Module 4: Introducing Polynomials

Section 4.1 Scientific Notation and Large Numbers

Looking Back 4.1

Two of God's most amazing creations are the eye and the optic nerve. When light hits the retina of the eye, a very complex system is set into motion that sends 125,000,000 signals along the optic nerve. An enormous number of cells must be turned on to initiate the process.

In mathematics, it is easy to make errors when working with large numbers such as this. Zeroes may accidently be added or dropped. This is unfortunate because 10,000 people living in a city is much smaller than 100,000. In fact, 100,000 is 10 times bigger than 10,000, which is quite big difference when working with large numbers.

Scientific notation is a compact way to write these numbers. In this module, we will learn about polynomials and polynomials that involve exponents. All we have learned about exponents in Module 3 will help us work with numbers in scientific notation.

Looking Ahead 4.1

Example 1:	Look at the lists below. What is the difference between the two columns that represent the numbers?

Uses Scientific Notation	Does Not Use Scientific Notation
2.6×10^4	26×10^3
7.64×10^6	76.2×10^5
6.13×10^7	61,300,000
4.021×10^8	402.1×10^6
1×10^3	1,000
9.2×10^3	92×10^2

- In the column to the left above that uses scientific notation, every number is being multiplied by a number with an exponent whose base is 10.
- In the column to the left above that uses scientific notation, in the first number (to the left of "×"), the decimal point in the number is always after the first non-zero digit.

Example 2:	Write the numbers in the column to the left above in standard form.

$$2.6 \times 10^4 =$$

$$7.64 \times 10^6 =$$

$$6.13 \times 10^7 =$$

$$4.021 \times 10^8 =$$

$$1 \times 10^3 =$$

$$9.2 \times 10^3 =$$

A number written in scientific notation is in the form $a \times 10^b$. The a is an integer that is always between 0 and 10 but may not be equal to 0 or 10.

The a is always multiplied by a power of 10. The b is the exponent of the 10. When b is positive, it is the number of places the decimal moves right. Zeroes may need to be added at the end of the number.

The standard notation number $30{,}000 = 3 \times 10^4$ is 3.0×10^4 in scientific notation.

The exponent 10^4 is $10 \times 10 \times 10 \times 10$, which is equal to $10{,}000$.

When we multiply a number by $10{,}000$, we move the decimal point four places to the right. The exponent tells us the number of places to move the decimal.

Standard Form: $33{,}000 = 3.3 \times 10^4$ in Scientific Notation

Remember, standard notation is the way we are used to seeing things, such as "30,000" or "33,000." Each is written as one number.

<u>Note</u>

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1{,}000$
$10^4 = 10{,}000$
10^b is 1 followed by b zeroes

Example 3: There are approximately $50{,}000{,}000{,}000{,}000$ (fifty trillion) cells in your body. There are approximately $25{,}000$ genes in each of your cells. That means there are $50{,}000{,}000{,}000{,}000 \times 25{,}000$ genes in your body. (Is the handiwork of God not amazing!?) Write the number of genes in your body in scientific notation. Write the non-zero numbers as a decimal first multiplied by a power of 10.

$$50{,}000{,}000{,}000{,}000 \times 25{,}000 = 1{,}250{,}000{,}000{,}000{,}000{,}000$$

$$1{,}250{,}000{,}000{,}000{,}000{,}000 = \underline{\hspace{6cm}}$$

Yet again, you are "fearfully and wonderfully" made by God.

Example 4: There are also approximately $93{,}000$ miles of nerves in your body. Using the conversion factor 1 mile $= 5{,}280$ feet, multiply the ratios and use the "cancel-keep" method to convert $93{,}000$ miles to feet.

Example 5: Convert the standard notation numbers to scientific notation.

a) 40,100,000

b) 406,000

c) 5,293,000,000

d) 22,109,000

Section 4.2 Scientific Notation and Small Numbers

Looking Back 4.2

Because of our vast and limitless universe, astronomers often work with large numbers. Maria Mitchell, a scientist from the 19th century, is recognized as the first female astronomer in the United States. Maria thought herself to be an ordinary student, but a very determined person.

