Recursion and financial modelling

Question 17

Consider the recurrence relation shown below.

$$A_0 = 3, \qquad A_{n+1} = 2A_n + 4$$

The value of A_3 in the sequence generated by this recurrence relation is given by

A. $2 \times 3 + 4$

B. $2 \times 4 + 4$

C. $2 \times 10 + 4$

D. $2 \times 24 + 4$

E. $2 \times 52 + 4$

Question 18

Geoff purchased a computer for \$4500. He will depreciate the value of his computer by a flat rate of 10% of the purchase price per annum.

A recurrence relation that Geoff can use to determine the value of the computer after n years, V_n, is

A. $V_0 = 4500, \quad V_{n+1} = V_n - 450$

B. $V_0 = 4500, \quad V_{n+1} = V_n + 450$

C. $V_0 = 4500, \quad V_{n+1} = 0.9\,V_n$

D. $V_0 = 4500, \quad V_{n+1} = 1.1\,V_n$

E. $V_0 = 4500, \quad V_{n+1} = 0.1\,(V_n - 450)$

Question 19

Manu invests \$3000 in an account that pays interest compounding monthly.

The balance of his investment after n months, B_n, can be determined using the recurrence relation

$$B_0 = 3000, \qquad B_{n+1} = 1.0048 \times B_n$$

The total interest earned by Manu's investment after the first five months is closest to

A. \$57.60

B. \$58.02

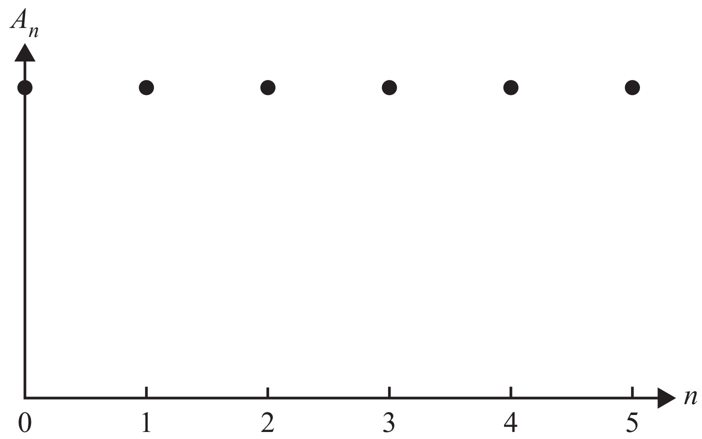
C. \$72.00

D. \$72.69

E. \$87.44

Question 20

The graph below represents the value A_n, in dollars, of an annuity investment for five time periods.



A recurrence relation that could match this graphical representation is

A. $A_0 = 200\,000, \quad A_{n+1} = 1.015A_n - 2500$

B. $A_0 = 200\,000, \quad A_{n+1} = 1.025A_n - 5000$

C. $A_0 = 200\,000, \quad A_{n+1} = 1.03A_n - 5500$

D. $A_0 = 200\,000, \quad A_{n+1} = 1.04A_n - 6000$

E. $A_0 = 200\,000, \quad A_{n+1} = 1.05A_n - 8000$

Question 21

Ray deposited \$7000 in an investment account earning interest at the rate of 3% per annum, compounding quarterly.

A rule for the balance, R_n, in dollars, after n years is given by

A. $R_n = 7000 \times 0.03^n$

B. $R_n = 7000 \times 1.03^n$

C. $R_n = 7000 \times 0.03^{4n}$

D. $R_n = 7000 \times 1.0075^n$

E. $R_n = 7000 \times 1.0075^{4n}$

Question 22

The value of a van purchased for \$45 000 is depreciated by k% per annum using the reducing balance method.

After three years of this depreciation, it is then depreciated in the fourth year under the unit cost method at the rate of 15 cents per kilometre.

The value of the van after it travels 30 000 km in this fourth year is \$26 166.24

The value of k is

A. 9

B. 12

C. 14

D. 16

E. 18

Question 23

Barb took out a personal loan, borrowing \$5000.

Interest for this loan compounds quarterly.

Barb makes no repayments in the first year and after one year she owes \$5325.14

The effective annual rate of interest for the first year of Barb's loan is closest to

A. 6.34%
B. 6.35%
C. 6.50%
D. 6.54%
E. 6.56%

Question 24

Twenty years ago, Hector invested a sum of money in an account earning interest at the rate of 3.2% per annum, compounding monthly.

After 10 years, he made a one-off extra payment of \$10 000 to the account.

