Geometry and Trigonometry Module 3 Foundations of Geometry

Section 3.1 Euclid's Postulates

Practice Problems

For Problem 1-5, solve the word problem.

1. Put two nickels on a table to represent two points. Pull a string taut from one nickel to the other. This represents the line segment. How many ways can you do this?

2. Below are two points. Connect them with a line segment. This means the line segment begins at one point and ends at the other. Use a straightedge or ruler so that your line segment is straight. How many ways can this be done? How many line segments can be drawn between the points?

3. Now put your straightedge or ruler over the points and extend the line segment across the page to the right and left so it touches the edges on both sides. If the paper kept unrolling and unrolling and getting bigger and bigger, how far could the line be extended?

4. Using a ruler, draw a line segment from the middle of the top of an index card to the middle of the bottom of the index card. This forms a perpendicular line because it meets both the bottom and the top at right angles. Cut the card in half along the line. Now, take one piece and lay it on the other piece. How do they fit? What do you notice about each of the corners? If we keep cutting these pieces in half will this continue to give us the same results? Why or why not?

5. Below are six dots. Draw line segments between each of the dots. What is the total number of line segments in the shape?

For Problem 6, fill in the table using the given information.

6. In the table below is a given number of points. Draw line segments between them when possible and put the total number of line segments possible in the rectangle at the bottom. The first one is done for you. No line segments can be drawn with only one point.

Let n be the number of points and see if you can come up with an expression that gives you the total number of line segments between each point.

1 Point	**2 Points**	**3 Points**	**4 Points**	**5 Points**
•	• •	• • •	• • • •	• • • • •
0 segments				

For Problem 7-10, follow the instructions to solve the problem.

7. Draw $\overline{AD}$.

8. Draw point G on $\overline{AD}$.

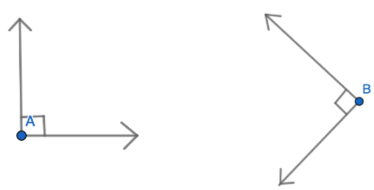
9. Complete the statement: $AD + GD =$ __________

10. What is another name for $\overline{AD}$?

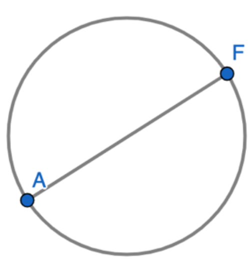
For Problem 11-15, tell whether the statement is true or false.

11. $DA - DG = GA$

12. $\angle A \ncong \angle B$



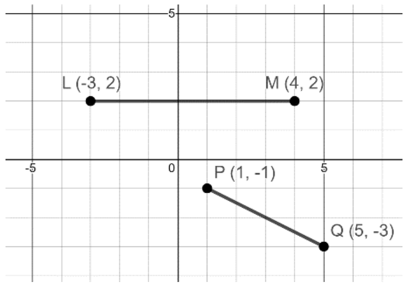
13. The radius of the circle is $\overline{AF}$. (Assume that $\overline{AF}$ goes through the center of the circle.)



14. If point L is the midpoint of $\overline{AF}$ in Problem 13, then the measure of AL is equal to the measure of LF.

15. Both $\overline{AL}$ and $\overline{LF}$ from Problem 14 are the radii of the circle.

For Problem 16-20, use the diagram below to solve the problem.



16. To find the length of the horizontal distance of LM, find the difference of the absolute value of the x-coordinates.

17. Fill in the blanks: To find the length of a ____________________ line segment on the coordinate grid, find the ____________________ of the absolute ____________________ of the y-coordinates.

18. Line segments are congruent if they have the same length. Tick marks are used to show when lines are congruent.

The symbol for congruency is "≅." We read "$\overline{MN} \cong \overline{PQ}$" as "line segment MN is congruent to line segment PQ."

$MN = PQ$ means that the measure of MN is equal to the measure of PQ. Perhaps both are one inch long.

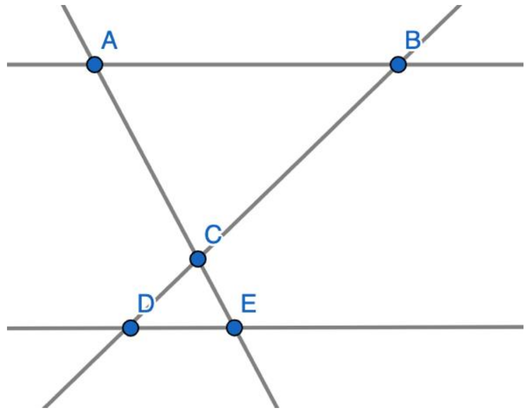
The midpoint of a line segment is in the middle and divides the line into two congruent segments. What is the midpoint of $\overline{LM}$?

19. The Midpoint Formula is the average of the x-coordinates and the average of the y-coordinates of the endpoints of the line segment. Find the midpoint of $\overline{PQ}$.

20. To find the distance of a line using the Pythagorean Theorem, we need the following distance formula: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Find the distance of PQ.

Section 3.2 Points and Lines
Practice Problems 3.2

For Problem 1-8, use the figure below to solve the problem.



1. Name two line segments and two rays. Use the correct letter order and symbols.

2. Name three collinear points.

3. Name two intersecting line segments and the point of intersection.

4. Name two line segments that appear to be parallel. How can you be sure?

5. Name two triangles that are formed where two intersecting lines meet.

6. How can you tell that there are no perpendicular lines in the figure?

7. If point C is lifted off the page and moved straight up with the lines still attached, is it coplanar or noncoplanar with the other points A, B, D, and E?

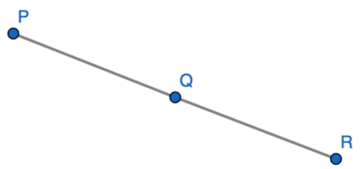
8. Stand in the middle of a living room or family room and find lines in the corners. Let the walls represent the planes. Find lines in the room that are parallel, perpendicular, and skew.

For Problem 9-11, solve the word problem.

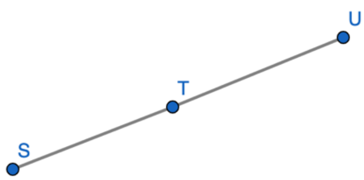
9. Given the endpoints of the line segments, find two line segments that are congruent.

$L(-5, 6)$ $M(-5, 4)$ $N(1, -2)$ $O(1, -4)$

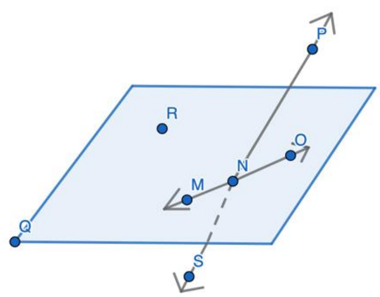
10. If $\cdot Q$ is the midpoint of $\overline{PR}$ and $PQ = 5$ cm., how long is QR?



11. If $\cdot T$ is the midpoint of $\overline{SU}$ and $SU = 13$ cm., how long is TU?



For Problem 12-20, use the figure to solve the problem. The shaded blue area is plane T.



12. Name three collinear points on plane T.

13. Name three collinear points, two of which are not on plane T.

14. Name two rays, one of which extends above plane T and one of which extends below plane T.

15. Name two angles containing point P.

16. Name a point on plane T that is not collinear with any other point.

17. Name a point that is an intersection of two lines.

18. Name the ray opposite $\overrightarrow{NO}$.

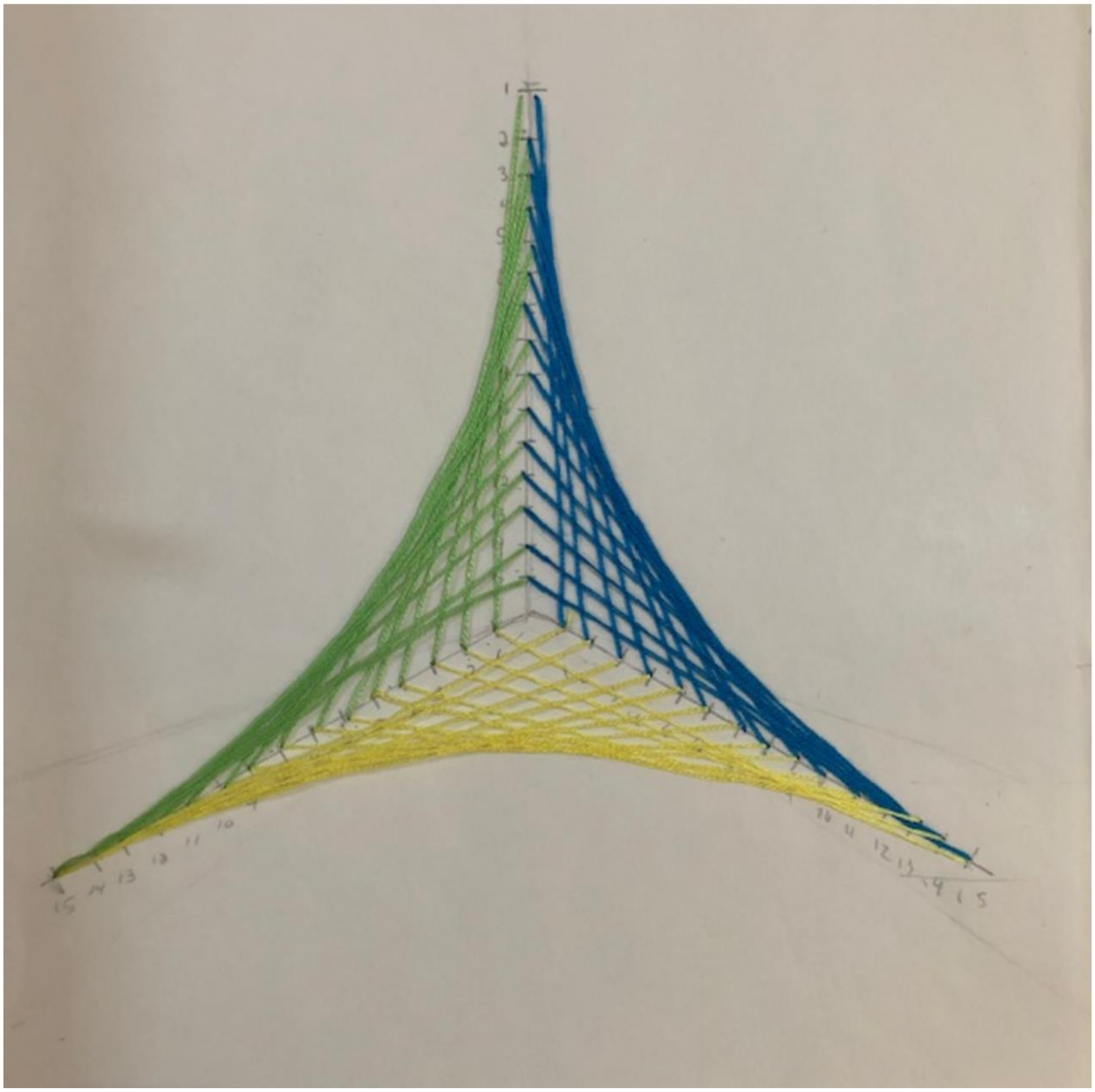
19. How many lines could be drawn to connect point R to point S?