Being of quaker upbringing meant that unlike many girls in the United States during her time, Maria would be educated at a local school. Along with school, she was tutored by her father, also an astronomer. Maria's father was the one who introduced astronomy to her as he considered it to be the evidence of God in the natural world. The two of them would use their knowledge of astronomy (and the proper equipment) to make important observations. Maria's most well-known achievement was her discovery of a comet! From 1865 until the end of her life, she did research and worked as a professor at Vassar Female College.

Maria believed in God and she believed God was good. She once wrote: "Scientific investigations, pushed on and on, will reveal new ways, in which God works, wholly unknown." She is quoted saying that if the revelations of the Bible and understanding of nature through science seem to conflict, then it is because they are misunderstood one to the other. To her, there were no contradictions between the Bible and science.

At her death, Maria Mitchell was considered one who always sought the truth.

Looking Ahead 4.2

Now, we can get a formal definition of scientific notation.

Scientific Notation is a number written as the product of two factors in the form $a \times 10^b$ in which $1 \le a < 10$ and a is an integer.

Let us look at how to use scientific notation for writing extremely small numbers. When we multiply a by a power of 10, the product gets bigger as b gets bigger. When we divide a by a power of 10, the number gets smaller as b gets bigger. When the number gets bigger, the decimal point moves to the right. When the number gets smaller, the decimal point moves to the left.

Example 1: Convert $30{,}000$ to scientific notation. Convert 0.0003 to scientific notation.

Note

$10^0 = 1$

$10^{-1} = \frac{1}{10}$

$10^{-2} = \frac{1}{100}$

$10^{-3} = \frac{1}{1{,}000}$

$10^{-4} = \frac{1}{10{,}000}$

10^{-b} where b is a positive integer is a decimal point followed by $b - 1$ zeroes

When you multiply 3 by $10{,}000$, it moves the decimal point four places to the right of 3. When you divide 3 by $10{,}000$, it moves the decimal point four places to the left of 3.

When you divide 3.2 by $10{,}000$, you get 0.00032, not 0.0032; move left from the decimal point, not the end of the number. For the number 3, the decimal point is at the end of the number.

Example 2: How would you write 0.000000602 in scientific notation?

The decimal point goes after the first non-zero number. All digits are included until after the last non-zero digit. There may be zeros in between the first and last non-zero.

Example 3: Below is a table of measurements equivalent to 1 year on the left and measurements equivalent to 1 second on the right. Convert the seconds to standard notation. There are $31{,}536{,}000$ seconds in one year.

Time	Units	Scientific Notation	Standard Notation
Years	1	$\frac{1}{31{,}536{,}000} = 3 \times 10^{-8}$	
Months	12	$\frac{12}{31{,}536{,}000} = 4 \times 10^{-7}$	
Weeks	52	$\frac{52}{31{,}536{,}000} = 2 \times 10^{-6}$	
Days	365	$\frac{365}{31{,}536{,}000} = 1 \times 10^{-5}$	
Hours	8,760	$\frac{8{,}760}{31{,}536{,}000} = 3 \times 10^{-4}$	
Minutes	525,600	$\frac{525{,}600}{31{,}536{,}000} = 2 \times 10^{-2}$	
Seconds	31,536,000	$\frac{31{,}536{,}000}{31{,}536{,}000} = 1 \times 10^{0}$	

Example 4: A nanosecond is one-billionth of a second (10^{-9} secs) ($\frac{1}{1{,}000{,}000{,}000}$ secs). To give you a ratio that makes sense, a nanosecond is to a second as a second is to 31.7 years. Can you visualize that!? The speed of a memory chip in a computer is so fast that it is measured in nanoseconds. If a computer command takes place in 3.02 nanoseconds (ns), how many seconds does it take place in?

