Pre-Calculus and Calculus Module 4 Polar Equations and Complex Numbers

Section 4.1 More on Parametric Equations

Practice Problems 4.1

For Problem 1-6, convert the parametric equations to rectangular equations.

1. $x = t - 1$
   $y = 2t$

2. $x = 2^t$
   $y = 3^t$

3. $x = \sqrt{t}$
   $y = t - 2$

4. $x = 2t + 1$
   $y = \frac{1}{3}t$

5. $x = \sqrt[3]{t}$
   $y = t - 5$

6. $x = 9t^2 - 6$
   $y = 3t$

For Problem 7-13, use the information from Problem 1-6 to solve the problem.

7. Graph the parametric and rectangular equations for Problem 1, but first write down what type of graph you expect and why.

8. Graph the parametric and rectangular equations for Problem 2, but first write down what type of graph you expect and why.

9. Graph the parametric and rectangular equations for Problem 3, but first write down what type of graph you expect and why.

10. Tell the type of graph for each problem given in parametric or rectangular equations.

	Parametric	**Rectangular**
Problem 4		
Problem 5		
Problem 6		

11. Complete the parametric table for Problem 4. Substitute the values of t in the first equation and solve for x. Substitute the values of x in the second equation and solve for y.

$\boldsymbol{t}$	$\boldsymbol{x}$	$\boldsymbol{y}$
-2		
-1		
0		
1		
2		

12. For Problem 5, substitute these x and y values in the parametric equations and solve for t. Do you get equivalent outcomes?

$\boldsymbol{t}$	$\boldsymbol{x}$	$\boldsymbol{y}$
	-2	-13
	-1	-6
	0	-5
	1	-4
	2	3

13. Complete the table for x and y given t for the parametric equations in Problem 6. Graph the equation for x and y. Check the equation and table using the graphing calculator.

t	x	y
-2		
-1		
0		
1		
2		

At the end of Geometry and Trigonometry, you studied conic sections. These will now be combined with parametric equations.

For Problem 14-20, set the graphing calculator to parametric mode with "$0 \leq t \leq 6.28$" and set the window for x over the interval $[-10, 10]$.

14. Plot the parametric equations:

$$x = \cos t$$
$$y = \sin t$$

What conic section results from these equations?

15. If you square both sides of the parametric equations and add them together, what equations and graphs result?

16. The circle is a special case of a(n) ____________________, which is also a conic section.

17. Plot the parametric equations:

$$x = 4\cos t$$
$$y = 2\sin t$$

What conic section results from these equations?

18. Comparing the graph in Problem 17 to the graph in Problem 14, there is a horizontal dilation by a scale factor of __________.

19. Comparing the graph in Problem 17 to the graph in Problem 14, there is a vertical dilation by a scale factor of __________.

20. Plot the parametric equations:

$$x = 4\cos t + 5$$
$$y = 2\sin t - 3$$

Compare this graph to the graph in Problem 17.

Section 4.2 The Helping Parameter

Practice Problems 4.2

For Problem 1-7, use the information given and from previous problems to solve the problem.

1. Solve the first equation for t.

$$x = \cos t$$
$$y = \sin t$$

2. Substitute the value for t from the first equation into the second equation for t.

$$x = \cos t$$
$$y = \sin t$$

3. This is where the inverse trigonometric identities can help you convert parametric equations to rectangular equations.

$$\sin(\cos^{-1} x) = \sqrt{1 - x^2}$$

Substitute $\sqrt{1 - x^2}$ in for $\sin(\cos^{-1} x)$ in the equation from Problem 2 and write the equation in standard form. This equation should look familiar. What is its shape?

4. This time, solve the second equation for t using the following equations:

$$x = \cos t$$
$$y = \sin t$$

5. Substitute the value for t from the second equation into the first equation for t.

$$x = \cos t$$
$$y = \sin t$$

6. Use the trigonometric identity $\cos(\sin^{-1} y) = \sqrt{1 - y^2}$ and substitute $\sqrt{1 - y^2}$ in for $\cos(\sin^{-1} y)$ in the equation from Problem 5 and write the equation in standard form. This equation should look familiar. What is its shape?

7. Complete the table for the parametric equations:

$$x = \cos t$$
$$y = \sin t$$

What shape do you think you will get? Why?

t	$\cos t$	$\sin t$
0	1	0
$\frac{\pi}{4}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$		
π		
$\frac{3\pi}{2}$		
2π		

Check it on the graphing calculator. Make sure the angle setting is in radians.

For Problem 8-13, use the given information to solve the problem.
A squirrel is in a tree 102 feet above the ground. He drops acorns to the ground as people walk by. The wind blows each acorn sideways at 20 feet per second. Each acorn falls faster and faster but is affected little by air resistance. The acorn height at time t is given by the equation $y = -16t^2 + 102$ from the effects of gravity ($-16t^2$).

8. What are the two functions for the x-coordinate (horizontal component) and y-coordinate (vertical component) of the acorn?

9. Complete the table for the first 3 seconds of the acorn fall.

t(seconds)	x(feet)	y(feet)
0		
0.5		
1		
1.5		
2		
2.5		
3		

10. Find the rectangular equation given the parametric functions and graph it. What type of curve do you get? Why?

11. When does the acorn hit the ground? How do you know?

12. What would the equation be if the wind were blowing at 35 feet per second?

13. What would the equation be if the squirrel were sitting in a tree 93 feet above ground given the original horizontal speed of 20 ft/s?

Let us again investigate parametric equations of conic sections.

For Problem 14-20, use the given information, graphing calculator, and the parametric equations below to solve the problem.

$$x = \sec t$$
$$y = \tan t$$

14. Plot the parametric equations. What conic section results?

15. What are the x-intercepts of the conic section?

16. What are the asymptotes of the graph?

17. Square both sides of the equations and subtract one from the other. What trigonometric identity is the result?

18. Plot the parametric equations:

$$x = 4\sec t$$
$$y = 2\tan t$$

What are the x-intercepts now? What else do you notice about the graph compared to the graph in Problem 14?

19. Plot the parametric equations:

$$x = 4\sec t + 5$$
$$y = 2\tan t - 3$$

What is the point of intersection of the asymptotes?

20. Plot the parametric equations:

$$x = \tan t$$
$$y = \sec t$$

How does it compare to the graph in Problem 14?

Section 4.3 Application Problems

Practice Problems 4.3

For Problem 1-4, find the work done by the force F in Joules to move an object from S to T.

1. $F = 2\boldsymbol{i} + 2\boldsymbol{j}$ $S(-1, 4)$ $T(5, 1)$

2. $F = 4\boldsymbol{i}$ $S(0, 0)$ $T(-3, 3)$

3. $F = \boldsymbol{i} - \boldsymbol{j}$ $S(1, 3)$ $T(2, 7)$

4. $F = \boldsymbol{i} - 3\boldsymbol{j}$ $S(-1, -3)$ $T(1, 5)$

For Problem 5-7, use the information given to solve the problem.

