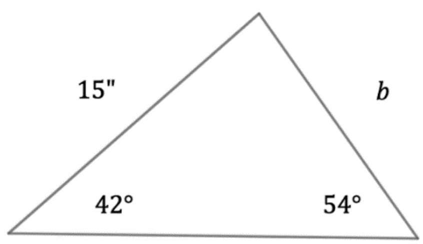
Geometry and Trigonometry Module 8 Trigonometric Identities and Conic Sections

Section 8.1 Law of Sines

Practice Problems 8.1

For Problem 1-3, find the missing side length in the triangle. Round answers to the nearest tenth.

1.



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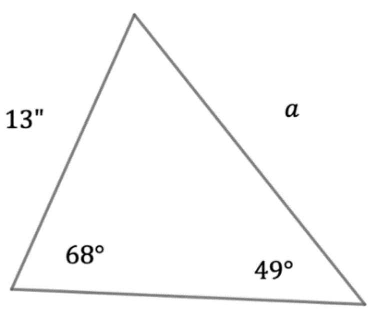

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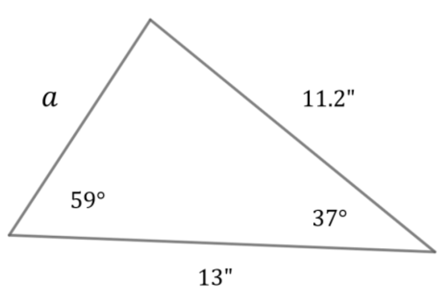
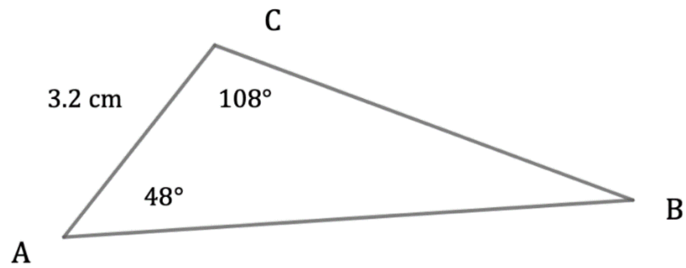


For Problem 4-7, use the triangle below to solve the problem.



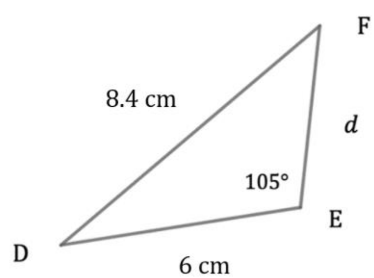
4. Find $m\angle$ B

5. Find AB.

6. Find BC.

7. Find and use the Law of Sines to solve for BC (which is different from Problem 6).

For Problem 8-10, use the triangle below to solve the problem.



8. Why can't you find $m\ \angle$D directly using the Law of Sines?

9. What angle measure would you have to find first before finding $m\angle$D? Find the measure of that angle.

10. Find $m\angle D$.

For Problem 11-13, use the information below to solve the problem.

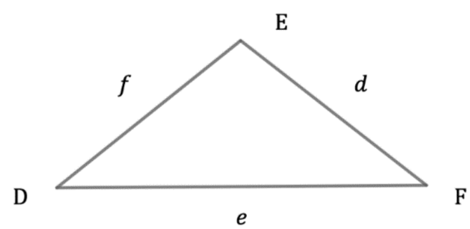
In an isosceles triangle, one of the base angles has a measure of $36°$. The length of each leg is 7.4 cm.

11. Use the Law of Sines to demonstrate that base angles of an isosceles triangle are equal.

12. Find the 3rd angle of the triangle.

13. What is the length of the base of the triangle?

For Problem 14-16, use the triangle below to fill in the blank(s).



14. $\frac{\sin D}{d} = \frac{\sin E}{\_\_}$

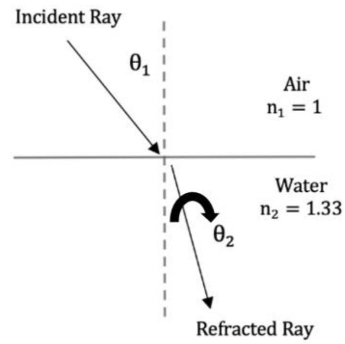
15. $\frac{\sin E}{\_\_} = \frac{\sin F}{f}$

16. $\frac{\sin D}{\_\_} = \frac{\sin F}{\_\_}$

For Problem 17-20, use the information below to solve the problem.

You did an experiment at the end of the last module involving the refraction of light. When light goes through a medium like water, it refracts (bends). Snell's Law of refraction states the following:

$$\frac{\sin \theta_1}{n_2} = \frac{\sin \theta_2}{n_1}$$



θ_1 = angle of incidence

θ_2 = angle of refraction

$n_1 = 1$ (index of refraction for air)

$n_2 = 1.33$ (index of refraction for water)

17. What is the angle of refraction if the angle of incidence is 0°?

18. What angle did the ray enter the water at if the angle of refraction from air to water is 40°?

19. What is the angle of refraction in water if the angle of incidence from air is 65°?

20. If the angle of incidence is 90°, what is the angle of refraction?

Section 8.2 Law of Cosines

Practice Problems 8.2

For Problem 1-4, given two points, find the distance of the line segment using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. $(3, 5)$ and $(-1, -4)$

2. $(9, -7)$ and $(-3, 3)$

3. $(8, 8)$ and $(14, 15)$

4. $(0, 2)$ and $(-1, -6)$

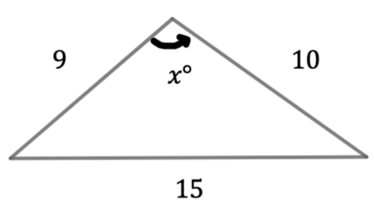
For Problem 5 and 6, use the given information to find the side length.

