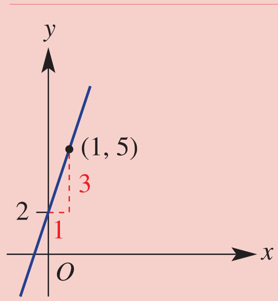


Hint: The gradient is the coefficient of x, which is the number multiplied by x. It does not include the x. Write the y-intercept, including the sign of c.

Example 20 Sketching a line using the y-intercept and gradient

Sketch the graph of $y = 3x + 2$ by considering the y-intercept and the gradient.

Solution



Explanation

Consider $y = mx + c$; the value of c is 2 and therefore the y-intercept is 2.

The value of m is 3 and therefore the gradient is 3 or $\frac{3}{1}$.

Start at the y-intercept 2 and, with the gradient of $\frac{3}{1}$, move 1 right (run) and 3 up (rise) to the point (1, 5).

Join the points in a line.

Now you try

Sketch the graph of $y = 4x - 1$ by considering the y-intercept and the gradient.

5 Sketch the graph of the following by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = 3x - 12$

c $y = x + 4$ **d** $y = -2x + 5$

e $y = -5x - 7$ **f** $y = -x - 4$

Hint: Plot the y-intercept first.
For a line with $m = -2$:
$m = -2 = \frac{-2}{1} = \frac{\text{down } 2}{\text{right } 1}$.
From the y-intercept, go right 1 then down 2 to plot the next point.

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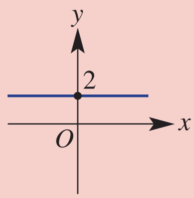
Example 21 Sketching special lines

Sketch the graphs of these equations.

a $y = 2$ **b** $x = -3$ **c** $y = -2x$

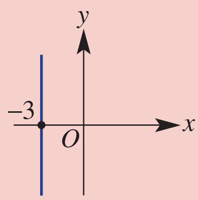
Solution | **Explanation**

a



Sketch a horizontal line passing through $(0, 2)$.

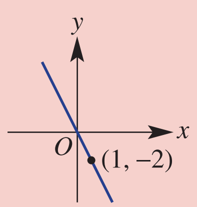
b



Sketch a vertical line passing through $(-3, 0)$.

c When $x = 0$, $y = -2(0) = 0$.

When $x = 1$, $y = -2(1) = -2$.



The line passes through the origin $(0, 0)$.

Use $x = 1$ to find another point.

Sketch the graph passing through $(0, 0)$ and $(1, -2)$.

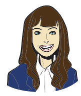
Now you try

Sketch the graphs of these equations.

a $y = -3$ **b** $x = 1$ **c** $y = 4x$

6 Sketch the following lines.

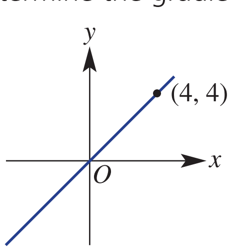
a $y = 4$ **b** $y = -2$ **c** $y = 5$
d $x = 5$ **e** $x = -2$ **f** $x = 9$
g $y = 3x$ **h** $y = 6x$ **i** $y = -2x$



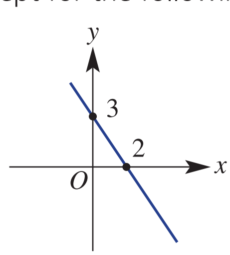
Hint: For the line equation $y = 2$, every point on the line has a y value of 2; e.g. $(-3, 2)$ $(0, 2)$ $(1, 2)$ $(3, 2)$. For the line equation $x = -3$, every point on the line has an x value of -3; e.g. $(-3, 1)$ $(-3, 0)$ $(-3, -4)$.

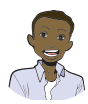
7 Determine the gradient and y-intercept for the following lines.

a



b





Hint: Use $m = \frac{\text{rise}}{\text{run}}$ for the gradient between two known points.

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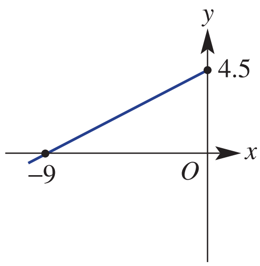
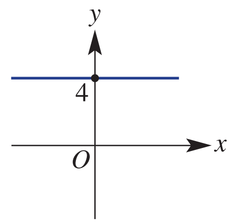
6G

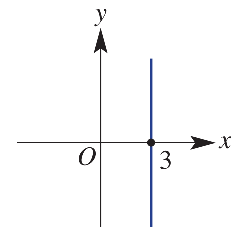
c



d



e

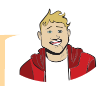


Problem-solving and reasoning

8, 9, 10(½), 12 | 8, 11–13

8 Match each of the following linear equations to one of the sketches shown.

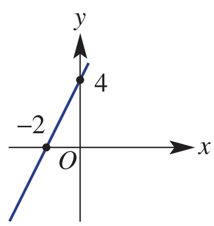
a $y = -\frac{2}{3}x + 2$ **b** $y = -x + 4$ **c** $y = x + 3$
d $y = 2x + 4$ **e** $y = 4$ **f** $y = 7x$
g $y = -3x + 6$ **h** $x = 2$ **i** $y = -3x$



Hint: A linear equation is an equation that gives a straight-line graph.

Hint: For a negative gradient, move the negative sign to the numerator.
$m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{down } 2}{\text{right } 3}$

i



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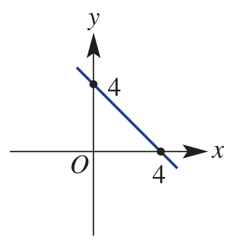

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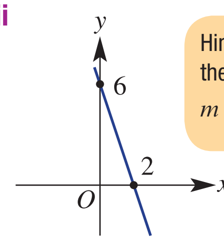

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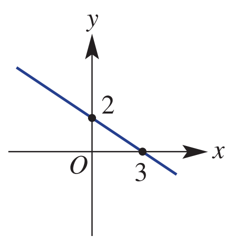

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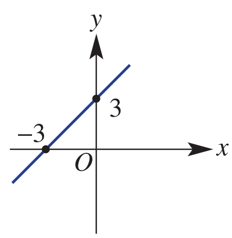

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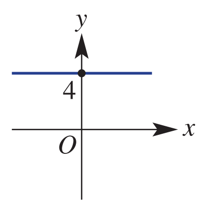

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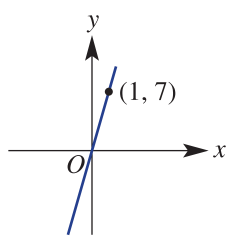

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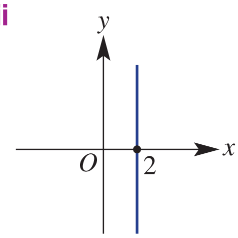

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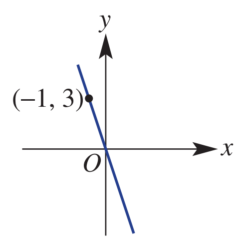

9 **a** Write down three different equations that have a graph with a y-intercept of 5.
b Write down three different equations that have a graph with a y-intercept of -2.

10 **a** Write down three different equations that have a graph with a gradient of 3.
b Write down three different equations that have a graph with a gradient of -1.
c Write down three different equations that have a graph with a gradient of 0.
d Write down three different equations that have a graph with an undefined gradient.

11 **a** Which of the following points lie on the line $y = 2$?
i $(2, 3)$ **ii** $(1, 2)$ **iii** $(5, 2)$ **iv** $(-2, -2)$
b Which of the following points lie on the line $x = 5$?
i $(5, 3)$ **ii** $(3, 5)$ **iii** $(1, 7)$ **iv** $(5, -2)$

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Example 22 Identifying points on a line

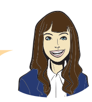
Does the point $(3, -4)$ lie on the line $y = 2x - 7$?

Solution	Explanation
$y = 2x - 7$	Copy the equation and substitute $x = 3$.
$y = 2 \times 3 - 7$	The y value for $x = 3$ is $y = -1$.
$y = -1$	Compare the y values.
$\neq -4$	The point $(3, -1)$ is *on* the line.
No, $(3, -4)$ is not on the line.	So $(3, -4)$ is *not* on the line.

Now you try

Does the point $(-2, 5)$ lie on the line $y = 3x + 2$?

12 **a** Does the point $(3, 2)$ lie on the line $y = x + 2$?
b Does the point $(-2, 0)$ lie on the line $y = x + 2$?
c Does the point $(1, -5)$ lie on the line $y = 3x + 2$?
d Does the point $(2, 2)$ lie on the line $y = x$?
e Does the line $y - 2x = 0$ pass through the origin?



Hint: Substitute the x value into the equation and compare the two y values. When the y values are the same, the point is on the line.

13 Draw each of the following on a number plane and write down the equation of the line.

a

x	0	1	2	3
y	4	5	6	7

b

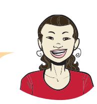
x	0	1	2	3
y	−1	0	1	2

c

x	−2	0	4	6
y	−1	0	2	3

d

x	−2	0	2	4
y	−3	1	5	9



Hint: Use your graph to find the gradient between two points (m) and locate the y-intercept (c). Then use $y = mx + c$.

Graphs using technology — 14, 15

14 Use technology to sketch a graph of these equations.

a $y = x + 2$ **b** $y = -4x - 3$ **c** $y = \frac{1}{2}x - 1$ **d** $y = 1.5x + 3$
e $y = 2x - 5$ **f** $y = 0.5x + 5$ **g** $y = -0.2x - 3$ **h** $y = 0.1x - 1.4$

15 **a** On the same set of axes, plot graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$ and $y = 2x - 3$, using a calculator.
Discuss what you see and describe the connection with the given equations.

b On the same set of axes, plot graphs of $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$, $y = \frac{1}{2}x - 1$ and $y = \frac{3}{4}x - 1$, using a calculator.
Discuss what you see and describe the connection with the given equations.

c The equations of families of graphs can be entered into a calculator using one line only. For example, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$ can be entered as $y = 2x + \{1, 2, 3\}$ using set brackets. Use this notation to draw the graphs of the rules in parts **a** and **b**.

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6H Parallel and perpendicular lines

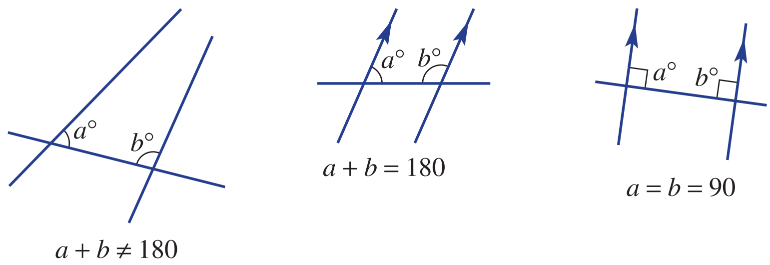
Learning intentions

- To know that parallel lines have the same gradient
- To know the relationship between the gradients of perpendicular lines
- To be able to determine if lines are parallel or perpendicular using their gradients
- To be able to find the equation of parallel or perpendicular lines

Key vocabulary: parallel, perpendicular, gradient, reciprocal, y-intercept

Euclid of Alexandria (300 BCE) was a Greek mathematician and is known as the 'Father of geometry'. In his texts, known as *Euclid's Elements*, his work is based on five simple axioms.

His fifth axiom, the Parallel Postulate, says that if cointerior angles do not sum to 180° then the two lines are not parallel. Furthermore, if the two interior angles are equal and also sum to 180°, then the third line must be perpendicular to the other two.

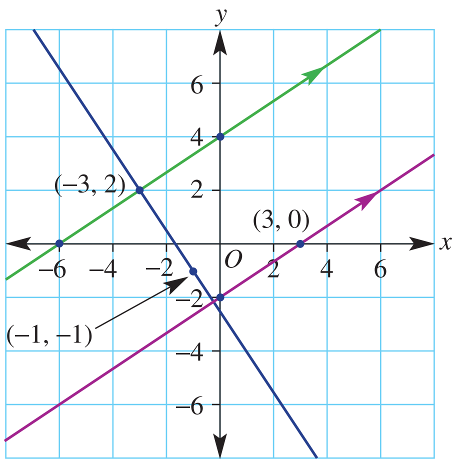


Parallel and perpendicular lines, including their gradient and rule, are the focus of this section.

Lesson starter: Gradient connection

Shown here is a pair of parallel lines and a third line that is perpendicular to the other two lines.

- Find the gradient of each line using the coordinates shown on the graph.
- What is common about the gradients for the two parallel lines?
- Is there any connection between the gradients of the parallel lines and the perpendicular line? Can you write down this connection as a formula?



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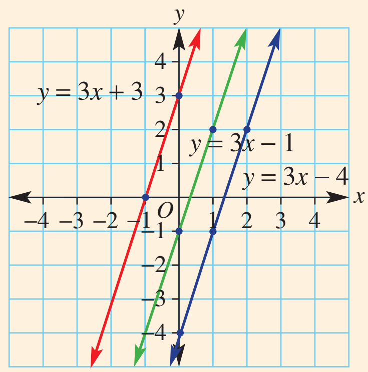
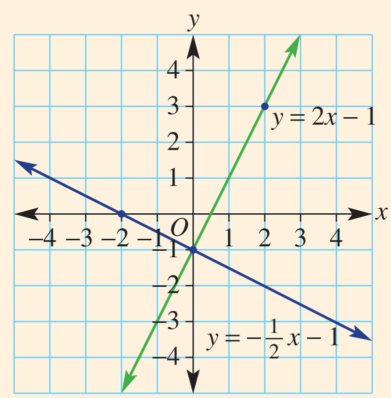
Key ideas

- Two **parallel lines** have the same gradient as they point in the same direction.
  For example, $y = 3x - 1$ and $y = 3x + 3$ have the same gradient of 3.



- Two **perpendicular lines** (lines that are at right angles to each other) with gradients m_1 and m_2 satisfy the following rule:
  $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$ (i.e. m_2 is the negative reciprocal of m_1).
  In the graph shown, $m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$.

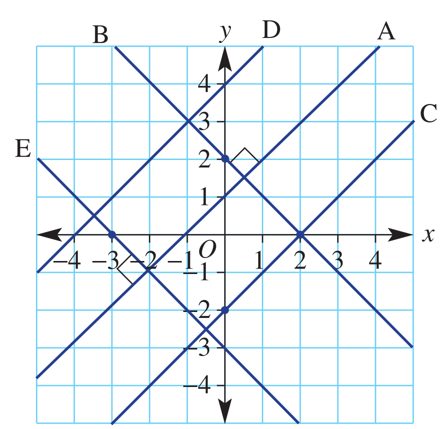


- Equations of parallel or perpendicular lines can be found by:
  - first finding the gradient (m)
  - then substituting a point to find c in $y = mx + c$

Exercise 6H

Understanding 1–3 3

1 For this diagram:



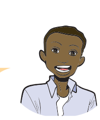
a Which lines, B, C, D or E, are parallel to line A?
b Which lines, B, C, D or E, are perpendicular to line A?
c Are lines C and D parallel?
d Are lines B and E parallel?
e Are lines E and C perpendicular?

