## Recursion and financial modelling

**Question 6** (4 marks)

Samuel owns a printing machine.

The printing machine is depreciated in value by Samuel using flat rate depreciation.

The value of the machine, in dollars, after $n$ years, $V_n$, can be modelled by the recurrence relation

$$V_0 = 120\,000, \qquad V_{n+1} = V_n - 15\,000$$

**a.** By what amount, in dollars, does the value of the machine decrease each year? 1 mark

**b.** Showing recursive calculations, determine the value of the machine, in dollars, after two years. 1 mark

**c.** What annual flat rate percentage of depreciation is used by Samuel? 1 mark

**d.** The value of the machine, in dollars, after $n$ years, $V_n$, could also be determined using a rule of the form $V_n = a + bn$.

Write down this rule for $V_n$. 1 mark

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**Question 7** (3 marks)

Samuel has a reducing balance loan.

The first five lines of the amortisation table for Samuel's loan are shown below.

| Payment number | Payment ($) | Interest ($) | Principal reduction ($) | Balance ($) |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 320 000.00 |
| 1 | 1600.00 | 960.00 | 640.00 | 319 360.00 |
| 2 | 1600.00 | 958.08 | 641.92 | 318 718.08 |
| 3 | 1600.00 | 956.15 | | 318 074.23 |
| 4 | 1600.00 | | | |

Interest is calculated monthly and Samuel makes monthly payments of $1600.

Interest is charged on this loan at the rate of 3.6% per annum.

**a.** **i.** Using the values in the amortisation table, calculate the principal reduction associated with payment number 3 1 mark

**ii.** Using the values in the amortisation table, calculate the balance of the loan after payment number 4 is made.

Round your answer to the nearest cent. 1 mark

**b.** Let $S_n$ be the balance of Samuel's loan after $n$ months.

Write down a recurrence relation, in terms of $S_0$, $S_{n+1}$ and $S_n$, that could be used to model the month-to-month balance of the loan. 1 mark

**Question 8** (3 marks)

Samuel now invests $500 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after $n$ months, $A_n$, can be modelled by a recurrence relation of the form

$$A_0 = 500\,000, \qquad A_{n+1} = kA_n - 2000$$

**a.** Calculate the balance of this annuity after two months if $k = 1.0024$ 1 mark

**b.** Calculate the annual compound interest rate percentage for this annuity if $k = 1.0024$ 1 mark

**c.** For what value of $k$ would this investment act as a simple perpetuity? 1 mark

**Question 9** (2 marks)

Some time later, Samuel takes out a new reducing balance loan.

The interest rate for this loan was 4.1% per annum, compounding monthly.

The balance of the loan after four years of monthly repayments is $329 587.25

The balance of the loan after seven years of monthly repayments is $280 875.15

Samuel will continue to make the same monthly repayment.

To ensure the loan is fully repaid, to the nearest cent, the required final repayment will be lower.

In the first seven years, Samuel makes 84 monthly repayments.

From this point on, how many more monthly repayments will Samuel make to fully repay the loan?

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## Recursion and financial modelling

**Question 5** (3 marks)

Arthur borrowed $30 000 to buy a new motorcycle.

Interest on this loan is charged at the rate of 6.4% per annum, compounding quarterly.

Arthur will repay the loan in full with quarterly repayments over six years.

**a.** How many repayments, in total, will Arthur make? 1 mark

The balance of the loan, in dollars, after $n$ quarters, $A_n$, can be modelled by the recurrence relation

$$A_0 = 30\,000, \qquad A_{n+1} = 1.016A_n - 1515.18$$

**b.** Showing recursive calculations, determine the balance of the loan after two quarters.
Round your answer to the nearest cent. 1 mark

**c.** The final repayment required will differ slightly from all the earlier repayments of $1515.18

Determine the value of the final repayment.
Round your answer to the nearest cent. 1 mark

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**Question 6** (4 marks)

Arthur invests $600 000 in an annuity that provides him with a monthly payment of $3973.00

Interest is calculated monthly.

Three lines of the amortisation table for this annuity are shown below.

| Payment number | Payment ($) | Interest ($) | Principal reduction ($) | Balance ($) |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 600 000.00 |
| 1 | 3973.00 | 2520.00 | 1453.00 | 598 547.00 |
| 2 | 3973.00 | 2513.90 | 1459.10 | 597 087.90 |

**a.** The interest rate for the annuity is 0.42% per month.

Determine the interest rate per annum. 1 mark

**b.** Using the values in the table, complete the next line of the amortisation table.

Write your answers in the spaces provided in the table below.

Round all values to the nearest cent. 1 mark

| Payment number | Payment ($) | Interest ($) | Principal reduction ($) | Balance ($) |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 600 000.00 |
| 1 | 3973.00 | 2520.00 | 1453.00 | 598 547.00 |
| 2 | 3973.00 | 2513.90 | 1459.10 | 597 087.90 |
| 3 | | | | |

**c.** Let $V_n$ be the balance of Arthur's annuity, in dollars, after $n$ months.

Write a recurrence relation in terms of $V_0$, $V_{n+1}$ and $V_n$ that can model the value of the annuity from month to month. 1 mark

**d.** The amortisation tables on page 11 show that the balance of the annuity reduces each month.

If the balance of an annuity remained constant from month to month, what name would be given to this type of annuity? 1 mark

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**Question 7** (5 marks)

Arthur takes out a new loan of $60 000 to pay for an overseas holiday.

Interest on this loan compounds weekly.