5. If $b = 12$, $c = 15$, and $\angle A = 52°$, find the length of side a.

6. If $a = 10$, $c = 20$, and $\angle B = 15°$, find the length of side b.

For Problem 7-10, use the Law of Cosines to find the missing side length or angle in the triangle (x).

7.



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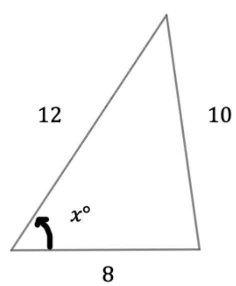

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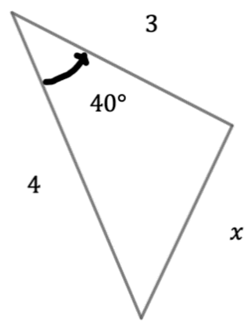

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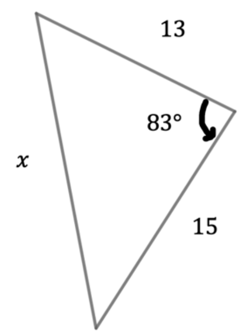


For Problem 11 and 12, solve the word problem.

11. Using the equation $b^2 = a^2 + c^2 - 2ac\ \cos \mathrm{B}$, solve for B.

12. Is $a^2 = b^2 + c^2 - 2ab\ \cos \mathrm{A}$ the same as $\cos \mathrm{A} = \frac{b^2-c^2-a^2}{2ab}$? Demonstrate whether it is or not.

For Problem 13-15, use the information to solve the problem.

Two cars leave a restaurant at the same time. One heads due east at 50 mph and the other heads due northwest at 65 mph. Assume northwest has a bearing of 315°.

13. Draw a diagram to show the distance between the two cars after h hours.

14. What will be the length of the distance each car travels after 2 hours? Label them on the diagram.

15. Find the distance between the two cars after two hours of traveling using the Law of Cosines.

For Problem 16 and 17, solve the word problem.

16. Given that in a triangle $a = 7$, $b = 8$, and $m\angle C = 30°$, find the length of side c.

17. In a tower at an amusement park, the start of the river rapids- the north end- can be seen from 600 feet away. The river rapids go straight through the park in the south end, which can be seen 300 feet away from the tower. The angle between the two lines of sight is $100°$. How long are the river rapids?

For Problem 18 and 19, fill in the blank.

18. $\cos^2\theta + \underline{\hspace{2cm}} = 1$

This is a fundamental trigonometric identity that is useful in trigonometry.

19. $1 - \underline{\hspace{2cm}} = \cos^2\theta$

For Problem 20, use the identities in Problem 18 and 19 to solve the problem.

20. What is one other way of stating the fundamental trigonometric identities in Problem 18 and 19? All these identities are used to solve problems involving trigonometric identities.

Section 8.3 Fundamental Trigonometric Identities

Practice Problems 8.3

For Problem 1-20, follow the instructions to solve the problem.

