Matrices

Question 25

The matrix below shows how five people, Anita (A), Beverly (B), Christy (C), Dion (D) and Eddie (E), can communicate with each other.

$$\begin{array}{cc} & \textit{receiver} \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \textit{sender} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A ‘1’ in the matrix shows that the person named in that row can send a message directly to the person named in that column.

For example, the ‘1’ in row 3 and column 4 shows that Christy can send a message directly to Dion.

Eddie wants to send a message to Beverly.

Which one of the following shows the order of people through which the message is sent?

A. Eddie – Beverly

B. Eddie – Christy – Beverly

C. Eddie – Christy – Anita – Beverly

D. Eddie – Christy – Dion – Beverly

E. Eddie – Christy – Dion – Anita – Beverly

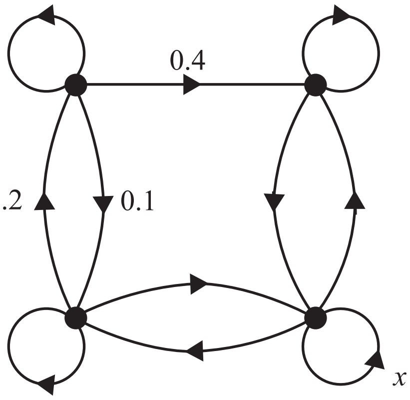
Question 26

A transition matrix, V, is shown below.

$$\begin{array}{cc} & \textit{this month} \\ & \begin{array}{cccc} L & T & F & M \end{array} \\ V = & \begin{bmatrix} 0.6 & 0.6 & 0.2 & 0.0 \\ 0.1 & 0.2 & 0.0 & 0.1 \\ 0.3 & 0.0 & 0.8 & 0.4 \\ 0.0 & 0.2 & 0.0 & 0.5 \end{bmatrix} \begin{array}{c} L \\ T \\ F \\ M \end{array} \textit{next month} \end{array}$$

The transition diagram below has been constructed from the transition matrix V.

The labelling in the transition diagram is not yet complete.



The proportion for one of the transitions is labelled x.

The value of x is

A. 0.1

B. 0.2

C. 0.5

D. 0.6

E. 0.8

Question 27

Four teams, A, B, C and D, competed in a round-robin competition in which each team played each of the other teams once. There were no draws.

The results are shown in the matrix below.

$$\begin{array}{ccc} & & \textit{loser} \\ & & \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{winner} & \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & f & 1 \\ 1 & 0 & 0 & 0 \\ 1 & g & 0 & 1 \\ 0 & 1 & 0 & h \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the team named in that row defeated the team named in that column.

For example, the '1' in row 2 shows that team B defeated team A.

In this matrix, the values of f, g and h are

A. $f = 0$, $g = 1$, $h = 0$

B. $f = 0$, $g = 1$, $h = 1$

C. $f = 1$, $g = 0$, $h = 0$

D. $f = 1$, $g = 1$, $h = 0$

E. $f = 1$, $g = 1$, $h = 1$

Question 28

Consider the matrix P, where $P = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$.

The element in row i and column j of matrix P is p_{ij}.

The elements in matrix P are determined by the rule

A. $p_{ij} = 4 - j$

B. $p_{ij} = 2i + 1$

C. $p_{ij} = i + j + 1$

D. $p_{ij} = i + 2j$

E. $p_{ij} = 2i - j + 2$

Question 29

The following Leslie matrix, L, can be used to model how a population of female animals of three age groups changes over time.

$$L = \begin{bmatrix} 0 & 0 & k \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}$$

where k is the average number of female offspring from a female animal in the third age group during one time period.

Let S_n represent the state matrix showing the population of each of the three age groups after n time periods. The matrix recurrence rule $S_{n+1} = LS_n$ is used to model this situation.

Given $S_0 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$, how many of the following statements are true?

- When $k = 1$, the population will remain at 100 over time.
- When $k = 2$, the population will decrease in the long term.
- When $k = 3$, the population will increase in the long term.
- When $k = 4$, the population will return to the same population age distribution every three years.

A. 0
B. 1
C. 2
D. 3
E. 4

Question 30

The table below shows information about three matrices, A, B and C.

Matrix	**Order**
A	2×4
B	2×3
C	3×4

The transpose of matrix A, for example, is written as A^T.
What is the order of the product $C^T \times (A^T \times B)^T$?

A. 2×3
B. 3×4
C. 4×2
D. 4×3
E. 4×4

Question 31

Matrix P has inverse matrix P^{-1}.

