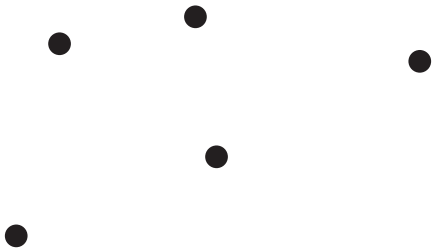
Networks and decision mathematics

Question 33

Consider the graph with five isolated vertices shown below.

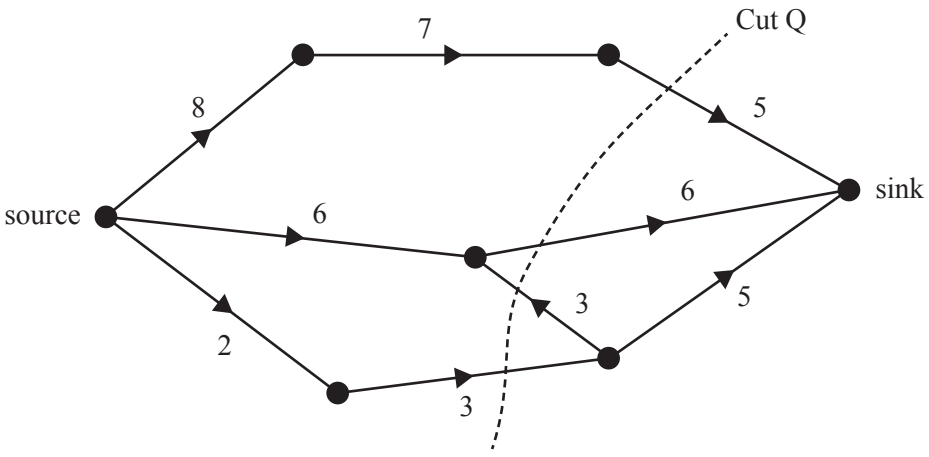


To form a tree, the minimum number of edges that must be added to the graph is

A. 1
B. 4
C. 5
D. 6
E. 10

Question 34

The flow of water through a series of pipes is shown in the network below.
The numbers on the edges show the maximum flow through each pipe in litres per minute.



The capacity of Cut Q, in litres per minute, is

A. 11
B. 13
C. 14
D. 16
E. 17

Question 35

A planar graph has five faces.

This graph could have

A. eight vertices and eight edges.

B. six vertices and eight edges.

C. eight vertices and five edges.

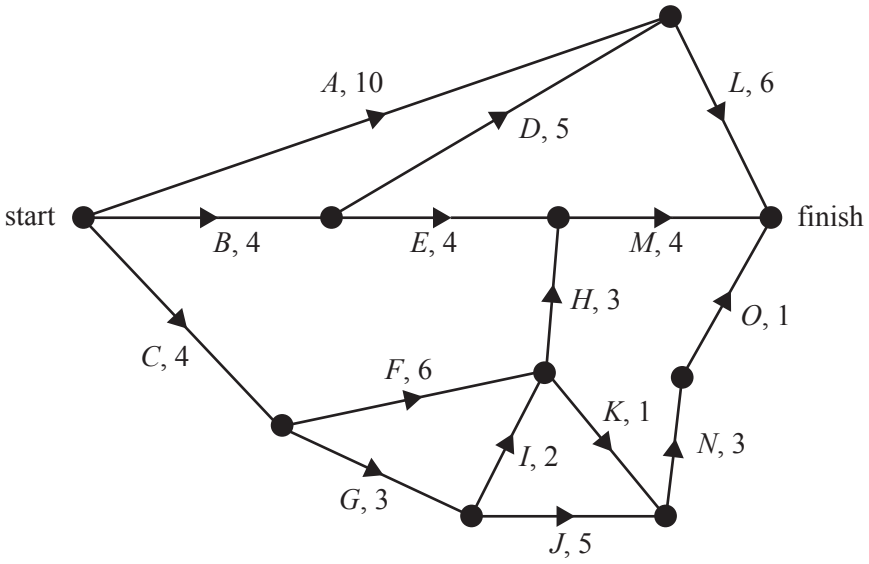
D. eight vertices and six edges.

E. five vertices and eight edges.

Use the following information to answer Questions 36 and 37.

The directed graph below shows the sequence of activities required to complete a project.

The time to complete each activity, in hours, is also shown.



