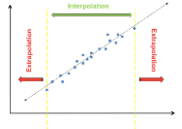
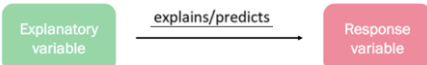
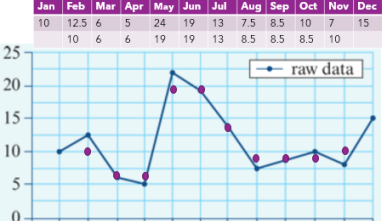
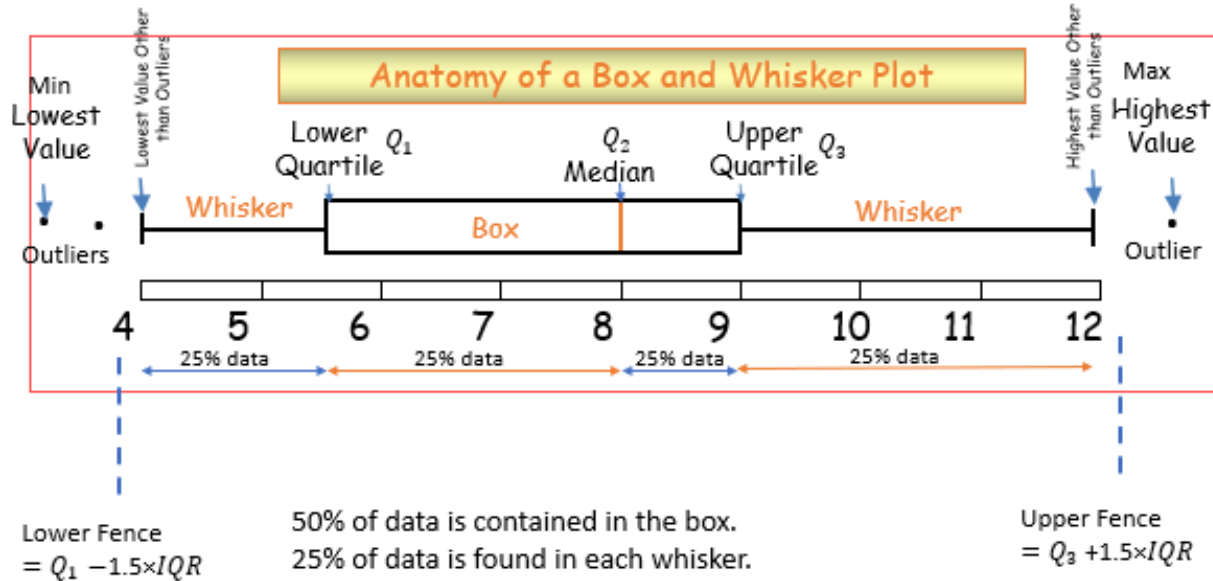


