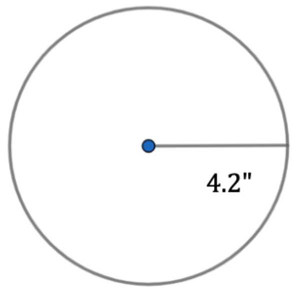
Geometry and Trigonometry Module 5 Circles

Section 5.1 Circumferences and Diameters of Circles

Practice Problems 5.1

For Problem 1-5, use the circle below to solve the problem.



1. What is the length of the radius?

2. What is the length of the diameter?

3. What is the circumference of the circle?

4. What is the area of the circle?

5. What is the area of the semicircle?

For Problem 6-10, solve the word problem.

6. A car tire has a diameter of 26 inches. How many revolutions will it make in 700 feet of travel?

7. The original height of the Navy Pier Ferris wheel in Chicago was 264 feet. This allowed the Ferris wheel to offer spectacular views of Lake Michigan. If one ride made 7 revolutions after all 42 gondolas were filled, how many linear feet would a person in a gondola travel while riding the Ferris wheel?

8. How many miles does a person in a gondola (from Problem 7) travel on one ride of the Navy Pier Ferris wheel?

9. A tree has a pair of growth rings (1 light and 1 dark) for a year of growth. If a slice of a tree trunk has 20 pairs of growth rings, each pair being about 0.73 cm., what is the circumference of the tree?

10. If a polygon is inscribed in a circle (which means all its vertices lie inside the polygon) and the number of sides of the polygon is increased, the ratio of the perimeter of the polygon to the diameter of the circle (diagonal of the polygon) approaches pi (3.14). This was used by Archimedes to approximate pi to $\frac{22}{7}$, which you learned about in Algebra 1. If a square is inscribed in a circle that has a circumference of 8π cm., what is the length of the diagonal of the square?

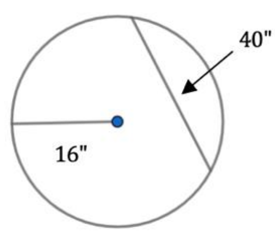
For Problem 11-13, answer true or false.

11. The diameter is the longest segment in a circle whose endpoints both touch the circumference of a circle.

12. The radius is the smallest segment in a circle whose endpoints both touch the circumference of a circle.

13. The diameter slices the circle into two sectors or semicircles whose areas are each $\frac{1}{2}(2\pi r)$; (or πr).

For Problem 14, use the diagram to solve the problem.



14. What is incorrect in the diagram and why?

For Problem 15-20, solve the word problem. Leave the answers in terms of π (exact form). Be sure to include the label.

15. If a circle has a diameter of 10 cm., what is the circumference?

16. If C $= 22$ m., find r.

17. If $r = 11.3$ in., find C.

18. If C $= 2\pi$ m., find d.

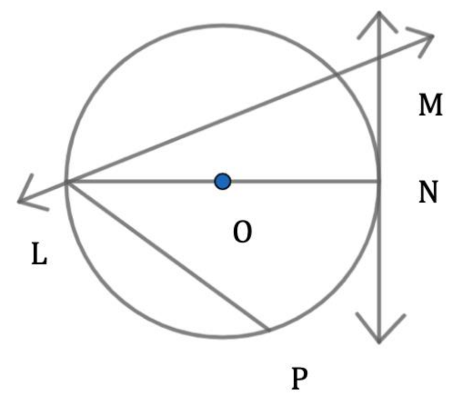
19. If $d = 2.7$ ft., find C.

20. If a circle has a circumference of 12π in., what is the diameter?

Section 5.2 Parts of a Circle

Practice Problems 5.2

For Problem 1-5, match the correct title to the given part of the circle.



Chord, Radius, Diameter, Secant, Tangent

1. $\overleftrightarrow{LM}$

2. $\overleftrightarrow{MN}$

3. $\overline{LP}$

4. $\overline{LN}$

5. $\overline{ON}$

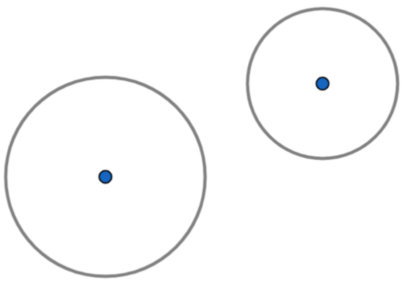
For Problem 6 and 7, draw the circles described.

6. Draw two circles that are not concentric and do not intersect.

7. Draw two circles that intersect in two points.

For Problem 8-11, draw all possible common internal tangents and common external tangents. Using colored pencils, draw all the internal with one color and all the external with another color.

8.



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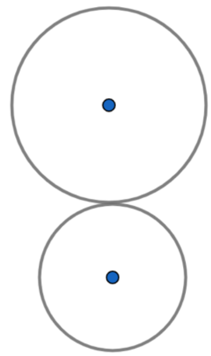

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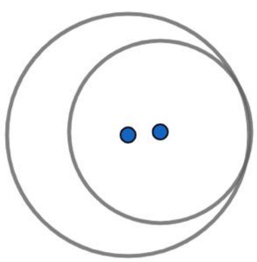

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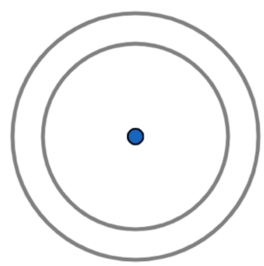
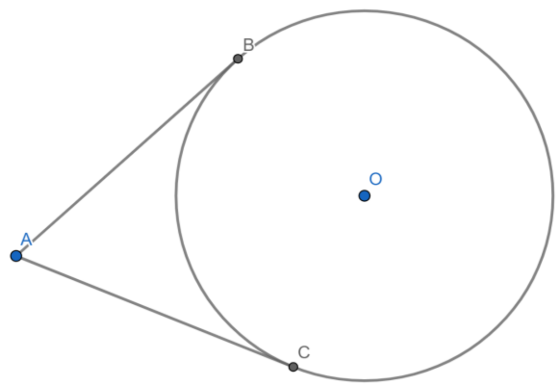


For Problem 12-17, use the diagram and information below to solve the problem.



$\overline{AB}$ and $\overline{AC}$ are tangent to $\odot$ O.

12. $\overline{OB}$ ___________ $\overline{OC}$

13. $\overline{OC}$ ___________ $\overline{AC}$

14. $\angle ABO$ is a(n) ____________________ angle

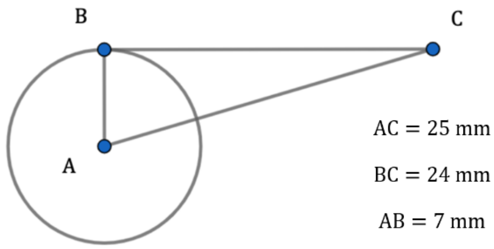
15. $\angle ACO$ is a(n) _____________________ angle

16. If $OC = 39$ mm. and $AO = 89$ mm., find the length of AC.

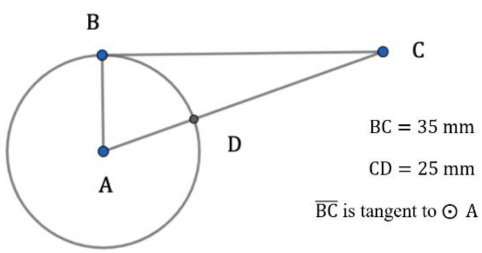
17. What is the length of AB? Explain why.

