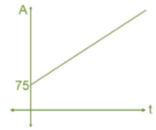
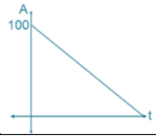
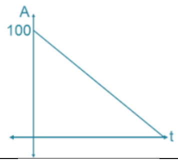
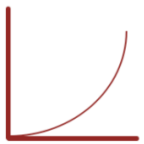
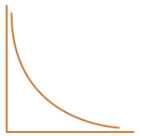
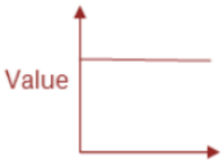
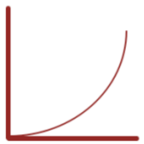
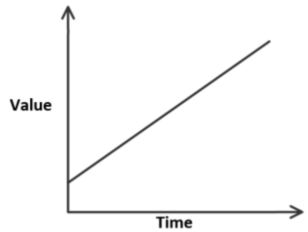
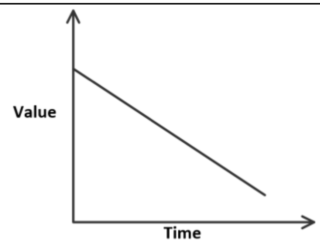
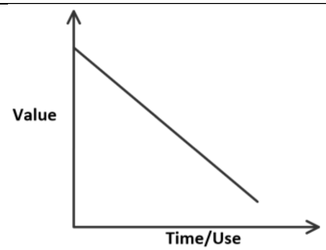
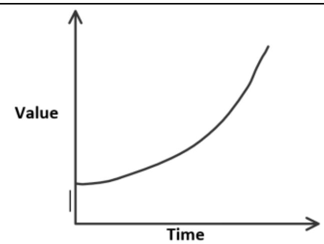
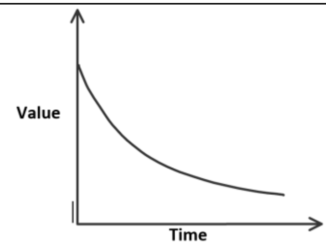
Financial Math

6.15 Amortisation FV↓ 6.18 Amortisation FV↑ 5.9 6.18 Effective Interest Rate

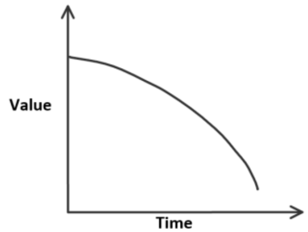
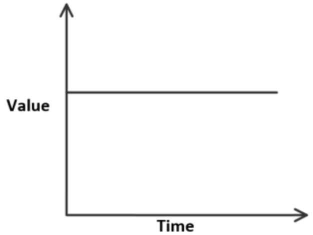
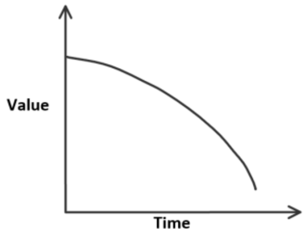
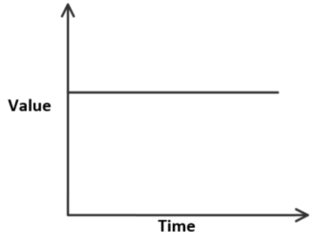
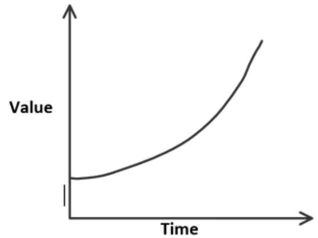
	Recurrence Relation	Application	Explicit Rules	Graphs	CAS Notes #
Arithmetic Sequences D Common difference	$v_0 = principal, v_{n+1} = v_n + D$ $D = \frac{r}{100} \times v_0$	Simple Interest 6.16.1	$v_n = v_0 + n * D$		solve Interest in $ 5.1 Sequence list 5.2 Future Value
	$v_0 = Initial\ Value, v_{n+1} = v_n - D$ $D = \frac{r}{100} \times v_0$	Flat Rate Depreciation 6.16.2	$v_n = v_0 - n * D$		solve depreciate $ 5.1 Sequence list 5.3 Future Value
	$v_0 = Initial\ Value, v_{n+1} = v_n - D$ D = *Unit cost in dollars*	Unit Cost Depreciation 6.16.3	$v_n = v_0 - n * D$		5.1 Sequence list 5.4 Future Value
Geometric Sequences R Common Ratio Growth Factor Decay factor	$v_0 = principal, v_{n+1} = R * v_n$ $R = 1 + \frac{r}{100}$	Compound Interest 6.16.4	$v_n = R^n * v_0$		solve Common Ratio 5.1 Sequence list 5.5 Future Value 5.7 Find r 5.8 Find n
	$v_0 = Initial\ Value, v_{n+1} = R * v_n$ $R = 1 - \frac{r}{100}$	Reduced Balance Depreciation 6.16.5	$v_n = R^n * v_0$		solve Common Ratio 5.1 Sequence list 5.6 Future Value 5.7 Find r 5.8 Find n
Combined Arithmetic & Geometric Sequences R Growth Factor D Repayment	$v_0 = Initial\ Value, v_{n+1} = R * v_n - D$ $R = 1 + \frac{r}{100*p}$ $D = \frac{r}{100*p} \times v_0$	FV → Interest Only Loan /Perpetuities 6.16.6			solve Common Ratio 6.1 Sequence list 6.13 Interest only 6.14 Perpetuities
	$v_0 = Initial\ Value, v_{n+1} = R * v_n - D$ $R = 1 + \frac{r}{100*p}$	FV↓ Reduced Balance Loans /Annuities 6.16.7	Compound Interest loan extra payment		solve Common Ratio 6.1, 6.2, 6.3 Sequence list 6.4 Financial Solver 6.6, 6.9 Analyse Loan 6.7, 6.10 Analyse Annuities 6.12 Condition change
	$v_0 = Initial\ Value, v_{n+1} = R * v_n + D$ $R = 1 + \frac{r}{100*p}$	FV↑ Compound Interest Investment Annuity Investment 6.16.8			solve Common Ratio 6.1 Sequence list 6.5, 6.8 Analyse investment 6.11 Condition change

Note: Yellow parts need real number, blue parts are formula to calculate, Letter P indicates Compounding monthly etc. Needing rate per month, Monthly Ratio etc. Green CAS Notes