2 Write down the gradient of a line that is parallel to the graph of these equations.

a $y = 4x - 6$ **b** $y = -7x - 1$
c $y = -\frac{3}{4}x + 2$ **d** $y = \frac{8}{7}x - \frac{1}{2}$

3 Use $m_2 = -\frac{1}{m_1}$ to find the gradient of the line that is perpendicular to the graphs of these equations.

a $y = 3x - 1$ **b** $y = -2x + 6$
c $y = \frac{7}{8}x - \frac{2}{3}$ **d** $y = -\frac{4}{9}x - \frac{4}{7}$



Hint: In part **a**, $m_1 = 3$ so find m_2, which is the perpendicular gradient. Note: $-\frac{1}{\left(\frac{7}{8}\right)} = -1 \div \frac{7}{8} = -1 \times \frac{8}{7}$

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6H

Fluency

4–6(½) 4–6(½)

Example 23 Deciding if lines are parallel or perpendicular

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = 4x + 2$ and $y = 4x - 6$

b $y = -3x - 8$ and $y = \frac{1}{3}x + 1$

Solution	Explanation
a $y = 4x + 2, m = 4$ (1) $y = 4x - 6, m = 4$ (2)	Note that both equations are in the form $y = mx + c$.
So the lines are parallel.	Both lines have a gradient of 4, so the lines are parallel.
b $y = -3x - 8, m = -3$ (1) $y = \frac{1}{3}x + 1, m = \frac{1}{3}$ (2)	Both equations are in the form $y = mx + c$.
$-3 \times \frac{1}{3} = -1$	Test $m_1 \times m_2 = -1$.
So the lines are perpendicular.	

Now you try

Decide if the graph of each pair of rules will be parallel, perpendicular or neither.

a $y = -3x + 2$ and $y = -3x - 7$

b $y = 4x - 2$ and $y = -\frac{1}{4}x + 1$

4 Decide if the line graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = 2x - 1$ and $y = 2x + 1$

b $y = 3x + 3$ and $y = -\frac{1}{3}x + 1$

c $y = 5x + 2$ and $y = 6x + 2$

d $y = -4x - 1$ and $y = \frac{1}{5}x - 1$

e $y = 3x - 1$ and $y = 3x + 7$

f $y = \frac{1}{2}x - 6$ and $y = \frac{1}{2}x - 4$

g $y = -\frac{2}{3}x + 1$ and $y = \frac{2}{3}x - 3$

h $y = -4x - 2$ and $y = x - 7$

i $y = -\frac{3}{7}x - \frac{1}{2}$ and $y = \frac{7}{3}x + 2$

j $y = -8x + 4$ and $y = \frac{1}{8}x - 2$

Hint: If the gradients are equal, then the lines are parallel. If $m_1 \times m_2 = -1$, then the lines are perpendicular.

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Example 24 Finding the equation of a parallel or perpendicular line when given the y-intercept

Find the equation of the line, given the following description.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 3.
b A line passes through $(0, -1)$ and is perpendicular to another line with gradient -2.

Solution	Explanation
a $m = 3$ and $c = 2$	A parallel line has the same gradient.
So $y = 3x + 2$.	Use $y = mx + c$ with $m = 3$ and y-intercept 2.
b $m = -\left(\frac{1}{-2}\right) = \frac{1}{2}$ and $c = -1$	Being perpendicular, use $m_2 = -\frac{1}{m_1}$.
So $y = \frac{1}{2}x - 1$.	Note also that the y-intercept is -1.

Now you try

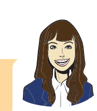
Find the equation of the line, given the following description.

a A line passes through $(0, 4)$ and is parallel to another line with gradient 2.
b A line passes through $(0, 3)$ and is perpendicular to another line with gradient 4.

5 Find the equation of the lines with the following description.

Hint: In $y = mx + c$, m is the gradient and c is the y-intercept.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 4.
b A line passes through $(0, 4)$ and is parallel to another line with gradient 2.
c A line passes through $(0, -3)$ and is parallel to another line with gradient -1.
d A line passes through $(0, 3)$ and is perpendicular to another line with gradient 2.
e A line passes through $(0, -5)$ and is perpendicular to another line with gradient 3.
f A line passes through $(0, -10)$ and is perpendicular to another line with gradient $\frac{1}{2}$.



Hint: $-\frac{1}{\left(\frac{1}{2}\right)} = -2$

g A line passes through $(0, 6)$ and is perpendicular to another line with gradient $\frac{1}{6}$.
h A line passes through $(0, -7)$ and is perpendicular to another line with gradient $-\frac{1}{4}$.

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6H

Example 25 Finding the equation of a parallel or perpendicular line

Find the equation of the line that is:

a parallel to $y = -2x - 7$ and passes through $(1, 9)$

b perpendicular to $y = \frac{1}{4}x - 1$ and passes through $(3, -2)$

Solution	Explanation
a $y = mx + c$ $m = -2$ $y = -2x + c$	Since the line is parallel to $y = -2x - 7$, $m = -2$. Write the equation $y = mx + c$ with $m = -2$.
Substitute $(1, 9)$: $9 = -2(1) + c$ $11 = c$ $\therefore y = -2x + 11$	Substitute the given point $(1, 9)$ where $x = 1$ and $y = 9$ and solve for c.
b $y = mx + c$ $m = -\frac{1}{\frac{1}{4}}$	The gradient is the negative reciprocal of $\frac{1}{4}$.
$= -1 \times \frac{4}{1}$ $= -4$ $y = -4x + c$	$-1 \div \frac{1}{4} = -1 \times \frac{4}{1}$.
Substitute $(3, -2)$: $-2 = -4(3) + c$ $-2 = -12 + c$ $c = 10$ $\therefore y = -4x + 10$	Substitute $(3, -2)$ and solve for c.

Now you try

Find the equation of the line that is:

a parallel to $y = 3x - 4$ and passes through $(2, 3)$

b perpendicular to $y = -\frac{1}{2}x + 2$ and passes through $(-1, 4)$

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6 Find the equation of the line that is:

a parallel to $y = x + 3$ and passes through $(1, 5)$

b parallel to $y = -x - 5$ and passes through $(1, 7)$

c parallel to $y = -4x - 1$ and passes through $(-1, 3)$

d parallel to $y = \frac{2}{3}x + 1$ and passes through $(3, -4)$

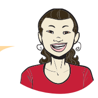
e perpendicular to $y = 2x + 3$ and passes through $(2, 5)$

f perpendicular to $y = -4x + 1$ and passes through $(-4, -3)$

g perpendicular to $y = \frac{2}{3}x - 4$ and passes through $(4, -1)$

h perpendicular to $y = -\frac{2}{7}x - \frac{3}{4}$ and passes through $(-8, 3)$

Hint: First find m. Then write $y = mx + c$. Substitute a point to find c; e.g. for $(1, 5)$ substitute $x = 1$ and $y = 5$.



Problem-solving and reasoning

7, 8	7, 9, 10

7 The lines on this grid are either parallel or perpendicular to each other.

a What is the gradient of each of these lines?

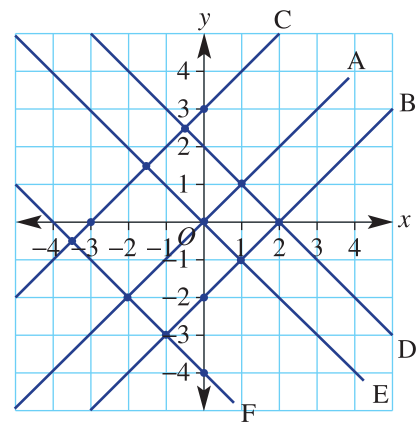
i A **ii** D

b What is the equation for the following lines?

i A **ii** B

iii C **iv** D

v E **vi** F



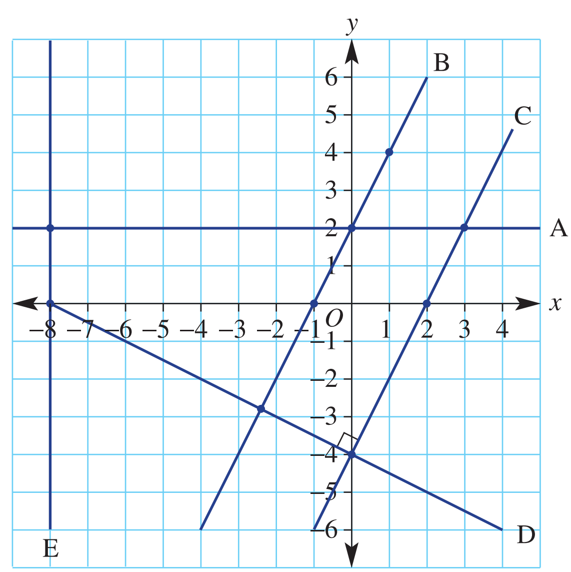
8 In its original position, the line A has equation $y = 2$, as shown.

a Line A is rotated to form line B. What is its new rule?

b Line B is shifted to form line C. What is its new rule?

c Line C is rotated 90° to form line D. What is its new rule?

d Line A is rotated 90° to form line E. What is its new rule?



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6H

9 Recall that the negative reciprocal of, say, $\frac{2}{3}$ is $-\frac{3}{2}$.

Use this to help find the equation of a line that:

a passes through $(0, 7)$ and is perpendicular to $y = \frac{2}{3}x + 3$

b passes through $(0, -2)$ and is perpendicular to $y = \frac{3}{2}x + 1$

c passes through $(0, 2)$ and is perpendicular to $y = -\frac{4}{5}x - 3$

d passes through $(1, -2)$ and is perpendicular to $y = -\frac{2}{3}x - 1$

10 Decide if the graphs of each pair of rules will be parallel, perpendicular or neither.

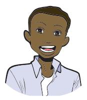
a $2y + x = 2$ and $y = -\frac{1}{2}x - 3$

b $x - y = 4$ and $y = x + \frac{1}{2}$

c $8y + 2x = 3$ and $y = 4x + 1$

d $3x - y = 2$ and $x + 3y = 5$

Hint: First write each rule in the form $y = mx + c$.



Perpendicular and parallel geometry — 11, 12

11 A quadrilateral, $ABCD$, has vertex coordinates $A(2, 7)$, $B(4, 9)$, $C(6, 3)$ and $D(8, 5)$.

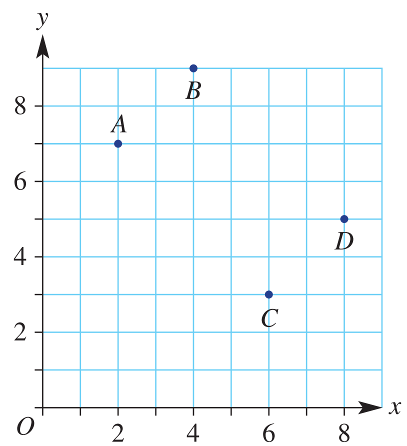
a Find the gradient of these line segments.

i AB ii CD

iii BD iv AC

b What do you notice about the gradient of the opposite sides?

c What type of quadrilateral is $ABCD$?



12 The vertices of triangle ABC are $A(0, 0)$, $B(3, 4)$ and $C\left(\frac{25}{3}, 0\right)$.

a Find the gradient of these line segments.

i AB ii BC iii CA

b What type of triangle is $\triangle ABC$?

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6I Sketching with x- and y-intercepts

Learning intentions

- To know that the x-intercept is the point where a graph crosses the x-axis
- To know how to find the x and y intercepts of a straight-line graph from its equation
- To know that only two points are needed to sketch a straight line
- To be able to sketch a linear graph using its x and y intercepts

Key vocabulary: x-intercept, y-intercept, x-axis, y-axis

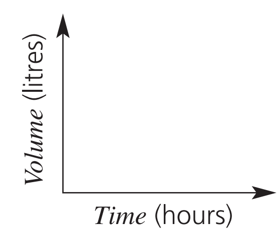
Only two points are required to define a straight line. Two convenient points are the x- and y-intercepts. These are the points where the graph crosses the x- and y-axis, respectively.

The axis intercepts are quite significant in practical situations. For example, imagine that 200 m^3 of dirt needs to be removed from a construction site before foundations for a new building can be laid. The graph of volume remaining versus time taken to remove the dirt has a y-intercept of 200, showing the total volume to be removed, and an x-intercept showing the time taken for the job.

Lesson starter: Leaking water

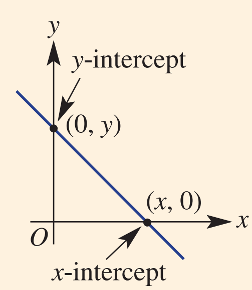
A family that is camping has 20 litres of water in a plastic container. The water begins to slowly leak at a rate of 2 litres per hour.

- Sketch two axes labelled 'Volume' and 'Time'.
- At $t = 0$, what is the volume of water in the container? Mark and label this point on the Volume axis.
- How long will it take the water container to be empty? Mark this point on the Time axis.
- Join these two points with a straight line.
- Write the coordinates of the Volume axis intercept and the Time axis intercept. Follow the order (i.e. Time, Volume).
- Can you suggest a rule for finding the volume of the water in the container after t hours?



Key ideas

- A straight line can be sketched by finding axis intercepts.
- The ***x*-intercept** (where the line cuts the x-axis) is where $y = 0$.
  Find the x-intercept by substituting $y = 0$ into the equation.
- The ***y*-intercept** is where $x = 0$.
  Find the y-intercept by substituting $x = 0$ into the equation.
- Once the axis intercepts are found, plot the points and join to form the straight-line graph.



Exercise 6I

Understanding | 1, 2 | 1

1 Copy and complete these sentences.

a The x-intercept is where _____ $= 0$. **b** The y-intercept is where _____ $= 0$.

2 Plot the following x- and y-intercept coordinates and join in a straight line to form the graph.

a $(0, 3)$ and $(-2, 0)$ **b** $(0, -1)$ and $(2, 0)$ **c** $(0, -4)$ and $(-1, 0)$ **d** $(0, 3)$ and $(5, 0)$

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6I

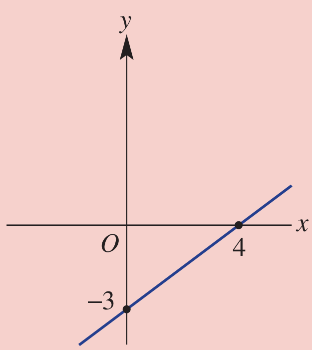
Fluency

3(½), 4, 5 | 3(½), 4, 5(½), 6

Example 26 Sketching lines in the form $ax + by = d$, using x- and y-intercepts

Sketch a graph of $3x - 4y = 12$ by finding the x- and y-intercepts.

Solution		Explanation
$3x - 4y = 12$		
x-intercept ($y = 0$):	$3x - 4(0) = 12$	Find the x-intercept by substituting $y = 0$.
	$3x = 12$	Simplify. Any number multiplied by 0 is 0.
	$x = 4$	Divide both sides by 3 to solve for x.
y-intercept ($x = 0$):	$3(0) - 4y = 12$	Find the y-intercept by substituting $x = 0$.
	$-4y = 12$	Simplify and retain the negative sign.
	$y = -3$	Divide both sides by -4 to solve for y.



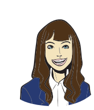
Sketch the graph by first marking the x-intercept $(4, 0)$ and the y-intercept $(0, -3)$ and join them with a line.

Now you try

Sketch a graph of $5x + 3y = 15$ by finding the x- and y-intercepts.

3 Sketch graphs of the following equations by finding the x- and y-intercepts.

- **a** $3x - 2y = 6$
- **b** $2x + 6y = 12$
- **c** $3x - 4y = 12$
- **d** $5x - 2y = 20$
- **e** $-2x + 7y = 14$
- **f** $-x + 3y = 3$
- **g** $-x - 2y = 8$
- **h** $-5x - 9y = 90$

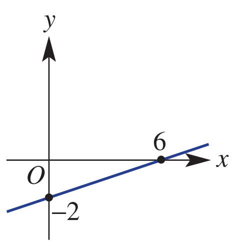


Hint: Two calculations are required: Substitute $y = 0$ and solve for x-intercept. Substitute $x = 0$ and solve for y-intercept.