For Problem 18-20, use the given diagram/information to solve the problem.

18. Is $\overline{BC}$ tangent to $\odot$ A? Demonstrate whether this is so or not.



19. Find the length of AB.



20. What is the length of AC? How do you know that $\overline{AC}$ is not tangent to the circle?

Section 5.3 Arcs and Arc Measures

Practice Problems 5.3

For Problem 1 and 2, given the central angle, name the degree of its intercepted arc.

1. $49°$

2. $157°$

For Problem 3-6, classify the arc as a semicircle, major arc, or minor arc.

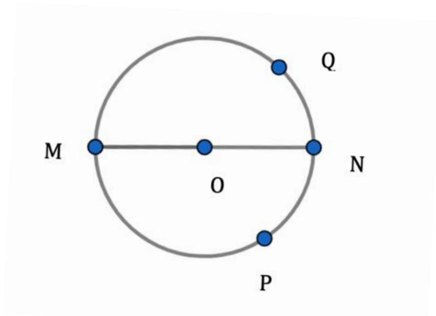
3. $22°$

4. $347°$

5. $180°$

6. $122°$

For Problem 7-9, use the circle below to solve the problem.

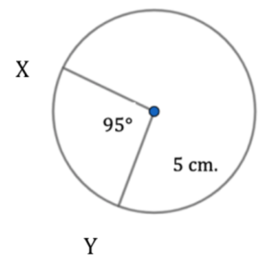


7. Name two semicircles.

8. Name two major arcs.

9. Name two minor arcs.

For Problem 10-15, use the circle to solve the problem.



10. $r =$ __________

11. $d =$ __________

12. $\mathrm{C} =$ __________

13. $m\,\widehat{\mathrm{XY}} =$ __________

14. Arc length XY $= r \cdot \frac{m(\mathrm{arcXY})}{360°} \cdot 2\pi$ means that the arc length associated with the central angle is proportional to the radius. What is the constant of proportionality?

15. This constant of proportionality is defined as the radian measure of the angle. What is the angle measure in radians?

For Problem 16, use the given information to solve the problem.

The radians are the measure of how many radii are equivalent to the arc length of the associated central angle. To convert degrees to radians, multiply the degree measure by $\frac{2\pi\,\mathrm{radians}}{360°}$ or $\frac{\pi\,\mathrm{radians}}{180°}$.

16. What would you multiply by to convert radians to degrees?

For Problem 17-20, use the given information to solve the problem.

A car tire has a rim measuring 13 inches and the tire itself is 5 inches.

17. Draw an illustration of the rim and tire and label the parts with dimensions.

18. What is the radius of a hubcap used to cover the rim?

19. What is the diameter of the tire and the rim?

20. How far does the car tire travel in feet when it has made 17 revolutions?

Section 5.4 Areas of Circle Sectors

Practice Problems 5.4

For Problem 1-6, fill in the blank.

1. The longest chord of a circle is the ____________________.

2. The radius is one-half of the ____________________.

3. The annulus is the region between two ____________________ circles.

4. The region between two radii and the ____________________ of a circle between the radii is a sector of the circle.

5. The region between a chord and arc of the circle is called the ____________________ of a circle.

6. The area of a segment of a circle is equivalent to the area of a sector of the circle minus the area of the ____________________.

For Problem 7-12, tell whether the statement is true or false.

7. If the radius of a circle is 6 inches, the diameter is 12 square inches.

8. If an intercepted arc is $36°$ degrees, then the central angle of the circle is $72°$ degrees.

9. The intercepted arc of a circle is one-half of the central angle.

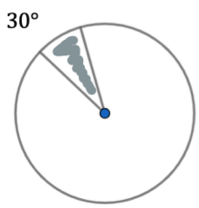
10. If the area of a circle is $36\pi\,\text{in.}^2$, then the radius is 6 inches.

11. The shaded region is the annulus of the circle.

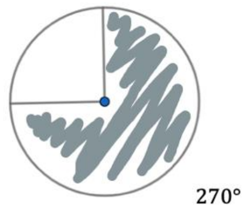
12. The shaded region is a segment of the circle.

For Problem 13-18, find the area of the shaded region of the circle.

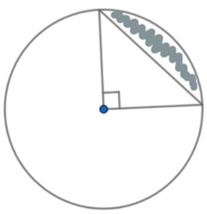
13. $r = 4$ cm.



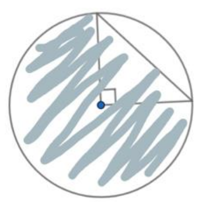
14. $r = 6$ cm.



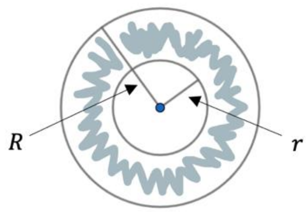
15. $r = 3$ in.



16. $r = 3$ in.



17. $R = 22$ cm.; $r = 10$ cm.

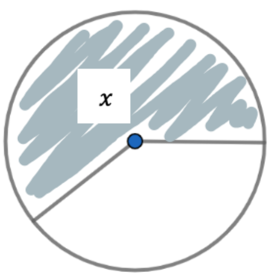


18. $d = 14$ cm. (for outermost circle)

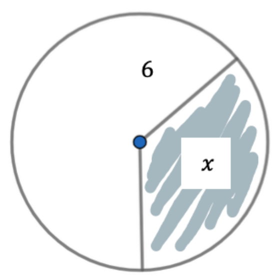
$r = 1$ cm.

For Problem 19 and 20, find x.

19. The shaded area is 60π cm.^{2} and $r = 10$ cm.



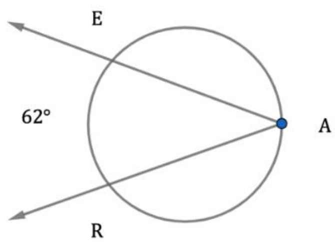
20. The shaded area is 12π cm.^{2} and $r = 6$ cm.



Section 5.5 Angles Inscribed in a Circle

Practice Problems 5.5

For Problem 1-5, use the circle below to solve the problem.



1. Arc ER subtends ___________

2. What arc subtends ∠REA?___________

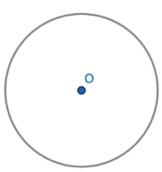
3. $m\angle EAR =$ ___________

4. If the measure of arc AR is 85° degrees, what is the measure of arc EA?

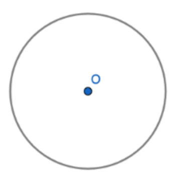
5. Name the inscribed angle.

For Problem 6-8, draw the demonstration for the case of the Inscribed Angle given.

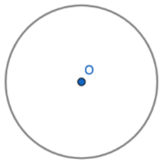
6. Center O is outside the inscribed angle.



7. Center O is on the inscribed angle.

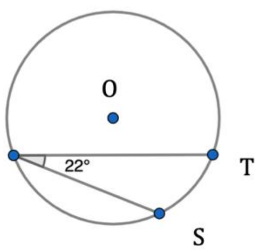


8. Center O is inside the inscribed angle.

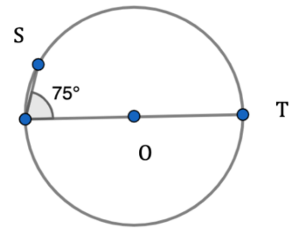


For Problem 9-11, find the measure of arc ST for the case given.

