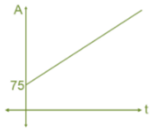
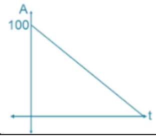
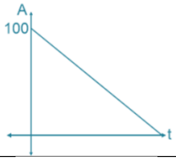
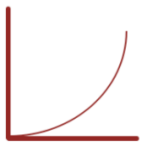
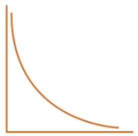
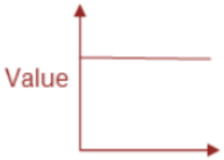
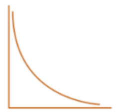
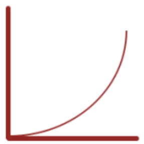
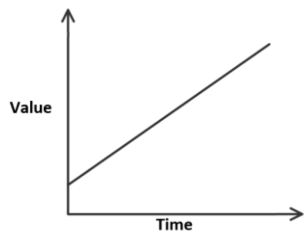
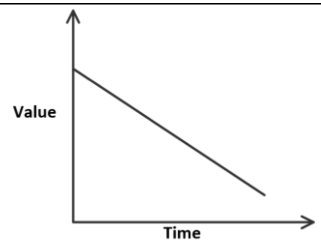
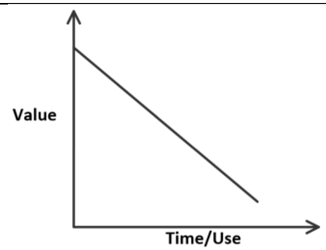
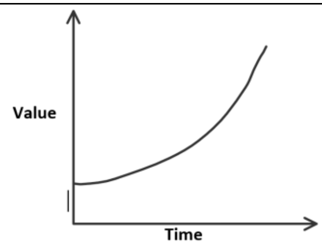
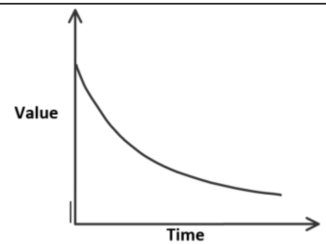
Financial Math

6.2 Amortisation FV↓ 5.3 Effective Interest Rate

	Recurrence Relation	Application	Explicit Rules	Graphs	CAS Notes #
Arithmetic Sequences D Common difference	$v_0 = principal, v_{n+1} = v_n + D$ $D = \frac{r}{100*p} \times v_0$	Simple Interest	$v_n = v_0 + n * D$		5.2 Interest in \$ 5.1 Sequence list 5.2 Future Value
	$v_0 = Initial\ Value, v_{n+1} = v_n - D$ $D = \frac{r}{100} \times v_0$	Flat Rate Depreciation	$v_n = v_0 - n * D$		5.2 depreciate \$ 5.1 Sequence list 5.2 Future Value
	$v_0 = Initial\ Value, v_{n+1} = v_n - D$ D = $Unit\ cost\ in\ dollars$	Unit Cost Depreciation	$v_n = v_0 - n * D$		5.1 Sequence list 5.2 Future Value
Geometric Sequences R Common Ratio Growth Factor Decay factor	$v_0 = principal, v_{n+1} = R * v_n$ $R = 1 + \frac{r}{100*p}$, $r_a = (R-1) * 100 * p$	Compound Interest	$v_n = R^n * v_0$		5.2 Common Ratio 5.1 Sequence list 5.2 Future Value 5.2 Find r 5.2 Find n
	$v_0 = Initial\ Value, v_{n+1} = R * v_n$ $R = 1 - \frac{r}{100}$, $r = (1-R) * 100$	Reduced Balance Depreciation	$v_n = R^n * v_0$		5.2 Common Ratio 5.1 Sequence list 5.2 Future Value 5.2 Find r 5.2 Find n
Combined Arithmetic & Geometric Sequences R Growth Factor D Repayment	$v_0 = Initial\ Value, v_{n+1} = R * v_n - D$ $R = 1 + \frac{r}{100*p}$, D = $\frac{r}{100*p} \times v_0$, $D = (R-1) * V_0$	FV → Interest Only Loan /Perpetuities			5.2 Common Ratio 6.1 Sequence list 5.2 Interest only 5.2 Perpetuities
	$v_0 = Initial\ Value, v_{n+1} = R * v_n - D$ $R = 1 + \frac{r}{100*p}$, $r_a = (R-1) * 100 * p$	FV↓ Reduced Balance Loans /Annuities ⬅	Compound Interest loan extra payment		5.2 Common Ratio 5.1 Sequence list 6.1 Financial Solver 6.1 Analyse Loan 6.1 Analyse Annuities 6.1 Condition change
	$v_0 = Initial\ Value, v_{n+1} = R * v_n + D$ $R = 1 + \frac{r}{100*p}$, $r_a = (R-1) * 100 * p$	FV↑ Compound Interest Investment Annuity Investment			5.2 Common Ratio 6.1 Sequence list 6.1 Analyse investment 6.1 Condition change