4 Match each of the following linear equations of the form $ax + by = d$ to one of the graphs shown.

- **a** $x + y = 3$
- **b** $2x - y = 4$
- **c** $x - 3y = 6$

i



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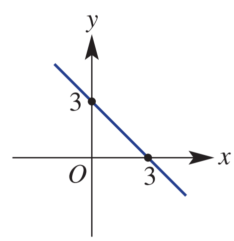

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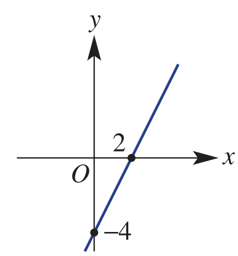

Hint: First find the x- and y-intercepts for the line equation.

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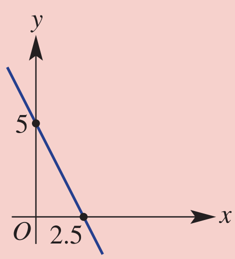
Example 27 Sketching lines in the form $y = mx + c$, using x- and y-intercepts

Sketch the graph of $y = -2x + 5$ by finding the x- and y-intercepts.

Solution		**Explanation**
$y = -2x + 5$		
x-intercept $(y = 0)$:	$0 = -2x + 5$	Substitute $y = 0$.
	$-5 = -2x$	Subtract 5 from both sides.
	$x = 2.5$	Divide both sides by -2.
y-intercept is $(x = 0)$:	$y = -2(0) + 5$	Substitute $x = 0$.
	$= 5$	Simplify.



Sketch the graph by first marking the x-intercept $(2.5, 0)$ and the y-intercept $(0, 5)$.

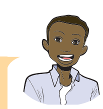
Now you try

Sketch the graph of $y = 3x - 9$ by finding the x- and y-intercepts.

5 Sketch graphs of the following equations by finding the x- and y-intercepts.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = -4x - 3$

d $y = -x + 2$ **e** $y = -\frac{1}{2}x + 1$ **f** $y = \frac{3}{2}x - 3$



Hint: **5e:** $0 = -\frac{1}{2}x + 1$

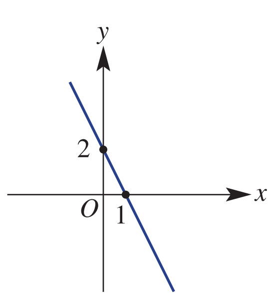
$\frac{1}{2}x = 1$

Now multiply both sides by 2.

6 Match each of the following linear equations of the form $y = mx + c$ to one of the sketches shown.

a $y = x + 1$ **b** $y = 3x - 1$ **c** $y = -2x + 2$

i



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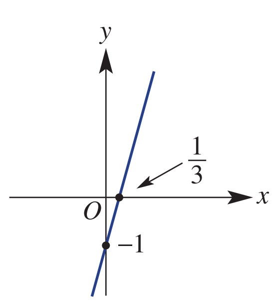

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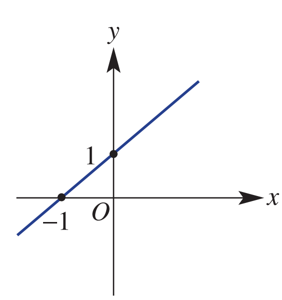

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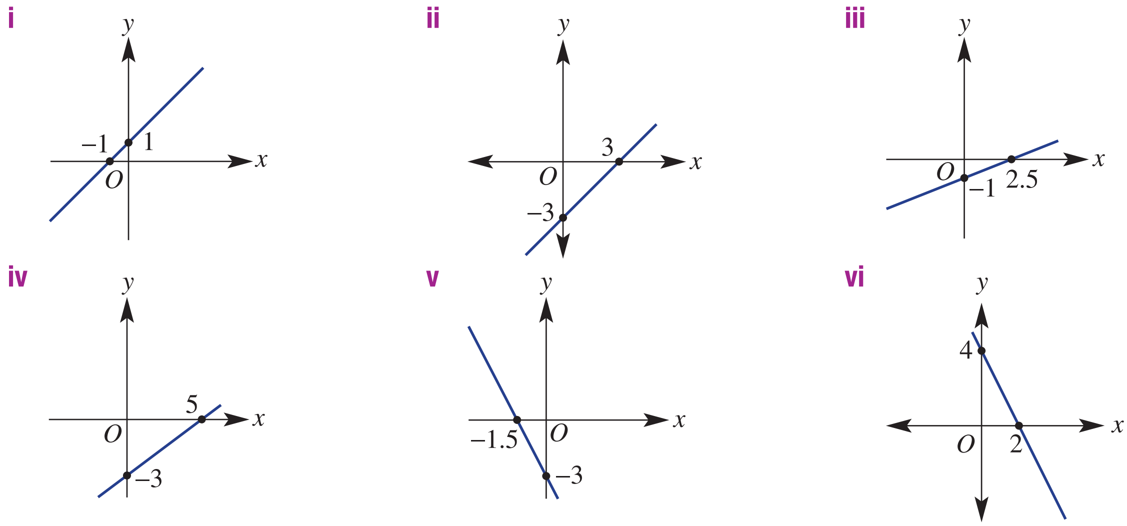
6I

Problem-solving and reasoning 7–9 8–11

7 Match each of the following linear equations to one of the graphs shown.

a $2x+y=4$ b $x-y=3$ c $y=x+1$

d $y=-2x-3$ e $3x-5y=15$ f $y=\frac{2}{5}x-1$



8 By first finding the x- and y-intercepts of the graphs of these equations, find the gradient in each case.

a $2x+y=4$ b $x-5y=10$ c $4x-2y=5$ d $-1.5x+3y=4$

Hint: A quick sketch of each line and the axis intercepts will help to show the rise and run.

9 For the graphs of each of the following equations, find:

i the x- and y-intercepts

ii the area of the triangle enclosed by the x- and y-axes and the graph of each equation

Remember that the area of a triangle is $A=\frac{1}{2}bh$.

a $2x-y=4$ b $-3x+3y=6$ c $y=-2x-3$ d $y=\frac{1}{2}x+2$

10 The height, h, in metres, of a lift above ground after t seconds is given by $h=90-12t$.

a How high is the lift initially (i.e. at $t=0$)?

b How long does it take for the lift to reach the ground (i.e. at $h=0$)?

11 If $ax+by=d$, find a set of numbers for a, b and d that give an x-intercept of $(2, 0)$ and y-intercept of $(0, 4)$.

Hint: Use trial and error to start.

Axes intercepts using technology — 12

12 For the following rules, use technology to sketch a graph and find the x- and y-intercepts.

a $y=2x-4$ b $y=-2x-10$ c $y=-x+1$

d $y+2x=4$ e $2y-3x=12$ f $3y-2x=2$

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6J Linear modelling

Learning intentions

- To be able to find the equation of a straight-line graph
- To be able to form a linear model from a word problem relating two variables and sketch its graph
- To be able to use a linear model to solve a problem

Key vocabulary: linear, gradient, y-intercept, modelling

When given at least two points, you can find the equation of a straight line.
If the relationship between two variables is linear, then:

- the graph of the relation is a straight line
- a rule can be written in the form $y = mx + c$.

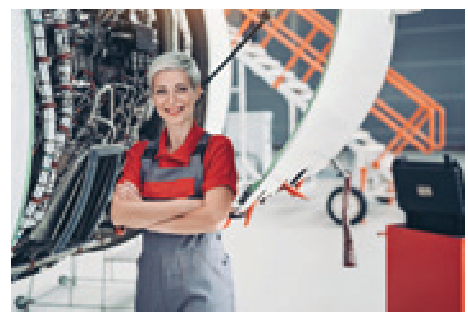
Lesson starter: Trainee pay

Isabella has a trainee scholarship to complete her apprenticeship as a mechanic. She is paid $\$50$ per week plus $\$8$/h for work at the garage.

Isabella's weekly wage can be modelled by the rule:

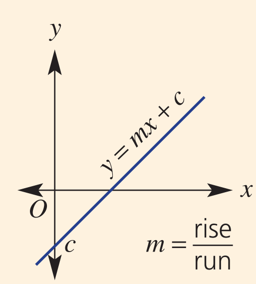
Wage $= 8t + 50$, where t is the number of hours worked in a week.

- Explain why the rule for Isabella's wage is Wage $= 8t + 50$.
- Show how the rule can be used to find Isabella's wage after $10, 20$ and 35 hours.
- Show how the rule can be used to find how long Isabella worked if she earned $\$114$, $\$202$ and $\$370$.



Key ideas

- The equation of a straight line can be determined using:
  - $y = mx + c$
  - gradient $= m = \frac{\text{rise}}{\text{run}}$
  - y-intercept $= c$



- If the y-intercept is not obvious, then it can be found by substituting a point.
- Vertical and horizontal lines:
  - vertical lines have the equation $x = k$, where k is the x-intercept
  - horizontal lines have the equation $y = c$, where c is the y-intercept
- **Modelling** may involve:
  - writing a rule linking two variables
  - sketching a graph
  - using the rule or the graph to help solve related problems

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6J

Exercise 6J

Understanding | 1, 2 | 2

1 Each week Ava gets paid $30 plus $15 per hour. Decide which rule shows the relationship between Ava's total weekly pay, P, and the number of hours she works, n.

A $P = 30 + n$ **B** $P = 15n$ **C** $P = 30 + 15$ **D** $P = 30 + 15n$

2 Riley is 100 km from home and is cycling home at 20 km/h. Decide which rule shows the relationship between Riley's distance from home, d km, and the number of hours he has been cycling, t.

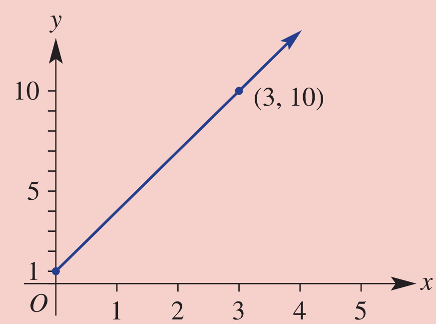
A $P = 100t$ **B** $P = 100 - 20$ **C** $P = 100 - 20t$ **D** $P = 100 + 20t$

Fluency | 3–6(½) | 3–7(½)

Example 28 Finding the equation of a line from a graph with a known *y*-intercept

A straight line passes through the points shown.

a Determine its gradient.
b Find the y-intercept.
c Write the equation of the line.



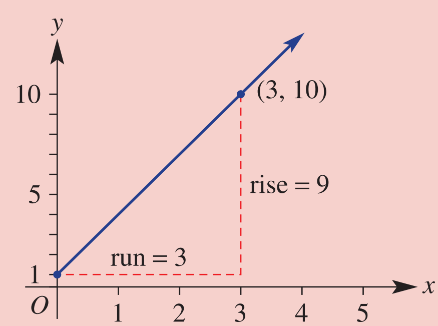
Solution | **Explanation**

a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{9}{3}$

$= 3$

Draw a triangle on the graph and decide whether the gradient is positive or negative.



b The y-intercept is 1, so $c = 1$.

Look at where the graph meets the y-axis.

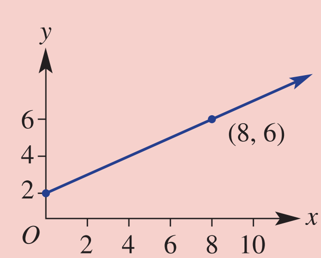
c $y = 3x + 1$

Substitute m and c into $y = mx + c$.

Now you try

A straight line passes through the points shown.

a Determine its gradient.
b Find the y-intercept.
c Write the equation of the line.



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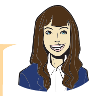
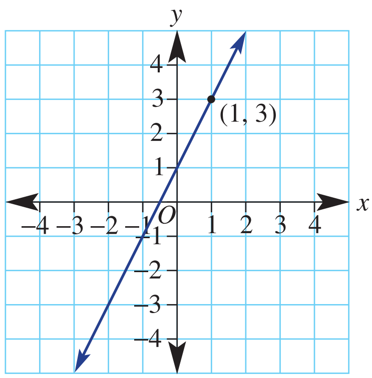
3 The graphs below show straight lines.
  i Determine the gradient of each.
  ii Find the y-intercept.
  iii Write the equation of the line.

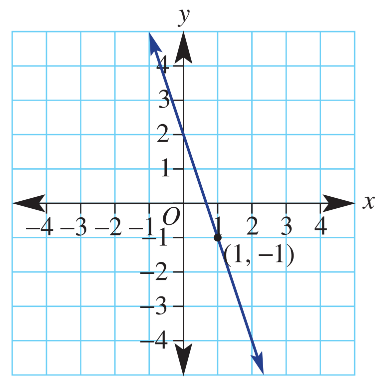


Hint: To find the rise and run, form a right-angled triangle using the y-intercept and the second point.

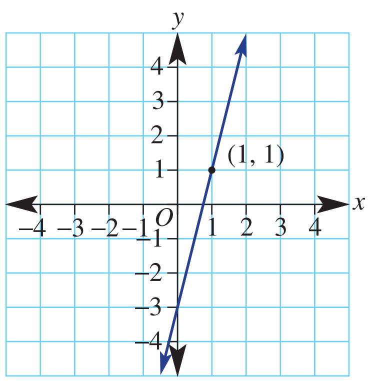
a



b



c



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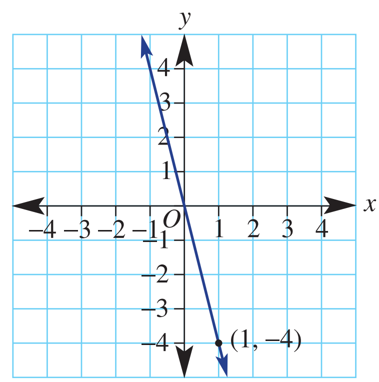
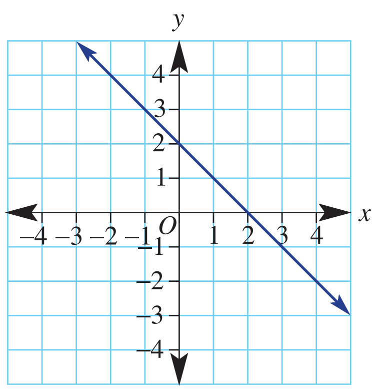
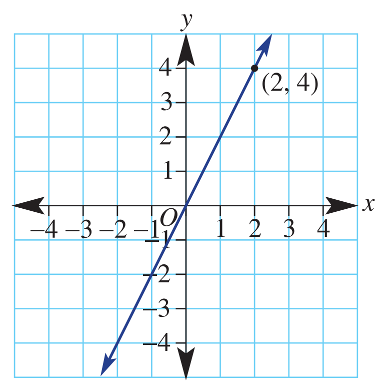


e

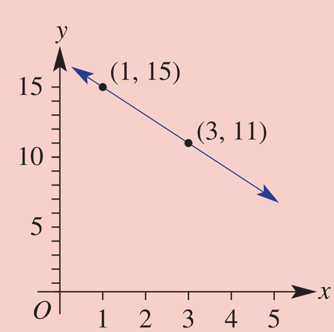


f



Example 29 Finding the equation of a line when given a graph with two known points

A straight line passes through the points shown.
a Determine the gradient.
b Find the y-intercept.
c Write the equation of the line.