Question 36

The earliest starting time, in hours, for activity *N* is

A. 3

B. 10

C. 11

D. 12

E. 13

Question 37

To complete the project in minimum time, some activities cannot be delayed.

The number of activities that cannot be delayed is

A. 2

B. 3

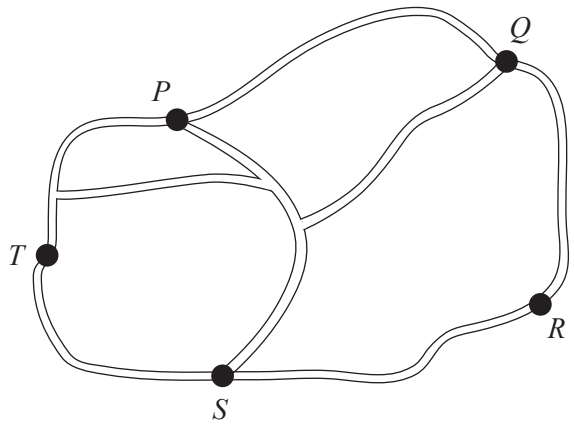
C. 4

D. 5

E. 6

Question 38

The map below shows all the road connections between five towns, P, Q, R, S and T.



The road connections could be represented by the adjacency matrix

A.

	P	Q	R	S	T
P	1	3	0	2	2
Q	3	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

B.

	P	Q	R	S	T
P	1	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

C.

	P	Q	R	S	T
P	0	3	0	2	2
Q	3	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

D.

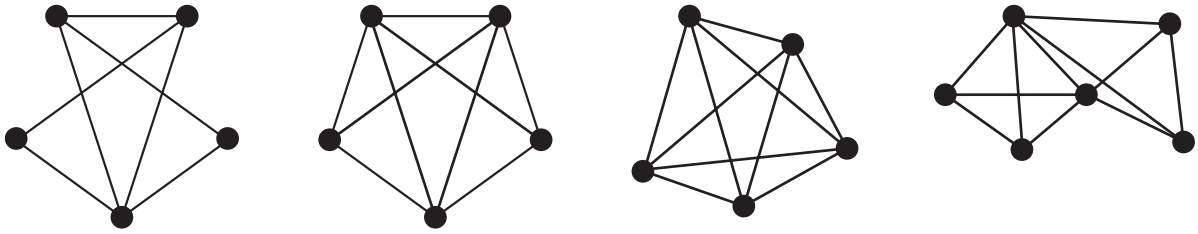
	P	Q	R	S	T
P	0	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	1	2
T	2	1	0	2	0

E.

	P	Q	R	S	T
P	1	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	1	1
T	2	1	0	1	0

Question 39

Consider the following four graphs.



How many of these four graphs are planar?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 40

Five children, Alan, Brianna, Chamath, Deidre and Ewen, are each to be assigned a different job by their teacher. The table below shows the time, in minutes, that each child would take to complete each of the five jobs.

	Alan	**Brianna**	**Chamath**	**Deidre**	**Ewen**
Job 1	5	8	5	8	7
Job 2	5	7	6	7	4
Job 3	9	5	7	5	9
Job 4	7	7	9	8	5
Job 5	4	4	4	4	3

The teacher wants to allocate the jobs so as to minimise the total time taken to complete the five jobs.

In doing so, she finds that two allocations are possible.

If each child starts their allocated job at the same time, then the first child to finish could be either

A. Alan or Brianna.

B. Brianna or Deidre.

C. Chamath or Deidre.

D. Chamath or Ewen.

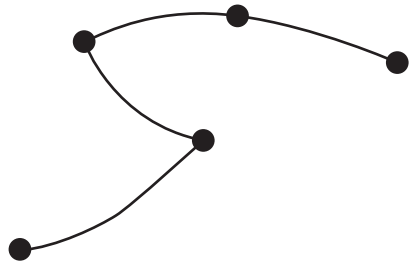
E. Deidre or Ewen.

END OF MULTIPLE-CHOICE QUESTION BOOK

Networks and decision mathematics

Question 33

Consider the following graph.



How many of the following five statements are true?

- The graph is a tree.
- The graph is connected.
- The graph contains a path.
- The graph contains a cycle.
- The sum of the degrees of the vertices is eight.

