Pre-Calculus and Calculus Module 1 Circular and Periodic Functions

Section 1.1 The Sine Function

Practice Problems 1.1

The seats of the Socasinusoid Ferris Wheel are located 15° apart. Draw the spokes at every 15° of the Ferris wheel below. Some of the degree angles of the spokes are drawn in for you, but 15° degrees, 75° degrees, 105° degrees, and 165° degrees are missing in Quadrants **I** and **II**. There are four more that need to be added in Quadrants **III** and **IV**.

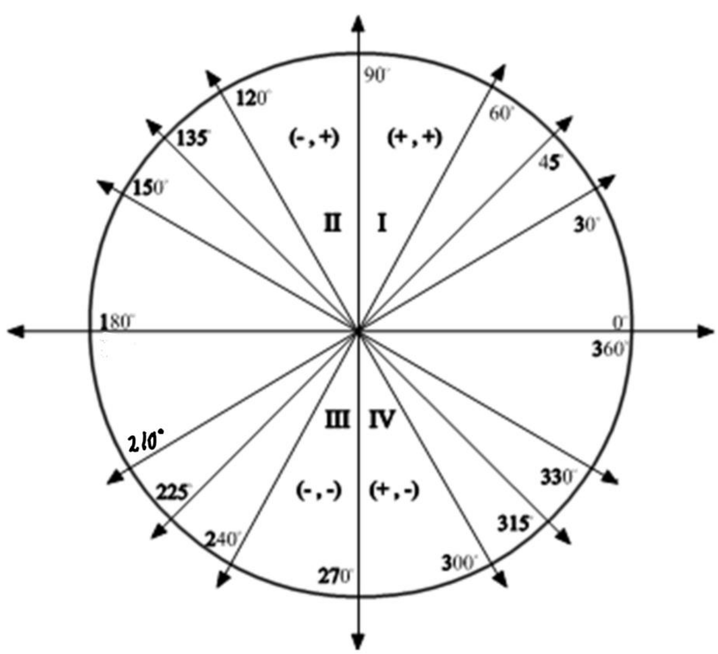
The 0° and 180° spokes are at ground level. Each individual rider enters at ground level, but they alternate between going to the 0° mark and the 180° mark to get in the seats of the Ferris wheel. The exit is also at the origin. The ride is above ground for the upper half of the ride and below ground for the bottom half of the ride. Color the outside of the circle below the x-axis blue (you can draw in sharks and fish if you wish as this part goes through an enclosed aquarium on either side of the Ferris wheel).

If the Ferris wheel suddenly stopped because of an emergency, the riders could walk directly up or down to get to the platform. At each seat, measure the vertical distance each rider would walk to get to the platform. Mark the distance above the water line positive and the distance below the water line negative. Use a centimeter ruler and measure to the nearest millimeter (55 mm. = 5.5 cm.).

Four distances are marked for you on the table of the Ferris wheel which has a diameter of 20 centimeters. The chart lists the reference angle of the seat, the vertical distance to the platform, the actual distance to the platform, and the approximate sine ratio for each angle. If 1 cm. = 10 ft, find the actual vertical distance the person would walk from the seat to the platform to leave the ride if it stopped due to an emergency.

Once you measure the distances for Quadrant **I**, you will be able to figure out the rest of the chart for Quadrants **II-IV**. Complete the chart and answer the questions.

1. On a white sheet of paper, draw a 360° circle with a radius of 10 centimeters. Open your compass to 10 centimeters. Use your protractor to mark every 15° degrees. Use this for all measurements to complete the table.

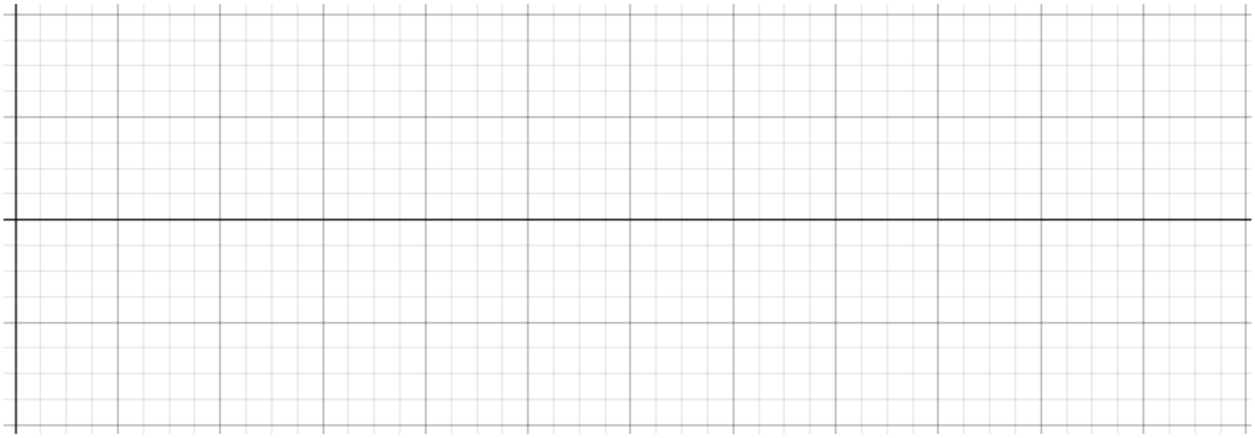


2.

Reference Angle of Seat from Center	Measured Vertical Distance to Platform (cm.)	Calculated Actual Walking Distance to Platform (ft.)	Approximation for the Sine of the Angle of the Seat
0°			
15°			
30°			
45°			
60°			
75°			
90°	10	100	1
105°			
120°			
135°			
150°			
165°			
180°	0	0	0
195°			
210°			
225°			
240°			
255°			
270°	-10.0	-100	-1
285°			
300°			
315°			
330°			
345°			
360°	0	0	0

3. How do the actual distances walked vertically from the seat to the platform replicate the sine function?

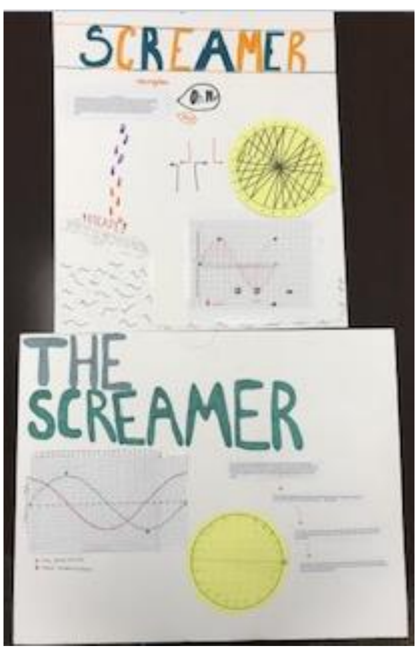
4. Draw the graph by locating ordered pairs from the chart where (x, y) represents (angle, measured vertical distance to platform). Mark the x-axis from 0° to 360° using a 15° interval. Mark the y-axis from -10 to 10. Connect the ordered pairs. What function does the graph model?

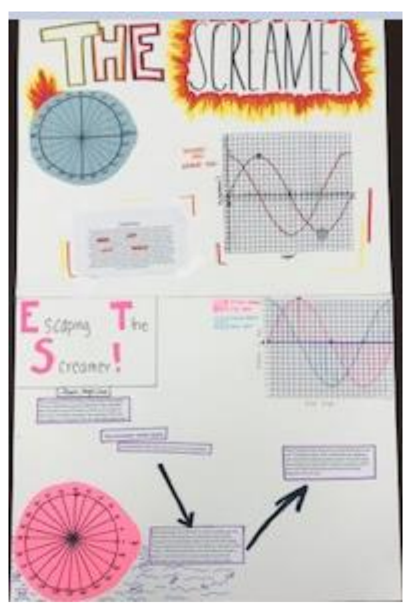


5. Would the y-axis change if (x, y) represented (angle, calculated actual walking distance to platform)?

6. At what two angles would the vertical distance measured be 0?

Below are two examples of groups of students' completed projects for the Socasinusoidal Ferris Wheel called "The Screamer."

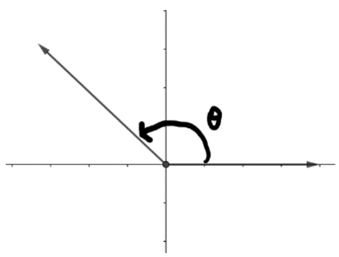




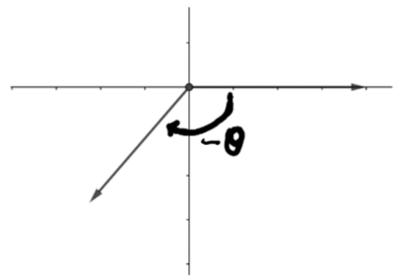
For Problem 7 and 8, use the information below to help you label the angle as a clockwise or counterclockwise rotation from the initial side. Mark the initial side "**I**." The terminal side is the final position of the side after the rotation. Mark the terminal side "**T**."

An angle is in standard position on the x-y coordinate plane with its initial side on the positive x-axis and its vertex at the origin.

7.



8.



