Chapter 4 Matrices Summary

NAME	DESCRIPTION	EXAMPLE	Mathematica
Row matrix	A matrix with only 1 row	$\begin{bmatrix}3 & 2 & 1-4\end{bmatrix}$	Nil
Column matrix	A matrix with only I column	$\begin{bmatrix}2\\3\end{bmatrix}$	Nil
Square matrix	the number of rows equals the number of columns	$\begin{bmatrix}5 & 4\\4 & 2\end{bmatrix}$ 2×2	Nil
Zero (Null) matrix	A matrix with all zero entries	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$	Nil
Transpose of a matrix	a new matrix that is formed by interchanging the rows and columns.	$A = \begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\\a_{31} & a_{32}\end{bmatrix}$ $\mathbf{A}^T = \begin{bmatrix}a_{11} & a_{21} & a_{31}\\a_{12} & a_{22} & a_{32}\end{bmatrix}$	2d
Summing matrix	A row or column matrix in which all the elements are 1. To sum the rows of an m × n matrix, post-multiply the matrix by an n × 1 Column summing matrix. To sum the columns of an m × n matrix, pre-multiply the matrix by a 1 × m row summing matrix.	$\begin{bmatrix}1\\1\\1\end{bmatrix}$ $\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 1\end{bmatrix}$ $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$	3a Or 8 Step by step

NAME	DESCRIPTION	EXAMPLE	Mathematica
Symmetric matrices	A matrix **A** is called symmetric if $\mathbf{A}^T = \mathbf{A}$	$\begin{bmatrix}2 & 3\\3 & 1\end{bmatrix}$ $\begin{bmatrix}2 & 3 & 4\\3 & 1 & 5\\4 & 5 & 3\end{bmatrix}$ $\begin{bmatrix}1 & 2 & 4 & 6\\2 & 1 & 5 & 7\\4 & 5 & 3 & 8\\6 & 7 & 8 & 5\end{bmatrix}$	2d
Diagonal matrices	if all of the elements off the leading diagonal are zero.	$\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}2 & 0 & 0\\0 & 1 & 0\\0 & 0 & 3\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 6 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 6\end{bmatrix}$	Nil
Identity matrices	This is denoted by the letter **I** and has zero entries except for 1's on the diagonal.	$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $I = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $I = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$	4a
Inverse matrices	A square matrix A has an inverse if there is a matrix A^{-1} such that: $AA^{-1} = I$	If $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then its inverse, A^{-1}, is given by $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix} = \frac{1}{\det(A)}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$ provided $\frac{1}{ad-bc} \neq 0$; that is, provided $\det(A) \neq 0$.	4b inverse 4c Complicated Inverse 4d Determinant
Binary matrices	A special kind of matrix that has only 1s and zeros as its elements.	$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ $\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{bmatrix}$	Nil

THE INVERSE OF A MATRIX

THIS SECTION FURTHER DEVELOPS THE ALGEBRA OF MATRICES. TO EGIN, CONSIDER THE REAL NUMBER EQUATION

$AX = B.$

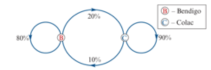
TO SOLVE THIS EQUATION FOR X, MULTIPLY EACH SIDE OF THE QUATION BY A^{-1} (PROVIDED THAT $A \neq 0$).

$AX = B$

$(A^{-1}A)X = A^{-1}B$

$(I)X = A^{-1}B$

$X = A^{-1}B$

NAME & Example	DESCRIPTION	EXAMPLE & key Points	Mathematica
Communication matrices Receiver: E W Y K; Speaker: E, W, Y, K $\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 1 & 1\\0 & 1 & 0 & 0\\0 & 1 & 0 & 0\end{bmatrix}$	A square binary matrix in which the 1s represent the links in a communication system.	All of the non-zero elements in the leading diagonal of a communication matrix, or its powers, represent redundant links in the matrix.	4b two way= power 2
Transition matrices	Used to describe the way in which transitions are made between two states. Recurrence Relation: S_0 = initial value, $S_{n+1} = T * S_n$ Explicit Rule: $S_n = T^n * S_0$ Steady State: determine values for a long run $S = T^{50} * S_0 = T^{51} * S_0$	 Rented in: Bendigo, Colac; Returned to: Bendigo, Colac $\begin{bmatrix}0.8 & 0.1\\0.2 & 0.9\end{bmatrix}$	7

Matrices Operations	Mathematica Commands	Matrices Operations	Mathematica Commands
Insert matrix	1	Power of a Matrix	3b
Adding, subtracting, scalar multiplication	2	Simultaneous Equations/ Matrices	5a or 5b 5c Matrices → Equations 5d Equations → Matrices
Two matrices multiplication	3a or 8	Solving unknow Matrix by given matrix equation	6

Matrix Multiplication

Multiplying Matrices together

→ Matrices can only be multiplied if the number of columns in the first is the same as the number of rows in the second.