A. 1
B. 2
C. 3
D. 4
E. 5

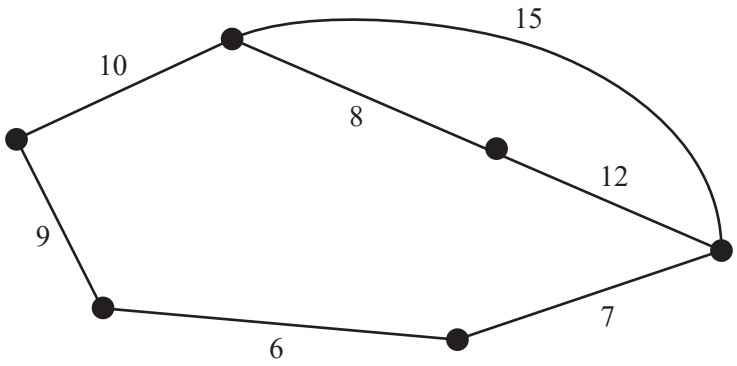
Question 34

A bipartite graph is typically used to display which one of the following?

A. the allocation of tasks on a construction site
B. the path used to visit five different construction sites
C. the total distance travelled between two construction sites
D. the critical path of activities to be completed in a construction project
E. the minimum length of cable required to connect six construction sites

Question 35

Consider the weighted graph shown below.



The weight of the minimum spanning tree is

A. 30

B. 32

C. 40

D. 42

E. 52

Question 36

Four employees, Anthea, Bob, Cho and Dario, are each assigned a different duty by their manager.

The time taken for each employee to complete duties 1, 2, 3 and 4, in minutes, is shown in the table below.

	Duty 1	**Duty 2**	**Duty 3**	**Duty 4**
Anthea	8	7	7	8
Bob	10	8	10	9
Cho	8	9	7	10
Dario	7	7	8	9

The manager allocates the duties so as to minimise the total time taken to complete the four duties.

The minimum total time taken to complete the four duties, in minutes, is

A. 29

B. 30

C. 31

D. 32

E. 33

Question 37

The adjacency matrix below represents a planar graph with five vertices.

$$\begin{array}{c} \begin{matrix} J & K & L & M & N \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 0 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} J \\ K \\ L \\ M \\ N \end{matrix} \end{array}$$

The number of faces on the planar graph is

A. 5

B. 7

C. 9

D. 15

E. 17

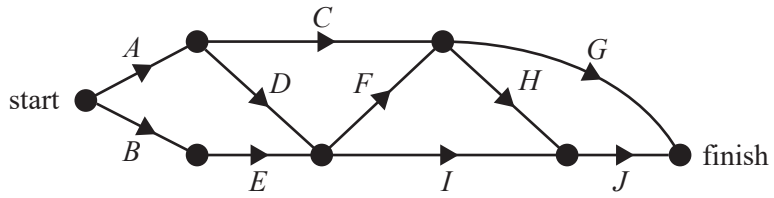
Question 38

A particular building project has ten activities that must be completed.
These activities and their immediate predecessor(s) are shown in the table below.

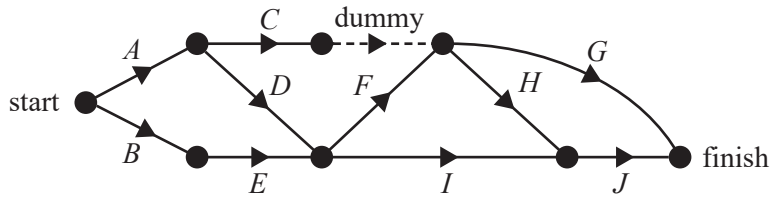
Activity	Immediate predecessor(s)
A	–
B	–
C	*A*
D	*A*
E	*B*
F	*D*, *E*
G	*C*, *F*
H	*F*
I	*D*, *E*
J	*H*, *I*

A directed graph that could represent this project is

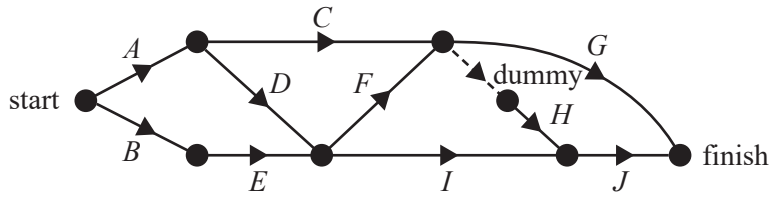
A.