For Problem 9-12, name the measure of each angle in degrees given each rotation. An angle rotated counterclockwise is positive and an angle rotated clockwise is negative. Tell whether the angle is positive or negative. If an angle makes more than one full rotation, it is more than 360° degrees.
Note: The measure of an angle of one full rotation counterclockwise is 360° and is measured in positive degrees. One full rotation clockwise would be $-360°$ degrees.

9. One-quarter turn clockwise

10. One-half turn counterclockwise

11. Three-fourths of a turn clockwise

12. One-eighth of a turn counterclockwise

For Problem 13-16, given the angle of rotation, name the angle of rotation going the other direction so that the initial sides and the terminal sides of the two angles meet. These angles are called coterminal. For example, 35° is a counterclockwise turn, so to get the angle going the other direction we would take $360° - |35°|$ degrees, which equals 325° clockwise, and is $-325°$ degrees.

13. 40°

14. $-180°$

15. $-22°$

16. 14°

For Problem 17-20, use the information below to sketch the standard position angle and name one coterminal angle.

Angles are coterminal if they share the same initial and terminal side. For example, $90°$ and $-270°$ are coterminal.

17. $-30°$

18. $45°$

19. $120°$

20. $22°$

Section 1.2 The Cosine Function

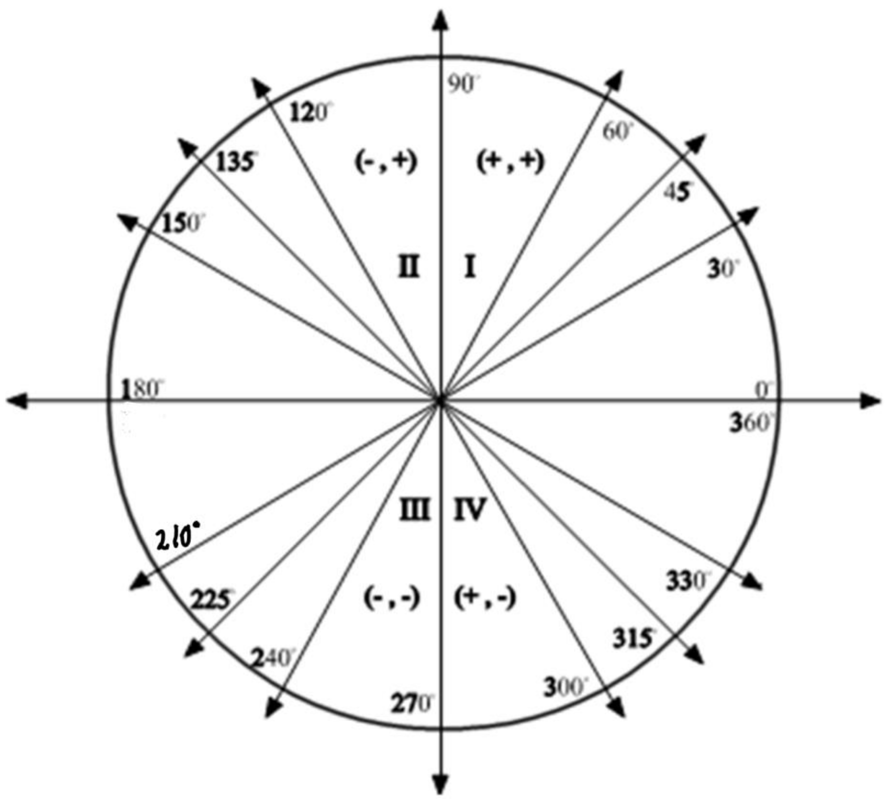
Practice Problems 1.2

Using the circle that you drew in the previous section, measure the horizontal distances walked to the center to exit once the horizontal platform is reached after first moving vertically down from the seat. Make sure you use the paper from the Practice Problems in Section 1 where you made a circle with the compass set at 10 cm., so the diameter was 20 cm. You used your protractor to mark every 15° degrees, so it looks like the circle below. All circles are similar. (You might remember that from Trigonometry.) The exit in an emergency is at the origin.

Mark the distances in Quadrant **I** with long arrows (→) to the right or left of where the seat touches the circle (where the rays of the seat angle intersect the circle) to the y-axis. Measure in centimeters. Use this information to complete the chart for the horizontal distances and the cosine of each seat angle. Two are done for you. Mark the horizontal distances coming from the right side of the y-axis as positive. Mark the horizontal distances coming from the left side of the y-axis as negative.

After completing the chart, answer the questions and complete a graph for the horizontal distances walked to the center to exit from each seat angle.

1.

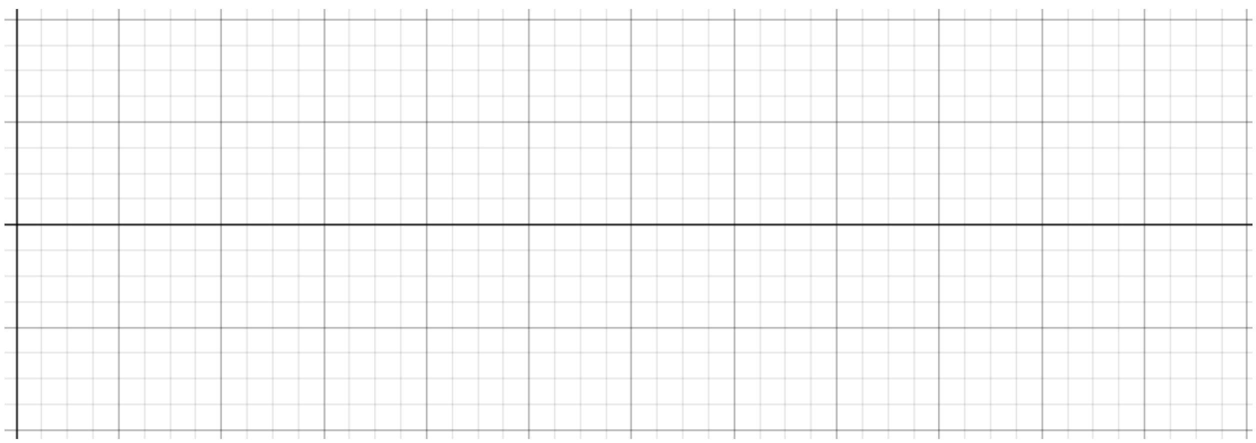


2.

Reference Angle of Seat from Center	**Measured Horizontal Distance to Center of Platform (cm.)**	**Calculated Actual Walking Distance to Center of Platform (ft.)**	**Approximation for the Cosine of the Angle of the Seat**
0°	10	100	1
15°			
30°			
45°			
60°			
75°			
90°	0	0	0
105°			
120°			
135°			
150°			
165°			
180°			
195°			
210°			
225°			
240°			
255°			
270°			
285°			
300°			
315°			
330°			
345°			
360°			

3. Why are the distances coming from the right of the center exit labeled as positive and those coming from the left of the center exit labeled as negative?

4. Draw the graph by locating ordered pairs from the chart where (x, y) represents (angle, measured horizontal distance to center of platform). Mark the x-axis from 0° to 360° using a 15° interval. Mark the y-axis from -10 to 10. Connect the ordered pairs. What function does the graph model?



5. What are the units for the (x, y) ordered pairs for the graph? How would this change for a graph of the actual distance walked?

6. How could you have found an approximate horizontal distance walked without measuring it?

For Problem 7-10, use the information below to find the complement of the angle.

Angles are complementary if they have a sum of 90° degrees.

7. 43°

8. 21°

9. 50°

10. 78°

For Problem 11-14, use the information below to find the supplement of each angle.

Angles are supplementary if they have a sum of 180° degrees.

11. 135°

12. 114°

13. 25°

14. 179°

For Problem 15 and 16, solve the word problem.

15. What is the complement of x?

16. What is the supplement of y?

For Problem 17-20, use the information below to name the angle given its measure.

Greek letters are used to represent angles. The most used Greek letter for angles is theta (θ), but alpha (α) and beta (β) are also used frequently. If $0° < \alpha < 90°$ the angle is said to be acute. If $90° < \beta < 180°$ then the angle is said to be obtuse. When $\theta = 90°$ the angle is said to be a right angle.

17. $\alpha = 90°$

18. β is a complementary angle

19. $\alpha = 98°$

20. $\beta = 23°$

Section 1.3 Socasinusoidal Ferris Wheel

Practice Problems 1.3

Mark 1-12 below on the sine curve of the Socasinusoid Ferris Wheel. Look back at your tables and graphs if you need to in order to answer the questions.

1. Put a red "x" on the graph where the riders initially get on the ride.
2. Put a blue "x" on the graph where the ride completes one rotation.
3. Put a green "x" on the positions on the graph where the riders would have to walk approximately 70 feet to get to the platform.
4. If the ride stops and a rider is in a seat at an angle of 350° degrees, approximately how far would he/she have to walk to get to the platform?