5. A skier is being pulled up a slope at an angle of 28° for 2 km. If the skier weighs 74.8 kg. (165 pounds), and is being pulled at a constant rate, what is the skier's gravitational force?

6. A crane is lifting a 2,000 kg. crate 40 meters to place it on a ramp. How much work is being done?

7. How much energy in kilowatt-hours (kW h) is being expended to lift the crate in Problem 6?

For Problem 8-13, use the given information to solve the problem.

A baseball player throws a baseball at an angle of 35° with the horizontal parallel to the ground. His arm is 8 feet above ground level when he releases the baseball for the throw (air resistance is negligible). The ball is thrown at 115 feet per second.

8. a) What is the initial speed of the baseball?

b) What is the angle of release of the baseball relative to the horizontal?

c) What are the coordinates of the initial position of the baseball?

d) Draw a diagram of the baseball throw.

9. Find the parametric equations for the baseball. Assume that acceleration due to gravity is $32\,\frac{\text{ft}}{s^2}$. Use the equation $y(t) = -\frac{1}{2}t^2 + v_0 \cdot sint + h_0$ where v_0 is the initial speed and h_0 is the initial height of the baseball.

$x(t) =$

$y(t) =$

10. a) How long does the baseball travel before it hits the ground (in seconds)?

b) How far does the baseball travel before it hits the ground (in feet)?

11. How would the equation for $y(t)$ change if the baseball were released from 6.5 feet above the ground?

12. How would the equation change if the angle of release were 48° with the horizontal?

13. How would the equation change if the initial velocity of the baseball thrown were 110 ft./sec.?

Let us now investigate parametric and rectangular equations when conics are rotated.

For Problem 14-20, use the given information to solve the problem.

Let matrix [N] equal $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

14. Multiply matrix $[\text{M}] = \begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$ by matrix [N].

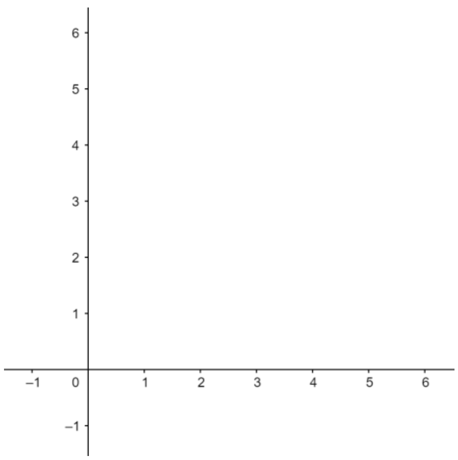
15. a) What are the dimensions of matrix [M]?

b) What are the dimensions of matrix [N]?

c) What are the dimensions of the product matrix $[\mathrm{M}] \cdot [\mathrm{N}]$?

16. Let matrix $[\mathrm{M}]$ equal $\begin{bmatrix} \cos 30° & \cos(30° + 90°) \\ \sin 30° & \sin(30° + 90°) \end{bmatrix}$, which is a $30°$ counterclockwise matrix. Let matrix $[\mathrm{N}]$ represent the point $(2, 3)$. Find the product matrix of $[\mathrm{M}] \cdot [\mathrm{N}]$.

17. Graph point N and draw a line to the origin. Graph point (x, y) from the rotated matrix and draw a line to the origin.



18. Measure the line segment from the origin to N. Measure the line segment from the origin to $[\mathrm{M}] \cdot [\mathrm{N}]$.. Are these line segments the same? Is a rotation a rigid motion?

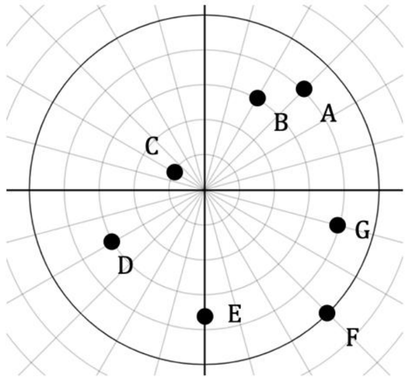
19. Measure the angle between the line segments. Does it match the angle of rotation?

20. What is the rotation matrix for a counterclockwise rotation of $50°$?

Section 4.4 The Polar Coordinate System

Practice Problems 4.4

For Problem 1 and 2, use the polar coordinate grid below to solve the problem.



1. Give the standard polar components of A, B, C, D, E, F, G on the polar coordinate grid. Let r be positive.

A

B

C

D

E

F

G

2. Give the components again for points A, B, C, D, E, F, G if r is negative?

A

B

C

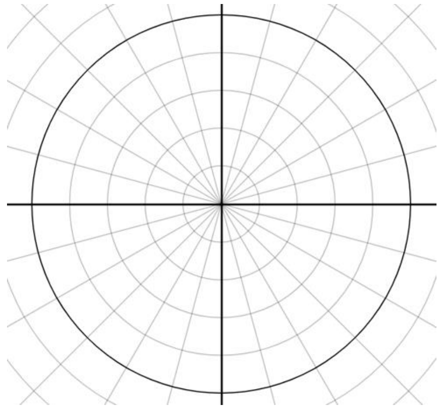
D

E

F

G

For Problem 3-6, use the grid below and information given to solve the problem.



3. Locate and label the points given on the coordinate plane above.

$A(-4, 45°)$ $B(-3.5, 120°)$ $C(3, 0°)$ $D(-3, 0°)$

$E(2, 210°)$ $F(-2, 180°)$ $G(1, 270°)$

4. What is the name of the origin of the rectangular coordinate system in polar coordinates?

5. The polar angle is undefined for the origin or how many other ways are there to name O?

6. Name $(5, 270°)$ another way.

For Problem 7-9, use the information given to solve the problem.

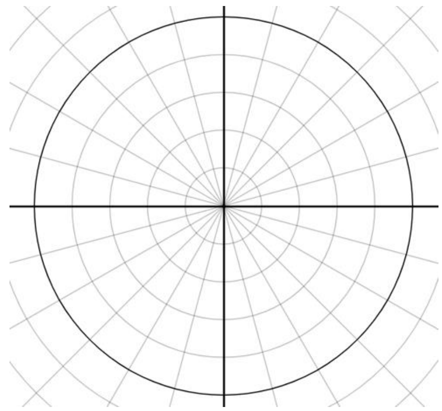
7. a) Name θ given the polar point $(-2, 135°)$.

b) Name θ given the polar point $(-4, \pi)$.

c) Name r given the polar point $(2, 45°)$.