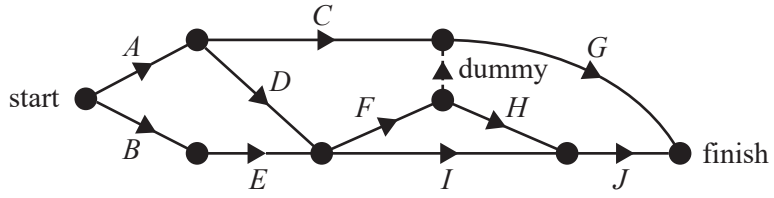
B.



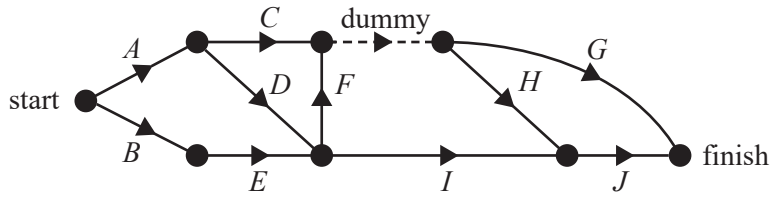
C.



D.

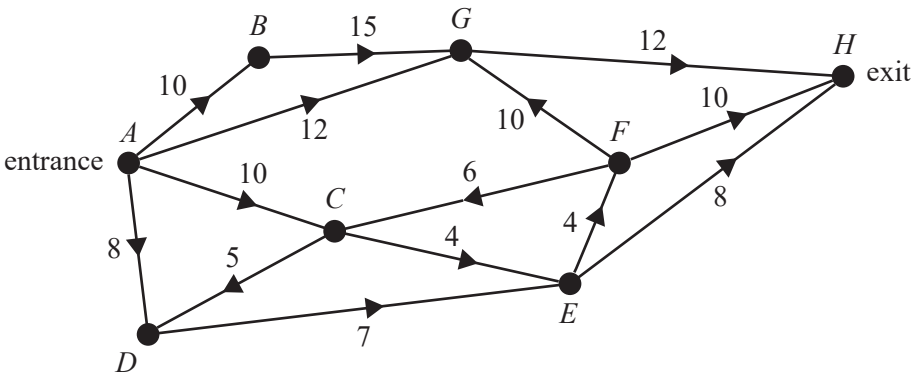


E.



Use the following information to answer Questions 39 and 40.

The network below shows the one-way paths between the entrance, *A*, and the exit, *H,* of a children's maze.
The vertices represent the intersections of the one-way paths.
The number on each edge is the maximum number of children who are allowed to travel along that path per minute.



Question 39

Cuts on this network are used to consider the possible flow of children through the maze.
The capacity of the minimum cut would be

A. 20
B. 23
C. 24
D. 29
E. 30

Question 40

One path in the maze is to be changed.
Which one of these five changes would lead to the largest increase in flow from entrance to exit?

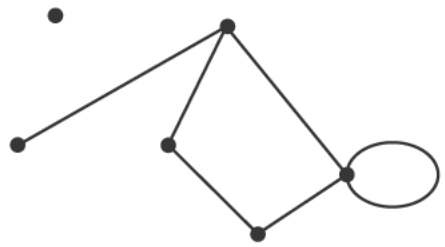
A. increasing the capacity of flow along the edge *CE* to 12
B. increasing the capacity of flow along the edge *FH* to 14
C. increasing the capacity of flow along the edge *GH* to 16
D. reversing the direction of flow along the edge *CF*
E. reversing the direction of flow along the edge *GF*

END OF MULTIPLE-CHOICE QUESTION BOOK

Networks and decision mathematics

Question 33

Consider the following graph.



The sum of the degrees of the vertices is

A. 10

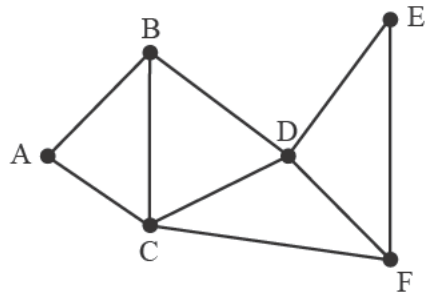
B. 11

C. 12

D. 13

Question 34

Consider the following graph.



A Eulerian trail through this graph could be

A. ABCDEF

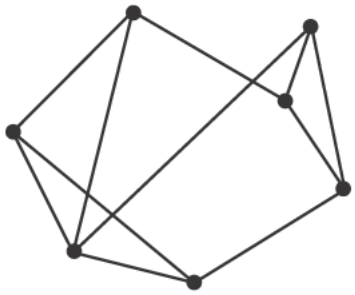
B. ACBDCFDEF

C. BACBDCFDEF

D. BDCABCDFCDEF

Question 35

Consider the following graph.



