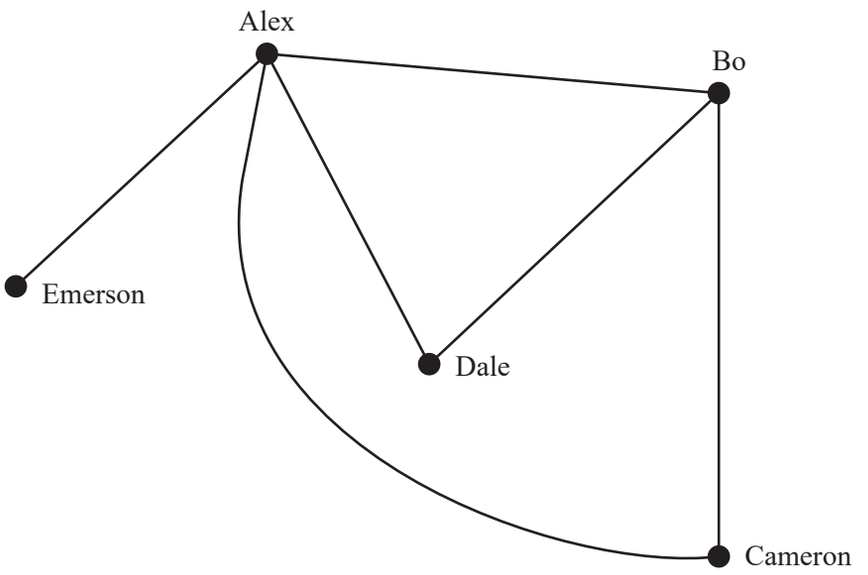
DO NOT WRITE IN THIS AREA

Networks and decision mathematics

Question 14 (2 marks)

The Sunny Coast Cricket Club has five new players join its team: Alex, Bo, Cameron, Dale and Emerson.

The graph below shows the players who have played cricket together before joining the team. For example, the edge between Alex and Bo shows that they have previously played cricket together.



a. How many of these players had Emerson played cricket with before joining the team? 1 mark

b. Who had played cricket with both Alex and Bo before joining the club? 1 mark

Question 15 (1 mark)

A cricket team has 11 players, who are each assigned to a batting position.

Three of the new players, Alex, Bo and Cameron, can bat in position 1, 2 or 3.

The table below shows the average scores, in runs, for each player for the batting positions 1, 2 and 3.

Player	Batting position 1	Batting position 2	Batting position 3
Alex	22	24	24
Bo	25	25	21
Cameron	24	25	19

Each player will be assigned to one batting position.

To which position should each player be assigned to maximise the team's score? Write your answer in the table below.

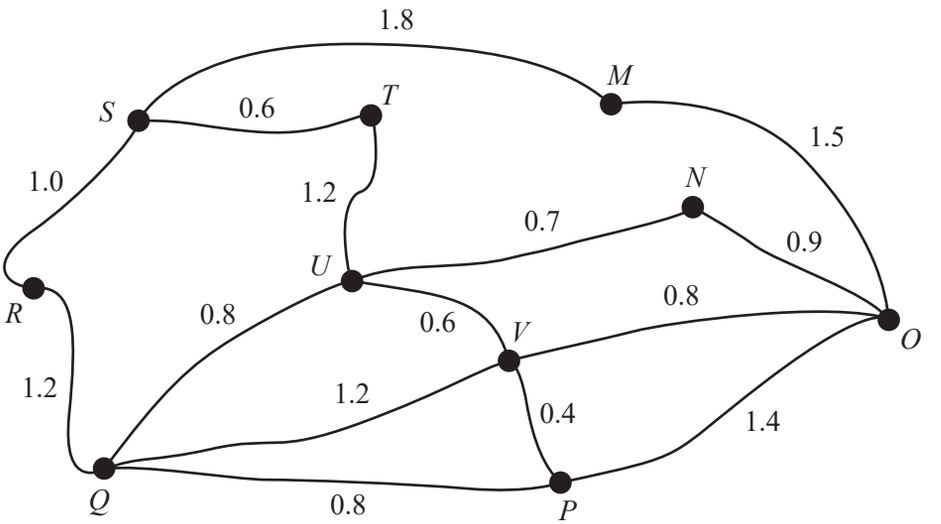
Player	Batting position
Alex	
Bo	
Cameron	

Question 16 (3 marks)

A local fitness park has 10 exercise stations: *M* to *V*.