1. Write two other identities for $1+\tan^2\theta=\sec^2\theta$.

2. Write two other identities for $1+\cot^2\theta=\csc^2\theta$.

3. Solve for $\tan\theta$ in $1+\tan^2\theta=\sec^2\theta$.

4. Solve for $\cos\theta$ in $\sin^2\theta=1-\cos^2\theta$.

5. Simplify $\tan\theta\ \sin\theta+\cos\theta$ to a one term trigonometric function.

6. Simplify $\csc\theta-\cos\theta\cot\theta$ to a one term trigonometric function.

7. Write $\cot\theta$ in terms of $\sin\theta$.

8. Prove that $\cos\theta\ \csc\theta\ \tan\theta = 1$.

9. Show that $\frac{1-\cos^2\theta}{\sin\theta\ \cos\theta}$ is equal to $\tan\theta$.

10. Prove that $\cot^2\theta\cos^2\theta = \cot^2\theta - \cos^2\theta$.

11. Write $\frac{\cot\theta}{\sin\theta}$ in terms of $\cos\theta$.

12. Simplify $\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta}$ to one term.

13. Simplify $\frac{1+\cot^2\theta}{\cot^2\theta}$.

14. Multiply $(1-\cos\theta)$ by $(1+\cos\theta)$.

15. Find the product of $(\sin\theta + 1)(\sin\theta - 1)$.

16. Simplify $\tan^2\theta - \frac{\sec\theta}{\cos\theta}$.

17. Simplify $\frac{\tan\theta+\cot\theta}{\sec^2\theta}$.

18. Write $\tan\theta\ \sec\theta$ in terms of $\cos\theta$.

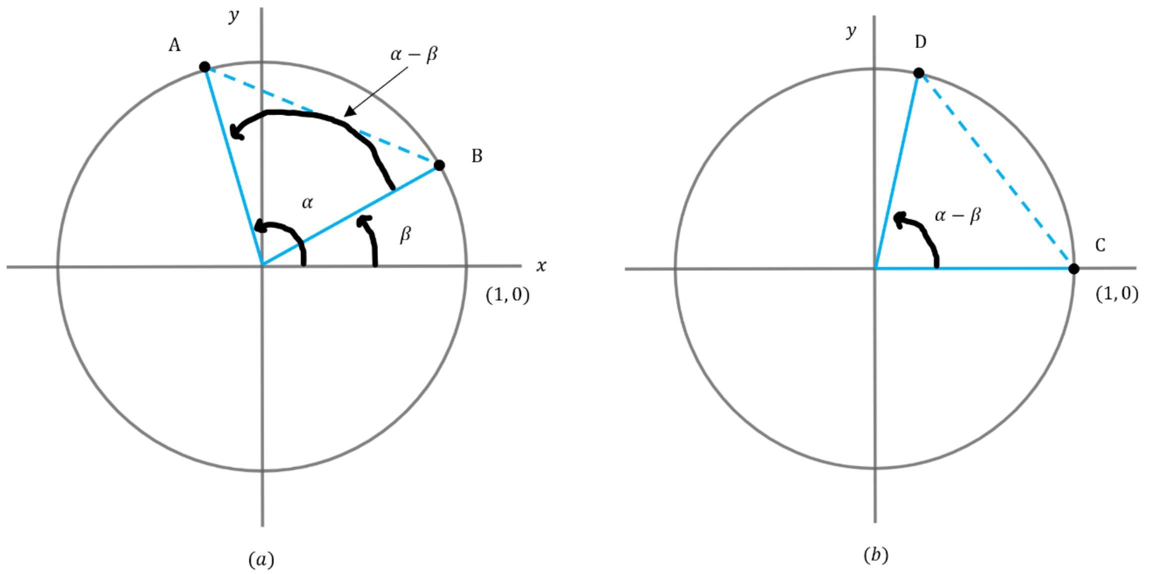
19. Simplify $\sin^2\theta\,(\tan^2\theta + 1)$.

20. Prove that $\sin\theta + \cos\theta\cot\theta = \frac{\sec\theta+\csc\theta}{1+\tan\theta}$.

Section 8.4 Sum and Difference Formulas

Practice Problems 8.4

For Problem 1-9, fill in the blanks to prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. Both central angles in circles O below have measures $\alpha - \beta$, so they are congruent.



A $(\cos\alpha, \sin\alpha)$

B $(\cos\beta, \sin\beta)$

C $(1, 0)$

D $(\cos(\alpha - \beta), \sin(\alpha - \beta))$

For circles O, angles α and β are in standard position on the left (a) and $\alpha - \beta$ is in standard position on the right (b).

Use the distance formula to complete the statement below. Since $\overset{\frown}{CD} \cong \overset{\frown}{AB}$, then $\overline{CD} \cong \overline{AB}$.

Firstly, find $(CD)^2$…

1. $(CD)^2 = [1 - \cos(\alpha - \beta)]^2 +$ ____________________

2. $(CD)^2 = 1 - 2\cos(\alpha - \beta) + \cos^2(\alpha - \beta) +$ __________________

3. $(CD)^2 = 2 -$ __________________________

Next, find $(\mathrm{AB})^2$…

4. $(\mathrm{AB})^2 = (\cos\alpha - \cos\beta)^2 -$ ____________________________

5. $(\mathrm{AB})^2 = \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta +$ ____

6. $(\mathrm{AB})^2 = (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) - 2\cos\alpha\cos\beta -$ ______________________________

7. $(\mathrm{AB})^2 = 2 - 2\cos\alpha\cos\beta -$ ____________________________

For Problem 8 and 9, set $d(\mathrm{CD})^2 = d(\mathrm{AB})^2$ and simplify.

8. ___________________________$=-2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

9. $\cos(\alpha - \beta)=$______________________________________

For Problem 10-12, follow the instructions to solve the problem.

10. To derive the formula for $\cos(\alpha + \beta)$, use the identity of opposites from Example 4 of the lesson notes.

$$\cos(\alpha + \beta) = \cos[\alpha - (\_\_\_\_\_\_\_\_)]$$
$$= \cos\alpha\cos(-\beta) + \sin\alpha\sin(\_\_\_\_\_\_\_\_)$$
$$= \cos\alpha\cos\beta - \sin\alpha\sin\_\_\_\_\_\_\_\_$$

11. To prove $\sin(\alpha + \beta)$, use the cofunction identities $\cos(90° - \theta) = \sin\theta$ and $\sin(90° - \theta) = \cos\theta$.

$$\sin(\alpha + \beta) = \cos[90° - (\alpha + \_\_\_\_\_\_\_\_)]$$
$$= \cos[(90° - \alpha) - \_\_\_\_\_\_\_\_]$$
$$= \cos(90° - \alpha)\cos\beta + \sin(90° - \alpha)\sin\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\cos\beta + \_\_\_\_\_\_\_\_\sin\beta$$

12. Prove that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$. Let $\alpha-\beta=\alpha+(-\beta)$.

For Problem 13-16, fill in the blank(s).