Note: Yellow parts need real number, blue parts are formula to calculate, Letter P indicates Compounding monthly etc. Needing rate per month, Monthly Ratio etc. Green CAS Notes

	SIMPLE INTEREST	FLAT RATE DEPRECIATION	UNIT COST DEPRECIATION	COMPOUND INTEREST	REDUCING BALANCE DEPRECIATION
Description	A constant amount of interest that is added to the principal at regular time periods. It is calculated as a percentage of the principal.	The value of an asset decreases by a constant amount each time period. This amount is a percentage of the initial value.	The value of an asset uses value after each unit of use.	Takes into account both the principal and any accumulated interest earned in previous periods. The value of the investment grows with increasing amounts each time period.	The value of an asset depreciated by a set percentage, rather than a set amount. The amount of depreciation decreases each time period.
Reccurence relation	V_0 = principal, $V_{n+1} = V_n + D$ $D = r/100 \times V_0$	V_0 = initial value of asset, $V_{n+1} = V_n - D$ $D = r/100 \times V_0$	V_0 = initial value of asset, $V_{n+1} = V_n - D$ D = the depreciation (cost) per unit of use	V_0 = principal, $V_{n+1} = RV_n$ $R = 1 + r/100 \times CP$	V_0 = principal, $V_{n+1} = RV_n$ $R = 1 - r/100 \times CP$
Rule	$V_n = V_0 + nD$ $D = r/100 \times V_0$	$V_n = V_0 - nD$ $D = r/100 \times V_0$	$V_n = V_0 - nD$ D = the depreciation (cost) per unit of use	$V_n = R^n \times V_0$ $R = 1 + r/100 \times CP$	$V_n = R^n \times V_0$ $R = 1 - r/100 \times CP$
Graph					

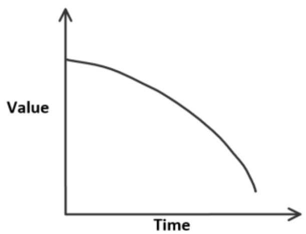
r = interest rate per annum.
CP/P = compound period. If there are no compound periods, then CP = 1
Recurrence relation: step by step process.
Rule: find values in the future in one step.

Standard compound periods:
1 = yearly
2 = biannually
4 = quarterly
6 = half yearly
12 = monthly
26 = fortnightly
52 = weekly
365 = daily

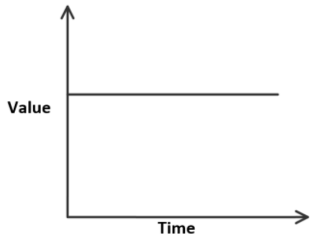
	REDUCING BALANCE LOANS	INTEREST ONLY LOANS	ANNUITY	PERPETUITY	ANNUITY INVESTMENT
Description	Has compound interest but also requires a fixed amount to be paid at regular intervals.	The borrower only repays interest that is charged. As a result, the principal must be paid back at the termination of the loan.	A form of investment, where compound interest is earned, and a fixed sum is paid to the investor periodically until the investment reaches $0.	Type of annuity that lasts indefinitely. The amount of money invested remains the same as the regular payment to the investor is only the interest earned.	An investment where interest is received, and regular payments are made. Money is not being paid out to the investor; the account is growing.
Recurrence Relation	V_0 = principal, $V_{n+1} = RV_n - D$ R = 1 + r/100 x CP D = regular payment	D = r/100 x V_0 D = payment	V_0 = principal, $V_{n+1} = RV_n - D$ R = 1 + r/100 x CP D = regular payment	D = r/100 x V_0 D = payment	V_0 = principal, $V_{n+1} = RV_n + D$ R = 1 + r/100 x CP D = regular payment

