Geometry and Trigonometry Module 1 Discrete Mathematics

Section 1.1 Graph Theory

Practice Problems 1.1

For Problem 1, solve the word problem.

1. Circle the topics that are included in the field of discrete mathematics from the following questions:
   a. How can electrical transactions at financial institutions be safeguarded?
   b. What are the most efficient routes for mail delivery and street cleaning?
   c. How many solutions are there for a quadratic equation?
   d. What is the fastest way to search computer storage?

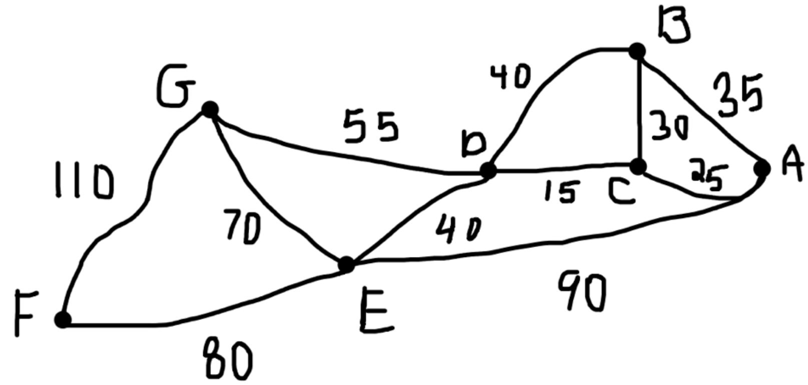
For Problem 2-4, use the Traveling Salesman Problem to solve the problem.

2. List all the possible permutations of the four cities in the Traveling Salesman Problem. Use a tree diagram if you need to. Have him start and end at each of the cities. In other words, find all the possible arrangements of ABCD.

3. Calculate the total distances for each route if the salesman returns to the city of origination. What do you notice? What is the shortest distance the salesman can travel?

4. Using the nearest-neighbor algorithm, find the total distance the salesman travels if he/she starts at A, then continues going to the closest city until coming back to A? Is this the shortest route starting from and returning to city A?

For Problem 5-9, use the diagram below to solve the problem.



5. Is there a route where each city could be visited just once if you start at city A and end at city A? If so, what is the total distance of that route?

6. What is another route that is the same distance as the route in Problem 5?

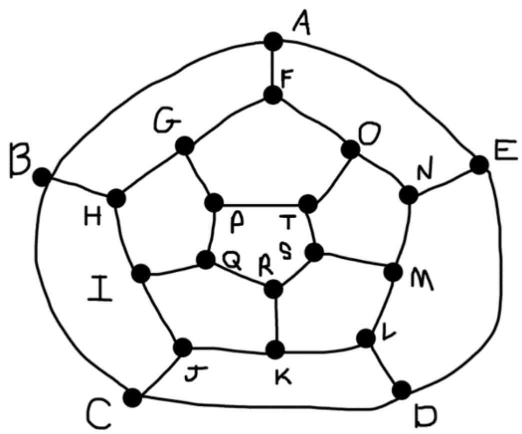
7. How many possible routes are there that start at city A and return to city A?

8. Find one other possible route from city A to city A that goes through city D twice and goes through every other city. What is the total distance of that route?

9. Is the route of the least distance always the one that visits each city only once?

For Problem 10, use the diagram below to solve the problem.

10. The diagram below shows all the cities in Hexcircleville. A traveling salesman started at city A and visited every city once and once only. He stayed all night in each city and returned to city A on the 20th day. On the same day, another traveling salesman left from city B with the same plan. He/she visited one city each day, stayed overnight there, and returned to city B on the 20th day after visiting each city only once. On the fifth day of their travels, the salesmen met at city L, and the following day they met again. What other cities did they meet at along the way?

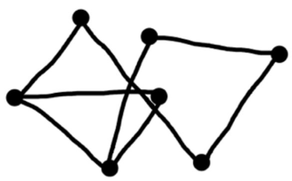


Section 1.2 Euler Paths and Circuits

Practice Problems 1.2

For Problem 1-10, name the number of edges and vertices in each diagram and tell the degree of each node. Also tell whether it is an Euler path, Euler circuit, or neither.

1.



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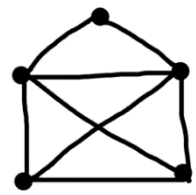

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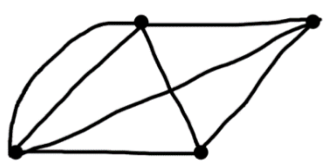

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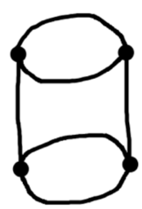

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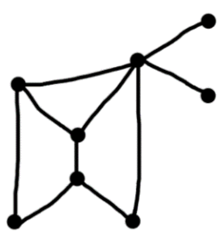

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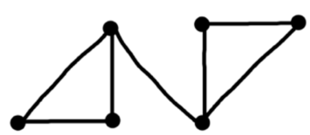

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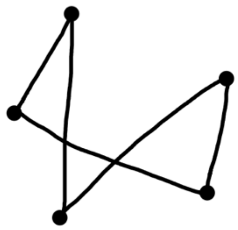

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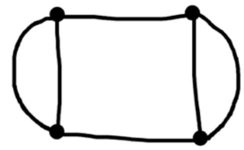

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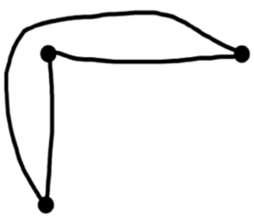

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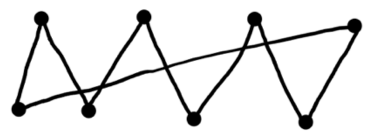
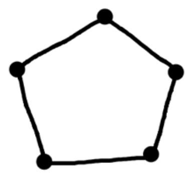


<u>Section 1.3 Hamiltonian Paths and Circuits</u>

<u>Practice Problems 1.3</u>

For Problem 1-6, name the number of vertices and the degree of each vertex for the shape. Tell whether each diagram is a Hamiltonian path, circuit, or neither.

1.