The balance of the loan, in dollars, after $n$ weeks, $V_n$, can be determined using a recurrence relation of the form

$$V_0 = 60\,000, \qquad V_{n+1} = 1.0015V_n - d$$

**a.** Show that the interest rate for this loan is 7.8% per annum. 1 mark

**b.** Determine the value of $d$ in the recurrence relation if

**i.** Arthur makes interest-only repayments 1 mark

**ii.** Arthur fully repays the loan in five years.

Round your answer to the nearest cent. 1 mark

**c.** Arthur decides that the value of $d$ will be 300 for the first year of repayments.

If Arthur fully repays the loan with exactly three more years of repayments, what new value of $d$ will apply for these three years?

Round your answer to the nearest cent. 1 mark

**d.** For what value of $d$ does the recurrence relation generate a geometric sequence? 1 mark

## Recursion and financial modelling

**Question 5** (4 marks)

Emi operates a mobile dog-grooming business.

The value of her grooming equipment will depreciate.

Based on average usage, a rule for the value, in dollars, of the equipment, $V_n$, after $n$ weeks is

$$V_n = 15\,000 - 60n$$

Assume that there are exactly 52 weeks in a year.

**a.** By what amount, in dollars, does the value of the grooming equipment depreciate each week? 1 mark

**b.** Emi plans to replace the grooming equipment after four years.

What will be its value, in dollars, at this time? 1 mark

**c.** $V_n$ is the value of the grooming equipment, in dollars, after $n$ weeks.

Write a recurrence relation in terms of $V_0$, $V_{n+1}$ and $V_n$ that can model this value from one week to the next. 1 mark

**d.** The value of the grooming equipment decreases from one year to the next by the same percentage of the original \$15 000 value.

What is this annual flat rate percentage? 1 mark

Do not write in this area.

**Question 6** (2 marks)

Emi invested profits of $10 000 into a savings account that earns interest compounding fortnightly, for one year.

The effective interest rate, rounded to two decimal places, is 5.07%.

Assume that there are exactly 26 fortnights in a year.

**a.** What is the nominal percentage rate of interest for the account?

Round your answer to two decimal places. 1 mark

**b.** Explain why the nominal interest rate appears lower than the effective interest rate. 1 mark

Do not write in this area.

**Question 7** (4 marks)

Emi decides to invest a $300 000 inheritance into an annuity.

Let $E_n$ be the balance of Emi's annuity after $n$ months.

A recurrence relation that can model the value of this balance from month to month is

$$E_0 = 300\,000, \qquad E_{n+1} = 1.003E_n - 2159.41$$

**a.** Showing recursive calculations, determine the balance of the annuity after two months. Round your answer to the nearest cent. 1 mark

**b.** For how many years will Emi receive the regular payment? 1 mark

**c.** Calculate the annual compound interest rate for this annuity. 1 mark

**d.** If Emi wanted the annuity to act as a perpetuity, what monthly payment, in dollars, would she receive? 1 mark

Do not write in this area.

## Recursion and financial modelling

**Question 7** (5 marks)

Cleo took out a reducing balance loan to buy an apartment.

Interest on this loan is charged monthly and the loan is scheduled to be repaid in full with monthly repayments over $20$ years.

The balance of Cleo's loan, in dollars, after $n$ months, $C_n$, can be modelled by the recurrence relation

$$C_0 = 560\,000 \qquad C_{n+1} = 1.005C_n - 4012$$

**a.** What amount, in dollars, did Cleo borrow? 1 mark

**b.** Determine the total value, in dollars, of the repayments made by Cleo in the first year of the loan. 1 mark

**c.** The interest rate for Cleo's loan is $6\%$ per annum.

Use this value in a calculation to show that the multiplication factor in the recurrence relation is $1.005$ 1 mark

**d.** Complete the next line in the amortisation table.

Write your answers in the spaces provided in the table below. 1 mark

| Payment number | Repayment ($) | Interest ($) | Principal reduction ($) | Balance ($) |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 560 000.00 |
| 1 | | | | |

**e.** The final monthly repayment required to fully repay the loan to the nearest cent will be slightly higher than all previous payments.

Determine the value of this final repayment.

Round your answer to the nearest cent. 1 mark

Do not write in this area.

**Question 8** (4 marks)

Cleo owns equipment that was purchased for $\$50\,000$.

She depreciates the value of the equipment using the unit cost method.

Let $V_n$ be the value of the equipment, in dollars, after $n$ units of use.

A recurrence relation that can model this value from one unit of use to the next is given by

$$V_0 = 50\,000, \qquad V_{n+1} = V_n - k$$

**a.** What does $k$ represent in this recurrence relation? 1 mark

**b.** If $k = 12.50$, determine the value of the equipment after one year if it is used twice per day on all $365$ days of the year. 1 mark

Another option for Cleo is to depreciate the value of the $\$50\,000$ equipment using the reducing balance method.

The value of the equipment, in dollars, after $n$ months, $V_n$, can be modelled by a recurrence relation of the form

$$V_0 = 50\,000, \qquad V_{n+1} = RV_n$$

**c.** If the depreciation rate per month was $1.5\%$, what would be the value of $R$ in this recurrence relation? 1 mark

**d.** For what value of $R$ would the equipment be valued at $\$42\,868.75$ after three months? 1 mark

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Do not write in this area.

**Question 9** (3 marks)

Cleo took out a loan of $\$35\,000$ to pay for an overseas holiday.

Interest is charged at the rate of $10\%$ per annum compounding quarterly.

For the first year of this loan, Cleo made quarterly repayments of $\$1722$.

**a.** Let $V_n$ be the balance of Cleo's loan, in dollars, after $n$ quarters.

Write a recurrence relation in terms of $V_0$, $V_{n+1}$ and $V_n$ that can model the value of the loan from quarter to quarter for the first year. 1 mark

For the second year of the loan, Cleo increased her quarterly repayments to $\$2000$.

**b.** Determine the total amount of interest Cleo paid in the first two years of the loan.

Round your answer to the nearest cent. 2 marks

Do not write in this area.