20. How many points lie between point N and point P?

Section 3.3 Line Designs (Project)

Practice Problems 3.3

Below is a completed deltoid line design. Thick cross stitch thread was used with tagboard and a thick needle. Thread the needle and come up through the number in the back of the tagboard and stitch down through the number in front of the tagboard. Use different colors of needlepoint thread for each angle.



Make your own line design. You could cover a foam board with felt and use straight pins to mark each point and wrap the colored string around the pins going from one point to the next to create your design. You could also paint a piece of wood in the background and put nails in for the tick marks and wrap the yarn around the nails. String works better as it does not get so loose.

Section 3.4 Tools of Geometry
Practice Problems 3.4

For Problem 1-7, use a Mira® to perform the constructions.

1. Given $\overline{AB}$, locate the midpoint and label it "0."

2. Draw a perpendicular bisector of $\overline{AB}$ through the midpoint drawn at 0.

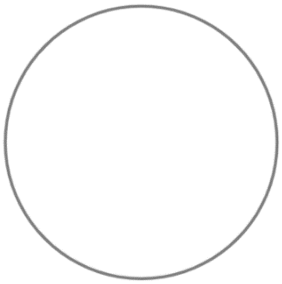
3. Draw the perpendicular bisector through point P, which is located on the line, but not at the midpoint.

4. Draw a line segment that is parallel to $\overline{AB}$.

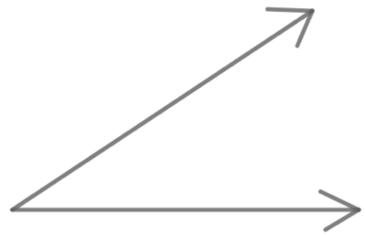
For Problem 5-7, use the given diagram to solve the problem.

5. Given $\overline{AB}$, draw the perpendicular bisector through point 0, which is not located on the line.

6. Given the circle below, construct the center of the circle.



7. Bisect the given angle.



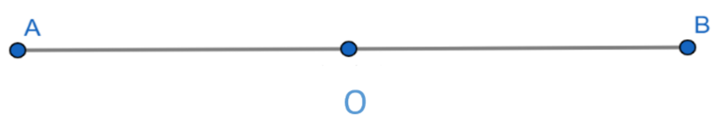
For Problem 8-10, use a compass to solve the problem.

8. Construct a perpendicular line to a line segment from a point on the line segment.

a) Draw arcs that are equidistant from point 0 to its right and left but are still on the line segment. Label the points where the arcs intercept the line segment as X and Y.

b) Open the compass a little more and draw an arc above the line segment from point X and draw another arc above the line segment from point Y. Make sure that each arc is long enough so they intercept.

c) Connect the intercepting arcs to point 0.

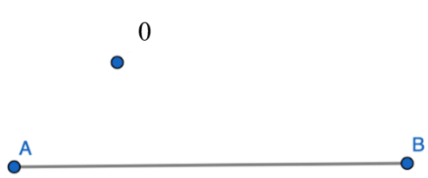


9. Construct a perpendicular line to a line segment from a point that is not on the line segment.

a) Draw an arc from point 0 so it crosses the line segment in two places. Label the points where the arc intercepts the line segment as X and Y.

b) Open the compass a little more and draw an arc below the point from point X and draw another arc below the point from point Y. Make sure that each arc is long enough so they intercept.

c) Connect the intercepting arcs through the line segment to point 0.

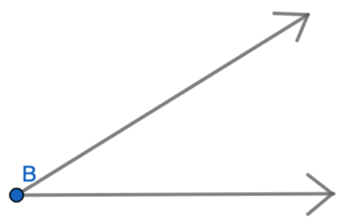


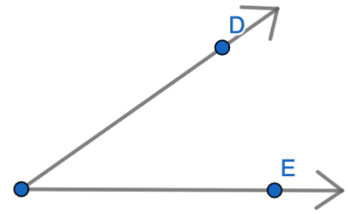
For Problem 10, use the information below to solve the problem.

An angle bisector is a ray that divides an angle into two angles that have the same measure.

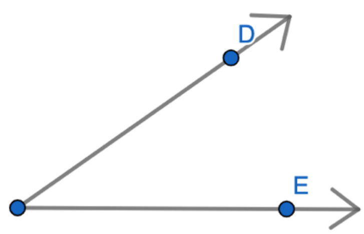
10. Construct an angle bisector of $\angle B$.

a) Open the compass so that it will cross the rays of the angle when the point of the compass is at point B. Draw an arc through the rays and mark the points of crossing D and E.

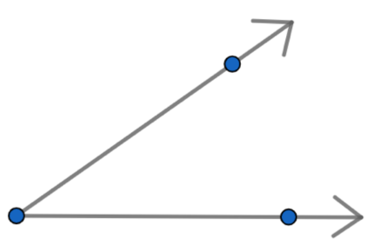




b) Place the point of the compass at point D and draw a small arc in the middle of the angle past the previous arc. With the same radius of the compass, draw another arc with the compass point at E. The arcs should cross.



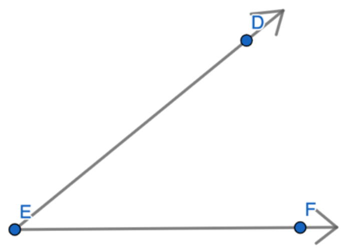
c) Use a ruler to draw a line from the angle vertex to the intercepting arcs.



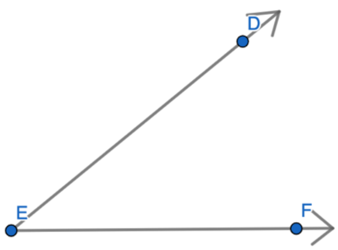
Section 3.5 Congruency

Practice Problems 3.5

For Problem 1-5, follow the steps to duplicate $\angle DEF$. We will call the duplication $\angle HGI$.

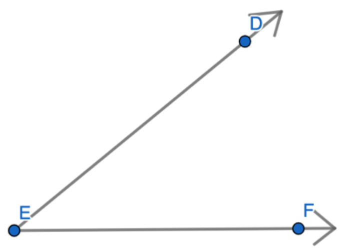


1. Put the point of your compass at E and draw an arc through $\overrightarrow{ED}$ and $\overrightarrow{EF}$. Does it matter how wide you open the compass?



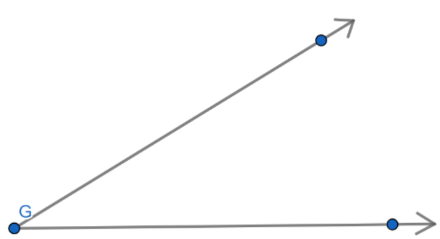
2. Draw point G and the extended ray and mark off the arc distance from point G. Why do we put point G first for the duplicate angle? Does it matter how long the ray extends?

3. Now we will go back to the original angle. Open the compass the distance of the arc created from where the arc intersects $\overrightarrow{EF}$ to where it crosses $\overrightarrow{ED}$. Make another arc here. What are we trying to accomplish with this step?



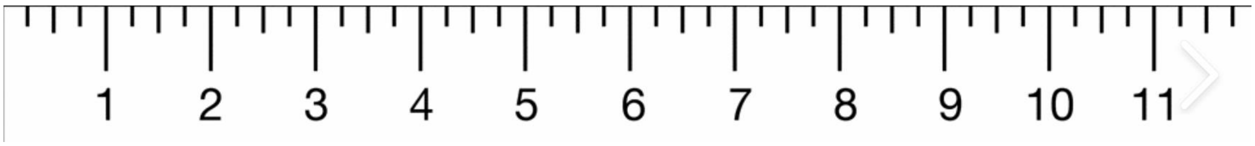
4. Mark the same arc from step 3 on the duplicate angle from the point where the original arc crosses the ray beginning at point G. What will be the final step?

5. Label the two new points created on the duplicate angle H and I. Lay one angle on top of the other to make sure they are equal. Write the new angle and the duplicate using the congruent symbol.



For Problem 6-8, draw the line segments. The ruler is for an example only and is not to scale.