Graph

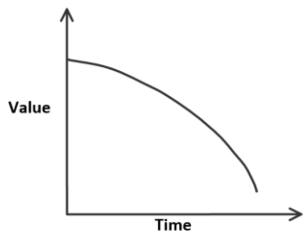
REDUCING BALANCE LOANS



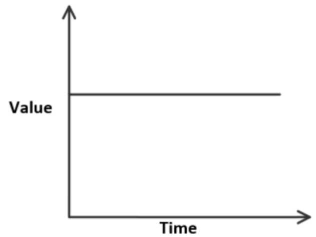
INTEREST ONLY LOANS



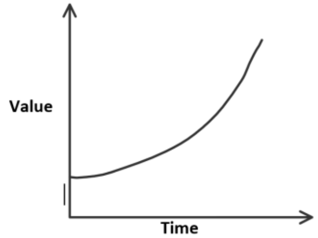
ANNUITY



PERPETUITY



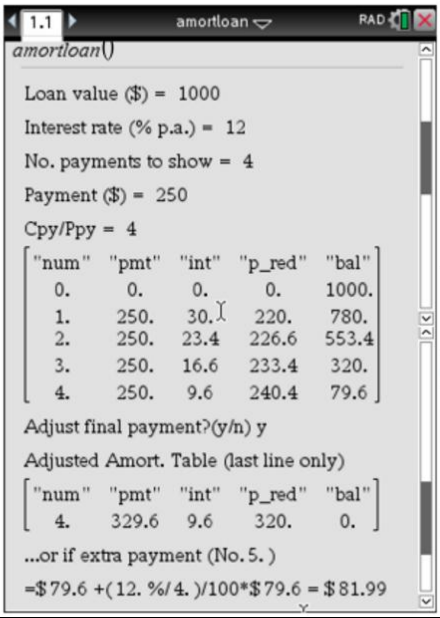
ANNUITY INVESTMENT

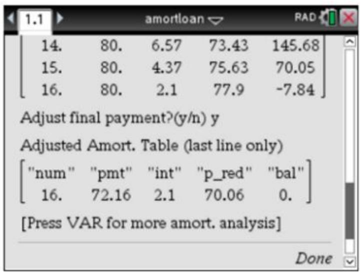


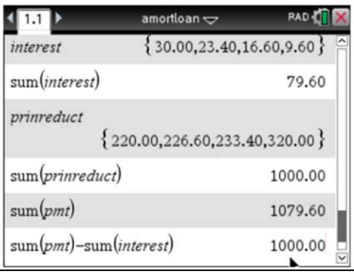
Finance Solver

	REDUCING BALANCE LOANS	INTEREST ONLY LOANS	ANNUITY	PERPETUITY	ANNUITY INVESTMENT
N		1		1	
I(%)					
PV	Positive	Positive	Negative	Negative	Negative
Pmt	Negative	Negative	Positive	Positive	Negative
FV	Positive Negative Zero	Negative value of the PV	Positive Zero	Positive value of the PV	Positive
PpY					
CpY					

CAS reference sheet: Finance

<table>
<tr><td>5.1 Using a calculator to generate a sequence of numbers from a rule</td><td>1. Ctrl N > 1 ↵
2. Enter initial data ↵
3. Enter the rule ↵ ↵ ↵ ↵ ↵ to show more terms</td></tr>
<tr><td>5.2 Solve</td><td>1. Calculator Page
2. Menu > 3 > 1 ↵
3. You write inside the brackets, there must be an equals sign (=), you must choose a variable (x) and then use the same variable at the end and do ‘comma variable’ (,x)
solve(10=x+2,x)   x=8.</td></tr>
<tr><td>5.3 Effective Interest Rate</td><td>1. Nominal Interest Rate → Effective Interest Rate
2. Menu > 8 > 5 > 2 ↵
3. eff (Nominal Interest Rate, Compounding periods)
4. Effective Interest Rate → Nominal Interest Rate
5. Menu > 8 > 5 > 1 ↵
6. nom (Effective Interest Rate, Compounding periods)</td></tr>
<tr><td>6.1 Financial Solver</td><td>1. Ctrl N > 1 > Menu > 8 > 1 ↵</td></tr>
<tr><td>6.2 Amortisation Table “Amortloan”</td><td>1. var > Amortloan()↵
Enter the following values when requested.
○ Loan value ($) = 1000
○ Interest rate (%p.a.) = 12
○ No. of payments to show = 4
○ Payments ($) = 250
○ Cpy/Ppy = 4
○ Adjust final payment (y/n)? y
2. sum(interest) ↵ to calculate total interests
3. sum(prinreduct) ↵ to calculate total principal reduction
4. sum (pmt) ↵ to calculate total payment





</td></tr>
</table>

5A Recurrence Relations & their graphs

Sequence: list of numbers written succession.

Term: each number in a sequence.

Ellipses (…) shows the sequence continues in the same fashion.

Recurrence Relation: formula that links each term in a pattern-based sequence to the next.