We have previously used a scientific notation number called $c \approx 3.0 \times 10^8$ m/sec. This is the speed of light as discovered by Albert Einstein. On any station in space, an observer would observe that light signals move through space with the same constant speed ($c = 299{,}792{,}458$ km/sec). This does not depend on the light source nor is it a coincidence. The speed of light is independent of the observer and is absolute, not relative. The speed of light travels at 670,616,629 mph. You could go around the earth 7.5 times in 1 second if you could travel at the speed of light.

However, it does not matter how strong an object is or even the forces acting upon it, it can never accelerate to the speed of light. Einstein explains this in his formula $E = mc^2$ (energy is equal to the mass of an object times the speed of light squared). Many thought this equation and Einstein's Theory of Relativity unlocked the mysteries of the universe. Therefore, Albert Einstein was awarded the Nobel Prize for his General Theory of Relativity.

Moreover, Einstein was a fighter for social justice. After fleeing Germany to Switzerland before World War II, he completed his doctorate and published four of his most influential works. The use of his atomic research for nuclear bombs in World War II was devastating to him. He studied the elements of the world to better the world, not to destroy it. In other ways, we can see how he has bettered the world. Much of his research has gone to creating helpful devices, for example, remote control devices, lasers, and even DVD players. Even nuclear energy does have positive uses as well, one such is powering systems that enable more efficient space travel.

Later in his life, Einstein immigrated to the United States where he was a professor of advanced studies at the prestigious Princeton University in New Jersey.

Albert Einstein is quoted as saying:

"The human mind is not capable of grasping the Universe. We are like a little child entering a huge library. The walls are covered to the ceilings with books in many different tongues. The child knows that someone must have written these books. It does not know who or how. It does not understand in which they are written. But the child notes a definite plan in the arrangement of the books- a mysterious order which it does not comprehend, but only dimly suspects."

We know this plan, this order, this arrangement, was orchestrated by God for us. We can see, as Albert Einstein suspected, the reality of God in the Universe!

Section 4.3 Multiplying and Dividing Scientific Notation Numbers

Looking Back 4.3

Galileo Galilei is a well-known astronomer of the 16^{th} century. During his time, it was thought that the Earth was the center of the universe as was passed down from Aristotle to the Roman Catholic church. Galilei was ostracized by society and eventually put on house arrest for teaching that in reality the planets, including the Earth, revolve around the Sun. As we now know, Galileo turned out to be right!

Not only was Galilei an astronomer, but a mathematician and an engineer. His work includes but is not limited to discovering the Earth and planets revolve around the Sun, building a telescope and discovering the Moon is not round and smooth but contains craters, and demonstrating the speed of a fall of a heavy object is not proportional to its weight by dropping objects of different weights from the Leaning Tower of Pisa.

When Galilei received a position at the University of Padua, he taught geometry, mechanics, and astronomy. It was during this time that he developed the universal law of acceleration which all objects in motion obey, just as Albert Einstein had discovered the speed of light. However, his pursuit of knowledge was stunted here as well when he ran into financial difficulties after the death of his father.

Though faced with many difficulties in his life, Galileo Galilei's pursuit of truth put his place in history as the "Father of Modern Science." He is quoted as saying: "Mathematics is the language with which God has written the universe."

We learned previously about the speed of light. When we divided powers of base 10 numbers with exponents, we subtracted the exponents. Using the rules of exponents, when we multiply with exponents or like or common bases, we add the exponents.

Example 1: An illustrator for a biology book wants to draw an amoeba. The microscopic organism is 1.3×10^{-3} meters long. The illustrator wants the actual diagram to be 6.5 centimeters long. How many times must the amoeba be magnified to reach the size needed for the drawing? First convert meters to centimeters.

Example 2: There are about 50 trillion cells in the human body and each has about 75,000 genes. Approximately how many genes are in the cells of the human body?

Example 3: Divide or multiply the numbers below which are written in scientific notation.