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



The Sunny Coast cricket coach designs three different training programs, all starting at exercise station *S*.

Training program number	Training details
1	The team must run to exercise station *O*.
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station *S*.

a. What is the shortest distance, in kilometres, covered in training program 1? 1 mark

b. At which exercise station would training program 2 finish? 1 mark

c. To complete training program 3 in the minimum distance, one track will need to be repeated.
Complete the following sentence by filling in the boxes provided. 1 mark

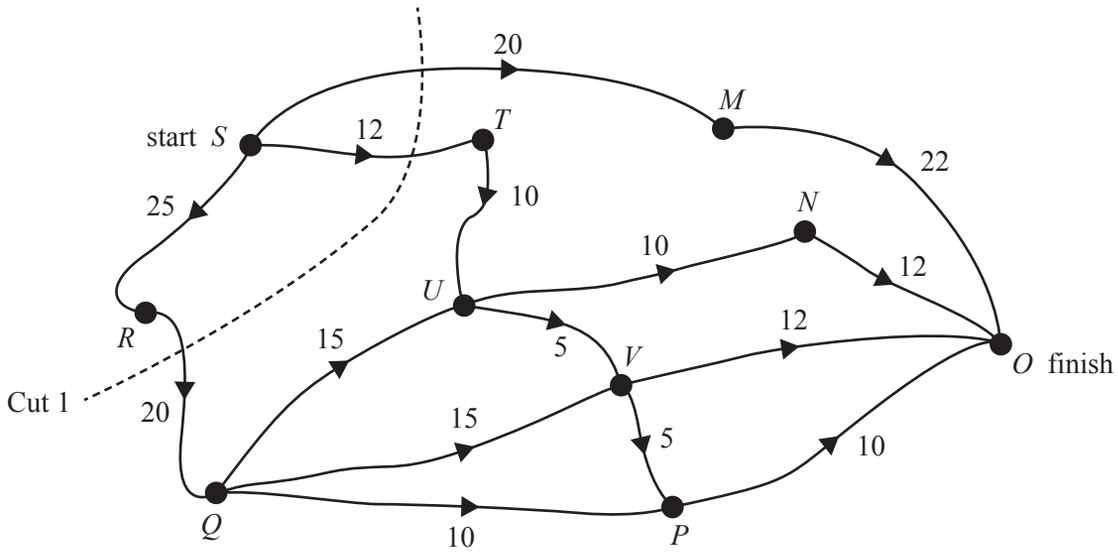
This track is between exercise station ☐ and exercise station ☐.

Question 17 (2 marks)

Training program 1 has the cricket team starting from exercise station *S* and running to exercise station *O*.

For safety reasons, the cricket coach has placed a restriction on the maximum number of people who can use the tracks in the fitness park.

The directed graph below shows the capacity of the tracks, in number of people per minute.



When considering the possible flow of people through this network, many different cuts can be made.

a. Determine the capacity of Cut 1, shown above. 1 mark

b. What is the maximum flow from *S* to *O*, in number of people per minute? 1 mark

DO NOT WRITE IN THIS AREA

Question 18 (4 marks)

The Sunny Coast cricket clubrooms and oval are undergoing a major works project.

This project involves nine activities: *A* to *I*.

The table below shows the earliest start time (EST) and duration, in months, for each activity.

The immediate predecessor(s) is also shown.

The duration for activity *C* is missing.

Activity	EST	Duration	Immediate predecessor(s)
A	0	2	–
B	0	5	–
C	5		*A*, *B*
D	7	7	*C*
E	7	9	*C*
F	5	3	*B*
G	14	4	*D*
H	8	9	*F*
I	18	2	*E*, *G*, *H*

The information in the table above can be used to complete a directed network.

This network will require a dummy activity.

a. Complete the following sentence by filling in the boxes provided. 1 mark

This dummy activity could be drawn as a directed edge from the end of activity

[ ] to the start of activity [ ] .

b. What is the duration, in months, of activity *C*? 1 mark

c. Name the four activities that have a float time of at least one month. 1 mark

d. The project is to be crashed by reducing the completion time of one activity only.