For the next 10 years, the account earned interest at the rate of 2.8% per annum, compounding monthly.

The balance of his account today is \$686 904.09

The sum of money Hector originally invested is closest to

A. \$355 000
B. \$370 000
C. \$377 000
D. \$384 000
E. \$385 000

Recursion and financial modelling

Question 17

A sequence of numbers is generated by the recurrence relation shown below.

$$T_0 = 5, \qquad T_{n+1} = -T_n$$

The value of T_2 is

A. -10
B. -5
C. 0
D. 5
E. 10

Use the following information to answer Questions 18 and 19.

Gus purchases a coffee machine for $15 000 and depreciates its value using the unit cost method.

The rate of depreciation is $0.04 per cup of coffee made.

A recurrence relation that models the year-to-year value G_n, in dollars, of the machine is

$$G_0 = 15000, \qquad G_{n+1} = G_n - 1314$$

Question 18

A rule for G_n, the value of the machine after n years is

A. $G_n = 15\,000 - 0.04n$
B. $G_n = 15\,000 + 0.04n$
C. $G_n = 15\,000 - 1314n$
D. $G_n = 1314 - 0.04n$
E. $G_n = 1314 + 0.04n$

Question 19

The number of cups made by the machine per year is

A. 1314
B. 13 686
C. 15 000
D. 31 536
E. 32 850

Use the following information to answer Questions 20 and 21.

For taxation purposes, Audrey depreciates the value of her \$3000 computer over a four-year period. At the end of the four years, the value of the computer is \$600.

Question 20

If Audrey uses flat rate depreciation, the depreciation rate, per annum is

A. 10%
B. 15%
C. 20%
D. 25%
E. 33%

Question 21

If Audrey uses reducing balance depreciation, the depreciation rate, per annum is closest to

A. 10%
B. 15%
C. 20%
D. 25%
E. 33%

Question 22

Timmy took out a reducing balance loan of \$500 000, with interest calculated monthly.

The balance of the loan, in dollars, after n months, T_n, can be modelled by the recurrence relation

$$T_0 = 500\,000, \qquad T_{n+1} = 1.00325T_n - 2611.65$$

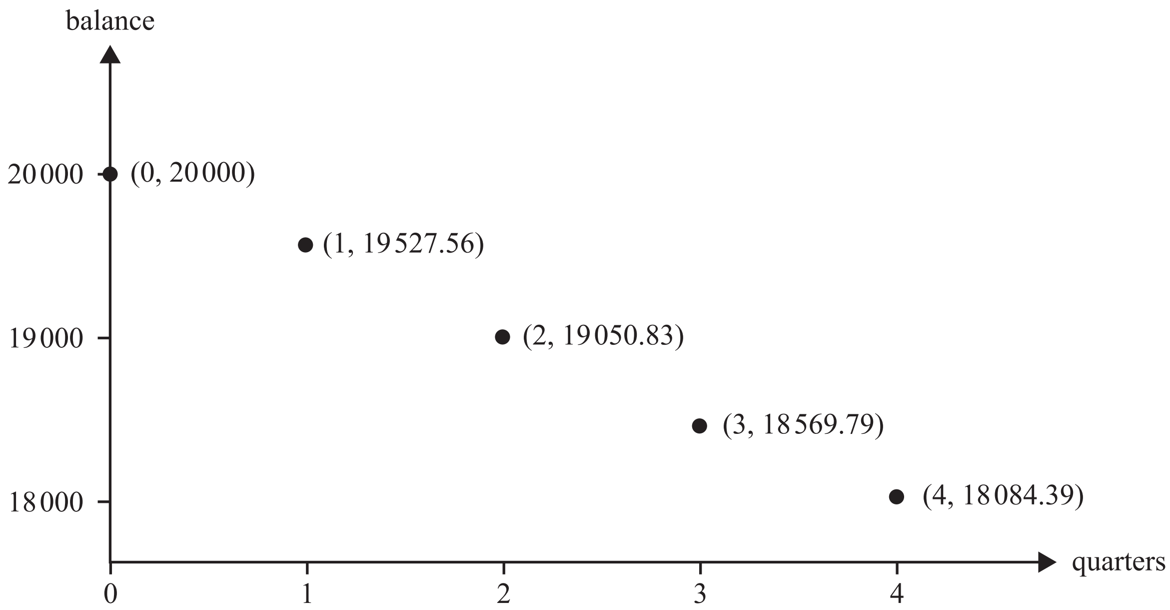
A final repayment that will fully repay the loan to the nearest cent is

A. \$2605.65
B. \$2609.18
C. \$2611.65
D. \$2614.12
E. \$2615.81

Question 23

Tavi took out a loan of \$20 000, with interest compounding quarterly. She makes quarterly repayments of \$653.65.