9.



10.



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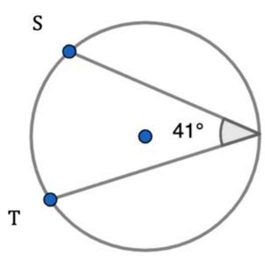


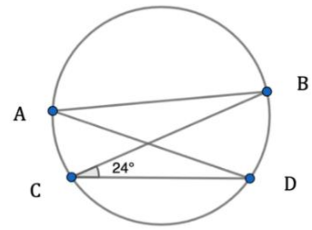
For Problem 12-14, find the measure of the inscribed angle given the intercepted arc that subtends the inscribed angle of a circle.

12. 54°

13. 86°

14. 266°

For Problem 15-17, use the circle below to solve the problem.

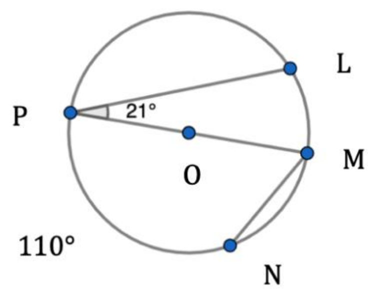


15. Find the measure of arc BD

16. $m\angle BAD =$ __________

17. If two inscribed angles of a circle intercept the same arc, do the angles have the same measure?

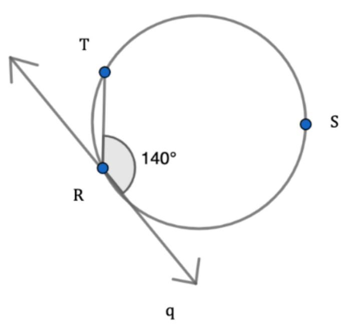
For Problem 18 and 19, use the circle below to solve the problem.



18. The measure of arc LM is __________

19. $m\angle PMN =$ __________

For Problem 20, use the circle below to solve the problem.



20. Line q is tangent to the circle. Find the measure of arc RST.

Section 5.6 Circumscribed Angles of a Circle

Practice Problems 5.6

For Problem 1 and 2, use the information given to solve the problem.

1. Draw a circle with your compass and draw diameter $\overline{MN}$ through the center of the circle. Draw two inscribed angles in the semicircle. Use your protractor to measure the angles. What type of angles are these?

2. Write a conjecture about angles inscribed in a semicircle. What proof explains your conjecture?

For Problem 3-10, fill in the blank(s).

3. The diameter is a ____________________ of the circle.

4. The diameter cuts a circle into two congruent ____________________.

5. A tangent to a circle is ____________________ to the radius at the point of tangency.

6. The central angle of a circle is the same measure as its ____________________ arc.

7. The inscribed angle is one-half of the measure of the ____________________ arc of a circle.

8. ____________________ angles inscribed in a semicircle are ____________________ angles.

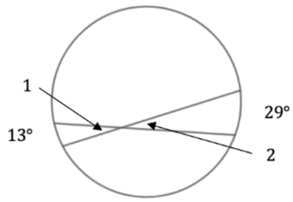
9. Inscribed angles that intercept the same arc in a circle are ____________________.

10. The angle formed by the intersection of a chord and a tangent to a circle at the point of tangency is

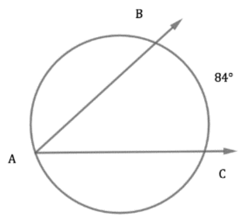
____________________ the measure of the intercepted arc.

For Problem 11-18, use the given circle to solve the problem.

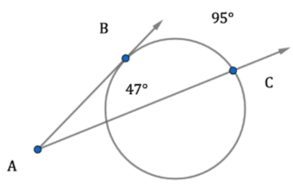
11. Find the measure of angles 1 and 2.



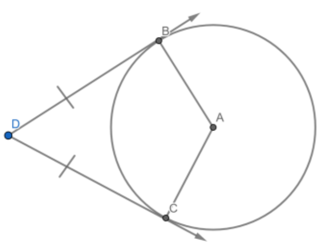
12. Find the measure of angle BAC.



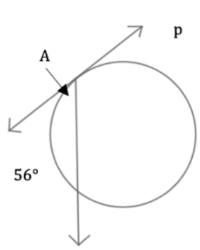
13. Find the measure of angle CAB.



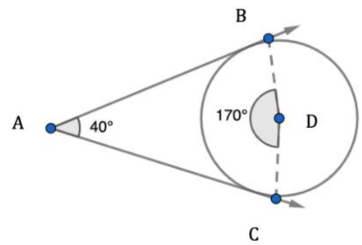
14. Find the measure of angle BDC if the measure of angle BAC is 125°.



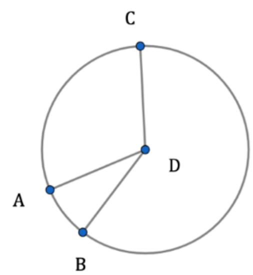
15. Find the measure of angle A; p is tangent to the circle.



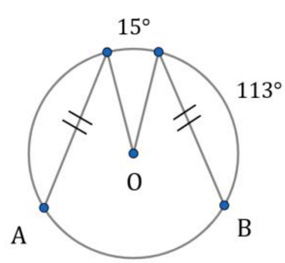
16. Explain why the diagram is not correct; $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangent to circle D.



17. Find the measure of $\angle$ADC and the measure of arc AC if the measure of arc AB is $30°$ and $m\angle\text{CDB} = 140°$ given circle D.



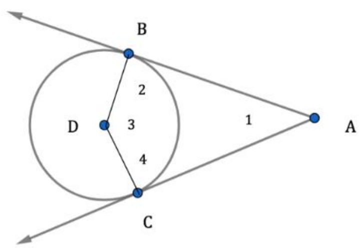
18. Find the measure of arc AB in circle O.



For Problem 19, fill in the blank and then use that information to solve Problem 20.

19. The measure of the angle formed by two intersecting tangents to a circle is __________ minus the measure of the intercepted arc.

20. Complete the reasons for the theorem of the intersecting tangents from Problem 19.



Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangent to circle O

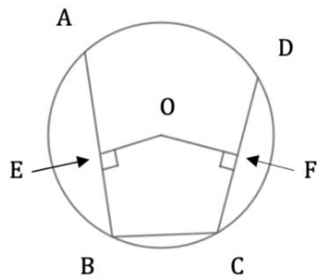
Prove: $m\angle 1 = 180° - m(\text{arc BC})$

Statement	Reasons
1. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangent to circle O	
2. $\overrightarrow{AB} \perp \overline{BD}$ and $\overrightarrow{AC} \perp \overline{CD}$	
3. $\angle 2$ and $\angle 4$ are right angles	
4. $m\angle 2 + m\angle 4 = 180°$	
5. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$	
6. $m\angle 1 + m\angle 3 = 180°$	
7. $m\angle 1 = 180° - m\angle 3$	
8. $m\angle 3 = m$ (arc BC)	
9. $m\angle 1 = 180° - m$ (arc BC)	

Section 5.7 Chords and Chord Theorems

Practice Problems 5.7

For Problem 1-6, use the circle with center O to solve the problem.