   ____

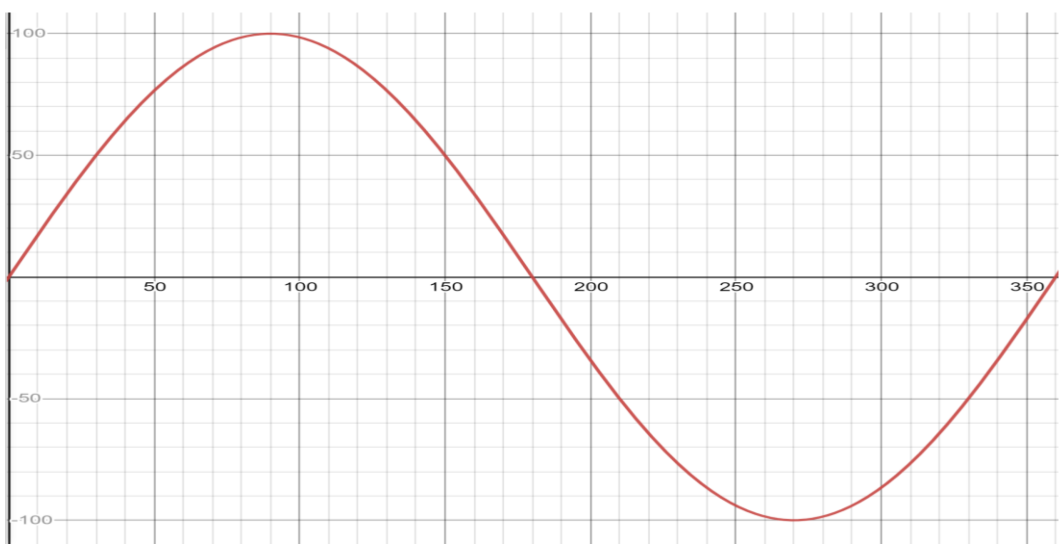
5. If the ride stops and a rider is at a seat angle of 230° degrees, is he/she above ground or in the aquarium?

   ____

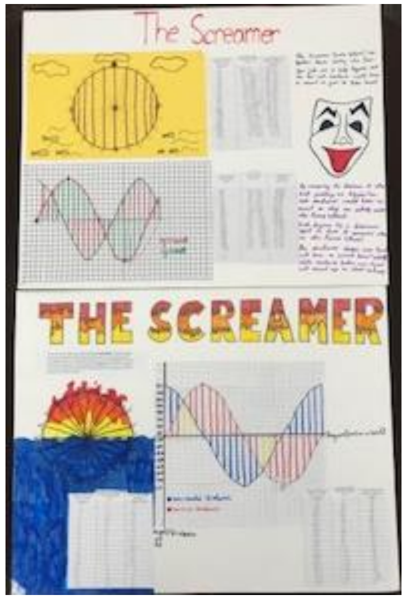
6. Put a green "H" on the graph where the rider is at the highest point.
7. Put a green "L" on the graph where the rider is at the lowest point.
8. Put a red circle on the graph to locate the riders that are halfway between the platform and the highest point on the ride.
9. Put a blue circle at the locations where a rider walks 50 feet to get to the platform should the ride stop.
10. If the ride stops and a rider is at an 80° angle, how many feet does the rider have to walk to get to the platform and follow the escape route? __________________________
11. What are the other three angles where riders have to walk the same distance as the rider in Problem10?

    ________________________, ______________________________,______________________________

12. Put a green circle where a rider gets off the ride after one complete rotation.



Now, try to make your own poster for the escape routes for the Sinusoidal Ferris Wheel. Add tables, graphs, equations, and diagrams to tell the story of "The Screamer." (Or you can come up with your own name for the ride.)



For Problem 13-16, tell in which quadrant the terminal side of the given angle (in standard position) would be located.

13. $275°$

14. $179°$

15. $-50°$

16. $-10°$

For Problem 17-20, tell the degree measure of each angle and sketch it in standard position.

17. one-eighth counterclockwise turn

18. One-fifth clockwise turn

19. one-fourth clockwise turn

20. One-half counterclockwise turn

Section 1.4 The Unit Circle

Practice Problems 1.4

For Problem 1-6, convert the degrees to radians.

1. $270°$
2. $180°$
3. $135°$
4. $60°$
5. $210°$
6. $330°$

For Problem 7-12, convert the radians to degrees.

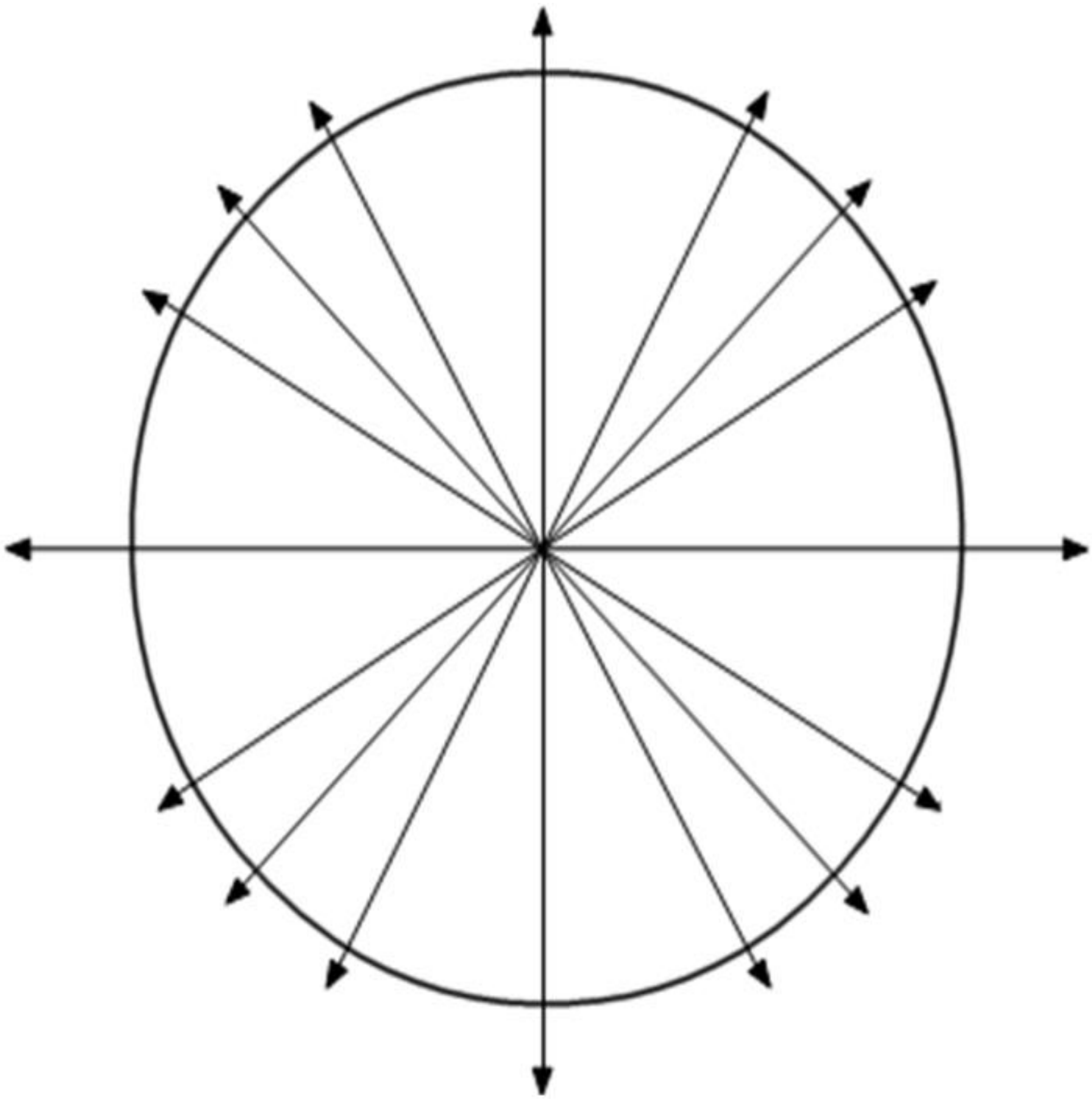
7. $\frac{\pi}{2}$
8. $\frac{7\pi}{4}$
9. 4π
10. $\frac{5\pi}{12}$
11. $\frac{\pi}{6}$
12. $\frac{5\pi}{6}$

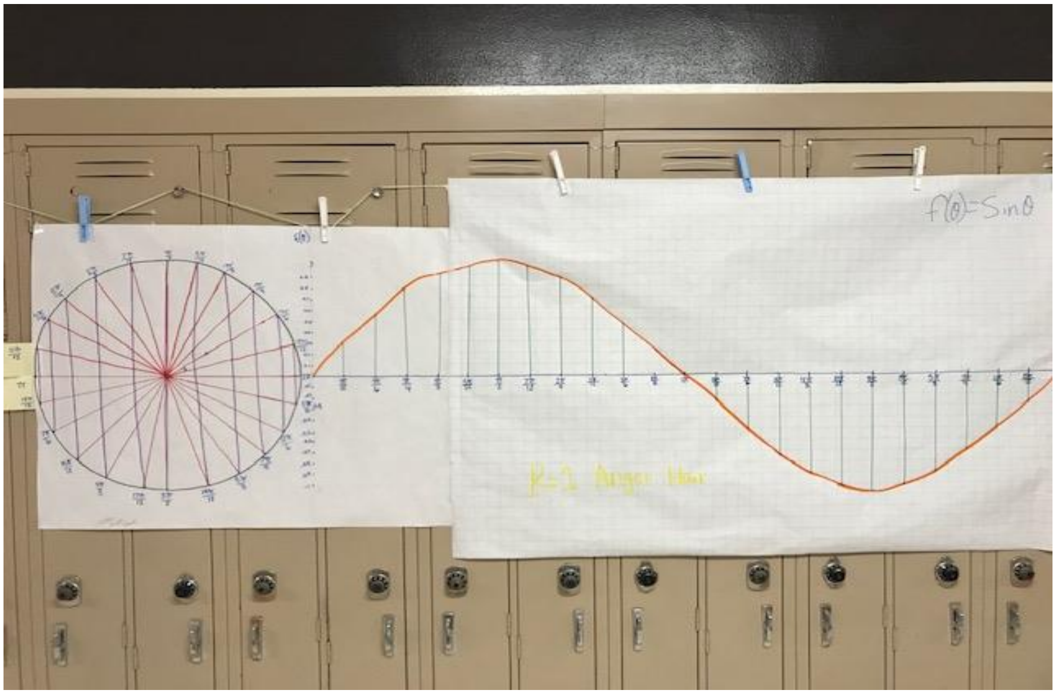
For Problem 13, complete the table below.