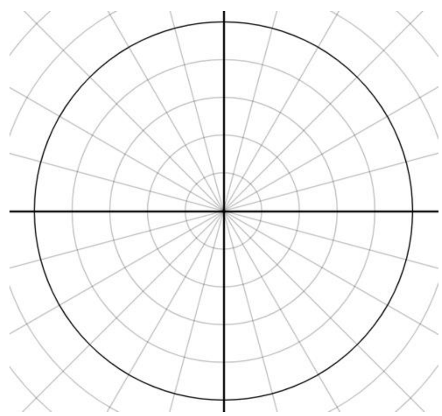
8. Let $r = \sin \theta$. Complete the table below and graph the points on the graph paper. What is the shape of the graph?

r	θ
0	0
0.5	$\frac{\pi}{6}$
	$\frac{\pi}{3}$
	$\frac{\pi}{2}$
	$\frac{2\pi}{3}$
	$\frac{5\pi}{6}$
	π
	$\frac{7\pi}{6}$
	$\frac{4\pi}{3}$
	$\frac{3\pi}{2}$
	$\frac{5\pi}{3}$
	$\frac{11\pi}{6}$
	2π



9. Let $r = \cos \theta$. Complete the table below. What do you think the graph will look like?

r	θ
1	0
0.86	$\frac{\pi}{6}$
	$\frac{\pi}{3}$
	$\frac{\pi}{2}$
	$\frac{2\pi}{3}$
	$\frac{5\pi}{6}$
	π
	$\frac{7\pi}{6}$
	$\frac{4\pi}{3}$
	$\frac{3\pi}{2}$
	$\frac{5\pi}{3}$
	$\frac{11\pi}{6}$
	2π



For Problem 10-20, use the information given to solve the problem.

10. What are the zeroes of $r = \sin\theta$?

11. What is the maximum point of $r = \sin\theta$?

12. What is the maximum point of $r = 2\sin\theta$?

13. What is the maximum point of $r = a\sin\theta$

14. Sketch a graph of $r = -\sin\theta$?

15. What are the zeroes of $r = \cos\theta$?

16. What is the maximum point of $r = \cos\theta$?

17. What if the maximum point of $r = 3\cos\theta$.

18. What is the maximum point of $r = a\cos\theta$?

19. Sketch a graph of $r = -\cos\theta$.

20. The (x, y) coordinates of a point are $(0, -4)$. What are the polar coordinates?

Section 4.5 Plotting Polar Coordinates

Practice Problems 4.5

For Problem 1-10, use the information given to solve the problem.

1. In Example 2, you used radians for θ rather than degrees to find the polar coordinates. See if you can complete the rule for finding the four forms of the polar coordinates using radians.

(r, θ) $r = |r|$

$(r, \theta - 2\pi)$

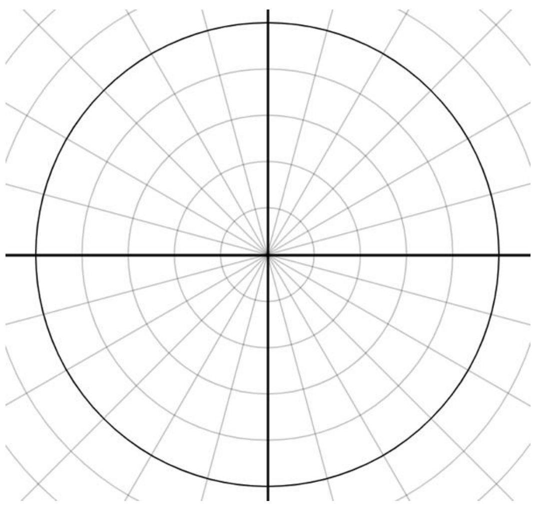
$(-r, \theta -$ ________$)$

$(-r, \theta +$ ________$)$

2. Plot the given points on the polar graph and label them by their letters.

$A(2, \frac{\pi}{4})$ $B(3, \pi)$ $C(-2, -\pi)$

$D(3, \frac{3\pi}{4})$ $E(2, \pi)$ $F(-3, -\frac{\pi}{4})$



3. Which pair of points in Problem 2 represent the same point?

4. Which pair of points in Problem 2 represent opposite points?

5. What is the total angle distance between opposite points in degrees? What is the total distance between opposite points in radians?

6. If you know a point is at an angle of $\frac{\pi}{3}$, what is the measure of the opposite point in radians? What is the angle of the opposite point in degrees?

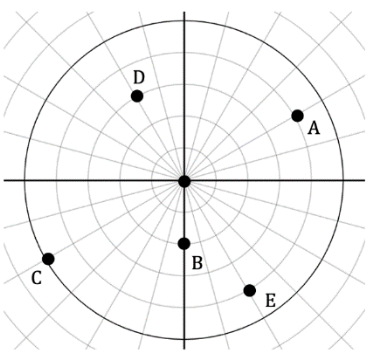
7. The equation $r = 2\cos\theta$ is a circle along the positive x-axis. What is the diameter of the circle?

8. The equation $r = -3\cos\theta$ is a circle with a diameter of -3. Which axis does it lie on?

a) Positive x-axis b) Negative x-axis

c) Positive y-axis d) Negative y-axis

9. Name the points on the polar graph using degrees.



10. Name each of the points in Problem 9 using radians.

For Problem 11-20, follow the steps in each Problem to graph the equation $r = 4\cos 2\theta$.

θ	**2θ**	**cos 2θ**	**4 cos 2θ**	**(r, θ)**
0°				
30°				
45°				
60°				
90°				
120°				
135°				
150°				
180°				

11. What type of symmetry does the equation demonstrate?

12. A rose curve graph is formed from the polar equation $r = a\sin n\theta$ or $r = a\cos n\theta$ in which $a \neq 0$ and/or $n > 1$, and n is an integer. What is the value of a in the equation? What is the value of n in the equation?

13. If n is an even integer, the rose will have $2n$ petals; if n is an odd integer, the rose will have n petals. How many petals will the rose have given the equation?

14. What is the length of each petal if it is determined by the value of a?

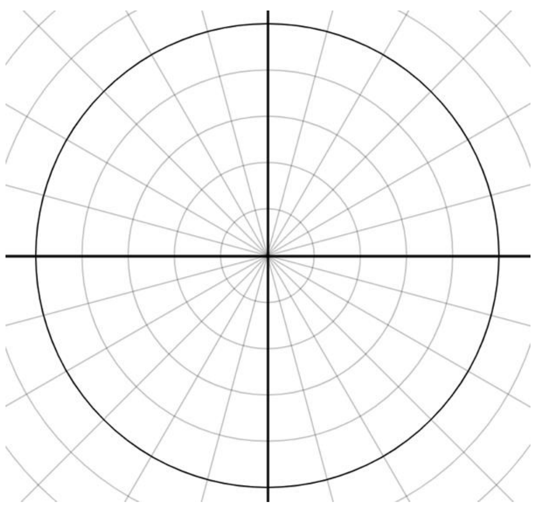
15. Complete the table for 2θ for $0° \leq \theta \leq 180°$.