- There are two parts:

1) The initial value (a) t_0=a
2) The pattern

Arithro-geometric

- Combination of linear and geometric
- Multiplication and addition / subtraction
- t_0 = a , t_{n+1} = R × t_n ± D

Example: ×2+1

1. Write RR: 11, 23, 47 …

t_0 = 11 , t_{n+1} = 2 × t_n + 1

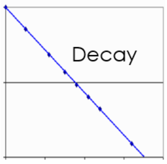
Linear growth / decay

- Increases or decreases by a constant amount.
- "common difference" = D

RR : t_0 = a , t_{n+1} = t_n ± D

first term — next term — previous term — Common difference

+ = increase / Growth
− = decrease / decay



Examples: t_0 t_1 t_2 t_3

1. Write the RR for: 30, 23, 16, 9 …

t_0=30 , t_{n+1}= t_n −7

$t_1 - t_0$: 23 − 30= −7

$t_2 - t_1$: 16 − 23= −7

2. Write the first 5 terms for:

t_0=150 , t_{n+1}= t_n +24

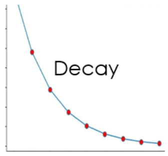
t_0	t_1	t_2	t_3	t_4
150	174	198	222	246

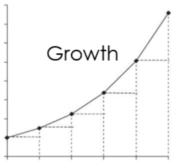
Geometric growth / decay

- Increases or decreases by a common ratio.
- "common ratio" =R

RR: t_0 = a , t_{n+1} = R × t_n

R > 1 = increase
R < 1 = decrease





Examples: : t_0 t_1 t_2

1. RR for: 100, 90, 81 …

t_0=100 , t_{n+1}= 0.9 × t_n

$\frac{t_1}{t_0}$: $\frac{90}{100}$ = 0.9

$\frac{t_2}{t_1}$: $\frac{81}{90}$ = 0.9

2. First 5 terms of: t_0=10 , t_{n+1}= 2.5 × t_n

10, 25, 62.5, 156.25, 390.63

5B – Unit cost & Flat rate depreciation

depreciation: loss of value of an asset

→ linear depreciation: flat rate + unit cost

flat rate: constant amount of depreciation per period

is a % of the principal / initial value

RR

V_0 = *principal*, V_{n+1} = V_n − D

next value — previous value

where: $D = \frac{r}{100} \times V_0$

r =depreciation rate (%) per period

to find r: r = $\frac{D}{V_0} \times 100$

unit cost: asset loses value after each use ($)

RR

V_0 = *principal*, $V_{n+1} = V_n - D$

where: D = cost per use ($)

Examples: Car costs $17,500 and depreciates at a rate of 3.8% per annum

a. write the RR

- $D = \frac{3.8}{100} \times 17500 = 665$
- $V_0 = 17500, V_{n+1} = V_n - 665$

b. using recurrence, show V_2

$V_{0+1} = V_0 - D$	$V_{1+1} = V_1 - D$
$V_1 = V_0 - D$	$V_2 = V_1 - D$
$= 17500 - 665$	$= 16835 - 665$
$= 16835$	$= 16170$

c. the car can also depreciate by kms

- It depreciates by 0.03c per km.
- write the RR: D

$V_0 = 17500, V_{n+1} = V_n - 0.03$

d. if a car drives 20,000 km per year, calculate the yearly depreciation.

$20000\ km \times 0.03c = \$600$

e. write the RR that models yearly depreciation

$V_0 = 17500,\ V_{n+1} = V_n - 600$

2. RR: $V_0 = 450,\ V_{n+1} = V_n - 12$

What is the rate of depreciation?

Solve (12=$\frac{r}{100} \times 450, r$) | rearrange(exam)

$\frac{12}{450} \times 100 = r$

r=2.67%

5C: Reducing balance depreciation

- depreciation is by a fixed ratio per period (rather than a fixed amount like flat-rate or unit cost)
- depreciates by a percentage of its previous value
- amount of depreciation **decreases** each time period as the value decreases

RR $V_0 = \textit{initial value}, V_{n+1} = R \times V_n$

$R = 1 - \frac{r}{100}$

r = depreciation rate per period (%)

V_{n+1} = next term

V_n = previous term

Example: Office furniture cost $6900 and depreciates by 6.4% per annum.

a) write the RR:

1. calculate **R**:

$$R = 1 - \frac{6.4}{100} = 0.936$$

R is below 1 ∴ geometric decay will occur

2. fill in **RR**:

$$V_0 = 6900, \quad V_{n+1} = 0.936 \times V_n$$

b) using recursion determine the value of the office furniture after 3 years:

$V_0 = 6900$

$V_1 = R \times V_0 = 0.936 \times 6900 = 6458.4$

$V_2 = R \times V_1 = 0.936 \times 6458.4 = 6045.06$

$V_3 = R \times V_2 = 0.936 \times 6045.06 = 5658.18$

c) In the 3-year period, how much has the office furniture depreciated by?

$$V_0 - V_3 = 6900 - 5658.18 = 1241.82$$

d) When will the office furniture first drop below $5000?

use CAS trick:

6900 → enter

× 0.936 → keep hitting enter until value drops below 5000

$$V_5 = 4957.41$$

→ *after 5 years.*

e) the printer is worth $3200 when new, given the RR to model depreciation, what is the depreciation rate?