Degrees	0°	30°		60°		120°	135°			210°	
Radians			$\frac{\pi}{4}$		$\frac{\pi}{2}$	$\frac{2\pi}{3}$		$\frac{5\pi}{6}$	π		$\frac{5\pi}{4}$

For Problem 14, complete the unit circle using any of the methods you have learned. Fill in the degrees, radians, and ordered pairs at each arrow.

14.





For Problem 15-20, solve the problem.

15. What is $\tan 90°$?

16. What is $\tan \frac{3\pi}{2}$?

17. What is $\cos 60°$?

18. What is $\sin 150°$?

19. What is $\cos \frac{5\pi}{4}$?

20. What is $\sin \frac{7\pi}{6}$?

Section 1.5 Toothpick Curves

Practice Problems 1.5

Toothpick Sine Curve Questions:

1. What is the radius of the circle in toothpick units?

2. What is the circumference of the circle in toothpick units?

3. Where would a triangle corresponding to $390°$ be constructed?

4. What is the period of the sine curve? That is, what is the wavelength? How many radians is it before the graph starts to repeat?

5. What is the period of the cosine curve? What is the wavelength, or at what radian does the curve start to repeat?

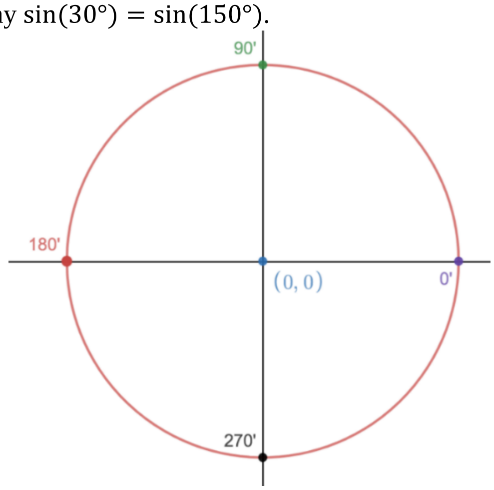
6. Compared to the radius, what is the height of the triangle at $60°$?

7. What is $\sin 60°$?

8. Compared with the radius, what is the height of the triangle at a) $150°$, b) $330°$, and c) $390°$?

9. If you build triangles only at the $15°$ angle marks, what is the smallest number of different triangles you need to form the lengths necessary to construct the graph of one period of the sine curve?

10. Explain why $\sin(30°) = \sin(150°)$.



For Problem 11-13, find the x-coordinates and y-coordinates on the unit circles by finding the cosine and sine of the given radian measures.

11. $(\cos\frac{\pi}{6}, \sin\frac{\pi}{6})$

12. $(\cos\frac{\pi}{4}, \sin\frac{\pi}{4})$

13. $(\cos\frac{\pi}{3}, \sin\frac{\pi}{3})$

For Problem 14-16, given $\csc\theta = \frac{1}{\sin\theta}$, find the value of the following.

14. $\csc\frac{\pi}{6}$

15. $\csc\frac{\pi}{4}$

16. $\csc\frac{\pi}{3}$

For Problem 17-20, given $\sec\theta = \frac{1}{\cos\theta}$, find the value of the following. Rationalize the denominator to eliminate any radicals in the denominator.

17. $\sec\frac{\pi}{6}$

18. $\sec\frac{\pi}{4}$

19. $\sec\frac{\pi}{3}$

For Problem 20, find the value of the following.

20. $a)\ \tan\frac{\pi}{6},$ b) $\tan\frac{\pi}{4}$ c) $\tan\frac{\pi}{3}$

Section 1.6 Trigonometric and Parametric Equations

Practice Problems 1.6

On your graphing calculator, go to the Home Page and scroll to the Graphs Page. Press "Menu," then "Enter," "#3: Graph Type," then "Enter," "#2: Parametric," then "Enter." Below is what you should see:

$$x\,1\,(t) =$$
$$y\,1\,(t) =$$
$$0 \le t \le 6.28 \qquad t \text{ step} = 0.13$$

Press "Menu," then "#4: Window/Zoom," and then "#1: Window Settings."
Tab around the table to enter the following settings:

x min.	-1.2
x max.	6.3
x scale	1
y min.	-2.5
y max.	2.5
y scale	1

Tab to "Ok" and click "Enter."

$$x\,1\,(t) = t$$
$$y\,1\,(t) = \sin(t)$$

1. What is the highest point of the sine curve on the graph?

2. What is the lowest point of the sine curve on the graph?

3. Complete the table below for the sine curve:

t	0	1.57	3.14	4.71	6.28
$\sin t$					

4. What are the radians for the decimal approximations in the table below?

t	0	$\frac{\pi}{2}$			2π
$\sin t$					

Go to the same page that has the sine wave on the calculator. Follow the instructions at the beginning of this section to switch from Functions mode to Parametric mode. The window settings should still be what you entered previously. Now enter:

$$x\,2\,(t) = t$$
$$y\,2\,(t) = \cos(t)$$

5. What is the highest point on the cosine curve?

6. What is the lowest point on the cosine curve?

7. Complete the table below for the cosine curve.

t	0	1.57 or $(\frac{\pi}{2})$	3.14 or (π)	4.71 or $(\frac{3\pi}{2})$	6.28 or (2π)
$\cos t$					

8. How far right of 0 would the cosine graph need to be shifted to align with the sine graph? Write the equation for this. How far left would the sine graph need to be shifted to align with the cosine graph? Write the equation for this as well.

The labels may be crowding the graph. Move the cursor (↖) over the label until the hand appears. Hold the click button in the center of the Navpad down until the hand closes. Move the label to a blank space and click "Enter" to place it there.

Go to a new Graphs page on the graphing calculator. Follow the instructions at the beginning of the section to switch from Functions mode to Parametric mode one last time and enter:

$$x\,3\ (t) = \cos(t)$$
$$y\,3\ (t) = \sin(t)$$

What do you think the graph will look like? Click "Enter."

9. What is the highest point on the unit circle?

10. What is the lowest point on the unit circle?

11. Fill in the blanks: The period of the sine curve is ____________________ full cycle(s) of the wave. The wavelength for the period of the sine function is the same as the ____________________ function. The period of both the sine and cosine function is ____________________. In the unit circle, the cosine function tracks the ______-coordinate of the ordered pair (x, y) and the y-coordinate of the ordered pair is tracked by the ____________________ function.

12. Complete the table for the parametric graph of the unit circle.

t	0	1.57 or $\frac{\pi}{2}$	3.14 or π	4.71 or $\frac{3\pi}{2}$	6.28 or 2π
$\cos t$					
$\sin t$					

Once again, in completing this chart you can see the consistent patterns in God's elegant creation and how each part of His creation relates to another.

For Problem 13-18, fill in the blanks.

13. Since the unit circle has a radius of 1, for all real numbers t, ___________ $\leq \sin t \leq$ ____________.

14. Each time a point goes around the unit circle one revolution is completed, which is a length of _______________ radians.

15. For each revolution, n, the points $P(\cos t$, ________) are repeated values so that $\sin t = (t \pm 2\pi n)$.

16. Since the unit circle has a radius of 1, for all real numbers t, ___________ $\leq \cos t \leq$ ____________.

17. Each time a point goes around the unit circle one revolution is completed, which is a rotation of __________ degrees.

18. For each revolution, n, the points $P($__________, $\sin t)$ are repeated values so that $\sin t = (t \pm 180°\, n)$.

For Problem 19 and 20, given that $t = \frac{13\pi}{6}$ and $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, fill in the blanks.

19. $\sin\frac{13\pi}{6} = \sin(2\pi + \{\_\}) = \sin\frac{\pi}{6} = \{\_\}$

20. $\cos\frac{13\pi}{6} = \cos(\{\_\} + \frac{\pi}{6}) = \cos\{\_\} = \frac{\sqrt{3}}{2}$

Section 1.7 Periodic Functions

Practice Problem 1.7

For Problem 1-3, state at which intervals the graph is positive and at which intervals the graph is negative for each function over the given interval $-2\pi \leq \theta \leq 2\pi$.

