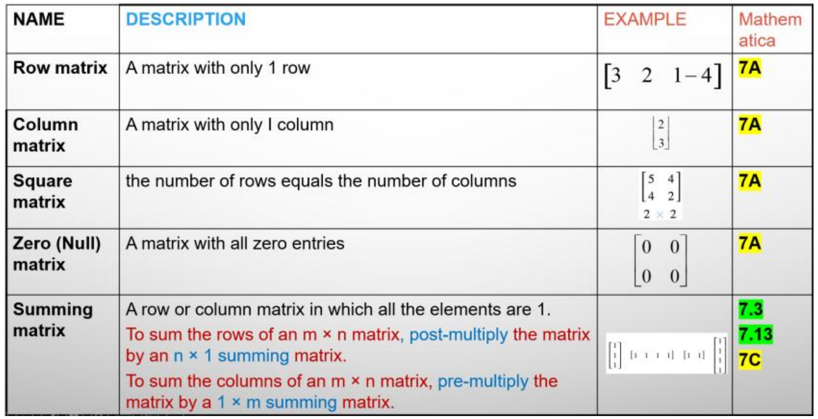
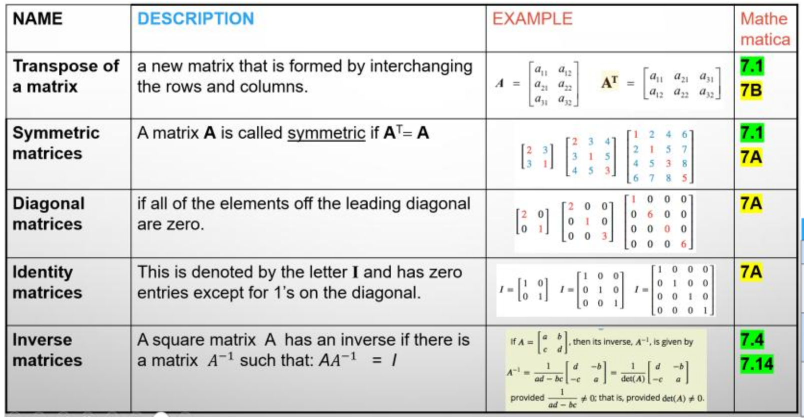
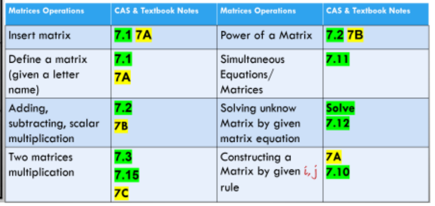


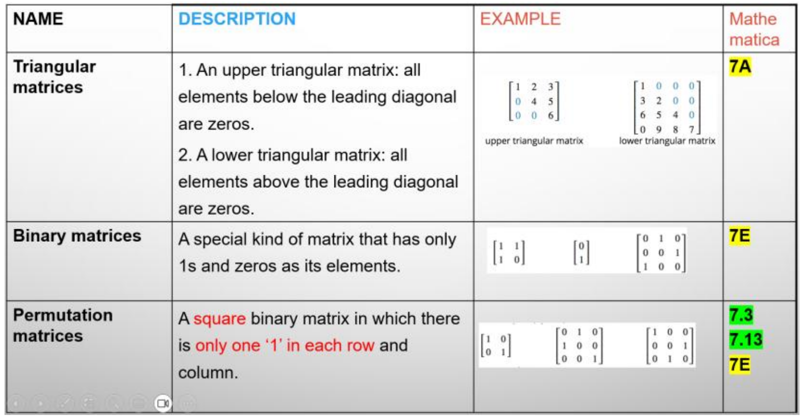
NAME	DESCRIPTION	EXAMPLE	Mathematica
Row matrix	A matrix with only 1 row	$\begin{bmatrix}3 & 2 & 1 -4\end{bmatrix}$	7A
Column matrix	A matrix with only I column	$\begin{bmatrix}2\\3\end{bmatrix}$	7A
Square matrix	the number of rows equals the number of columns	$\begin{bmatrix}5 & 4\\4 & 2\end{bmatrix}$ 2 × 2	7A
Zero (Null) matrix	A matrix with all zero entries	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	7A
Summing matrix	A row or column matrix in which all the elements are 1. To sum the rows of an m × n matrix, post-multiply the matrix by an n × 1 summing matrix. To sum the columns of an m × n matrix, pre-multiply the matrix by a 1 × m summing matrix.	[illegible]	7.3 7.13 7C



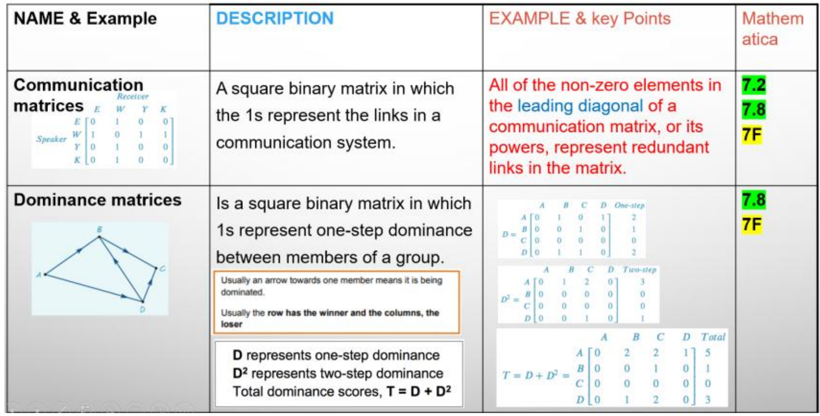
NAME	DESCRIPTION	EXAMPLE	Mathematica
Transpose of a matrix	a new matrix that is formed by interchanging the rows and columns.	$A = \begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\\a_{31} & a_{32}\end{bmatrix}$ $\mathbf{A}^{\mathrm{T}} = \begin{bmatrix}a_{11} & a_{21} & a_{31}\\a_{12} & a_{22} & a_{32}\end{bmatrix}$	7.1 7B
Symmetric matrices	A matrix **A** is called symmetric if $\mathbf{A}^{\mathrm{T}} = \mathbf{A}$	$\begin{bmatrix}2 & 3\\3 & 1\end{bmatrix}$ $\begin{bmatrix}2 & 3 & 4\\3 & 1 & 5\\4 & 5 & 3\end{bmatrix}$ $\begin{bmatrix}1 & 2 & 4 & 6\\2 & 1 & 5 & 7\\4 & 5 & 3 & 8\\6 & 7 & 8 & 5\end{bmatrix}$	7.1 7A
Diagonal matrices	if all of the elements off the leading diagonal are zero.	$\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 3\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 6 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 6\end{bmatrix}$	7A
Identity matrices	This is denoted by the letter **I** and has zero entries except for 1's on the diagonal.	$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $I = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $I = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$	7A
Inverse matrices	A square matrix A has an inverse if there is a matrix A^{-1} such that: $AA^{-1} = I$	If $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then its inverse, A^{-1}, is given by $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix} = \frac{1}{\det(A)}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$ provided $\frac{1}{ad - bc} \neq 0$; that is, provided $\det(A) \neq 0$.	7.4 7.14