Topics	Data Types 1A		Display/Analyse Tools		Report/Explain/Interpret/Describe		
Univariate Data	Categorical variables 1B	Nominal data Ordinal data	Bar chart, Pie Chart, Frequency Table 1a, Segmented bar chart		Mode/ Modal Value Frequency types, Frequency % = $\frac{Count}{Total\ Count} \times 100\%$		
Chapter 2	Numerical variables 1C	Discrete data	Boxplots 1c 1.2, Grouped Frequency Tables	Stem plot 1b, dot plot 1b 1.2	Shape → Centre → Spread → Outliers → 1F	Symmetric  Mean $\bar{x}$ 1.1 Standard Deviation S 1G 1.1  68-95-99.7% rule 1H 1I $Z\text{-score}=Z=\frac{x-\bar{x}}{S}$ 1I $x = \bar{x} + Z * S$	Skewed  1.1 Median M or Q_2 IQR, Range { Lower Fence = $Q_1 - 1.5 * IQR$ Upper Fence = $Q_3 + 1.5 * IQR$ 1.1 5-figure summary: Min, Q_1, Q_2, Q_3 , Max IQR= $Q_3 - Q_1$, Range=Max–Min
		Continuous data		Stem plot 1b, histogram 1b 1.1, log 1.2 log-histogram 1.2			
Bivariate Data	Two categorical variables 2A		Segmented bar chart, two-way frequency table 2a, parallel bar chart 2.1		Mode/ Modal Value Frequency types		
Chapter 2	One categorical, one numerical variable 2B		Back-to-back stem plots, parallel dot plots, parallel box plots 2b 2.2		Shape → Centre → Spread →	Symmetric  1.1 Mean $\bar{x}$ Standard Deviation S	Skewed  1.1 Median M or Q_2 IQR, Range
	Two numerical variables 2C  Interpolation Extrapolation The assumptions for fitting a least squares line 1. the data is numerical 2. the association is linear 3. there are no clear outliers.		Scatterplot 2c 3.2  residual = actual data value y – predicted 3.3 value y residual = $y - \hat{y}$ Nil pattern residual 3.2 plot 2f = Linear relation Curved/ patterned residual plot ≠ linear relation		Strength → Direction → Form → 3A 3B 3C 2e LSRL $y=a+bx$ 3.1 2e slope $b=\frac{r s_y}{s_x}$ 2e y-intercept $a = \bar{y} - b\bar{x}$	Strong/Moderate/Weak (Check r value) 2D 3.1 Positive / Negative  Linear / Non-linear  Reporting 2d on Coefficient of Determination r^2 3.1 Almost [r^2 in %] of [RV y] can be explained / predicted by [EV x].	
Time Series 4A 4E: LSRL	Features 4A	Moving smoothing 4.1			Seasonal Index S.I. 4D	Deseasonalising 4D	
	Trend  Cycles  Seasonality Structure change Outliers 	Moving Mean 4B 4.1 3/5 moving mean 	2/4 Moving mean 	Moving Median 4C 	$S.I.=\frac{Value\ for\ Season}{Yearly\ Average}$ $Yearly\ Average=\frac{Sum\ of\ Season\ Values}{No.of\ season\ per\ year}$ Correct S.I. = $(\frac{1}{S.I.} - 1) \times 100$ + means ↑, – means ↓	Deseasonalised Figure = $\frac{Actual\ Figure}{S.I.}$ = Actual Figure * $\frac{1}{S.I.}$ Actual figure= Deseasonalised Figure * S.I.	

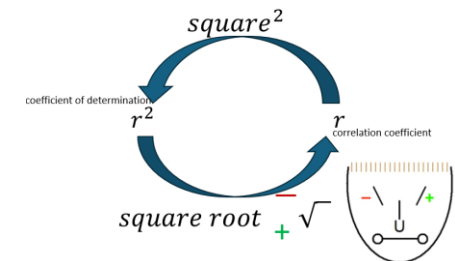
Note: Textbook Summary Notes Section #, Report Instruction Notes #, CAS Instruction Notes #

Box and Whisker Plots

Box plots are graphical representations of 5 number summary.



$0.75 \leq r \leq 1$
Strong, positive, linear association
$0.5 \leq r < 0.75$
Moderate, positive, linear association
$0.25 \leq r < 0.5$
Weak, positive, linear association
$-0.25 < r < 0.25$
No association
$-0.5 < r \leq -0.25$
Weak, negative, linear association
$-0.75 < r \leq -0.5$
Moderate, negative, linear association
$-1 \leq r \leq -0.75$
Strong, negative, linear association



Name of Variation	Symbol of Variation	Equation of Variation	CAS	
			Slope/Equation	Plot/Equation
Direct Variation	$y \propto x$	$y = kx$	3.1	$y = kx + c$
Inverse Variation	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$	3.1	$y = \frac{k}{x} + c$
Logarithm Variation	$y \propto \log x$	$y = k \log x$	3.1	$y = k \log x + c$
Variation involving powers	$y \propto x^2$	$y = kx^2$	3.1	$y = kx^2 + c$
Other variations	$y \propto \sqrt{x}$	$y = k\sqrt{x}$	3.1	
	$y \propto x^3$	$y = kx^3$		
	$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$		

1A: Types of data

Categorical: characteristics/qualities

- Nominal: grouped according to characteristics
- Ordinal: can be grouped and ordered

Numerical: numbers/quantities

- Discrete: whole numbers, can be counted
- Continuous: is measured

1B: displaying categorical data

Count frequency: number of times the category appears in the data

Percentage frequency: $\frac{\text{count frequency}}{\text{total count}} \times 100$

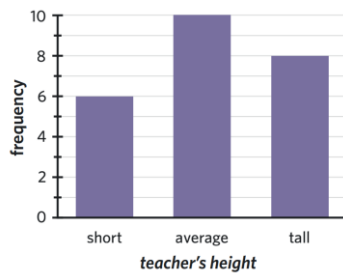
Mode: most frequently occurring value or category