What is the minimum time, in months, in which the project can be completed? 1 mark

END OF QUESTION AND ANSWER BOOK

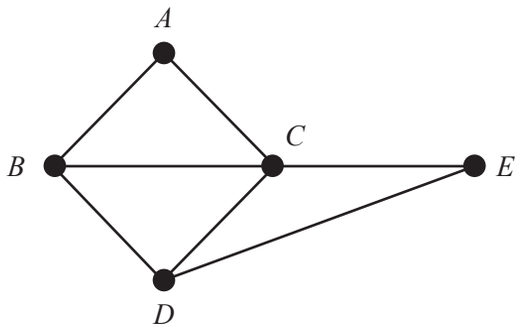
Networks and decision mathematics

Question 12 (4 marks)

A country has five states, *A, B, C, D* and *E*.

A graph can be drawn with vertices to represent each of the states.

Edges represent a border shared between two states.



a. What is the sum of the degrees of the vertices of the graph above? 1 mark

b. Euler's formula, $v + f = e + 2$, holds for this graph.

i. Complete the formula by writing the appropriate numbers in the boxes provided below. 1 mark

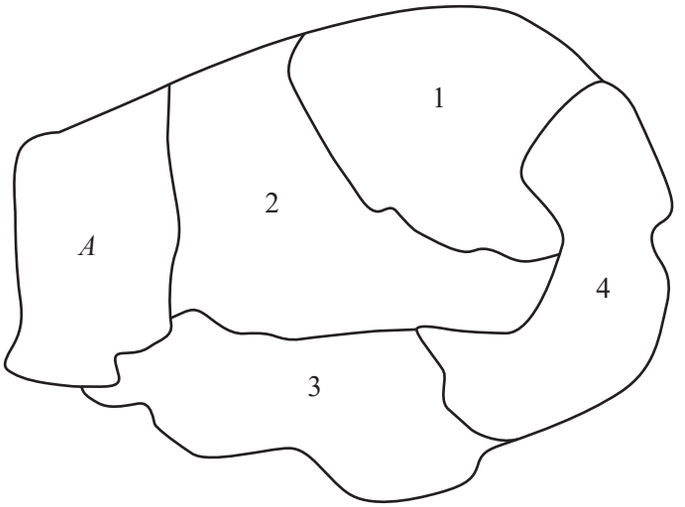
	+		=		+	2
v		*f*		*e*		

ii. Complete the sentence by writing the appropriate word in the box provided below. 1 mark

Euler's formula holds for this graph because the graph is connected and ☐ .

DO NOT WRITE IN THIS AREA

c. The diagram below shows the position of state *A* on a map of this country.
The four other states are indicated on the diagram as 1, 2, 3 and 4.



Use the information in the graph on page 20 to complete the table below. Match the state (*B*, *C*, *D* and *E*) with the corresponding state number (1, 2, 3 and 4) given in the map above. 1 mark

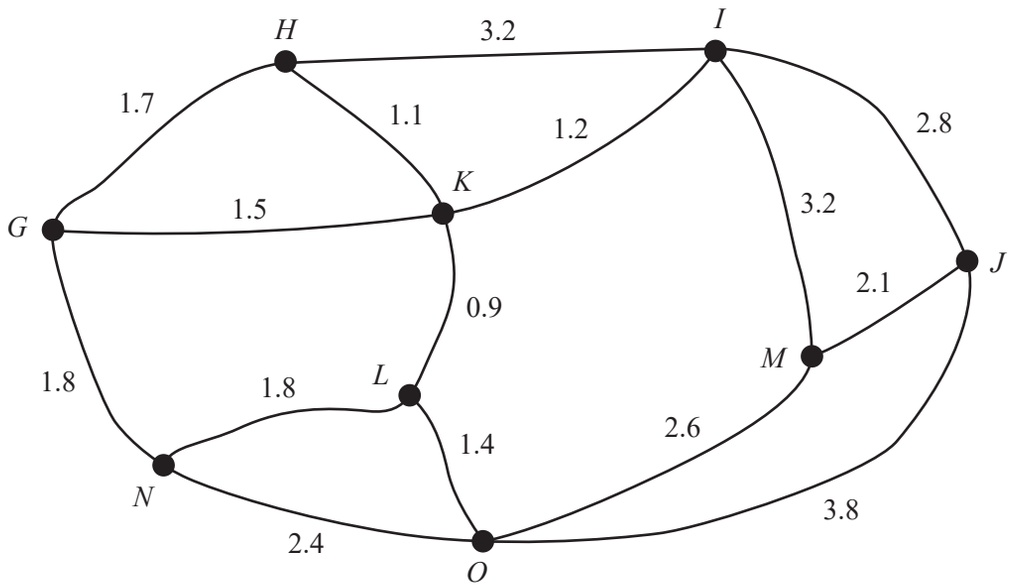
State	State number
B	
C	
D	
E	

Question 13 (3 marks)

The state *A* has nine landmarks, *G, H, I, J, K, L, M, N* and *O*.