The graph below represents the balance in dollars of Tavi's loan at the end of each quarter of the first year of the loan.



The effective interest rate for the first year of Tavi's loan is closest to

A. 3.62%

B. 3.65%

C. 3.66%

D. 3.67%

E. 3.68%

Question 24

The following recurrence relation models the value, P_n, of a perpetuity after n time periods.

$$P_0 = a, \qquad P_{n+1} = RP_n - d$$

The value of R can be found by calculating

A. $a + d$

B. $\dfrac{a+d}{a}$

C. $\dfrac{a+d}{d}$

D. $1+\dfrac{a+d}{a}$

E. $1+\dfrac{a+d}{d}$

Recursion and financial modelling

Question 17

A first-order linear recurrence relation of the form

$$u_0 = a, \qquad u_{n+1} = Ru_n + d$$

generates the terms of a sequence. A geometric sequence will be generated if

A. $R = 1$ and $d = -1$

B. $R = 1$ and $d = 1$

C. $R = 4$ and $d = -1$

D. $R = 2$ and $d = 0$

Question 18

Trevor took out a reducing balance loan of \$400 000, with interest calculated weekly.

The balance of the loan, in dollars, after n weeks, T_n, can be modelled by the recurrence relation

$$T_0 = 400\,000, \qquad T_{n+1} = 1.00075T_n - 677.55$$

Assume that there are exactly 52 weeks in a year.

The interest rate, per annum, for this loan is

A. 0.75%

B. 3.9%

C. 4.5%

D. 7.5%

Question 19

Liv bought a new car for \$35 000. The value of the car will be depreciated by 18% per annum using the reducing balance method.

A recurrence relation that models the year-to-year value of her car is of the form

$$L_0 = 35\,000, \qquad L_{n+1} = k \times L_n$$

The value of k is

A. 0.0082

B. 0.18

C. 0.82

D. 1.18

Question 20

Dainika invested \$2000 for three years at 4.4% per annum, compounding quarterly.

To earn the same amount of interest in three years in a simple interest account, the annual simple interest rate would need to be closest to

A. 4.60%

B. 4.68%

C. 4.84%

D. 4.98%

Question 21

Lee took out a loan of \$121 000, with interest compounding monthly. He makes monthly repayments of \$2228.40 for five years until the loan is repaid in full.

The total interest paid by Lee is closest to

A. \$4434

B. \$5465

C. \$10 539

D. \$12 704

Use the following information to answer Questions 22 and 23.

Stewart takes out a reducing balance loan of $240 000, with interest calculated monthly.

Stewart makes regular monthly repayments.

Three lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	240 000.00
1	2741.05	960.00	1781.05	238 218.95
2	2741.05			

Question 22

The principal reduction associated with Payment number 2 is closest to

A. $1773.93

B. $1781.05

C. $1788.17

D. $2741.05

Question 23

The number of years that it will take Stewart to repay the loan in full is closest to

A. 9

B. 10

C. 11

D. 12

Question 24

André invested \$18 000 in an account for five years, with interest compounding monthly.

He adds an extra payment into the account each month immediately after the interest is calculated.

For the first two years, the balance of the account, in dollars, after n months, A_n, can be modelled by the recurrence relation

$$A_0 = 18\,000, \qquad A_{n+1} = 1.002A_n + 100$$

After two years, André decides he would like the account to reach a balance of \$30 000 at the end of the five years.

He must increase the value of the monthly extra payment to achieve this.

The minimum value of the new payment for the last three years is closest to

A. \$189.55

B. \$195.45

C. \$202.35

D. \$246.55

Recursion and financial modelling

Question 17

Mel bought a new car for $60 000. She will depreciate the value of the car using the reducing balance method. A recurrence relation that models the year-to-year value of her car, M_n, is

$$M_0 = 60\,000, \qquad M_{n+1} = 0.85\,M_n$$

An equivalent rule to determine the value of the car after n years is

A. $M_n = 60\,000 - 0.85n$

B. $M_n = 60\,000 + 0.85n$

C. $M_n = 60\,000 + 0.85^n$

D. $M_n = 60\,000 \times 0.85^{n-1}$

E. $M_n = 60\,000 \times 0.85^n$

Question 18

A sequence of numbers is generated by a recurrence relation of the form

$$T_0 = 5, \qquad T_{n+1} = k - T_n$$

All terms of the sequence have the same value.