Matrix P is multiplied by the scalar $w\,(w \neq 0)$ to form matrix Q.

Matrix Q^{-1} is equal to

A. $\frac{1}{w}P^{-1}$

B. $\frac{1}{w^2}P^{-1}$

C. wP^{-1}

D. w^2P^{-1}

E. P^{-1}

Question 32

Consider the matrix recurrence relation below.

$$S_0 = \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}, \qquad S_{n+1} = T\,S_n \qquad \text{where } T = \begin{bmatrix} j & 0.3 & l \\ 0.2 & m & 0.3 \\ 0.4 & 0.2 & n \end{bmatrix}$$

Matrix T is a regular transition matrix.

Given the information above and that $S_1 = \begin{bmatrix} 42 \\ 28 \\ 20 \end{bmatrix}$, which one of the following is true?

A. $m > l$

B. $j + l = 0.7$

C. $j = n$

D. $j > m$

E. $l = m + n$

Matrices

Question 25

The daily maximum temperature at a regional town for two weeks is displayed in the table below.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
Week 1	20 °C	17 °C	23 °C	20 °C	18 °C	19 °C	30 °C
Week 2	29 °C	27 °C	28 °C	21 °C	20 °C	20 °C	22 °C

This information can also be represented by matrix *M,* shown below.

$$M = \begin{bmatrix} 20 & 17 & 23 & 20 & 18 & 19 & 30 \\ 29 & 27 & 28 & 21 & 20 & 20 & 22 \end{bmatrix}$$

Element m_{21} indicates that

A. the temperature was 29 °C on Monday in week 2.

B. the temperature was 17 °C on Tuesday in week 1.

C. the lowest temperature for these two weeks was 17 °C.

D. the highest temperature for these two weeks was 29 °C.

E. week 2 had a higher average maximum temperature than week 1.

Question 26

Matrix P is a permutation matrix and matrix Q is a column matrix.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} t \\ e \\ a \\ m \\ s \end{bmatrix}$$

When Q is multiplied by P, which three letters change position?

A. t, e, a

B. e, a, m

C. a, m, s

D. m, s, t

E. e, a, s

Question 27

The following transition matrix, T, models the movement of a species of bird around three different locations, M, N and O from one day to the next.

$$\begin{array}{cc} & \textit{this day} \\ & \begin{array}{ccc} M & N & O \end{array} \\ T = & \begin{bmatrix} \frac{1}{3} & 0 & \frac{9}{10} \\ \frac{1}{3} & 1 & \frac{1}{10} \\ \frac{1}{3} & 0 & 0 \end{bmatrix} \begin{array}{c} M \\ N \\ O \end{array} \quad \textit{next day} \end{array}$$

Which one of the following statements best represents what will occur in the long term?

A. No birds will remain at location M.

B. No birds will remain at location N.

C. All of the birds will end up at location M.

D. All of the birds will end up at location O.

E. An equal number of birds will be at all three locations.

Question 28

Four table tennis teams played in a round-robin tournament.

Each team played each other team once and there were no draws.

The overall ranking of each team at the end of the tournament, based on number of wins, is shown in the table below.

First	Unicorns (U)
Second	Vampires (V)
Third	Scorpions (S)
Fourth	Titans (T)

A dominance matrix can display the results of each game, where a '1' in the matrix shows that the team named in that row defeated the team named in that column.

The dominance matrix for this tournament could be

A.
$$\text{winner}\quad \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

B.
$$\text{winner}\quad \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

C.
$$\text{winner}\quad \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

D.
$$\text{winner}\quad \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

E.
$$\text{winner}\quad \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\begin{array}{c} \textit{loser} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Question 29

Matrix K is a 3×2 matrix.

The elements of K are determined by the rule $k_{ij} = (i - j)^2$.

Matrix K is

A. $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 & 4 \\ 1 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 4 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 4 & 1 \end{bmatrix}$

Question 30

How many of the following statements are true?

- All square matrices have an inverse.
- The inverse of a matrix could be the same as the transpose of that matrix.
- If the determinant of a matrix is equal to zero, then the inverse does not exist.
- It is possible to take the inverse of an identity matrix.

A. 0
B. 1
C. 2
D. 3
E. 4

Question 31

A species of bird has a life span of three years.

The females in this species do not reproduce in their first year but produce an average of four female offspring in their second year, and three in their third year.