$V_0 = 3200, V_{n+1} = 0.872 \times V_n$ → *R*

solve $(0.872 = 1 - \frac{r}{100}, \text{r})$ $r = 12.8\%$

5D depreciation – finding a rule for the n[th] term

the rule allows us to find any term in a sequence in one step.

flat rate: $V_n = V_0 - n \times D$

V_0 → principal

V_0 → first / starting value

$D = \frac{r}{100} \times V_0$

r = % dep.

unit cost: $V_n = V_0 - n \times D$

value per use

reducing bal. dep: $V_n = V_0 \times R^n$

$R = 1 - \frac{r}{100}$

r = % dep.

n= time period

examples:

1. the following rule models flat rate dep. V_0

$V_n = 12000 - n \times 675$ ← dep. Amount

a. calc. the value after 10 yrs. ← n

$V_{10} = 12000 - 10 \times 675 = 5250$

b. how many years b4 it first drops below $10000? ← V_n

solve (10000 = 12000 − n × 675, n) menu >3>1

n = 2.96 = 3 years.

2. A printer cost $1200, it depreciates by 8% of it's value each year. r V_0

a. write a rule

$D = \frac{r}{100} \times V_0 = \frac{8}{100} \times 1200 = 96$

$V_n = V_0 - n \times D$

$V_n = 1200 - n \times 96$

b. what is the value after 10 years?

$V_{10} = 1200 - (10 \times 96)$

$= 1200 - 960 = 240$

c. how many years before it first drops below $500?

solve (500 = 1200 − n × 96, n)

n = 7.3 years

3. A hairdryer cost $850, it depreciates by 25¢ per hour.

a. write a rule (n = hours)

$V_n = V_0 - nD$

$V_n = 850 - n \times 0.25$

b. what is the value after 12 hours of use?

$V_{12} = 850 - 12 \times 0.25$

$= 847$

c. what is the value if it is used 8.5 hours a day, 6 days a week for 5 weeks?

8.5 × 6 × 5 = 255 hours (n)

$V_{255} = 850 - 255 \times 0.25$

$= 786.25$

4. calculate the rate of depreciation for a computer that reduces by the following rule.

$V_n = 0.8^n \times 9500$

$V_n = R^n \times V_0$

$R = 1 - \frac{r}{100}$

Solve$(0.8 = 1 - \frac{r}{100}, \text{r})$

r = 20%

5E: Simple Interest

Interest:

- the cost of borrowing money
- loans: banks charge lenders a fee for borrowing their money
- investments: investor receives interest/ payment as the bank is borrowing their money

simple interest:

- a constant amount of interest that is paid or earned each period
- calculated as a percentage of the principal

recurrence relation: next value previous value

$V_0 =$ principal, $V_{n+1} = V_n + D$

$D = \frac{r}{100} \times V_0$ interest

r = interest rate (%)

rule: any future value

$V_n = V_0 + n \times D$

$D = \frac{r}{100} \times V_0$

r = interest rate (%)

examples:

1. Joseph invests $12000 into a simple interest account earning 4.3% p.a.

a. write the recurrence relation:

$V_0 = 12000$, $V_{n+1} = V_n + 516$

$D = \frac{4.3}{100} \times 12000 = 516$

b. write the rule:

$V_n = 12000 + n \times 516$

c. calculate the value after 3 years using recurrence and the rule:

recurrence: step by step

$V_0 = 12000$

$V_1 = V_0 + D = 12000 + 516 = 12516$

$V_2 = V_1 + D = 12516 + 516 = 13032$

$V_3 = V_2 + D = 13032 + 516 = 13548$

rule: $V_3 = 12000 + 3 \times 516$

$= 13548$

2. Lulu opens a simple interest loan to purchase a new laptop, the rule is: $V_n = 2000 + n \times 121$

a. how much was borrowed?

$2000

b. what is the interest rate?