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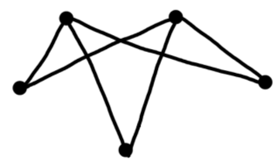

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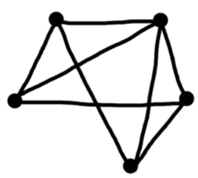

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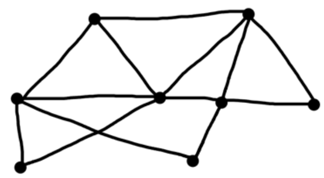

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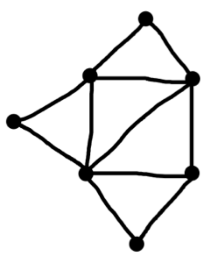

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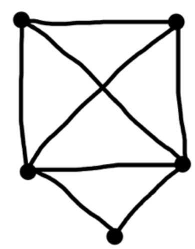
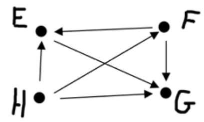


For Problem 7-10, solve the word problem.

7. Four basketball teams played in a tournament. The games played and the winners of the games can be shown using a graph.

Game	E-F	E-G	E-H	F-G	F-H	G-H
Winner	F	E	H	F	H	H



This is a diagraph, a graph with direction. The arrows point to the team defeated in each game. Follow the Hamiltonian graph beginning with the winner (H defeated all three other teams) to find which place each team finished in the tournament.

8. Draw a diagraph (a graph with direction) for five teams in which team E beats all the other teams, team F beats all the teams but E, team G is beaten by all the other teams, and team H beats team I.

9. Use the diagraph for Problem 8 to determine the outcome of the tournament.

10. a) Draw a diagraph for a tournament in which one team beats all the other teams.

b) Draw a diagraph for a tournament with three teams in which each team wins one game.

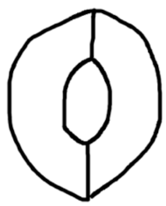
c) Draw a diagraph for a tournament with three teams in which one team loses all the games they play.

Section 1.4 Graph Coloring and Planarity

Practice Problems 1.4

For Problem 1-4, use the given information to solve the problem.
In Euler's polyhedron formula, every map contains a digon, a triangle, a square, or a pentagon in its configuration. Show that each diagram can be colored using four colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color.

1.



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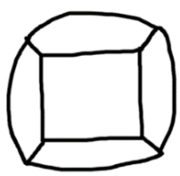

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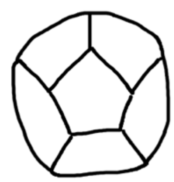
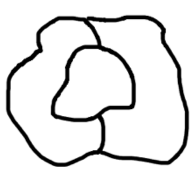


For Problem 5-7, color the maps using the rules stated in Problem 1-4 and draw the planar graphs represented by the maps. The four-color theorem states that any planar map can be colored with only four colors in such a way that two adjacent regions have different colors.

5.



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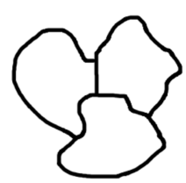

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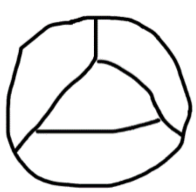
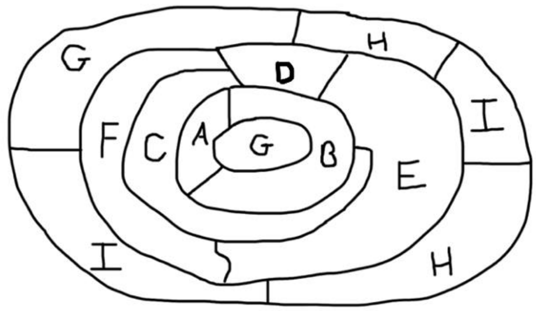


For Problem 8-10, use the map below and the four colors red, yellow, blue, and green to solve the problem.



8. If A is red and B is green, what are the possible colors C could be?

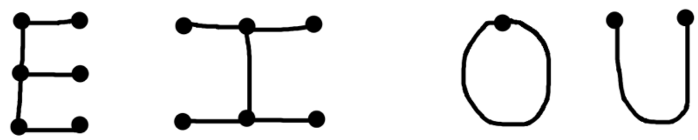
9. Given the possible colors for C in Problem 8, what are the possible colors G could be?

10. Use the map above to test the four-color theorem. Play this game with a partner: Firstly, color A blue. Next, have your partner color B using any of the four colors so that no two regions sharing a common boundary (other than a single point) share the same color. Continue taking turns coloring the next letter in alphabetical order until the map is filled. If regions have the same letter, they must have the same color. See if you only use four colors. Play several times making different combinations.

Section 1.5 Polyhedral Formula and Doodle Drawings

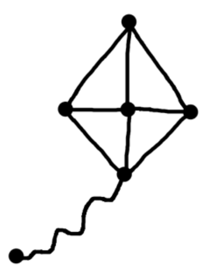
Practice Problems 1.5

1. Use the formula $V + F - 1 = E$ with the vertices and faces in each letter to find the edges. Verify it by counting. We have already done the letter "A" in the work text. (Notice that the letter "O" is a loop and the letter "U" is a link. Therefore, the letter "U" has one more vertex than the letter "O.")



For Problem 2-6, use the formula $V + F - 1 = E$ to find the number of edges of the following doodle drawings. Verify it by counting.

2.



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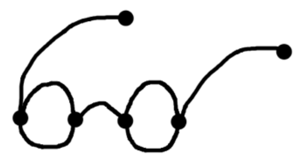

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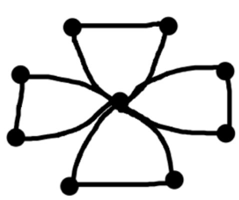

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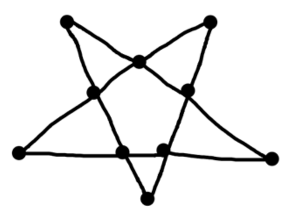

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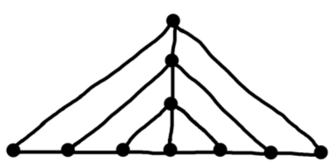


For Problem 7 and 8, complete the table for the circle doodles. Let n be the number of circles.

7.