6. $4\frac{1}{8}$ inches 7. $3\frac{1}{2}$ inches 8. $2\frac{5}{16}$ inches



For Problem 9-11, measure the line segments you drew in Problem 6-8 in millimeters and convert them to centimeters.

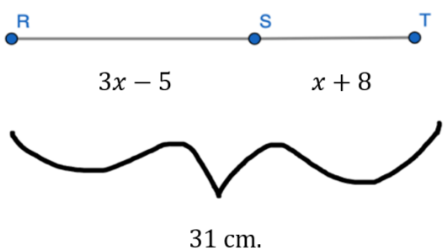
7. __________mm. __________cm.

8. __________mm. __________cm.

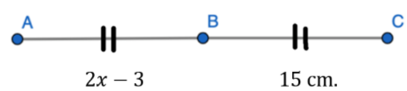
9. __________mm. __________cm.

For Problem 12 and 13, use the Segment Addition Postulate to solve the problem.

12. Find the length of RS and ST.

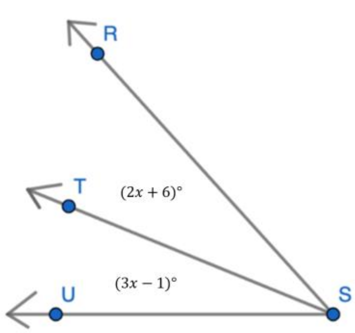


13. Find x and the length of AC.

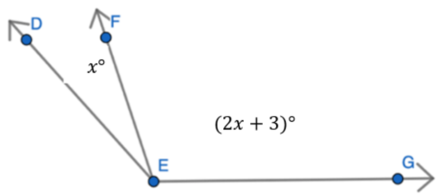


For Problem 14 and 15, use the Angle Addition Postulate to solve the problem.

14. Find the measure of $\angle RST$ and $\angle TSU$ ($m\angle RSU = 85°$).



15. Find the measure of $\angle DEF$ and $\angle FEG$ ($m\angle DEG = 105°$).



For Problem 16 and 17, tell whether the statement is true or false.

16. An axiom and a postulate are both rules accepted without proof.

17. An angle is a set of points consisting of two different rays and two different endpoints.

For Problem 18-20, match the definition to the term.

18. Two coplanar lines whose points are equidistant from one another.

a) Skew Lines

19. Two lines that are not coplanar and do not intersect.

b) Parallel Lines

20. Two coplanar lines that intersect at right angles.

c) Perpendicular Lines

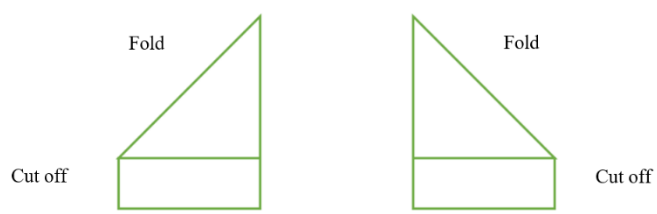
Section 3.6 Constructions Using Technology
Practice Problems 3.6

Kaleidoscope Name Design Project

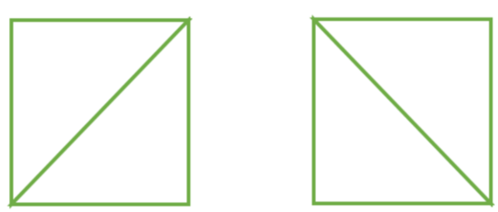
"There is a name in the kaleidoscope hanging in the background"

You have constructed a square using a geometer's technology utility and now you are going to construct a square using paper-folding. This square paper and Mira® will be used to create a symmetrical art design of your name.

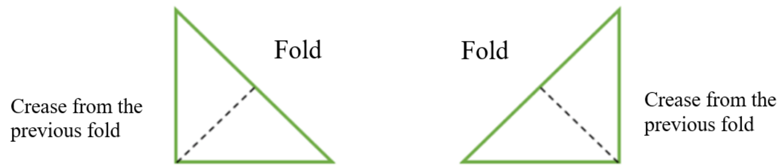
1. Get a piece of white paper. Fold the top left corner onto the right side so the edges match and then fold it. Score it along the diagonal. Cut off the strip of paper left at the bottom. (You can also do this from right to left; either way will work.)



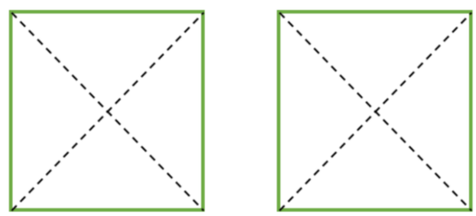
2. Once you cut off the strip at the bottom, open the paper back up. You will have a square whose sides are equal to the top edge of the paper (8" x 8").



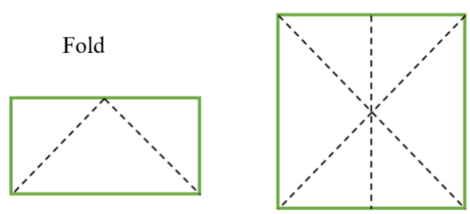
3. Now, match the corners away from the crease and make a new fold.



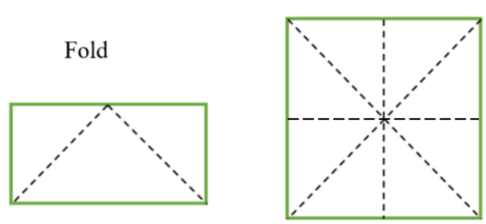
4 When you open the square up again, you will have folds along both diagonals.



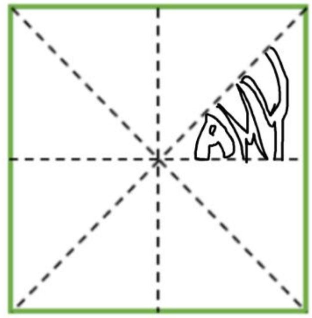
5. Now, fold the paper horizontally top to bottom and score it. When you open the paper again you should see 6 triangles.



6. Rotate the paper and fold it top to bottom one more time. When you open the paper again you should see 8 equal triangles.



7. Write your name in one of the triangles so that it fills the triangle. The top and the bottom of each letter should touch the sides of the triangle. The name Amy is used in the example below.



8. You will be reflecting your name all the way around the triangles that make up the square using the hypotenuse of each right triangle as the line of reflection.
You can reflect your name one of three ways:

1) Flip the triangle left or down onto the next triangle and trace the name on the back of it. Copy it onto the front when you open it (it should show through the paper). You will now have your name on both the front and the back, but you only need it on the front.

2) Copy your name on a piece of tracing paper and flip that over or down onto the next triangle. Put it behind the next triangle and trace it.

3) Put a Mira® along the hypotenuse of the given triangle. It acts as a see-through mirror. The beveled edge must be facing the design and touching the paper. Trace the image reflected through the Mira®. This is the easiest method; when you use the Mira®, you can see the image through it and copy it directly (however, a Mira® must be ordered through an online mathematics store so getting it can take longer and cost more than the other methods).

9. When your design is complete, you can color it in with pencils, markers, or even paint. You will see a design like what you see when you look through a kaleidoscope, but your name will appear in it.

(This example uses Grace instead of Amy, but it illustrates the same principle.)

The symmetry of God's designs in the universe can be seen in the work of artists from all generations. Constructive geometric designs deal with patterns and relationships. Many of these are derived from the Greeks' classical ideas of balance and symmetry. The Greek sculptures were a great example of their affinity for beauty; they used the golden ratio to make them balanced, symmetrical, and pleasing to the eye.

The kaleidoscope design incorporates balance and symmetry through reflection. Perhaps now you will want to paint your name design and frame/mount it to give as a gift to someone.

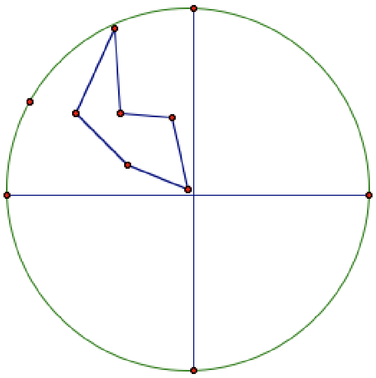
Now, we will be making a hand-drawn kaleidoscope.

Step 1: The ends of the kaleidoscope are circular. Draw a circle and mark the center of it.

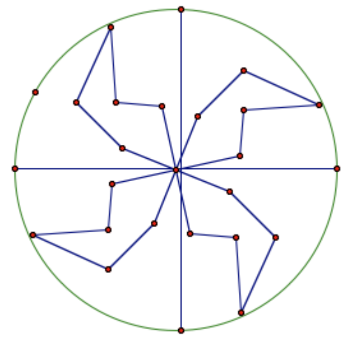
Step 2: Before drawing a design, decide how many sections there are going to be in the circle. If you want 6 sections, divide 360° by 6 and they will each be 60° degrees. If you want 4 sections, divide 360° by 4 and they will each be 90° degrees. If you want 9 sections, divide 360° by 9 and they will each be 40° degrees. By now, I think you get the idea. Make sure whatever number you choose is a multiple of 360° degrees.

Step 3: Put the protractor in the center of the circle. Make marks at every multiple of 90° if you want to create 4 sections. Make dots at 90° and 180° on top and do the same thing on the bottom of the circle by moving the protractor around the circle. Draw lines from each dot to the opposite side to make that number of sectors.

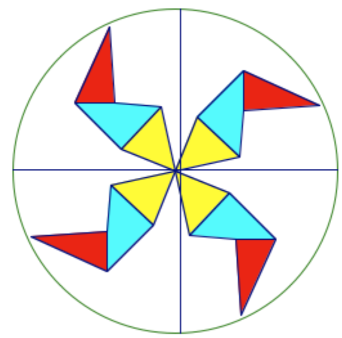
Step 4: Draw a many-sided figure in one of the sections that extends from the center of the circle to the circumference of the circle.