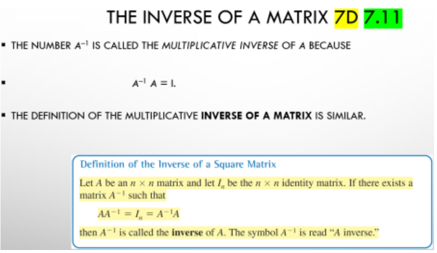
Matrices Operations	CAS & Textbook Notes	Matrices Operations	CAS & Textbook Notes
Insert matrix	7.1 7A	Power of a Matrix	7.2 7B
Define a matrix (given a letter name)	7.1 7A	Simultaneous Equations/ Matrices	7.11
Adding, subtracting, scalar multiplication	7.2 7B	Solving unknow Matrix by given matrix equation	Solve 7.12
Two matrices multiplication	7.3 7.15 7C	Constructing a Matrix by given i, j rule	7A 7.10



NAME	DESCRIPTION	EXAMPLE	Mathematica
Triangular matrices	1. An upper triangular matrix: all elements below the leading diagonal are zeros. 2. A lower triangular matrix: all elements above the leading diagonal are zeros.	$\begin{bmatrix}1 & 2 & 3\\0 & 4 & 5\\0 & 0 & 6\end{bmatrix}$ upper triangular matrix $\begin{bmatrix}1 & 0 & 0 & 0\\3 & 2 & 0 & 0\\6 & 5 & 4 & 0\\0 & 9 & 8 & 7\end{bmatrix}$ lower triangular matrix	7A
Binary matrices	A special kind of matrix that has only 1s and zeros as its elements.	$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ $\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$	7E
Permutation matrices	A square binary matrix in which there is only one '1' in each row and column.	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}$	7.3 7.13 7E



NAME & Example	DESCRIPTION	EXAMPLE & key Points	Mathematica
Communication matrices $\begin{array}{cc|cccc} & & \multicolumn{4}{c}{Receiver} \\ & & E & W & Y & K \\ \hline & E & 0 & 1 & 0 & 0 \\ Speaker & W & 1 & 0 & 1 & 1 \\ & Y & 0 & 1 & 0 & 0 \\ & K & 0 & 1 & 0 & 0 \end{array}$	A square binary matrix in which the 1s represent the links in a communication system.	All of the non-zero elements in the leading diagonal of a communication matrix, or its powers, represent redundant links in the matrix.	7.2 7.8 7F
Dominance matrices	Is a square binary matrix in which 1s represent one-step dominance between members of a group. Usually an arrow towards one member means it is being dominated. Usually the **row has the winner and the columns, the loser** **D** represents one-step dominance **D²** represents two-step dominance Total dominance scores, **T = D + D²**	$D = \begin{array}{c|cccc|c} & A & B & C & D & One\text{-}step \\ \hline A & 0 & 1 & 0 & 1 & 2 \\ B & 0 & 0 & 1 & 0 & 1 \\ C & 0 & 0 & 0 & 0 & 0 \\ D & 0 & 1 & 1 & 0 & 2 \end{array}$ $D^2 = \begin{array}{c|cccc|c} & A & B & C & D & Two\text{-}step \\ \hline A & 0 & 1 & 2 & 0 & 3 \\ B & 0 & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ D & 0 & 0 & 1 & 0 & 1 \end{array}$ $T = D + D^2 = \begin{array}{c|cccc|c} & A & B & C & D & Total \\ \hline A & 0 & 2 & 2 & 1 & 5 \\ B & 0 & 0 & 1 & 0 & 1 \\ C & 0 & 0 & 0 & 0 & 0 \\ D & 0 & 1 & 2 & 0 & 3 \end{array}$	7.8 7F



THE INVERSE OF A MATRIX 7D 7.11

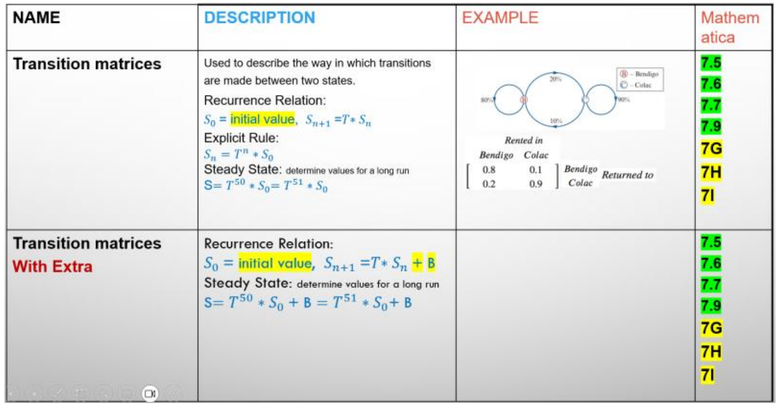
- THE NUMBER A^{-1} IS CALLED THE *MULTIPLICATIVE INVERSE* OF *A* BECAUSE
- $A^{-1}A = I.$
- THE DEFINITION OF THE MULTIPLICATIVE **INVERSE OF A MATRIX** IS SIMILAR.

Definition of the Inverse of a Square Matrix

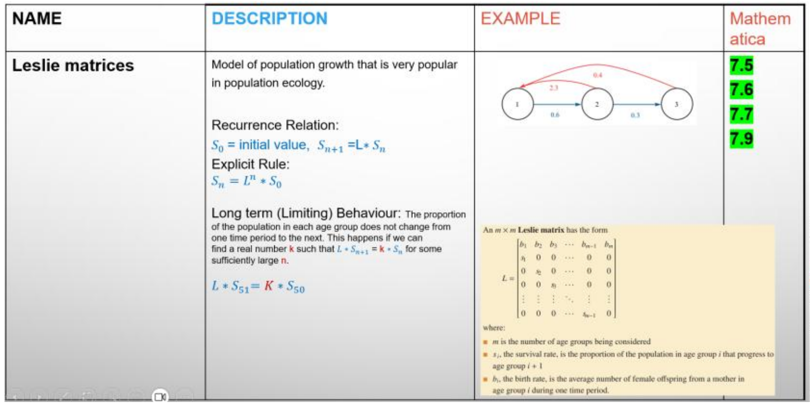
Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that

$$AA^{-1} = I_n = A^{-1}A$$

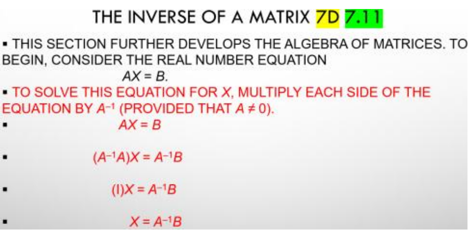
then A^{-1} is called the **inverse** of A. The symbol A^{-1} is read "A inverse."