Frequency Table:

teacher's height	frequency	
	number	%
short	6	25.0
average	10	41.7
tall	8	33.3
total	24	100.0

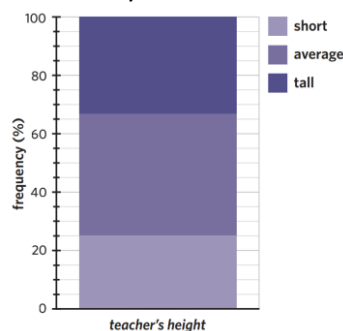
Bar chart:

- Must have gaps between bars



Segmented bar chart:

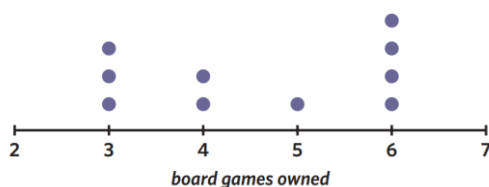
- Can be count or percentage frequency
- Must have a key



1C: Displaying Numerical data

Dot plot

- Discrete data
- Small data sets



Stem and leaf plot

- Needs a key
- Can have class intervals (splitting the stem in two if it is really large)

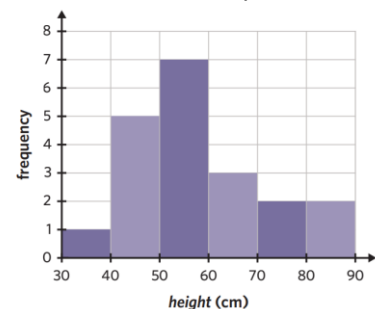
No intervals		With class intervals of 5	
Key: 1 2 = 1.2		Key: 1 2 = 12	
1	3 3 4 6 8	0	1 1 2 3 4
2	0 4 9	0	5 6 6 8 8 9
3	1 1 1 4 5 8	1	2 3 3
4	2	1	6 7 7 8 9 9

Grouped frequency tables

height (cm)	frequency	
	number	%
30-<40	1	5
40-<50	5	25
50-<60	7	35
60-<70	3	15
70-<80	2	10
80-<90	2	10
total	20	100

Histogram:

- Continuous data
- Intervals – no gaps between bars
- No gaps between bars
- X-axis markers are always a whole number

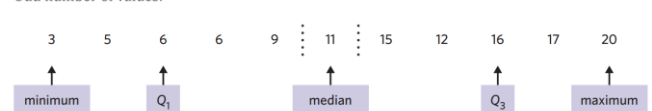


1E: the five-number summary and boxplots

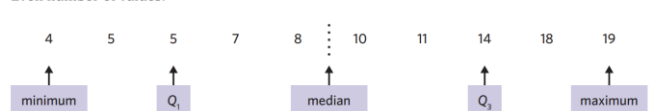
5 number summary:

- Minimum: smallest value in data set
- Q1: median of the lower half
- Median: middle value in an ordered data set
- Q3: median of the upper half
- Maximum: largest value in data set

Odd number of values:



Even number of values:



Spread: refers to how variable the data set is

Range = maximum – minimum

Interquartile range: measure of spread of the middle 50% of a data set. Accurate measure of spread when outliers are present.

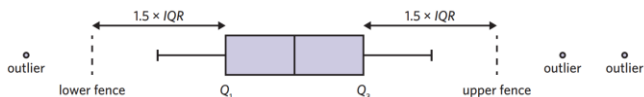
$$IQR = Q_3 - Q_1$$

Outliers: values which fall outside of what is 'normal'. Outliers are still the minimum and maximum value!