$(5 \times 10^8)(2.1 \times 10^3)$ $\frac{6\times10^7}{2\times10^9}$

Section 4.4 Finding the Greatest Monomial Factor

Looking Back 4.4

In *Pre-Algebra*, we learned about factors and multiples. We listed out the factors of numbers to find the Greatest Common Factor (GCF) of a pair of numbers. We used the same method to list all the factors of a pair of monomials to find the Greatest Monomial Factor (GMF). When we studied Number Theory and Exponents, we found the prime factors of a number by making a factor tree. This is called prime factorization. Now, in *Algebra 1*, we will use this method to find the Greatest Common Factor of two or more monomials.

Looking Ahead 4.4

Example 1: Find the prime factorization of $32x^2y$.

Example 2: Find the GMF of $16x^2y$ and $24xy^2$ using prime factorization.

Example 3: If $8xy$ is one factor of $24xy^2$, find another factor of $24xy^2$. Check to make sure the factor is correct.

Note
When you multiply monomials with a common base, add the exponents. When you divide monomials with a common base, subtract the exponents.

Section 4.5 Using Operations with Monomials

Looking Back 4.5

In *Pre-Algebra*, we learned that when we multiply monomial terms, we add the exponents of the common bases. We learned the inverse as well: when we divide monomial terms, we subtract the exponents of the common bases. We will be using these rules of exponents along with the distributive property to multiply a monomial by a binomial.

A monomial is one term that is a number, a variable, or a combination of both a number and one or more variables. When a number is next to a variable it means the number is multiplied by the variable.

A monomial is one term, and a binomial is two terms. Can you guess the name for three terms? Terms are separated by a plus or minus sign.

Looking Ahead 4.5

Example 1:	Tell whether or not the terms below are monomial terms.

a) $\frac{d}{5}$

b) $\frac{d}{m}$

c) $3x$

d) x^2

e) $\sqrt{x}$

Monomial terms with a variable must have exponents that are non-negative integers. The degree of each monomial term is the sum of the exponents of the variables in the term.

Example 2:	Name the degree of each monomial below.

a) -3

b) $\frac{1}{3}x^2y^2$

c) $6z$

d) $0.2xy^3$

Two monomial terms being added or subtracted are called a binomial. "Bi" means "two" when used as a prefix, which explains why a "bi"-nomial represents two terms. A real-world example of this is a bicycle, which has two wheels. To add or subtract binomials, the bases and exponents of the terms must be the same.

Monomial	Binomial	Trinomial
$2x$	$3y + 2x$	$-x - 4y + 3$
"Sad and Lonely"	"A fun Pair"	"Tries and Tries, but Cannot Get It Together"

Example 3: Add the binomials below. Combine the like terms.

$$(3x^2 + 2x) + (4x^2 - 8x)$$

Example 4: Subtract the binomials below. Combine the like terms.

$$(3x^2 + 2x) - (4x^2 - 8x)$$

Example 5:	Multiply the monomial by the binomial below using the distributive property.

$$-3(x^2y + xyz)$$

Example 6:	Multiply the monomial by the binomial below using the distributive property.

$$5m^2n(mn - 9n)$$

Section 4.6 Factoring Binomials Using the Greatest Monomial Factor

Looking Back 4.6

There are many ways to factor binomials and we will learn several of these methods in the next module. For now, we will investigate a binomial as two monomials being added or subtracted and determine if there is a common factor between them. If there is, each term can be divided by this common factor; we will "undo" the multiplication with division. It is as if we "undo" the distributive property.

In the previous section, we saw:

$$a(b+c)=ab+ac$$

We used the distributive property to multiply the monomial a by the binomial $b+c$. In this section, we will see:

$$ab+ac=a(b+c)$$

We will take out the common factor of a and once we divide, we will have the remaining terms $b+c$.

Looking Ahead 4.6

To simplify $15(4+3x)$ means to use the distributive property and multiply to get a binomial.

$$15(4+3x)=15(4)+15(3x)$$

$$15(4+3x)=60+45x$$

In the following examples, we will start with the binomial and find the Greatest Monomial Factor (GMF).

Example 1:	Factor $60+45x$. Use the distributive property to check your solution.