NAME	DESCRIPTION	EXAMPLE	Mathematica
Transition matrices	Used to describe the way in which transitions are made between two states. Recurrence Relation: S_0 = initial value, $S_{n+1} = T * S_n$ Explicit Rule: $S_n = T^n * S_0$ Steady State: determine values for a long run $S = T^{50} * S_0 = T^{51} * S_0$	$\begin{array}{cc} \multicolumn{2}{c}{Rented\ in} & \\ Bendigo & Colac & \\ \end{array}$ $\begin{bmatrix}0.8 & 0.1\\0.2 & 0.9\end{bmatrix} \begin{array}{l}Bendigo\\Colac\end{array}$ Returned to	7.5 7.6 7.7 7.9 7G 7H 7I
Transition matrices With Extra	Recurrence Relation: S_0 = initial value, $S_{n+1} = T * S_n + B$ Steady State: determine values for a long run $S = T^{50} * S_0 + B = T^{51} * S_0 + B$		7.5 7.6 7.7 7.9 7G 7H 7I



NAME	DESCRIPTION	EXAMPLE	Mathematica
Leslie matrices	Model of population growth that is very popular in population ecology. Recurrence Relation: S_0 = initial value, $S_{n+1} = L * S_n$ Explicit Rule: $S_n = L^n * S_0$ Long term (Limiting) Behaviour: The proportion of the population in each age group does not change from one time period to the next. This happens if we can find a real number k such that $L * S_{n+1} = k * S_n$ for some sufficiently large n. $L * S_{51} = K * S_{50}$	An $m \times m$ **Leslie matrix** has the form $L = \begin{bmatrix}b_1 & b_2 & b_3 & \cdots & b_{m-1} & b_m\\s_1 & 0 & 0 & \cdots & 0 & 0\\0 & s_2 & 0 & \cdots & 0 & 0\\0 & 0 & s_3 & \cdots & 0 & 0\\\vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\0 & 0 & 0 & \cdots & s_{m-1} & 0\end{bmatrix}$ where: ■ m is the number of age groups being considered ■ s_i, the survival rate, is the proportion of the population in age group i that progress to age group $i+1$ ■ b_i, the birth rate, is the average number of female offspring from a mother in age group i during one time period.	7.5 7.6 7.7 7.9



THE INVERSE OF A MATRIX 7D 7.11

- THIS SECTION FURTHER DEVELOPS THE ALGEBRA OF MATRICES. TO BEGIN, CONSIDER THE REAL NUMBER EQUATION

$$AX = B.$$

- TO SOLVE THIS EQUATION FOR X, MULTIPLY EACH SIDE OF THE EQUATION BY A^{-1} (PROVIDED THAT $A \neq 0$).
- $AX = B$
- $(A^{-1}A)X = A^{-1}B$
- $(I)X = A^{-1}B$
- $X = A^{-1}B$

7A: introduction to matrices

A matrix is an array of numerical values

- Values are arranged into ROWS and COLUMNS

ROWS: are horizontal

- Number rows from top to bottom

COLUMNS: are vertical

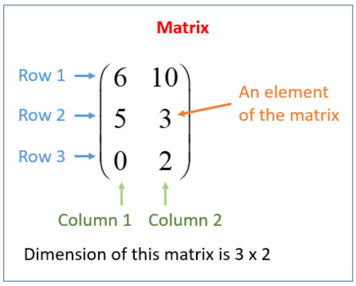
- Number columns from left to right

ORDER OF A MATRIX:

- Way to describe the dimensions (size) of a matrix

ORDER = ROWS × COLUMNS

- × is ‘by’



Types of Matrices:

Column: only one column, any number of rows	Row: only one row, any number of columns	Zero: every element is ‘0’. Can be any size
$\begin{bmatrix}42\\56\\74\end{bmatrix}$	$[8\ \ 17\ \ 42\ \ 52]$	$\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$

Square matrices: have the same number of rows and columns.

Diagonal matrix: all values apart from the leading diagonal are zero	$\begin{bmatrix}25&0&0\\0&9&0\\0&0&16\end{bmatrix}$
Identity matrix: diagonal matrix, where the leading diagonals elements are all ‘1’	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
Symmetric matrix: unchanged by transposition, the elements above the leading diagonal are a mirror image of the elements below	$\begin{bmatrix}3&6&5\\6&0&1\\5&1&2\end{bmatrix}$
Upper triangular matrix: all elements below the leading diagonal are zero	$\begin{bmatrix}4&5&7\\0&1&6\\0&0&8\end{bmatrix}$
Lower triangular matrix: all elements above the leading diagonal are zero	$\begin{bmatrix}3&0&0\\2&3&0\\5&4&2\end{bmatrix}$

Elements and notation:

- Matrices are labelled with a capital letter
- The values within a matrix are called **elements**
- Elements are labelled with lowercase letters
- For matrix A, element a_{mn} refers to the entry in the mth row and nth column

$A=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{22}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\ddots&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{bmatrix}$	$A=\begin{bmatrix}5&2\\3&1\\1&9\end{bmatrix}$ Eg. Element a_{21} = 3

x^{ij} – matrices can be constructed using element rules. Eg. Matrix C is a 2×2 matrix with the element rule

c_{ij} = i+j. Create the matrix.

$$A=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}\quad A=\begin{bmatrix}1+1&1+2\\2+1&2+2\end{bmatrix}=\begin{bmatrix}2&3\\3&4\end{bmatrix}$$

7B Operations with matrices

ADDITION AND SUBTRACTION:

- Matrices must have the same order
- Add/subtract elements in the same position

Matrix addition

$$\begin{bmatrix}a&b\\c&d\end{bmatrix}+\begin{bmatrix}e&g\\f&h\end{bmatrix}=\begin{bmatrix}a+e&b+g\\c+f&d+h\end{bmatrix}$$

Matrix subtraction

$$\begin{bmatrix}a&b\\c&d\end{bmatrix}-\begin{bmatrix}e&g\\f&h\end{bmatrix}=\begin{bmatrix}a-e&b-g\\c-f&d-h\end{bmatrix}$$

SCALAR MULTIPLICATION:

- Multiply each element in the matrix by a scalar

$$k\times\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}k\times a&k\times b\\k\times c&k\times d\end{bmatrix}$$

Transpose: swapping a matrices rows and columns. The transpose of matrix A is A^T. First row becomes first columns, etc.

$$\begin{bmatrix}a&b&c&d\\e&f&g&h\end{bmatrix}^T=\begin{bmatrix}a&e\\b&f\\c&g\\d&h\end{bmatrix}$$

7C: Advanced operations with matrices

MATRIX MULTIPLICATION: Involves both multiplication and addition

- **Post multiplication:** AB, matrix A is ‘post multiplied’ by matrix B
- **Pre multiplication:** AB, matrix B is ‘pre multiplied’ by matrix A

You must check that matrices can be multiplied first:

- Multiplication criteria: the number of columns in the first matrix MUST EQUAL the number of rows in the second matrix.
    - ‘defined’ = can be multiplied
    - ‘undefined’ = cant be multiplied

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad B = \begin{bmatrix} e \\ f \end{bmatrix}$$

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e \\ f \end{bmatrix} \qquad BA = \begin{bmatrix} e \\ f \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Order: } 2 \times \underbrace{2 \quad 2}_{\text{equal}} \times 1 \qquad \text{Order: } 2 \times \underbrace{1 \quad 2}_{\text{not equal}} \times 1$$

AB is defined, BA is not defined.

Matrix product: the resulting matrix when two or more matrices are multiplied. The size of this matrix is determined by the outside numbers when writing the matrices orders. AB will produce a 2×1

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e \\ f \end{bmatrix}$$

$$\text{Order: } \underbrace{2 \times 2 \quad 2 \times 1}_{2 \times 1}$$

Multiplication by hand: multiply the rows of the first matrix by the columns of the second matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \times e + b \times g & a \times f + b \times h \\ c \times e + d \times g & c \times f + d \times h \end{bmatrix}$$

Summing matrices: a row or column matrix where all elements are 1.