The edges on the graph represent the roads between the landmarks.

The numbers on each edge represent the length, in kilometres, along each road.



Three friends, Eden, Reynold and Shyla, meet at landmark *G*.

a. Eden would like to visit landmark *M*.

What is the minimum distance Eden could travel from *G* to *M*? 1 mark

b. Reynold would like to visit all the landmarks and return to *G*.

Write down a route that Reynold could follow to minimise the total distance travelled. 1 mark

c. Shyla would like to travel along all the roads.
To complete this journey in the minimum distance, she will travel along two roads twice.
Shyla will leave from landmark *G* but end at a different landmark.

Complete the following by filling in the boxes provided. 1 mark

The two roads that will be travelled along twice are the roads between:

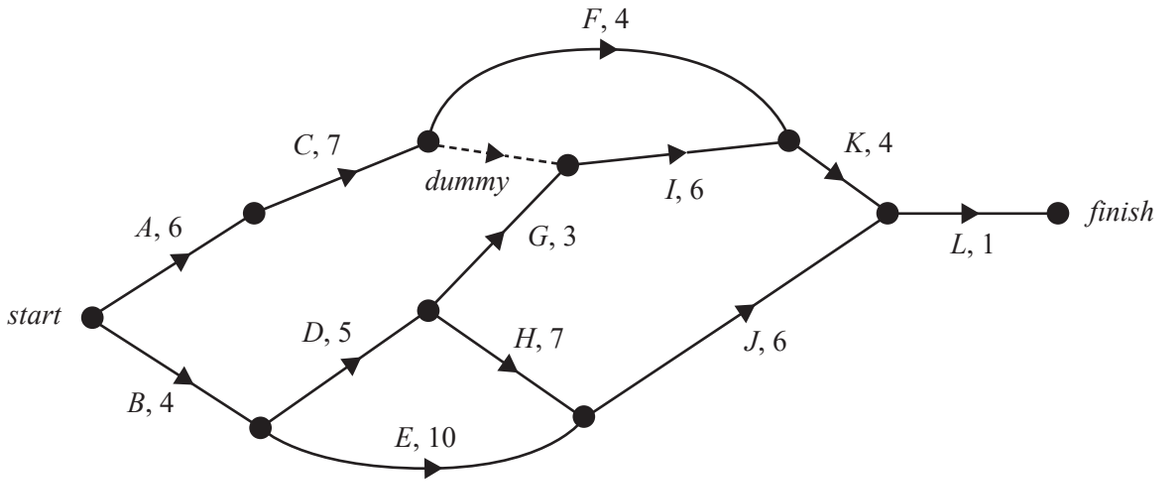
- vertex ☐ and vertex ☐
- vertex ☐ and vertex ☐

DO NOT WRITE IN THIS AREA

Question 14 (5 marks)

One of the landmarks in state *A* requires a renovation project.

This project involves 12 activities, *A* to *L*. The directed network below shows these activities and their completion times, in days.



The table below shows the 12 activities that need to be completed for the renovation project.

It also shows the earliest start time (EST), the duration, and the immediate predecessors for the activities.

The immediate predecessor(s) for activity *I* and the EST for activity *J* are missing.

Activity	EST	Duration	Immediate predecessor(s)
A	0	6	–
B	0	4	–
C	6	7	*A*
D	4	5	*B*
E	4	10	*B*
F	13	4	*C*
G	9	3	*D*
H	9	7	*D*
I	13	6	
J		6	*E, H*
K	19	4	*F, I*
L	23	1	*J, K*

DO NOT WRITE IN THIS AREA

a. Write down the immediate predecessor(s) for activity *I*. 1 mark

b. What is the earliest start time, in days, for activity *J*? 1 mark

c. How many activities have a float time of zero? 1 mark

The managers of the project are able to reduce the time, in days, of six activities.
These reductions will result in an increase in the cost of completing the activity.
The maximum decrease in time of any activity is two days.