13. $\cos 20°\cos 60°-\sin 20°\sin 60°=\cos$ ________

14. $\cos 60°\cos 40°+\sin 60°\sin 40°=\cos$ ________

15. $\sin\frac{\pi}{2}\cos\frac{\pi}{3}+\cos\frac{\pi}{2}\sin\frac{\pi}{3}=$ ________ $\frac{5\pi}{6}$

16. $\sin\frac{\pi}{2}\cos\frac{\pi}{3}-\cos\frac{\pi}{2}\sin\frac{\pi}{3}=\sin$ ________

For Problem 17-20, follow the instructions to solve the problem.

17. Find the exact value of $\sin 15°$ using $\sin(\alpha-\beta)$.

18. Find the exact value of $\cos 75°$ using $\cos(\alpha+\beta)$.

19. Find the exact value of $\tan 15°$ using $\tan(\alpha-\beta)$.

20. Prove the identity $\cos(90°-x)=\sin x$ using the Difference Formula.

Section 8.5 More Trigonometric Theorems

Practice Problems 8.5

For Problem 1 and 2, follow the instructions to solve the problem.

1. Draw a unit circle to demonstrate that $\cos(20°) = \cos(-20°)$.

2. On the same unit circle, demonstrate that $\sin(-20°) = -\sin\theta$.

For Problem 3-6, use the trigonometric theorems to solve the problem for all numbers θ such that $0 < \theta < \frac{\pi}{2}$.

3. Let $\cos\theta = \frac{1}{8}$; find $\cos(180° - \theta)$.

4. Let $\sin\theta = \frac{1}{8}$; find $\sin(90° - \theta)$.

5. Let $\tan\theta = 0.404$; find $\tan(\pi - \theta)$.

6. Let $\tan\theta = 0.404$; find $\tan(-\theta)$.

For Problem 7-10, tell whether the statement is true or false.

7. $\cos(\frac{\pi}{4}) = \cos(\frac{3\pi}{4})$

8. $\sin(-85°) = \sin 85°$

9. $\tan 42° = \tan 24°$

10. $\tan(-\frac{3\pi}{4}) = -\tan(\frac{3\pi}{4})$

For Problem 11-13, name the theorem given the equation.

11. $\sin x = \cos(\frac{\pi}{2} - x)$

12. $-\cos x = \cos(\pi - x)$

13. $\tan(-x) = -\tan x$

For Problem 14 and 15, tell the exact value of the given function on the unit circle.

14. $\cos(-\frac{\pi}{6})$

15. $\sin(\frac{2\pi}{3})$

For Problem 16 and 17, name the radian for the degree angle on the unit circle.

16. $270°$

17. $-45°$

For Problem 18-20, follow the instructions to solve the problem.

18. Use $\frac{\pi}{2}$ for θ to demonstrate that $\sin(\pi - \theta) \neq \sin \pi - \sin \theta$.

19. Use $\frac{\pi}{2}$ for θ to demonstrate that $\cos(\pi - \theta) \neq \cos \pi + \cos \theta$.

20. If $\sin \theta = \frac{7}{25}$, find two values of $\cos \theta$ using the Pythagorean Identity.

Section 8.6 Double-Angle Formulas

Practice Problems 8.6

For Problem 1-12, write each trigonometric function in terms of one function.

1. $1 - 2\sin^2 x$

2. $2\cos^2 x - 1$

3. $2\sin x \cos x$

4. $\cos^2 x - \sin^2 x$

5. $2\sin 4x \cos 4x$

6. $\cos^2 9x - \sin^2 9x$

7. $\cos^2 50° - \sin^2 50°$

8. $1 - 2\sin^2 16°$

9. $2\cos^2 2\pi - 1$

10. $2\ \cos 4\theta\ \sin 4\theta$

11. $2\ \sin 110°\ \cos 110°$

12. $\cos^2 3t - \sin^2 3t$

For Problem 13-16, find the trigonometric function.

13. Let $\sin\theta = \frac{4}{5}$, such that $\frac{\pi}{2} < \theta < \pi$; find $\sin 2\theta$.

14. Let $\sin\theta = \frac{4}{5}$, such that $\frac{\pi}{2} < \theta < \pi$; find $\cos 2\theta$.

15. Let $\sin\theta = \frac{3}{5}$; find $\sin 2\theta$ for $0 < \theta < \frac{\pi}{2}$.

16. Let $\sin\theta = \frac{3}{5}$; find $\cos 2\theta$ for $0 < \theta < \frac{\pi}{2}$.

For Problem 17-20, find the exact value of the trigonometric function.

17. $2\ \sin 15° \cos 15°$

18. $1 - 2\sin^2 30°$

19. $\cos^2 \frac{\pi}{12} + \sin^2\ \frac{\pi}{12}$

20. $2\cos^2 \frac{\pi}{4} - 1$

Section 8.7 Half-Angle Formulas

Practice Problems 8.7

For Problem 1-4, name the sine and cosine signs for the angle given.

1. An angle in Quadrant IV.

2. An angle such that $180° < \theta < 270°$.

3. An angle such that $270° < \theta < 360°$.

4. An angle such that $0 < \theta < \frac{\pi}{2}$.

For Problem 5-8, name the sine and cosine signs of the half-angles $\frac{\theta}{2}$.

5. An angle θ in Quadrant II.

6. An angle θ such that $90° < \theta < 180°$.

7. An angle θ such that $\frac{3\pi}{2} < \theta < 2\pi$.

8. An angle such that $180° < \theta < 270°$.

For Problem 9-12, simplify the trigonometric identity to a trigonometric function of a single angle. Do not evaluate it.