1. Is $\overline{AE} \cong \overline{BE}$? Why or why not?

2. Is $\overline{CF}$ congruent to any other line segment(s) in the circle?

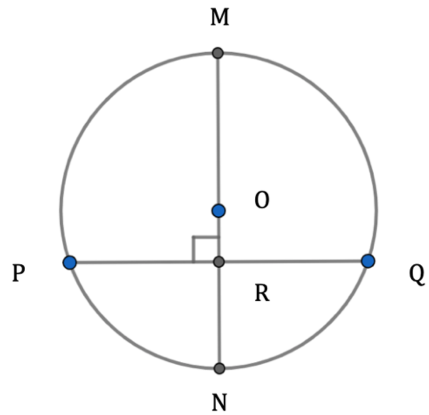
3. Is $m\angle EOF \cong m(\text{arc BC})$? Why or why not?

4. Is $\angle EOF$ an inscribed angle? Why or why not?

5. Name two inscribed angles. Name the arc that is double the measure of each inscribed angle.

6. Is pentagon BEOFC inscribed in the circle? Explain why or why not.

For Problem 7-10, use the circle with center O to solve the problem.



7. Which chord is the diameter, $\overline{MN}$ or $\overline{PQ}$?

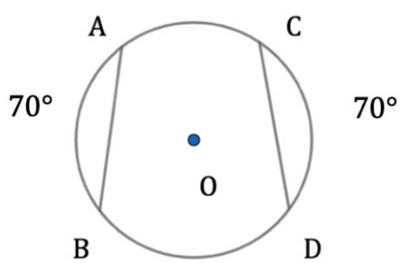
8. If PR = 14 cm., what is the length of RQ?

9. If $m(\text{arc QN}) = 44°$ degrees, what is the measure of arc PN?

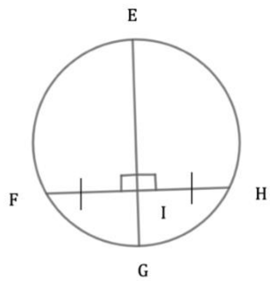
10. Find the measure of arc PNQ if $m(\text{arc NQ}) = 3x°$ and $m(\text{arc NP}) = (x + 50)°$.

For Problem 11-14, tell what conclusion you can make for the diagram and what theorem justifies your conclusion.

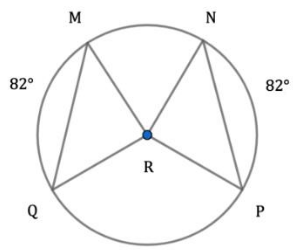
11. Circle O



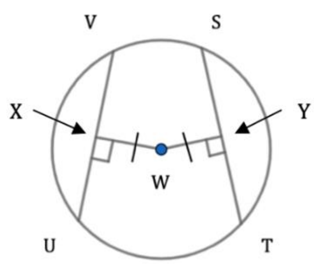
12.



13. Circle R

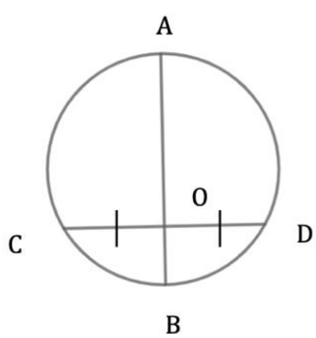


14. Circle W

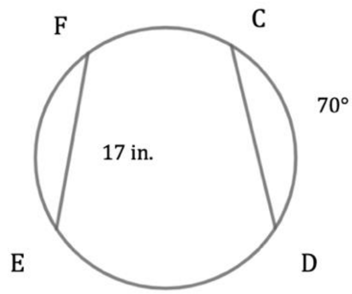


For Problem 15-18, some incorrect conclusions are given based on the diagrams. Explain the errors. Use theorems to support your reasoning.

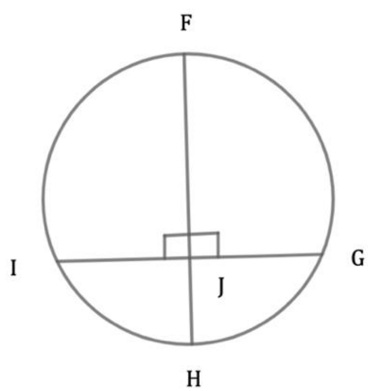
15. $\overline{AB}$ is the diameter of circle O



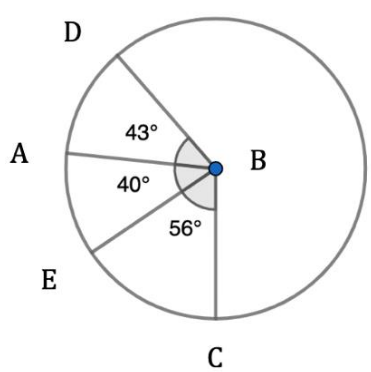
16. $m\,(\text{arc EF}) = 70°$



17. $\overline{\text{IJ}} \cong \overline{\text{GJ}}$

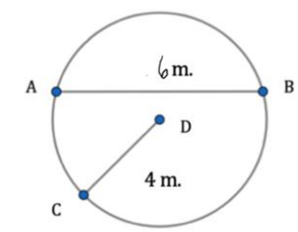


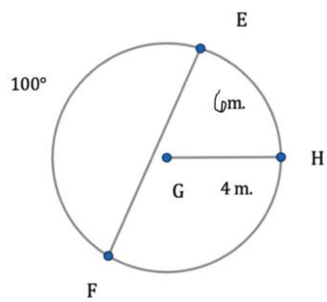
18. $m(\text{arc AC}) = 84°$



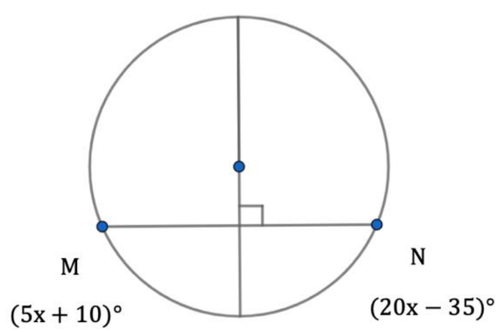
For Problem 19 and 20, find the measure of the given arc.

19. Minor arc AB





20. Minor arc MN given that the unmarked point is the center of the circle



Section 5.8 Tangent Theorems

Practice Problems 5.8

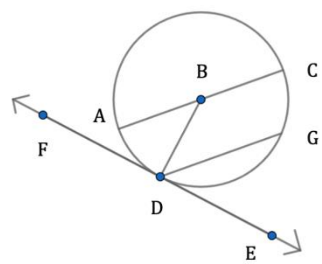
For Problem 1 and 2, solve the word problem.

1. If two circles are concentric and tangent lines are drawn to the inner circle, what types of lines will these be to the outer circle?

2. If the chord that is perpendicular to a tangent line of a circle is the diameter, what is the sum of the angles formed?

For Problem 3-6, use the information and diagram below to solve the problem.

Given: $\overline{AC} \parallel \overline{DG}$; $\overleftrightarrow{FE}$ is tangent to circle B



3. Name two angles that have the same measure as their intercepted arcs.

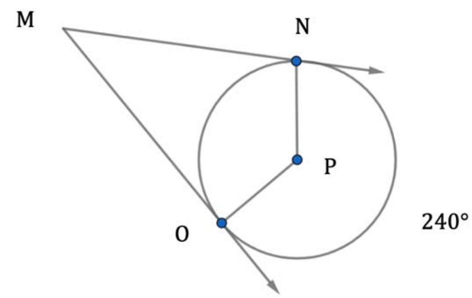
4. Name one angle that is one-half the measure of its intercepted arc.