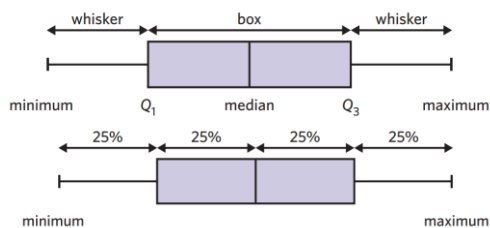
Fence: defines the boundary of what is an outlier. If a value is less than the lower fence or greater than the upper fence it is considered to be an outlier.

$$\text{lower fence} = Q_1 - (1.5 \times IQR)$$

$$\text{upper fence} = Q_3 + (1.5 \times IQR)$$

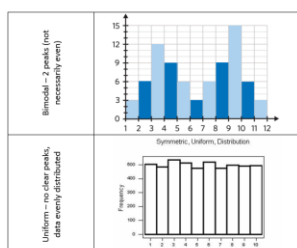
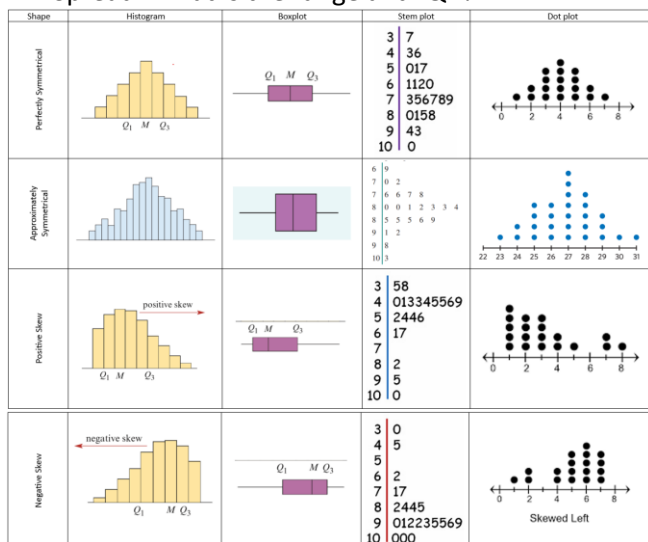


Boxplots:



1F: describing numerical data

- Shape: is the data symmetrical, skewed or have any outliers?
- Centre: What is the median value?
- Spread: What is the range and IQR?

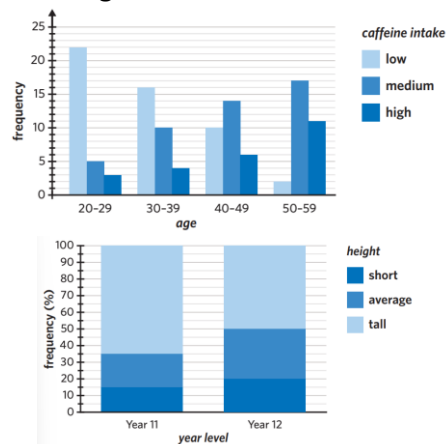


2A: association between 2 variables

Two-way frequency table:

- Columns = EV, Rows = RV
- Percentage frequency is used for greater accuracy when making comparisons if sample sizes are different

Grouped and segmented bar charts:



Describing the association between two variables:

- Whether or not an association between the two variables exists
- Appropriate percentages to support findings

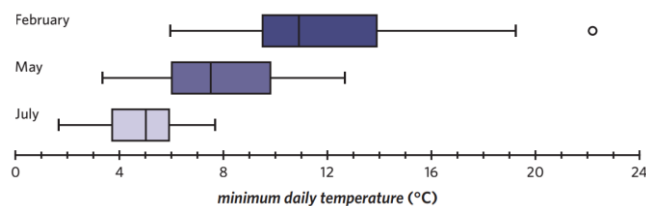
2B: association between numerical and categorical variables

- Back to back stem plot

Key: 3 | 7 = 37 points

Crocodiles	Zebras
4 0	2
8 7 5 2	3 7
7 4 4	4
8 2	5 2 8
3 1	6 0 1 5 9
	7 0 3 5 8
0	8 1 3 4

- Parallel boxplot



- Making comparisons: refer to 1F and compare shape, centre and spread of the two categories

2C: association between two numerical variables

Response variable: RV, may be explained or predicted by changes in the explanatory variable.

Explanatory variable: EV, used to explain or predict the changes observed in the response variable.