9. $\sqrt{\frac{1+\cos 100°}{2}}$

10. $\sqrt{\frac{1-\cos 2\pi}{2}}$

11. $\frac{\sin 90°}{1+\cos 90°}$

12. $\frac{\sin \pi}{1+\cos \pi}$

For Problem 13-16, fill in the blank(s) to demonstrate that $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$ since $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

13. $\tan\theta = \frac{2\ \sin\theta \_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\ \cos\theta}$

14. $\tan\theta = \frac{2\ \sin^2 \_\_\_\_\_\_\_}{2\ \sin\theta \_\_\_\_\_\_\_}$

15. $\tan\theta = \frac{1-\cos 2\theta}{\sin \_\_\_\_\_\_\_\ \theta}$

16. $\tan\frac{\theta}{2} = \frac{1-\cos 2\frac{\theta}{2}}{\sin \_\_\_\_ \frac{\theta}{2}}$

For Problem 17-20, use the Half-Angle Formula to solve the problem.

17. Find the exact value of $\sin 165°$.

18. Find the exact value of $\cos\frac{7\pi}{8}$.

19. If $\cos\theta = \frac{5}{17}$ and $180° < \theta < 360°$, find $\cos\frac{\theta}{2}$

20. Find the exact value of $\tan\frac{7\pi}{8}$.

Section 8.8 Restrictions on the Trigonometric Functions

Practice Problems 8.8

For Problem 1 and 2, give the measure of the angle in radians and degrees.

1. $\cos^{-1}(\frac{\sqrt{2}}{2})$

2. $\sin^{-1}(\frac{\sqrt{3}}{2})$

For Problem 3-20, complete the tables for $\cos x$ and $\sin x$ and their inverses.

	$y = \cos x$	$y = \cos^{-1} x$
3.	$(0, 1)$	$(1,$ ________$)$
4.	$(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$	$(\frac{\sqrt{3}}{2},$ ________$)$
5.	$(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$	$($________ $, \frac{\pi}{4})$
6.	$(\frac{\pi}{3},$ ________$)$	$(\frac{1}{2}, \frac{\pi}{3})$
7.	$($________ $, 0)$	$(0, \frac{\pi}{2})$
8.	$(\frac{2\pi}{3}, -\frac{1}{2})$	$($________ , ________$)$
9.	$($________ , ________$)$	$(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4})$
10.	$(\frac{5\pi}{6},$ ________$)$	$($________ , ________$)$
11.	$(\pi,$ ________$)$	$($________ , ________$)$

	$y = \sin x$	$y = \sin^{-1} x$
12.	$(1, 0)$	$(0,$ ______$)$
13.	$(\frac{\pi}{6}, \frac{1}{2})$	$($______$, \frac{\pi}{6})$
14.	$(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$	$($______ , ______$)$
15.	$(\frac{\pi}{3},$ ______$)$	$(\frac{\sqrt{3}}{2},$ ______$)$
16.	$($______ $, 1)$	$(1, \frac{\pi}{2})$
17.	$(\frac{2\pi}{3},$ ______$)$	$(\frac{\sqrt{3}}{2},$ ______$)$
18.	$(\frac{3\pi}{4},$ ______$)$	$($______$, \frac{3\pi}{4})$
19.	$(\frac{5\pi}{6},$ ______$)$	$(\frac{1}{2},$ ______$)$
20.	$($______ $, 0)$	$($______ $, \pi)$

Section 8.9 Solving Trigonometric Equations

Practice Problems 8.9

For Problem 1-10, solve the problem for the equation $\sin \theta = 0.6$.

1. Find the solution between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

2. What is the Supplements Theorem for the sine function?

3. Use the Supplements Theorem for the sine function to find another solution for $\sin \theta = 0.6$ such that $0 \leq \theta \leq 2\pi$.

4. Which quadrant is the solution for Problem 3 in?

5. Are there any other solutions for $\sin \theta = 0.6$ in the interval $0 \leq \theta \leq 2\pi$? Why or why not?

6. What is the periodicity for sine?

7. What is the general solution for all real numbers for $\sin \theta = 0.6$?

8. Use the graphing calculator to draw the graphs of $y = \sin x$ and $y = 0.6$ on the same set of axes. Using the trace button, what is the y-coordinate when $x \approx 0.644$?

9. Using the graph from Problem 8, what is the y-value when $x = 2.498$?

10. What do the points of intersection for Problem 8 and Problem 9 represent?

For Problem 11-20, follow the instructions to solve the problem.