$D = \frac{r}{100} \times V_0$

solve $(121 = \frac{r}{100} \times 2000, r)$

r= 6.05%

c. when will the loan reach $2700?

rule, to find n using the given value

solve(2700=2000+n x 121, n)

n= 5.79

= 6 years

5F Compound Interest

- most common form of interest
- interest is calculated taking into account the principal and any interest earned
- the amount of interest increases each time period

recurrence relation:

$V_0 =$ **principal**, $V_{n+1} = R \times V_n$

$R = 1 + \frac{r}{100}$

r = interest rate per compound period

rule: $V_n = V_0 \times R^n$

$R = 1 + \frac{r}{100}$

r = interest rate per compound

Compound periods

- interest rates are often given as a yearly rate/ per annum
- accounts can compound more frequently than this

❖the annual interest rate must be converted to the interest rate per compound period

Type	Compound periods per year
annual	1
biannual	2
quarterly	4
monthly	12
fortnightly	26
weekly	52
daily	365

Divide the annual interest rate by the compound period

e.g.

6% pa, compounding monthly

$\frac{6\%}{12} = 0.5\%$ monthly

examples:

1. Trent opens a compound interest account with $6500. The bank pays 5.7% interest p.a. compounding quarterly.

a. write the recurrence relation

$R = 1 + \frac{5.7/4}{100} = 1.01425$

$V_0 = 6500$, $V_{n+1} = 1.01425 \times V_n$

b. write the rule

$V_n = 1.01425^n \times 6500$

c. determine the value after 1 year using recursion (4 comp periods)

$V_1 = 1.01425 \times 6500 = 6592.63$

$V_2 = 1.01425 \times 6592.63 = 6686.57$

$V_3 = 1.01425 \times 6688.57 = 6781.85$

$V_4 = 1.01425 \times 6781.85 = 6878.49$

d. Determine the value after 4 years

$V_{16} = 6500 \times (1.01425)^{16} \approx 8151.43$

e. How much interest was earned in 4 years?

$V_{16} - V_0 = 8151.43 - 6500 = 1651.43$

2. .The rule models a compound interest account (compounding monthly)

$V_n = 8710 \times (1.0075)^n$

a. What is the annual interest rate?

Solve(1.0075= $1 + \frac{r}{100}, r)$ $r = 0.75 \times 12$

= 9%

Solve(1.0075= $1 + \frac{r/12}{100}, r)$ r = 9%

b. When will the account reach $10,000?

Solve (1000=8710 × $1.0075^n, n)$

$n \approx 18.48$ =19 months

c. How much interest was earned in the 3rd year?

V_0 → 1st yr → V_{12} → 2nd yr → V_{24} → 3rd yr → V_{36}

$V_{24} = 8710 \times (1.0075)^{24} = 10{,}420.76$

$V_{36} = 8710 \times (1.0075)^{36} = 11{,}398.30$

$V_{36} - V_{24} = 977.46$

5G Nominal and Effective Interest Rates

Nominal: The quoted interest rate for a loan/investment, a.k.a. the annual interest rate

Effective: The adjusted or "real" rate, Takes into account the number of compounding periods

→Used to compare different loan/investment options

Changing Nominal to Effective

$$r_{\text{eff}} = \left(\left(1 + \frac{r}{100n}\right)^n - 1\right) \times 100$$

Where:

- r= nominal interest rate
- n= compounding periods

CAS

Nominal → Effective

$eff(\text{nominal interest rate}, \text{compound periods})$

menu → 8 → 5 → 2

If you have a **loan**:
Borrower wants the **smallest** effective interest rate

If you have an **investment**:
Investor wants the **highest** effective interest rate

Effective → Nominal

nom(_%_ , _n_)

menu → 8 → 5 → 1

Examples

1. Harper takes out a loan. They have 2 options. Which is the best option?

1. 3.7% p.a. compounding biannually

2. 3.68% p.a. compounding weekly

eff(3.7, 2) = **3.74%**

eff(3.68, 52) = **3.75%**

∴ **Option 1**

2. Billy is planning to invest $10,000. Which option is best?

1. 8.4% p.a. compounding fortnightly

eff(8.4, 26) = **8.75%**

2. 8.41% p.a. compounding quarterly

eff(8.41, 4) = **8.68%**

∴ **Option 1**

6A Financial Applications

Amortisation Tables:

- shows step by step calculations for balances of loans or annuities
- shows consecutive compound periods

example: can be: addition, total of annuity, investment

payment number	payment	interest	principal reduction	balance of loan/investment
number of payments been made/received	the amount paid to or from	amount added to loan or investment	change in balance from payment/interest	the balance from previous balance and principal red.
0	0	0	0	V_0 (principal / starting value)
1				

finance solver

CAS: menu > 8 > 1

N: number of payments (years x compound periods)
I%: annual interest rate
PV: present value (current value)
Pmt: payment per compound period
FV: future value, after n compound periods
PpY: payments received per year
CpY: compound periods per year *(same)*
PmtAt: when the payment is made @ beginning or end; **always END for us**

the direction of money exchange must be specified when using finance solver:
the PV, Pmt and FV are either positive or negative

- positive: individual receives money
- negative: individual pays money

6B Reducing Balance Loans

- compound interest loan ŵ repayments made @ regular intervals
- the payment is greater than the amount of interest charged

V_0 = **principal** , V_{n+1} = **R** V_n − **D**

R = 1 + $\frac{r}{100}$

r = % per compound period

D = regular payment

amortisation table rules:

interest = $\frac{r}{100}$ × previous loan balance

prin reduction = payment − interest

balance of loan = previous loan balance− prin reduction

finance solver:

PV:	**POSITIVE: as we borrow money**
Pmt:	**NEGATIVE**: make the payment
FV:	**NEGATIVE**: money still owed Or **ZERO**: loan repaid

Examples

1. Kat has a $30,000 loan with $2,500 quarterly repayments and must be paid in full in 7 years. What interest rate allows this?

- $N = 7 \times 4 = 28$
- $I \approx$ **28.47%** ←
- $PV = 30{,}000$
- $Pmt = -2{,}500$
- $FV = 0$
- $Cpy = 4$
- $Ppy = 4$

2. Ellie has a $260,000 home loan. Interest is charged at 5.65% p.a., compounded monthly. Payments are calculated over a 20-year loan period. What is monthly payment?

- $N = 20 \times 12$
- $I = 5.65$
- $PV = 260{,}000$
- $Pmt =$ **−1,810.61** ←
- $FV = 0$
- $Cpy = 12$
- $Ppy = 12$

Payment is $1810.61 per month.

3. Joan takes out a $35,000 loan. Interest rate: 5.4% p.a., compounded monthly. Monthly repayment: $864

a. Write the recurrence relation

$$R = 1 + \frac{5.4}{12 \times 100} = 1.0045$$

$$V_0 = 35{,}000, V_{n+1} = 1.0045V_n - 864$$

b. Model the first 4 payments in an amortisation table

$$\frac{5.4}{12 \times 100} \times 35000 = 157.50, \frac{5.4}{12 \times 100} \times 34293.5 = 154.32,$$

Payment #	Payment	Interest	Prin. Red.	Balance
0	0	0	0	35,000
1	864	157.50	706.50	34,293.50
2	864	154.32	709.68	33,583.82
3	864	—	—	—
4	864	—	—	—

c. How long will it take Joan to pay off the loan?

- $N =$ **44.82** ≈ 45 payments ←
- $I = 5.4$
- $PV = 35{,}000$
- $PMT = -864$
- $FV = 0$
- $Cpy = 12$
- $Ppy = 12$

d. After 2 years, interest rate increases to 6.8% p.a. To repay the loan in the **same time**, what is the new monthly payment?

2 years	**Remaining loan**
$N = 2 \times 12 = 24$	$N = 45 - 24 = 21$
$I = 5.4$	$I = 6.8$
$PV = 35{,}000$	$PV = 17{,}136.87$
$Pmt = -864$	$Pmt =$ **−867.87**
$FV =$ **−17,136.61**	$FV = 0$
$Cpy = 12$	$Cpy = 12$
$Ppy = 12$	$Ppy = 12$

Monthly payment is $867.87

$Interest = \frac{r}{100}$ ×prev. balance

$Interest = r/CP/100$ ×prev. balance

$Interest = \frac{r}{100 \times CP}$ ×previous balance

6C. Interest only loans

- borrower only repays the interest charged
- principal doesn't change
- principal paid back @ termination of loan
- payment = interest charged

V_0 = **principal** , V_{n+1} = **R** V_n − **D**

R = 1 + $\frac{r}{100}$ interest / payment

D = $\frac{r}{100}$ × V_0

r = interest rate per compounding period

- **finance solver**

N	always 1
PV	POSITIVE
Pmt	NEGATIVE
FV	NEGATIVE

**PV and FV are the SAME number*

Example

1. Georgia borrows $100,000, if the interest rate is 7.35% compounding quarterly, what are the quarterly repayments?

N	1
I	7.35
PV	100,000
Pmt	**−1837.50**
FV	−100,000
Cpy/Ppy	4

Finding V_0

If you are asked to find the V_0 / starting value / amount borrowed you **MUST** use the **D Rule** and solve for V_0!!