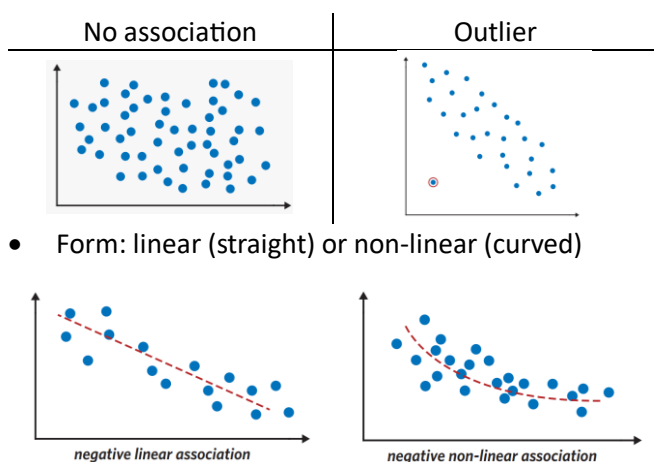
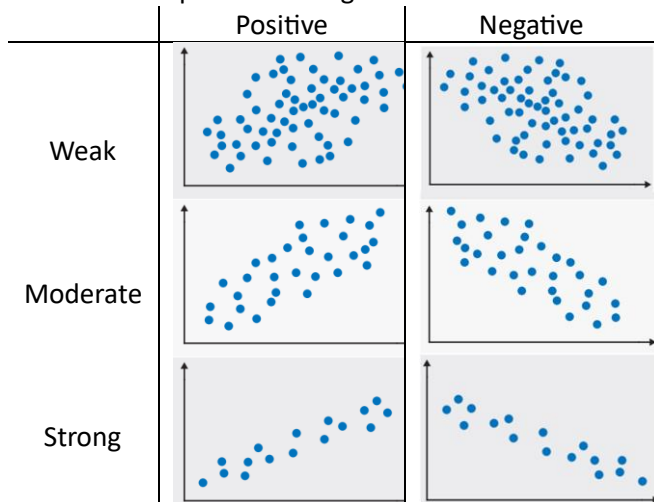
- 'EV explains the RV'

Scatterplots:

- EV = x axis, RV = y axis

Describing relationship/analysing scatterplots:

- Strength: how close the data points are together
- Direction: positive or negative



2D: Correlation and causation

Pearson's correlation coefficient (r): numerical value that determines strength and direction between two numerical variables, assuming:

- Data is linear
- Data is numeric
- No outliers present

$0.75 \leq r \leq 1$	Strong, positive, linear association
$0.5 \leq r < 0.75$	Moderate, positive, linear association
$0.25 \leq r < 0.5$	Weak, positive, linear association
$-0.25 < r < 0.25$	No association
$-0.5 < r \leq -0.25$	Weak, negative, linear association
$-0.75 < r \leq -0.5$	Moderate, negative, linear association
$-1 \leq r \leq -0.75$	Strong, negative, linear association

3A: fitting a least squares regression line

Least squares regression line (LSRL): is the line which creates the minimum sum of the squares of residuals. There are assumptions:

- Data is numerical

- The relationship between variables is linear
- There are no clear outliers present

The line is used to show the general trend in the data and is given by the equation:

$$y = a + bx$$

Intercept Slope

Determining LSRL from a graph: Find the intercept (a) and the slope (b).

- Intercept: read directly from the graph when the EV is 0
- Slope: choose two points on the line that you can clearly read the coordinates. Use the rule:

$$b = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Drawing the LSRL on a graph: Sub in the first value on the x-axis and the last value on the x-axis into the equation. Plot the two points, join the line using a ruler.

3B: Interpreting LSRL: use the following statements, fill in EV and RV and values of a and b.

y-intercept: when the EV is 0, the RV is a.

Slope: for every one-unit increase in the EV, the RV increases/decreases by b. (If b is positive, increases, if b is negative, decreases)

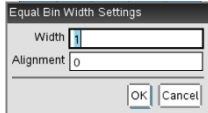
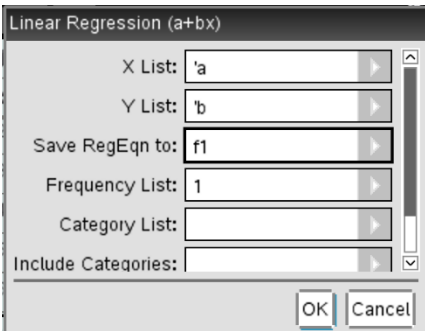
Making predictions: the LSRL can be used to predict the value of the RV from the EV.