- **To sum the rows** of an m×n matrix, **post multiply** by an n×1 summing matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 2 \times 1 + 3 \times 1 \\ 4 \times 1 + 5 \times 1 + 6 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 15 \end{bmatrix}$$

- **To sum the columns** of an m×n matrix, **pre multiply** by a 1×m summing matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 3 & 3 \\ 7 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 16 \end{bmatrix}$$

Raising matrices to a power: only SQUARE matrices can be raised to a power. The power indicates how many times the matrix is multiplied by itself.

7D: Inverse Matrices

The determinant:

- Used to identify if an inverse exists
- Must be a positive or negative number
- If the determinant equals ZERO, there is no inverse and the matrix is said to be ‘singular’

Calculating the determinant (2x2 by hand):

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = ad - bc$$

The inverse:

- Only square matrices have an inverse
- Inverse means opposite
- Something multiplied by its inverse is equal to 1, in this case, the IDENTITY MATRIX
- A^{-1} = the inverse of matrix A
- A × A^{-1} = I (identity matrix)

Calculating the inverse:

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- b and c multiply by -1
- a and d swap positions
- Multiply by the scalar so that the answer is just the matrix

****if you are given the inverse of a matrix and need to find the original****

- Put the inverse to the power of -1
- $(A^{-1})^{-1} = A$

7E: Binary and Permutation Matrices

Binary matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Permutation matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Binary: only has 1’s and 0’s as its elements (any size)
Permutation matrix: a type of binary matrix that has **only one 1 in each row and column.** It is always square and is used to rearrange elements.

- Column permutation: rearranges columns.
    - Post multiply (Q × P)
- Row permutation: rearranges rows.
    - Pre multiply (P× Q)

7F: Communication and Dominance Matrices

Communication: square binary matrix

- 1's represent a direct line of communication
- Communication goes both ways
- C = one step (direct communication)
- C2 = two step (communication via someone else)
- C + C2 = TC (total communication, showing all one and two step links)

Dominance:

- Used to display hierarchy
- One-way connections
- 1's are used for 'winners', 0's for 'losers'
- D = one step
- D2 = two step
- D + D2 = TD (total dominance)
- Directed, arrows point to loser

Determining dominance when given the sum of one step and two step dominance:

- One step dominance tells you how many times they won
- Two step dominance tells you the total sum of the one step dominances

7G: Introduction to transition matrices

State matrix S_n

- Column matrix
- Population at a given time

Initial state matrix

- Starting population
- S_0

Transition matrix

- Square matrix
- Studies change over time
- Elements are decimal numbers (0-1) that represent percentages
- Each column must sum to 1
- Other language: 'now/next', 'today/tomorrow', 'from/to'

$$\boxed{\begin{array}{cccc} & \multicolumn{2}{c}{\textit{This time}} & \\ & P & I & \\ T = & \multicolumn{2}{c}{\left[\begin{array}{cc} & \\ & \end{array}\right]} & \begin{array}{c} P \\ I \end{array} \;\textit{Next time} \end{array}}$$

Calculating state matrices:

- Used to find the next state
- Use recursion (step by step)

$$\boxed{S_0 = \textit{initial state matrix}, \quad S_{n+1} = T \times S_n}$$

S_n is the current state matrix
S_{n+1} is the next state matrix
T is the transition matrix.

- Use the rule (find future values, n, faster)

$$\boxed{S_n = T^n \times S_0}$$

*****Note: Finding a previous state:** say you are given T and S_4, how could you find a previous state? For example, if you need to find S_3 take the inverse of T and multiply is by S_4.

7H: The Equilibrium matrix

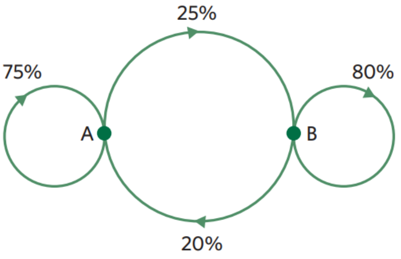
- 'Steady state'
- 'In the long term'
- Over time the populations settle to where there is no visible change
- This is when two consecutive matrices are equal (2dp for accuracy)
- Consecutive means finding S_{17} and S_{18} to be the same

7I: Applications of transition matrices

Transition diagram: a visual representation of how a transition matrix functions.

$$T = \begin{array}{c} \text{today} \\ \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.75 & 0.2 \\ 0.25 & 0.8 \end{bmatrix} \end{array} \begin{array}{c} \text{A} \\ \text{B} \end{array} \text{ tomorrow}$$

can be represented by the following transition diagram.



Culling and Restocking:

- Populations are subject to change over time
- Considers external forces affecting the population

$$S_{n+1} = TS_n + B$$

Culling: reduction/removed, negative number in matrix B

Restocking: addition, positive number in matrix B

Keeping a population constant: all future state matrices are equal to the initial. This can be determined by calculating $S_0 - S_1$

7.1 Entering a matrix into a CAS calculator

CAS 1: How to enter a matrix on the TI-Nspire CAS

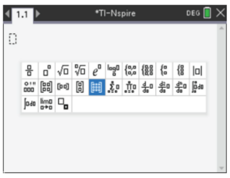
Enter the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ and determine its transpose (A^T).

Steps

1 Press `ctrl` + `N`. Select **Add Calculator**.

2 Press `⊞` and use the cursor ▼ ▶ arrows to highlight the matrix template shown. Press `enter`.

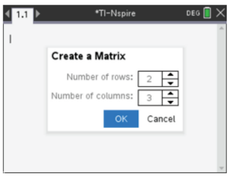
Note: **Math Templates** can also be accessed by pressing `ctrl` + `menu` **>Templates**.



3 Use the ▼ arrow to select the **Number of rows** required (number of rows in this example is 2).

Press `tab` to move to the next entry and repeat for the **Number of columns** (the number of columns in this example is 3).

Press `tab` to highlight **OK**; press `enter`.

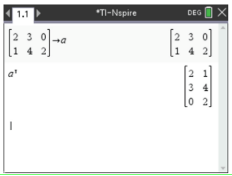


Create a Matrix

Number of rows: 2

Number of columns: 3

4 Type the values into the matrix template. Use `tab` to move to the required position in the matrix to enter each value. When the matrix has been completed, press `tab` to move outside the matrix, press `ctrl` `var`, followed by `A`. Press `enter`. This will store the matrix as the variable a.



$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} \to a$ $\quad \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$

a^{T} $\quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \\ 0 & 2 \end{bmatrix}$

5 When you type ***A*** (or ***a***) it will paste in the matrix $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. Press `enter` to display.

6 To find a^T, type in *a* (for matrix A) and then `menu` **>Matrix & Vector>Transpose >** `enter` as shown.

Note: Superscript T can also be accessed from the symbols palette (`ctrl` `⊞`).

7.2 To perform matrix addition, subtraction, and scalar multiplication

CAS 2: How to add, subtract and scalar multiply matrices using the TI-Nspire CAS

If $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix}$, find:

a $A + B$ **b** $A - B$ **c** $3A - 2B$

Steps

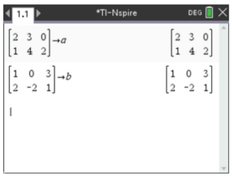
1 Press `ctrl` + `N`. Select **Add Calculator**.