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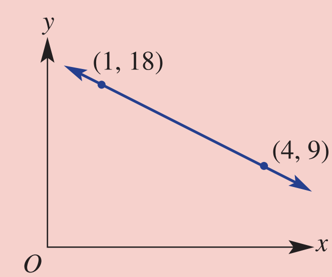
Solution	Explanation
a $m = \frac{\text{rise}}{\text{run}}$ $= \frac{-4}{2}$ $= -2$	The gradient is negative. Run $= 3 - 1 = 2$ Rise $= 11 - 15 = -4$ A 'fall' of 4 means rise $= -4$. Simplify.
b $y = -2x + c$ $11 = -2(3) + c$ $11 = -6 + c$ $17 = c$, so the y-intercept is 17.	Write $y = mx + c$ using $m = -2$. Substitute a chosen point into $y = -2x + c$, (use $(3, 11)$ or $(1, 15)$). Here, $x = 3$ and $y = 11$. Simplify and solve for c.
c $y = -2x + 17$	Substitute $m = -2$ and $c = 17$ into $y = mx + c$.

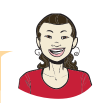
Now you try

A straight line passes through the points shown.

a Determine the gradient.
b Find the y-intercept.
c Write the equation of the line.

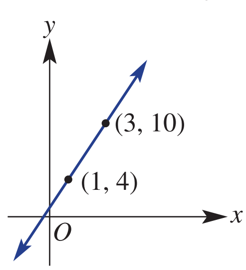


4 Straight lines passing through two points are shown below.
- **i** Determine the gradient.
- **ii** Find the y-intercept.
- **iii** Write the equation of the line.

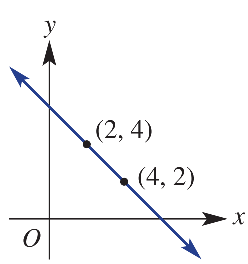


Hint:
For m, find the rise and run between the two given points.
Choose either point to substitute when finding c. If $m = 4$ and $(3, 5) = (x, y)$:
$y = mx + c$.
$5 = 4 \times 3 + c$
Solve for c.

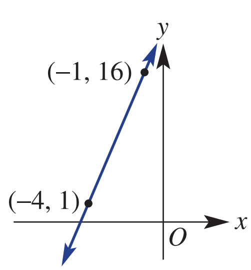
a



b



c



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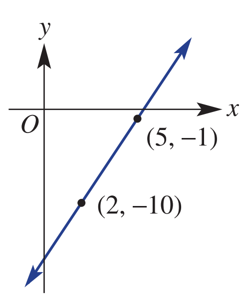

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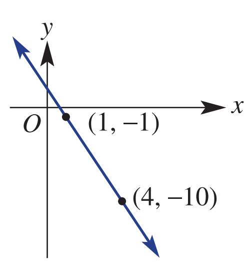

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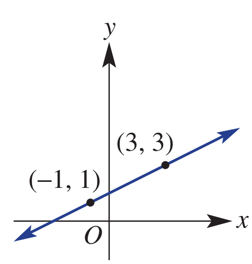

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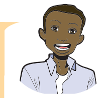
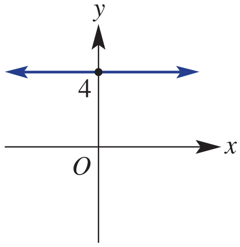
5 Determine the equation of each of the following lines. Remember from Section 6G that vertical and horizontal lines have special equations.

Hint: Vertical lines cut the x-axis and have an equation such as $x = 3$. Horizontal lines cut the y-axis and have an equation such as $y = -4$.



a



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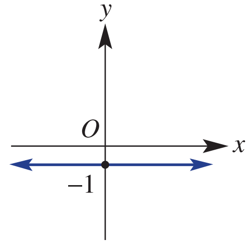

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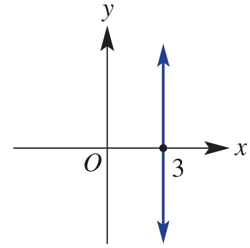

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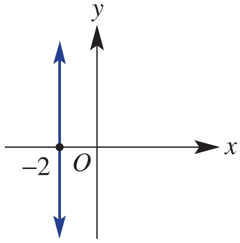

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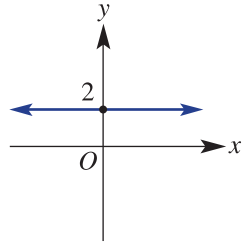

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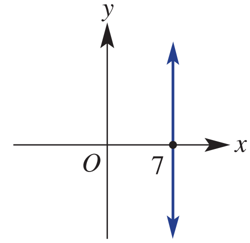
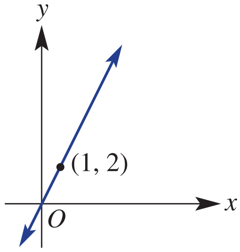


6 Remember that equations of graphs that pass through the origin are of the form $y = mx$ (since $c = 0$). Find the equation of each of these graphs.

a



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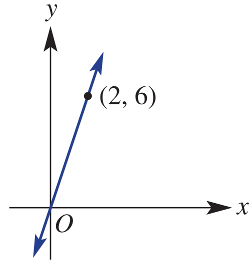

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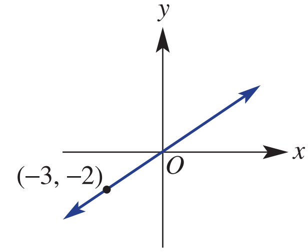

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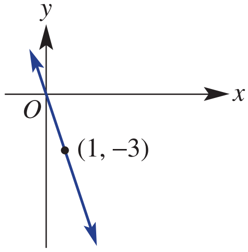

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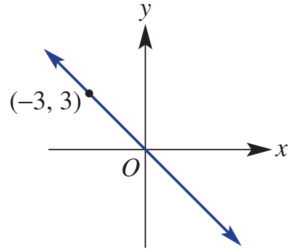

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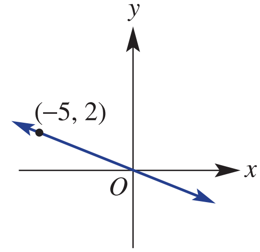

7 For the line joining the following pairs of points, find:

i the gradient **ii** the equation of the line

Hint: Substitute the gradient and a point in $y = mx + c$ to find c in part **ii**.

a $(0, 0)$ and $(1, 7)$ **b** $(0, 0)$ and $(2, -3)$
c $(-1, 1)$ and $(1, 3)$ **d** $(-2, 3)$ and $(2, -3)$
e $(-4, 2)$ and $(7, 2)$ **f** $(3, -3)$ and $(3, 1)$

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6J

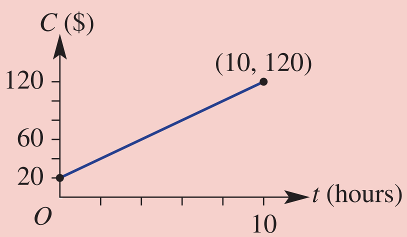
Problem-solving and reasoning

8, 9 | 8, 10, 11

Example 30 Modelling with linear graphs

An employee gets paid \$20 plus \$10 for each hour of work. If he earns \$$C$ for t hours of work, complete the following.

a Write a rule for C in terms of t.
b Sketch a graph, using t between 0 and 10.
c Use your rule to find:
  i the amount earned after working 8 hours
  ii the number of hours worked if \$180 is earned

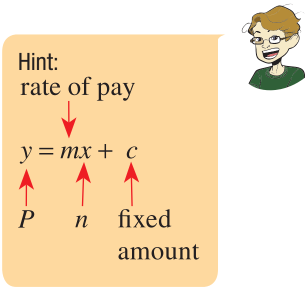
Solution	Explanation
a $C = 10t + 20$	\$10 is earned for each hour and \$20 is a fixed amount.
b 	20 is the y-intercept and the gradient is $10 = \frac{10}{1}$. For $t = 10$, $C = 10(10) + 20 = 120$.
c i $C = 10(8) + 20$	Substitute $t = 8$ into $C = 10t + 20$.
$= 100$	Simplify.
\$100 is earned.	Write your answer in words.
ii $180 = 10t + 20$	Substitute $C = 180$ into $C = 10t + 20$.
$160 = 10t$	Subtract 20 from both sides.
$t = 16$	Divide both sides by 10.
16 hours of work is completed.	Write your answer in words.

Now you try

A tennis court costs \$100 to hire plus \$20 per hour of use. If \$$C$ is the cost for t hours of hire, complete the following.

a Write a rule for C in terms of t.
b Sketch a graph, using t between 0 and 5.
c Use your rule to find:
  i the cost of hiring for 3 hours
  ii the number of hours of hire if the cost is \$260

8 A seasonal worker gets paid \$10 plus \$2 per kg of tomatoes that they pick. If the worker earns \$$P$ for n kg of tomatoes picked, complete the following.
  a Write a rule for P in terms of n.
  b Sketch a graph of P against n for n between 0 and 10.
  c Use your rule to find:
    i the amount earned after picking 9 kg of tomatoes
    ii the number of kilograms of tomatoes picked if the worker earns \$57



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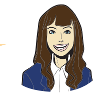
9 An architect charges $100 for the initial consultation plus $60 per hour thereafter. If the architect earns $\$A$ for t hours of work, complete the following.



> Hint:
> Draw a line between the points at $t = 0$ and $t = 15$.

- **a** Write a rule for A in terms of t.
- **b** Sketch a graph of A against t for t between 0 and 15.
- **c** Use your rule to find:
  - **i** the amount earned after working for 12 hours
  - **ii** the number of hours worked if the architect earns $700

10 A man's weight when holding two empty buckets of water is 80 kg. 1 kg is added for each litre of water poured into the buckets. If the man's total weight is W kg with l litres of water, complete the following.

- **a** Write a rule for W in terms of l.
- **b** Sketch a graph of W against l for l between 0 and 20.
- **c** Use your rule to find:
  - **i** the man's weight after 7 litres of water are added
  - **ii** the number of litres of water added if the man's weight is 109 kg

11 The amount of water (W litres) in a leaking tank after t hours is given by the rule $W = -2t + 1000$.

- **a** State the gradient and y-intercept for the graph of the rule.
- **b** Sketch a graph of W against t for t between 0 and 500.
- **c** State the initial water volume at $t = 0$.
- **d** Find the volume of water after:
  - **i** 320 hours **ii** 1 day **iii** 1 week
- **e** Find the time taken, in hours, for the water volume to fall to:
  - **i** 300 litres **ii** 185 litres

Production lines — 12

12 An assembly plant needs to order some new parts. Three companies can supply them but at different rates.

- Mandy's Millers charge: set-up fee $\$0 + \$1.40$ per part
- Terry's Turners charge: set-up fee $\$3000 + \$0.70$ per part
- Lenny's Lathes charge: set-up fee $\$4000 + \$0.50$ per part

a Complete a table of values similar to the following for each of the companies.

No. of parts (p)	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Cost (C)										

b Plot a graph of the total cost against the number of parts for each company on the same set of axes. Make your axes quite large as there are three graphs to complete.

c Use the graphs to find the lowest price for:
- **i** 1500 parts
- **ii** 1000 parts
- **iii** 6500 parts
- **iv** 9500 parts

d Advise the assembly plant when it is best to use Mandy's, Terry's or Lenny's company.

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6K Direct proportion

Learning intentions

- To understand what it means if two variables are directly proportional
- To know the form of the equation and graph of two variables in direct proportion
- To be able to form and use an equation for two directly proportional variables
- To be able to find the constant of proportionality for two variables in direct proportion

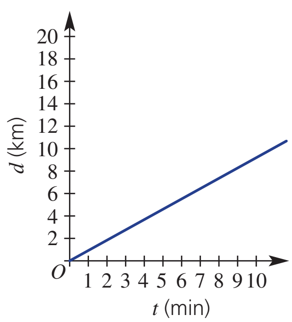
Key vocabulary: directly proportional, constant of proportionality

Two variables are said to be directly proportional when the rate of change of one variable with respect to the other is constant. So if one variable increases, then the other also increases and at the same rate. For example, distance is directly proportional to speed because, in a given time, if the speed is doubled then the distance travelled is doubled also.

Lesson starter: Discovering the features of direct variation

Here are five different travel graphs showing how the distance from home varies with time.

A



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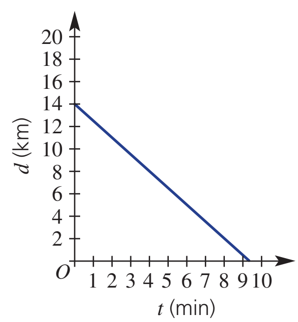

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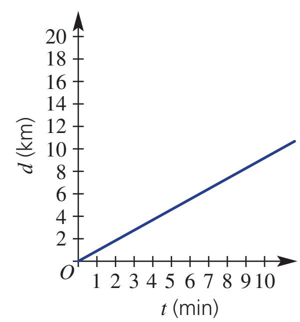

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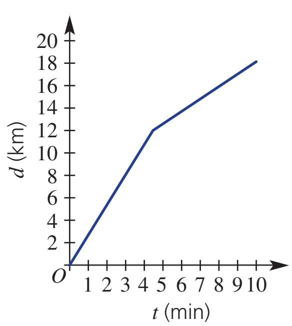

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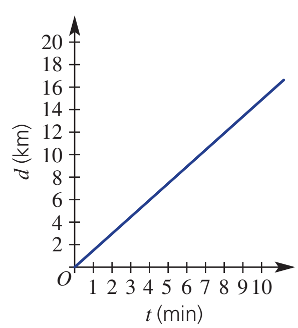

List all the graphs that show the following features.

- The distance from home increases as the time increases.
- The distance from home decreases as the time increases.
- The person travels at a constant speed throughout the trip.
- The person starts from home.
- The person starts from home and also travels at a constant speed throughout the trip.
- The graph starts at $(0, 0)$ and the gradient is constant.
- The distance is directly proportional to the time (i.e. the equation is of the form $y = mx$).

List three features of graphs that show when two variables are directly proportional to each other.

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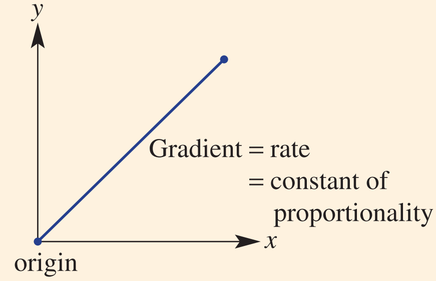
Key ideas

- For two variables that are **directly proportional**:
  - Both variables will increase together or decrease together at the same rate. For example, the cost of buying some sausages is directly proportional to the weight of the sausages. If the weight increases, then the cost increases; when the weight decreases, the cost decreases.

Graph of direct variation



  - The rate of change of one variable with respect to the other is constant.
  - The graph is a straight line passing through the origin $(0, 0)$; i.e. the rule is of the form $y = mx$.
  - The rule is usually written as $y = kx$, where k is the constant of proportionality.
  - The **constant of proportionality**, k, is the gradient with units, it is the same as the rate.

Exercise 6K

Understanding 1–3 2

1 Write in the missing words for these statements: *increases* or *decreases*.

a As the volume of fuel decreases, the distance a car can travel _______.

b As the volume of fuel decreases, the cost of filling the tank _______.