11. Let $2\sin\theta = 1$. Find the exact value of θ.

12. Use the Supplements Theorem to find another solution for $2\sin\theta = 1$.

13. Give the general solution over all real numbers for $2\sin\theta = 1$.

14. Substitute the solution from Problem 11 into the equation $2\sin\theta = 1$ to verify it is valid.

15. Substitute the solution from Problem 12 into the equation $2\sin\theta = 1$ to verify it is valid.

16. Solve $3\cos\theta + 1 = 0$ for θ over the interval $0 \leq \theta \leq \pi$.

17. Find another solution using the Theorem of Opposites over the interval $0 \leq \theta \leq 2\pi$.

18. Find the general solutions for all real numbers for $3\cos\theta + 1 = 0$.

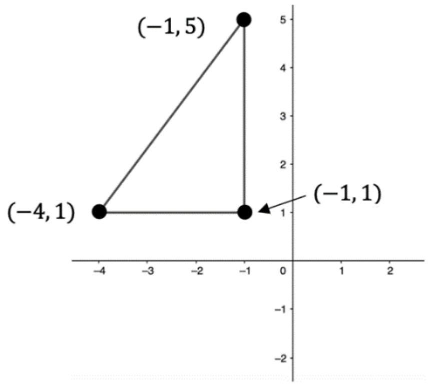
19. On the same set of axes, graph $y = 3\cos x$ and $y = -1$ and trace or intersect to find the two solutions from Problem 14 and Problem 15. What are the ordered pairs?

20. Substitute 1.91 and -1.91 in the equation $3\cos\theta + 1 = 0$ to verify they are solutions.

Section 8.10 Conic Sections and Circles

Practice Problems 8.10

For Problem 1-12, use the diagram to solve the problem given the points: A(-4,1), B(-1,1) and C(-1,5).



1. What is the distance of AB?

2. Confirm the distance of AB by using the distance formula.

3. What is the distance of BC?

4. Confirm the distance of BC by using the distance formula.

5. What is the distance of AC? Find it by using the Pythagorean Theorem.

6. What is the distance of AC? Find it by using the distance formula.

7. Find the midpoint of $\overline{AB}$.

8. Find the midpoint of $\overline{BC}$.

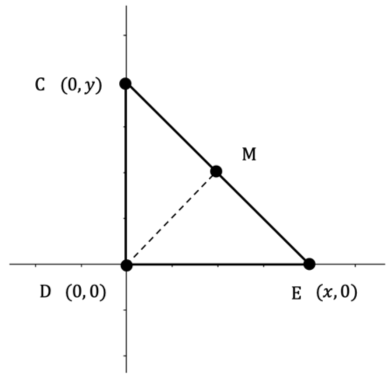
9. Find the midpoint of $\overline{AC}$.

10. Draw the median of each side of the triangle from each vertex to the midpoint of the opposite side. The point of concurrency is the centroid. Estimate the coordinates of the centroid. The centroid has a distance to a vertex that is twice the distance from the centroid to the midpoint of the opposite side. Check this using the distance formula.

11. Find the distance between a point $C(5, 8)$ and a point $D(-3, -6)$.

12. Find the midpoint between point C and point D from Problem 11.

For Problem 13-17, use triangle CDE to prove that the midpoint of the hypotenuse of a right triangle is the same distance to each of the vertices of the triangle, in this case: C, D, and E. Let M be the midpoint of CE.



13. Find the distance of CE using the Pythagorean Theorem. What is the distance of CM and ME?

14. What are the coordinates of M, the midpoint of $\overline{CE}$?

15. Use the distance formula to find the distance of CM.

16. Use the distance formula to find the distance of ME.

17. Use the distance formula to find the distance of DM.

For Problem 18-20, follow the instructions to solve the problem.

18. Given the equation $x^2 + (y - 6)^2 = 49$, find the center of the circle and the radius to graph the circle.

19. Find the equation of a circle that has a center of $(-4, -6)$ and a radius of $\sqrt{3}$.

20. Find the equation of a circle that has a diameter with endpoints $(0, 2)$ and $(3, 6)$. (Hint: The midpoint of the diameter is the center of the circle and the radius is the distance between the center and either endpoint.)

Section 8.11 Conic Sections and Parabolas

Practice Problems 8.11

For Problem 1 and 2, given the vertex and focus of a parabola, find the directrix.

1. $\mathrm{V}(0,6)$; $\mathrm{F}(0,8)$

2. $\mathrm{V}(-5,1)$; $\mathrm{F}(7,1)$

For Problem 3 and 4, given the vertex and directrix of a parabola, find the focus.

3. $\mathrm{V}(-1,2)$; $x=4$

4. $\mathrm{V}(3,-1)$; $y=-2$

For Problem 5 and 6, given the focus and the directrix of a parabola, find the vertex.

5. $\mathrm{F}(3,-3)$; $x=-5$

6. $\mathrm{F}(0,2)$; $y=6$

For Problem 7-12, follow the instructions to solve the problem.

7. Because $|c|$ is the distance between the vertex and the focus, find $|c|$ from Problem 1.

8. Using the formula $a = \frac{1}{4c}$, find a for Problem 2 and state whether the graph opens up, down, right, or left.

9. Find the equation for the parabola in Problem 3.

10. Find the equation for the parabola in Problem 4 and sketch its graph.

11. Find $|c|$ and a for Problem 5 and tell whether the graph opens up, down, right, or left.

12. Find $|c|$ and a for Problem 6. Tell whether the graph opens up, down, right, or left. Find the equation and sketch its graph.

For Problem 13 and 14, use the given information to find the equation of the parabola.

13. Find the equation of the parabola given the vertex $\mathrm{V}(-1,-3)$ and the directrix $y=-7$.

14. Find the equation of the parabola given the focus $\mathrm{F}(0,-2)$ and the directrix $x=4$.

For Problem 15 and 16, given the equation, find the vertex, the distance from the vertex to the focus ($|c|$), and tell whether the graph opens up, down, right, or left. Let $c=\frac{1}{4a}$.