Activity	*A*	*B*	*F*	*H*	*I*	*K*
Daily cost ($)	1500	2000	2500	1000	1500	3000

d. If activities *A* and *B* have their completion time reduced by two days each, the overall completion time of the project will be reduced.

What will be the maximum reduction time, in days? 1 mark

e. The managers of the project have a maximum budget of $15 000 to reduce the time for several activities to produce the maximum reduction in the project's overall completion time.

Complete the table below, showing the reductions in individual activity completion times that would achieve the earliest completion time within the $15 000 budget. 1 mark

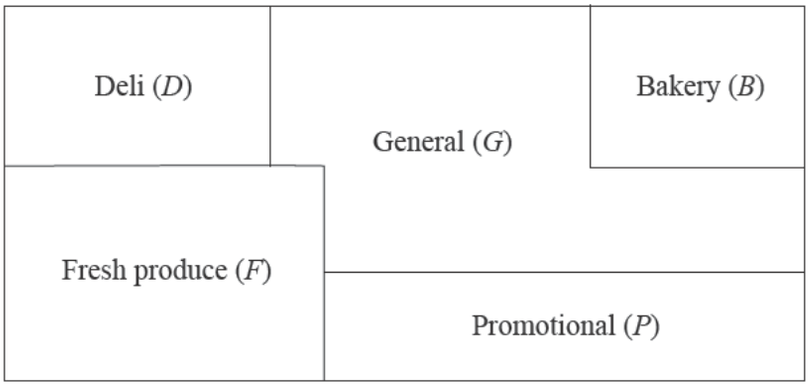
Activity	**Reduction in completion time (0, 1 or 2 days)**
A	
B	
F	
H	
I	
K	

END OF QUESTION AND ANSWER BOOK

Networks and decision mathematics

Question 13 (4 marks)

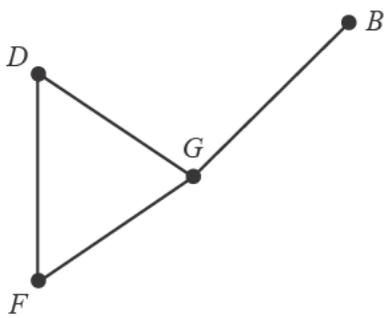
A supermarket has five departments, with areas allocated as shown on the floorplan below.



The floorplan is represented by the graph below.

On this graph, vertices represent departments and edges represent boundaries between two departments.

This graph is incomplete.



a. Draw the missing vertex and missing edges on the graph above. Include a label. 1 mark

Karla is standing in the Promotional department.

She wants to visit each department in the supermarket once only.

b. **i.** In which department will she finish? 1 mark

---

ii. What is the mathematical name for this type of journey? 1 mark

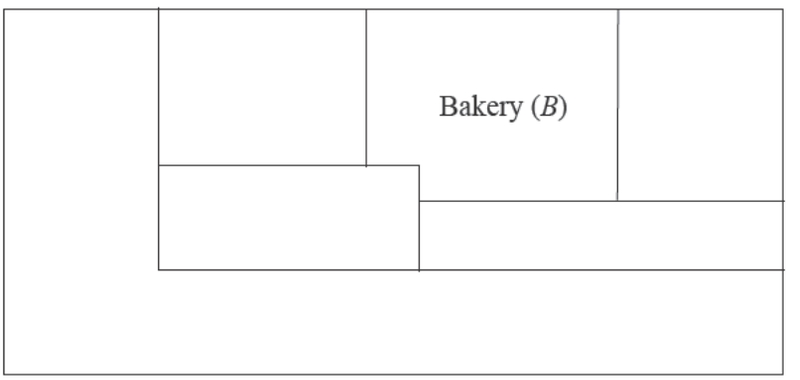
---

c. The supermarket adds a new Entertainment department (E), and the floorplan is rearranged.