2 **a** Which of these equations show that y is directly proportional to x?

i $y = 2x + 4$ **ii** $y = 3x$ **iii** $y = x - 2$
iv $y = 8x + 5$ **v** $y = 70x$

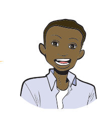
Hint: The direct proportion rule is $y = kx$, where k is the constant of proportionality.

b State the value of k, the constant of proportionality, for each of these direct proportion equations.

i $y = 5x$ **ii** $y = 12x$ **iii** $d = 40t$
iv $V = 22t$ **v** $C = 7.5w$

3 For each of the following graphs of direct variation, determine the value of k and write the equation in the form $y = kx$.

Hint: $k = \text{gradient} = \frac{\text{rise}}{\text{run}}$



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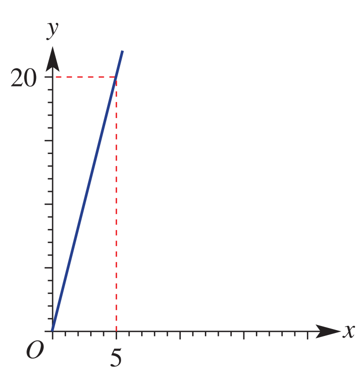

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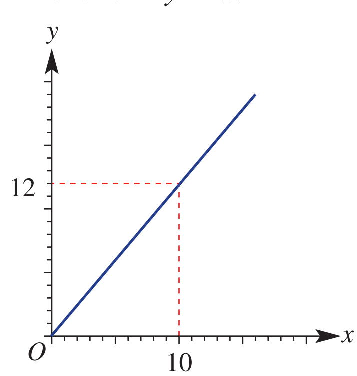

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6K

Fluency

4–6, 8 | 5–7, 8(½)

Example 31 Forming direct proportion equations when given the constant of proportionality, *k*

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres.

a Write the direct proportion equation, given that $k = \$1.45/\text{L}$.
b Use this equation to calculate the cost of 63 L.

Solution	Explanation
a $C = 1.45n$	$y = kx$ becomes $C = kn$, where $k = 1.45$.
b $C = 1.45 \times 63$	Substitute $n = 63$ into the equation.
$= \$91.35$	Write the answer in dollars.

Now you try

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres of fuel pumped.

a Write the direct proportion equation, given that $k = \$1.38/\text{L}$.
b Use this equation to calculate the cost of 48 L.

4 For a fixed rate of pay, wages (W) in dollars are directly proportional to the number (n) of hours worked.

a Write the direct proportion equation, given that $k = \$11.50/\text{h}$.
b Use this equation to calculate the wages earned for 37.5 hours worked.

5 At a fixed flow rate, the volume (V) in litres of water flowing from a tap is directly proportional to the amount of time (t) the tap has been turned on.

a Write the direct proportion equation, given that $k = 6$ L/min.
b Use this equation to calculate the volume of water, in litres, flowing from a tap for 4 hours.
c Change the constant of proportionality to units of L/day and rewrite the equation with this new value of k.
d Use this equation to calculate the volume of water, in litres, flowing from a tap for 1 week.

Hint: There are $60 \times 24 = 1440$ minutes in a day.

Example 32 Forming direct proportion equations from given information

The amount of wages Sonali earns is in direct proportion to the number of hours she works.

a Find the constant of proportionality, k, given that Sonali earned \$166.50 in 18 hours.
b Write the direct proportion equation relating Sonali's wages (W) in dollars and the number of hours (n) that she worked.
c Calculate the wages earned for 8 hours and 45 minutes of work.
d Calculate the number of hours Sonali must work to earn \$259.

Continued on next page

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Solution	Explanation
a $k = \dfrac{166.50}{18}$ $= \$9.25/\text{h}$	The constant of proportionality, k, is the rate of pay. Include units in the answer.
b $W = 9.25n$	$y = kx$ becomes $W = kn$, where $k = 9.25$.
c $W = 9.25n$ $= 9.25 \times 8.75$ $= \$80.94$	Write the equation. 45 min $= 45 \div 60 = 0.75$ h. Substitute $n = 8.75$. Write \$ in the answer and round to two decimal places.
d $W = 9.25n$ $259 = 9.25n$ $\dfrac{259}{9.25} = n$ $n = 28$ ∴ Sonali must work for 28 h. Check: $W = 9.25 \times 28 = \$259$	Write the equation. Substitute $W = 259$. Divide both sides by 9.25. Write the answer in words. Check that your answer is correct.

Now you try

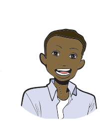
Daniel's wages earned are in direct proportion to the hours he works at the local service station.

a Find the constant of proportionality, k, given that Daniel earned \$200 in 16 hours.
b Write the direct proportion equation relating Daniel's wages (W) in dollars and the number of hours (n) worked.
c Calculate the wage earned for 6 hours of work.
d Calculate the number of hours Daniel must work to earn a wage of \$237.50.

6 The amount that a farmer earns from selling wheat is in direct proportion to the number of tonnes harvested.
- **a** Find the constant of proportionality, k, given that a farmer receives \$8296 for 34 tonnes of wheat.
- **b** Write the direct proportion equation relating selling price (P) in dolllars and number of tonnes (n).
- **c** Calculate the selling price of 136 tonnes of wheat.
- **d** Calculate the number of tonnes of harvested wheat that is sold for \$286 700.

7 When flying at a constant speed, the distance that an aeroplane has travelled is in direct proportion to the time it has been flying.
- **a** Find the constant of proportionality, k, given that the plane flies 1161 km in 1.5 h.
- **b** Write the direct proportion equation relating distance (d) in km and time (t) in hours.
- **c** Calculate the time taken, in hours, for the aeroplane to fly from Sydney to Perth, a distance of around 3300 km. Round your answer to two decimal places.
- **d** Calculate the distance that the plane would fly in 48 minutes.

Hint: Convert 48 mins to hours in part **d**.



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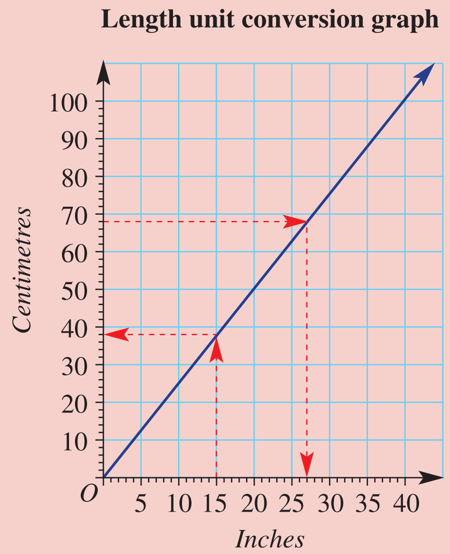
6K

Example 33 Using a unit conversion graph

A length, measured in centimetres, is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 15 inches to cm **b** 68 cm to inches

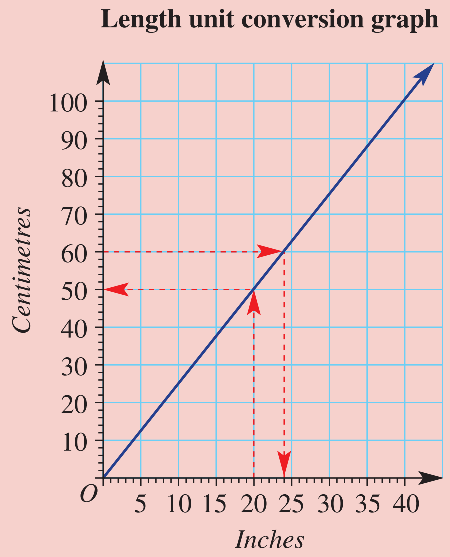


Solution	Explanation
a 38 cm	Start at 15 inches. Now move up to the line and then across to the centimetre scale.
b 27 inches	Start at 68 cm. Now move across to the line and then down to the inches scale. Round your answer to the nearest whole number.

Now you try

A length, measured in centimetres, is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 20 inches to cm **b** 60 cm to inches



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8 Use the graph in Example 33 to make the following unit conversions. Round your answers to the nearest whole number.
 a 19 inches to cm
 b 25 cm to inches
 c 1 foot (12 inches) to cm
 d 1 hand (4 inches) to cm
 e the height, in inches, of the world's shortest living man, who is 54.6 cm tall
 f the height, in cm, of a miniature pony that is 7.5 hands high
 g the height, in cm, of the world's shortest living woman, who is about 2 feet and 1 inch tall
 h the length, in cm, of a giant Australian earthworm that is 39.5 inches long

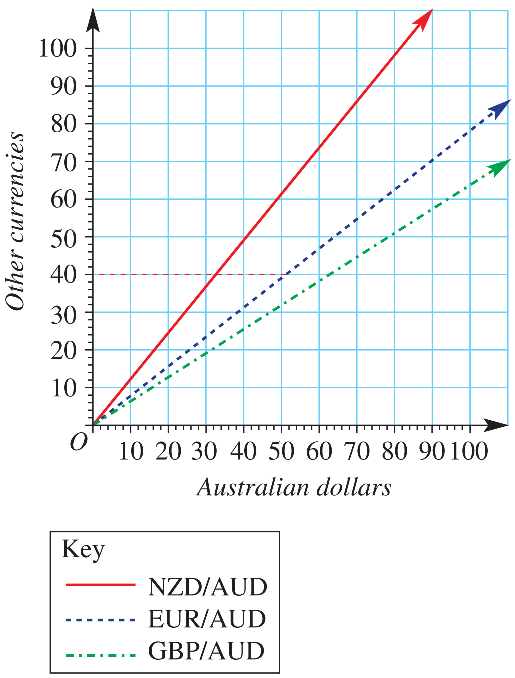
Problem-solving and reasoning | 9–11 | 10–12, 13(½), 14

9 Use the length unit conversion graph in Example 33 to answer these questions.
 a Convert 94 cm to inches and use these values to find the gradient of the line, to two decimal places.
 b State the conversion rate in cm/inch, to two decimal places.
 c State the value of k, the constant of proportionality, to two decimal places.
 d Write the direct proportion equation between centimetres (y) and inches (x).
 e Use the equation to calculate the number of centimetres in 50 inches.

Hint: Gradient = $\frac{\text{rise}}{\text{run}} = \frac{\text{cm}}{\text{inches}}$
k = rate
 = gradient with units

10 At any given time, an amount in Australian dollars is directly proportional to that amount in a foreign currency. This graph shows the direct variation (at a particular time) between Australian dollars (AUD) and New Zealand dollars (NZD), European euros (EUR) and Great British pounds (GBP).
 a Use the graph to make these currency conversions.
  i 80 AUD to NZD
  ii 80 AUD to EUR
  iii 80 AUD to GBP
  iv 50 NZD to AUD
  v 32 EUR to AUD
  vi 26 GBP to AUD
 b Answer these questions, using the line that shows the direct variation between the euro (EUR) and the Australian dollar (AUD).
  i Find 40 EUR in AUD and, hence, find the gradient of the line, to one decimal place.
  ii State the conversion rate in EUR/AUD, to one decimal place.
  iii State the value of k, the constant of proportionality, to one decimal place.
  iv Write the direct proportion equation between EUR (y) and AUD (x).
  v Use the equation to calculate the value in euros of 625 Australian dollars.

Currency conversion graph



Hint: The rate in EUR/AUD = ? euros, per 1 Australian dollar. The rate = the constant of proportionality, k.

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6K

11 The cost (C) of buying fuel is directly proportional to the volume (V) of fuel pumped.

a Copy and complete this table for the cost of diesel, using the rule $C = 1.5V$.

Volume (V) of diesel, in litres	0	10	20	30	40	50
Cost (C), in dollars						

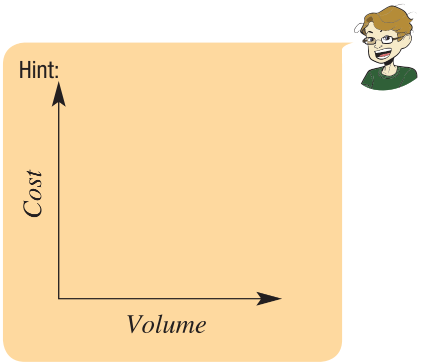
b Plot these points and then use a ruler to join them to form a neat graph of Volume (V) vs Cost (C).

c Find the gradient, m, of the line.

d At what rate is the cost of fuel increasing, in \$/L?

e What is the constant of proportionality, k, between cost and volume?

Hint:



12 For each of the following pair of variables, describe, using sentences, why the variables are in direct proportion to each other or why they are not in direct proportion.

a The number of *hours* worked and *wages* earned at a fixed rate per hour.

b The *cost* of buying tomatoes and the *number of kilograms* at a fixed price per kilogram.

c The *speed* and *time* taken to travel a certain distance.

d The *size* of a movie file and the *time* taken to download it to a computer at a constant rate of kB/s.

e The *cost* of a taxi ride and the *distance* travelled. The cost includes flag fall (i.e. a starting charge) and a fixed rate of \$/km.

13 Convert the following rates to the units given in brackets.

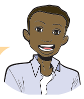
a \$9/h (cents/min) **b** \$24/h (cents/min)

c 10.8 L/h (mL/s) **d** 18 L/h (mL/s)

e 72 km/h (m/s) **f** 18 km/h (m/s)

g \$15/kg (cents/g) **h** \$32/kg (cents/g)

i 400 g/month (kg/year) **j** 220 g/day (kg/week)



Hint:
$9/h = 900 cents in 1 hour
= 900 cents in 60 mins
= 15 cents in 1 min
= 15 cents/min

14 For a fixed speed, the distance (d) that a car travels is directly proportional to time (t).

a Write the direct proportion equation, given that $k = 90$ km/h.

b Change the constant of proportionality to units of m/s and rewrite the equation with this new value of k.

c Use this equation to calculate the distance, in metres, that a car would travel in 4 seconds.

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Currency conversions

	—	15

15 At any given time, an amount of money in a foreign currency is in direct proportion to the corresponding amount in Australian dollars.
For example, if 8 Hong Kong dollars is equivalent to 1 Australian dollar, the conversion rate is HK \$8/AUD and the direct proportion equation is HK $= 8 \times$ AUD.

- To change A\$24 to Hong Kong dollars, we must substitute 24 for AUD:
  $$\text{HK} = 8 \times 24$$
  $$= \$192$$
- To change HK \$24 to Australian dollars, we substitute 24 for HK:
  $$24 = 8 \times \text{AUD}$$
  $$\frac{24}{8} = \text{AUD}$$
  $$\text{AUD} = \$3$$

Follow the example above to complete the following questions.

a Singapore dollar (SGD)
  i Write the direct proportion equation, given the conversion rate is 1.2 SGD/AUD.
  ii Convert AUD 240 to SGD.
  iii Convert SGD 240 to AUD.

b Chinese yuan (CNY)
  i Write the direct proportion equation, given the conversion rate is 6.47 CNY/AUD.
  ii Convert AUD 75 to CNY.
  iii Convert CNY 75 to AUD.

c South African rand (ZAR)
  i Write the direct proportion equation, given the conversion rate is 9.5 ZAR/AUD.
  ii Convert AUD 50 to ZAR.
  iii Convert ZAR 50 to AUD.

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6L Inverse proportion

Learning intentions

- To understand what it means if two variables are inversely proportional
- To be able to find the constant of proportionality for two inversely proportional variables
- To be able to form and use an equation for inversely proportional variables

Key vocabulary: inversely proportional, constant of proportionality, variables

Two variables are said to be inversely proportional when an increase in one variable causes the other variable to decrease. For example, when a pizza is shared equally, the size of each pizza slice is inversely (or indirectly) proportional to the number of people sharing it. If the number of people sharing increases, then the size of each pizza slice decreases.