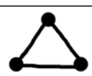
Doodle Drawing	n	V	F	E
	2	2		4
	3		5	
	4			12

8.

n circles	n	$n + (n - 2)$ Or $2n - 2$	$n + (n - 1)$ Or	$n + (n - 2)$ + $n + (n - 2)$ Or

For Problem 9, complete the table for the triangle doodle drawing. Let n be the number of triangles.

9.

Doodle Drawings	n	V	F	E
	1			
	2	8		
	3		5	
	4			24

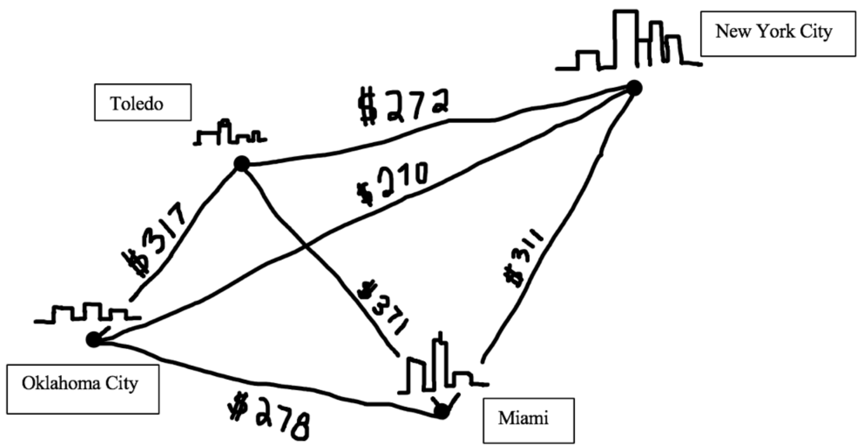
For Problem 10, solve the word problem.

10. If there is a given number of n triangles for Problem 9, write the formulas to find the number of vertices, faces, and edges.

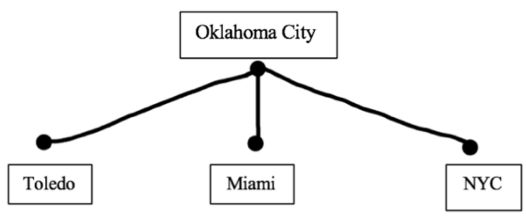
Section 1.6 Spanning Trees

Practice Problems 1.6

For Problem 1-4, use the diagram below, which shows the costs of flights to different cities, to answer the problem.



1. Complete the tree diagram for all possible flights from Oklahoma City, visiting each city once and then returning to Oklahoma City. It has been started for you.



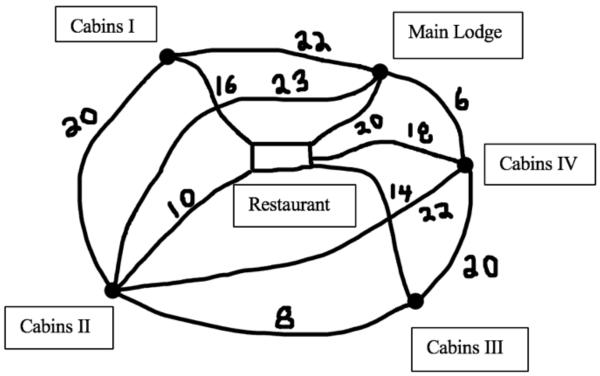
2. Label each branch of the tree diagram with the cost of the flight.

3. Put the total cost of the round trip at the end of each final branch of the tree diagram. What is the minimum cost to start and finish at Oklahoma City and visit all the three other cities once in between?

4. How do you know how many flights are possible? How many different routes are there and why?

For Problem 5, solve the word problem.

5. A parks and recreation board bought a 50-square mile property out in the country. They hired developers to build roads to join the lodges, restaurants, and cabins on the property. The roads and the amounts they cost, in thousands of dollars, are listed. Find the minimum cost of building enough roads to connect all the locations using the minimum spanning tree algorithm.

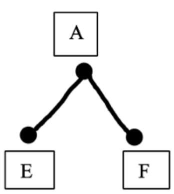


For Problem 6-10, use the information below to solve the problem.

A company that manufactures chemicals wants to ship its chemicals to various locations by rail. However, there are legal guidelines that make it so chemicals cannot be shipped together in the same railway cars.

Chemical Label	A	B	C	D	E	F
Incompatible Chemicals	B, C, D	A, C, E	A, B, D	A, C	B, F	E

6. Make tree diagrams to show which chemicals may be stored in the same railcar. The first one is done for you.



7. Can just two railcars be used to ship all the chemicals?

8. Draw a connected graph to show the chemicals that are not compatible with one another.

9. Use color-coding to color each vertex that must be in a different railcar. Chemicals (vertices) that are not compatible must use a different color. What is the minimum number of railcars needed to ship the chemicals? (What is the minimum number of colors used?)

10. List two possible arrangements of chemicals that can be shipped in each railcar.

Section 1.7 Decision-Making

Practice Problems 1.7

The table below outlines the activities it takes to finish a stained-glass project.
For Problem 1-5, use the table below to solve the problem.

Activity	**Duration**	**Prerequisite**
A. Design Project	1	None
B. Pick Colors	1	A
C. Order Glass	3	B
D. Cut Glass	2	C
E. Color Design	1	B
F. Arrange Pieces	2	D, E
G. Attach Lead	5	D, E
H. Solder and Clean Pieces	3	G, F

1. Draw the graph and label the vertices and edges for the stained-glass project.

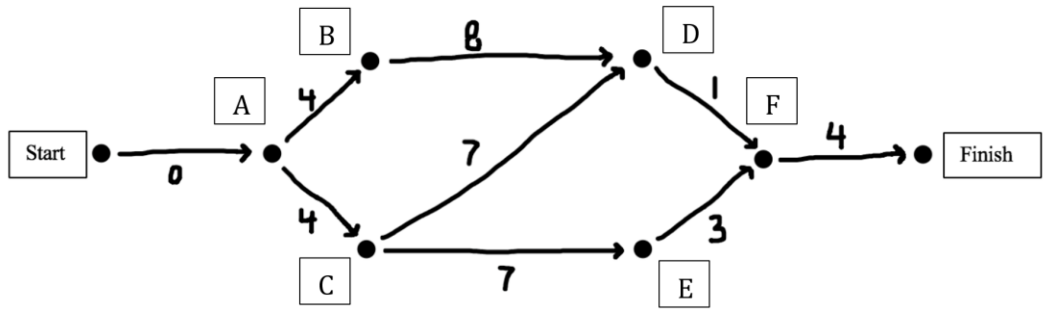
2. Put a number in parenthesis above each vertex for the estimated starting time for each activity. What is the minimum number of days needed to finish the stained-glass project? When would the project have started if it was completed in the minimum number of days?