15. $y=\frac{1}{8}(x+5)^2$

16. $x=-\frac{1}{12}y^2-6$

For Problem 17 and 18, given the equation, find the vertex, focus, directrix, and axis of symmetry. Sketch the parabola and check it using a graphing calculator.

17. $y=-\frac{1}{36}(x-3)^2-4$

18. $x=-(y-1)^2$

For Problem 19 and 20, follow the instructions to solve the problem.

19. A parabola with vertex V(h, k) and a vertical axis of symmetry has a focus F$(h, k + c)$ and a line $y = k - c$ as a directrix. Use the point P(x, y) not on the directrix and point D$(x, k - c)$ on the directrix to find the equation of the parabola, which is $y = \frac{1}{4c}(x - h)^2 + k$. Let the distance PD = PF to begin.

$$\sqrt{(x - x)^2 + (y - (k - c))^2} = \sqrt{(x - h)^2 + (y - (k + c))^2}$$

20. The general equation for a parabola is $y = a(x - h)^2 + k$. The equation from Problem 19 is $y = \frac{1}{4c}(x - h)^2 + k$. By substitution, what is a equal to?

Section 8.12 Ellipses and Their Transformations

Practice Problems 8.12

For Problem 1-6, given the ellipse $\frac{x^2}{25}+\frac{y^2}{36}=1$, answer the question.

1. What are the coordinates of the center?

2. What are the coordinates of the endpoints of the major axis?

3. How long is the major axis in units?

4. What are the coordinates of the endpoints of the minor axis?

5. How long is the minor axis in units?

6. What are the coordinates of the foci?

For Problem 7 and 8, given the ellipse $\frac{x^2}{36}+\frac{y^2}{25}=1$, answer the question.

7. What are the coordinates of the endpoints of the major axis?

8. What are the coordinates of the endpoints of the minor axis?

For Problem 9 and 10, use the ellipse $\frac{(x-2)^2}{16}+\frac{(y+5)^2}{4}=1$ to solve the problem.

9. What is the center of the ellipse?

10. Find the endpoints of the major and minor axis and sketch a graph of the ellipse.

For Problem 11 and 12, use the ellipse $\frac{(x+9)^2}{9}+\frac{(y-3)^2}{64}=1$ to solve the problem.

11. What is the center of the ellipse?

12. Find the value of c.

For Problem 13-20, given the eccentricity and mean distance of the planet from the sun, find the equation for the orbit of the planet.

	Planet	**Eccentricity**	**Mean Distance from Sun (Au)**	**Equation for the Orbit of the Planet**
13.	Mercury	0.206	0.387	
14.	Venus	0.007	0.723	
15.	Earth	0.017	1.000	
16.	Mars	0.093	1.524	
17.	Jupiter	0.049	5.203	
18.	Saturn	0.056	9.539	
19.	Uranus	0.047	19.182	
20.	Neptune	0.009	30.058	

Section 8.13 Hyperbolas and Their Transformations

Practice Problems 8.13

For Problem 1-9, use the equation $\frac{x^2}{9} - \frac{y^2}{16} = 1$ to answer the question.

1. What is a?

2. What are the coordinates of the x-intercepts?

3. What is the length between vertices?

4. What is b?

5. What does b represent?

6. What is c?

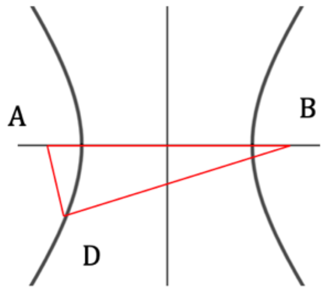
7. What are the coordinates of the foci?

8. What are the equations of the asymptotes?

9. Sketch a graph of the hyperbola.

For Problem 10-15, use the information below to solve the problem.

Hyperbolas can track objects that emit sound, such as animal calls.



The diagram to the left shows when a dolphin makes a sound and underwater sonar devices are at A and B and are 8,000 feet apart.

The sound from a dolphin (D) is received 0.2 seconds later at point B than point A. The speed of sound in water is 5,000 feet per second.

10. The dolphin can be located on the hyperbola. If the hyperbola is centered at $(0, 0)$, what are the coordinates of the foci at A and B?

11. What is the focal constant (k)?

12. What is a if $2a = k$?

13. What is c?

14. What is b?

15. Find the equation for the hyperbola on which the dolphin lies.

For Problem 16-20, solve the word problem.

16. Find the equation of the hyperbola with foci $(2,-3)$ and $(2,7)$ and difference of focal radii of 8.

17. Given the hyperbola $\frac{x^2}{25}-\frac{y^2}{4}$, find the center and the foci.

18. Given the hyperbola $\frac{(y+2)^2}{1}-\frac{(x-1)^2}{4}$, find the center and the foci.

19. The equation for a hyperbola with foci (a,a) and $(-a,-a)$ and difference of focal radii $2a$ is $xy=\frac{a^2}{2}$. This special case is a type of equation you learned about in Algebra 2. Name the parent function and sketch the graph.

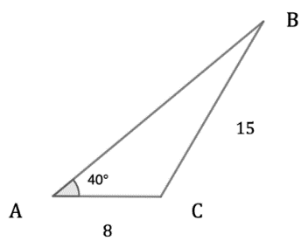
20. The equation for a hyperbola with foci $(-a,a)$ and $(a,-a)$ and difference of focal radii $2a$ is $xy=-\frac{a^2}{2}$. This special case has the same parent function as Problem 19. Sketch the graph for the equation and in this one name the asymptotes.