Lesson starter: Bushwalking age groups

Imagine you are planning a 12 km bush hike for people of various age groups. You estimate that the average speed for each group is as follows.

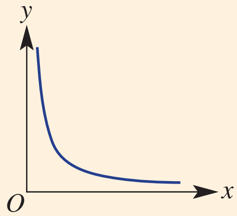
Age (years)	Speed (km/h)
20–35	6
36–50	4
51–65	3
66+	2

- How long will it take for each group to complete the hike?
- Plot points to form a graph of time (t hours) to complete the hike for different age group speeds (s km/h). Use t hours on the vertical axis.
- Do you think that the variables t and s are inversely proportional? Explain why.

Key ideas

- For two **variables** that are **inversely** or **indirectly proportional**:
  - When one variable increases, then the other variable decreases.
  - The graph is a curve showing that as x increases, then y decreases. For example, speed is indirectly or inversely proportional to travelling time. As the speed decreases, the time taken to travel a particular distance increases; as the speed increases, the time taken to travel a particular distance decreases.
  - The rule is usually written as $y = \frac{k}{x}$, where k is the constant of proportionality.

Graph of indirect or inverse variation



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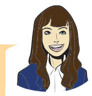
Exercise 6L

Understanding (1–3 | 2, 3)

1 For each part, state whether the variables are directly or indirectly (i.e. inversely) proportional.
- **a** As the number of questions correct increases, the total mark for the test increases.
- **b** As the speed decreases, the time taken to travel a particular distance increases.
- **c** As the number of hours worked increases, the pay for that work increases.
- **d** As the size of a computer file decreases, the time required to transfer it decreases.
- **e** As the rate of typing words per minute increases, the time needed to type an assignment decreases.

2 Which of the following equations shows inverse proportion?
- **a** $y = \frac{2}{x}$
- **b** $y = 3x$
- **c** $T = \frac{2.7}{s}$



Hint: Inverse proportion equation is of the form $y = \frac{k}{x}$.

3 Determine the value of k in $y = \frac{k}{x}$ when:
- **a** $x = 2$ and $y = 3$
- **b** $y = 2$ and $x = 0.5$

Fluency (4, 5 | 4–6)

Example 34 Working with inverse proportion

The length of a rectangle, l metres, with a fixed area of 2 m^2, varies inversely with the width, w metres, such that $l = \frac{2}{w}$.

a Complete this table of values.

w	1	2	3	4
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
c Find the length of the rectangle if the width is 5 m.
d Find the width of the rectangle if the length is 0.5 m.

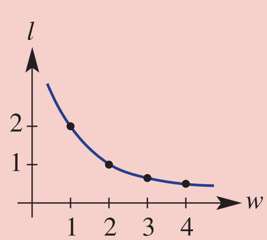
Solution | **Explanation**

a

w	1	2	3	4
l	2	1	$\frac{2}{3}$	$\frac{1}{2}$

Substitute each value of w into the rule $l = \frac{2}{w}$.

b



Plot the points from the table and join the points in a smooth curve.

c
$l = \frac{2}{w}$

$l = \frac{2}{5}$

length = 0.4 metres

Substitute $w = 5$ into the given rule.

Continued on next page

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6L

d $l = \frac{2}{w}$

$0.5 = \frac{2}{w}$

$0.5w = 2$

$w = 4$

width = 4 metres

Substitute $l = 0.5$ into the given rule then solve for w. Note that $2 \div 0.5 = 4$.

Now you try

The length of a rectangle, l metres, with a fixed area of $12\ \text{m}^2$, varies inversely with the width, w metres, such that $l = \frac{12}{w}$.

a Complete this table of values.

w	1	3	6	12
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
c Find the length of the rectangle if the width is 4 m.
d Find the width of the rectangle if the length is 2.5 m.

4 The length of a rectangle, l metres, with a fixed area of $6\ \text{m}^2$, varies inversely with the width, w metres, such that $l = \frac{6}{w}$.

a Complete this table of values.

w	1	2	3	6
l				

b Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
c Find the length of the rectangle if the width is 8 m.
d Find the width of the rectangle if the length is 1.5 m.

5 The time taken (t days) to paint a house is inversely proportional to the number of painters (n) such that $t = \frac{16}{n}$.

a Find the time taken to paint a house for the following number of painters, n.
- **i** 1
- **ii** 4
- **iii** 8

b Sketch a graph of t vs n with n on the x-axis.
c Find how long it will take for 10 painters to paint a house.
d Find the number of painters required to paint a house in 8 days.

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6 y is inversely proportional to x such that $y = \frac{20}{x}$.

a Find the value of y if:

i $x = 5$

ii $x = 8$

b Find the value of x if:

i $y = 10$

ii $y = 40$

Problem-solving and reasoning 7–9 7–10

Example 35 Finding and using the constant of proportionality

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 2$, $y = 3$.

b Find the value of y if $x = 6$.

Solution	Explanation
a $y = \frac{k}{x}$ $3 = \frac{k}{2}$ $k = 2 \times 3$ $= 6$	Substitute $x = 2$ and $y = 3$ into the rule and solve for k.
b $y = \frac{6}{x}$ $y = \frac{6}{6}$ $y = 1$	Write the rule using $k = 6$. Substitute $x = 6$ and simplify to find the value of y.

Now you try

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 4$, $y = 2$.

b Find the value of y if $x = 1$.

7 y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 5$, $y = 10$.

b Find the value of y if $x = 2$.

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6L

8 For each of the following determine the constant of proportionality, k, if $y = \frac{k}{x}$. Then complete the table of values.

a

x	1	2	3	4
y	12	6		

b

x	0.5	1	2	4
y	8		2	

9 The volume, V cm^3, of gas in a container is inversely proportional to the pressure, P kg/cm^3. When the pressure is 2 kg/cm^3 the volume is 20 cm^3.

a Find the constant of proportionality k if $V = \frac{k}{P}$.

b Find the volume if the pressure is 4 kg/cm^3.

c Find the pressure if the volume is 16 cm^3.

10 The cost to rent a house for a week is \$500.

a Find the cost per person if:

i 2 people rent the house

ii 5 people rent the house

b Find a rule linking the cost per person, \$$C$, with the number of people, n.

c Find the number of people sharing the house if the cost per person is \$62.50.

Electrical current — 11

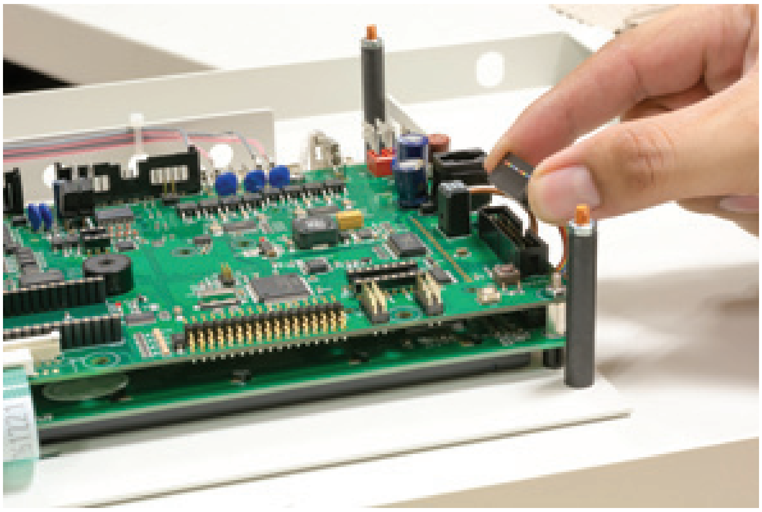
11 The current (I amps) that flows from a device is inversely proportional to the resistance (R ohms). When the current is 4 amps, the resistance is 60 ohms.

a Find a rule linking I and R.

b Find the current if the resistance is 40 ohms.

c Find the resistance if the current is 120 amps.

d What percentage increase in resistance causes a 50% reduction in current?



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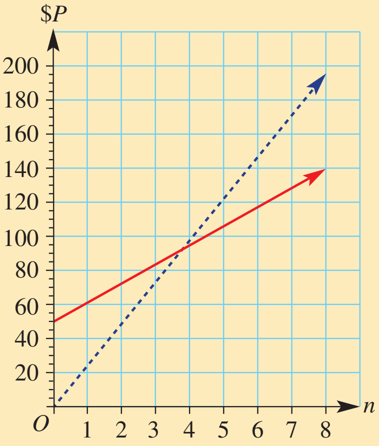
Maths@Work: Accountant or small business owner

Accountants and small business owners need to have an understanding of operating costs and income. It is important to understand break-even points, which are when the costs or expenses equal the income. This is essential knowledge for small business owners, given that many start-up businesses fail in their first 3 years.

Reading and drawing graphs helps accountants and business owners understand break-even points. Many business situations can be displayed using linear functions.

Complete these questions that an accountant or small business owner may face in their day-to-day job.

1 Chloe buys and sells necklaces at a local community weekend market. She sells each necklace for $25 and her costs, C, are calculated using the formula $C = 50 + 12n$, where n is the number of necklaces Chloe buys and $50 is the fixed cost of hiring the stand each weekend.



- **a** For the graph shown at right, which line represents Chloe's costs and which line represents Chloe's income?
- **b** State the possible numbers of necklaces to be sold each weekend that will result in Chloe making a loss.
- **c** What is the loss at the break-even point? What is the profit at the break-even point?
- **d** State the minimum number of necklaces to be sold each weekend for Chloe to start making a profit.
- **e** Sketch a copy of the graph and shade the region that represents the area of profit.
- **f** Write down a rule, in terms of n, for calculating Chloe's profit, $\$P$, each weekend.
- **g** How much profit is made by selling:
  - **i** 10 necklaces?
  - **ii** 15 necklaces?
  - **iii** 30 necklaces?
  - **iv** 50 necklaces?

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2 Consider this graph for a company creating retro drinking glasses.

a What is the value of the y-intercept for the costs relationship, and what could it represent in this situation?

b What is the wholesale price of each glass?

c What is the gradient of the income line and what does this represent?

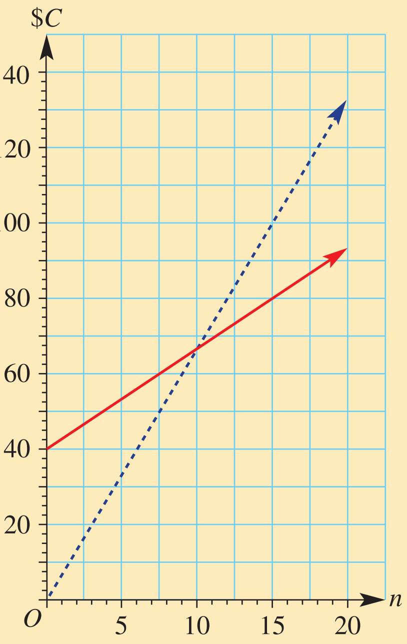
d How many glasses must be sold for the company to break even?

e Write an equation for the:

i cost

ii income

iii profit



Using technology

3 An industrial plant produces car parts, which they sell to car manufacturers for \$80 per car part. The costs of production for the plant are \$200 plus \$50 per car part produced each day.

a Write an equation for the costs ($\$C$) of production of n car parts.

b Write an equation for the income ($\$I$) generated.

c Use a graphics calculator or digital graphing tool to graph these two equations.

d From the graphs, what is the break-even point for the plant each day?

e Determine the profit when 32 car parts are sold on any given day.

4 Alex has a start-up company. He estimates his costs, including staff, rent, electricity and insurance, to be \$3000 per month. He buys computer parts at \$4 per part and sells them on at a retail price of $\$a$ per part.

Use a graphics calculator or a digital graphing tool to answer these questions.

a By drawing pairs of graphs, find the number of parts, x, and Alex's costs, at each break-even point when the retail price, $\$a$, per part is:

i \$10 **ii** \$12 **iii** \$14 **iv** \$16

Hint: Enter all the graph equations into the graphics calculator or computer graphing tool and then select two equations at a time to graph.

b Alex buys a batch of 500 parts. Write an equation for Alex's profit when these parts are sold at the retail price $\$a$ per part.

c Calculate the profit made when the parts are sold at each value of $\$a$ given in part **a** above.

d Give one reason why Alex must limit the retail price charged per part.

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1 What is not so devious? Solve the puzzle to find the answer.
Match the letter beside each question to the answers below.

Find where each line cuts the x-axis:

- **O** $y = 3x - 24$
- **!** $y = -\frac{3}{2}x - 9$
- **N** $4x - 2y = -20$
- **I** This line joins $(0, 4)$ to $(5, -1)$.
- **S** $x = -7$

Find the gradient of each line:

- **E** $y = 3x - 4$
- **L** $y = -\frac{5}{2}x + 7$
- **P** This line joins the origin to $(3, -6)$.
- **G** This line joins $(2, 5)$ to $(-4, 11)$.
- **T** $y = 5$

$\overline{-2}\ \overline{-2.5}\ \overline{8}\ \overline{0}\ \overline{0}\ \overline{4}\ \overline{-5}\ \overline{-1}\quad \overline{-2.5}\ \overline{4}\ \overline{-5}\ \overline{3}\ \overline{-7}\ \overline{-6}$

2 Solve the wordfind below.

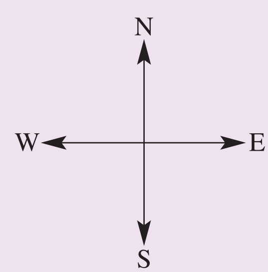
T	F	Z	T	V	M	V	Z	J	H	E	R
O	W	I	M	J	G	R	J	O	A	L	A
T	M	G	T	E	R	K	R	U	T	B	T
E	C	N	A	T	S	I	D	N	E	A	E
S	T	M	P	R	Z	A	E	S	I	I	F
R	P	H	G	O	X	M	E	O	Z	R	E
D	Y	E	N	Z	G	J	U	R	X	A	F
H	G	T	E	E	J	Q	W	G	C	V	H
I	A	L	S	D	R	U	S	G	U	N	A
L	P	G	R	A	P	H	A	P	R	P	I

- DISTANCE
- GRAPH
- HORIZONTAL
- INCREASE
- RATE
- SEGMENT
- SPEED
- TIME
- VARIABLE

3 Cooper and Sophie are in a cycling orienteering competition.

- From the starting point, Cooper cycles 7 km east, then 3 km south to checkpoint 1. From there, Cooper cycles 5 km east and 8 km north to checkpoint 2.
- Sophie cycles 10 km north from the starting point to checkpoint 3.

Use calculations to show that the distance between where Sophie and Cooper are now is the same as the direct distance that Cooper is now from the starting point.



4 Lucas and Charlotte want to raise money for their school environment club, so they have volunteered to run a strawberry ice-cream stall at their town's annual show.
It costs \$200 to hire the stall and they make \$1.25 profit on each ice-cream sold.

a How many ice-creams must be sold to make zero profit (i.e. not a loss)?
b If they make \$416.25 profit, how many ice-creams were sold?

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Chapter summary

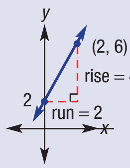
Straight-line graphs

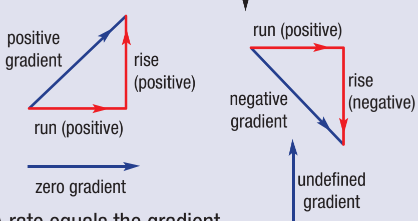
Gradient of a line

Gradient measures the slope of a line

Gradient, $m = \frac{\text{rise}}{\text{run}}$ e.g.