3. Use tick marks to label the critical path of the stained-glass project. Name the critical path using ordered vertices.

4. What is the last day E can start for the stained-glass project to be finished in the minimum number of days?

5. When must activity G begin for the project to be finished in the minimum number of days?

For Problem 6-8, use the diagram to solve the problem.



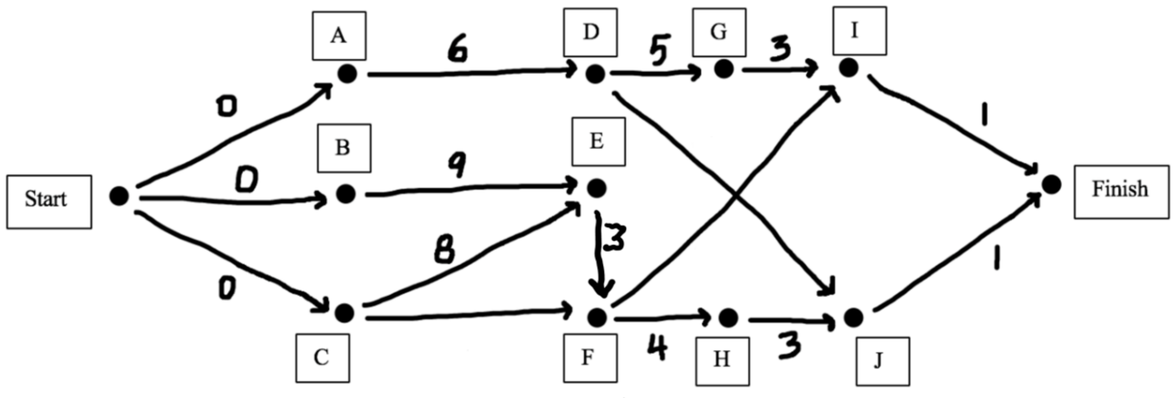
6. Complete the table for the project.

Activity	Duration	Pre-Requisite

7. Label the earliest starting time for each activity. What is the minimum number of days needed to complete the project?

8. Use tick marks to label the critical path of the project. Name the critical path using the letters between the "Start" and "Finish."

For Problem 9 and 10, use the diagram below to solve the problem.



9. Find the earliest starting time and label each vertex with the minimum number of days it will take to complete the task. Name the critical path of the project and use tick marks to label it.

10. What is the minimum number of days needed to complete the project? Name the latest starting time for activity D so that the project is completed in the minimum number of days?

Section 1.8 Fair Division

Practice Problems 1.8

For Problem 1-3, use Example 1 from the Looking Ahead section to answer the problem.

1. If Taryn cut the cake right down the middle so one half is all chocolate and the other half is all vanilla, would each girl still get a fair share?

2. If Gracie cut the cake in half right down the middle, what would guarantee each girl gets a fair share?

3. What would make the shares unfair in Problem 2?

For Problem 4-7, use Example 2 from the Looking Ahead section to answer the problem.

4. Will the other half of the pile Sara made be a fair share for David?

5. Find another method for finding the amount of the second pile in Example 2.

6. If Sara likes Hershey® bars, Snickers® bars, and Reese's® cups equally, will either pile be a fair share for her?

7. If Sara prefers two candy bars over the other, name a value for each candy bar so that she will get a fair share if David picks the remaining pile and Sara gets the pile she originally gave to David.

8. Faylynn bids $60,000.00 on an estate property and Lilly Jean bids $70,000.00 on the same property. The estate also bequeaths $50,000.00 to be divided fairly among the siblings. Who gets the house and how much money does each receive or owe?

The members of a student council include: 21 freshmen, 13 sophomores, 10 juniors and 10 seniors. The school principal is forming an advisory council with 12 members and would like each grade represented fairly from the student council members. Use this information to answer Problem 9 -10.

9. a) Find the ideal ratio for each class using the algorithm: $Ideal\ Ratio = \frac{Total\ Population}{Number\ of\ Seats}$.

b) Find the class quota for fair representation using the algorithm: $Class\ Quota = \frac{Class\ Number}{Ideal\ Ratio}$.

10. a) Use the Hamilton method to find the number apportioned from each class by truncating the decimal portion.

b) Use the Webster method to find the number apportioned from each class by rounding from the arithmetic mean (dividing the sum of the two integers the quota lies between by 2).

c) Use the Hill method to find the number apportioned from each class by rounding from the geometric mean (taking the square root of the product of the two integers the quota lies between).

d) Choose the method that you think is most fair and explain why.

e) What makes the Hamilton method unfair? Could this method be made fair?

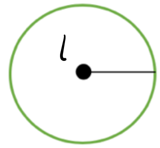
<u>Section 1.9 Recursive Thinking</u>

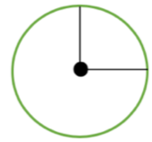
<u>Practice Problems 1.9</u>

For Problem 1-10, solve the word problem.

1. A radius is a line segment that joins the center of a circle to any point on the circumference of the circle. Complete the table below for the maximum number of regions given the number of radii that divide the circle. To find the number of regions for r, determine the relationship between the number of radii and the number of regions and write the explicit formula.

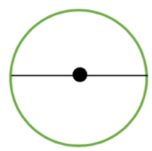
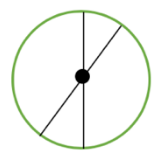
Number of Radii (r)	**Number of Regions (n)**
1	1
2	2
3	
4	
5	
r	





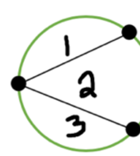
2. A diameter is a line segment that connects points on opposite sides of a circle and runs through the center of the circle. Complete the table below for the maximum number of regions given the number of diameters that divide the circle. To find the number of regions for d, determine the relationship between the number of diameters and the number of regions and write the explicit formula.