5. Name two arcs that are congruent.

6. Name one radius and one diameter of circle B.

For Problem 7-13, use the information and diagram below to solve the problem.

$\overrightarrow{MN}$ and $\overrightarrow{MO}$ are tangent to circle P



7. If $NP = 12''$ and the perimeter of MNPO is $44,''$ what is the length of MN?

8. What is the measure of major arc NO and minor arc NO?

9. What is the measure of ∠NPO?

10. What is the name of ∠NPO?

11. What is the measure of ∠M?

12. If a satellite orbit remains at point M and circle P is the rotating earth, what percentage of the equator is visible from the satellite?

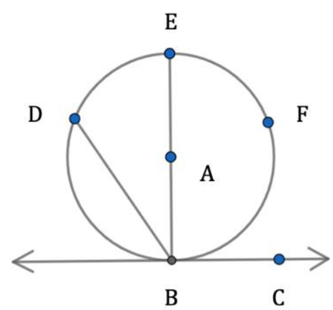
13. Mark any congruent segments or arcs in the diagram.

For Problem 14, fill in the blanks.

14. The Tangent – Chord Theorem states that the measure of an angle formed by the intersection of a tangent with a chord is ___________-___________ of the intercepted arc.

For Problem 15, complete the steps to prove the Tangent – Chord Theorem.

15. Given: $\overrightarrow{BC}$ is tangent to circle A Prove: $m\angle DBC = \frac{1}{2}(m(\text{arc DFB}))$



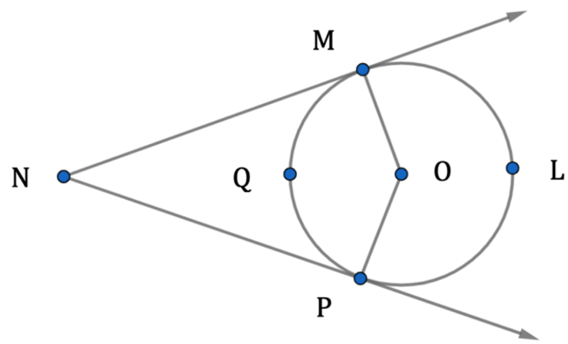
Statements	Reasons
1.	1. Given
2. $m\angle EBC =$ ___________	2. Tangent Lines of a Circle Theorem
3. $m\angle DBE = \frac{1}{2}$ ___________	3. Inscribed Angle Theorem
4. $m(\text{arc EFB}) =$ ___________	4. Definition of a Semicircle
5. $m\angle EBC = \frac{1}{2}$ ___________	5. ____
6. $m\angle DBC = m\angle DBE + m\angle EBC$	6. ____
7. $m\angle DBC = \frac{1}{2}(m(\text{arc DE})) + \frac{1}{2}(m(\text{arc EFB}))$	7. ____
8. $m\angle DBC = \frac{1}{2}$ (________ + ________)	8. Distributive Property
9. $m(\text{arc DE}) + m(\text{arc EFB}) = m(\text{arc DFB})$	9. ______________________ Addition Postulate
10. $m\angle DBC = \frac{1}{2}(m(\text{arc DFB}))$	10. ____

Section 5.9 Secant Theorems

Practice Problems 5.9

For Problem 1-10, use the information and diagram below to solve the problem.

Given: $\overrightarrow{NM}$ and $\overrightarrow{NP}$ are tangent to circle O.



1. The Circumscribed Angle of a Circle Theorem states that the measure of the circumscribed angle is 180° minus the measure of the central angle of the intercepted arc. Write this theorem using the letters from the diagram.

2. Write the theorem from Problem 1 with the intercepted arc substituted for the central angle.

3. Find $m\angle MNP$ if $m\angle MOP = 100°$ degrees.

4. What is the measure of arc MP from Problem 3?

5. What is the measure of arc MLP from Problem 4?

6. The Angle Outside of a Circle Theorem states that two tangents, or two secants, or a tangent and a secant

form an angle that is __________-___________________ the ___________________ of the intercepted arcs.

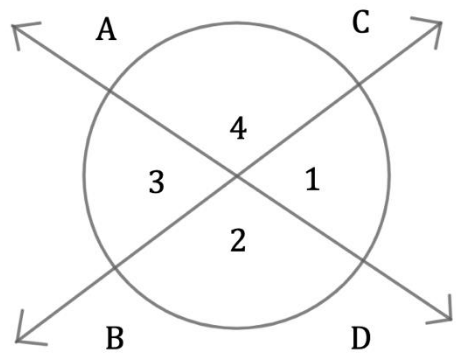
7. Write the Angle Outside of a Circle Theorem using the letters from the diagram.

8. Use the theorem from Problem 7 to find the measure of ∠MNP given the measure of ∠MOP is $100°$ degrees.

9. The answer for Problem 8 should be the same as the answer you got for Problem 3. Why is this so?

10. What is the sum of $m\angle PNM + m\angle NMO + m\angle MOP + m\angle OPN$?

For Problem 11-20, use the diagram below to solve the problem.



11. If $m\ \angle 1 = 52°$ and $m(\text{arc BD}) = 118°$ degrees, find $m(\text{arc AC})$.

12. Find $m\ \angle 3$ and $m\ \angle 4$ from Problem 11. What theorem did you use?

13. What is the sum of the measure of angles 1-4? Check and see if your answers to Problem 11 and 12 are correct.

14. If $m(\text{arc AB}) = 35°$ and $m(\text{arc CD}) = 41°$ degrees, what is the measure of $\angle 3$?

15. What are the measures of $\angle 1$, $\angle 2$, and $\angle 4$ in Problem 14?

16. If $m(\text{arc AC}) = 120°$, and $m\angle 4 = 100°$ what is $m(\text{arc BD})$?

17. What are the two secants in the diagram?

18. What two chords lie on the secants?

For Problem 19 and 20, fill in the blanks.

19. Complete the theorem for the diagram from Problem 11-18. If two chords intersect

____________________ a circle, then the ____________________ of each ____________________ is one-half of

the ____________________ of the arcs intercepted by the angle and its ____________________ angle.

20. The Angle Inside of a Circle Theorem holds true for two intersecting chords or ____________________

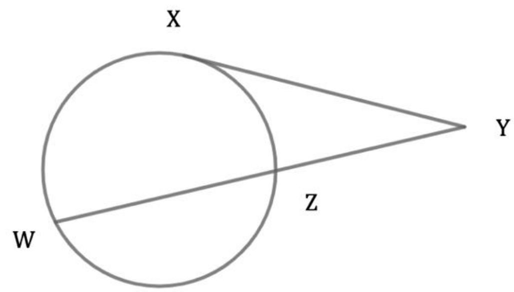
that intersect inside a circle.

Section 5.10 Chords, Tangents, and Secants

Practice Problems 5.10

For Problem 1-7, use the information and diagram below to solve the problem.

Given: $\overline{XY}$ is a tangent segment; $\overline{WY}$ is a secant segment



1. Fill in the blanks: The Tangent-Secant Segment Theorem states that $XY^2 =$ ________ $\cdot$ ________

2. True or false: $\frac{XY}{ZY} = \frac{WZ}{WY}$

3. If $XY = 12$ and $YZ = 9$, find the length of WY.

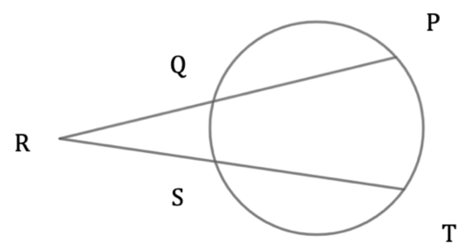
4. If $XY = 24$ and $WY = 48$, find YZ.