- D = $\frac{r}{100}$ × V_0

Example:

2. Jamie takes out a loan that requires them to pay $339.58 interest monthly. The interest rate is 8.15% compounding monthly. How much did Jamie borrow?

Solve(339.58 = $\frac{8.15/12}{100}$ * x, x)

x = $49,999.51

∴ Jamie borrowed $50,000

6D Amortising Annuities

annuity:
- investment where investor gets a regular payment
- earns compound interest
- account closes when balance reaches zero

V_0 = *principal* , V_{n+1} = R V_n − D

$R = 1 + \frac{r}{100}$

r = interest rate per compound period

D = payment

amortisation table:
- same as RBL
- see handout

finance solver:

PV : NEGATIVE

Pmt : POSITIVE

FV : POSITIVE or ZERO

examples:

1. The following table displays Karen's annuity account. Payments and interest are monthly.

Pmt #	Pmt $	interest	Prin red.	balance
0	0	0	0	120,000
1	1300	699.60	600.40	119,399.60
2	1300	696.10	603.90	118,795.70
3	1300	692.97	607.03	118,188.67

a. calculate the monthly and annual interest rate

$\frac{699.60}{120\,000} \times 100 = 0.583$ monthly

$\times 12 = 6.996 \approx 7\%$ annual

b. write the RR

$R = 1 + \frac{7/12}{100} = 1.00583$

$V_0 = 120\,000$, $V_{n+1} = 1.00583 \times V_n - 1300$

c. complete the third line of the amortisation table.

interest: $\frac{7/12}{100} \times 118795.70 = 692.97$

d. determine how many months the annuity will last.

N	132.93 → 133 months.
I	7
PV	−120000
Pmt	1300
FV	0
Cpy/Ppy	12/12

∴ annuity will last 133 months.

6E: Perpetuities

- type of annuity
- payment received = interest earned
- lasts indefinitely

V_0 = *principal* , V_{n+1} = R V_n − D

$R = 1 + \frac{r}{100}$	**D = payment**
r = interest rate per compound period	$D = \frac{r}{100} \times V_0$

Finance solve

N always 1

PV NEGATIVE *

Pmt POSITIVE

FV POSITIVE *

- PV and FV must be the same value*

Example:

a. Em opens a perpetuity account with $320,000. It earns 3.7% p.a, compounding monthly. What are the monthly payments?

N	1
I	3.7
PV	−320000
Pmt	986.67
FV	320000
Cpy/Ppy	12/12

or

$D = \frac{r}{100} \times V_0$

$= \frac{3.7/12}{100} \times 320{,}000$

$= 986.67$

b. write the recurrence relation:

$R = 1 + \frac{r}{100} = 1 + \frac{3.7/12}{100} = 1.0031$

$V_0 = 320000$, $V_{n+1} = 1.0031 V_n - 986.67$

*** if you are asked to find V_0, you must use the D rule and solve for V_0***

Example: maths @ TC set up an **annual endeavour award.** The award is $840, the interest rate is 6.7% p.a. How much does TC **invest** in a perpetuity account?

$D = \frac{r}{100} \times V_0$

solve($840 = \frac{6.7}{100} \times x, x$)

x = $12537.31

6F Annuity Investments

- type of annuity
- money is invested, it earns compound interest, investor adds an additional payment @ regular intervals
- investment will continue to increase

V_0 = *principal* , $V_{n+1} = R\,V_n + D$

$R = 1 + \frac{r}{100}$	D = payment
r = interest rate per compound period	

- **amortisation table**
- see handout
- **finance solver**
- PV NEGATIVE
  Pmt NEGATIVE
  FV POSITIVE

Example

1. Tony invests $35,000 into an annuity investment. He receives 4.6% p.a interest, compounding monthly. He makes a monthly payment of $500.

a. write the recurrence relation

$R = 1 + \frac{4.6/12}{100} = 1.0038$

$V_0 = 35000$,
$V_{n+1} = 1.0038 \times V_n + 500$

b. model the first 3 payments in an amortisation table

Pmt #	Pmt $	interest	Prin add..	balance
0	0	0	0	+ 35000,00
1	500 +	134.17	= 634.17	= 35634.17
2	500	136.60	636.60	36270.77
3	500			

$\frac{4.6/12}{100} \times 35000 = 134.17$

$\frac{4.6/12}{100} \times 35634.17 = 136.60$

c. after how many months will the account first reach $50,000?

FV

N	22.68	→ **23 months!**
I	4.6	
PV	−35000	
Pmt	−500	
FV	50,000	
CPY/PPY	12/12	