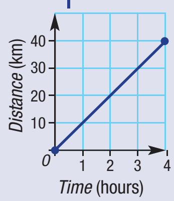
$m = \frac{4}{2} = 2$





A rate equals the gradient with units.

e.g. Speed = $\frac{40}{4}$ = 10 km/h



Equation of a line

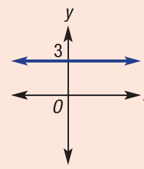
$y = mx + c$

gradient y-intercept

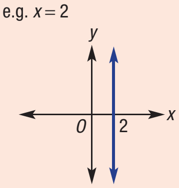
- The rule is a linear equation.
- The graph is made up of points in a straight line.

Special lines

Horizontal lines e.g. $y = 3$



Vertical lines e.g. $x = 2$

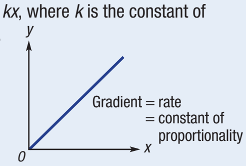


Parallel and perpendicular lines

- Parallel-lines have the same gradient; e.g. $y = 3x - 4$ and $y = 3x + 1$
- For perpendicular lines, the product of their gradients is -1, so $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

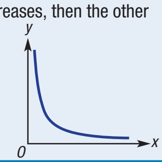
For two variables that are directly proportional

- Both variables will increase or decrease together at the same rate.
- The rule is $y = kx$, where k is the constant of proportionality.



For two variables that are inversely (or indirectly) proportional

- When one variable increases, then the other variable decreases.
- The graph is a curve.



Sketching a line

Plotting straight-line graphs:

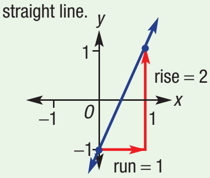
- Complete a table of values.
- Plot points and join them to form a straight line.

Using the y-intercept and gradient:

- Plot the y-intercept (c).
- Use the gradient to plot the next point.
- Join points to form a straight line.

e.g. $y = 2x - 1$

$c = -1$ $m = \frac{2}{1}$



Midpoint of a line segment

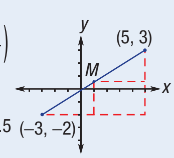
Find the average of the end point coordinates.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$x = \frac{-3 + 5}{2} = \frac{2}{2} = 1$

$y = \frac{-2 + 3}{2} = \frac{1}{2} = 0.5$

$\therefore M = (1, 0.5)$



Length of a line segment

Use Pythagoras' theorem.

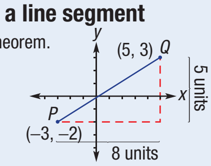
$PQ^2 = 8^2 + 5^2$

$PQ^2 = 64 + 25$

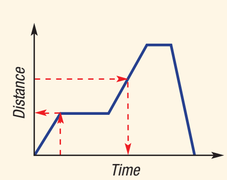
$PQ^2 = 89$

$PQ = \sqrt{89}$

$\sqrt{89}$ is an exact length.



Distance–time graph



- Flat segment means the object is at rest.

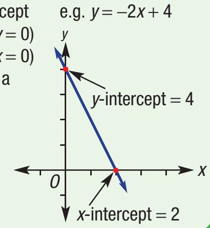
Reading from a graph:

- Start on given distance; move across to line and then down to time scale (or in reverse).

Using the axes intercepts

- Plot each axis intercept
  x-intercept (when $y = 0$)
  y-intercept (when $x = 0$)
- Join points to form a straight line.

e.g. $y = -2x + 4$



Linear modelling

- Find a rule in the form $y = mx + c$, using the appropriate pronumerals.
- Sketch a graph.
- Apply the rule to solve problems.
- Answer the problem in words.

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Chapter checklist

A version of this checklist that you can print out and complete can be downloaded from your Interactive Textbook.

✔

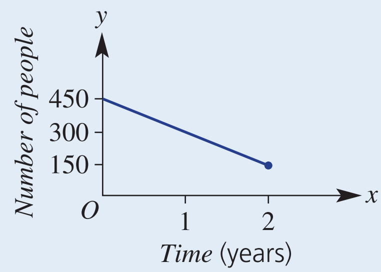
6A

1 I can interpret information from a graph.

e.g. The number of people living on a small island has decreased over recent years according to the graph shown.

a How many people were there to begin with?

b How many people left the island during the 2-year period?



6A

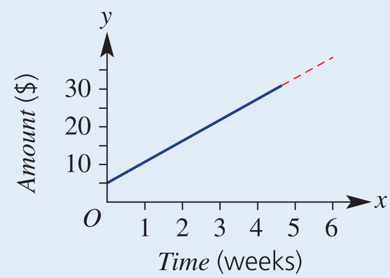
2 I can read off a graph using interpolation and extrapolation.

e.g. This graph shows the increase in Chloe's pocket money savings over 4 weeks.

a How much has she saved over the 4 weeks?

b Use the graph to find out how much she saved after 2 weeks.

c After how long does the graph suggest she will have saved $35?



6B

3 I can interpret a distance–time graph.

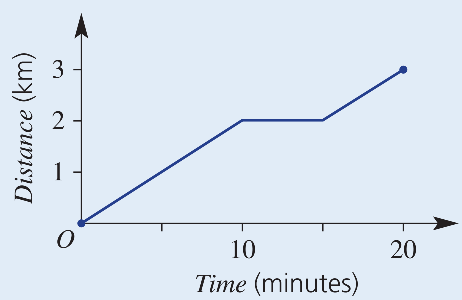
e.g. The distance–time graph shows a student's bike ride from school, to the corner store for an ice cream and then to home.

Determine:

a the total distance covered

b how long the student was stopped at the store

c the total distance travelled after 17.5 minutes



6B

4 I can sketch a distance–time graph.

e.g. Sketch a distance–time graph displaying all the following information.

- total distance covered is 12 km in 3 hours
- 6 km covered in the first hour
- a half-hour rest stop after the first hour

6C

5 I can plot a graph from a rule.

e.g. Plot the graph of $y = 3x - 2$ by first completing the table of values.

x	-1	0	1
y			

6C

6 I can construct a table and graph and interpret it.

e.g. A tv technician charges $110 for a service call and $70 per hour for labour. Complete the table of values and plot a graph of cost against number of hours.

No. of hours (n)	0	1	2	3	4
Cost (C)					

Use the graph to determine:

a the cost for 3.5 hours of work

b how long the technician worked on a job that cost $285

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Chapter checklist

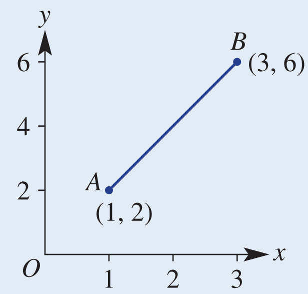
✔

6D

7 I can find the midpoint of a line segment from a graph and from coordinates.

e.g. Find the midpoint of the line segment joining the following points.

a A and B **b** $(-3, 6)$ and $(5, 1)$

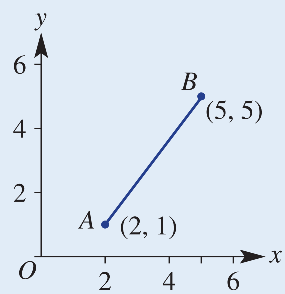


6D

8 I can find the length of a line segment from a graph and from coordinates.

e.g. Find the distance between the following pairs of points.

a A and B **b** $(-2, 3)$ and $(2, 8)$

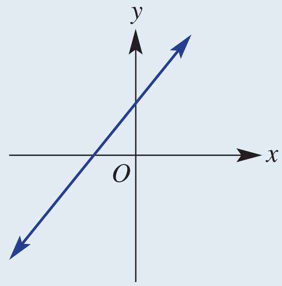


6E

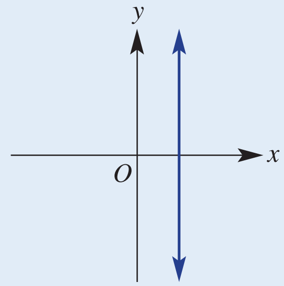
9 I can describe the gradient of a graph.

e.g. Describe the gradients of these lines as positive, negative, zero or undefined.

a



b

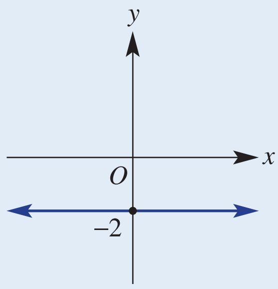


6E

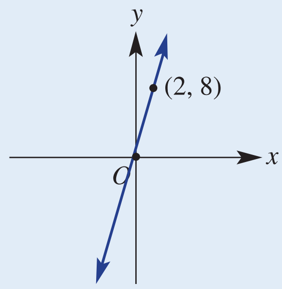
10 I can calculate the gradient from a graph.

e.g. Find the gradient of the following lines and line segments.

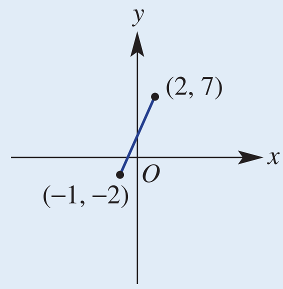
a



b



c



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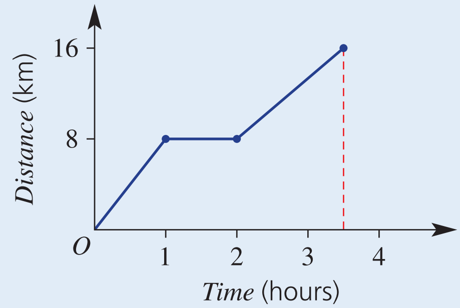
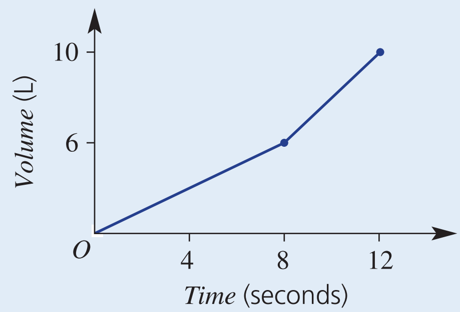
Chapter checklist

		✔
6F	**11 I can calculate speed from a distance–time graph.** e.g. A cyclist completes a journey which is described by this graph. Find how fast the cyclist was travelling during the: **a** first hour **b** second section **c** third section 	
6F	**12 I can calculate a rate from a graph.** e.g. A kettle is being filled with water from a tap as shown. How fast (in L/s) is the kettle being filled in the first 4 seconds, final 4 seconds and between the 4- and 8-second mark? 	
6G	**13 I can determine the gradient and y-intercept from an equation.** e.g. For $y = \frac{1}{2}x - 4$, state the gradient and the y-intercept.	
6G	**14 I can sketch a line using the y-intercept and gradient.** e.g. Sketch the graph of $y = 2x + 3$ by considering the y-intercept and gradient.	
6G	**15 I can sketch horizontal and vertical lines.** e.g. Sketch the graph of the equations: **a** $x = 4$ **b** $y = -3$.	
6G	**16 I can sketch lines passing through the origin.** e.g. Sketch the graph of $y = -3x$.	
6H	**17 I can determine if lines are parallel or perpendicular.** e.g. State whether the following pair of lines are parallel, perpendicular or neither: $y = -2x - 3$ and $y = \frac{1}{2}x + 1$	
6H	**18 I can find the equation of a line that is parallel or perpendicular to another line given the y-intercept.** e.g. A line passes through $(0, -3)$. Give the equation of the line if it is: **a** parallel to a line with gradient 2 **b** perpendicular to another line with gradient 3	
6H	**19 I can find the equation of a line that is parallel or perpendicular to a line.** e.g. A line passes through $(2, -3)$. Find the equation of the line if it is: **a** parallel to the line with equation $y = -3x + 1$ **b** perpendicular to the line with equation $y = \frac{1}{2}x + 1$	

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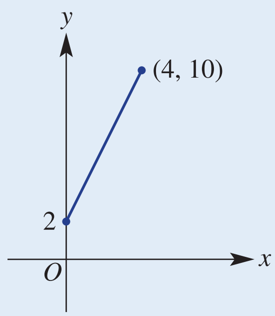
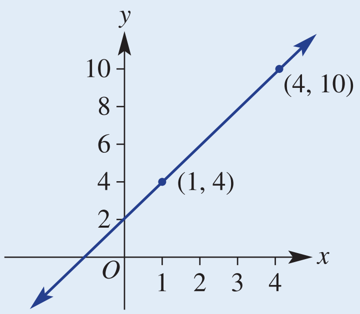
Chapter checklist

		✔
6I	**20 I can sketch lines in the form $ax + by = d$ using x- and y-intercepts.** e.g. Sketch a graph of $2x - 5y = 15$ by finding the x- and y-intercepts.	
6I	**21 I can sketch lines in the form $y = mx + c$ using x- and y-intercepts.** e.g. Sketch a graph of $y = -3x + 12$ by finding the x- and y-intercepts.	
6J	**22 I can find the equation of a line from a graph with a known y-intercept.** e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line. 	
6J	**23 I can find the equation of a line from a graph given two points.** e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line. 	
6J	**24 I can form a linear model and graph for a problem.** e.g. An employee gets paid $\$50$ plus $\$15$ for each hour of work. If she earns $\$C$ for t hours of work, write a rule for C in terms of t and sketch the graph for t between 0 and 8. Use the rule to find: **a** the amount earned after working 6 hours **b** the number of hours worked if $\$200$ is earned	
6K	**25 I can determine the constant of proportionality and find a rule connecting two variables which are directly proportional.** e.g. The volume of water in a bucket is in direct proportion to the number of seconds it has been filled for. The bucket was filled with 6 L of water in 12 seconds. **a** Find the constant of proportionality, k. **b** Use this to write the direct proportion equation relating the volume (V litres) of water in the bucket and the number of seconds (s) it is filled for. **c** Use the rule to find the volume after 8 seconds and the number of seconds to fill the bucket with 20 L of water.	
6L	**26 I can use an equation connecting inversely proportional variables to sketch a graph and find the value of an unknown.** e.g. In a rectangle of area 8 cm^2, the length, l cm, is inversely proportional to the width, w cm, such that $l = \frac{8}{w}$. Sketch a graph of l vs w and find the length if $w = 2.5$.	
6L	**27 I can determine the constant of proportionality and find a rule connecting two variables which are inversely proportional.** e.g. If y is inversely proportional to x and when $x = 3$, $y = 4$, find a rule linking y and x. Then find the value of x if $y = 6$.	

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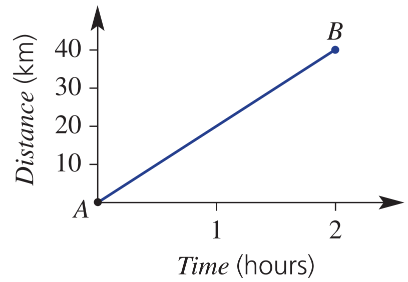
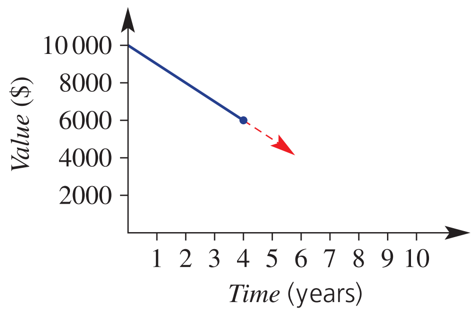
Short-answer questions

6A/B **1** This graph shows the journey of a cyclist from place A to place B.