1. $f(\theta) = \sin\theta$

2. $f(\theta) = \cos\theta$

3. $f(\theta) = \tan\theta$

For Problem 4-9, tell which of the functions (sine, cosine, tangent) the statement holds true for.

4. The graph is symmetric about the x-axis.

5. The graph is symmetric about the y-axis.

6. The domain is the set of all real numbers.

7. The range is the set of all real numbers.

8. The period is π.

9. The period 2π.

For Problem 10-12, solve the word problem.

10. Find three other values of x that work for $\sin x \approx 0.8624$ and check your solutions.

11. In Example 1, $\cos 0.9991 \approx 0.5410\ldots\ 0.9991 + 3\pi\ \approx -0.5410\ldots\ 0.9991 - 3\pi \approx -0.5410$. Explain why.

12. Fill in the blanks: When $\sin\theta$ and $\cos\theta$ are both positive or both negative, then $\tan\theta$ is ____________________. When either $\sin\theta$ or $\cos\theta$ is positive, and the other is negative, then $\tan\theta$ is ____________________. When $\sin\theta = 0$, then $\tan\theta =$____________________. When $\cos\theta = 0$, then $\tan\theta =$____________________.

For Problem 13-20, use the information below to solve the problem.

In a unit circle, the central angle equals the length of the intercepted arc. Given $c = 2\pi\mathrm{r}$, then $\frac{\theta}{2\pi} = \frac{x}{2\pi\mathrm{r}}$ and $x = r\theta$.

Find the intercepted arc length, x ,of a circle given the radius, r,and the central angle θ. Firstly, convert degrees to radians.

13. Find the arc of a circle with a radius of 3 and a central angle of $\frac{\pi}{2}$. (Firstly, convert degrees to radians.)

14. Given $r = 4$ and $\theta = 30°$, find the arc x.

15. Given $r = 15$ and $\theta = 30°$, find the arc x.

16. Given $r = 6$ and $\theta = \frac{\pi}{6}$, find the arc x.

17. Given $r = 3.4$ and $\theta = 24°$, find the arc x.

18. Given $r = 2.6$ and $\theta = 45°$, find the arc x.

19. Given $r = 1.8$ and $\theta = 180°$, find the arc x.

20. A wheel with a radius of 16 inches is rotating at 650 rpm (revolutions per minute). Convert the rpm to radians per second to find the angular speed of the wheel (1 rpm = 2π rad/min.). Once you know the angular speed, find the linear speed in feet per second using the formula $x = r\theta$.

Section 1.8 Trigonometric Identities

Practice Problems 1.8

For Problem 1-4, use $\cos\theta = 0.707107$ and the trigonometric identities to solve the problem.

1. $\cos(-\theta)$
2. $\cos(\pi - \theta)$
3. $\cos(\frac{\pi}{2} - \theta)$
4. $\tan\theta$

For Problem 5-8, solve the problem without using a calculator. Use the trigonometric identities given $\cos\theta = \frac{2}{7}$.

5. $\sin(\frac{\pi}{2} - \theta)$
6. $\cos(\pi - \theta)$
7. $\cos(-\theta)$
8. If $\cos\theta = \frac{2}{7}$, find $\sin\theta$.

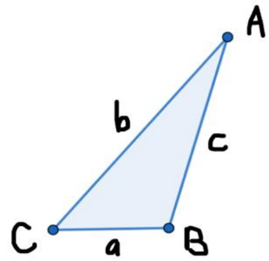
For Problem 9-11, tell whether the equation is true or false. Use a calculator to verify your answer.

9. $\cos(\pi - 2) = -\cos 2$
10. $\sin(\frac{\pi}{2} - 2) = \cos 2$
11. $\tan(\pi - 2) = \tan 2$

For Problem 12, fill in the blanks.

12. In any triangle, the ratio of the sine of the angles to the __________ __________ __________ __________ __________ __________ are equal.

For Problem 13-16, tell if the statements are equivalent for the Laws of Sines for ΔABC.



13. $\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$

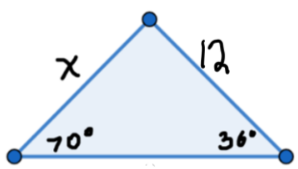
14. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

15. $\frac{a}{b} = \frac{\sin A}{\sin B}, \frac{b}{c} = \frac{\sin B}{\sin C}, \frac{a}{c} = \frac{\sin A}{\sin C}$

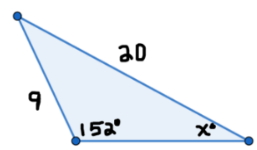
16. $ab \sin C = bc \sin A = ac \sin B$

For Problem 17 and 18, use the Law of Sines to find x.

17.



18.



For Problem 19 and 20, use the information below to solve the problem.

The Costas Family is thinking of buying a plot of land. The plot is triangular with one side on the lakefront. The other two sides are 320 feet long and 452 feet long. The angle between the lakeshore and the 320 ft. side length is 53°.

19. Find the measure of the angle between the lakefront and 452 ft. side length.

20. About how many feet of lakeshore frontage does the plot have?

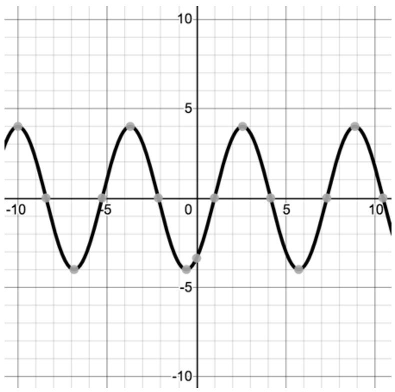
Section 1.9 Transformations of Periodic Functions

Practice Problems 1.9

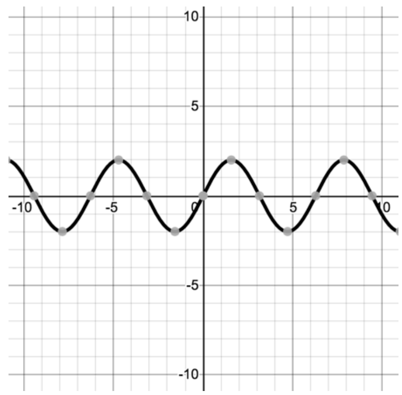
Below are four graphs numbered 1-4. One is the function $f(x)$ and the other three, $(f(2x), f(x-1)$, and $\frac{1}{2}f(x))$, are transformations of it.

For Problem 1-4, write the function that defines the graph in the blank.

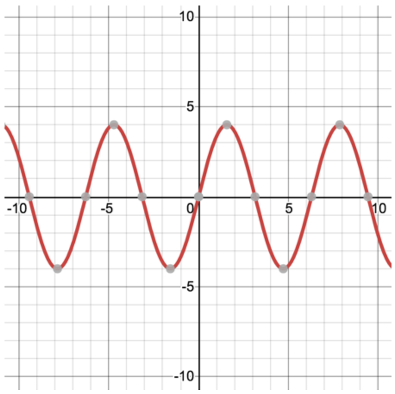
1. ____________________



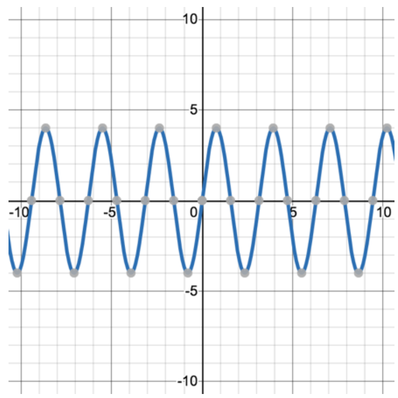
2. ____________________



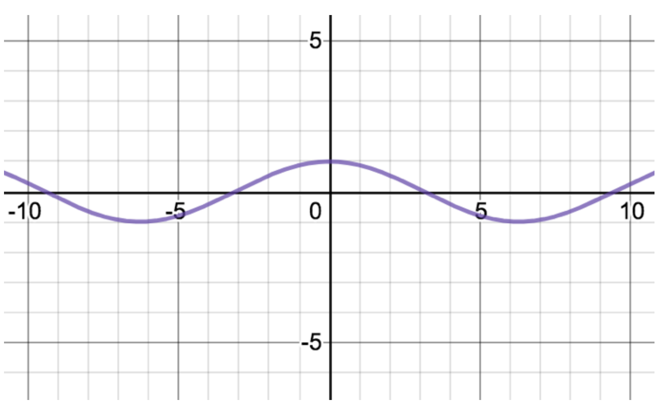
3. ____________________



4. ____________________



For Problem 5-7, given the equation of $y = \cos\frac{1}{2}x$, state how the transformation will change the graph and sketch the new graph.



5. $y = 3\cos\frac{1}{2}x$

6. $y = \cos\frac{1}{2}x + 4$

7. $y = f(2x)$

For Problem 8-11, solve the word problem.