	SIMPLE INTEREST	FLAT RATE DEPRECIATION	UNIT COST DEPRECIATION	COMPOUND INTEREST	REDUCING BALANCE DEPRECIATION
Description	A constant amount of interest that is added to the principal at regular time periods. It is calculated as a percentage of the principal.	The value of an asset decreases by a constant amount each time period. This amount is a percentage of the initial value.	The value of an asset uses value after each unit of use.	Takes into account both the principal and any accumulated interest earned in previous periods. The value of the investment grows with increasing amounts each time period.	The value of an asset depreciated by a set percentage, rather than a set amount. The amount of depreciation decreases each time period.
Reccurence relation	V_0 = principal, $V_{n+1} = V_n + D$ $D = r/100 \times V_0$	V_0 = initial value of asset, $V_{n+1} = V_n - D$ $D = r/100 \times V_0$	V_0 = initial value of asset, $V_{n+1} = V_n - D$ D = the depreciation (cost) per unit of use	V_0 = principal, $V_{n+1} = RV_n$ $R = 1 + r/100 \times CP$	V_0 = principal, $V_{n+1} = RV_n$ $R = 1 - r/100 \times CP$
Rule	$V_n = V_0 + nD$ $D = r/100 \times V_0$	$V_n = V_0 - nD$ $D = r/100 \times V_0$	$V_n = V_0 - nD$ D = the depreciation (cost) per unit of use	$V_n = R^n \times V_0$ $R = 1 + r/100 \times CP$	$V_n = R^n \times V_0$ $R = 1 - r/100 \times CP$
Graph					

r = interest rate per annum.
CP/P = compound period. If there are no compound periods, then CP = 1
Recurrence relation: step by step process.
Rule: find values in the future in one step.

Standard compound periods:
1 = yearly
2 = biannually
4 = quarterly
6 = half yearly
12 = monthly
26 = fortnightly
52 = weekly
365 = daily

	REDUCING BALANCE LOANS	INTEREST ONLY LOANS	ANNUITY	PERPETUITY	ANNUITY INVESTMENT
Description	Has compound interest but also requires a fixed amount to be paid at regular intervals.	The borrower only repays interest that is charged. As a result, the principal must be paid back at the termination of the loan.	A form of investment, where compound interest is earned, and a fixed sum is paid to the investor periodically until the investment reaches $0.	Type of annuity that lasts indefinitely. The amount of money invested remains the same as the regular payment to the investor is only the interest earned.	An investment where interest is received, and regular payments are made. Money is not being paid out to the investor; the account is growing.
Recurrence Relation	V_0 = principal, $V_{n+1} = RV_n - D$ R = 1 + r/100 x CP D = regular payment	D = r/100 x V_0 D = payment	V_0 = principal, $V_{n+1} = RV_n - D$ R = 1 + r/100 x CP D = regular payment	D = r/100 x V_0 D = payment	V_0 = principal, $V_{n+1} = RV_n + D$ R = 1 + r/100 x CP D = regular payment
Graph					

Finance Solver	REDUCING BALANCE LOANS	INTEREST ONLY LOANS	ANNUITY	PERPETUITY	ANNUITY INVESTMENT
N		1		1	
I(%)					
PV	Positive	Positive	Negative	Negative	Negative
Pmt	Negative	Negative	Positive	Positive	Negative
FV	Positive Negative Zero	Negative value of the PV	Positive Zero	Positive value of the PV	Positive
PpY					
CpY					

5.1 Using a calculator to generate a sequence of numbers from a rule

Example 3 Generating a sequence of numbers with a calculator

Use a calculator to generate the first five terms of the sequence with a starting value of 5 and the rule 'double and then subtract 3'.

Explanation

Steps

1 Start with a blank computation screen.

2 Type 5 and press enter or EXE.

3 Next type **×2 − 3** and press enter or EXE to generate the next term in the sequence. The computation generating this value is shown as '5·2−3' on the TI-Nspire and 'ans × 2 − 3 on the ClassPad (here 'ans' represents the answer to the previous calculation).

5 State your answer.

Solution

TI-Nspire

5	5
5· 2−3	7
7· 2−3	11
11· 2−3	19

The first five terms are 5, 7, 11, 19, 35

Example 5 Using a calculator to generate sequences from recurrence relations

A sequence is generated by the recurrence relation

$$V_0 = 300, \quad V_{n+1} = 0.5V_n - 9$$

Use your calculator to generate this sequence and determine how many terms at the start of the sequence are positive.

Explanation

1 Start with a blank computation screen.

2 Type **300** and press enter (or EXE).

3 Next type ×0.5 − 9 and press enter (or EXE) to generate the next term in the sequence.

4 Continue to press enter (or EXE) until the first negative term appears.

5 Write your answer.

Solution

300	300.
300 · 0.5 − 9	141.
141 · 0.5 − 9	61.5
61.5 · 0.5 − 9	21.75
21.75 · 0.5 − 9	1.875
1.875 · 0.5 − 9	−8.625

The first five terms of the sequence are positive.

5.2 Simple interest loans and investments

Example 9 Using a recurrence relation to analyse a simple interest investment

Cheryl's simple interest investment is modelled by

$$V_0 = 5000, \quad V_{n+1} = V_n + 240$$

where V_n is the value of the investment after n years.

a Use the recurrence relation to show that the value of Cheryl's investment after 3 years is $5720.

b When will Cheryl's investment first exceed $6000, and what will its value be then?

Explanation

a Calculate V_0, V_1, V_2 and V_3.

Solution

$V_0 = 5000$

$V_1 = 5000 + 240 = 5240$

$V_2 = 5240 + 240 = 5480$

$V_3 = 5480 + 240 = 5720$

Thus, after three years, the value of Cheryl's investment is $5720.

b i On a blank calculation screen, type **5000** and press enter (or EXE).

ii Type **+240** and press enter (or EXE) until the value of the investment first exceeds $6000.

iii Count the number of times that 240 was added. Write your answer.

5000	5000.
5000. + 240	5240.
5240. + 240	5480.
5480. + 240	5720.
5720. + 240	5960.
5960. + 240	6200.

After 5 years; $6200.

5.3 Flat rate depreciation

Example 11 Using a recurrence relation to analyse flat rate depreciation

The flat rate depreciation of a car is modelled by

$$V_0 = 24\,000, \quad V_{n+1} = V_n - 4800$$

where V_n is the value of the car after n years.

a Use the model to determine the value of the car after 2 years.

b If the car was purchased in 2023, in what year will the car's value depreciate to zero?

c What was the percentage depreciation rate?

Explanation

a i Write down the recurrence relation.

ii On a blank calculation screen, type **24 000** and press enter (or EXE).

iii Type **−4800** and press enter (or EXE) twice to obtain the value of the car after 2 years' depreciation. Write your answer.

b i Continue pressing enter (or EXE) until the car has no value.

ii Write your answer.

c Use the amount of depreciation and initial value.