The boundaries between the departments are represented in the adjacency matrix below, where a '1' indicates a boundary between the departments.

$$\begin{array}{c} \\ B \\ D \\ E \\ F \\ G \\ P \end{array} \begin{array}{c} \begin{array}{cccccc} B & D & E & F & G & P \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Use the adjacency matrix to complete the floorplan below by labelling each department. The Bakery (B) is already labelled. 1 mark



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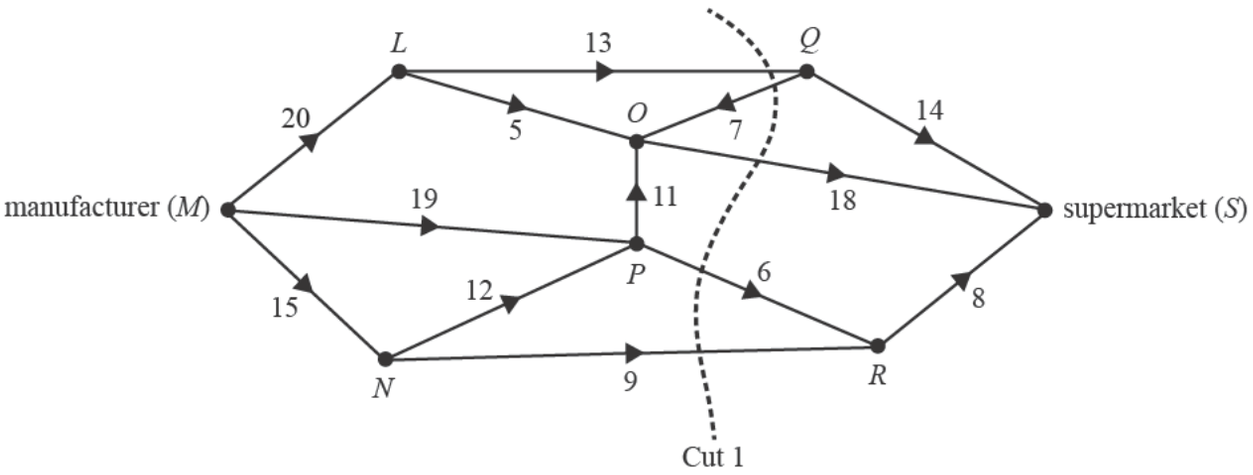
Question 14 (3 marks)

A manufacturer (M) makes deliveries to the supermarket (S) via a number of storage warehouses, L, N, O, P, Q and R. These eight locations are represented as vertices in the network below.

The numbers on the edges represent the maximum number of deliveries that can be made between these locations each day.



a. When considering the possible flow of deliveries through this network, many different cuts can be made.

Determine the capacity of Cut 1, shown above. 1 mark

---

b. Determine the maximum number of deliveries that can be made each day from the manufacturer to the supermarket. 1 mark

---

c. The manufacturer wants to increase the number of deliveries to the supermarket.

This can be achieved by increasing the number of deliveries between one pair of locations.

Complete the following sentence by writing the locations in the boxes provided:

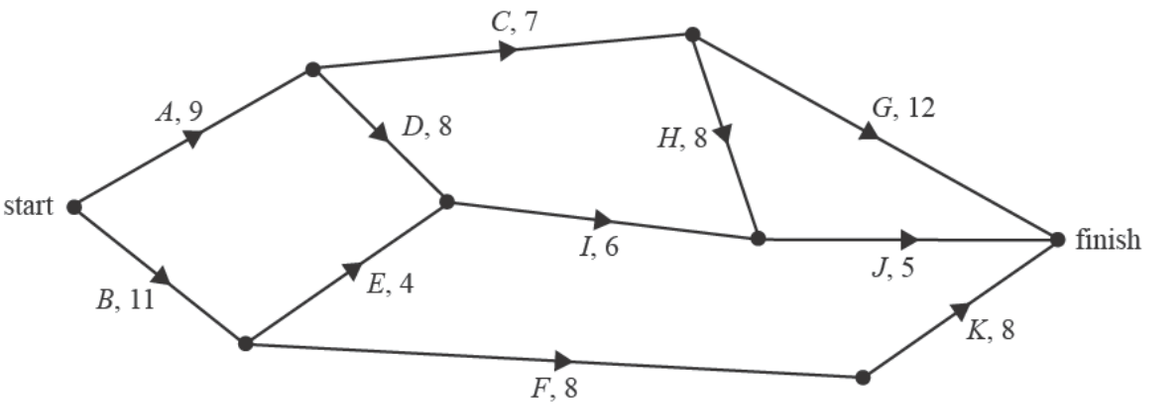
To maximise this increase, the number of deliveries should be increased between locations ☐ and ☐. 1 mark

Question 15 (5 marks)

An upgrade to the supermarket requires the completion of 11 activities, *A* to *K*.

The directed network below shows these activities and their completion time, in weeks.