The constant k is equal to

A. -10

B. -5

C. 0

D. 5

E. 10

Use the following information to answer Questions 19 and 20.

Edo invests $\$10\,000$ in an account earning 3% interest per annum compounding monthly.

Question 19

The value, V_n, of Edo's investment after n months is given by

A. $V_n = 10\,000 \times 1.0025^n$

B. $V_n = 10\,000 \times 1.003^n$

C. $V_n = 10\,000 \times 1.03^n$

D. $V_n = 10\,000 \times 1.003^{12n}$

E. $V_n = 10\,000 \times 1.03^{12n}$

Question 20

The effective interest rate for Edo's investment is closest to

A. 2.96%

B. 2.98%

C. 3.00%

D. 3.02%

E. 3.04%

Question 21

Yolanda purchased a motorcycle for $\$30\,000$. She explores two options for predicting the value of the motorcycle after four years.

Option 1: For the first two years, the value of the motorcycle is depreciated by 10% per annum using flat rate depreciation. For the next two years, the value of the motorcycle is depreciated by 10% per annum using reducing balance depreciation.

Option 2: The value of the motorcycle is depreciated using reducing balance depreciation with a constant depreciation rate per annum for four years.

For both options to predict the same value after four years, the rate per annum used for Option 2 is closest to

A. 9.4%

B. 9.7%

C. 10.0%

D. 10.3%

E. 10.6%

Question 22

The recurrence relation below models the value, P_n, in a financial context after n time periods.

$$P_0 = a, \qquad P_{n+1} = RP_n - d$$

All constants, a, R and d, are greater than 1.

Four options of what the value of P_n could represent are listed below.

- a reducing balance loan
- an annuity
- an asset depreciated using the unit cost method
- a perpetuity

How many of these four options could be represented by the recurrence relation?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 23

Todd invested $450 000 in an annuity at the start of 2024.

The interest rate for this annuity is 3.75% per annum compounding monthly.

He will receive regular monthly payments for the 15-year life of the annuity.

In which year will the balance of the annuity first fall below $350 000?

A. 2027

B. 2028

C. 2029

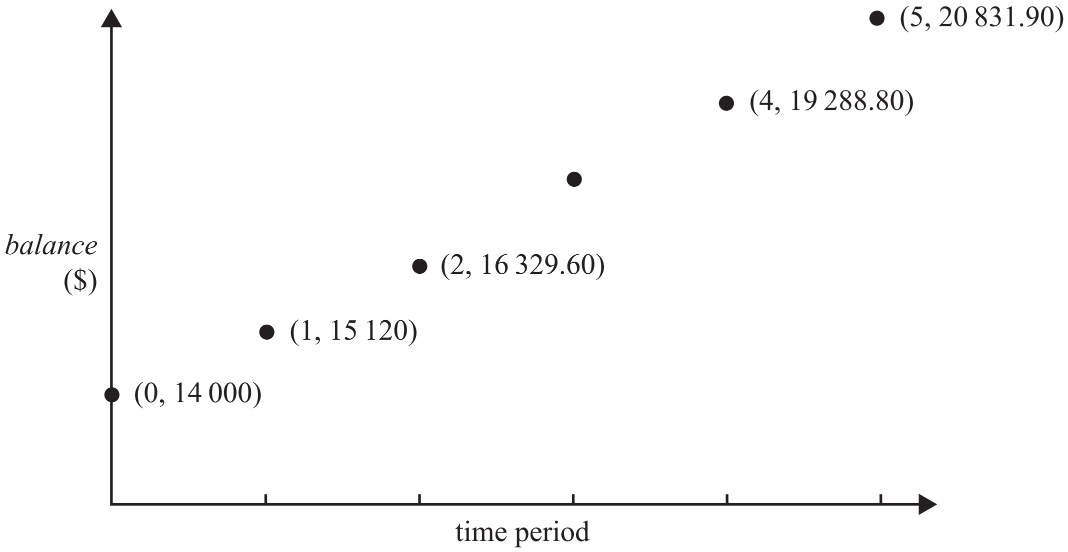
D. 2030

E. 2031

Question 24

Jarryd invested $14 000 into an account earning compound interest at a fixed rate per time period.

The graph below shows the balance of the account for four of the first five time periods after the initial investment. The information for time period 3 is not shown.



Immediately after the interest was calculated for time period 3, Jarryd added an extra one-off amount into the account.

This amount was closest to

A. $224.03

B. $225.97

C. $228.62

D. $229.38

E. $231.46