Section 8.14 Module Review

For Problem 1 and 2, follow the instructions to solve the problem.

1. Fill in the blanks: The Law of Sines states that $\frac{\sin A}{\_\_} = \frac{\sin B}{\_\_} = \frac{\sin C}{\_\_}$.

2. Find $m\angle B$ using using the Law of Sines.



For Problem 3-5, tell whether the equation is true or false.

3. $1 + \tan^2\theta = \csc^2\theta$

4. $\sec\theta = \frac{1}{\cos\theta}$

5. $\tan(90° - \theta) = \cot\theta$

For Problem 6, follow the instructions to solve the problem.

6. Write two other identities for $1 + \cot^2\theta = \csc^2\theta$.

For Problem 7-9, match to make the equations work.

7. $\sin(\alpha + \beta) =$

8. $\cos(\alpha - \beta) =$

9. $\tan(\alpha + \beta) =$

a) $\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

b) $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

c) $\cos\alpha\cos\beta + \sin\alpha\sin\beta$

For Problem 10 and 11, follow the instructions to solve the problem.

10. Given the equation $(x+2)^2 + (x-8)^2 = 16$, what is the center of the circle and the radius?

11. Write the equation for a circle that has center $(2, -7)$ and a radius of 6.

For Problem 12-14, given the equation, determine whether the graph is an ellipse, parabola, or hyperbola.

12. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

13. $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{9} = 1$

14. $y = \frac{1}{8}(x-2)^2$

For Problem 15-20, follow the instructions to solve the problem.

15. Given the parabola $x - 1 = \frac{1}{8}(y+1)^2$, answer the following questions and sketch the parabola.

a) What is the vertex?

b) What is the focus?

c) What is the directrix?

d) What is the axis of symmetry?

16. Given the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$, what is the center? Locate four extreme points and sketch the graph of the ellipse.

17. Given the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$, find the four intercepts. Let x be 0 and solve for y; then let y be 0 and solve for x. Sketch the graph of the ellipse.

18. Given the hyperbola $\frac{x^2}{25}-\frac{y^2}{36}=1$, find a and b and the foci.

19. What are the equations for the asymptotes of the hyperbola in Problem 18?

20. Sketch a graph of the hyperbola in Problem 18.

Section 8.15 Module Test

For Problem 1 and 2, follow the instructions to solve the problem.

1. The Law of Cosines states that:

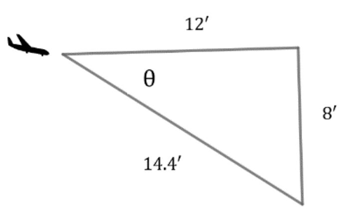
$$a^2 = \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ - 2bc\cos \mathrm{A}$$

$$b^2 = a^2 + c^2 - \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\_\_\_\_\_\_\_\_ = a^2 + b^2 - 2ab\cos \mathrm{C}$$

2. Solve the problem using the Law of Cosines:

A toy airplane is 8 feet above the ground and 12 feet from the barn. It starts its $14.4'$-foot descent toward the barn. What is its angle of descent?



For Problem 3-5, tell whether the equation is true or false.

3. $\tan(\frac{\pi}{2} - \theta) = \cot\theta$

4. $\cos\theta \csc\theta = 1$

5. $\cos^2\theta = 1 + \sin^2\theta$

For Problem 6, write two other identities for the equation.

6. $1 + \tan^2\theta = \sec^2\theta$

For Problem 7-9, match to make the equation work.

7. $\sin(\alpha - \beta) =$

8. $\cos(\alpha + \beta) =$

9. $\tan(\alpha - \beta) =$

a) $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

b) $\sin\alpha\cos\beta - \cos\alpha\sin\beta$

c) $\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

For Problem 10 and 11, follow the instructions to solve the problem.

10. Given the equation $(x-1)^2 + (x-4)^2 = 4$, what is the center of the circle and the radius?

11. Write the equation for a circle that has center $(-3, -4)$ and a radius of 5.

For Problem 12-14, given the equation, determine whether the graph is an ellipse, parabola, or hyperbola.

12. $x = \frac{1}{2}(y+2)^2$

13. $\frac{x^2}{16} + \frac{(y-1)^2}{4} = 1$

14. $\frac{(x-1)^2}{4} - \frac{(y-3)^2}{36} = 1$

For Problem 15-20, follow the instructions to solve the problem.

15. Given the parabola $y - 1 = -\frac{1}{12}(x-2)^2$, answer the following questions and sketch the graph of the parabola.

a) What is the vertex?

b) What is the focus?

c) What is the directrix?

d) What is the axis of symmetry?

16. Given the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$, what is the center? Locate four extreme points and sketch the graph of the ellipse.

17. Given the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$, find the four intercepts, two on the x-axis and two on the y-axis, and sketch the graph of the ellipse.

18. Given the hyperbola $\frac{y^2}{9}-\frac{x^2}{25}=1$, find a and b and the foci.

19. What are the equations for the asymptotes of the hyperbola in Problem 18?

20. Sketch a graph of the hyperbola in Problem 18.