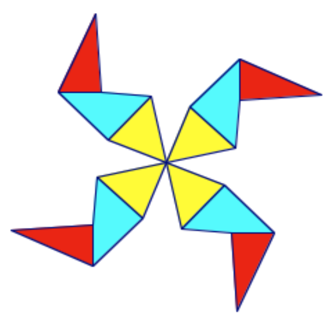


Step 5: Use tracing paper, copy the circle, sector lines, and design. Move it clockwise or counterclockwise around the circle so the lines meet and copy the design again until all the sections contain the design.



Step 6: Now that you have the kaleidoscope, color in the design. You may erase the outer circle and lines, or you may leave them.





Now, use technology (GeoGebra®) to make a kaleidoscope. Use the animation tools on GeoGebra® to make the design spin.

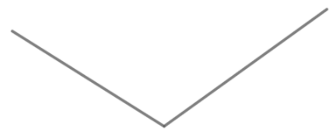
Section 3.7 Measuring and Drawing Angles

Practice Problems 3.7

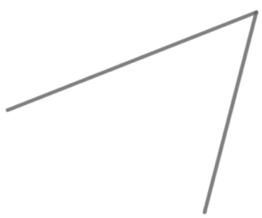
For Problem 1-6, measure the angle.

1.

2.

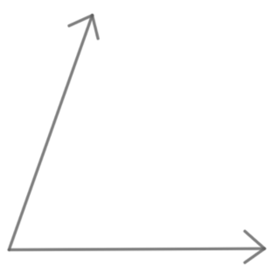


3.

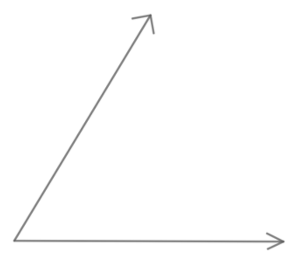


4.

5.



6.



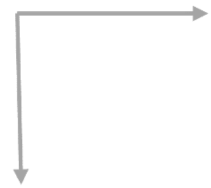
Estimate the measure of each angle for Problem 7-10. Check by measuring with a protractor.

7.

8.

9.

10.



For Problem 11-20, given the measure, draw the angle.

11. 108°

12. 26°

13. 49°

14. 45°

15. 110°

16. 31°

17. 95°

18. 18°

19. 2°

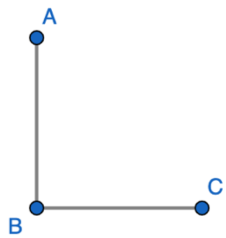
20. 90°

Section 3.8 Naming and Classifying Angles

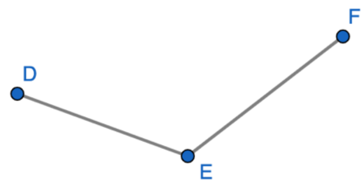
Practice Problems 3.8

For Problem 1-5, name the vertex of each angle, give the angle two different names (using symbols), and tell whether it is right, acute, or obtuse.

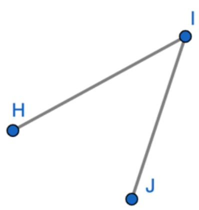
1.



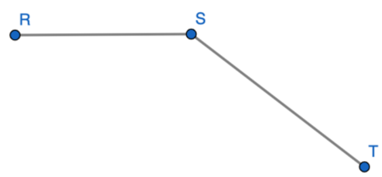
2.



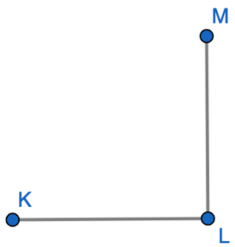
3.



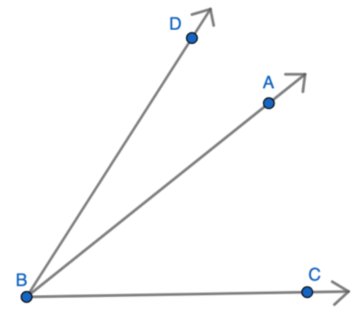
4.



5.



For Problem 6-10, use the figure below to solve the problem.



6. Name all three angles in the figure using symbols and letter names.

For Problem 7 and 8, use the figure above as well as the Angle Addition Postulate to fill in the blanks:

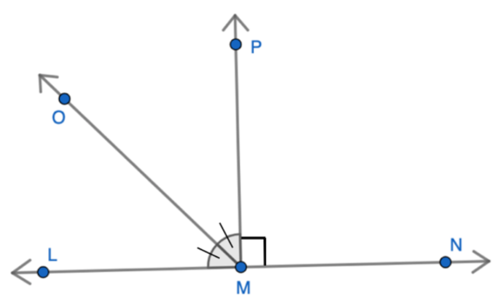
7. $m\angle DBA + m$________ $= m\angle DBC$

8. If $m\,\angle DBC = 89°$, and $m\,\angle ABC = 62°$, $m\,\angle DBA =$ ________.

9. If $m\,\angle DBC = 85°$ and the $m\,\angle DBA$ is one-half of 55° less than $m\,\angle ABC$, what is the measure of $\angle DBA$ and $\angle ABC$?

10. If $m\,\angle DBA$ is 15° and $m\,\angle ABC$ is 5° more than three times $m\,\angle DBA$, what is the measure of $\angle DBC$?

For Problem 11-16, use the diagram below to answer the questions if $\angle LMN$ is a straight angle.



11. What can you conclude about $\angle LMO$ and $\angle OMP$?

12. What can you conclude about $\angle PMN$?

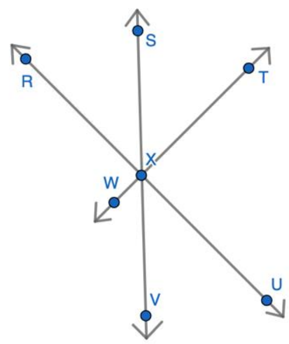
13. What is the measure of $\angle LMO$ and $\angle OMP$?

14. Name two pairs of supplementary angles.

15. Name one angle that is 180° degrees.

16. Name one pair of complementary angles.

For Problem 17-20, use the diagram to answer the problem.



17. Name three sets of collinear points that make three straight lines in the plane.

18. Name three pairs of vertical angles.

19. Can you conclude that $\angle RXS \cong \angle SXT$? Explain why or why not.

20. Given that $\angle RXS$ and $\angle SXT$ are complementary and $\angle RXW$ and $\angle RXT$ are a linear pair, complete the reasons for the proof that $\angle RXW$ is a right angle.

Statements	Reasons
1. $\angle RXS$ and $\angle SXT$ are complementary	1.
2. $\angle RXW$ and $\angle RXT$ are a linear pair	2.
3. $m\angle RXW + m\angle RXT = 180°$	3.
4. $m\angle RXS + m\angle SXT = 90°$	4.
5. $\angle RXS + \angle SXT = \angle RXT$	5.
6. $90° + m\angle RXW = 180°$	6.
7. $m\angle RXW = 90°$	7.
8. $\angle RXW$ is a right angle	8.

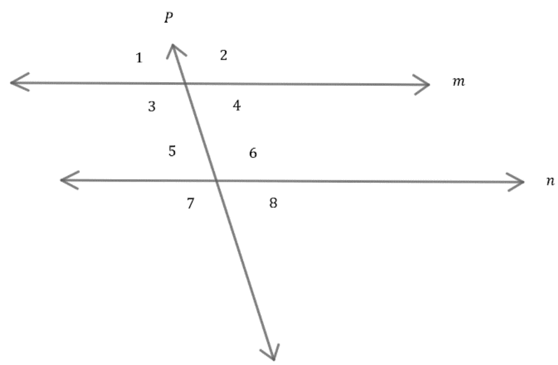
<u>Section 3.9 Angle Relationships</u>

<u>Practice Problems 3.9</u>

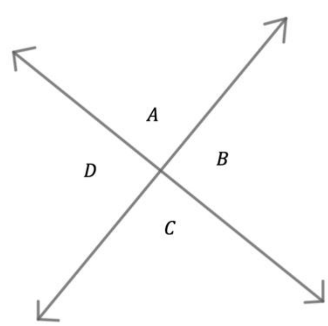
For Problem 1-3, use the given diagram to solve the problem.

1. In the diagram below, lines m and n are parallel. They are cut by the transversal P. Use tracing paper to trace $\angle 1$ and slide it down the left side of the transversal to cover $\angle 5$. This will demonstrate they are equal. Color these angles the same color. These are called corresponding angles because they have corresponding positions. They are both on the left side of the transversal and above the parallel lines.

There are three other pairs of corresponding angles in the diagram. Find them and color the corresponding pairs the same color. Make each pair a different color.

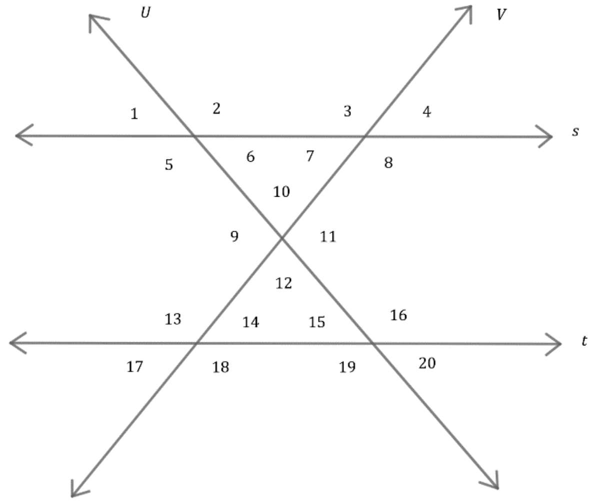


2. Take a piece of tracing paper. Fold the left corner up and crease it to make a line. Fold the right corner up and crease it to make a line that intersects the first line. Label the angles A, B, C, and D and darken the lines. Fold $\angle A$ onto $\angle C$. What do you notice? Fold $\angle B$ onto $\angle D$. What do you notice? Why is this so? (This is something we have previously learned.) What is each pair of angles called?