The number of faces is

A. 5

B. 6

C. 7

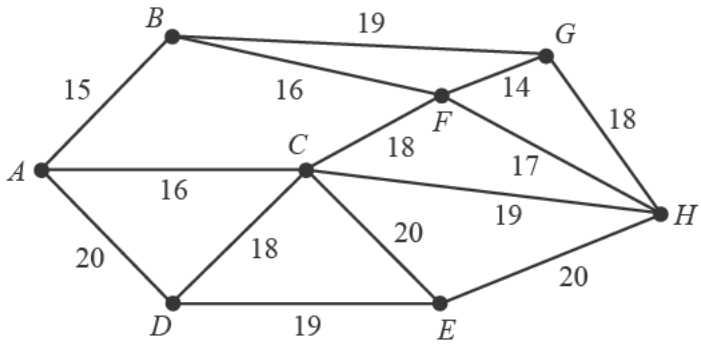
D. 8

Question 36

Eight houses in an estate are to be connected to the internet via underground cables.
The network below shows the possible connections between the houses.

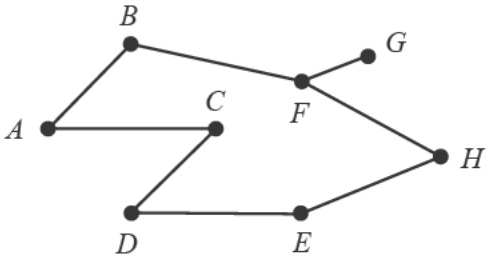
The vertices represent the houses.

The numbers on the edges represent the length of cable connecting pairs of houses, in metres.

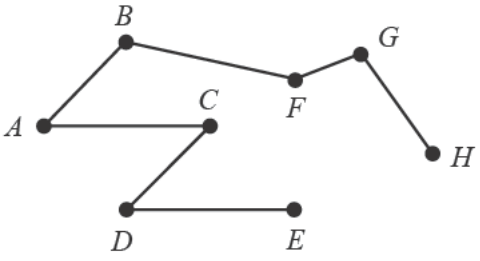


The graph that represents the minimum length of cable needed to connect all the houses is

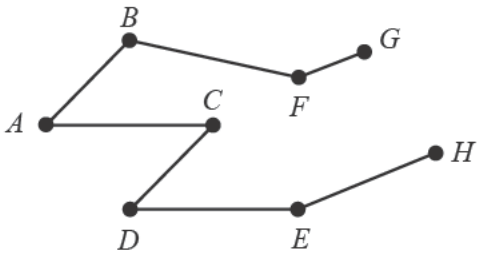
A.



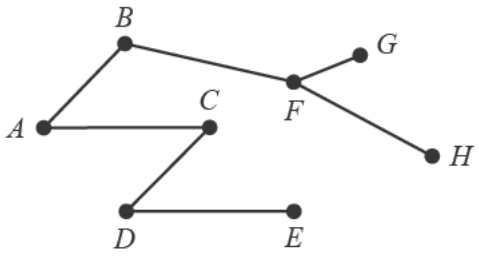
B.



C.



D.

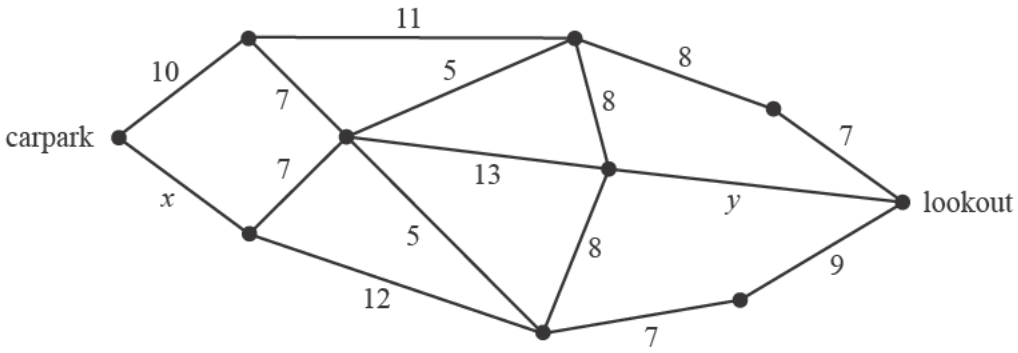


Do not write in this area.