5. True or false: $XY^2 = WZ \cdot ZY$

6. Name a chord of the circle.

7. Is the chord of the circle also a diameter?

For Problem 8-14, use the figure below to solve the problem.



8. Correct the error for the Secants-Segments Theorem: $RQ \cdot QP = RS \cdot ST$

9. Fill in the blanks for the theorem from Problem 8:

If ___________ secant segments share a common external endpoint of a(n) _____________________, then the _____________________ of the lengths of one secant segment and its_____________________ segment is equal to the product of the _____________________ of the other ____________________ segment and its external segment.

10. If $RQ = 18$, $QP = 22$, and $RS = 20$, find ST.

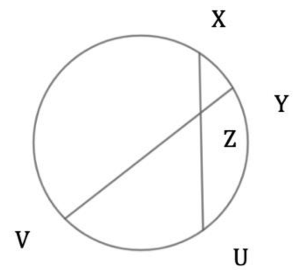
11. Name two chords of the circle.

12. Name two secant segments of the circle.

13. Name the common external point of the secants.

14. Fill in the blanks: $m\,\angle QRS = \frac{1}{2}\,m\,($________ $-$ ________$)$

For Problem 15-20, use the figure below to solve the problem. Point Z is the intersection of $\overline{XU}$ and $\overline{VY}$.



15. Name two chords of the circle.

16. True or False: $\angle$XZY is a central angle.

17. What angle is congruent to $\angle$XZY?

18. If XZ $= x + 3$, ZU $= 3x$, VZ $= x + 13$, and ZY $= x$, find the lengths of XU and VY.

19. If m(arc UY) $= 33°$ degrees and m(arc)VX $= 127°$ degrees, find the measure of $\angle$YZU.

20. If $m\ \angle$XZY $= 48°$ degrees and m(arc VU) $= 52°$ degrees, find the measure of arc XY.

Section 5.11 Standard Equation of a Circle

Practice Problems 5.11

For Problem 1-10, fill in the blank(s).

1. ____________________ symmetry means that one-half of a graph is the mirror image of the other half of the graph.

2. If __________ is on the graph whenever (x, y) is on the graph, the graph is symmetric about the x-axis.

3. If $(-x, -y)$ is on the graph whenever (x, y) is on the graph, then the graph is symmetric about the ___________________.

4. If $(-x, y)$ is on the graph whenever (x, y) is on the graph, then the graph is ____________________ about the y-axis.

5. In a circle, the graph is symmetric about both the __________ and __________ axis.

6. In the circle $x^2 + y^2 = 36$, all lines passing through the ____________________ are lines of symmetry.

7. The graph of $x^2 + y^2 = 36$ is the pair of graphs ___________________ and $y = -\sqrt{36 - x^2}$.

8. The radius of $x^2 + y^2 = 36$ is ___________.

9. The x-intercepts of $x^2 + y^2 = 36$ are __________ and __________.

10. The y-intercepts of $x^2 + y^2 = 36$ are __________ and __________.

For Problem 11-13, demonstrate whether the relation is symmetric about the x-axis, the y-axis, or the origin.

11. $y = -2x^3 + 3x$

12. $y = 3x^4 + x^2$

13. $y^2 = x^4 - 2x^2$

14. $y^2 = x - 3$

For Problem 15, find the x-intercepts and y-intercepts of the equation.

15. $2x^2 + y^2 = 4$

For Problem 16 and 17, find the center and radius of the circle given the equation.

16. $(x + 7)^2 + (y - 5)^2 = 144$

17. $(x - 10)^2 + (y + 11)^2 = 19$

For Problem 18 and 19, write an equation for the circle given its center and radius.

18. $(1, -8)$ and $r = 11$

19. $(-6, -3)$ and $r = 9$

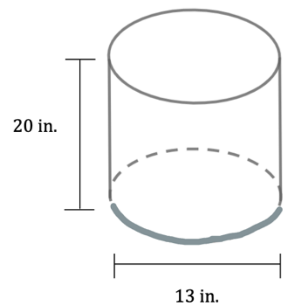
For Problem 20, complete the square to find the center and radius of the circle given the equation.

20. $x^2 + 4x + y^2 - 2y + 4 = 0$

Section 5.12 Surface Area and Circles

Practice Problems 5.12

For Problem 1-4, use the diagram below to solve the problem.



1. Draw the net for the cylinder below and label the radius of the circle and sides of the rectangle.

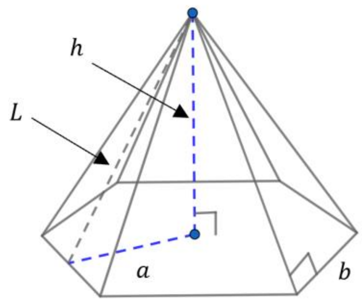
2. Find the surface of the cylinder.

3. Write a formula for surface area of a cylinder.

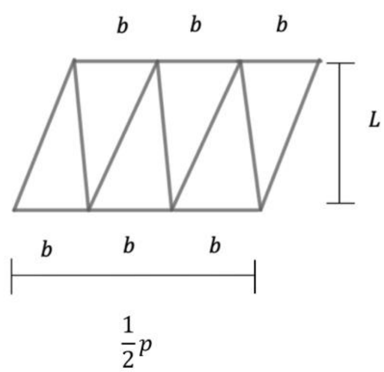
4. Find the surface area of a cylinder whose base has a radius of 4 inches and whose height is 6.1 inches.

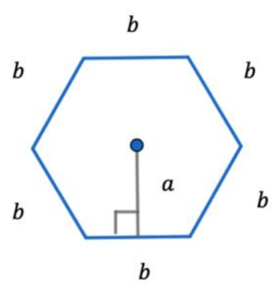
For Problem 5-10, use the given information to solve the problem.

5. The surface area of a pyramid is the sum of the area of the base and the area of the triangular faces. The height of the pyramid is h, and the slant height of the face is L. Draw the net for the hexagonal pyramid.



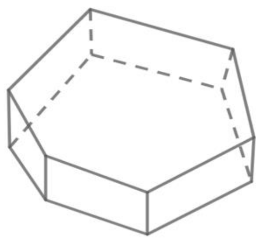
6. If the triangular faces are placed right side up and upside down, the shape approximates a parallelogram. Let n be the number of sides, a be the apothem (perpendicular to the sides of the base), and b be the length of the side of the base. Find the area for any triangular pyramid in terms of n, a, b. Simplify what you find using p for perimeter.



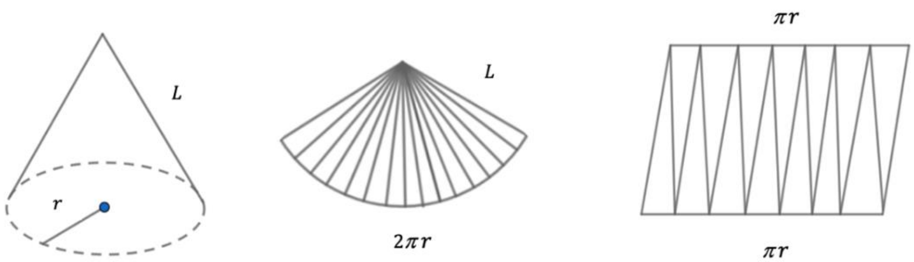


7. Find the surface area of a hexagonal pyramid with apothem $a = 11$ in., side $s = 16$ in., and height of lateral face $L = 15$ in.