3. In Problem 2, $\angle A$ and $\angle C$ are called vertical angles. They are formed when two lines cross. They form a "V" and they intersect at the same point. They are not adjacent because they do not share a common side, but they do share a common point. Find all the pairs of vertical angles in the diagram in Problem 1.

For Problem 4-8, use the diagram to fill in the blanks or solve the problem. (Line s is parallel to t. Both are cut by transversal U and transversal V.)



4. a) $\angle 4 \cong \angle$________ by corresponding angles b) $\angle 14 \cong \angle$________ by vertical angles

c) If $\angle 4 \cong \angle$________ and $\angle 14 \cong \angle 17$, then $\angle 4 \cong \angle$________ because when two angles are congruent to the same ____________________, then the two angles are ____________________ to each other.

5. a) Both $\angle 19$ and $\angle 20$ are ____________________ because they have a sum of 180°.

b) Both $\angle 19$ and __________ are ____________________ because they have a sum of 180°.

c) If $\angle 20$ is acute, so is $\angle$________ because they are congruent by vertical ____________________.

d) When two lines in a plane ____________________ each other, they form vertical angles and vertical angles are always ____________________.

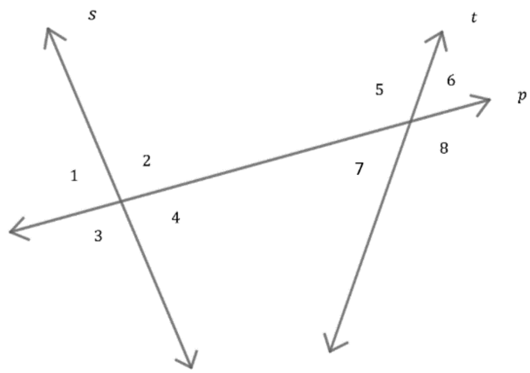
e) If $\angle 20$ and $\angle 15$ are acute, $\angle 19$ must be ____________________.

6. What is the sum of $m\angle 6 + m\angle 7 + m\angle 10$?

7. Name three other acute angles that have a sum of 180°.

8. Why aren't there any complementary angles?

For Problem 9-16, use the diagram below to solve the problem. (Transversal *p* cuts through non-parallel lines *s* and *t*).



9. Will lines *s* and *t* eventually intersect in the plane? Explain why of why not.

10. Name all the pairs of alternate interior angles.

11. Name all the pairs of alternate exterior angles.

12. Name all the pairs of consecutive interior angles.

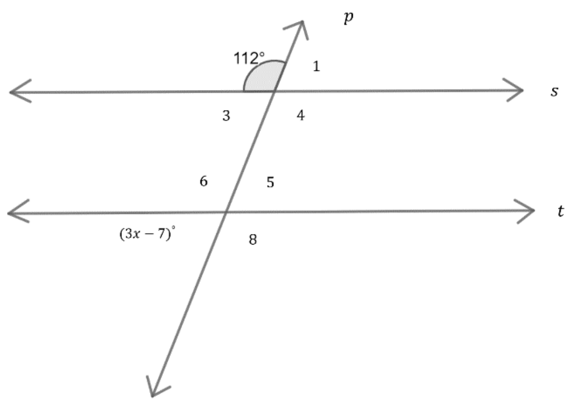
13. Name all the pairs of corresponding angles.

14. Are the pairs of corresponding angles supplementary? Explain why or why not.

15. Are the pairs of alternate interior angles congruent? Are the pairs of alternate exterior angles congruent? Explain why or why not.

16. Name all the pairs of vertical angles. Are they congruent?

For Problem 17-20, use the diagram below to solve the problem. (Transversal p cuts through parallel lines *s* and *t*).



17. What is the measure of $\angle 1$ and why?

18. What is the value of x?

19. What is the measure of angle $(3x - 7)°$ and $\angle 8$?

20. What are the measures of $\angle 3$, $\angle 4$, and $\angle 5$ and why?

Section 3.10 Theorems of Parallel and Perpendicular Lines

Practice Problems 3.10

For Problem 1, fill in the blanks to complete the Corresponding Angles Theorem.

1. If two __________ lines are cut by a ____________________, then the __________ of ____________________ angles are ____________________.

For Problem 2-4, follow the given directions to solve the problem.

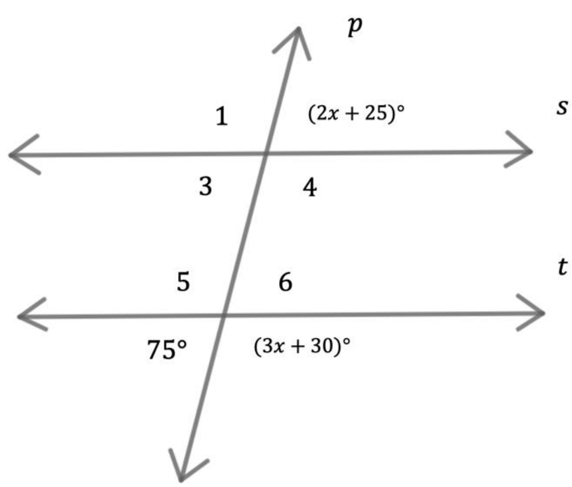
2. Write the converse of the Corresponding Angles Theorem.

3. The following statement has an error. Correct the error to make the statement true:

"If two lines are cut by a transversal so the consecutive interior angles are complementary, then the lines are parallel."

4. Name the Theorem in Problem 3.

For Problem 5-11, use the diagram below to solve the problem.



5. Fill in the blanks: The Straight Angle Postulate states that an angle that lies on a ____________ line is __________°.

6. Use the postulate from Problem 5 to solve for x.

7. Use the 75° angle to demonstrate that $s \parallel t$.

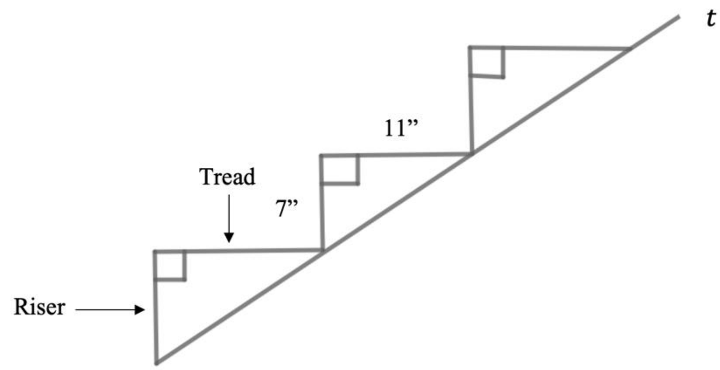
8. What is the name of the Theorem used in Problem 7?

9. Find the measure of $\angle 1$.

10. What other angles are congruent to $\angle 1$ and why are they congruent?

11. Which two theorems can be used to find the measure of $\angle 6$?

For Problem 12-15, use the step diagram to solve the problem.



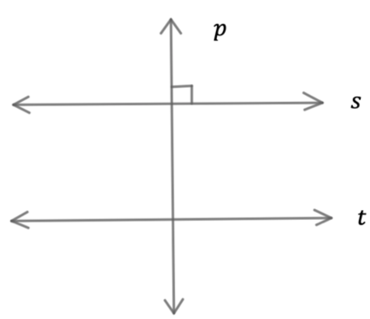
12. According to the international building code, all the risers of the steps are to be 7” inches and all the treads of the steps are to be 11”. Which Theorem states that the tread of the bottom step is parallel to the tread of the top step?

13. How do you know the riser of the bottom step is perpendicular to the tread of the bottom step?

14. If the riser is perpendicular to the tread of the bottom step, what can be concluded about all the treads and risers of the steps?

15. What is the recommended slope for steps according to the International Building Code?

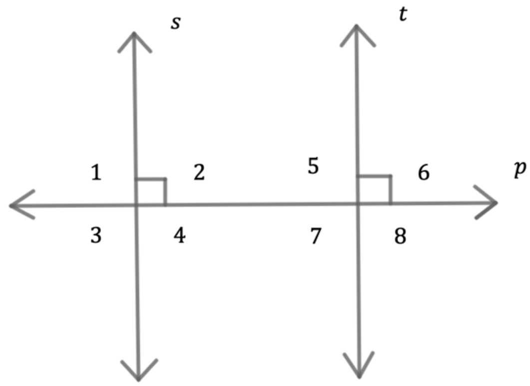
For Problem 16, fill in the blanks using the diagram for the Perpendicular Transversal Theorem.



16. If a transversal in a plane is ____________________ to one of the two parallel lines, then the ____________________ is perpendicular to the other ____________________.

If $s \parallel t$, and $p \perp s$, then $p \perp$ _______.

For Problem 17-21, use the diagram below to answer the problem.



17. What information can you conclude about the diagram by looking at it?

18. What can you conclude about lines s and t? Why can you make this conclusion?

19. What is the measure of angle 2 and what is the measure of angle 6?

20. The Linear Pair Perpendicular Theorem states that if two lines intersect in such a way that the linear pairs are congruent, then the lines are perpendicular. Name at least two linear pairs in the diagram.

For Problem 21, complete the Linear Pair Perpendicular Theorem Proof.