Question 37

The network below represents paths through a park from the carpark to a lookout.

The vertices represent various attractions, and the numbers on the edges represent the distances between them in metres.



The shortest path from the carpark to the lookout is 34 m.

This can be achieved when

A. $x = 8$ and $y = 8$

B. $x = 9$ and $y = 7$

C. $x = 10$ and $y = 6$

D. $x = 11$ and $y = 5$

Question 38

A connected graph has six vertices and six edges.

How many of the following four statements must **always** be true?

- the graph has no vertices of odd degree
- the graph contains a Eulerian trail
- the graph contains a Hamiltonian path
- the sum of the degrees of the vertices is 12

A. 1

B. 2

C. 3

D. 4

Question 39

Anush, Blake, Carly and Dexter are workers on a construction site. They are each allocated one task.

The time, in hours, it takes for each worker to complete each task is shown in the table below.

	Task 1	**Task 2**	**Task 3**	**Task 4**
Anush	12	8	16	9
Blake	10	7	15	10
Carly	11	10	18	12
Dexter	10	14	16	11

The tasks must be completed sequentially and in numerical order: Task 1, Task 2, Task 3 and then Task 4.

Management makes an initial allocation of tasks to minimise the amount of time required, but then decides that it takes the workers too long.

Another worker, Edgar, is brought in to complete one of the tasks.

His completion times, in hours, are listed below.

	Task 1	**Task 2**	**Task 3**	**Task 4**
Edgar	9	5	14	8

When a new allocation is made and Edgar takes over one of the tasks, the minimum total completion time compared to the initial allocation will be reduced by

A. 1 hour.

B. 2 hours.

C. 3 hours.

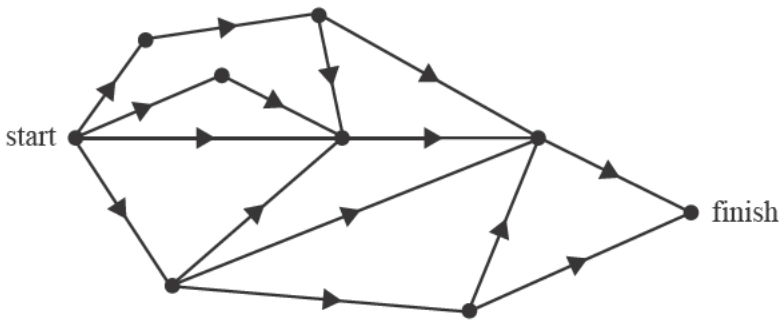
D. 4 hours.

Question 40

A project has 15 activities, *A*–*O*, that need to be completed.

The directed network that represents this project is shown below.

The activities are not labelled.



The activity table that could represent this project is

A.

Activity	Immediate predecessor(s)
A	–
B	–
C	–
D	–
E	*A*
F	*B*
G	*E, F, C, H*
H	*D*
I	*A*
J	*D*
K	*H*
L	*D*
M	*L*
N	*I, G, K, L*
O	*L*

B.

Activity	Immediate predecessor(s)
A	*C*
B	*A*
C	–
D	–
E	*A*
F	*B, K, M, L*
G	*D*
H	–
I	–
J	*I*
K	*E, G, H, J*
L	*N*
M	*I*
N	*I*
O	*N*

Do not write in this area.

C.

Activity	Immediate predecessor(s)
A	*C*
B	*A*
C	–
D	–
E	*A*
F	*B, K, M, L*
G	*D*
H	–
I	–
J	*I*
K	*D, G, H, J*
L	*N*
M	*I*
N	*I*
O	*N*

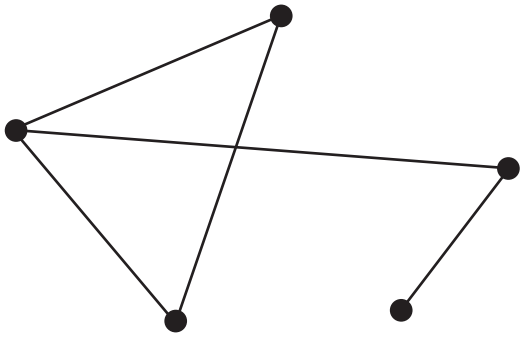
D.