8. What is the original function, $f(x)$, in Problem 1-4. Name the amplitude, period and frequency.

9. Why is the minimum and maximum value 1 and -1 for $y = \sin 3x$?

10. What value of b in $y = a\cos bx$ gives a horizontal compression? What value of b gives a horizontal stretch?

11. What value of a in $y = a\sin bx$ gives a vertical compression? What value of a gives a vertical stretch?

For Problem 12-14, solve the multiple-choice problem.

12. Which other equation is equivalent to $y = 3\sin 2x$?

a) $3\sin(-2x)$ b) $-3\sin 2x$ c) $-3\sin(-2x)$

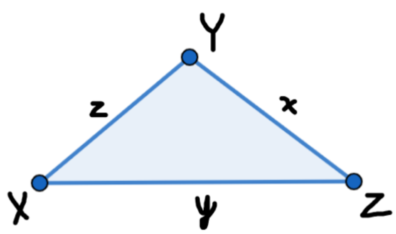
13. Which other equation is equivalent to $y = -4\sin 2x$?

a) $y = 4\sin 2x$ b) $y = 4\sin(-2x)$ c) $y = -4\sin(-2x)$

14. Given x, y, and z in ΔXYZ, which formula can be used to find the $m\angle Y$ directly?

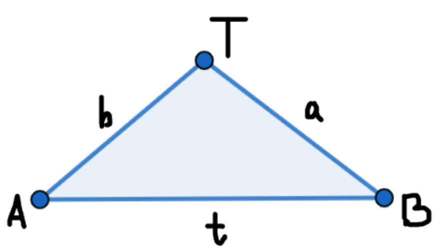
a) $x^2 = y^2 + z^2 - 2yz\cos X$ b) $y^2 = x^2 + z^2 - 2xz\cos Y$

c) $z^2 = x^2 + y^2 - 2xy\cos Z$



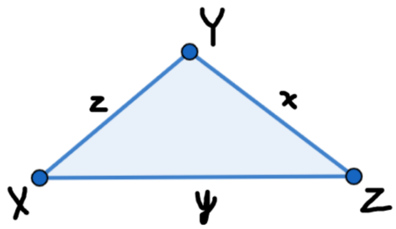
For Problem 15-20, solve the word problem.

15. Use the Law of Cosines to write a formula for t in terms of $\angle T$ and sides a and b.



16. Suppose $a = 6$, $b = 5$, and $m\angle T = 120°$. Find the exact length of t, and the approximate length of t.

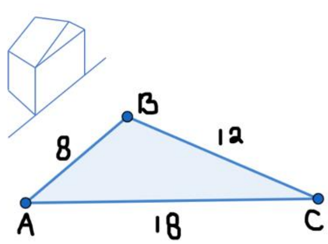
17. In ΔXYZ the measure of the side lengths are $x = 10$, $y = 15$, and $z = 20$. Find a) $m\,\angle Z$ and b) $m\,\angle X$.



18. Is the triangle from Problem 17 an accurate rendering of ΔXYZ? If not, sketch a more accurate rendering.

19. A baseball diamond has bases that are the vertices of a square with sides of 90 feet. Suppose that an infielder is standing 20 feet behind second base. To the nearest foot, how far must that infielder throw the ball to get a player out who is running to third base?

20. A triangular plot of land is to be used as a garden. The measures of the sides of the plot are $AB = 8$ m., $BC = 12$ m., and $AC = 18$ m. Benches will be placed on each side of ∠A. Use the Law of Cosines to find the measure of ∠A.



Section 1.10 Compositions of Trigonometric Functions

Practice Problems 1.10

For Problem 1-3, sketch the graph for the function.

1. Let $f(x) = \sin x$ and $h(x) = \tan x$. Is $(f \circ h)(x)$ the same as $(h \circ f)(x)$? Sketch the graphs using the graphing calculator.

2. Let $f(x) = \sin x$ and $g(x) = \cos x$. Sketch the graph of $(f \circ f)x$.

3. Let $f(x) = \sin x$ and $g(x) = \cos x$. Sketch the graph of $(g \circ g)x$.

Use the graph in Example 3 to find the composition of functions.

4. $(g \circ f \circ g)(2)$

5. $(f \circ g)(4)$

6. $(f \circ f \circ f)(0)$

7. $(g \circ g)(-2)$

8. $(g \circ g \circ f)(1)$

9. $(g \circ f \circ g)(4)$

10. Is $y = \tan(\cos x)$ a translation of $y = \tan(\sin x)$?

For Problem 11 and 12, find the composition of the function when $f(x) = x + 1$ and $g(x) = \sqrt{x}$.

11. $f(g(x))$

12. $g(f(x))$

For Problem 13-20, let $f(x) = x^2 + 1$ and $g(x) = \sqrt[2]{x}$.

13. Fill in the blanks: The composition of $g(f(x))$ is the set of all x's in the domain of the function f such that $f(x)$ is in the domain of the function g. Therefore, the range of function _________ must be in the domain of the function _________.

14. What is the range of f?

15. What is the domain of g?

16. Is the range of f in the domain of g? If so, find $g(f(x))$. What is the domain and range of $g \circ f$?

17. What is the range of g?

18. What is the domain of f?

19. Is the range of g in the domain of f? If so, find $f(g(x))$.

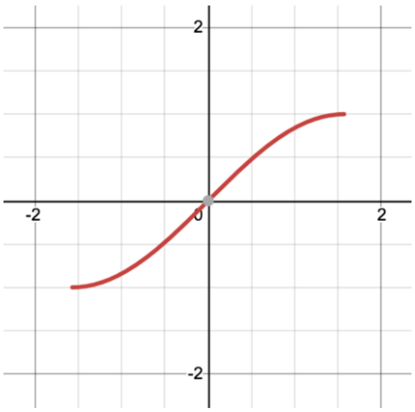
20. What is the domain and range of $f \circ g$? Is the domain and range the same as that of $g \circ f$?

Section 1.11 Inverse Circular Functions

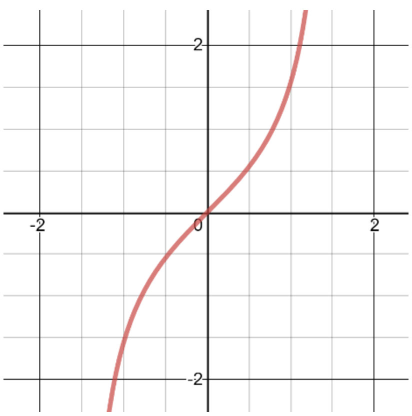
Practice Problems 1.11

For Problem 1 and 2, use the graph to complete the instructions.

1. The restricted domain and range for $y = \sin x$ is given by the graph. Draw the inverse, $y = \sin^{-1}x$, and state the domain and range.



2. The restricted domain and range for $y = \tan x$ is given by the graph. Draw the inverse, $y = \tan^{-1}x$, and state the domain and range.



3. Complete the table for the coordinates for $y = \cos^{-1}x$ given the coordinates for $y = \cos x$.

(x, y) for $y = \cos x$	$(0, 1)$	$(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$	$(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$	$(\frac{\pi}{3}, \frac{1}{2})$	$(\frac{\pi}{2}, 0)$
(x, y) for $y = \cos^{-1}x$					

4. Evaluate $\tan(\cos^{-1}0.6)$

5. What is a formula that can be used to evaluate $\tan(\sin^{-1}\frac{b}{c})$ given that b and c are not equal to zero?

6. What is a formula that can be used to evaluate $\tan(\cos^{-1}\frac{a}{c})$ given that a and c are not equal to zero?

7. Use inverse trigonometric functions to evaluate $\sin(\sin^{-1}x)$ for x such that $-1 \leq x \leq 1$.

8. Complete the table for the coordinates for $y = \sin^{-1}x$ given the coordinates for $y = \sin x$.

(x, y) $y = \sin x$	$(0, 0)$	$(\frac{\pi}{6}, \frac{1}{2})$	$(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$	$(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$	$(\frac{\pi}{2}, 1)$
(x, y) $y = \sin^{-1}x$					

9. Complete the table for the coordinates for $y = \tan^{-1}x$ given the coordinates for $y = \tan x$.

(x, y) $y = \tan x$	$(\frac{\pi}{4}, 1)$	$(\frac{\pi}{3}, \sqrt{3})$	$(\frac{2\pi}{3}, -\sqrt{3})$	$(\frac{3\pi}{4}, 1)$
(x, y) $y = \tan^{-1}x$				

10. Use the tables to evaluate $\tan(\sin^{-1}\frac{\sqrt{3}}{2})$.

For Problem 11-18, let $f(x) = \sin x$ such that $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $-1 \leq f(x) \leq 1$. Therefore, $(f^{-1} \circ f)(x) = x$ for all x's in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $(f \circ f^{-1})(x) = x$ for all x's in $[-1, 1]$.

11. What does $\sin^{-1}(\sin x)$ equal for all x's in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$?

12. What does $\sin(\sin^{-1} x)$ equal for all x's in the interval $[-1, 1]$?

13. What is the interval for the domain and range of $f(x) = \sin^{-1}(\frac{1}{2}x)$? (Hint: Use the domain of the inverse of the sine function, which is $[-1, 1]$.)