16. Complete the table for $\cos 2\theta$ for $0° \leq \theta \leq 180°$.

17. Complete the table for $4 \cos 2\theta$ for $0° \leq \theta \leq 180°$.

18. Complete the table for the ordered pairs for (r, θ) for $0° \leq \theta \leq 180°$.

19. Graph the point (r, θ) for $0° \leq \theta \leq 180°$ from the table on a polar graph. Use symmetry to complete the petals. What is the complete graph?



20. How many petals do you think the equation $r = 3 \cos 3\theta$ has?

Section 4.6 Converting from Rectangular to Polar Coordinates

Practice Problems 4.6

For Problem 1-4, convert the polar coordinate to a rectangular coordinate. Give the exact answer.

1. $(2, 30°)$

2. $(-3, -150°)$

3. $(4, -45°)$

4. $(3, 60°)$

For Problem 5-8, convert the rectangular coordinate to a polar coordinate.

5. $(5, 0)$

6. $(0, -2)$

7. $(-3, 3)$

8. $(-\sqrt{2}, -\sqrt{2})$

For Problem 9 and 10, convert the equation given to solve the problem.

9. Convert $x^2 + y^2 = 2y$ to a polar equation and solve for r. (Remember, $\cos^2\theta + \sin^2\theta = 1$.)

10. Convert $r = 2\cos\theta + \sin\theta$ to a rectangular equation. (Hint: Multiply both sides of the equation by r first.)

For Problem 11-14, convert the equation in rectangular coordinates to an equation using polar coordinates.

11. $x = 4$

12. $y = -7$

13. $y = -x$

14. $x^2 + y^2 = 25$

For Problem 15-19, convert the equation in polar coordinates to an equation using rectangular coordinates.

15. $r = 6$

16. $\theta = 270°$

17. $\sin\theta = \cos\theta$

18. $r\sin\theta = 5$

19. $r\cos\theta = -3$

For Problem 20, use the information given to solve the problem.

20. Does $r = 4\sin 3\theta$ make a three-leaf rose or a four-leaf rose?

Section 4.7 Converting Between Polar and Rectangular Equations

Practice Problems 4.7

For Problem 1- 9, convert the rectangular-coordinate equation to a polar-coordinate equation.

1. $x^2 + y^2 = -2x$

2. $y = -3$

3. $-x = y$

4. $5x - 3y = 4$

5. $4y - y^2 = x^2$

6. $x^2 + y^2 = -5$

7. $x^2 + y^2 = 25$

8. $x^2 + y^2 = 4 + 4x + x^2$

9. $x^2 + y^2 = 9x$

Convert the polar-coordinate equation to a rectangular-coordinate equation for Problem 10

10. $r = -2\,r\cos\theta$

For Problem 11-15, use the rectangular coordinates $(-2, -3)$ to solve the problem.

11. Plot the point and draw a segment from the origin to the point. Let the segment be the terminal side of an angle drawn in standard position.

12. Draw a right triangle by extending a segment from the point perpendicular to $x -$ axis and use the Pythagorean Theorem to solve for r (the terminal segment).

13. Use an inverse trigonometric function to find one value of the angle θ.

14. Use the trigonometric identity $\theta \approx \theta \pm 180°n$ to find the value of θ for the third quadrant.

15. Name the polar coordinates for the rectangular coordinates $(-2, -3)$.

For Problem 16-17, given (x, y) in the rectangular system is the same point as $[r, \theta]$ in the polar system, tell whether the statement is true or false.

16. For all integers n, $[r, \theta + 2\pi n]$ represents the point $[r, \theta]$.

17. For all integers n, $[-r, \theta + 2\pi n + 1]$ represents the same point $[r, \theta]$.

Problem 18 below, answer question a)-j).

a) The polar coordinates of a point are $(2, 90°)$. What are the (x, y) rectangular coordinates on the x-y coordinate plane?

b) The polar coordinates of a point are $(4, -90°)$. What are the (x, y) rectangular coordinates on the x-y coordinate plane?

c) The Cartesian coordinate grid gives points with rectangular coordinates. What is the radius of point $(4, 4)$ on the polar grid?

d) What is the angle on the polar grid for the rectangular point $(4, 4)$?

e) Name the polar coordinates for the rectangular coordinates for the point $(4, 4)$.

f) What is the radius of the rectangular coordinates $(-3, -3)$ on the polar graph?

g) What is the angle on the polar grid for the rectangular point $(-3, -3)$?

h) Name the polar coordinates for the rectangular coordinates $(-3, -3)$.

For Problem 19 and 20, use trigonometric identities to simplify the trigonometric expressions.

19. $r^2 + r^2\tan^2\theta$

20. $1 + \frac{\cos^2\theta}{\sin^2\theta}$

Section 4.8 Graphs of Polar Equations

Practice Problems 4.8

For Problem 1-10, use the information given to solve the problem.

1. Describe what the graph of $\theta = \frac{\pi}{6}$ would look like.

2. Name equations that are equivalent to $\theta = \frac{\pi}{6}$.

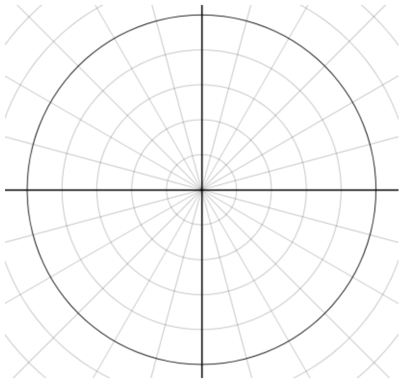
3. How is the graph of $r = 2$ similar to the graph of $r = 3$? How is it different?

4. What do you think the graph of $r = 2\theta$ would look like? Why?

5. What would the graph of $r = \cos 2\theta$ look like compared to the graph of $r = \sin 2\theta$? First, complete the table below.

$\boldsymbol{\theta}$	$\mathbf{\cos 2\theta}$	$\mathbf{\sin 2\theta}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
60°		$\frac{\sqrt{3}}{2}$
90°		0
120°		$-\frac{\sqrt{3}}{2}$
150°		$-\frac{\sqrt{3}}{2}$
180°		0

6. What would the graph of $r = 2\cos 2\theta$ look like? First, sketch it, then make a table and draw it. Extend the table: $0 \le \Theta \le 360°$.



7. How does $r = 2\cos(\theta)$ transform the graph of $r = \cos(\theta)$?

8. How does $r = \cos(2\theta)$ transform the graph of $r = \cos(\theta)$?

9. How would changing the polar equation $r = \cos(2\theta)$ to $r = 2\cos(2\theta)$ change the graph?

10. How would changing the equation $r = 2\cos(\theta)$ to $r = -2\cos(\theta)$ change the polar graph?

For Problem 11-15, use the polar equation $r = 3\sin\theta$ to solve the problem.