Example 2:	Factor the binomial $2m^2-10m^4$. Use the distributive property to check your solution.

Example 3:	Factor $-3x + 9x^2y$. Find the GMF first.

Example 4:	Factor $4mn - 20m^2n^2$. Find the GMF first.

Section 4.7 Multiplying a Monomial and a Trinomial

Looking Back 4.7

In a previous section of Practice Problems, we multiplied a monomial by a binomial. We used the distributive property to distribute a through the parenthesis to multiply by b and c when given $a(b+c)$. In other words, we multiplied the monomial by the first term and then the second term in parenthesis. This is because there are two terms in a binomial expression separated by an addition or subtraction sign. The same process can be used to multiply a monomial by a trinomial. In a trinomial, $a(b+c+d)$, a is distributed through three terms in the parenthesis.

Three monomial terms being added or subtracted are called a trinomial. It is called a trinomial because there are three terms between addition and/or subtraction signs ("tri—" means *three*).

Looking Ahead 4.7

Example 1: Multiply $3x(x+4)$ using the distributive property and long multiplication.

Example 2: Multiply the monomial and the trinomial below using the distributive property and long multiplication.

$$-2m(m^2+2m-6)$$

Example 3: Multiply the monomial and the trinomial below using the distributive property and long multiplication.

$$y(3y^3-5y+6)$$

Section 4.8 Multiplying Two Binomials Using the Distributive Property

Looking Back 4.8

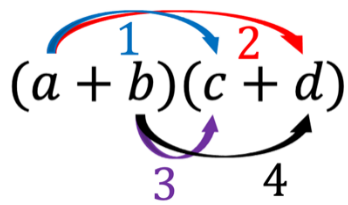
We have been multiplying a monomial by another monomial, a monomial by a binomial, and a monomial by a trinomial. We have applied the rules of exponents in doing so. Multiplying a binomial by another binomial is a little more complicated.

In mathematics, there are often many ways to solve problems. In the next three sections, we will learn at least three different methods to multiply two binomials together, and in the future, you can pick the one that is easiest for you to use.

Looking Ahead 4.8

When multiplying two binomials, the principle is the same as multiplying using parenthesis. Every term in the first set of parentheses must be multiplied by every term in the second set of parentheses using the distributive property. Again, terms are separated by an addition or subtraction sign.

Use the distributive property to multiply $a + b$ by $c + d$.



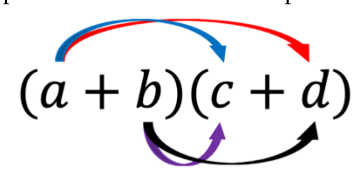
1. The first term in the first parenthesis gets multiplied by the ____________________ term in the second parenthesis.

2. Then the first term in the first parenthesis gets multiplied by the ____________________ term in the second parenthesis.

3. Next, the second term in the first parenthesis gets multiplied by the ____________________ term in the second parenthesis.

4. Finally, the second term in the first parenthesis gets multiplied by the ____________________ term in the second parenthesis.

At that point, every term in the first parenthesis has been multiplied by every term in the second parenthesis.



$$ac + ad + bc + bd$$

The final step is to simplify or add any like terms (those that have common bases and the same exponent). All the addition and subtraction are done using the coefficients.

$$a\underline{c} + a\underline{\underline{d}} + b\underline{c} + b\underline{\underline{d}}$$

Using the commutative property, the like bases can be put next to each other.

$$ac + bc + ad + bd$$
$$(a + b)c + (a + b)d$$

You can see that coefficients that have common bases may be added together.

Example 1:	Multiply $(x+2)$ by $(x+3)$ using the distributive property.

Example 2: There is an acronym that is often used to remember how to multiply two binomials. It is FOIL: Multiply the First; Multiply the Outer; Multiply the Inner; Multiply the Last.

1. F _________________ terms in both sets of parentheses
2. O __________________ terms in both sets of parentheses
3. I __________________ terms in both sets of parentheses
4. L __________________ terms in both sets of parentheses

$$(x+2)(x-4)$$

Example 3: Does FOIL always work? Try multiplying $(a + b)$ by $(c + d + e)$. (This is multiplying a binomial by a trinomial.)