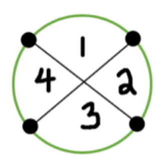
Number of Diameters (d)	**Number of Regions (n)**
1	2
2	4
3	
4	
5	
d	

3. A chord is a segment that connects two points on a circle. It may or may not run through the center of the circle. If it does it is a diameter. Complete the table for all the possible regions until the maximum number is reached given the number of chords that divide the circle. To find the number of regions for c chords, determine the relationship between the number of chords and the number of regions and write the explicit formula.

Number of Chords (c)	**Number of Regions (n)**
1	2
2	3, 4
3	
4	
c	





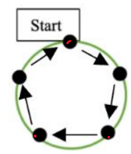
For Problem 4, use the information below to solve the problem.

4. A star polygon is a non-simple polygon formed when equidistant points (n) on the circumference of a circle are connected and form a star.

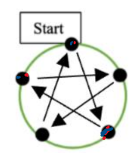
The notation $\begin{Bmatrix} n \\ m \end{Bmatrix}$ is used to create star polygons, with n being the total number of points on the circumference of a circle, any of which may be the start point, with every m point clockwise from the start being connected. For example, if the notation $\begin{Bmatrix} 5 \\ 3 \end{Bmatrix}$ is given, then you will start at any of the 5 points on the circumference of the circle and move to the third point clockwise around the circle and connect those points with a line. You will continue moving three points clockwise and connect a line from the previous point until you return to where you started.

For the figures below, $n = 5$ and a number for m is given.

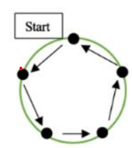
$\begin{Bmatrix} 5 \\ 1 \end{Bmatrix}$
$m = 1$



$\begin{Bmatrix} 5 \\ 2 \end{Bmatrix}$
$m = 2$

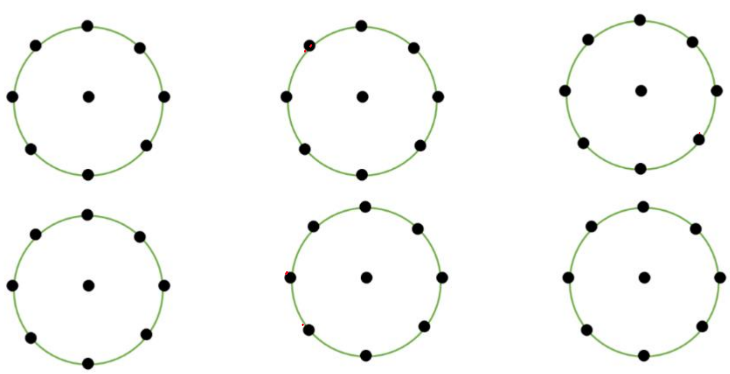


$\begin{Bmatrix} 5 \\ 4 \end{Bmatrix}$
$m = 4$

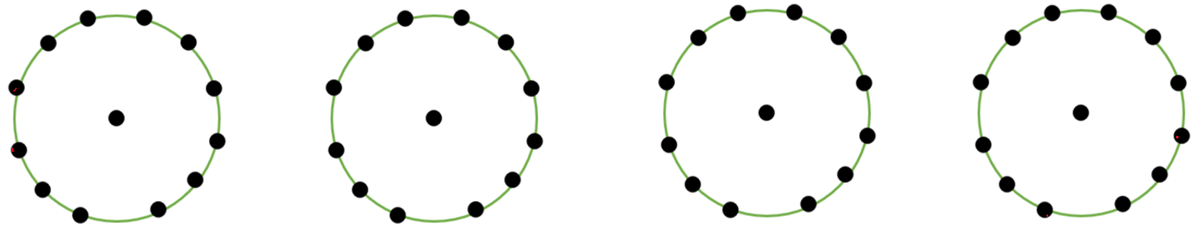


What would be a repeat of $\begin{Bmatrix} 5 \\ 2 \end{Bmatrix}$? How many unique 5-star polygons are there (a polygon that creates a star)?

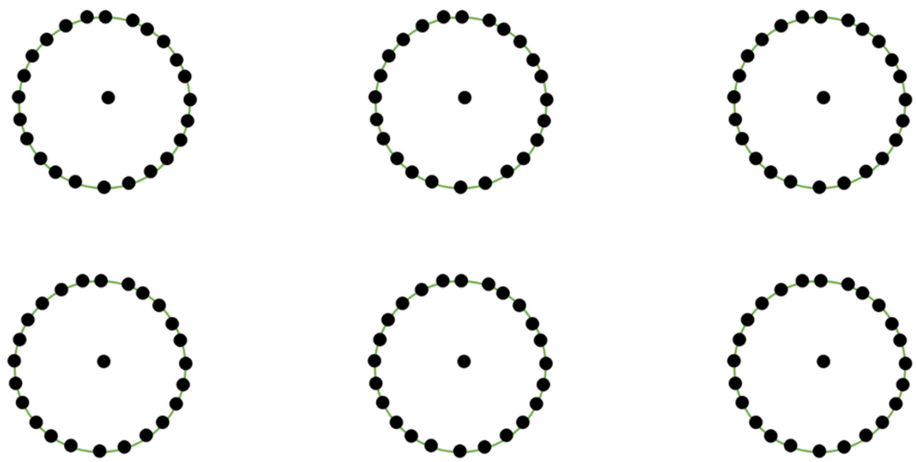
5. The star polygons $\begin{Bmatrix} 5 \\ 2 \end{Bmatrix}$ and $\begin{Bmatrix} 5 \\ 3 \end{Bmatrix}$ are called complements. Use the circles below to find the unique 8-star polygons when $n = 8$. How many 8-star polygons are there?



6. Use the circles below to find all the unique 12-star polygons.



7. Do you think there is only one unique star polygon for any number of dots (n) on the circumference of a circle. Use the circles below to find all the unique star polygons and their complements.



8. Is there a numerical relationship between "n" and "m" in $\left\{ {n \atop m} \right\}$? Which star polygons exist given n? What is the relationship between "n" and "m" in the complements?

9. Which other numerical symbol for $\left\{ {n \atop m} \right\}$ is the same star polygon as $\left\{ {10 \atop 7} \right\}$?