Statement	**Reasons**
1. $\angle 2$ is a right angle	1. ____________________
2. $\angle 1$ and $\angle 2$ are a linear pair	2. Definition of a ____________________ Pair
3. $m\angle 2 = 90°$	3. Definition of ____________________ angles
4. $m\angle 1 + m\angle 2 = 180°$	4. ____________________ Angle Postulate
5. $m\angle 1 + 90° = 180°$	5. ____________________ Property of Equality
6. $m\angle 1 = 90°$	6. ____________________ Property of Equality
7. $m\angle 1 = m\angle 2$	7. Substitution ____________________ of ____________________
8. $\angle 1 \cong \angle 2$	8. ____________________ of Angles
9. $s \perp t$	9. Definition of ____________________ Lines

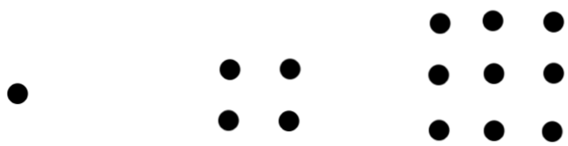
Section 3.11 Inductive and Deductive Reasoning

Practice Problems 3.11

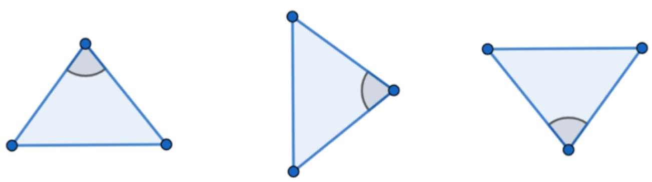
For Problem 1-3, draw the next figure in each series.

1.

2.



3.



For Problem 4-6, make a conjecture based on the given information.

4.

$3 \cdot 1 = 3$	$5 \cdot 1 = 5$	$7 \cdot 1 = 7$	$9 \cdot 1 = 9$
$3 \cdot 2 = 6$	$5 \cdot 2 = 10$	$7 \cdot 2 = 14$	$9 \cdot 2 = 18$

5. Lily loves to draw pink flowers. All pink flowers are beautiful.

6. Sebastian is a good Algebra 2 student. All good Algebra 2 students are also good at Geometry.

For Problem 7 and 8, make a conjecture from the given information and then test the conjecture by giving a few examples.

7. What can you conclude about the sign of the product of an even number of negative integers?

8. What can you conclude about the sum of any three consecutive negative integers?

For Problem 9 and 10, try to prove the conjecture is false by finding a counterexample.

9. The value of y^3 is always greater than y^2.

10. The difference of two numbers is always less than the sum.

For Problem 11-14, tell whether inductive or deductive reasoning was used to reach the conclusion.

11. Each time Malerie goes to the store she buys chocolate. The next time Malerie goes to the store she will come home with chocolate.

12. The product of an odd number of negative numbers is negative. Three negative numbers are multiplied. The product of the three negative numbers is negative.

13. You will reap what you sow. If you give kindness, you will receive kindness. If you are unkind to others, others will be unkind to you.

14. If a polygon has all sides congruent, then it is a regular polygon. If it is a regular polygon, then it has all interior angles congruent.

For Problem 15-17, given the information below, underline the hypothesis with one line and underline the conclusion with two lines.

"If P, then Q" is a conditional statement. The hypothesis is P and the conclusion is Q.

15. If an angle is 90°, then it is a right angle.

16. If an angle is less than 90°, then it is a complementary angle.

17. If two lines are parallel to the same line, then the lines are parallel to each other.

For Problem 18-20, given the information below, use the Law of Syllogisms to write a conditional statement that follows from the given conditional statements.

A syllogism states that if P, then Q, and if Q, then R, then if P, then R. If the first two statements are true, then the last statement is also true.

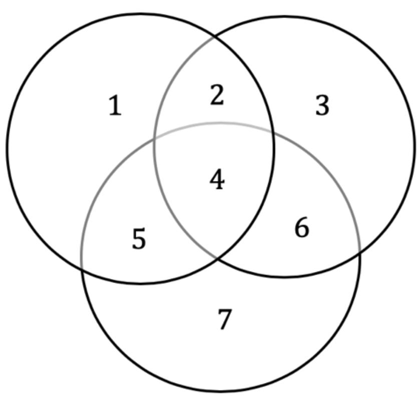
18. Kyndra is a quilter in Darke County. She is making a quilt for the fair. All fair quilts are donated to the local children's hospital after the fair.

19. If Faylynn gets an A on the test, she gets to go shopping. If she goes shopping, she gets to buy a dress.

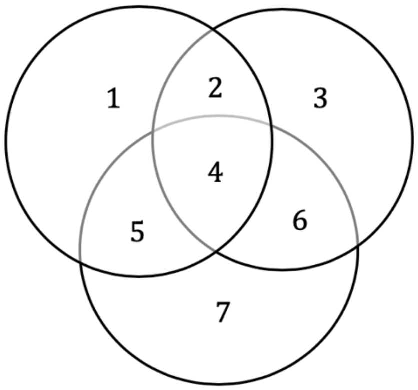
20. All people who love God love others. All people who love others forgive others.

For Problem 21-28, tell if the following syllogisms are VALID or INVALID. Complete the Venn Diagram and explain why those you decided were invalid do not have a logical conclusion based on the premise.

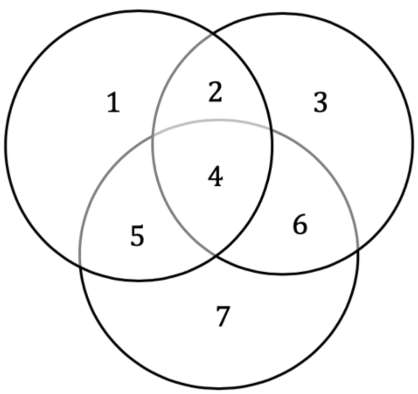
21. All stuffed animals are soft; all soft things are fluffy; therefore, all stuffed animals are fluffy.



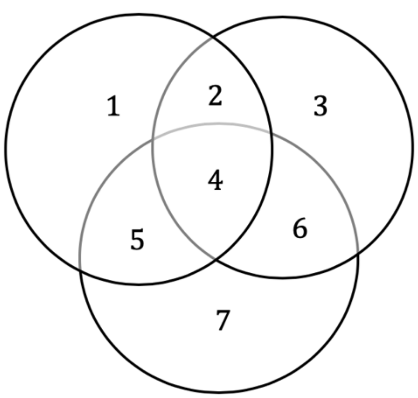
22. All tea is hot; nothing hot is chocolate; therefore, no tea is chocolate.



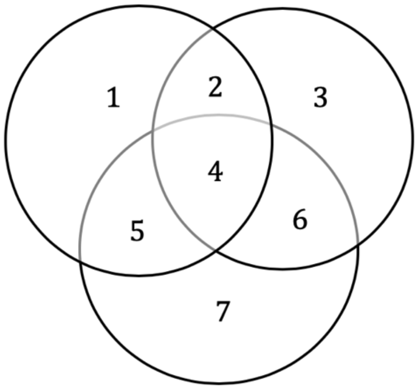
23. Some light things are pink; all powder is pink; therefore, some pink things are powder.



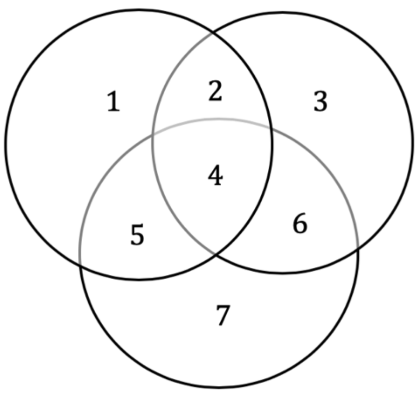
24. All pre-algebra is fun; no trigonometry is pre-algebra; therefore, no trigonometry is fun.



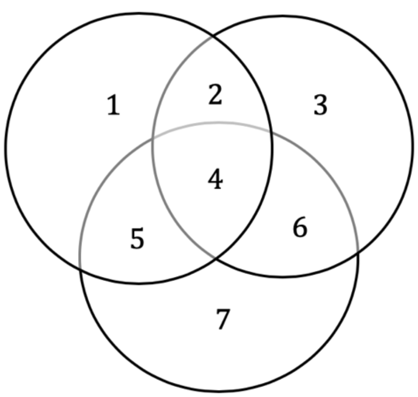
25. All apples are red; all red fruit is sweet; therefore, no apples are sweet.



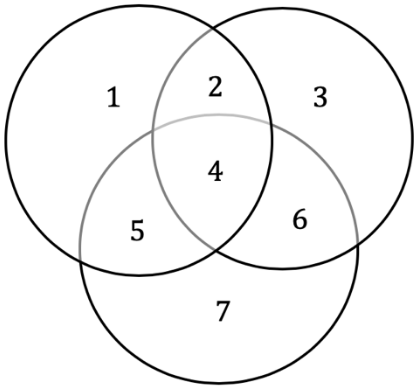
26. All adults watch television; no television viewers are bears; therefore, no adults are bears.