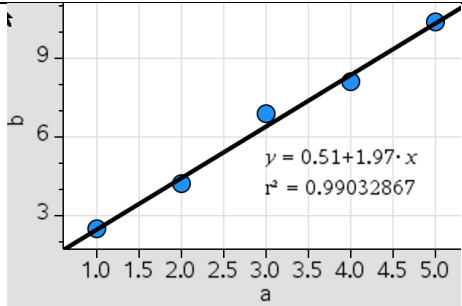
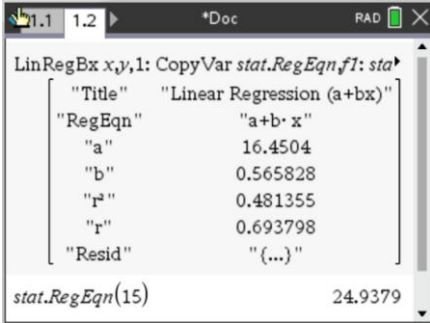
Interpolation: predicting within the range of data.

Extrapolation: predicting outside the range of data. Less reliable.

How to predict: sub the EV value that you are predicting for into the LSRL equation to predict the RV.

CAS reference sheet: Data

1.1 Univariate Data Statistics: to find the 5 number summary, mean and standard deviation	- Ctrl n > 4, Enter data in 'list and spreadsheets' (ensure a title) - Menu > 4 > 1 > 1 OR ctrl i > 1 > menu > 6 > 1 > 1 in a new page - Select the title you gave the data in the 'X-List' <table border="1" data-bbox="432 344 719 607"> <tr><td>MinX</td><td>1.</td></tr> <tr><td>Q₁X</td><td>1.5</td></tr> <tr><td>MedianX...</td><td>3.</td></tr> <tr><td>Q₃X</td><td>4.5</td></tr> <tr><td>MaxX</td><td>5.</td></tr> </table> <table border="1" data-bbox="1129 344 1449 607"> <tr><td>$\bar{x}$</td><td>3.</td></tr> <tr><td>Σx</td><td>15.</td></tr> <tr><td>Σx^2</td><td>55.</td></tr> <tr><td>$s_x := s_n - \dots$</td><td>1.58113...</td></tr> </table>	MinX	1.	Q ₁ X	1.5	MedianX...	3.	Q ₃ X	4.5	MaxX	5.	$\bar{x}$	3.	Σx	15.	Σx^2	55.	$s_x := s_n - \dots$	1.58113...
MinX	1.																		
Q ₁ X	1.5																		
MedianX...	3.																		
Q ₃ X	4.5																		
MaxX	5.																		
$\bar{x}$	3.																		
Σx	15.																		
Σx^2	55.																		
$s_x := s_n - \dots$	1.58113...																		
1.2 Graphing Univariate Data: dot plot, boxplots, histograms	- Ctrl n > 4, Enter data in 'list and spreadsheets' (ensure a title) - Ctrl doc OR ctrl i OR home menu > 5 to open a new page and select 'data and statistics' - Click to add data to the x-axis - Menu > 1: choose the plot type you are after Histogram settings: - Menu > 2 > 2 > 2 > 1 - Width is where you put your interval size. Alignment is where the graph will start on the x-axis - Zoom: menu > 5 > 2 - Frequency or Percent: menu > 2 > 2 > 1 																		
2.1 Graphing Bivariate Data: Grouped Bar Chart	- Create first Bar Chart - Menu > 2 > 9 > select second set of data																		
2.2 Graphing Bivariate Data: parallel boxplots	- Create first boxplot - Menu > 2 > 5 > select second set of data																		
3.1 Bivariate Data Statistics: a, b, r, r ²	- Ctrl n > 4, Enter data in 'list and spreadsheets' (ensure you give both columns a title) - Menu > 4 > 1 > 4 (Spreadsheets) OR ctrl i > 1 > Menu > 6 > 1 > 4 (Calculator) - Select your EV in the 'X-List' and your RV in the 'Y-List'  <table border="1" data-bbox="879 1368 1230 1697"> <tr><th>RegEqn</th><th>a+b*x</th></tr> <tr><td>a</td><td>0.51</td></tr> <tr><td>b</td><td>1.97</td></tr> <tr><td>r²</td><td>0.99032...</td></tr> <tr><td>r</td><td>0.99515...</td></tr> </table>	RegEqn	a+b*x	a	0.51	b	1.97	r ²	0.99032...	r	0.99515...								
RegEqn	a+b*x																		
a	0.51																		
b	1.97																		
r ²	0.99032...																		
r	0.99515...																		
3.2 Scatterplots	- Ctrl n > 4, Enter data in 'list and spreadsheets' (ensure you give both columns a title) - Ctrl doc OR ctrl i OR home menu > 5 to open a new page and select 'data and statistics' - Click to add the EV to the x-axis and the RV to the y-axis Scatterplot Least Squares Regression Line: - Menu 4 > 6 > 2 OR ctrl i > 1 > Menu > 6 > 1 > 4 (Calculator)																		