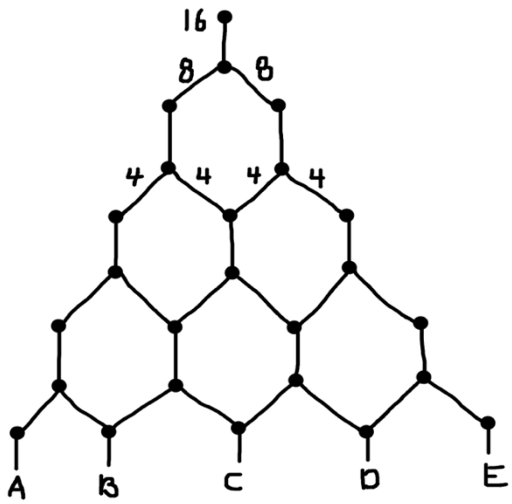
10. What is the shape of $\left\{ {6 \atop 1} \right\}$? What other numerical symbol $\left\{ {n \atop m} \right\}$ gives the same shape?

Section 1.10 Pascal's Triangle

Practice Problems 1.10

For Problem 1, solve the word problem.

1. Sixteen marbles are dropped from the top of Pascal's Maze. How many of the marbles will roll out of the maze at A? How many of the marbles will roll out at B? C? D? E?



For Problem 2 and 3, use the information below to solve the problem.

The first three powers of 11 are $11^0 = 1$, $11^1 = 11$, and $11^2 = 121$.

1

1 1

1 2 1

2. Find the expanded form of the powers of 11: 11^3, 11^4, 11^5, 11^6, and 11^7. Does the Pascal's Triangle pattern continue?

3. Explain how to use Pascal's Triangle to find the expanded form of 11^5, 11^6 and 11^7.

For Problem 4-10, solve the word problem.

4. How many ways can the word “ON” be spelled moving from top to bottom?

O
$N \quad N$

5. How many ways can the word “WON” be spelled moving from top to bottom?

W
$O \quad O$
$N \quad N \quad N$

6. How many ways can the words below (“HEAR” and “HEART”) be spelled moving from top to bottom?

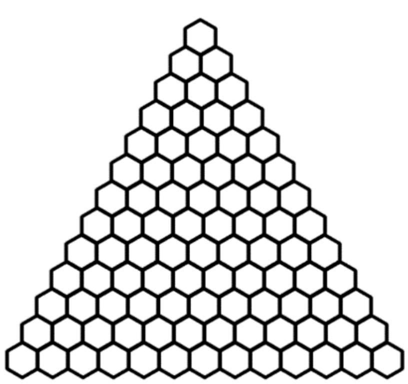
H
$E \quad E$
$A \quad A \quad A$
$R \quad R \quad R \quad R$

H
$E \quad E \quad E$
$A \quad A \quad A \quad A$
$R \quad R \quad R \quad R \quad R$
$T \quad T \quad T \quad T \quad T \quad T$

7. Without counting, how many ways can PASCAL be spelled moving from top to bottom?

P
$A \quad A$
$S \quad S \quad S$
$C \quad C \quad C \quad C$
$A \quad A \quad A \quad A \quad A$
$L \quad L \quad L \quad L \quad L \quad L$

8. Fill in Pascal’s Triangle for Even and Odd Sums. Put E for Even and O for Odd. It is not necessary to calculate each row, just look at the two numbers above any number and remember that $E + E = E$ and $O + O = E$, but $E + O = O$. Complete the triangle below using E’s and O’s.



For Problem 9 and 10, use the information below to solve the problem.

In Modulo 2 arithmetic, when any number is divided by two, there is a remainder of only 0 or 1. When the sum of two numbers are divided by two, the remainder again is 0 or 1.

If the sum of the two numbers is even, the remainder is 0.

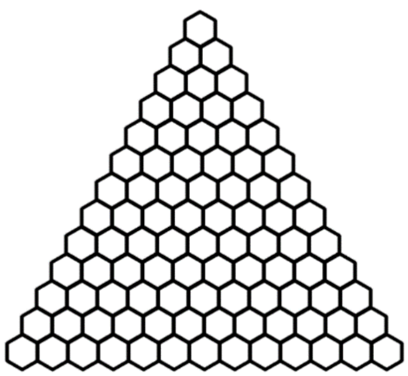
Example: $6 + 4 = 10$
$\frac{10}{2} = 5$ remainder 0.
The remainder is 0.

If the sum of the two numbers is odd, the remainder is 1.

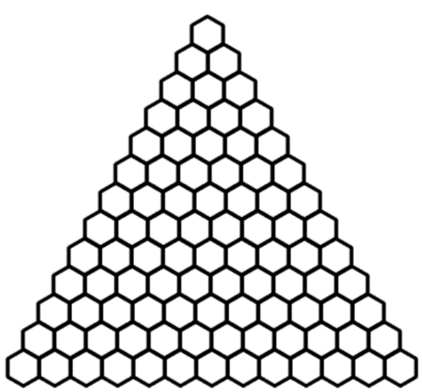
Example: $4 + 1 = 5$
$\frac{5}{2} = 2$ remainder 1.
The remainder is 1.

Therefore, every sum Modulo 2 is 0 if the sum is even and 1 if the sum is odd.

9. Rewrite Pascal's Triangle replacing odd numbers with 1 and even numbers with 0.



10. Now use a colored pencil and shade in all the 1's (Odd numbers) and leave all the 0's (Even Numbers) unshaded in the triangle. The pattern that emerges is a Modulo 2 Pascal Triangle.



Section 1.11 Counting Techniques

Practice Problems 1.11

For Problem 1-10, solve the word problem.

1. How many ways can the letters O and N be arranged? Show all the possibilities using any method.

2. How many ways can the letters W, O, and N be arranged? Show all the possibilities using a tree diagram.

3. How many ways can the word BLUES be arranged? Use a permutation to answer the question.

4. How many different ways can the word MATH be arranged to create all possible one-letter arrangements, two-letter arrangements, three-letter arrangements, etc.?

5. Which of the following is the same as $P(8,6)$?

$$\frac{8!}{6!} \text{ or } \frac{6!}{8!} \text{ or } \frac{8!}{2!}$$

6. Calculate the permutation from Problem 5.

7. You are playing a card game using only face cards. If you can only pick one card to be the wild card, how many possibilities are there for cards you can select?