11. Make a table of values for $[r, \theta]$ for $0 \le \theta \le \pi$.

$\boldsymbol{\theta}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\boldsymbol{r}$									

12. Is θ the independent or dependent variable? Is r the independent or dependent variable?

13. Plot the points on the polar graph. What conic section results?

14. As θ increases from 0 to $\frac{\pi}{2}$, r ___________________from 0 to ___________.

15. As θ increases from $\frac{\pi}{2}$ $to\ \pi, r$ ___________________from ___________ to 0.

16. The maximum value of $|r|$ on the polar graph $r = \sin 3\theta$ is ___________.

17. True or False: The equation $r = \sin 3\theta$ causes a dilation of $r = \sin\theta$ by a scale factor of 3.

18. Explain the transformation of $r = \sin 3\theta$ from $r = \sin\theta$.

19. Explain the transformation of $r = 3\sin\theta$ from $r = \sin\theta$.

20. Explain the transformation of $r = 3\sin 3\theta$ from $r = \sin\theta$.

Section 4.9 Shifts of Polar Graphs

Practice Problems 4.9

For Problem 1-3, compare and contrast the given polar equations.

1. $r = 2\cos\theta$ and $r = 4\cos\theta$

2. $r = 4\cos\theta$ and $r = 4\sin\theta$

3. $r = \cos 2\theta$ and $r = \cos 3\theta$

For Problem 4-20, use the information given to solve the problem.

4. How many petals does each graph have?

a) $r = \cos 2\theta$

b) $r = \cos 3\theta$

c) $r = \cos 4\theta$

d) $r = \cos 5\theta$

5. The graph of $r = 4\sin\theta$ is a circle and $r = 4\sin(\theta + \frac{\pi}{2})$ is rotated 90° clockwise from $r = 4\sin\theta$. Write the equation to move the original graph 90° counterclockwise.

6. Can you see a pattern to determine the number of petals for $r = \cos a\theta$ that depends on whether a is even or odd?

7. What is the new equation when the graph of $r = 2 + 4\sin\theta$ is rotated an angle of $\frac{\pi}{3}$ clockwise?

8. How do you rotate $r = 7\cos\theta$ to obtain the graph of $r = 7\cos(\theta - \frac{\pi}{2})$?

9. Compared to the graph of $r = \sin 7\theta$, what will the graph of $r = \sin[\,7(\theta - \frac{\pi}{6})]$ looks like?

10. How many petals will $r = \sin 9\theta$ have?

11. What is a if $r = \sin a\theta$ has 12 petals?

12. Graph the polar equation $r = \theta$ for $0 \leq \theta \leq 6\pi$. What is the largest value of r as you trace around the arithmetic spiral?

13. The spiral of Archimedes is $r = c\theta$ in which c is a constant and θ is measured in radians. Graph $r = 5\theta$. What graph results? Is it wider or more narrow than $r = \theta$?

14. Write a polar equation that will stretch the graph of $r = 5\theta$.

15. Write a polar equation that will shrink the graph of $r = 5\theta$.

16. Graph the polar equation $r = \frac{1}{\theta}$ in which $0.2 \leq \theta \leq 4\pi$. What graph results?

17. The spiral in Problem 16 is of the form $r = \frac{c}{\theta}$ and is called the hyperbolic spiral. In what Quadrant does it seem to infinitely extend?

18. Graph the polar equation $r = 2^{\theta}$ in which $-\pi \leq \theta \leq \pi$. What graph results?

19. The spiral in Problem 18 is of the form $r = cb^{\theta}$ in which c is a constant and b is also a constant. It is called a logarithmic spiral. Trace the graph in Problem 18. What is the greatest value of r? How many radians is θ when r is at its greatest value?

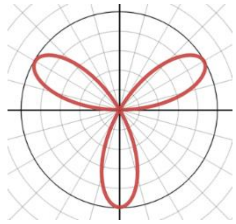
20. Try the integers $-3, -2, 2$, and 3 for $r = cb^{\theta}$ when $-\pi \leq \theta \leq \pi$. What is one point contained in all the curves for $b > 0$?

Section 4.10 Graphing Polar Functions Using Technology

Practice Problems 4.10

For Problem 1 and 2, use the given information to solve the problem.

The graph of $r = \sin 3\theta$ looks like this:



1. Sketch what you think the graph of $r = \sin(-3\theta)$ will look like and tell why.

2. Consider the polar coordinate curves $r = 2 + 3\sin\theta$, $r = 3 + 7\sin\theta$, and $r = 2 + 5\sin\theta$. If you changed sin to cos in each of these equations, how would the graph change? What would stay the same?

For Problem 3, use the information given to solve the problem.

3. Two examples of cardioids are $r = 3 + 3\sin\theta$ and $r = 3 + 3\cos\theta$. How are cardioids different than limaçons where $0 < \frac{a}{b} < 1$? Discuss the equations and the graphs. What is $\mathrm{a} - \mathrm{b}$ or $|\mathrm{b} - \mathrm{a}|$ and what effect does this have on the graph?

For Problem 4 and 5, circle the statement that is true for the equations $r = \mathrm{b}\sin(\mathrm{n}\theta)$ or $r = \mathrm{b}\cos(\mathrm{n}\theta)$ in which n is an integer.

4. a) b determines the number of petals

b) $\mathrm{b} + \mathrm{n}$ determines the number of petals

c) n determines the number of petals

d) $|\mathrm{b} - \mathrm{n}|$ determines the number of petals

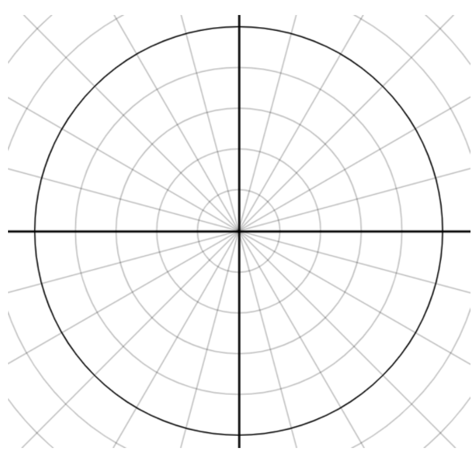
5. a) b acts as a stretching or shrinking factor

b) $\mathrm{b} + \mathrm{n}$ acts as a stretching or shrinking factor

c) n acts as a stretching or shrinking factor

d) $|\mathrm{b} - \mathrm{n}|$ acts as a stretching or shrinking factor

For Problem 6-8, graph the equation $r = 2 + 3\cos 2\theta$ and then tell whether the statement given is true or false.