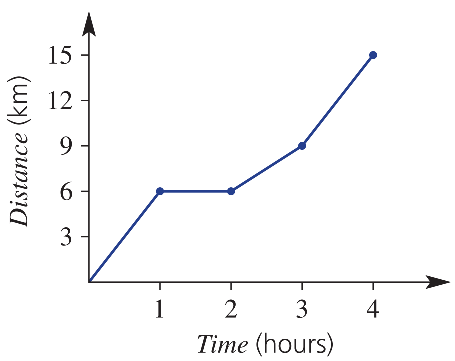
- **a** How far does the cyclist travel?
- **b** How long does it take the cyclist to complete the journey?
- **c** If the cyclist were to ride from A to B and then halfway back to A, how far would the journey be?

6A **2** The value of a poor investment has decreased according to this graph.



- **a** Find the value of the investment after:
  - **i** 4 years **ii** 2 years
  - **iii** 1 year
- **b** Extend the graph and use it to estimate the value of the investment after:
  - **i** 8 years **ii** 6 years
  - **iii** 5 years
- **c** After how many years will the investment be valued at $0?

6B **3** The distance travelled by a walker is described by this graph.



- **a** What is the total distance walked?
- **b** For how long does the person actually walk?
- **c** How far has the person walked after:
  - **i** 1 hour? **ii** 2 hours?
  - **iii** 3 hours? **iv** 4 hours?
- **d** How long does it take the walker to walk a distance of 12 km?

6B **4** Sketch a graph to show a journey described by:

- a total distance of 60 m in 15 seconds
- 30 m covered in the first 6 seconds
- a 5 second rest after the first 6 seconds

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Chapter review

6C **5** Francene delivers burgers for a fast-food outlet. She is paid \$10 a shift plus \$5 per delivery.

a Complete the table of values.

No. of deliveries (*d*)	0	5	10	15	20
Pay (*P*)					

b Plot a graph of amount paid against number of deliveries.

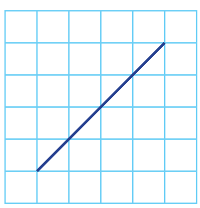
c Use the graph to determine:

i the amount of pay for 12 deliveries

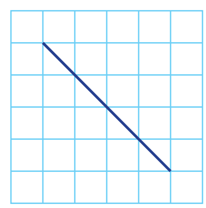
ii the number of deliveries made if Francene is paid \$95

6E **6** Find the gradient of the following lines.

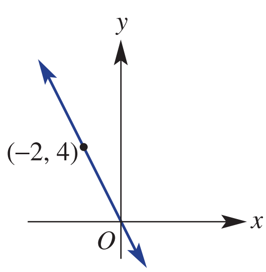
a



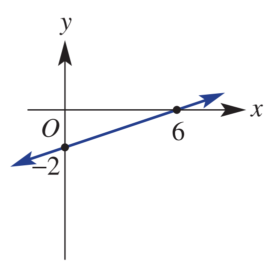
b



c

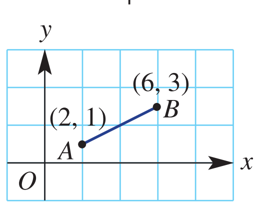


d



6D **7** Find the midpoint of each line segment.

a

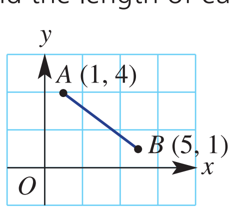


b $P(5, 7)$ to $Q(-1, -2)$

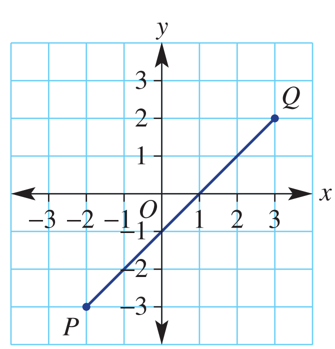
c $G(-3, 8)$ to $H(6, -10)$

6D **8** Find the length of each line segment.

a



b



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6G **9** State the gradient and y-intercept of the following lines.

a $y = 3x + 4$ **b** $y = -2x$

6G **10** Sketch the following lines by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = -4x$ **c** $y = 2$ **d** $x = -1$

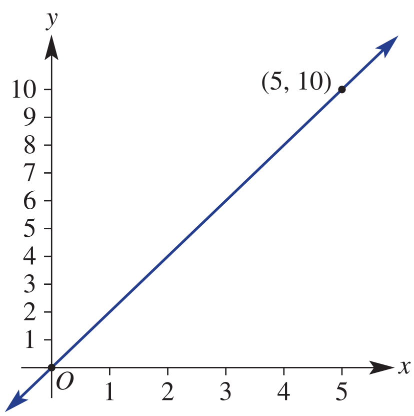
6I **11** Sketch the following lines by considering the x- and y-intercepts.

a $3x + 4y = 12$ **b** $2x - y = 6$ **c** $y = 3x - 9$

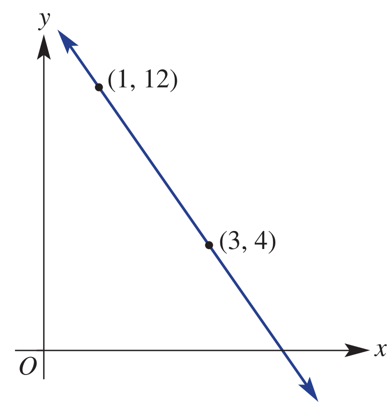
6G **12** For each of the straight lines shown:

i Determine the gradient.
ii Find the y-intercept.
iii Write the equation of the line.

a



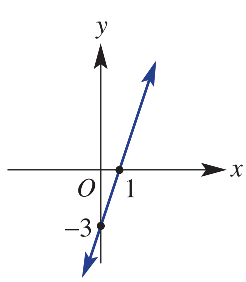
b



6G/I **13** Match each of the linear equations to the lines shown.

a $y = 3x - 3$ **b** $y = 5x$ **c** $5x + 4y = 20$
d $x = 2$ **e** $-2x + 5y = 10$ **f** $y = -4$

i



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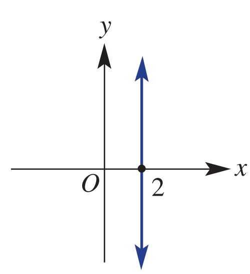

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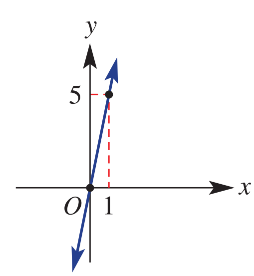

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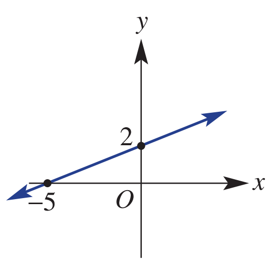

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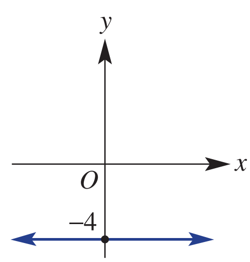

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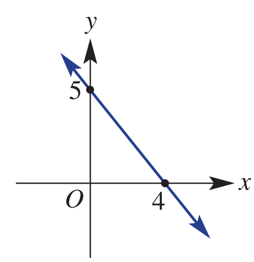

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Chapter review

6J

14 A fruit picker earns $\$50$ plus $\$20$ per bin of fruit picked. If the picker earns $\$E$ for n bins picked, complete the following.

- **a** Write a rule for E in terms of n.
- **b** Sketch a graph for n between 0 and 6.
- **c** Use your rule to find:
  - **i** the amount earned after picking four bins of fruit
  - **ii** the number of bins of fruit picked if $\$160$ is earned

6H

15 Find the equation of the lines with the given description.

- **a** A line passes through $(0, 3)$ and is parallel to another line with gradient 2.
- **b** A line passes through $(0, -1)$ and is parallel to another line with gradient $\frac{1}{2}$.
- **c** A line passes through $(0, 2)$ and is perpendicular to another line with gradient 1.
- **d** A line passes through $(0, -7)$ and is perpendicular to another line with gradient $\frac{3}{4}$.
- **e** A line passes through $(1, 2)$ and is parallel to another line with gradient -4.
- **f** A line passes through $(-2, 5)$ and is perpendicular to another line with gradient 2.

6K

16 An employee earns wages in direct proportion to the hours he works.

- **a** Find the constant of proportionality, k, given that he earned $198 in 12 hours.
- **b** Write the direct proportion equation relating the wages (W) in dollars and the number of hours (n) worked.
- **c** Use the rule in part **b** to calculate:
  - **i** the wages earned for 8 hours of work
  - **ii** the number of hours the employee must work to earn a wage of $\$264$

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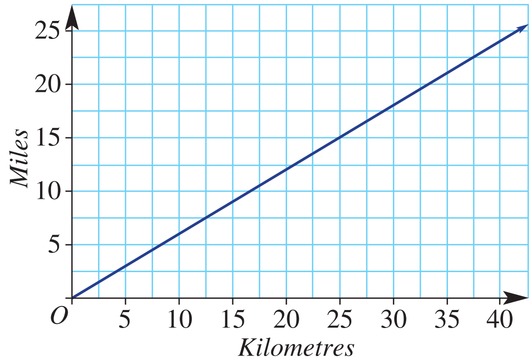
6L **17** This graph shows the direct proportional relationship between miles and kilometres.



- **a** Use the graph to convert 5 miles to kilometres.
- **b** Use the graph to convert 35 kilometres to miles.
- **c** Given that 15 miles is 24.14 km, find the gradient, to three decimal places.
- **d** State the conversion rate in miles/km, to three decimal places.
- **e** Determine the constant of proportionality, k, to three decimal places.
- **f** Write the direct proportion equation between miles (y) and kilometres (x).
- **g** Use this equation to find the number of miles in 100 km.
- **h** Use this equation to find the number of kilometres in 100 miles.

6K **18** State whether these variables are in direct or indirect proportion and give a reason why.

- **a** Cost of buying cricket balls and the number of balls.
- **b** Cost per person of renting a beach house for a week and the number of people sharing it.

6K **19** The length of a rectangle, l metres, with a fixed area, varies inversely with the width, w metres, such that $l = \frac{k}{w}$.

- **a** If when $w = 3$, $l = 5$ find the value of k, the constant of proportionality.
- **b** Complete this table of values.

w	1	3	5	15
l				

- **c** Sketch a graph of l vs w using your table of values to help. Use w on the x-axis.
- **d** Find the length of the rectangle if the width is 10 m.
- **e** Find the width of the rectangle if the length is 4 m.

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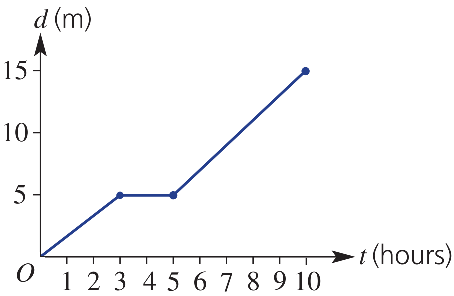
Chapter review

Multiple-choice questions

Questions **1–4** refer to the following graph of the movement of a snail.



6A **1** The total number of hours the snail is at rest is:

A 2 **B** 4 **C** 5
D 6 **E** 10

6B **2** The distance travelled in the first 3 hours is:

A 3 m **B** 3 hours **C** 7 m
C 4 m **D** 5 m

6F **3** The speed of the snail in the last 5 hours is:

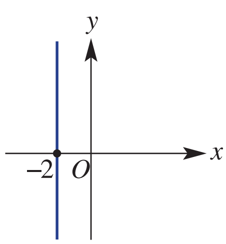
A 5 hours **B** 10 m **C** 10 m/h **D** 2 m/h **E** 5 m/h

6B **4** The total distance travelled by the snail is:

A 15 m **B** 10 m **C** 5 m **D** 12 m **E** 8 m

6G **5** The equation of the line shown at the right is:

A $x = 2$ **B** $x = -2$ **C** $y = 1$
D $y = -2$ **E** $y = -2x$



6G **6** The graph of $C = 10t + 5$ would pass through which of the following points?

A $(1, 10)$ **B** $(1, 20)$ **C** $(2, 20)$ **D** $(4, 50)$ **E** $(5, 55)$

6E **7** The gradient of the line joining $(0, 0)$ and $(2, -6)$ is:

A 2 **B** 3 **C** -3 **D** 6 **E** -6

6E **8** A vertical line has gradient:

A undefined **B** zero **C** positive **D** negative **E** 1

6E **9** A line passes through $(-2, 7)$ and $(1, 2)$. The gradient of the line is:

A -3 **B** $-\frac{5}{3}$ **C** 3 **D** $\frac{5}{3}$ **E** $-\frac{3}{5}$

6I **10** The x- and y-intercepts of the graph of the rule $3x - y = 4.5$ are, respectively:

A $(0, 3.5)$ and $(4.5, 0)$ **B** $(-1.5, 0)$ and $(4.5, 0)$ **C** $(1.5, 0)$ and $(0, 4.5)$
D $(1.5, 0)$ and $(0, -4.5)$ **E** $(0, 3.5)$ and $(-4.5, 0)$

6G **11** Which of the following equations has a gradient of 2 and a y-intercept of -1?

A $2y + x = 2$ **B** $y - 2x = 1$ **C** $y = -2x + 1$ **D** $y = 2x - 1$ **E** $2x + y = 1$

6I **12** A line has x- and y-intercepts of, respectively, 1 and 2. Its equation is:

A $2x - y = 2$ **B** $y = -x + 2$ **C** $y = 2x + 2$ **D** $x + 2y = 1$ **E** $y = -2x + 2$

6H **13** The gradient of the line that is perpendicular to the line with equation $y = -2x - 5$ is:

A 2 **B** -2 **C** $\frac{1}{2}$ **D** $-\frac{1}{2}$ **E** $\frac{1}{5}$

6K **14** Which equation shows that y is directly proportional to x?

A $y = 5x - 6$ **B** $y = \frac{6}{x}$ **C** $y = 2x + 4$ **D** $y = 12x$ **E** $y = 20 - 3x$

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Extended-response questions

1 David and Kaylene travel from Melton to Moorbank army base to watch their daughter's march-out parade. The total distance for the trip is 720 km, and they travel an average of 90 km per hour.

a Complete the table of values below from 0 to 8 hours.

Time in hours (t)	0	2	4	6	8
km from Moorbank	720				

b Plot a graph of the number of kilometres from Moorbank army base against time.

c David and Kaylene start their trip at 6 a.m. If they decide to stop for breakfast at Albury and Albury is 270 km from Melton, what time would they stop for breakfast?

d If the car they are driving needs refilling every 630 km, how long could they drive for before refilling the car?

e What would be the total driving time if they didn't stop at all?

f If the total number of breaks, including food and petrol stops, is 2 hours, when would they arrive at the army base?

2 A young maths whiz in the back seat of a car is counting down the distance to the nearest town, which initially is 520 km away. The car is travelling at an average speed of 80 km per hour.

a Find the distance to the town after:

i 1 hour **ii** 3 hours

b D km is the distance to the town after t hours.

i Write a rule for D in terms of t.

ii Sketch a graph for t between 0 and 6.5.

c Use your rule to find:

i the distance to the town after 4.5 hours

ii the time it takes for the distance to the town to be 340 km

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