Solution

$V_0 = 24\,000, \quad V_{n+1} = V_n - 4800$

24000	24000.
24000. − 4800	19200.
19200. − 4800	14400.
14400. − 4800	9600.
9600. − 4800	4800.
4800. − 4800	0.

a $14 400

b In 2028

c $\frac{4800}{24000} \times 100\% = 20\%$

The percentage depreciation rate is 20%

5.4 Unit cost depreciation

Example 12 Modelling unit cost depreciation with a recurrence relation

A professional gardener purchased a lawn mower for $270. The mower depreciates in value by $3.50 each time it is used.

a Model the depreciating value of this mower using a recurrence relation of the form:

$$V_0 = \text{initial value}, \quad V_{n+1} = V_n - D$$

where D is the depreciation in value per use and V_n is the value of the mower after being used to mow n lawns.

b Use the model to determine the value of the mower after it has been used three times.

c How many times can the mower be used until its depreciated value is first less than $250?

Explanation

a 1 Write down the value of V_0. Here, V_0 is the value of the mower when new.

Solution $V_0 = 270$

2 Write down the unit cost rate of depreciation, D.

$D = 3.50$

3 Write your answer.

$V_0 = 270, \quad V_{n+1} = V_n - 3.50$

b 1 Write down the recurrence relation.

$V_0 = 270, \quad V_{n+1} = V_n - 3.50$

2 On a blank calculation screen, type **270** and press enter (or EXE). Type **−3.50** and press enter (or EXE) three times to obtain the value of the mower after three mows.

270	270.
270. − 3.5	266.5
266.5. − 3.5	263.
263. − 3.5	259.5
259.5 − 3.5	256.
256. − 3.5	252.5
252.5 − 3.5	249.

3 Write your answer.

$259.50

c 1 Continue pressing enter (or EXE) until the value of the lawn mower is first less than $250.

2 Write your answer.

After six mows

5.5 Compound interest investments and loans

Example 20 Modelling compound interest with a recurrence relation

The following recurrence relation can be used to model a compound interest investment of $2000 paying interest at the rate of 7.5% per annum.

$$V_0 = 2000, \quad V_{n+1} = 1.075 \times V_n$$

In the recurrence relation, V_n is the value of the investment after n years.

a Use the recurrence relation to show that the value of the investment after 3 years is $2484.59.

b Determine when the value of the investment will first exceed $2500.

Explanation

a 1 Write down the principal, V_0.

2 Use the recurrence relation to calculate V_1, V_2 and V_3 and round to the nearest cent.

Solution

$V_0 = 2000$

$V_1 = 1.075 \times 2000 = 2150$

$V_2 = 1.075 \times 2150 = 2311.25$

$V_3 = 1.075 \times 2311.25 = 2484.59$

b 1 Type '2000'. Press enter (or EXE).

2 Type **×1.075**.

3 Count how many times you press enter (or EXE) until the term value is greater than 2500.

2000	2000.
2000. · 1.075	2150.
2150. · 1.075	2311.25
2311.25 · 1.075	2484.59375
2484.59375 · 1.075	2670.93828125

4 Write your answer.

The investment will first exceed $2500 after 4 years.

5.6 Reducing balance depreciation

Example 22 Using reducing balance depreciation with recurrence relations

The following recurrence relation can be used to model the value of office furniture with a purchase price of \$9600, depreciating at a reducing-balance rate of 7% per annum.

$V_0 = 9600, \quad V_{n+1} = 0.93 \times V_n$

In the recurrence relation, V_n is the value of the office furniture after n years.

a Use the recurrence relation to find the value of the office furniture, correct to the nearest cent, after 1, 2 and 3 years.

b If the office furniture was initially purchased in 2023, at the end of which year will the value of the investment first be less than \$7000?

Explanation

a 1 Write down the purchase price of the furniture, V_0.

2 Use the recurrence relation to calculate V_1, V_2 and V_3. Use your calculator if you wish.

Solution

$V_0 = 9600$

$V_1 = 0.93 \times 9600 = 8928$
$V_2 = 0.93 \times 8928 = 8303.04$
$V_3 = 0.93 \times 8303.04 = 7721.83$

b Steps

1 Type **9600** and press enter or EXE.

2 Type **× 0.93**.

3 Count how many times you press enter until the term value is less than 7000.

9600	9600.
9600. · 0.93	8928.
8928. · 0.93	8303.04
8303.04 · 0.93	7721.8272
7721.8272 · 0.93	7181.299296
7181.299296 · 0.93	6678.708345

4 Write your answer.

The value of the furniture first drops below \$7000 after 5 years. Thus, it is first worth less than \$7000 at the end of 2028.

5.7 Geometric growth and decay problems to find n

Example 26 Using a calculator to solve geometric growth and decay problems to find n

How many years will it take for an investment of \$5000, paying compound interest at 6% per annum, to grow above \$8000? Write your answer correct to the nearest year.

Explanation

1 Write down the values of V_0, V_n and R.

$V_0 = 5000, \quad R = 1 + \frac{6}{100} = 1.06$
$V_n = 8000$

2 Substitute into the rule for the particular term of a sequence.

$V_n = R^n \times V_0$
$8000 = 1.06^n \times 5000$

3 Solve this equation for n using a CAS calculator.

$\text{solve}(8000 = (1.06)^n \cdot 5000,\ n)$
$n = 8.06611354799$

4 Write your answer, rounding up as interest is paid at the end of the year. After 8 years, the value is \$7969.24.

The value of the investment will grow above \$8000 after 9 years.

5.8 Geometric growth and decay problems to find r

Example 27 Using a calculator to solve geometric growth and decay problems to find r

An industrial weaving company purchased a new loom at a cost of \$56 000. It has an estimated value of \$15 000 after 10 years of operation. If the value of the loom is depreciated using a reducing balance method, what is the annual rate of depreciation? Write your answer correct to one decimal place.

Explanation

1 Write down the values of V_0, V_n, R and n.

$V_0 = 56\,000,\ V_n = 15\,000,\ n = 10$
$R = 1 - \frac{r}{100}$

2 Substitute into the rule for the nth term of a sequence.

$V_n = R^n \times V_0$
$V_{10} = \left(1 - \frac{r}{100}\right)^{10} \times V_0$
$15\,000 = \left(1 - \frac{r}{100}\right)^{10} \times 56\,000$

3 Solve this equation for r using a CAS calculator.

$\text{solve}\left(15000 = \left(1 - \frac{r}{100}\right)^{10} \cdot 56000,\ r\right)$
$r = 12.3422491484$ or
$r = 187.657750852$

Note: there are two answers. Choose the more appropriate of the two.