8. How many possible ways are there to arrange the letters of the word HELLO?

9. How many ways can three one-digit numbers be arranged?

10. How many ways can five different books be placed on a shelf if only three fit on the shelf at a time?

Section 1.12 Probability and Combinatorics

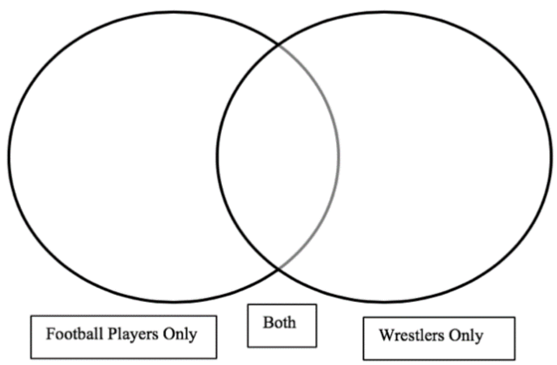
Practice Problems 1.12

For Problem 1-3, use the information below to solve the problem.
Keep in mind that if the addition principle is being used, two events cannot have common members.

A school has 61 football players and 24 wrestlers. Of those, 14 are on both the football team and wrestling team. Only one player on either team can get the Most Valuable Player award.

1. Are there 85 ways to choose one player from either team to get the MVP award?

2. Complete the Venn Diagram below for the situation in Problem 1.



3. How many possible ways are there to pick one player from either team?

For Problem 4-8, solve the word problem.

4. What are the binomial coefficients of $(x + y)^5$?

5. Which row in Pascal's Triangle is given by the binomial coefficients below?

$$\binom{4}{0} \binom{4}{1} \binom{4}{2} \binom{4}{3} \binom{4}{4}$$

6. Which of the following is the same as the binomial coefficient $\binom{9}{5}$?

$$\frac{9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \quad \text{or} \quad \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

7. Which is smaller, $C(5,2)$ or $C(8,4)$?

8. Which is larger, $C(6,4)$ or $C(6,2)$?

For Problem 9 and 10, use the information below to solve the problem.

There are seven young women and six young men on a student council. The council must choose a committee of four members that is made up of any combination of young men and young women from the council.

9. How many ways can the committee be selected?

10. If the committee on student council must be made of two young women and two young men, how does this change the number of ways the committee can be selected?

a) Find $C(7,2)$ to find the number of ways to select the young women for the committee.

b) Find $C(6,2)$ to find the number of ways to select the young men for the committee.

c) Use the multiplication rule to find the number of ways to form the committee.

d) If four committee members are selected at random, what is the probability that the committee will consist of two young men and two young women?

Section 1.13 Dependent and Independent Events

Practice Problems 1.13

For Problem 1, use the information to answer the questions.

1. Let sample space S be all the outcomes for the roll of a dice. Let A be an even-numbered roll, and B be a roll that is greater than 4.

a. What is sample space S?

b. What is the sample space for event A?

c. What is the sample space for event B?

d. What is the probability of A occuring?

e. What is the probability of B occurring?

f. What is the probability of $A \cap B$ occurring and what does it mean?

g. What is the probability of $A \cup B$ occurring and what does it mean?

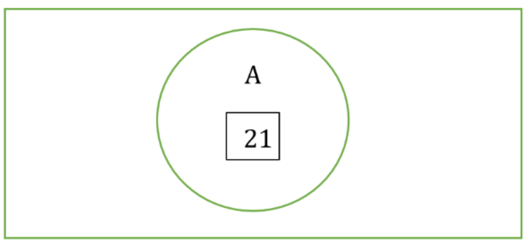
For Problem 2-7, solve the word problem.

2. If the probability that you will get 8 hours of sleep tonight is 0.85, what is the probability that you will not get 8 hours of sleep?

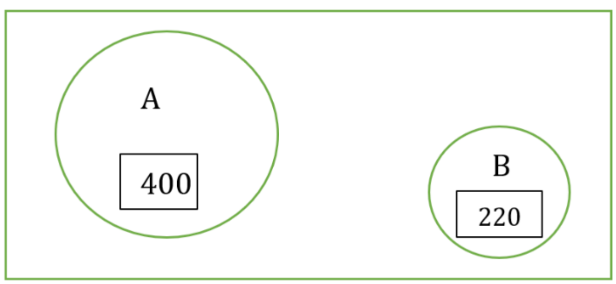
3. If the $P(A) = 0.45$, the $P(B) = 0.30$, and the $P(A \cup B) = 0.55$, what is the $P(A \cap B)$? Are the events mutually exclusive or not?

4. If the $P(A) = 0.7$, the $P(B) = 0.25$, and the $P(A \cap B) = 0$, what is the $P(A \cup B)$? Are the events mutually exclusive or not?

5. If a sample space has 93 outcomes, find the probability of the complement of A (or $P(A')$) given the diagram below.



6. If a sample space has 720 outcomes, find $P(A \cup B)$ given the diagram below.



What is $P(A \cap B)$?

7. Of families chosen in a survey conducted in Skylar County, the probability they have a desktop computer is 0.37, the probability they have a laptop is 0.47, and the probability they have both is 0.26. What is the probability that a family has neither a desktop nor laptop computer? (Find $(P(A \cup B)')$.

For Problem 8, use the information and matrix below to solve the problem.

There are 22 elementary schools in an urban district. One will be randomly chosen to close due to shortages in funds. The number of students and age of the building are shown in the matrix below.

Number of Students	< 300	$300 - 500$	> 500
Schools less than 100 Years Old	3	7	4
Schools more than 100 Years Old	2	5	1

8. What is the probability that the building that will be closed has less than 300 children or is more than 100 years old?

9. Find the probability that two rolls of a dice will not be the same number.

a) Find the sample space or possible outcomes.

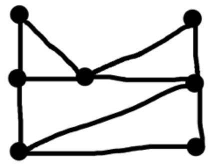
b) Let P(A) be the probability that the two rolls of a dice will be the same number and find $P(A')$.

10. If two friends meet at a birthday party, what is the probability that they share the same birthday?

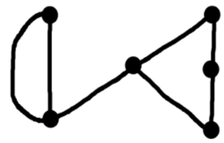
Section 1.14 Module Review

For Problem 1 and 2, tell whether the graph is an Euler path, Euler circuit, or neither. Explain why.