2 Enter and store the matrices A and B into your calculator.

a To determine $A + B$, type ***a*** + ***b***. Press `enter` to evaluate.

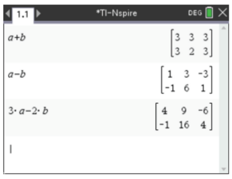
b To determine $A - B$, type ***a*** – ***b***. Press `enter` to evaluate.

c To determine $3A - 2B$, type **3*a*** – **2*b***. Press `enter` to evaluate.



$\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} \to a$ $\quad \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix} \to b$ $\quad \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix}$



$a+b$ $\quad \begin{bmatrix} 3 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix}$

$a-b$ $\quad \begin{bmatrix} 1 & 3 & -3 \\ -1 & 6 & 1 \end{bmatrix}$

$3\cdot a-2\cdot b$ $\quad \begin{bmatrix} 4 & 9 & -6 \\ -1 & 16 & 4 \end{bmatrix}$

7.3 To multiply two matrices

CAS 3: How to multiply two matrices using the TI-Nspire CAS

If $C = \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix}$ and $D = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find the matrix CD.

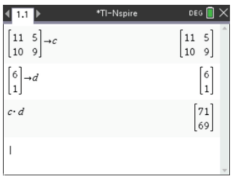
Steps

1 Press `ctrl` + `N`. Select **Add Calculator**.

2 Enter and store the matrices C and D into your calculator.

3 To calculate matrix CD, type ***c*** × ***d***. Press `enter` to evaluate.

Note: You must put a multiplication sign between the *c* and *d*.



$\begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix} \to c$ $\quad \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 1 \end{bmatrix} \to d$ $\quad \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

$c\cdot d$ $\quad \begin{bmatrix} 71 \\ 69 \end{bmatrix}$

7.4 To determine the determinant and inverse of an n × n matrix

CAS 4: How to find the determinant and inverse of a matrix using the TI-Nspire CAS

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix}$, find det($A$) and A^{-1}.

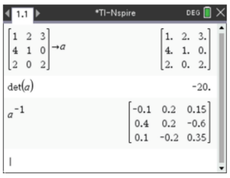
Steps

2 Press `ctrl` + `N`. Select **Add Calculator**.

3 Enter the matrix A into your calculator.

4 To calculate det(A), type **det(*a*)** and press `enter` to evaluate.

Note: det() can also be accessed using `menu`>**Matrix & Vector>Determinant**.



$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} \to a$ $\quad \begin{bmatrix} 1. & 2. & 3. \\ 4. & 1. & 0. \\ 2. & 0. & 2. \end{bmatrix}$

det(a) $\quad$ -20.

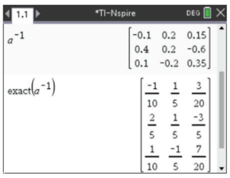
a^{-1} $\quad \begin{bmatrix} -0.1 & 0.2 & 0.15 \\ 0.4 & 0.2 & -0.6 \\ 0.1 & -0.2 & 0.35 \end{bmatrix}$

4 To calculate the inverse matrix A^{-1} type **a ∧ – 1** and press `enter` to evaluate. If you want to see the answer in fractional form, enter as **exact (a∧ –1)** and press `enter` to evaluate.

Note:

1 Long strings of decimals can be avoided by asking for an exact inverse. Type in **exact(a^{-1})**.

2 If the matrix has no inverse, the calculator will respond with the error message **Singular matrix**.

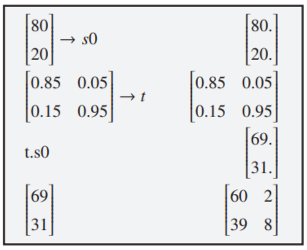


a^{-1} $\quad \begin{bmatrix} -0.1 & 0.2 & 0.15 \\ 0.4 & 0.2 & -0.6 \\ 0.1 & -0.2 & 0.35 \end{bmatrix}$

exact(a^{-1}) $\quad \begin{bmatrix} \frac{-1}{10} & \frac{1}{5} & \frac{3}{20} \\ \frac{2}{5} & \frac{1}{5} & \frac{-3}{5} \\ \frac{1}{10} & \frac{-1}{5} & \frac{7}{20} \end{bmatrix}$

7.5 Transition matrices – using recursion

Calculator hint: In practice, generating matrices recursively is performed on your CAS calculator as shown opposite for the calculations performed in Example 11.



$\begin{bmatrix} 80 \\ 20 \end{bmatrix} \to s0$ $\quad \begin{bmatrix} 80. \\ 20. \end{bmatrix}$

$\begin{bmatrix} 0.85 & 0.05 \\ 0.15 & 0.95 \end{bmatrix} \to t$ $\quad \begin{bmatrix} 0.85 & 0.05 \\ 0.15 & 0.95 \end{bmatrix}$

t.s0 $\quad \begin{bmatrix} 69. \\ 31. \end{bmatrix}$

$\begin{bmatrix} 69 \\ 31 \end{bmatrix}$ $\quad \begin{bmatrix} 60 & 2 \\ 39 & 8 \end{bmatrix}$

and so on.

7.6 Transition – the n^{th} state of a system using the rule $s_n = T^n s_0$

1 Write down the transition matrix, T, and initial state matrix, S_0. Enter the matrices into your calculator. Use T and S.

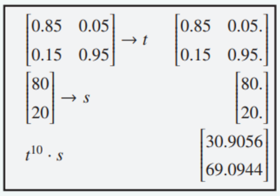
$$T = \begin{bmatrix} 0.85 & 0.05 \\ 0.15 & 0.95 \end{bmatrix} \quad S_0 = \begin{bmatrix} 80 \\ 20 \end{bmatrix}$$

2 To find out how many machines are in operation and how many are broken after 10 days, write down the rule $S_n = T^n S_0$ and substitute $n = 10$ to give $S_{10} = T^{10} S_0$.

$$S_n = T^n S_0$$

$$\therefore S_{10} = T^{10} S_0$$

3 Enter the expression $T^{10}S$ into your calculator and evaluate.



$\begin{bmatrix} 0.85 & 0.05 \\ 0.15 & 0.95 \end{bmatrix} \to t$ $\quad \begin{bmatrix} 0.85 & 0.05. \\ 0.15 & 0.95. \end{bmatrix}$

$\begin{bmatrix} 80 \\ 20 \end{bmatrix} \to s$ $\quad \begin{bmatrix} 80. \\ 20. \end{bmatrix}$

$t^{10} \cdot s$ $\quad \begin{bmatrix} 30.9056 \\ 69.0944 \end{bmatrix}$

4 Write down your answer in matrix form and then in words.

$$S_{10} = \begin{bmatrix} 30.9 \\ 69.1 \end{bmatrix}$$

7.7 Transition matrices – the steady state solution

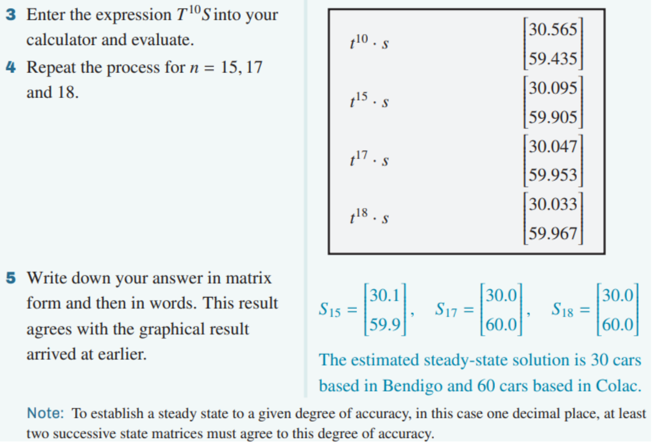
1 Write down the transition matrix T and initial state matrix S_0. Enter the matrices into your calculator. Use T and S.