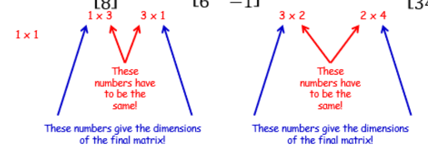
These must match. $m \times n \cdot n \times p$ The answer is size $m \times p$.

$$\begin{bmatrix}6 & 5 & -2\end{bmatrix}\begin{bmatrix}5\\5\\8\end{bmatrix} = \begin{bmatrix}39\end{bmatrix}$$

1 × 3, 3 × 1, 1 × 1

$$\begin{bmatrix}3 & 2\\2 & 5\\6 & -1\end{bmatrix}\begin{bmatrix}6 & 1 & 5 & -4\\2 & 3 & 7 & 3\end{bmatrix} = \begin{bmatrix}22 & 9 & 29 & -6\\22 & 17 & 45 & 7\\34 & 3 & 23 & -27\end{bmatrix}$$

3 × 2, 2 × 4, 3 × 4



Multiplying a Matrix by Another Matrix

"Dot Product"

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} \times \begin{bmatrix}7 & 8\\9 & 10\\11 & 12\end{bmatrix} = \begin{bmatrix}58 & \end{bmatrix}$$

The "Dot Product" is where we **multiply matching members**, then sum up:

(1, 2, 3) • (7, 9, 11) = 1×7 + 2×9 + 3×11 = 58

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} \times \begin{bmatrix}7 & 8\\9 & 10\\11 & 12\end{bmatrix} = \begin{bmatrix}58 & 64\end{bmatrix}$$

(1, 2, 3) • (8, 10, 12) = 1×8 + 2×10 + 3×12 = 64

(4, 5, 6) • (7, 9, 11) = 4×7 + 5×9 + 6×11 = 139

(4, 5, 6) • (8, 10, 12) = 4×8 + 5×10 + 6×12 = 154

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} \times \begin{bmatrix}7 & 8\\9 & 10\\11 & 12\end{bmatrix} = \begin{bmatrix}58 & 64\\139 & 154\end{bmatrix} ✓$$

Create a matrix equation

$3a - 5b + 2c = 9$

$4a + 7b + c = 3$

$2a \quad - c = 12$

$$\begin{bmatrix}3 & -5 & 2\\4 & 7 & 1\\2 & 0 & -1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}9\\3\\12\end{bmatrix}$$

THE INVERSE OF A MATRIX

- THE NUMBER A^{-1} IS CALLED THE *MULTIPLICATIVE INVERSE* OF A BECAUSE
- $A^{-1} A = I.$
- THE DEFINITION OF THE MULTIPLICATIVE **INVERSE OF A MATRIX** IS SIMILAR.

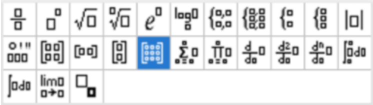
Definition of the Inverse of a Square Matrix

Let A be an $n \times n$ matrix and let I_n be the $n \times n$ identity matrix. If there exists a matrix A^{-1} such that

$AA^{-1} = I_n = A^{-1}A$

then A^{-1} is called the **inverse** of A. The symbol A^{-1} is read "A inverse."

CAS reference sheet: Matrices

7.1 Enter a Matrix	1. Ctrl N > 1 > |□| {□ ↵ 2. ***OR*** Ctrl N > 1 > Ctrl > Menu > 8 ↵  3. Select No. of rows & columns ↵ 4. Type each element, using **[tab]** to move.
7.2 Define a Matrix	1. Name of the matrix > ctrl > |□| {□ > enter full matrix ↵ $a:=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$ $\begin{bmatrix}1. & 2. & 3.\\4. & 5. & 6.\\7. & 8. & 9.\end{bmatrix}$ 2. ***OR*** Enter full matrix > ctrl var > name of the matrix↵ $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}\rightarrow b$ $\begin{bmatrix}1. & 2. & 3.\\4. & 5. & 6.\\7. & 8. & 9.\end{bmatrix}$
7.3 Transpose Matrix	1. Name of the matrix > Menu > 7> 2↵ a^{T} $\begin{bmatrix}1. & 4. & 7.\\2. & 5. & 8.\\3. & 6. & 9.\end{bmatrix}$
7.4 Determinant	1. Menu > 7 > 3> name of the matrix↵ $\det(a)$ 0.
7.5 Inverse	1. Name of the matrix > Λ > **Negation key (-) > 1** ↵ $c:=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ $\begin{bmatrix}1. & 2.\\3. & 4.\end{bmatrix}$ c^{-1} $\begin{bmatrix}-2. & 1.\\1.5 & -0.5\end{bmatrix}$ 2. Decimal → fraction $\text{exact}(c^{-1})$ $\begin{bmatrix}-2 & 1\\\frac{3}{2} & \frac{-1}{2}\end{bmatrix}$
7.6 Forming matrix by i, j rule	1. Menu > 7 > 1 >A > ij rule, i, j, row size, column size ↵ $\text{constructMat}(2\cdot i+3\cdot j,i,j,2,3)$ $\begin{bmatrix}5. & 8. & 11.\\7. & 10. & 13.\end{bmatrix}$