Activity	Immediate predecessor(s)
A	*C*
B	*A*
C	–
D	–
E	*A*
F	*B, K, J, L*
G	*D*
H	–
I	–
J	*I*
K	*E, G, H, J*
L	*N*
M	*N*
N	*I*
O	*I*

End of Multiple-Choice Question Book

Networks and decision mathematics

Question 33



In the graph shown above, the number of vertices of even degree is:

A. 0

B. 1

C. 2

D. 3

E. 4

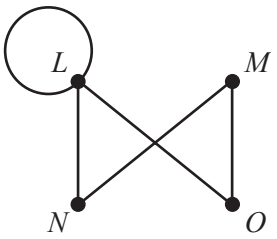
Question 34

The adjacency matrix below represents the road connections between four towns, labelled L, M, N and O.

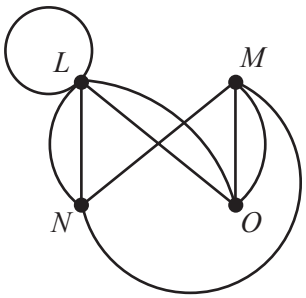
$$\begin{array}{c} \\ L \\ M \\ N \\ O \end{array} \begin{array}{c} \begin{array}{cccc} L & M & N & O \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

A graph that represents all the road connections in the adjacency matrix is:

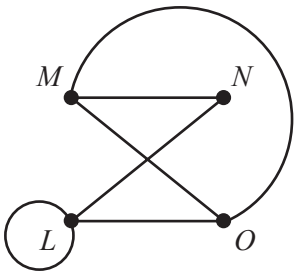
A.



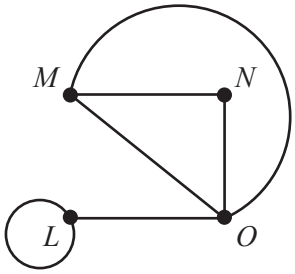
B.



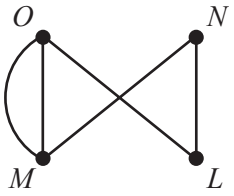
C.



D.



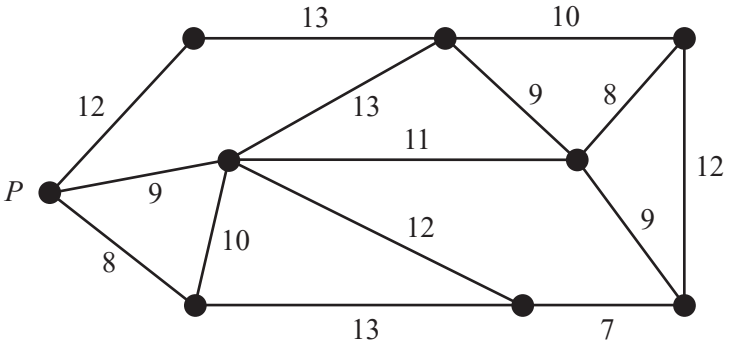
E.



Do not write in this area.

Question 35

The vertices of the graph below represent cabins in a holiday park, and the water pump (P) that will supply them. The numbers on the edges show the length, in metres, of water pipe required to connect the cabins and the pump.



The water pipes will cost $\$52$ per metre. What is the minimum cost to link all the cabins to the water pump (P)?

A. $\$3744$

B. $\$3796$

C. $\$3848$

D. $\$3900$

E. $\$3952$

Question 36

Four students, Peggy, Quincy, Radley and Sarah, are grouped together to complete a project. The project is in four parts, labelled W, X, Y and Z. Each student must complete one part of the project.

The table below shows each student's estimate of the score they will receive if they complete each section.

	Peggy	Quincy	Radley	Sarah
W	12	19	18	16
X	16	15	15	16
Y	10	16	17	15
Z	19	20	18	18

Based on the estimates, which allocation of project parts will maximise the students' group score on the project?

A.

W	Quincy
X	Sarah
Y	Radley
Z	Peggy

B.

W	Radley
X	Peggy
Y	Quincy
Z	Sarah

C.

W	Sarah
X	Quincy
Y	Peggy
Z	Radley

D.

W	Radley
X	Peggy
Y	Sarah
Z	Quincy

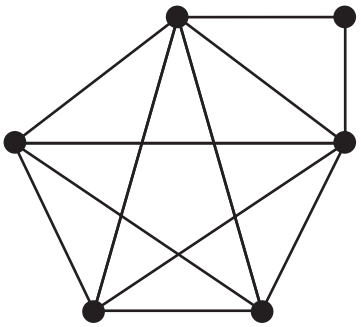
E.