14. Explain the transformation from $y = \sin^{-1} x$ to $f(x) = \sin^{-1}(\frac{1}{2}x)$.

15. Sketch the graph of $y = \sin^{-1} x$ and $f(x) = \sin^{-1}(\frac{1}{2}x)$ over the given intervals.

16. What is the interval for the domain and range of $g(x) = \sin^{-1}(3x)$?

17. Explain the transformation from $y = \sin^{-1} x$ to $g(x) = \sin^{-1}(3x)$?

18. Sketch the graph of $y = \sin^{-1} x$ and $g(x) = \sin^{-1}(3x)$ over the given intervals.

For Problem 19 and 20, let $f(x) = \cos x$ such that $-1 \leq x \leq 1$ and $0 \leq f(x) \leq \pi$. Therefore, $\cos^{-1}(\cos x) = x$ for all x's in $[-1, 1]$ and $\cos(\cos^{-1} x) = x$ for all x's in $[0, \pi]$.

19. For the function $f(x) = \cos^{-1}(\frac{1}{4}x)$…

a) … find the domain and range.

b) … describe the transformation from $y = \cos^{-1} x$.

c) … sketch the graphs of $y = \cos^{-1} x$ and $f(x) = \cos^{-1}(\frac{1}{4}x)$.

20. For the function $g(x) = \cos^{-1}(2x)$…

a) … find the domain and range.

b) … describe the transformation from $y = \cos^{-1} x$.

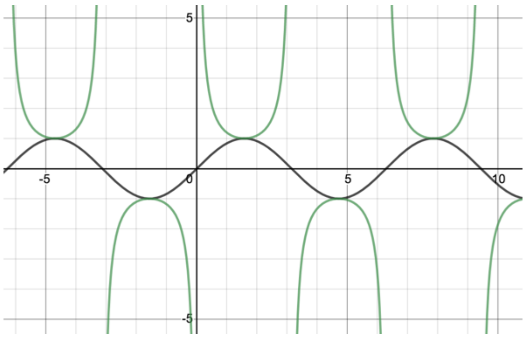
c) … sketch the graphs of $y = \cos^{-1} x$ and $g(x) = \cos^{-1}(2x)$.

Section 1.12 Reciprocal Trigonometric Functions

Practice Problems 1.12

For Problem 1-3, let $y = \sin x$ be in black on the graph and $y = \csc x$ be in green on the graph.

1. Identify the domain and range of $\csc x$.



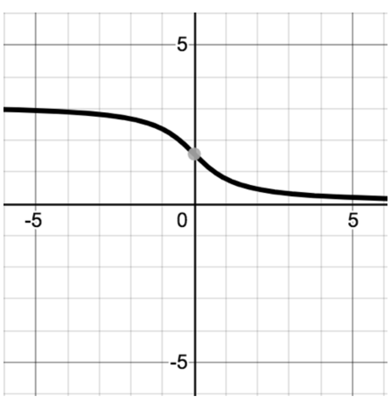
2. What are the relative maximums and minimums of $y = \csc x$?

3. What is the period of $y = \csc x$?

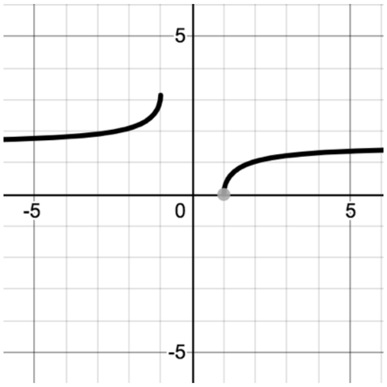
There are inverse functions for secant, cosecant, and cotangent given by the calculator keys $\sec^{-1}$, $\csc^{-1}$, and $\cot^{-1}$. Their graphs are the restricted reflection of the parent graph over the line $y = x$.

For Problem 4-6, name the three graphs appropriately given the titles $y = \sec^{-1}x$, $y = \csc^{-1}x$, and $y = \cot^{-1}x$.

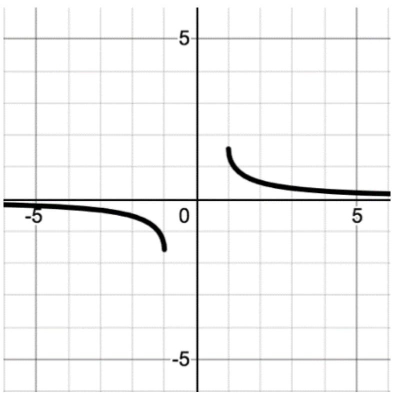
4. Domain: all real numbers; Range: $0 < y < \pi$



5. Domain: $|x| \geq 1$; Range: $0 \leq y \leq \pi$; $y \neq \frac{\pi}{2}$

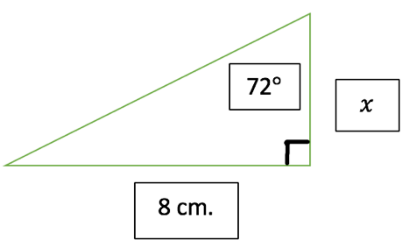


6. Domain: $|x| \geq 1$; Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$; $y \neq 0$

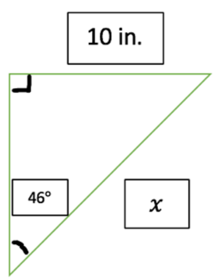


For Problem 7-9, solve for x.

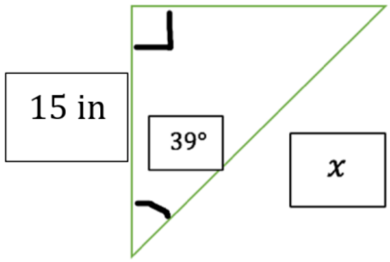
7. Use cotangent to solve for x.



8. Use cosecant to solve for x.



9. Use secant to solve for x.



For Problem 10, solve the word problem.

10. Find the remaining side and angle in the right triangle in Problem 9.

For Problem 11-14, use the tangent ratio to solve the problem.

11. Dallas is building a handicap ramp for his grandmother. The door is 3 ft. off the ground and the walkway that is to be covered extends 17 ft. from the front of the house. Draw a diagram of the problem.

12. The maximum slope for a ramp should be 1.5 inches of rise to every 12 inches of tread or an angle of about 7° degrees. Is the ramp safe according to these safety standards?

13. Use the inverse of cotangent to find the angle directly.

14. What would be an appropriate length for the distance of the ramp from the house to be safe?

For Problem 15-17, use the tangent ratio to solve the problem.

15. Drake is building a cellphone tower and needs to put a guy wire on the south side for safety. One end of the wire needs to be staked in the ground at an angle of 65° to the top of the tower, which is 120 feet high. Draw a diagram of the problem.

16. How far does the wire need to be staked from the base of the tower in order to meet the safety standards from Problem 12?

17. Now use cotangent to solve Problem 16. Did you get the same answer as when you used tangent?

For Problem 18-20, use the tangent ratio to solve the problem.

18. A safety light is being installed above a school entrance that is 30 feet tall. It is designed to turn on when it detects someone at least 3 feet tall approaching as far as 40 feet away from the entrance. Draw a diagram of the problem.

19. At what angle must the light be placed above the entrance in to function according to safety specifications?

20. Use the inverse of cotangent to solve Problem 19 directly.

Section 1.13 Matrices and More Trigonometric Identities

Practice Problems 1.13

For Problem 1-10, solve the word problem.

1. A rotation of $180°$ degrees can be considered a $90°$ rotation followed by another $90°$ rotation. Find the matrix for a $180°$ rotation.

2. How would you find the matrix for a $270°$ rotation around the origin?

3. Find the matrix for rotating through the angle $120°$.

4. The coordinates of a parallelogram are $(2, 1)$, $(4, 1)$, $(3, 2)$, and $(5, 2)$. What are the coordinates of the image after a $90°$ rotation?

5. If $\alpha = 30°$ and $\beta = 45°$, show that $\cos(\alpha + \beta) \neq \cos\alpha + \cos\beta$ and $\sin(\alpha + \beta) \neq \sin\alpha + \sin\beta$.

6. Use the Addition Identity for Sines and Cosines and other Trigonometric Identities from previous sections and courses to derive a formula for $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$ in terms of sines and cosines of α and β.

7. Write $2\sin 15° \cos 15°$ as a single argument of sine or cosine using the Double Angle Identity and solve. Check your answer.

8. How far will a baseball travel horizontally to the ground if it is thrown at a speed of 30 m/sec. at an angle that is $45°$ to the ground?

9. Find $\sin 2\theta$ if $\theta = 30°$. Use $2\sin\theta\cos\theta$.

10. Using a calculator…
a) What is $2\sin 30°$?

b) What is $\sin 30°$? What is double that?

c) What is $\sin 60°$?

d) Which of a, b, or c is the answer from Problem 9?

For Problem 11 and 12, solve the problem for a, b, and c.