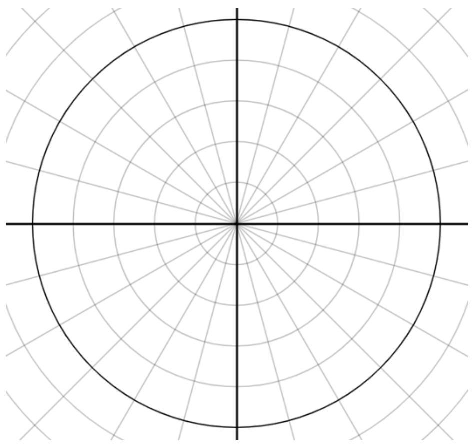


6. As one petal stretches, the alternating petals stretch as well.

7. The sum of the longest lengths of the alternating petals is equal to 2b.

8. The length of the longer petal is a − b.

9. Sketch the graph of $r = 2 + 3\sin 2\theta$ if it is a 45° counterclockwise rotation of $r = 2 - 3\cos 2\theta$.



For Problem 10, use the given equations to solve the problem.

$$r = \frac{k}{a + b\sin\theta} \qquad r = \frac{k}{a + b\cos\theta}$$

10. Substitute values for k, a, and b in the equations and see if you can make any conjectures about the properties of the graph when a = b.

For Problem 11-20, use the polar equation $r = 2 + 3\sin\theta$ to solve the problem.

11. Complete the table for $0° \leq \theta \leq 360°$ by $15°$ increments. Solve for r and round to the tenths place. Put the equation in "Lists and Spreadsheets" on the calculator and set the calculation mode to "Approximation."

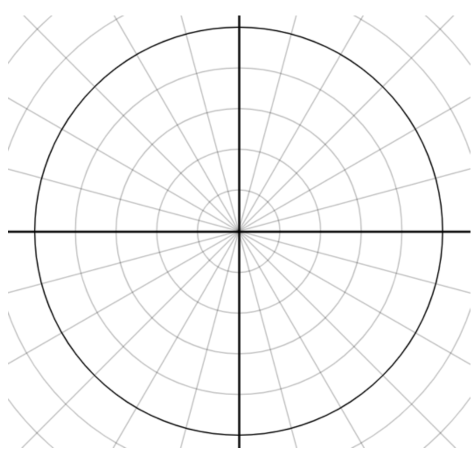
θ	r
0°	
15°	
30°	
45°	
60°	
75°	
90°	

θ	r
105°	
120°	
135°	
150°	
165°	
180°	

θ	r
195°	
210°	
225°	
240°	
255°	
270°	

θ	r
285°	
300°	
315°	
330°	
345°	
360°	

12. Plot the polar points on the polar graph.



13. Solve the equation $r = 2 + 3\sin\theta$ when $\theta = 170°$. Is this a polar point on the polar curve?

14. What is the longest inner length of the smaller petal from Problem 12?

15. What is the longest inner length of the larger petal from Problem 12?

16. What is the polar point of r when θ is π?

17. Let $r = 0$ in the polar equation and solve for θ. What is a general solution for θ?

18. At which two values of θ in the interval $[0, 360°]$ does the polar graph go through the pole?

19. At which values of θ does r equal 2? Explain why.

20. This is called a limaçon of Pascal as well (pronounced "lɪməsɒn (lee-ma-son)"). Limaçon is the French word for 'snail.' Graphs of polar functions can also be conic sections (as we have seen). Is the graph $r = \frac{7}{4-3\cos\theta}$ a limaçon or a conic section? If it is a conic section, tell what type of conic section it is.

Section 4.11 Complex Numbers and the Complex Plane

Practice Problems 4.11

For Problem 1-5, add, subtract, multiply, or divide and simplify the imaginary number.

1. $\frac{1}{\sqrt{-3}}$

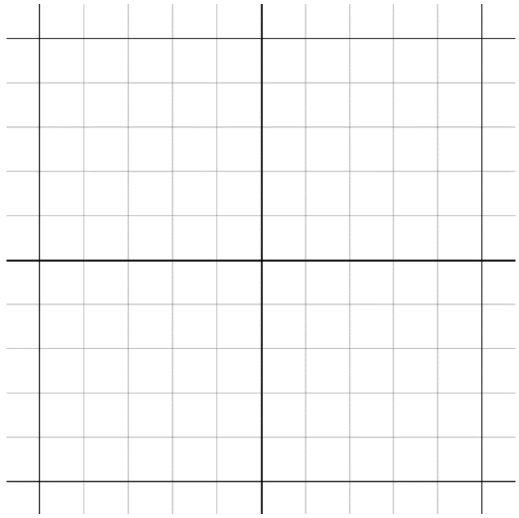
2. $(2\sqrt{-5}) \cdot (-3\sqrt{20})$

3. $5i \cdot -4i$

4. $\sqrt{-9} + \sqrt{-27}$

5. $2\sqrt{-54} - 3\sqrt{24}$

For Problem 6-10, graph the complex number given on the complex plane below.



6. $z = 4i$

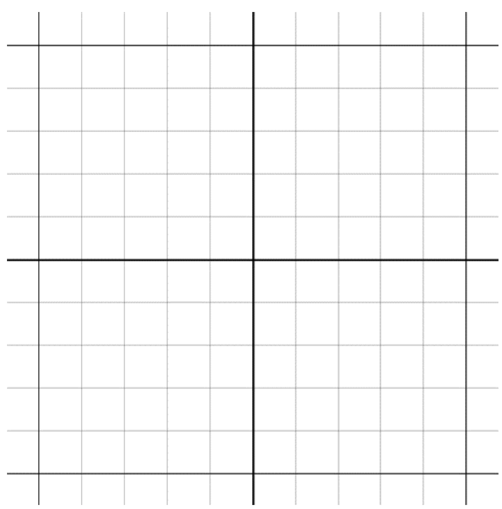
7. $z = 3 + 2i$

8. $z = 1 - 4i$

9. $z = -5i$

10. $z = -3 - 2i$

For Problem 11-15, simplify the complex numbers and graph them on the complex plane below.



11. $(2 + 7i) + (3 - 2i)$

12. $\frac{3i}{-15i}$

13. $(3 - 5i) - (6 - 3i)$

14. $(1 + 2i)(1 - 2i)$

15. Find the reciprocal of $2 - i$.

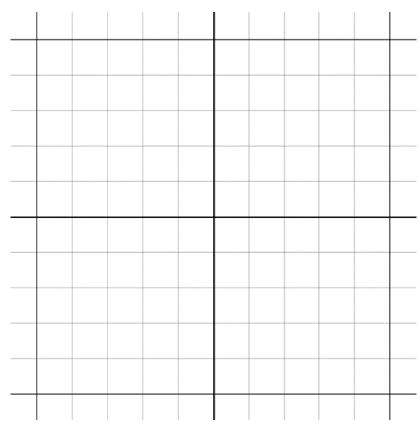
For Problem 16 and 17, use the information given to solve the problem.