$$T = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}, \ S_0 = \begin{bmatrix} 50 \\ 40 \end{bmatrix}$$

2 Use the rule $S_n = T^n S_0$ to write down the expression for the nth state for $n = 10$.

$$S_n = T^n S_0$$

$$\therefore S_{10} = T^{10} S_0 = \begin{bmatrix} 30.6 \\ 59.4 \end{bmatrix}$$



3 Enter the expression $T^{10}S$ into your calculator and evaluate.

4 Repeat the process for $n = 15, 17$ and 18.

$t^{10} \cdot s$ $\begin{bmatrix}30.565\\59.435\end{bmatrix}$

$t^{15} \cdot s$ $\begin{bmatrix}30.095\\59.905\end{bmatrix}$

$t^{17} \cdot s$ $\begin{bmatrix}30.047\\59.953\end{bmatrix}$

$t^{18} \cdot s$ $\begin{bmatrix}30.033\\59.967\end{bmatrix}$

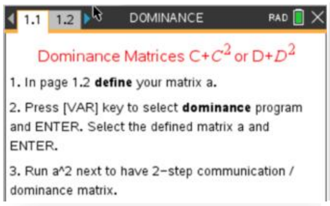
5 Write down your answer in matrix form and then in words. This result agrees with the graphical result arrived at earlier.

$S_{15} = \begin{bmatrix}30.1\\59.9\end{bmatrix}, \quad S_{17} = \begin{bmatrix}30.0\\60.0\end{bmatrix}, \quad S_{18} = \begin{bmatrix}30.0\\60.0\end{bmatrix}$

The estimated steady-state solution is 30 cars based in Bendigo and 60 cars based in Colac.

Note: To establish a steady state to a given degree of accuracy, in this case one decimal place, at least two successive state matrices must agree to this degree of accuracy.

7.8 Using “Dominance Communication” Template

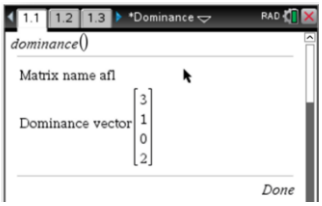


Dominance Matrices C+C^2 or D+D^2

1. In page 1.2 **define** your matrix a.

2. Press [VAR] key to select **dominance** program and ENTER. Select the defined matrix a and ENTER.

3. Run a^2 next to have 2−step communication / dominance matrix.

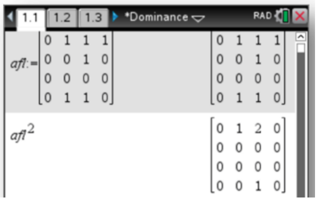


dominance()

Matrix name afl

Dominance vector $\begin{bmatrix}3\\1\\0\\2\end{bmatrix}$

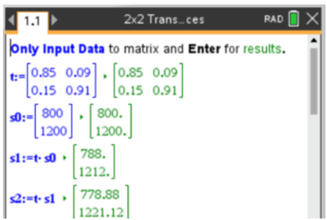
Done



$afl:=\begin{bmatrix}0&1&1&1\\0&0&1&0\\0&0&0&0\\0&1&1&0\end{bmatrix}$ $\begin{bmatrix}0&1&1&1\\0&0&1&0\\0&0&0&0\\0&1&1&0\end{bmatrix}$

afl^2 $\begin{bmatrix}0&1&2&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix}$

7.9 Using “Transition Leslie Matrices” Template



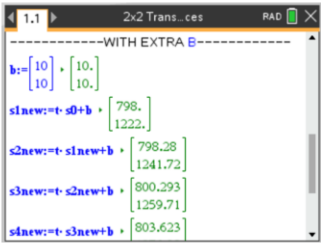
Only Input Data to matrix and **Enter** for results.

$t:=\begin{bmatrix}0.85&0.09\\0.15&0.91\end{bmatrix} ▸ \begin{bmatrix}0.85&0.09\\0.15&0.91\end{bmatrix}$

$s0:=\begin{bmatrix}800\\1200\end{bmatrix} ▸ \begin{bmatrix}800.\\1200.\end{bmatrix}$

$s1:=t \cdot s0 ▸ \begin{bmatrix}788.\\1212.\end{bmatrix}$

$s2:=t \cdot s1 ▸ \begin{bmatrix}778.88\\1221.12\end{bmatrix}$



------------WITH EXTRA B------------

$b:=\begin{bmatrix}10\\10\end{bmatrix} ▸ \begin{bmatrix}10.\\10.\end{bmatrix}$

$s1new:=t \cdot s0+b ▸ \begin{bmatrix}798.\\1222.\end{bmatrix}$

$s2new:=t \cdot s1new+b ▸ \begin{bmatrix}798.28\\1241.72\end{bmatrix}$

$s3new:=t \cdot s2new+b ▸ \begin{bmatrix}800.293\\1259.71\end{bmatrix}$

$s4new:=t \cdot s3new+b ▸ 803.623$

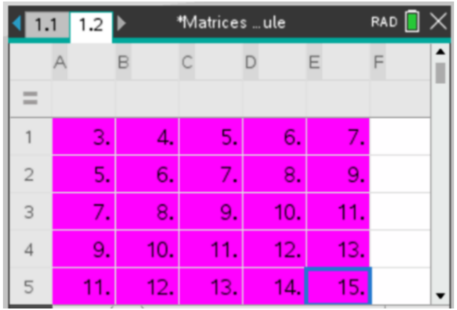
---------FIND PREVIOUS MATRIX---------

$t^{-1} \cdot (s2new-b) ▸ \begin{bmatrix}798.\\1222.\end{bmatrix}$

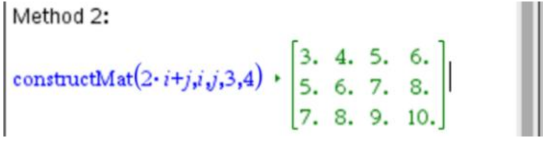
7.10 Using “Matrices by i j rule” Template

$rule(i,j):=2 \cdot i+j ▸ Done$

Method 1: Only Input Rule above and Enter for results in page 1.2.



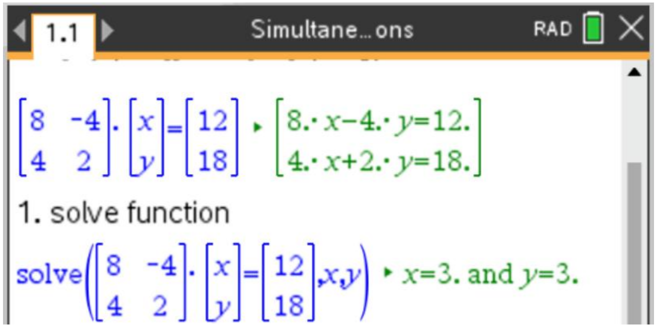
	A	B	C	D	E	F
=						
1	3.	4.	5.	6.	7.	
2	5.	6.	7.	8.	9.	
3	7.	8.	9.	10.	11.	
4	9.	10.	11.	12.	13.	
5	11.	12.	13.	14.	15.	