11. If $0 \leq \theta \leq \frac{\pi}{2}$, what must be true of the signs of the functions below? Are they positive or negative?

a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$

12. Write each value as a trigonometric function in terms of $\sin$, $\cos$, and $\tan$.

a) $\csc\theta$ b) $\sec\theta$ c) $\cot\theta$

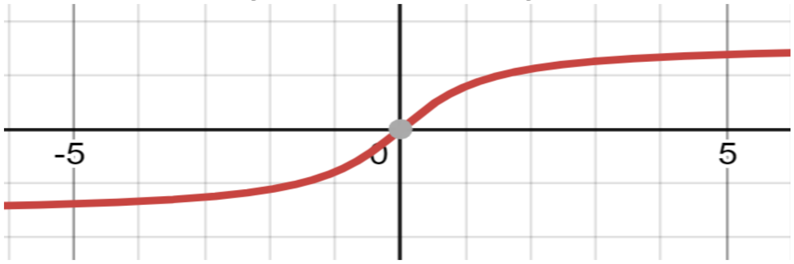
For Problem 13 and 14, fill in the blanks.

13. The functions $\sin, \cos,$ and $\tan$ as well as $\sin^{-1}, \cos^{-1},$ and $\tan^{-1}$ are called ____________________.

14. The functions sine, cosine, and tangent as well as cosecant, secant, and cotangent are called ____________________.

For Problem 15-17, solve the word problem.

15. Is the graph below the inverse tangent function or the cotangent function?

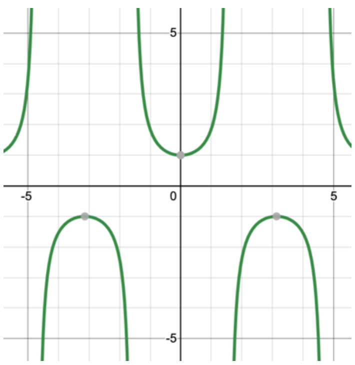


16. What is the domain of the function in Problem 15?

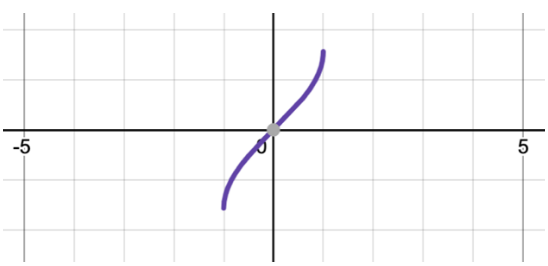
17. What is the range of the function in Problem 15?

For Problem 18-20, solve the word problem.

18. Name the parent function of the graph below. Draw the reciprocal function in it and name it.



19. Name the inverse function shown on the graph below.



20. What is the domain and range of the function in Problem 19?

Section 1.14 Module Review

For Problem 1 and 2, convert the degrees to radians.

1. $90°$

2. $135°$

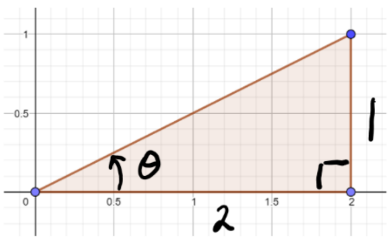
For Problem 3 and 4, convert the radians to degrees.

3. $\frac{\pi}{6}$

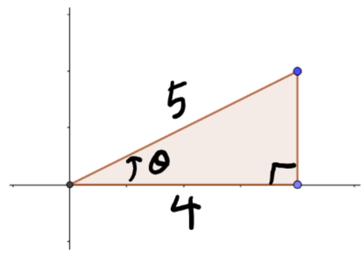
4. $\frac{2\pi}{3}$

For Problem 5 and 6, find the values of θ using cosine and sine. The terminal angle θ is in standard position.

5.



6.



For Problem 7-10, complete the table.

$\sin\theta$	$\frac{3}{5}$	$\frac{\sqrt{3}}{2}$	9.	$\frac{\sqrt{2}}{3}$	$\frac{12}{13}$
$\cos\theta$	7.	8.	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{7}}{3}$	10.

For Problem 11 and 12, evaluate the trigonometric identities if $\cos\theta = \frac{1}{9}$ and $0 < \theta < \frac{\pi}{2}$ without using a calculator.

11. $\cos(180° - \theta)$

12. $\sin(90° - \theta)$

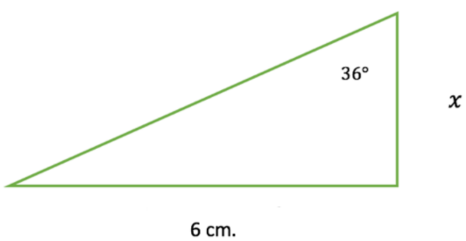
For Problem 13-15, find the exact value of the trigonometric function.

13. $\csc\frac{\pi}{6}$

14. $\tan\frac{3\pi}{4}$

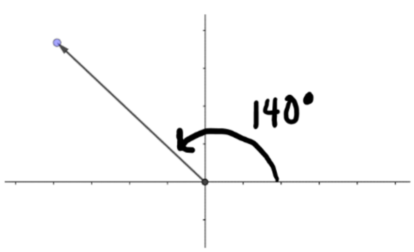
For Problem 15, use cotangent to solve for x.

15.

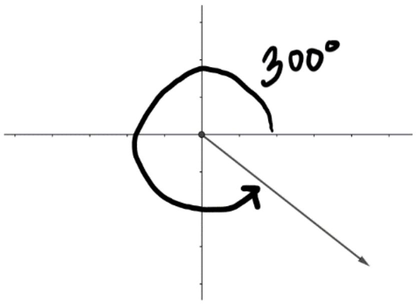


For Problem 16-20, solve the word problem.

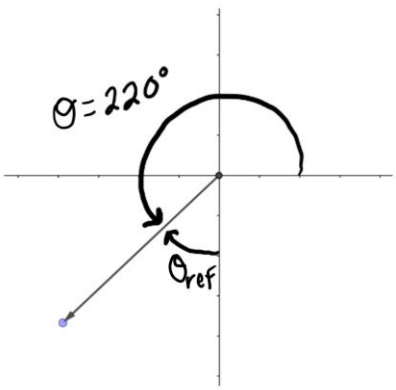
16. Angle $\theta = 140°$ is in standard position. The reference angle (θ_{ref}) is measured counterclockwise from the terminal side of θ to the horizontal axis nearest to it. Draw θ_{ref}. What is its measure?



17. Draw the reference angle in the figure below and calculate its measures.



18. What is incorrect in the figure below according to the definition of reference angle?



19. What is θ_{ref} for an angle whose measure is $90°$?

20. What is θ_{ref} for an angle whose measure is $-120°$?

Section 1.15 Module Test

For Problem 1 and 2, convert the degrees to radians.

1. $120°$

2. $35°$

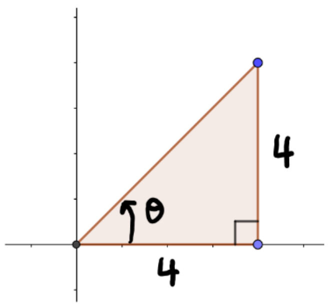
For Problem 3 and 4, convert the radians to degrees.

3. $\frac{\pi}{2}$

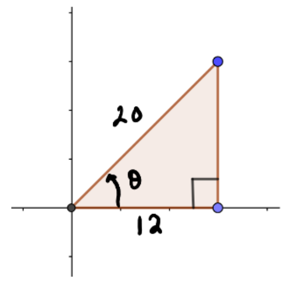
4. $\frac{7\pi}{4}$

For Problem 5 and 6, find the values of θ using cosine and sine. The terminal angle θ is in standard position.

5.



6.



For Problem 7-10, complete the table.

$\sin\theta$	7.	8.	$\frac{\sqrt{2}}{3}$	$\frac{12}{13}$	$\frac{\sqrt{15}}{4}$
$\cos\theta$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{\sqrt{7}}{3}$	9.	10.

For Problem 11 and 12, evaluate the trigonometric identities if $\sin\theta = \frac{2}{7}$ and $\cos\theta > 0$ without using a calculator.

11. $\cos(\pi - \theta)$

12. $\sin(-\theta)$

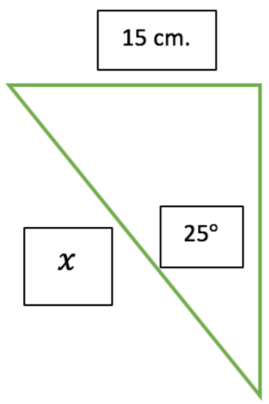
For Problem 13 and 14, find the exact value of the trigonometric function.

13. $\sec\frac{\pi}{4}$

14. $\tan\frac{\pi}{4}$

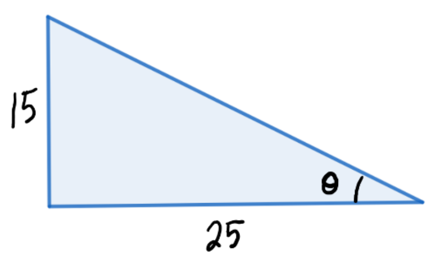
For Problem 15, use cosecant to solve for x.

15.



For Problem 16, use cotangent to find θ.

16.



For Problem 17, make the sketch.

17. Sketch an angle of 135° in standard position.

For Problem 18-20, solve the word problem.

18. What is the measure of the reference angle in Problem 16?

19. What are the radians for the standard angle and reference angle in Problem 17 and 18?

20. The terminal side of the reference angle is 2 units long. Draw a segment from the point perpendicular to the horizontal axis to form a right triangle. Find the lengths of the two legs of the triangle.