16. What is the distance of the point $z = 5 - 2i$ from the origin?

17. What is the absolute value of the point $z = -6 + 3i$?

For Problem 18-20, use the graph you make in Problem 18 and point z_1 and z_2 to solve the problem.

18. Plot the point $z_1 = 2i + 1$ and $z_2 = 3i - 3$ on the imaginary plane.



19. What is the distance from z_1 to z_2?

20. a) What is the modulus of point z_1?

b) What is the magnitude of point z_2?

Section 4.12 Polar Form of Complex Numbers

Practice Problems 4.12

For Problem 1-12, use the information given to solve the problem.

1. Given the complex number $z = 2 + (2\sqrt{3})i$, what is r?

2. Given the complex number $z = 2 + (2\sqrt{3})i$, what is θ?

3. Find a and b in polar form (a, b) for the complex number $z = 2 + (2\sqrt{3})i$ and graph it on the complex plane.

4. What is the absolute value of $z = 2 + (2\sqrt{3})i$?

5. Write the complex number $z = 2 + (2\sqrt{3})i$ in standard form as a complex polar number.

6. Convert the complex number in polar form, $z = 6(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$, to a complex number in standard form.

7. Convert the complex number to polar form: $z = \sqrt{2} + \sqrt{6}i$.

8. Locate $z = 5\sqrt{2}(\cos\frac{\pi}{4} + i\ \sin\frac{\pi}{4})$ on the complex grid.

9. Find the rectangular form of $z = 5(\cos\left(\frac{3\pi}{2}\right) + i\ \sin(\frac{3\pi}{2}))$.

10. Transform the polar equation $r = -2\sec\theta$ into rectangular form.

11. Expand $(3 - 3i)^2$ and simplify it.

12. For the complex number $z = -4(\cos\left(\frac{\pi}{4}\right) + i\ \sin\left(\frac{\pi}{4}\right))$, find its rectangular form.

For Problem 13-20, use $z_1 = 4 - 3i$ and $z_2 = 1 + 2i$ to solve the problem.

13. Which has a greater magnitude, z_1 or z_2?

14. What are the rectangular coordinates for z_1 and z_2 on the complex plane?

15. What is the complex number in polar form for z_1?

16. What is the complex number in polar form for z_2?

17. What is $z_1 + z_2$?

18. What is $z_1 - z_2$?

19. What is $z_1 \cdot z_2$?

20. What is $\frac{z_1}{z_2}$?

Section 4.13 Finding Roots and Powers of Complex Numbers

Practice Problems 4.13

For Problem 1-5, let $f(x) = x^3 - 1$.

1. Verify that 1 is a root of the polynomial $f(x)$?

2. Write the monomial factor (of the form $(x - r)$) corresponding to the root 1.

3. Find the quotient of $f(x)$ when divided by the factor in Problem 2.

4. Use the quadratic formula with the quotient in Problem 3 to find two other roots of $f(x)$.

5. Write the three roots of $f(x)$.

For Problem 6-9, write the given complex number in polar form: $r(\cos\theta + i\sin\theta)$.

6. $-3 + i$

7. -4

8. $-i$

9. $2 + i\sqrt{3}$

For Problem 10-13, write the given polar number in complex form: $a + bi$.

10. $5(\cos 0 + i \sin 0)$

11. $3\,(\cos 120° + i \sin 120°)$

12. $4(\cos 270° + i \sin 270°)$

13. $\sqrt{2}(\cos 45° + i \sin 45°)$

For Problem 14-17, let $z_1 = 4 + 3i$ and $z_2 = 1 - i$.

14. Find $z_1 z_2$.

15. Find $\frac{z_1}{z_2}$.

16. Find ${z_1}^3$.

17. Find z^6 for $z = 3(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$.

For Problem 18-20, use the information given to solve the problem.

18. De Moivre's Theorem may also be used to find roots of complex numbers in polar form. For any complex number $z = r(\cos\theta + i\ \sin\theta)$ where $r > 0$, and positive integer n, valid representations of n distinct nth roots of z may be obtained using the following expression:

$$\sqrt[n]{z} = \sqrt[n]{r}[\cos(\frac{\theta}{n} + \mathrm{k}\frac{360°}{n}) + i\ \sin\frac{\theta}{n} + \mathrm{k}\frac{360°}{n})]$$

Find the general formula for the fourth roots of $z = 1 + i\sqrt{3}$. From Example 4, $z^4 = 16[(\cos 120° + i\ \sin 120°)]$ in polar form.

19. Let k take on the value of any non-negative integer less than n: e.g., for any $n = 4$, then $\mathrm{k} = 0, 1, 2, 3$, and find the fourth roots of $z = 1 + i\sqrt{3}$ in polar form.

20. Write the fourth roots of $z = 1 + i\sqrt{3}$ in $a + bi$ form.

Section 4.14 Module Review

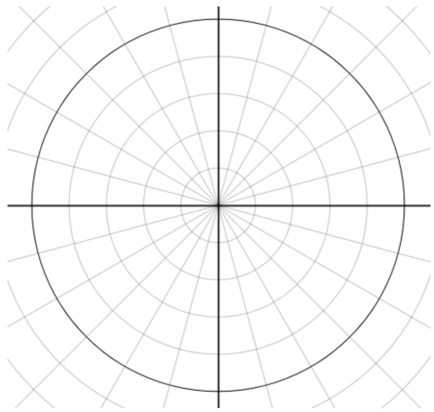
For Problem 1 and 2, use the instructions given and the graph in Problem 1 to solve the problem.

1. Graph the given points on the polar grid.

a) $(2, 120°)$

b) $(-3, 90°)$

c) $(1, -45°)$



2. Name three other polar coordinates for each polar coordinate in Problem 1.

For Problem 3 and 4, convert the given point to solve the problem.

3. Convert the polar coordinates $\mathrm{J}(-3, 120°)$ to rectangular coordinates.

4. Convert the rectangular coordinates $\mathrm{L}(5, -5)$ to polar coordinates.

For Problem 5 and 6, transform the given equation to solve the problem.

5. Transform $3r^2 - 9\cos\theta = 12 + 3\sin^2\theta$ to a rectangular-coordinate equation.

6. Transform $y = x^2$ to a polar-coordinate equation.

For Problem 7-20, use the information given to solve the problem.

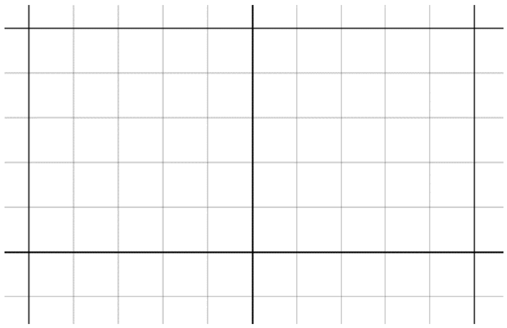
7. Write the polar equation for $y = x^2$ with a rotation of $120°$ counterclockwise.