Method 2:

$constructMat(2 \cdot i+j,i,j,3,4) ▸ \begin{bmatrix}3.&4.&5.&6.\\5.&6.&7.&8.\\7.&8.&9.&10.\end{bmatrix}$

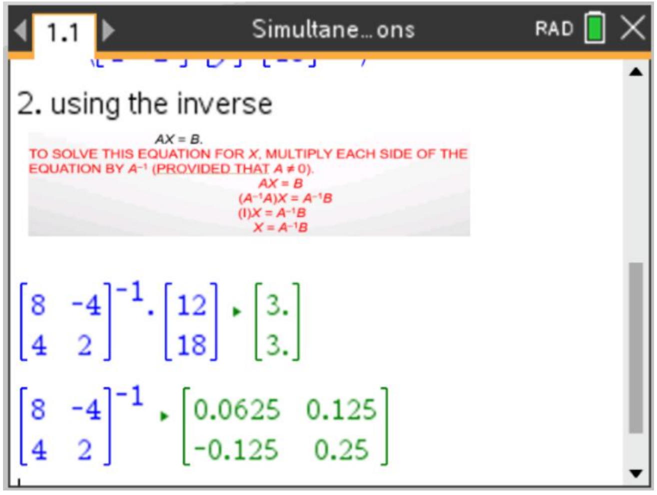
7.11 Using “Simultaneous Equations” Template



$\begin{bmatrix}8&-4\\4&2\end{bmatrix} \cdot \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}12\\18\end{bmatrix} ▸ \begin{bmatrix}8. \cdot x-4. \cdot y=12.\\4. \cdot x+2. \cdot y=18.\end{bmatrix}$

1. solve function

$solve\left(\begin{bmatrix}8&-4\\4&2\end{bmatrix} \cdot \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}12\\18\end{bmatrix},x,y\right) ▸ x=3. \text{ and } y=3.$



2. using the inverse

$AX = B.$

TO SOLVE THIS EQUATION FOR X, MULTIPLY EACH SIDE OF THE EQUATION BY A^{-1} (PROVIDED THAT $A \neq 0$).

$AX = B$

$(A^{-1}A)X = A^{-1}B$

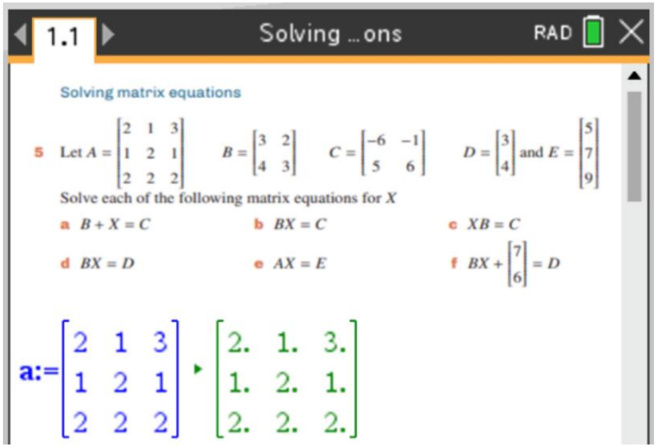
$(I)X = A^{-1}B$

$X = A^{-1}B$

$\begin{bmatrix}8&-4\\4&2\end{bmatrix}^{-1} \cdot \begin{bmatrix}12\\18\end{bmatrix} ▸ \begin{bmatrix}3.\\3.\end{bmatrix}$

$\begin{bmatrix}8&-4\\4&2\end{bmatrix}^{-1} ▸ \begin{bmatrix}0.0625&0.125\\-0.125&0.25\end{bmatrix}$

7.12 Using “Solving Matrices Equations” Template



Solving matrix equations

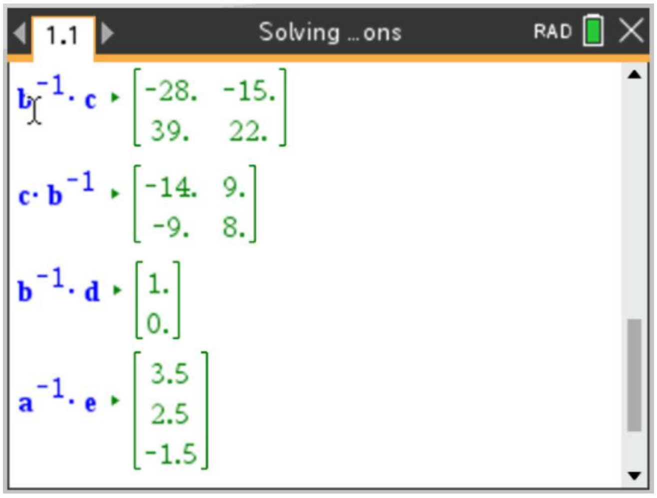
5 Let $A = \begin{bmatrix}2&1&3\\1&2&1\\2&2&2\end{bmatrix}$ $B = \begin{bmatrix}3&2\\4&3\end{bmatrix}$ $C = \begin{bmatrix}-6&-1\\5&6\end{bmatrix}$ $D = \begin{bmatrix}3\\4\end{bmatrix}$ and $E = \begin{bmatrix}5\\7\\9\end{bmatrix}$

Solve each of the following matrix equations for X

a $B + X = C$ **b** $BX = C$ **c** $XB = C$

d $BX = D$ **e** $AX = E$ **f** $BX + \begin{bmatrix}7\\6\end{bmatrix} = D$

$a:=\begin{bmatrix}2&1&3\\1&2&1\\2&2&2\end{bmatrix} ▸ \begin{bmatrix}2.&1.&3.\\1.&2.&1.\\2.&2.&2.\end{bmatrix}$



$b^{-1} \cdot c ▸ \begin{bmatrix}-28.&-15.\\39.&22.\end{bmatrix}$

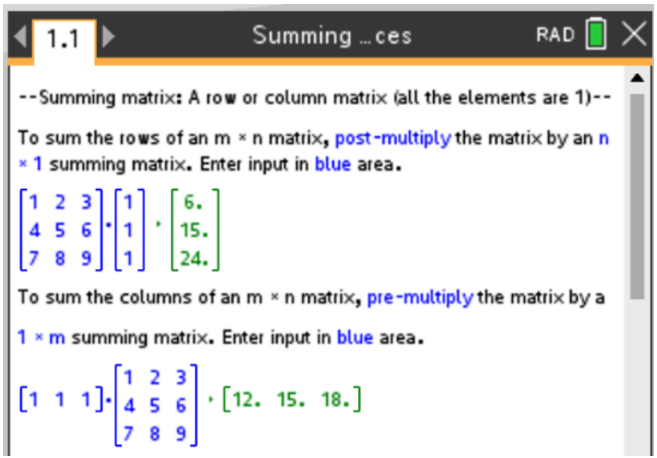
$c \cdot b^{-1} ▸ \begin{bmatrix}-14.&9.\\-9.&8.\end{bmatrix}$