4 Write your answer.

The annual rate of depreciation is 12.3%, correct to one decimal place.

5.9 Effective interest rate

Example 32 Calculating effective interest rates using a CAS calculator

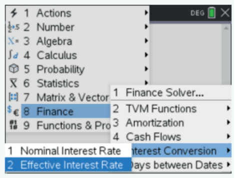
Marissa has \$10 000 to invest. She chooses an account that will earn compounding interest at the rate of 4.5% per annum, compounding monthly.

Use a CAS calculator to find the effective interest rate for this investment, correct to three decimal places.

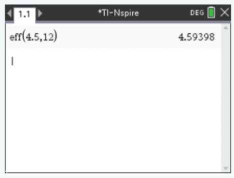
Explanation

Steps

1 Press menu and then select
8: Finance ▶
5: Interest Conversion ▶
2: Effective interest rate
to paste in the **eff(. . .)** command.
The parameters of this function are eff(nominal rate, number of times the interest compounds each year).



2 Enter the nominal rate (4.5) and number of times the interest compounds each year (12) into the function, separated by a comma. Press enter to get the effective interest rate.



Write your answer.

The effective interest rate for this investment is 4.594%.

6.1 Compound interest investments with regular additions to the principal

Example 4 Using a recurrence relation to analyse compound interest investments with additions to the principal

Albert has an investment that can be modelled by the recurrence relation

$V_0 = 400, \quad V_{n+1} = 1.005V_n + 30$

where V_n is the value of the investment after n months.

a State the value of the initial investment.

b Determine the value of the investment after Albert has made three extra payments. Round your answer to the nearest cent.

c What will be the value of his investment after 6 months? Round your answer to the nearest cent.

d Plot the points for the value of the investment after 0, 1, 2 and 3 months on a graph.

Explanation

a Note that $V_0 = 400$.

The initial investment was \$400.

b Perform the calculations.

$V_0 = \$400$
$V_1 = 1.005 \times 400 + 30 = \$432$
$V_2 = 1.005 \times 432 + 30 = \$464.16$
$V_3 = 1.005 \times 464.16 + 30 = \$496.48$
The value of Albert's investment is \$496.48.

c i Either continue performing the calculations or use your CAS by:

ii Type 400 and press enter (or EXE).

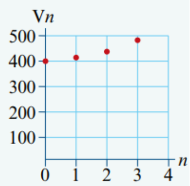
iii Type × 1.005 + 30 and press enter (or EXE) six more times.

400	400
400 · 1.005 + 30	432
432 · 1.005 + 30	464.16
464.16 · 1.005 + 30	496.48
496.4808 · 1.005 + 30	528.96
528.9632 · 1.005 + 30	561.61
561.6180 · 1.005 + 30	594.42

ii Write your answer.

The value of Albert's investment is \$594.42.

d Plot each of the points on the graph.



6.2 Using a recurrence relation to analyse a reducing balance loan

Example 8 Using a recurrence relation to analyse a reducing balance loan

Alyssa's loan can be modelled by the recurrence relation:

$V_0 = 1000, \quad V_{n+1} = 1.0125V_n - 257.85$

a Use your calculator to find the balance of the loan after four payments.

b Find the balance of the loan after two payments have been made. Round your answer to the nearest cent.

Explanation

a **i** Write down the recurrence relation.

ii Type '1000' and press 'enter' or 'EXE'.

iii Type '× 1.0125-257.85' and press 'enter' (or EXE) 4 times to obtain the screen opposite.

b Read the third line of the calculator.

Solution

$V_0 = 1000, V_{n+1} = 1.0125V_n - 257.85$

1000	1000
1000 ·1.0125 – 257.85	754.65
754.65·1.0125 – 257.85	506.23
506.2331·1.0125 – 257.85	254.71
254.7110·1.0125 – 257.85	0.044927

Balance $0.04 (to the nearest cent).

$506.23 (to the nearest cent)

6.3 Annuities

Example 9 Modelling an annuity with a recurrence relation

Reza invests $12 000 in an annuity that earns interest at the rate of 6% per annum, compounding monthly, providing him with a monthly income of $2035.

a Model this annuity using a recurrence relation of the form

V_0 = the principal, $\quad V_{n+1} = RV_n - D$

where V_n is the value of the annuity after n months.

b Use your calculator to find the value of the annuity after the first four months. Round your answer to the nearest cent.

Explanation

a **i** State the value of V_0 and D.

ii Determine the value of R using the formula $R = 1 + \frac{r}{100 \times p}$.

iii Use the values of V_0, R and D to write down the recurrence relation.

b **i** Type **12000** and press enter or EXE.

ii Type **× 1.005-2035** and press enter or EXE four times to obtain the screen opposite.

Solution

$V_0 = 12\,000$ and $D = 2035$

$R = 1 + \frac{6}{100 \times 12} = 1.005$

$V_0 = 12\,000, \quad V_{n+1} = 1.005V_n - 2035$

	12000.
12000. · 1.005 – 2035	10025.
10025. · 1.005 – 2035	8040.125.
8040.125. · 1.005 – 2035	6045.326.
6045.326. · 1.005 – 2035	4040.55.

6.4 Using a finance solver to find the balance and final payment

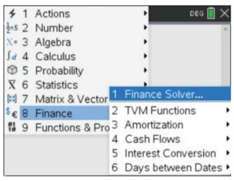
Using the Finance Solver on the TI-Nspire CAS

Steps

1 Press ctrl + N

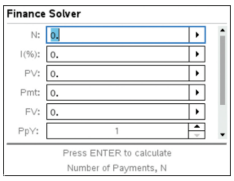
2 Select **Add Calculator**.

Press menu>**Finance**> **Finance Solver**.



3 To use Finance Solver you need to know the meaning of each of its symbols.

- **N** is the total number of payments.
- **I(%)** is the annual interest rate.
- **PV** is the present value of the loan/investment.
- **Pmt** is the amount paid at each payment.

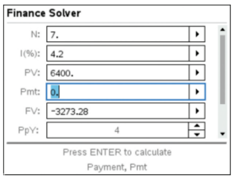


- **FV** is the future value of the loan/investment.
- **PpY** is the number of payments per year.
- **CpY** is the number of times the interest is compounded per year. (It is the same as **PpY**.)
- **PmtAt** is used to indicate whether the interest is compounded at the end or at the beginning of the time period. Ensure this is set at **END**.