27. All A is B; all B is C; therefore, all A is C.

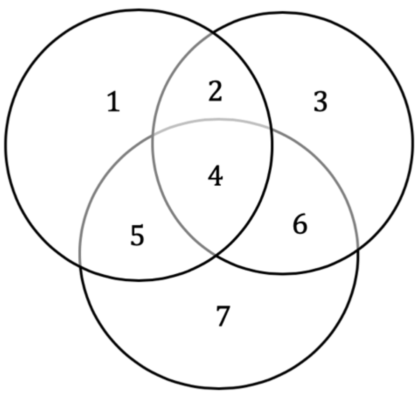


28. All pre-algebra is difficult; no pre-calculus is pre-algebra; therefore, no pre-calculus is difficult.

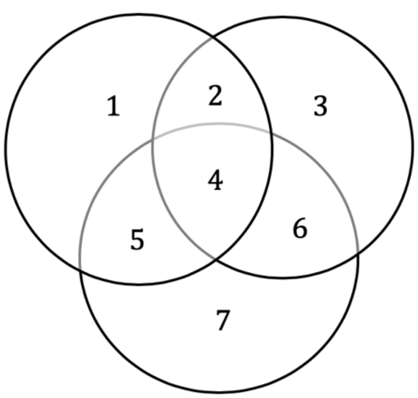


For Problem 29 and 30, tell whether the syllogisms are VALID or INVALID. Then complete the Venn Diagram.

29. All math teachers are nice; all nice people are smart; therefore, all math teachers are smart.



30. All good students read well; no math students are good students; therefore, no math students read well.



Section 3.12 Conditional Statements and Truth Tables

Practice Problems 3.12

For Problem 1-4, draw one line under the hypothesis, and two lines under the conclusion.

1. If it is raining outside, Alyssa will not ride her bike to the park.

2. If two lines are perpendicular, then they form a 90° angle.

3. If an angle is less than 90°, then it is an acute angle.

4. If an angle is an acute angle, then it is less than 90°.

For Problem 5-9, write the statements as conditional statements in "if-then" form.

5. Jessie is a musician, and she plays piano.

6. Collin plays guitar, and he is an excellent musician.

7. When you cry out to God, then He hears your prayers.

8. All birds have wings.

For Problem 9-12, use the equation $3x + 9 = 21$ and the solution $x = 2$ to solve the problem.

9. If p is $3x + 9 = 21$, what is q?

10. Write the equation and solution as a conditional statement in "if-then" form.

11. Write the contrapositive of the conditional statement in "if-then" form, as "∼q → ∼p."

12. a) Is p → q a true statement? b) Is ∼q → ∼p a true statement?

For Problem 13-19, use the statement below to solve the problem.
Two angles are complementary whose measures have a sum of 90°.

13. Write the conditional statement and tell whether it is true or false.

14. Write the converse of the conditional statement and tell whether it is true or false.

15. Write the negation of the hypothesis of the conditional statement.

16. Write the negation of the conclusion of the conditional statement.

17. Write the inverse of the conditional statement and tell whether it is true or false.

18. Write the contrapositive of the conditional statement and tell whether it is true or false.

19. Write the biconditional statement and tell whether it is true or false.

For Problem 20, use the statement to solve the problem. Let p be "two angle measures have a sum of 180°" and let q be "two angles are supplementary."

20. a) Write the contrapositive statement. Equivalent statements can be one of two things: both true; both false. Is the contrapositive equivalent to the conditional statement?

b) Write the biconditional statement.

Section 3.13 Formal Geometric Proofs

Practice Problems 3.13

For Problem 1 and 2, complete the given proof.

1. Given: B is the midpoint of $\overline{AC}$

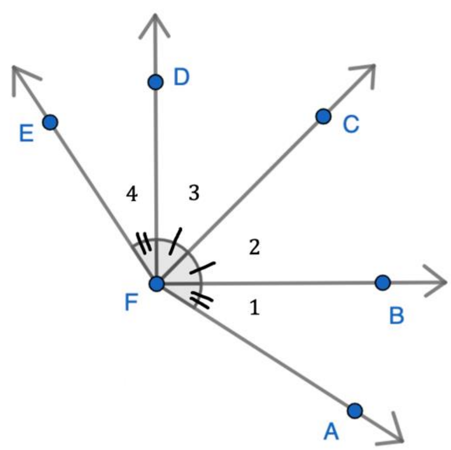
Prove: $\overline{BC} \cong \overline{AB}$

Statement	Reason
1. B is the midpoint of $\overline{AC}$	1.
2. $AB = BC$	2.
3. $\overline{AB} \cong \overline{BC}$	3.
4. $\overline{BC} \cong \overline{AB}$	4.

2. Given: $m\angle 1 = m\angle 4$

$m\angle 2 = m\angle 3$

Prove: $m\angle AFC = m\angle EFC$



Statement	Reason
1. $m\angle 1 = m\angle 4$	1.
2. $m\angle 2 = m\angle 3$	2.
3. $m\angle AFC = m\angle 1 + m\angle 2$	3.
4. $m\angle EFC = m\angle 3 + m\angle 4$	4.
5. $m\angle AFC = m\angle 3 + m\angle 4$	5.
6. $m\angle EFC = m\angle AFC$	6.

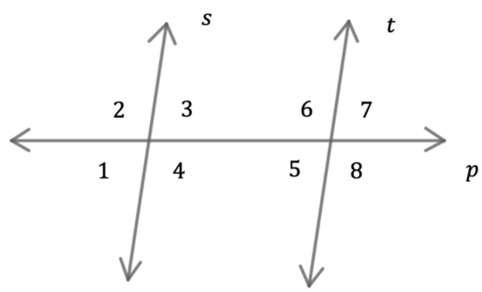
For Problem 3, solve the equation and use the information below to complete the proof.

3. Solve $2(3x - 5) = 14$ for x. Let the steps be the statements and list the property applied to each step as the reason.

Statement	**Reason**
1.	1. Given
2. $6x - 10 = 14$	2. Distributive Property
3.	3. Addition Property of Equality
4. $x = 4$	4.

For Problem 4 and 5, use the diagram below to complete the proof.

4.



Given: $\angle 5$ and $\angle 6$ are supplementary
Given: $\angle 2$ and $\angle 5$ are supplementary
Prove: $m\angle 2 = m\angle 6$

Statement	**Reason**
1. $\angle 5$ and $\angle 6$ are supplementary	1.
2.	2. Given
3. $m\angle 5 + m\angle 6 = 180°$	3.
4. $m\angle 2 + m\angle 5 = 180°$	4.
5. $m\angle 5 + m\angle 6 = m\angle 2 + m\angle 5$	5.
6.	6.

5. Given: $\angle 1 \cong \angle 5$
Prove: $\angle 3 \cong \angle 7$

Statement	**Reason**
1. $\angle 1 \cong \angle 5$	1.
2. $\angle 1 \cong \angle 3$	2.
3. $\angle 5 \cong \angle 3$	3.
4. $\angle 5 \cong \angle 7$	4.
5. $\angle 3 \cong \angle 7$	5.

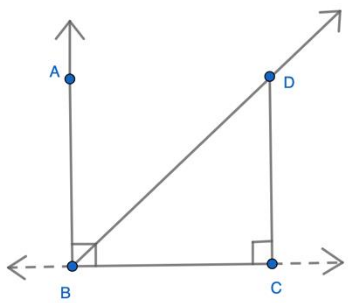
For Problem 6, apply the given property to the given information.

6. a) Given $\angle A$, apply the Reflexive Property of Angle Congruence.

b) Given $\angle A \cong \angle B$, apply the Symmetric Property of Angle Congruence.

c) Given $\angle A \cong \angle B$ and $\angle B \cong \angle C$, apply the Transitive Property of Angle Congruence.

For Problem 7-9, use the diagram below and the given information to complete the proof.



Given: $\angle ABC$ is a right angle

Given: $\angle DCB$ is a right angle

Given: $\overrightarrow{BD}$ bisects $\angle ABC$

7. Prove: $\angle ABD \cong \angle BDC$

Statement	Reason
1.	1. Given
2.	2. Given
3.	3. Definition of a Right Angle
4.	4. Definition of a Right Angle
5.	5. Reflexive Property of Segment Congruence
6.	6. Lines Perpendicular to a Transversal Theorem
7.	7. Given
8.	8. Alternate Interior Angles Congruence Theorem

8. Prove: $\angle CBD \cong \angle BDC$

Statement	Reason
1. $\angle ABC$ is a right angle	1.
2.	2. Given
3. $\overrightarrow{BA} \perp \overleftrightarrow{BC}$	3.
4.	4. Definition of Perpendicular Lines
5. $\overline{BC} \cong \overline{CB}$	5.
6.	6. Lines Perpendicular to a Transversal Theorem
7. $\angle ABD \cong \angle BDC$	7.
8.	8. Given
9. $\angle ABD \cong \angle CBD$	9.
10.	10. Transitive Property of Angle Congruence

9. Prove: $\angle DBC$ and $\angle BDC$ are complementary

(Continue the steps from the proof in Problem 8)

Statement	Reason
11. $m\angle ABC = 90°$	11.
12. $m\,\angle ABD + m\,\angle DBC = m\,\angle ABC$	12.
13. $m\,\angle ABD + m\,\angle DBC = 90°$	13.
14. $\angle ABD$ and $\angle DBC$ are complementary	14.
15. $\angle CBD$ and $\angle BDC$ are complementary	15.

For Problem 10, fill in the blanks.

10. A property that is accepted without proof is called a(n) _____________________ or a _____________________.

A type of reasoning that investigates patterns is called _____________________ reasoning.

A(n) _____________________ has a premise and a conclusion. It is either VALID or _____________________. A true hypothesis can sometimes lead to a false _____________________ when using truth tables.

The logical reasoning used to complete formal proofs is called _____________________ reasoning. Once a property or rule is proven, it is called a(n) _____________________.

Section 3.14 Module Review

This module began with Euclid's five postulates. There are many more, some of which you have learned throughout this module. Just as clubs have rules and organizations have a code of conduct to ensure people get along, there are postulates that make sure everyone is on the same page mathematically.

For Problem 1-10, fill in the blanks concerning the five postulates.