8. Find the surface area of a hat box (with missing a lid) whose bases have the dimensions from Problem 7 but that has a height of 5 inches.



9. As the number of faces of a cone increases, the cone starts to look like a parallelogram.



a) What is the area of the base?

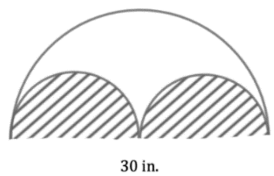
b) Write a proportion in terms of L and r for the lateral surface area of the cone.

c) The area of the sector is $\frac{2\pi r}{2\pi L}$ (the portion) times the area of the whole circle πl^2. Simplify this.

d) Write a formula for surface area of the cone using the sum of parts a and c.

10. Find the surface area of a right cone with a radius of 4 cm. and a slant height of 9 cm. Round the answer to the nearest tenths place.

For Problem 11 and 12, use the diagram and information below to solve the problem.

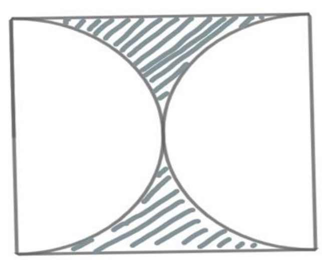


All the circles are semicircles.

11. Find the area of the shaded region.

12. Find the area of the non-shaded region.

For Problem 13-14, use the diagram and information below to solve the problem.



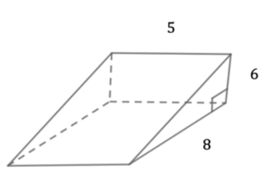
The circles are semicircles. The quadrilateral is a square with side lengths of 18 inches.

13. Find the area of the non-shaded region.

14. Find the area of the shaded region.

For Problem 15, use the information given to solve the problem.

15. A company orders 7 handicap ramps that need to have all sides painted except the base. If paint costs $16.99 a gallon and one gallon of paint covers 400 square feet, how much must be built into the costs to account for the paint?



For Problem 16-19, match the formula with the surface area that is shaded.

16.

17.

18.

19.

a) $SA = \pi r^2 + \pi r L$

b) $SA = 2\pi r h + 2\pi r^2$

c) $A = \pi R^2 - \pi r^2$

d) $A = \frac{a}{360}\pi r^2$

For Problem 20, use the information given to solve the problem.

20. Mars is the fourth planet from the sun and the second smallest planet after Mercury.

a) If the Mars Orbiter had a 30° view of Mars at a stationary point, what percentage of the planet could be viewed?

b) The diameter of Mars is 4,212.3 miles and the diameter of Mercury is 3,031.9 miles. How much more surface area does Mars have than Mercury?

Section 5.13 Volume and Circles

Practice Problems 5.13

For Problem 1-6, round your answer to the nearest tenths place.

1. A right hexagonal prism has a side length of $5\sqrt{3}$ centimeters, and a height of 10 centimeters. What is the volume of the prism?

2. What is the volume of a right hexagonal pyramid that has a congruent base and is equal to the height of the hexagonal prism in Problem 1?

3. A right cylinder has a diameter of 4 inches and a height of 5.5 inches. What is the volume of the cylinder?

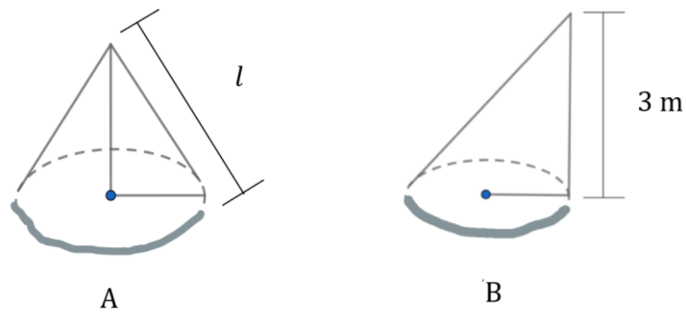
4. A right cone has the same diameter and height as the right cylinder in Problem 3. What is the volume of the cone?

5. An oblique cylinder has a base congruent to the base of the right cylinder in Problem 3. The perpendicular height is equal to the height of the cylinder in Problem 3 as well. What is the volume of the oblique cylinder?

6. Fill in the blanks: If two solids have the same cross-sectional area throughout, then their

___________________ is the same. This is called Cavalieri's principle because of his discovery.

For Problem 7-16, use the cones below to solve the problem.



7. Fill in the blank: Cone A is called a(n) ___________________ cone because the height is from the vertex perpendicular to the center of the base.

8. Fill in the blank: In Cone A, l represents the ____________________ height.

9. Fill in the blank: Cone B is called a(n) ___________________ cone because l represents the lateral side height and is perpendicular from the vertex to the side of the base.

10. Fill in the blank: In Cone B, if the radius is one-third of the height, then the diameter is __________ m.

11. What is the volume of Cone B given the radius in Problem 10?

12. Cone A has a height of 3 m. and a radius of 1 m. The cone is to be filled with ice cream and with another half scoop placed on top, which is 0.5 m. high. How much ice cream is needed for the giant ice cream in a cone?

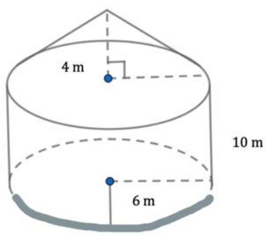
13. Fill in the blanks: The giant ice cream in a cone is called a composite solid since it is made up of two solids, a ____________________ and a ___________________.

14. Homemade ice cream is to be used to fill the giant cone for the opening of a new ice cream factory on Pi Day. Cups will be filled for all who come on a tour until the homemade ice cream is gone from the giant cone. If 1 cubic meter is equal to approximately 264.2 US liquid gallons, how many liquid gallons of ice cream will need to be made to fill the cone? (Assume any extra expansion from the liquid when it freezes will be placed on top of the cone.)

15. If the average customer eats 2 cups of ice cream, how many customers will be served (1 gallon is equal to 16 cups)?

For Problem 16-20, use the given information to solve the problem.

16. What is the volume of a corn silo that is 10 meters high from base to base with a conic roof above the base that has a height of 4 meters (shown below)?



17. Slushies are made in a cylinder cup that is 7 inches high and has a diameter of 5 inches. This option is \$3.25 for one slushy. Slushies also come in cones with the same dimensions for \$1.75. Which is a better deal?

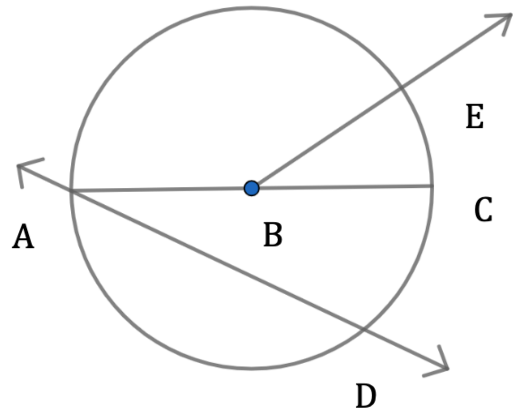
18. If the slushy in the cup from Problem 18 is \$3.75, what should be the cost of the slushy in a cone so each is of equal value?