Note: Use tab or ▼ to move down boxes, press ▲ to move up. For **PpY** and **CpY** press tab to move down to the next entry box.

4 When using Finance Solver to solve loan and investment problems, there will be one unknown quantity. To find its value, move the cursor to its entry field and press enter to solve.

In the example shown, pressing enter will solve for **Pmt**.



6.5 Using a financial solver to analyse a compound interest investment with regular additions to the principal

Using Finance solver for a compound interest investment with regular additions to the principal

In finance solver:

- **PV:** Negative: you make an investment by giving the bank some money.
- **PMT:** Negative: you make regular payments to the bank.
- **FV:** Positive: after the payment is made and the investment matures, the bank will give you the money.

Example 18 Determining the value of an investment with regular additions made to the principal using a financial solver

Lars invests $500 000 at 5.5% per annum, compounding monthly. He makes a regular deposit of $500 per month into the account. What is the value of his investment after 5 years? Round your answer to the nearest cent.

Explanation

1 Open Finance Solver and enter the information below, as shown opposite.

- **N**: 60 (5 years)
- **I%**: 5.5
- **PV**: –500 000 (you give this to the bank)
- **PMT**: –500 (you give this to the bank)
- **FV**: to be determined
- **Pp/Y**: 12 payments per year
- **Cp/Y**: 12 compounding periods per year

2 Solve for FV and write your answer, rounding to the nearest cent. Note that this is positive as the bank will give this money to you.

Solution

N:	60
I%:	5.5
PV:	–500000
Pmt or PMT:	–500
FV:	692292.297...
Pply or P/Y:	12
Cp/Y or C/Y:	12

After 5 years, Lars' investment will be worth $692 292.30.

6.6 Using finance solver for a reducing balance loan

Finance solver for a reducing balance loan

In finance solver:

- **PV:** Positive: the bank gives you money through a loan.
- **PMT:** Negative: you repay the loan by making regular repayments to the bank.
- **FV:** Negative, zero or positive: after the payment is made:
  - you still owe the bank money (FV negative),
  - the loan is fully repaid (FV zero), or
  - you have overpaid your loan and the bank now owes you money (FV positive).

Example 19 Determining the balance and final payment of a reducing balance loan after a given number of payments

Andrew borrows $20 000 at an interest rate of 7.25% per annum, compounding monthly. This loan will be repaid over 4 years with regular payments of $481.25 each month for 47 months followed by a final payment to fully repay the loan.

a How much does Andrew owe after 3 years? Round your answer to the nearest cent.

b What is the final payment amount that Andrew must make to fully repay the loan within 4 years (48 months)? Round your answer to the nearest cent.

Explanation

a **1** Open Finance Solver and enter the following:

- **N**: 36 (number of months in 3 years)
- **I%**: 7.25 (annual interest rate)
- **PV**: 20000 (positive to indicate that this is money received by Andrew from the lender)
- **Pmt** or **PMT**: –481.25 (negative as Andrew is giving this back to the lender)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solution

N:	36
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	
Pply or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press to [enter] solve.
- *ClassPad:* Tap on the **FV** entry box and tap 'Solve'. The amount –5554.3626 . . . now appears in the **FV** entry box.

Note: A negative FV indicates that Andrew will still owe the lender money after the payment has been made.

N:	36
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	-5554.3626
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer, correct to the nearest cent

Andrew owes $5554.36.

b **1** Enter the information below.

- **N**: 48 (number of months in 4 years)
- **I%**: 7.25 (annual interest rate)
- **PV**: 20000
- **Pmt** or **PMT** (the payment amount is negative): −481.25
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	48
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press [enter] to solve.
- *ClassPad:* Tap on the **FV** entry box and tap **Solve** The amount 0.1079. . . (11 cents) now appears in the **FV** entry box.

N:	48
I%:	7.25
PV:	20000
Pmt or PMT:	-481.25
FV:	0.107924
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

Since FV is positive (+11 cents), the bank owes Andrew 11 cents so we **subtract** this from the regular payment.

3 Write your answer.

Final payment
= $481.25 − $0.11
= $481.14
Andrew's final payment will be $481.14.

6.7 Using a finance solver to analyse an annuity

Finance solver for an annuity

In finance solver:

- **PV:** Negative: you buy an annuity by giving the bank some money.
- **PMT:** Positive: you receive regular payments from the bank.
- **FV:** Positive or zero: after the payment is made:
  - the bank still owes you money (FV positive),
  - the annuity is fully paid out (FV zero)

Note: An annuity should never have a negative FV as a bank would never overpay the individual.

Example 20 Determining the balance of an annuity using a finance solver

Charlie invests $300 000 into an annuity, paying 5% interest per annum, compounding monthly. Over the next ten years, Charlie receives a payment of $3182 per month from the annuity for each month except the final month.

a Find the value of the annuity after five years. Round your answer to the nearest cent.

b Find the final payment from the annuity. Round your answer to the nearest cent.

Explanation

a **1** Open Finance Solver and enter the following:

- **N**: 60 (number of monthly payments in 5 years)
- **I%**: 5.00 (annual interest rate)
- **PV**: −300000 (negative to indicate that this is money paid by Charlie to the bank)
- **Pmt** or **PMT**: 3182 (positive to indicate that the bank is paying back to Charlie)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solution

N:	60
I%:	5.00
PV:	-300000
Pmt or PMT:	3182
FV:	
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press to [enter] solve.
- *ClassPad:* Tap on the **FV** entry box and tap 'Solve'. The amount 168612.24795 . . . now appears in the **FV** entry box.

Note: A positive FV indicates that Charlie is still owed money from the annuity.

N:	60
I%:	5.00
PV:	-300000
Pmt or PMT:	3182
FV:	168612.247951
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

The balance of the annuity is $168 612.25

b **1** Find the value of the annuity after 120 payments. Enter the information below, as shown opposite.

- **N**: 120 (number of monthly payments in 10 years)
- **I%**: 5.00
- **PV**: −300000
- **Pmt** or **PMT**: 3182
- **Pp/Y**: 12
- **Cp/Y**: 12

N:	120
I%:	5.00
PV:	-300000
Pmt or PMT:	3182
FV:	
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (FV). On the:

- *TI-NspireCAS:* Move the cursor to the **FV** entry box and press [enter] to solve.
- *ClassPad:* Tap on the **FV** entry box and tap **Solve** The amount −5.36 (−$5.36) now appears in the **FV** entry box.

Note: The FV is negative ($5.36). This means that Charlie owes the annuity $5.36. To compensate, Charlie's final payment will be decreased by $5.36.