The Leslie matrix, L, below is used to model the female population distribution of this species of bird.

$$L = \begin{bmatrix} 0 & 4 & 3 \\ 0.2 & 0 & 0 \\ 0 & 0.4 & 0 \end{bmatrix}$$

The element in the second row, first column states that on average 20% of this population will

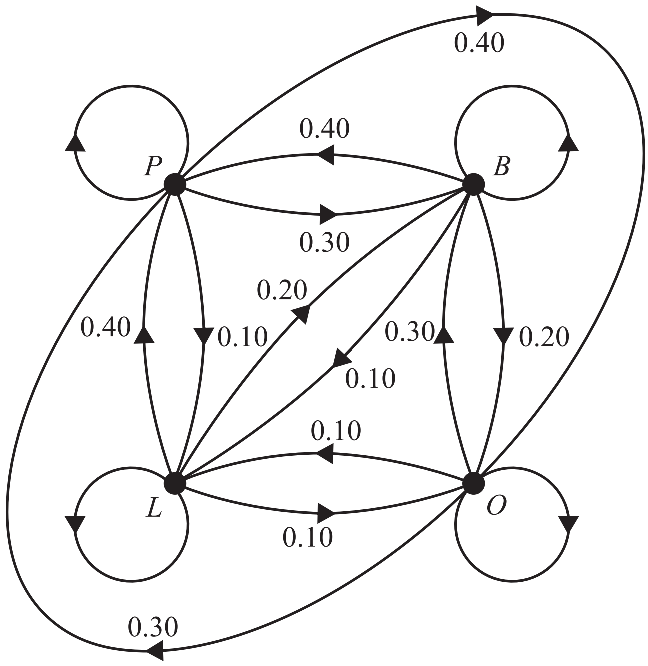
A. be female.
B. never reproduce.
C. survive into their second year.
D. produce offspring in their first year.
E. live for the entire lifespan of three years.

Question 32

For one particular week in a school year, students at Phyllis Island Primary School can spend their lunch break at the playground (P), basketball courts (B), oval (O) or the library (L).

Students stay at the same location for the entire lunch break.

The transition diagram below shows the proportion of students who change location from one day to the next.



The transition diagram is incomplete.

On the Monday, 150 students spent their lunch break at the playground, 50 students spent it at the basketball courts, 220 students spent it at the oval, and 40 students spent it in the library.

Of the students expected to spend their lunch break on the oval on the Wednesday, the percentage of these students who also spent their lunch break on the oval on Tuesday is closest to

A. 27%

B. 30%

C. 33%

D. 47%

E. 52%

Matrices

Question 25

Matrix J is a 2×3 matrix.

Matrix K is a 3×1 matrix.

Matrix L is added to the product JK.

The order of matrix L is

A. 1×3

B. 2×1

C. 2×3

D. 3×2

Question 26

A market stall sells three types of candles.

The cost of each type of candle is shown in matrix C below.

$$C = \begin{bmatrix} 25 & 32 & 43 \end{bmatrix}$$

Towards the end of the day, the cost of each item is discounted by 15%.

Which one of the following expressions can be used to determine each discounted price?

A. $0.15C$

B. $0.85C$

C. $8.5C$

D. $15C$

Question 27

Consider the following matrix, where $h \neq 0$.

$$\begin{bmatrix} 4 & g \\ 8 & h \end{bmatrix}$$

The inverse of this matrix does **not** exist when g is equal to

A. $-2h$

B. $\frac{h}{2}$

C. h

D. $2h$

Question 28

A primary school is hosting a sports day.

Students represent one of four teams: blue (B), green (G), red (R) or yellow (Y).

Students compete in one of three sports: football (F), netball (N) or tennis (T).

Matrix W shows the number of students competing in each sport and the team they represent.

$$\begin{array}{cc} & \begin{array}{cccc} B & G & R & Y \end{array} \\ W = & \begin{bmatrix} 85 & 60 & 64 & 71 \\ 62 & 74 & 80 & 64 \\ 63 & 76 & 66 & 75 \end{bmatrix} \begin{array}{c} F \\ N \\ T \end{array} \end{array}$$

Matrix W is multiplied by the matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ to produce matrix X.

Element x_{31} indicates that

A. 210 students represent the blue team.

B. 210 students compete in netball.

C. 280 students compete in tennis.

D. 280 students compete in football.

Question 29

A tennis team consists of five players: Quinn, Rosie, Siobhan, Trinh and Ursula.

When the team competes, players compete in the order of first, then second, then third, then fourth.

The fifth player has a bye (does not compete).

On week 1 of the competition, the players competed in the following order.