The minimum completion time for the project is 29 weeks.



a. Write down the critical path. 1 mark

b. Which activity can be delayed for the longest time without affecting the minimum completion time of the project? 1 mark

Question 15 continues on the next page.

Use the following information to answer parts c–e.

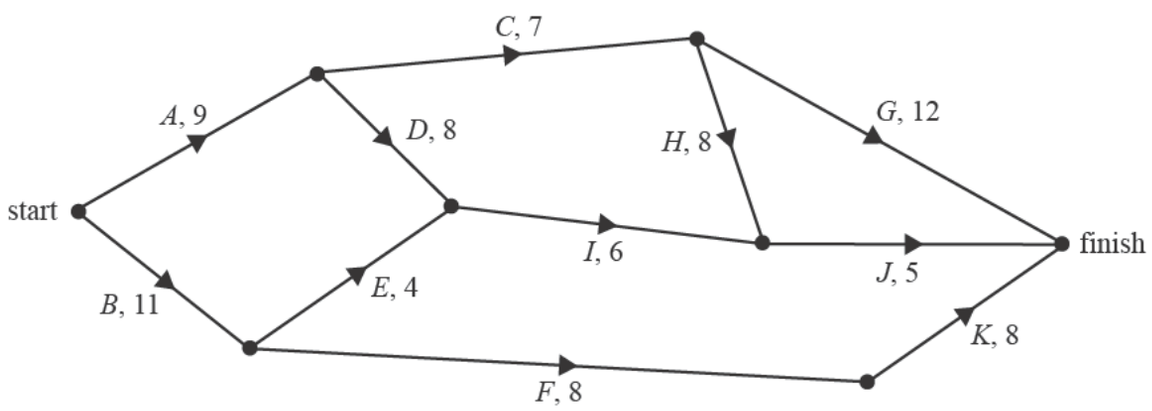
A change is made to the order of activities.

The table below shows the activities and their new latest starting times in weeks.

Activity	Latest starting time (weeks)
A	0
B	2
C	10
D	9
E	13
F	14
G	18
H	17
I	19
J	25
K	22

A dummy activity is now required in the network.

c. On the directed network below, draw a directed edge to represent the dummy activity. Include a label. 1 mark



d. What is the new minimum completion time of the project? 1 mark

_____

e. The owners of the supermarket want the project completed earlier.

They will pay to reduce the time of some of the activities.

A reduction in completion time of an activity will incur an additional cost of $10 000 per week.

Activities can be reduced by a maximum of two weeks.

The minimum number of weeks an activity can be reduced to is seven weeks.

What is the minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible? 1 mark

_____

End of Question and Answer Book

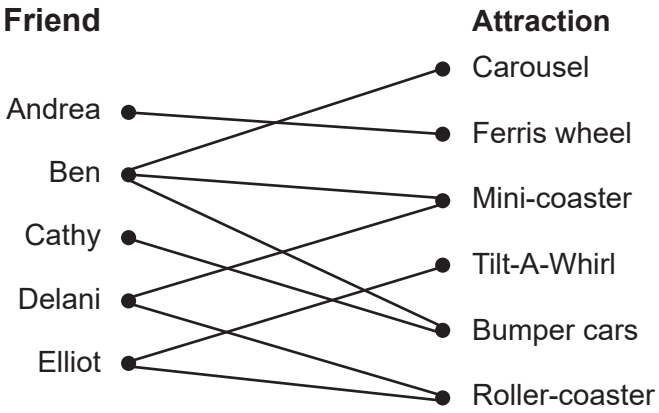
Networks and decision mathematics

Question 14 (3 marks)

Andrea, Ben, Cathy, Delani and Elliot are a group of friends.

They are visiting a theme park that has six attractions: a Carousel, a Ferris wheel, a Mini-coaster, a Tilt-A-Whirl, Bumper cars and a Roller-coaster.

The following bipartite graph illustrates the attractions each friend plans to visit.



a. Which friend plans to visit the most attractions? 1 mark

b. How many attractions have only one friend planning to visit? 1 mark

c. When the friends arrive at the theme park, there is only time for each friend to visit one of the attractions that they had planned to visit.