N:	120
I%:	5.00
PV:	-300000
Pmt or PMT:	3182
FV:	-5.36388
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer.

Final payment
= $3182 − $5.36
= $3176.64
Charlie's final payment will be $3176.64.

6.8 Using a finance solver to find interest rates, time taken and regular payments

Example 21 Finding the interest rate for an investment with additional payments

Mingjia puts $20 000 into a compound interest investment where interest compounds monthly. She adds $50 per month. She wants her investment to reach $40 000 in 10 years.

Find the annual interest rate required for this to occur. Round your answer to two decimal places.

Explanation

1 Open finance solver and enter the following:

- **N**: 120 (10 years)
- **PV**: −20000
- **PMT**: −50
- **FV**: 40000 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

2 Solve for **I**.

3 Write your answer.

Solution

N:	120
I%:	4.807676
PV:	-20000
Pmt or PMT:	-50
FV:	40000
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

Minjia would require an interest rate of 4.81% per annum.

Example 22 Finding the regular monthly payment and time taken for an investment with additions to the principal

Winston puts $20 000 into an investment, paying 5.1% interest per annum, compounding monthly.

a If Winston wants his investment to be worth at least $40 000 in 5 years, what is the minimum he will need to add each month?

b If Winston invests $1000 each month immediately after interest is calculated, what is the minimum number of months required for his investment to at least triple in value?

Explanation

a **1** Open finance solver and enter the following:

- **N**: 60 (5 years)
- **I%**: 5.1 (annual interest rate)
- **PV**: −20000
- **FV**: 40000 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

2 Solve for **Pmt** or **PMT**.

Note: The sign of Pmt or PMT is negative, because it is money that Winston invests.

Solution

N:	60
I%:	5.1
PV:	-20000
Pmt or PMT:	-208.341646
FV:	40000
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer, noting that $208.34 per month is insufficient as it gives a balance of $39999.89. . .

Pmt or PMT:	-208.34
FV:	-39999.8877

Winston will add $208.35 each month to the investment.

b **1** Change the payment **Pmt** or **PMT** to −1000 and the **FV** to 60 000 and solve for **N**.

N:	34.3211
I%:	5.1
PV:	-20000
Pmt or PMT:	-1000
FV:	60000
Ppl/y or P/Y:	12
Cp/Y or C/Y:	12

2 Write your answer, noting that 34 months has a **FV** of $59598.147. . . so we need to round up

The value of Winston's investment will take 35 months to triple.

6.9 Analysing a reducing balance loan with financial solver

Example 23 Determining the payment amount, total repayment and total amount of interest paid for a reducing balance loan

Sipho borrows $10 000 to be repaid in 59 equal monthly payments followed by a 60th payment of less than one dollar more than the regular payment. Interest is charged at the rate of 8% per annum, compounding monthly.

a Find the regular monthly payment amount. Round your answer to the nearest cent.
b Find the final payment. Round your answer to the nearest cent.
c Find the total of the repayments on the loan. Round your answer to the nearest cent.
d Find the total amount of interest paid. Round your answer to the nearest cent.

Explanation | **Solution**

a 1 Open Finance Solver and enter the following:
- **N**: 60 (number of monthly payments in 5 years, assuming 60 equal payments)
- **I%**: 8 (annual interest rate)
- **PV**: 10000
- **FV**: 0 (the balance will be zero when the loan is repaid)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	60
I%:	8
PV:	10000
Pmt or PMT:	
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (Pmt or PMT). On the:
- *TI-Nspire:* Move the cursor to the **Pmt** entry box and press enter to solve.
- *ClassPad:* Tap on the **PMT** entry box and tap **Solve**.

The amount –202.7639… now appears in the **Pmt** or **PMT** entry box.

Note: The sign of the payment is negative to indicate that this is money Sipho is giving back to the lender.

N:	60
I%:	8
PV:	10000
Pmt or PMT:	-202.7639
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer.

Sipho repays $202.76 as the regular payment.

b To find the final payment:

1 Find the final value after 60 payments of $202.76.

Final payment
= 202.76 + 0.29
= 203.05

2 Since **FV** is –0.289, the final payment is 0.29 more than the regular payment.

Final payment is $203.05

c Total of repayments of the loan = 59 × regular payment + final payment

Total of repayments = 59 × 202.76 + 203.05 = $12 165.89

d Total interest = total repayments – the principal

Interest paid = 12 165.89 – 10 000 = $2165.89

6.10 Analysing an annuity with financial solver

Example 24 Finding the interest rate, time taken and regular payment for an annuity

Joe invests $200 000 into an annuity, with interest compounding monthly.

a What interest rate would allow Joe to withdraw $2500 each month for 10 years? Round your answer to one decimal place.
b Assume the interest rate is 5% per annum and that Joe receives a regular monthly payment of $3000. For how many months will Joe receive a regular payment?
c Assume that the interest rate is 5% per annum and that Joe wishes to be paid monthly payments for 10 years. How much will he regularly receive each month?
d If Joe receives the regular monthly payment found in part **c** for 119 months, what will his final payment be? Round your answer to the nearest cent.

Explanation | **Solution**

a 1 Open finance solver and enter the following:
- **N**: 120 (10 years)
- **PV**: –200000
- **PMT**: 2500
- **FV**: 0 (exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Solve for I%.

N:	120
I%:	8.68922416
PV:	-200000
Pmt or PMT:	2500
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Write your answer.

Joe would require an interest rate of 8.7% per annum to make monthly withdrawals of $2500 for 10 years.

b 1 Change the payment **Pmt** or **PMT** to 3000 and solve for **N**.

N:	78.2639745
I%:	5
PV:	-200000
Pmt or PMT:	3000
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Write your answer, rounding down as we are only counting regular payments.

Joe will receive a regular payment for 78 months.

c 1 Open the finance solver on your calculator and enter the information below, as shown.
- **N**: 120 (10 years)
- **I%**: 5 (annual interest rate)
- **PV**: –200000
- **FV**: 0 (the annuity will be exhausted after 10 years)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	120
I%:	5
PV:	-200000
Pmt or PMT:	
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for **Pmt** or **PMT**.

Note: The sign of Pmt or PMT is positive, because it is money received.

N:	120
I%:	5
PV:	-200000
Pmt or PMT:	2121.3103
FV:	0
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 Write your answer.

Joe will receive $2121.31 each month from the annuity.

d Find the final value after 120 months of $2121.31.

N:	120
I%:	5
PV:	–200000
Pmt or PMT:	2121.31
FV:	0.047
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

Since **FV** is 0.047…, the final payment is 0.05 more than the regular payment.