First	Second	Third	Fourth	Bye
Quinn	Rosie	Siobhan	Trinh	Ursula

This information can be represented by matrix G_1, shown below.

$$G_1 = \begin{bmatrix} Q & R & S & T & U \end{bmatrix}$$

Let G_n be the order of play in week n.

The playing order changes each week and can be determined by the rule $G_{n+1} = G_n \times P$

where $P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Which player has a bye in week 4?

A. Quinn

B. Rosie

C. Siobhan

D. Trinh

Question 30

Data has been collected on the female population of a species of mammal located on a remote island.

The female population has been divided into three age groups, with the initial population (at the time of data collection), the birth rate, and the survival rate of each age group shown in the table below.

	Age group (years)		
	0–2	2–4	4–6
Initial population	2100	6400	4260
Birth rate	0	1.8	1.2
Survival rate	0.7	0.6	0

The Leslie matrix (L) that may be used to model this particular population is

A. $L = \begin{bmatrix} 0 & 1.8 & 0 \\ 0.7 & 0 & 1.2 \\ 0 & 0.6 & 0 \end{bmatrix}$

B. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.7 & 0 & 0 \\ 0 & 0.6 & 0 \end{bmatrix}$

C. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.7 & 0.6 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

D. $L = \begin{bmatrix} 2100 & 6400 & 4260 \\ 0 & 1.8 & 1.2 \\ 0.7 & 0.6 & 0 \end{bmatrix}$

Question 31

The matrix below shows the results of a round-robin chess tournament between five players: *H, I, J, K* and *L*. In each game, there is a winner and a loser.

Two games still need to be played.

$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} H & I & J & K & L \end{array} \\ \textit{winner} \begin{array}{c} H \\ I \\ J \\ K \\ L \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & \ldots & 1 & \ldots \\ 1 & \ldots & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & \ldots & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the player named in that row defeated the player named in that column. For example, the 1 in row 4 shows that player *K* defeated player *L*.

A '…' in the matrix shows that the player named in that row has not yet competed against the player in that column.

At the end of the tournament, players will be ranked by calculating the sum of their one-step and two-step dominances.

The player with the highest sum will be ranked first. The player with the second-highest sum will be ranked second, and so on.

Which one of the following is **not** a potential outcome after the final two games have been played?

A. Player *I* will be ranked first.

B. Player *I* will be ranked fifth.

C. Player *J* will be ranked first.

D. Player *J* will be ranked fifth.

Question 32

A large sporting event is held over a period of four consecutive days: Thursday, Friday, Saturday and Sunday.

People can watch the event at four different sites throughout the city: Botanical Gardens (G), City Square (C), Riverbank (R) or Main Beach (M).

Let S_n be the state matrix that shows the number of people at each location n days after Thursday.

The expected number of people at each location can be determined by the matrix recurrence rule

$$S_{n+1} = TS_n + A$$

where

$$T = \begin{array}{c} \textit{this day} \\ \begin{array}{cccc} G & C & R & M \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0.4 & 0 \\ 0.4 & 0.1 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ G \\ C \\ R \\ M \end{array} \textit{next day} \quad \text{and} \quad A = \begin{bmatrix} 300 \\ 200 \\ 100 \\ 300 \end{bmatrix} \begin{array}{c} G \\ C \\ R \\ M \end{array}$$

Given the state matrix $S_3 = \begin{bmatrix} 5620 \\ 6386 \\ 4892 \\ 6902 \end{bmatrix} \begin{array}{c} G \\ C \\ R \\ M \end{array}$

the number of people watching the event at the Botanical Gardens (G) from Thursday to Sunday has

A. decreased by 162

B. decreased by 212

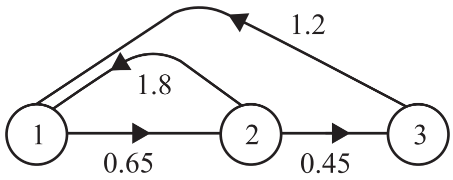
C. increased by 124

D. increased by 696

Matrices

Use the following information to answer Questions 25 and 26.

The following life cycle transition diagram shows changes in a female population of mammals with three age groups (1, 2 and 3).



Question 25

On average, what percentage of the female population from group 2 will survive to group 3?