1. Line Postulate: Exactly one line can be constructed through __________ points.
2. Midpoint Postulate: There is exactly one midpoint that lies in the __________ of any line _______________ that divides it into _______________ equal parts.
3. Segment Duplicate Postulate: Only one line _______________ can be constructed that is _______________ to another _______________ segment.
4. Segment Addition Postulate: If point A is on the line segment MP and in between points M and P, then $\mathrm{MA} + \mathrm{AP} =$__________.
5. Line Intersection Postulate: Two lines intersect in exactly __________ point(s).
6. Parallel Line Postulate: There is only one line that can be constructed parallel to any given line through a _______________ not on the given line.
7. Perpendicular Line Postulate: There is only __________ line(s) that can be constructed perpendicular to any given line through a point not on the given line.
8. Angle Bisector Postulate: Only __________ angle bisector(s) can be constructed in any __________.
9. Angle Duplicate Postulate: Only one angle can be constructed _______________ to any given angle.
10. Angle Addition Postulate: If point H lies on the interior ray of $\angle$__________ then $m\angle GEH + m\angle HEF = m\angle GEF$.

For Problem 11-20, use your tools of geometry to draw examples of the ten postulates listed in Problem 1-10.

11. Line Postulate
12. Midpoint Postulate
13. Segment Duplicate Postulate
14. Segment Addition Postulate

15. Line Intersection Postulate

16. Parallel Line Postulate

17. Perpendicular Line Postulate

18. Angle Bisector Postulate

19. Angle Duplicate Postulate

20. Angle Addition Postulate

Section 3.15 Module Test

For Problem 1-10, select the correct answer from the multiple choices.

1. Let p be “you are sixteen years old” and let q be “you are able to obtain a driver’s license in the United States of America.” What is the conditional statement? Tell whether it is true or false.

a) If you are not able to obtain a driver’s license in the USA, then you are not sixteen years old.
b) If you are able to obtain a driver’s license in the USA, then you are sixteen years old.
c) If you are not sixteen years old, then you are not able to obtain a driver’s license in the USA.
d) If you are sixteen years old, then you are able to obtain a driver’s license in the USA.

2. Let p be “you are a point guard” and let q be “you are on a basketball team.” What is the converse? Tell whether it is true or false.

a) If you are on a basketball team, then you are a point guard.
b) If you are a point guard, then you are on a basketball team.
c) If you are not a point guard, then you are not on a basketball team.
d) If you are not on a basketball team, then you are not a point guard.

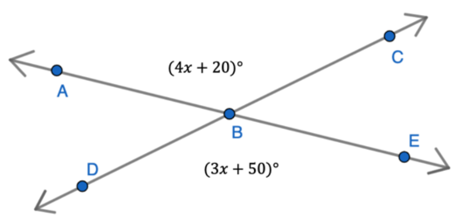
3. Let p be “a polygon is a rectangle” and let q be “a polygon is a square.” What is the inverse? Tell whether it is true or false.

a) If a polygon is not a square, then the polygon is not a rectangle.
b) If a polygon is not a rectangle, then the polygon is not a square.
c) If a polygon is a rectangle, then the polygon is a square.
d) If a polygon is a square, then the polygon is a rectangle.

4. Let p be “two adjacent angles have a sum of 180°” and let q be “two angles are a linear pair.” What is the contrapositive? Tell whether it is true of false.

a) If two adjacent angles have a sum of 180°, then the two angles are a linear pair.
b) If two adjacent angles do not have a sum of 180°, then the angles are not a linear pair.
c) If two adjacent angles are not a linear pair, then the two adjacent angles do not have a sum of 180°.
d) If two angles are a linear pair, the two adjacent angles have a sum of 180°.

5. What is the value of x and the measure of $\angle ABC$?



a) $x = 20, m\angle ABC = 70°$

b) $x = 50, m\angle ABC = 120°$

c) $x = 30, m\angle ABC = 140°$

d) $x = 20, m\angle ABC = 130°$

6. What is the measure of $\angle DBE$ in the figure in Problem 5?

a) $m\ \angle DBE = 70°$
b) $m\ \angle DBE = 120°$
c) $m\ \angle DBE = 140°$
d) $m\ \angle DBE = 130°$

7. Given the premises for the syllogism, write the conclusion:

All mice are small; All cats are small

a) Some cats are small
b) No mice are big
c) No cats are big
d) All mice are cats

8. Given the syllogism, circle the correct statement:

All 180° angles form a straight angle; All straight angles form a straight line; Therefore, all 180° angles form a straight line

a) INVALID conclusion
b) VALID conclusion
c) No Hypothesis
d) No conclusion

9. Rewrite the given statement in if-then form:

All 23° angles are acute

a) If an angle is 23°, then it is not obtuse.
b) If an angle is acute, then it is 23° degrees.
c) If an angle is not 23°, then it is not acute.
d) If an angle is 23°, then it is an acute angle.

10. Circle the choice that represents the contrapositive for "if p, then q."

a) p → q
b) ~q → ~p
c) ~p → ~q
d) q → p

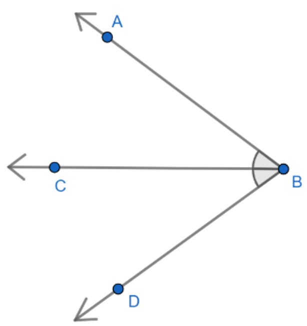
For Problem 11-14, tell whether the reasoning is inductive or deductive and explain why.

11. All angles with a measure greater than 90° but less than 180° are obtuse angles. Angle D is 147° degrees; Angle D is obtuse.

12. Wyatt likes chocolate. He eats a piece of chocolate every day. On Sunday, Wyatt will eat one piece of chocolate.

13. When Briane is late to soccer practice, she must run two extra laps. The next practice Briane is late she will run two laps.

14. In the given figure, $m\angle ABC + m\angle CBD = m\angle DBA$.



For Problem 15-17, choose the correct Property or Theorem.

15. If $\overline{FG} \cong \overline{GH}$ and $\overline{GH} \cong \overline{HI}$, then $\overline{FG} \cong \overline{HI}$.

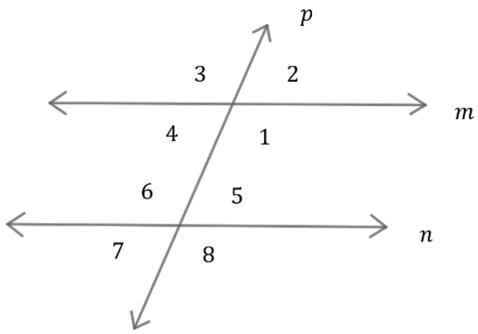
a) Reflexive Property of Segment Congruence

b) Substitution Property of Angle Congruence

c) Symmetric Property of Segment Congruence

d) Transitive Property of Segment Congruence

16.



$m \parallel n$
p is a transversal
$\angle 1$ and $\angle 5$ are supplementary

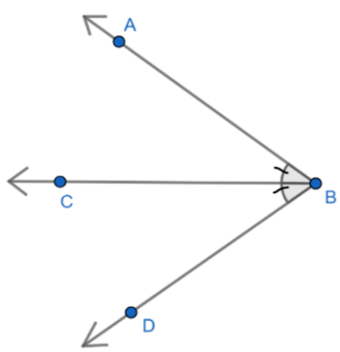
a) Consecutive Interior Angles Theorem

b) Corresponding Angles Congruence Theorem

c) Linear Pair Postulate

d) Parallel Line Postulate

17.



$\overrightarrow{BC}$ bisects $\angle ABD$
$\angle ABC \cong \angle CBD$

a) Linear Pair Postulate

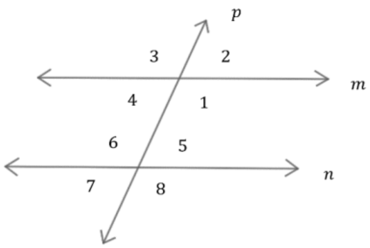
b) Angle Addition Theorem

c) Corresponding Angles Congruence Theorem

d) Angle Bisector Theorem

For Problem 18-20, use the given directions to solve the problem.

18. Find all the angles in the diagram below given that $m\angle 1 = 113°$ and $m \parallel n$, which is cut by the transversal p.

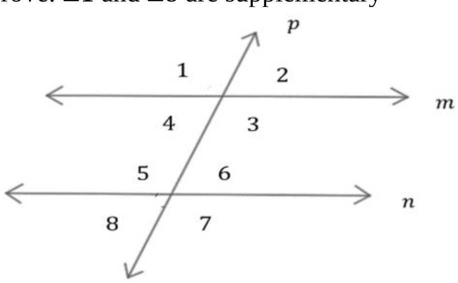


19. Complete the statements for the theorems.

a) The Consecutive Exterior Angles Theorem states that "If two parallel lines are cut by a transversal then the consecutive exterior angles are supplementary." Write the Consecutive Exterior Angles Converse Theorem.

b) The Complements Congruence Theorem states that "If two angles are complementary to the same angle or to congruent angles, then they are congruent to each other. Write the Supplements Congruence Theorem.

20. Complete the Reasons to Prove the Consecutive Exterior Angles Theorem: "If two parallel lines are cut by a transversal, then the consecutive exterior angles are supplementary."
Given: $m \parallel n$ (cut by transversal p)
Prove: $\angle 1$ and $\angle 8$ are supplementary



Statement	Reason
1. $m \parallel n$	1.
2. $\angle 1 \cong \angle 3$	2.
3. $\angle 3$ and $\angle 4$ are supplementary	3.
4. $\angle 1$ and $\angle 4$ are supplementary	4.
5. $\angle 4 \cong \angle 8$	5.
6. $\angle 1$ and $\angle 8$ are supplementary	6.