Final payment is $2121.36

6.11 Changing the regular payment to an investment

Example 25 Finding the value of an investment when the regular payment changes

Derek invests $50 000 into a compound interest investment paying 6.1% per annum, compounding annually. Derek invests an additional $8000 per year immediately after interest is calculated.

After five years, Derek increases his additional investment to $10 000 per year.

Calculate the value of Derek's investment after twelve years (in total).

Explanation | **Solution**

1 Open finance solver and enter the following:
- **N**: 5 (5 years before the change)
- **I%**: 6.1 (annual interest rate)
- **PV**: -50000 (value of initial investment)
- **PMT**: -8000 (additional amount added)
- **Pp/Y**: 1 (annual payment)
- **Cp/Y**: 1 (interest compounds annually)

Solve for **FV**.

N:	5
I%:	6.1
PV:	–50000
Pmt or PMT:	–8000
FV:	112414.364
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

2 Change the following in finance solver:
- **N**: 7 (7 years after the change)
- **PV**: -112414.364…(copied from FV above)
- **PMT**: -10000 (additional amount added)

Solve for **FV**

N:	7
I%:	6.1
PV:	–112414.3641
Pmt or PMT:	–10000
FV:	254343.79745
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

3 Write your answer.

The value of Derek's investment is $254 343.80

6.12 Changing the interest rate

Example 26 **Finding the final payment of a reducing balance loan when the interest rate changes**

Adrian borrows $150 000 for 25 years at an interest rate of 6.8% per annum, compounding monthly.

For the first three years, Adrian repays $1041.11 each month.

After 3 years, the interest rate rises to 7.2% per annum. Adrian still wishes to pay off the loan in 25 years so makes 263 monthly payments of $1076.18 followed by a final payment.

Calculate the final payment to ensure the loan is fully repaid at the end of 25 years. Round your answer to the nearest cent.

Explanation

1 Open the finance solver on your calculator and enter the information below, as shown opposite.

- **N**: 36 (number of monthly payments in 3 years)
- **I%**: 6.8 (annual interest rate)
- **PV**: 150000 (initial value of loan)
- **Pmt**: −1041.11 (monthly repayments)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

Note: You can enter **N** as 3 × 12 (3 years of monthly payments). The finance solver will calculate this as 36 for you.

Solution

N:	36
I%:	6.8
PV:	150000
Pmt or PMT:	-1041.11
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for **FV**.

N:	36
I%:	6.8
PV:	150000
Pmt or PMT:	-1041.11
FV:	-142391.8359
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

3 If the loan is still to be repaid in 25 years, there are still 22 years left.
Change:

- **N** to 22 × 12 or 264 payments
- **I(%)** to 7.2 (the new interest rate)
- **PV** to 142391.83593707 (the balance after 3 years)
- **Pmt** to −1076.18

N:	264
I%:	7.2
PV:	-142391.8359
Pmt or PMT:	-1076.81
FV:	
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

4 Solve for **FV**.

N:	264
I%:	7.2
PV:	-142391.8359
Pmt or PMT:	-1076.18
FV:	0.11735068
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

5 Since **FV** is 0.1173…, the final payment decreases by $0.12.

Final payment =
$1076.18 – $0.12 = $1076.06
Thus, the final payment will be $1076.06

Note: It is important that you do not round prematurely or you will get the incorrect answer of $1076.08

6.13 Interest-only loans

Example 27 **Finding the regular payment for an interest-only loan**

Jane borrows $50 000 to buy some shares. Jane negotiates an interest-only loan at an interest rate of 9% per annum, compounding monthly. What is the monthly amount Jane will be required to pay?

Explanation

Calculation method

Use the rule $D = \frac{r}{100 \times p} \times V_0$.

1 V_0 is the amount borrowed = $50 000

2 Calculate the interest payable where $r = 9$ and $p = 12$.

3 Evaluate the rule for these values and write your answer.

Solution

$V_0 = 50\,000$

$D = \frac{r}{100 \times p} \times V_0$

$D = \frac{9}{100 \times 12} \times 50\,000$

$D = 375$

Jane will need to repay $375 every month on this interest-only loan.

Finance solver method

Consider one compounding period because all compounding periods will be identical.

1 Open Finance Solver and enter the following.

- **N**: 1 (one compounding period)
- **I%**: 9 (annual interest rate)
- **PV**: 50000
- **FV**: −50000 (the amount owing will be the same after one payment)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

N:	1
I%:	9
PV:	50000
Pmt or PMT:	
FV:	–50000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

2 Solve for the unknown future value (Pmt or PMT). On the:

- *TI-Nspire:* Move the cursor to the **Pmt** entry box and press enter to solve.
- *ClassPad:* Tap on the **PMT** entry box and tap **Solve**.

The amount –375 now appears in the **Pmt** or **PMT** entry box.

N:	1
I%:	9
PV:	50000
Pmt or PMT:	–375
FV:	–50000
Pp/y or P/Y:	12
Cp/Y or C/Y:	12

6.14 Perpetuities

Example 32 **Calculating the interest rate of a perpetuity**

A university mathematics faculty has $30 000 to invest. It intends to award an annual mathematics prize of $1500 with the interest earned from investing this money in a perpetuity.

What is the minimum interest rate that will allow this prize to be awarded indefinitely?

Explanation

We will consider just one compounding period because all compounding periods will be identical.

Calculation method

1 Use the rule $D = \frac{r}{100 \times p} \times V_0$ and solve the equation for r.

2 Write your answer.

Solution

$1500 = \frac{r}{100 \times 1} \times 30\,000$

$1500 = r \times 300$

$r = \frac{1500}{300} = 5$

The minimum annual interest rate to award this prize indefinitely is 5%.

Financial solver

1 Open Finance Solver and enter the following.

- **N**: 1 (one payment)
- **PV**: –30 000
- **Pmt or PMT**: 1500 (prize is $1500 each year)
- **FV**: 30 000 (the balance will be the same after each payment)
- **Pp/Y**: 1 (yearly payment)
- **Cp/Y**: 1 (interest compounds yearly)

N:	1
I%:	
PV:	-30000
Pmt or PMT:	1500
FV:	30000
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

2 Solve for the unknown interest rate (**I%**). On the:

- *TI-Nspire:* Move the cursor to the **I%** entry box and press enter to solve.
- *ClassPad:* Tap on the **I%** entry box and tap **Solve** The amount **5** now appears in the **I%** entry box.