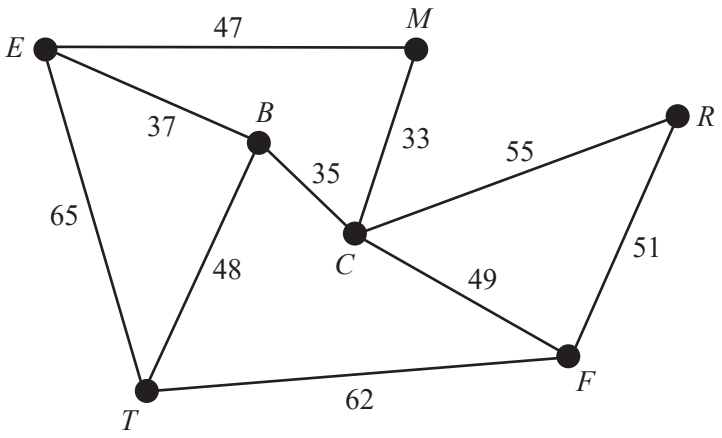
If the minimum number of different attractions are visited, list the names of the attractions that are visited. 1 mark

Question 15 (4 marks)

The theme park entrance (E), Roller-coaster (R), Mini-coaster (M), Bumper cars (B), Carousel (C), Tilt-A-Whirl (T), and Ferris wheel (F) are represented by the vertices of the graph below.

The edges on the graph represent the paths within the theme park.

The numbers on each edge represent the length, in metres, of each path.



a. Rowan works for the theme park collecting money from the ticket booths at each of the attractions.

To do this, he visits each attraction once, starting and finishing at the entrance.

i. What is the mathematical term used to describe this route? 1 mark

ii. Write down one possible route Rowan could take. 1 mark

An inspector is called to check all the paths in the theme park. The inspector would like to walk a route that travels along all the paths once only, starting at the entrance.

b. With reference to the degrees of the vertices on the graph on page 22, explain why the inspector is not able to walk such a route. 1 mark

The inspector decides that the path between the Bumper cars (B) and the Tilt-A-Whirl (T) needs to be closed for maintenance.

The next day Rowan will again need to collect money from the ticket booths at each of the attractions, starting and finishing at the entrance. To do this, Rowan may need to visit one or more attractions twice.

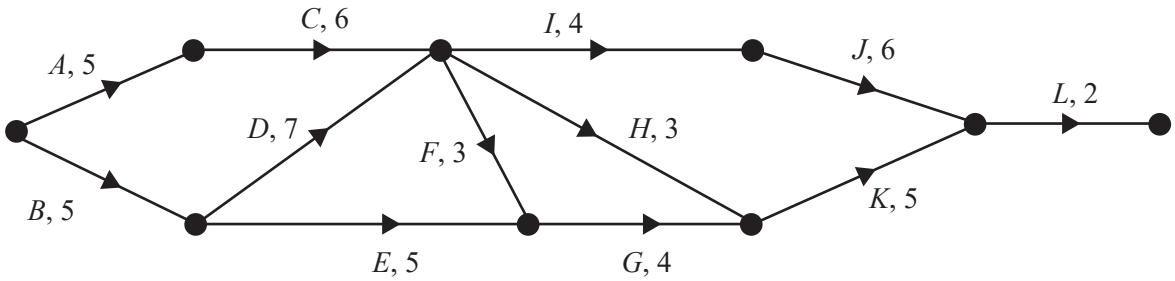
c. If Rowan visits each attraction, starting and finishing at the entrance, what is the length of the shortest path Rowan can take, in metres? 1 mark

Question 16 (5 marks)

The Tilt-A-Whirl attraction is being upgraded. It will be closed for the duration of the upgrade.

The upgrade involves 12 activities, A to L.

The directed network below shows the activities and their completion times, in days.



a. List the activities that have exactly one immediate predecessor. 1 mark

b. What is the minimum number of days the Tilt-A-Whirl will need to be closed to complete this project? 1 mark

c. What is the latest starting time, in days, for activity I? 1 mark

d. Which activity has the longest float time? 1 mark

The management of the theme park decides that the Tilt-A-Whirl attraction will be closed for too long to complete this project. They give the project manager a budget to reduce the overall completion time.

The project manager is able to hire extra people to reduce the time of some activities, represented in the table below. Each of the activities can be reduced by a maximum of two days.

Activity	Daily cost
A	$2000
B	$2000
D	$2500
E	$1000
G	$1500
H	$1200

e. Complete the table below, showing the reductions in individual activity times that would achieve the maximum reduction in completion time for the minimum cost. 1 mark

Activity	Reduction in completion time (0, 1 or 2 days)
A	
B	
D	
E	
G	
H	

End of examination questions