W	Sarah
X	Peggy
Y	Radley
Z	Quincy

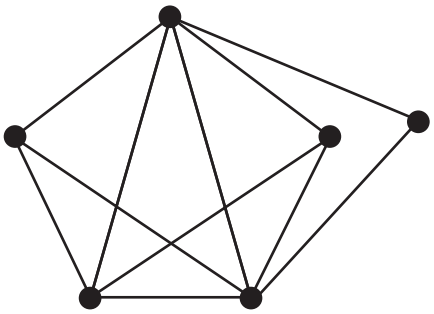
Question 37

Euler's formula can be applied to which of the following graphs?

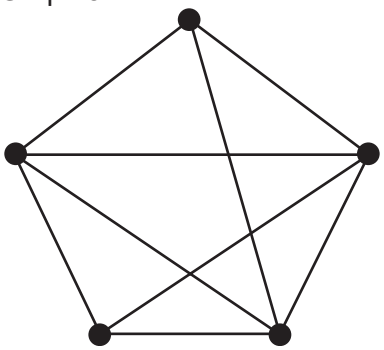
Graph 1



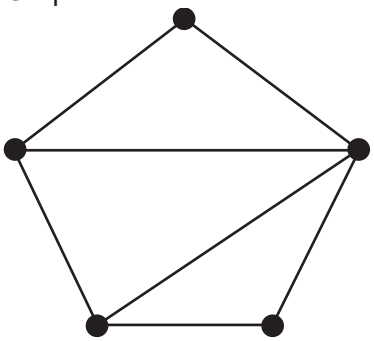
Graph 2



Graph 3



Graph 4



A. Graph 4 only

B. Graphs 1 and 2 only

C. Graphs 1, 2, 3 and 4

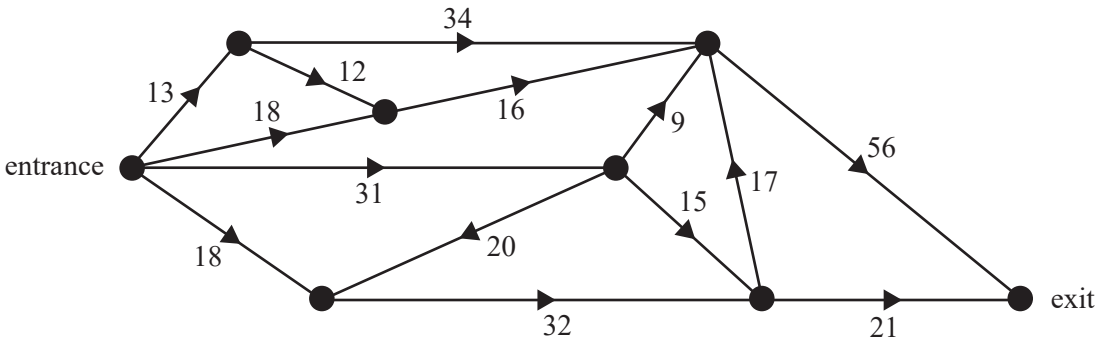
D. Graphs 3 and 4 only

E. Graphs 2, 3 and 4 only

Use the following information to answer Questions 38 and 39.

The following directed graph represents the one-way paths between attractions at an historical site. The entrance, exit and attractions are represented by vertices.

The numbers on the edges represent the maximum number of visitors allowed along each path per hour.



Question 38

What is the maximum number of visitors able to walk from the entrance to the exit each hour?

A. 75

B. 76

C. 77

D. 88

E. 96

Question 39

A group of students set out from the entrance and walk to the exit. The students all walk together and travel along the same route. They are the only people visiting the site that hour. What is the maximum number of students that could be in the group?

A. 9

B. 15

C. 16

D. 20

E. 31

Question 40

A project has 10 activities, labelled A to J. The table below shows the immediate predecessor(s) for each activity. Each activity has a duration of at least one day.

Activity	Immediate predecessor(s)
A	–
B	–
C	A
D	B
E	B
F	D
G	D, E
H	C, F
I	E
J	G, H

Which **one** of the following statements about this project is **not** true?

A. The earliest starting time of activity H could be two days.

B. In the network for this project, there would be a dummy activity from the end of activity D to the start of activity G.

C. One of the paths through the network of this project is $BDGJ$.

D. The latest starting time of activity E could be three days.

E. The network for this project would require two dummy activities.

End of examination questions