	
3.3 Prediction by LSRL	- Above 3.2 step1-3 to entre EV and RV. - ctrl i > 1 > Menu > 6 > 1 > 4 (Calculator) - var > stat.regeqn > x value ↵ 
4.2 Solve	- Calculator Page - Menu > 3 > 1 - You write inside the brackets, there must be an equals sign (=), you must choose a variable (x) and then use the same variable at the end and do 'comma variable' (,x) solve(10=x+2,x) x=8.
4.3 Back to top of List	Ctrl > 7

1a. Univariate Categorical Data: Frequency Table

The [types of categories] of [total frequency] [frequency type] were classified as [list of categories].

Modal Category

The majority of [frequency type], [modal percentage], were found to be [modal category].

Of the remaining [frequency types], [percentage X] were found to be [category X], while [percentage Y] were found to be [category Y], and while [etc.].

Equal Categories

The [frequency types] all had roughly the same percentages where [category X] had [percentage X], [category Y] had [percentage Y], [etc.].

1b. Univariate Numerical Data: Histogram, Dot Plot, Stem Plot

The shape of the distribution is [symmetric/positively skewed/negatively skewed]

Refer to 1F: Describing numerical data

The distribution has a [standard dev./range/IQR] of [value]

The distribution has a [mean/median/mode] of [value]

The distribution [has #/has no] outliers.

1c. Univariate Numerical Data: Box Plot

The distribution is [positively skewed/negatively skewed] with [outliers/no outliers]. The distribution is centred at [value], the median value.

The spread of the distribution, is measured by the IQR, is [value] and, as measured by the range [value]. If outliers present: There are [value] many outliers: [list of outliers]

2a. Bivariate Data (Both Categorical): Two-way Frequency Table

Worked example: Is there an association between interest in sports and age group?

Yes, the percentage of males with a high level of interest in sport steadily decreases with age group from 56.5 % for the 'under 18 years' age group, to 35.0% for the '36-50 years' age group.



Interest in sport	Age group (%)			
	Under 18 years	19-25 years	26-35 years	36-50 years
High	56.5	50.2	40.7	35.0
Medium	30.1	34.4	36.8	43.8
Low	13.4	13.4	22.5	20.3
Total	100.0	100.0	100.0	100.0

2b. Bivariate Data (one categorical, One Numerical): Comparing two boxplots:

The distributions at [variable name] are [symmetric/positively skewed/negatively skewed] for both [boxplot variables]. There [are/are no] outliers. The median [variable name] is higher for [boxplot 1], (M= value), than [boxplot 2], (M= value). The IQR is also greater for [boxplot 1], (IQR= value), than [boxplot 2], (IQR= value). The range of [variable name] is also greater for [boxplot 1], (R= value), than [boxplot 2], (R= value). Yes, there is an association between [variable name1] [variable name 2] as their median change from (M= value) to (M= value).

2c. Bivariate Data (Both Numerical): Scatter Plot

There is a [strong/moderate/weak], [positive/negative], [linear/non-linear] relationship between [response variable y] and [explanatory variable x]. There [are/are no] clear outliers.

2d. The coefficient of determination (r^2):

The coefficient of determination indicates that [$r^2 \times 100$] % of the variation in [response variable] is explained by the variation in [explanatory variable] and [remaining %] is explained by other factors and not explained by [explanatory variable].

2e. Least squares line:

The equation of the regression line is: [response variable] = [a] + [b] x [explanatory variable]

Slope (b):

On average, [response variable] [increases/decreases] by [b units] for every one [unit] increase in [explanatory variable].

y- intercept (a):

When [explanatory variable] is 0, [response variable] is predicted to be [a units].

2f. Residual Plot

The residual plot shows a [random scatter/ curved pattern] indicating there is a [linear/non- linear] relationship between [response variable] and [explanatory variable].

2g. Prediction Reliability

The prediction using [explanatory variable] of [value of EV] is [reliable/ unreliable] as it is [within/ outside] the data range and is therefore [interpolation/extrapolation].