19. What is the volume of a ball that has a radius of 2 inches?

20. The diameter of a soccer ball is 9 inches. What is the volume of the soccer ball?

Section 5.14 Module Review

For Problem 1-8, name the parts of the circle.



1. Radius

2. Chord

3. Secant

4. Minor Arc

5. Inscribed Angle

6. Semicircle

7. Central Angle

8. Name of the Circle

For Problem 9-12, use the circle above to answer the question.

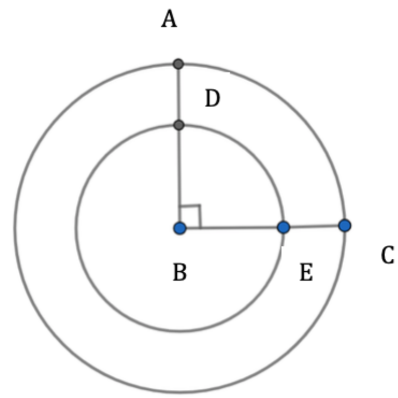
9. If $m(\text{arc EC}) = 22°$ degrees, what is the measure of $\angle$EBC?

10. If $m(\text{arc EC}) = 22°$ degrees and $m(\text{arc ECD}) = 48°$ degrees, what is the measure of $\angle$CAD?

11. If $m(\text{arc EC}) = 22°$ degrees, what is the measure of $\angle$EBA?

12. If $m\ \angle\text{EBC} = 15°$ and $\angle\text{CAD} = 14°$ degrees, find the measure of each arc of the circle.

For Problem 13-20, use the circle below to solve the problem.



13. The outer circle and the inner circle are called ____________________ circles.

14. The center of both circles is point __________.

15. If the inner circle were dilated, point D maps to point __________, and point E maps to point __________.

16. What is the measure of the circle sector at arc DE?

17. Use symbols for the area of the annulus of the circle (the outer ring).

18. Are the circles congruent or similar?

19. The measure of the rate at which an object revolves around an axis is angular velocity and is measured in degrees per second. If a ball spins along the inner circle and takes 5 seconds for one revolution, what is the angular velocity? What would be the angular velocity for a ball spinning along the inner circle?

20. The measure of the distance an object travels along a circular path in a given amount of time is called tangential velocity. It is measured in meters per second (like speed).

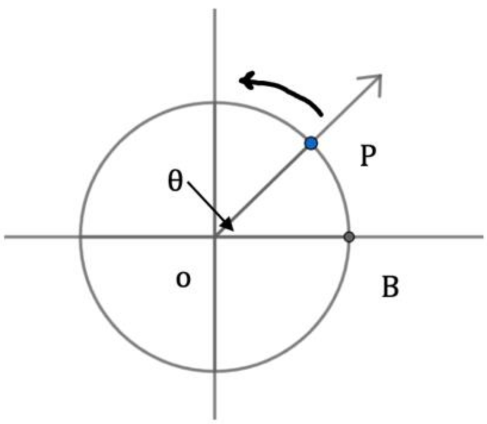
If spinning balls are 3 meters from the center of the circle along the inner ring and 4 meters from the center of the circle on the outer ring, what is the tangential velocity of each? (Each ball completes 1 revolution in 5 seconds.) Why are the tangential velocities different, but the angular velocities the same?

Section 5.15 Module Test

For the test, you will be using the figure and formulas below to answer questions regarding pendulums. You will need the following materials: a washer, tape, string, and a pencil.

Below are the formulas we will be using in our work with pendulums. Let R be the radius and θ be the central angle.

You may remember studying pendulums in Algebra 2; this is an extension of that study.



Formulas

Sector Area: $A = \frac{1}{2}\theta R^2$

Arc Length: $s = R\theta$

Linear Speed (V): $V = \frac{s}{t}$ (where s is arc length and t is time);
$V = \frac{s}{t} = \frac{R\theta}{t}$ (where θ is in radians); $V = \frac{R\theta}{t} = R \cdot \frac{\theta}{t} = R\,\omega$
(relates linear speed to angular speed)

Angular Speed: $\omega = \frac{\theta}{t}$

The table below will be completed as you progress through the activity.

Length of Pendulum	**Angle for one Period (Degrees & Radians)**	**Average Time for one Period (Seconds)**	**Arc Length (Distance Washer Travels)**	**Angular Speed (Radians per Second)**	**Linear Speed (Inches Per Second)**

1. Tie a washer on one end of the string and a pencil on the other end so there is 10 inches of string between them.

a) Pull the string back at an amplitude of 45° degrees. Let it go and time one period out and back. Do this 3 times and find the average of the 3 trials. Record it in the table.

b) Find the angle in degrees and convert it to radians for one period (one full swing out and back). Record the angle in the table.

c) Find the distance the washer (bob) traveled for one full swing. This is the arc length for one period. Record the distance in the table.

d) Find the angular speed of the washer, which is measured in radians per second. Record the angular speed in the table.

e) Find the linear speed of the washer, which is measured in inches per second. Record the linear speed in the table.

2. What do you think would happen to the distance and time it takes the washer to travel one period if you made the length of the pendulum (string) twice as long (20 inches)? Try it and test your hypothesis. Follow steps a)-e) from Problem 1 and record your conclusions in the table.

a) Pull the string back at an amplitude of 45° degrees. Let it go and time one period out and back. Do this 3 times and find the average of the 3 trials. Record it in the table.

b) Find the angle in degrees and convert it to radians for one period (one full swing out and back). Record the angle in the table.

c) Find the distance the washer (bob) traveled for one full swing. This is the arc length for one period. Record the distance in the table.

d) Find the angular speed of the washer, which is measured in radians per second. Record the angular speed in the table.

e) Find the linear speed of the washer, which is measured in inches per second. Record the linear speed in the table.

3. Were your predictions accurate?

4. What do you think would happen to the distance and time it takes to travel one period if you made the length of the pendulum (string) half as long as the original (5 inches)? Try it and test your hypothesis. Follow steps a)-e) from Problem 1 and record your conclusions in the table.

a) Pull the string back at an amplitude of 45° degrees. Let it go and time one period out and back. Do this 3 times and find the average of the 3 trials. Record it in the table.

b) Find the angle in degrees and convert it to radians for one period (one full swing out and back). Record the angle in the table.

c) Find the distance the washer (bob) traveled for one full swing. This is the arc length for one period. Record the distance in the table.

d) Find the angular speed of the washer, which is measured in radians per second. Record the angular speed in the table.

e) Find the linear speed of the washer, which is measured in inches per second. Record the linear speed in the table.

5. What would have to change so the 5-inch long pendulum and the 20-inch long pendulum travel the same distance in the same amount of time?

6. Calculate the change you predicted in Problem 5. Test it with the washers. Keep re-testing and re-calculating if necessary.

7. How far would the washer on the 20-inch long pendulum have to travel so it travels as far as the washer on the 10-inch long pendulum? What angle is formed?

8. How far would the washer on the 10-inch long pendulum have to travel so it travels as far as the washer on the 20-inch long pendulum? What angle is formed?

9. Use the linear speed in the last column to calculate the time it would take the 5, 10, and 20-inch long string washer to move through a complete circle? What is θ for a complete circle? Solve for time, t, using the linear speed formula.

a)

b)

c)

10. Why isn't linear speed the most accurate way to calculate the travel of a pendulum in a complete circle?