1.

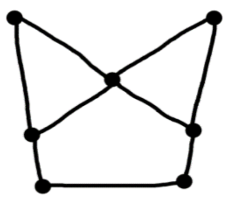


2.

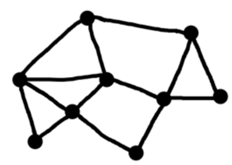


For Problem 3 and 4, tell whether the graph is a Hamiltonian path, Hamiltonian circuit, or neither. Explain why.

3.



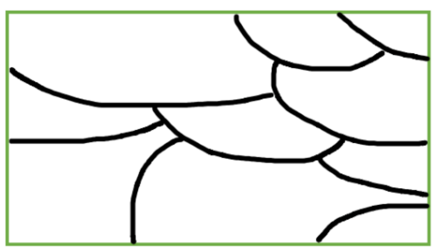
4.



For Problem 5-10, solve the word problem.

5. What is the maximum number of colors needed to color in the graph in Problem 1 so that no two colors are touching on adjacent sides or adjacent vertices?

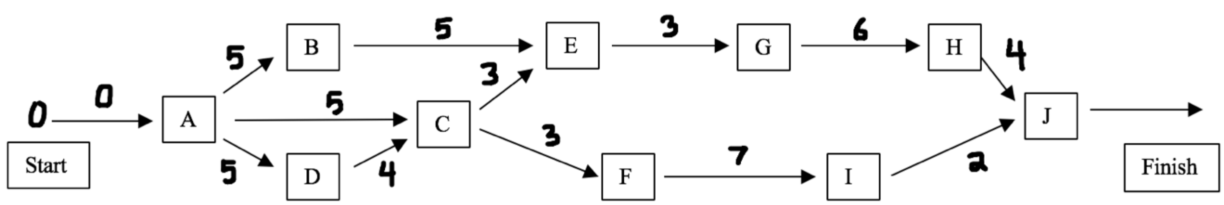
6. Color the following map using only three colors so no two colors are touching on adjacent sides or adjacent vertices.



7. A student council is meeting to form a homecoming committee. What is the minimum number of days to meet so each member can attend? A member cannot attend two meetings on the same day. Each committee consists of the following members:

Advertising: Peyton, Dalton, Dakota, Hannah, Morris, Rayven
Deejay: Peyton, Zach, Paige, Kenny, Jonah
Decorating: Jonah, Teresa, Michael, Collin, Alyssa
Tickets: Drake, Jessie, Kaleb, Hannah, Daytona
Food: Teresa, Hannah, Melanie, Kelsey, Jacob

8. For the graph below, list each vertex with the earliest start time for each activity and determine the minimum amount of time it will take to complete the project.



9. How many permutations are there of the word CABIN? What is the probability that vowels and consonants will not alternate? (Hint: There are three consonants and two vowels so consonants be in the 1st, 3rd, and 5th position and vowels will be in the 2nd and 4th position.) Find $P(E)$ before finding $P(E')$.

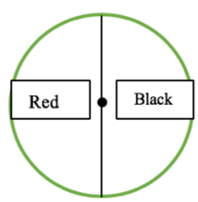
10. Find the following probabilities using Pascal's Triangle:

a) Flipping a coin 5 times and getting 1 head and 4 tails

b) Having 6 children and getting no girls and 6 boys

c) Choosing 5 numbers at random from numbers marked 1 to 100 and getting 5 odd numbers and no even numbers

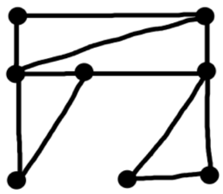
d) Spinning the spinner below 4 times and getting two reds and two blacks



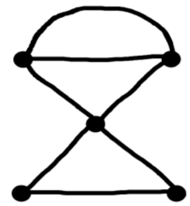
Section 1.15 Module Test

For Problem 1 and 2, tell whether the graph is an Euler path, Euler circuit, or neither. Explain why.

1.

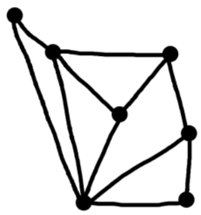


2.

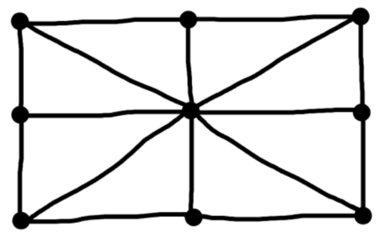


For Problem 3 and 4, tell whether the graph is a Hamiltonian path, Hamiltonian circuit, or neither. Explain why.

3.



4.



For Problem 5-10, solve the word problem.

5. What is the minimum number of colors needed to graph Problem 3 so that no two colors are touching on adjacent sides or adjacent vertices?

6. Color the map. What is the minimum number of colors needed to color the map so no two adjacent sides are the same color?

7. At a school, students are signing up for classes. Each class is one block. How many blocks are needed to offer courses so each student can attend class? What is the minimum number if the students can only attend one class per block?

Discrete Math: Emmalie, Zoe, Caeley, Macey
Chinese: Thomas, Rylee, Melody, Kylie
Guitar: Alexis, Emmalie, Zoe
Probability & Statistics: Macey, Rylee, Murton, Jenna

8. Draw a graph representing the given tasks below and determine the minimum number of days needed to finish the project.

Activity	**Duration**	**Prerequisite Activity**
Start	0	-
A	3	None
B	2	A
C	4	A
D	3	B
E	2	B
F	1	C
G	3	D
H	6	D, E, F
I	2	G, H
Finish		-

9. How many ways are there to roll three die so that at least two of them show the same number? (Hint: This means the event has failed if all three die show different numbers.) It is easier to find $P(E')$ first and then find $P(E)$.

10. Find the following probabilities using Pascal's Triangle:

a) Flipping a coin 5 times and getting 2 heads and 3 tails

b) Having 8 children and getting 3 boys and 5 girls

c) Choosing 7 numbers one at a time from a bowl with numbers marked 1 to 100 and getting 7 odd numbers and no even numbers.

d) Answering a 10-question true-false test at random and getting 10 false answers