N:	1
I%:	5
PV:	-30000
Pmt or PMT:	1500
FV:	30000
Pp/y or P/Y:	1
Cp/Y or C/Y:	1

3 Write your answer, rounding as required.

The minimum annual interest rate to award this prize indefinitely is 5%.

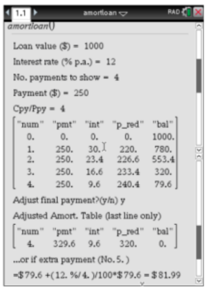
6.15 Using “amortloan” template

Creating an amortisation table with “amortloan”

On the TI-Nspire CAS

Program output from "amortloan"

- Open the TI-Nspire file **"amortloan.tns"**
- Press [var] and select **amortloan()** to run the program
- Enter the following values when requested.
  - Loan value ($) = **1000**
  - Interest rate (%p.a.) = **12**
  - No. of payments to show = **4**
  - Payments ($) = **250**
  - Cpy/Ppy = **4**
  - Adjust final payment (y/n)? **y**



Observe that the program has created a table, and provided two options for fully amortising the loan (if relevant to the scenario).

- An *adjusted* (increased) 4th payment, **or**
- An *extra* (5th) payment

What is "principal reduction"?

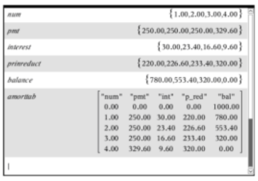
There is a column in the table that needs further explanation. The **"p_red"** column refers to "*principal reduction*", which is the amount by which the original loan value (or principal) has been reduced in each quarter. As you can see from the table, the principal reduction is calculated as the difference between the repayment and the value of the interest in that quarter. In general, for each time period:

principal reduction = repayment – interest

Further analysis of the amortisation table

Once the program has been run, each column of values in the amortisation table is stored for further analysis. Press **VAR** to access them at any time. Here's what each variable represents.

- **num** number of loan balances over loan period
- **pmt** fixed repayment
- **interest** interest payment over the loan period
- **prinreduct** principal reduction amount over loan period
- **balance** loan balance over the loan period
- **amorttab** Final amortisation table (including adjusted final payment if relevant)



The screen shows the output when [var] is pressed, and each of the variables is selected.
[Note: If you select **"amortloan"** it will just relaunch the program]

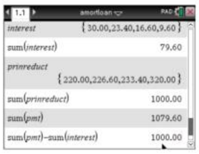
If you have not yet done so, type **"amortloan()"** to run the program. Enter the values as per the example on page 2, and type 'y' to ask the program to adjust the final payment for full amortisation in 4 payments. Then use the [var] key to verify that you get the same values as shown on the screen above right.

Now answer the following questions.

Question 3.

Type **sum(interest)** and then [enter]. What was the result, and what does it represent?

The total interest paid over the life of the loan ($79.60).



Question 4.

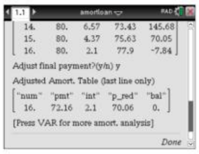
Type **sum(prinreduct)** and then [enter]. What was the result, and what does it represent?

The total amount that the principal has been reduced over the life of the loan (from $1000 to $0).

Question 5.

Type **sum(pmt)** and then [enter]. What was the result, and what does it represent?

The total value of payments made over the life of the loan ($1079.60).
Note that it will always be true by definition that
sum(prinreduct)+sum(interest) = sum(pmt)



Question 6.

Use the program to find out the number of payments required to amortise the above loan if the payment was only $80/quarter.

It will take 16 payments, or 4 years. The final payment will be slightly less than $80 ($72.16) *[Note that when using the program, if you enter a larger number of payments than required, it will ignore extra payments if it is able to amortise using fewer payments.]*

Footnote: the 'amortTbl' command

Finally, there is a built in function for generating amortisation tables as a matrix. It returns a matrix with columns in this order: Payment number, amount paid to interest, amount paid to principal, and balance. Its syntax is as follows:

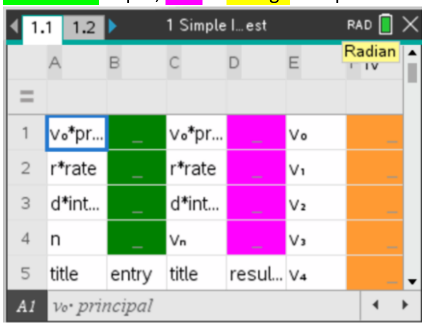
amortTbl(NPmt,N,I,PV,Pmt,FV,PpY,CpY,PmtAt,roundValue)

For example, the command to generate the amortisation table for Question 11 is as follows (see calculator manual for more detail!)

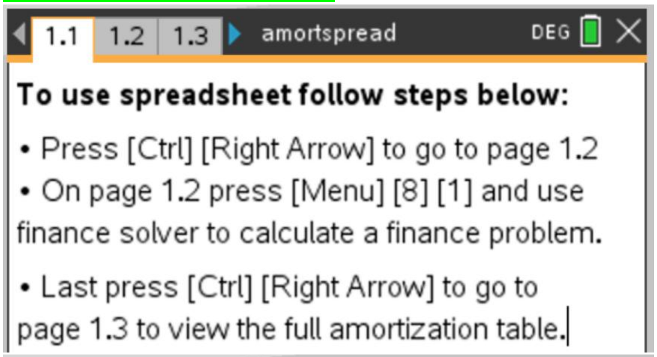
amortTbl(240,240,4.75,300000, –1939, 0, 12, 12, 0, 2)

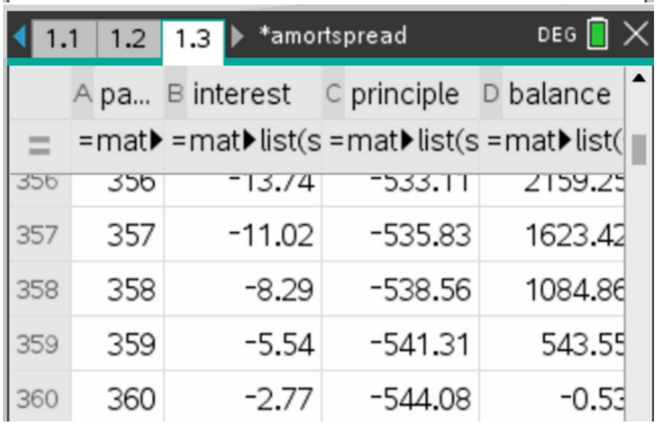
6.16 Using" 1-8 financial products" template

Green area=input; Pink & Orange= output



6.17 Using" amortspread" template





6.18 Using" Effective Interest Rate" template

Can compare 3 sets of nominal interest rates