A. 12%

B. 18%

C. 45%

D. 50%

E. 65%

Question 26

The associated Leslie matrix, L, for the above transition diagram is

A. $L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.8 & 1.2 \\ 0 & 0.65 & 0.45 \end{bmatrix}$

B. $L = \begin{bmatrix} 1 & 1.8 & 1.2 \\ 0 & 0.65 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}$

C. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.65 & 0 & 0 \\ 0.45 & 0 & 0 \end{bmatrix}$

D. $L = \begin{bmatrix} 1.8 & 1.2 & 0 \\ 0 & 0.65 & 0.45 \\ 0 & 0 & 0 \end{bmatrix}$

E. $L = \begin{bmatrix} 0 & 1.8 & 1.2 \\ 0.65 & 0 & 0 \\ 0 & 0.45 & 0 \end{bmatrix}$

Question 27

Matrix V is an $n \times n$ matrix with a determinant equal to 1.

The product of $V \times V^{-1}$ will result in

A. an identity matrix.

B. a Leslie matrix.

C. a column matrix.

D. a zero matrix.

E. a row matrix.

Question 28

Matrix D is a 2×2 matrix where each element is given by d_{ij}

Which rule will result in a binary matrix?

A. $d_{ij} = i + j$

B. $d_{ij} = i - j$

C. $d_{ij} = i \times j$

D. $d_{ij} = i \div j$

E. $d_{ij} = (i - j)^2$

Question 29

Matrix J is a row matrix of order $1 \times n$.

Matrix K is a column matrix of order $n \times 1$.

Matrix J^T is the transpose of Matrix J.

Matrix K^T is the transpose of Matrix K.

Consider the following matrix products where n is a whole number greater than or equal to 2:

- J^2
- JK
- KJ
- $J^T K^T$
- $K^T J^T$

How many of the above matrix products are defined?

A. 1

B. 2

C. 3

D. 4

E. 5

Question 30

Matrix R is a column matrix.

$$R = \begin{bmatrix} T \\ A \\ L \\ L \\ Y \end{bmatrix}$$

A permutation matrix, P, is multiplied by matrix R to form the product matrix $Q = PR$.

If Q is equal to R, how many different permutation matrices could have been used?

A. 1

B. 2

C. 3

D. 4

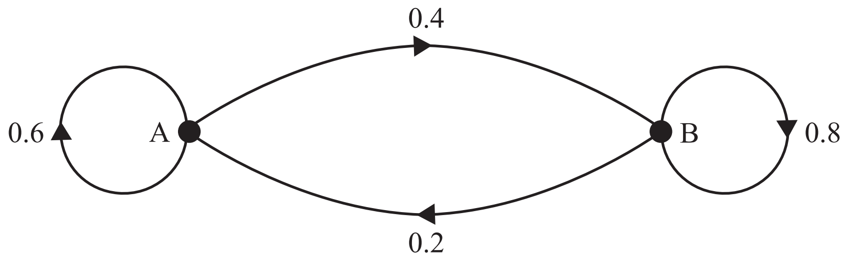
E. 5

Question 31

A group of meerkats lives in an enclosure at a zoo.

The meerkats sleep during the night in one of two chambers, chamber A or chamber B.

The transition diagram below shows the proportion of meerkats that stay in the same sleeping location or change sleeping location from night to night.



Every night there are a meerkats in chamber A.

Every night there are b meerkats in chamber B.

Of the meerkats sleeping in chamber A on Friday night, eight had slept in chamber B on the previous night.

How many meerkats live in the enclosure?

A. 20

B. 30

C. 40

D. 50

E. 60

Question 32

An online shop offers monthly subscriptions for protein powder.

The shop offers protein powder in three flavours: vanilla (V), chocolate (C) and malt (M).

Let P_n be the state matrix that shows the expected number of subscribers for each flavour n months after sales of the protein powder began.

The expected number of subscribers for each flavour can be determined by the matrix recurrence rule

$$P_{n+1} = TP_n + K$$

where

$$T = \begin{array}{c} \textit{this month} \\ \begin{array}{ccc} V & C & M \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0.4 & 0.6 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ V \\ C \\ M \end{array} \textit{next month} \quad \text{and} \quad K = \begin{bmatrix} 93 \\ 59 \\ 9 \end{bmatrix} \begin{array}{l} V \\ C \\ M \end{array}$$

The state matrix, P_2, below shows the expected number of subscribers for each flavour two months after sales began.

$$P_2 = \begin{bmatrix} 147 \\ 137 \\ 199 \end{bmatrix}$$

The increase in the expected number of subscribers for vanilla (V) between the initial sales, P_0, and the first month after sales began, P_1, is equal to

A. 27

B. 54

C. 60

D. 87

E. 93