$b^{-1} \cdot d ▸ \begin{bmatrix}1.\\0.\end{bmatrix}$

$a^{-1} \cdot e ▸ \begin{bmatrix}3.5\\2.5\\-1.5\end{bmatrix}$

7.13 Using “Summing Permutation Matrices” Template

Enter into Blue area



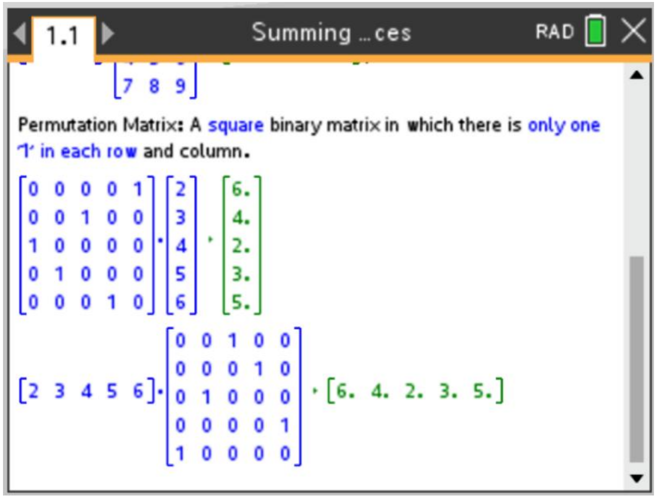
--Summing matrix: A row or column matrix (all the elements are 1)--

To sum the rows of an m × n matrix, post-multiply the matrix by an n × 1 summing matrix. Enter input in blue area.

$$\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\end{bmatrix} \blacktriangleright \begin{bmatrix}6.\\15.\\24.\end{bmatrix}$$

To sum the columns of an m × n matrix, pre-multiply the matrix by a 1 × m summing matrix. Enter input in blue area.

$$\begin{bmatrix}1&1&1\end{bmatrix}\cdot\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix} \blacktriangleright \begin{bmatrix}12.&15.&18.\end{bmatrix}$$



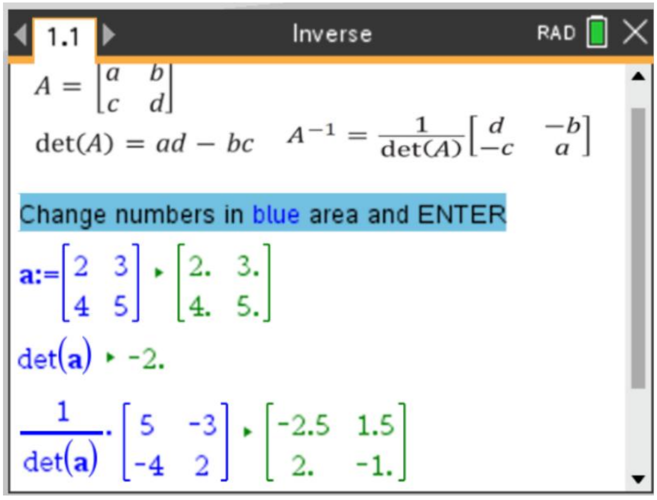
7 8 9

Permutation Matrix: A square binary matrix in which there is only one '1' in each row and column.

$$\begin{bmatrix}0&0&0&0&1\\0&0&1&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&0&1&0\end{bmatrix}\cdot\begin{bmatrix}2\\3\\4\\5\\6\end{bmatrix} \blacktriangleright \begin{bmatrix}6.\\4.\\2.\\3.\\5.\end{bmatrix}$$

$$\begin{bmatrix}2&3&4&5&6\end{bmatrix}\cdot\begin{bmatrix}0&0&1&0&0\\0&0&0&1&0\\0&1&0&0&0\\0&0&0&0&1\\1&0&0&0&0\end{bmatrix} \blacktriangleright \begin{bmatrix}6.&4.&2.&3.&5.\end{bmatrix}$$

7.14 Using “Inverse” Template



$$A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$$

$$\det(A) = ad - bc \qquad A^{-1} = \frac{1}{\det(A)}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$

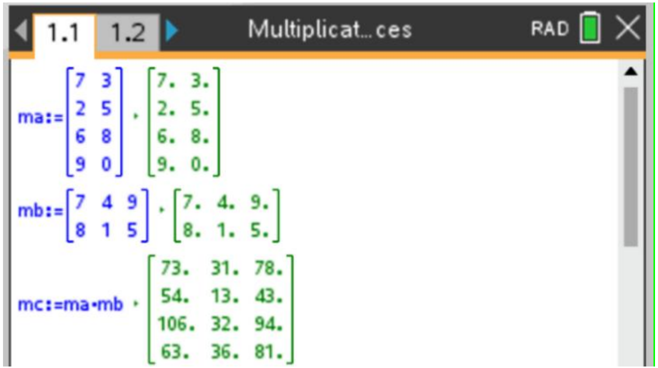
Change numbers in blue area and ENTER

$$a:=\begin{bmatrix}2&3\\4&5\end{bmatrix} \blacktriangleright \begin{bmatrix}2.&3.\\4.&5.\end{bmatrix}$$

$$\det(a) \blacktriangleright -2.$$

$$\frac{1}{\det(a)}\cdot\begin{bmatrix}5&-3\\-4&2\end{bmatrix} \blacktriangleright \begin{bmatrix}-2.5&1.5\\2.&-1.\end{bmatrix}$$

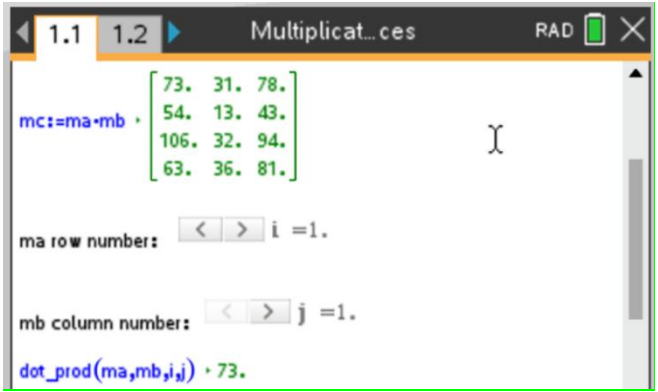
7.15 Using “Multiplication Two Matrices” Template



$$ma:=\begin{bmatrix}7&3\\2&5\\6&8\\9&0\end{bmatrix} \blacktriangleright \begin{bmatrix}7.&3.\\2.&5.\\6.&8.\\9.&0.\end{bmatrix}$$

$$mb:=\begin{bmatrix}7&4&9\\8&1&5\end{bmatrix} \blacktriangleright \begin{bmatrix}7.&4.&9.\\8.&1.&5.\end{bmatrix}$$

$$mc:=ma\cdot mb \blacktriangleright \begin{bmatrix}73.&31.&78.\\54.&13.&43.\\106.&32.&94.\\63.&36.&81.\end{bmatrix}$$



$$mc:=ma\cdot mb \blacktriangleright \begin{bmatrix}73.&31.&78.\\54.&13.&43.\\106.&32.&94.\\63.&36.&81.\end{bmatrix}$$

ma row number: < > i =1.

mb column number: < > j =1.

dot_prod(ma,mb,i,j) ▸ 73.