8. How many petals are on the graph of the rose $r = 4\sin 3\theta$?

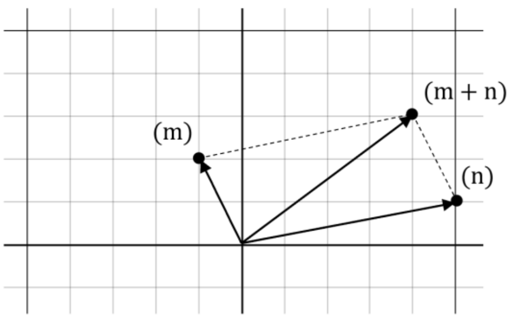
9. If $z = x + yi$ is any complex number, what is the conjugate of z? Demonstrate by showing the product is a real number.

10. What is the magnitude of $3 + 4i$?

11. Plot $\mathrm{m} = -1 + 2i$, $\mathrm{n} = 5 - i$, and $\mathrm{m} + \mathrm{n}$ on the complex plane.



12. Given the complex plane, what are the rectangular coordinates of $\mathrm{m} + \mathrm{n}$?



13. Simplify $\left(\frac{-5}{3i}\right)\left(\frac{4}{2i}\right)$.

14. Write the complex number $z = 3 - 5i$ in polar or trigonometric form. Round to the nearest tenth of a degree.

15. Check Problem 14 and see if the polar equation is equivalent to $z = 3 - 5i$ in complex form.

16. What is the modulus of $z = 7 - 2i$?

17. Given $\mathrm{m} = 2 + 3i$ and $n = -1 + 5i$, find a)-d).

a) Find $\mathrm{m} + \mathrm{n}$.

b) Find $\mathrm{m} - \mathrm{n}$.

c) Find $\mathrm{m} \cdot \mathrm{n}$.

d) Find $\frac{\mathrm{m}}{\mathrm{n}}$.

18. Convert the complex number in polar form to a standard form complex number: $z = 3(\cos(\frac{\pi}{3}) + i\ \sin(\frac{\pi}{3}))$.

19. Convert the complex number $z = 1 - 2i$ to polar or trigonometric form.

20. Use De Moivre's Theorem to find z^3 in polar form if $z = 1 - 2i$.

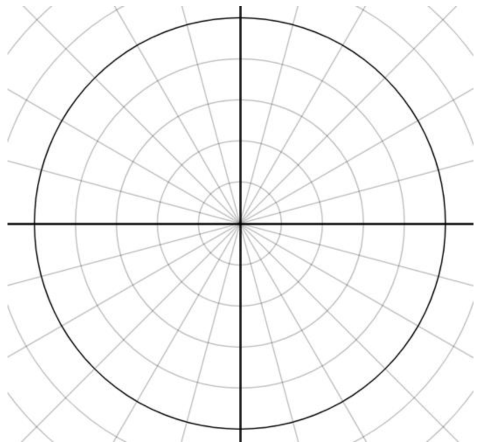
Section 4.15 Module Test

For Problem 1 and 2, use the instructions given and the graph in Problem 1 to solve the problem.

1. Graph the given points on the polar grid.

a) $(-3, -30°)$ b) $(2, 45°)$

c) $(1, -120°)$



2. Name three other polar coordinates for each polar coordinate in Problem 1.

For Problem 3 and 4, convert the given point to solve the problem.

3. Convert the polar coordinates B$(2, 315°)$ to rectangular coordinates.

4. Convert the rectangular coordinates E$(2, -2)$ to polar coordinates.

For Problem 5 and 6, transform the given equation to solve the problem.

5. Transform $r(1 + \sin\theta) = 6$ to a rectangular-coordinate equation.

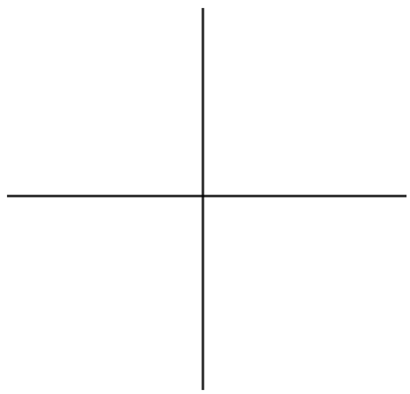
6. Transform $x^2 + y^2 = 4x$ to a polar-coordinate equation.

For Problem 7-20, use the information given to solve the problem.

7. Write the polar equation for $x^2 + y^2 = 4x$ with a rotation of $\frac{\pi}{3}$ counterclockwise.

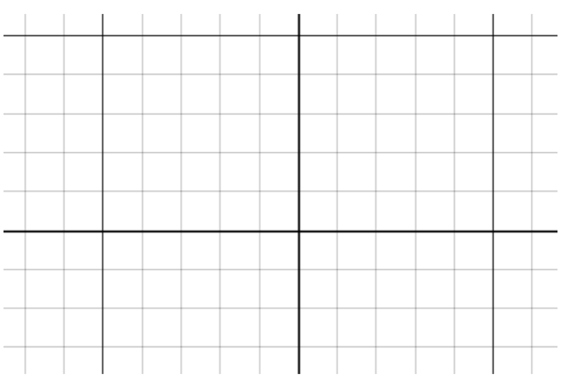
8. How many petals are on the graph of the rose $r = 2\cos 2\theta$?

9. If $z = x + yi$ is any complex number, what is $-z$? Demonstrate this on the complex plane below.



10. What is the magnitude of $-3 + 2i$?

11. Plot $s = -3 - i$, $t = 4 - 2i$, and $s - t$ on the complex plane.



12. From Problem 11, what are the rectangular coordinates of $s - t$.

13. Simplify $(\frac{i}{\sqrt{2}})(\frac{3}{\sqrt{-2}})$.

14. Write $z = -2 + 3i$ in polar or trigonometric form. Round to the nearest tenth of a degree.

15. Check Problem 14 and see if the polar equation is the equivalent to $z = -2 + 3i$ in complex form.

16. What is the modulus of $z = 3 + 3i$?

17. Given $m = 1 + 2i$ and $n = -1 - 2i$ find a)-d).

a) Find $m + n$.

b) Find $m - n$.

c) Find $m \cdot n$.

d) Find $\frac{m}{n}$.

18. Convert the complex number in polar form to a standard form complex number: $z = 5(\cos\frac{\pi}{2} + i\ \sin\frac{\pi}{2})$.

19. Convert the complex number $1 + 4i$ to polar or trigonometric form.

20. Use De Moivre's Theorem to find z^2 in polar form if $z = 1 + 4i$.