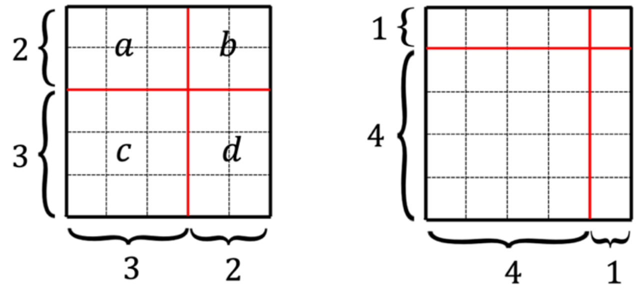
The FOIL acronym only works for multiplying binomials. It is best to just remember to use the distributive property!

Example 4: Multiply $(x + 5)$ by $(x - 5)$.

Section 4.9 Multiplying Two Binomials Using Geometric Models

Looking Back 4.9

The exponential 5^2 is the same as $5 \cdot 5$. It can be written as two binomials: $(3+2)(3+2)$ or $(4+1)(4+1)$. Because these are squares with side lengths of 5, they can be drawn as shown below:



The area of a is $3 \cdot 2 = 6$ sq. units

The area of b is $2 \cdot 2 = 4$ sq. units

The area of c is $3 \cdot 3 = 9$ sq. units

The area of d is $2 \cdot 3 = 6$ sq. units

The total area is $6 + 4 + 9 + 6 = 25$ sq. units

Multiply the binomials: $(3+2)(3+2) = (5)(5) = 25$ sq. units

In this section, we will also use geometric models with binomials that include variable terms to solve problems.

Looking Ahead 4.9

Example 1:	Make an area model to multiply $(6-3)$ by $(6-3)$.

Example 2: Make an area model to multiply $(x + 2)$ by $(x - 3)$.

Example 3: Make an area model to multiply $(x - 4)$ by $(x + 6)$.

Section 4.10 Multiplying Two Binomials Using Long Multiplication

Looking Back 4.10

We have already learned how long multiplication can be used to multiply a monomial and a binomial or to find the product when one factor is a trinomial; we simply use the same process as multiplying real numbers. To multiply two ten-digit numbers, we multiply the ones digit of the first factor by the ones digit of the second factor, then multiply the tens digit of the first factor by the ones digit of the second factor. After that, we use the same process to multiply the ones digit of the first factor by the tens digit of the second factor and then the tens digit of the first factor by the tens digit of the second factor. Make sure to line up one's place-value-digits with one's place-value-digits and ten's place-value-digits with ten's place-value-digits so that we can add them for a total value.

$$\begin{array}{rcc} & 2 & 4 \\ \times & 3 & 6 \\ \hline \end{array}$$

That same process is used to multiply two binomials together. To multiply $(x + 1)$ by $(x + 5)$, put one binomial on top and the other on bottom.

Example 1:	Multiply the two binomials below.

$$\begin{array}{rc} & x + 1 \\ (\times) & x + 5 \\ \hline \end{array}$$

Example 2:	Multiply the two binomials below using long multiplication.

$$(x - 3)(x - 4)$$

Example 3: Try the problem in Example 2 using the geometric model below.

	x	-4
x	x^2	$-4x$
-3	$-3x$	$+12$

Example 4: Multiply the two binomials below using long multiplication.

$$(x+2)(x-5)$$

Example 5: Try the problem in Example 4 using the geometric model.

	x	$+2$
x	x^2	$+2x$
-5	$-5x$	-10

Section 4.11 Introducing Polynomials

Looking Back 4.11

We have been working with polynomials throughout this module. A monomial is a polynomial with one term, a binomial is a polynomial with two terms, and a trinomial is a polynomial with three terms. The degree of a monomial is the sum of the exponents of the term.

Now, we will learn about the degree of a polynomial that is not a monomial. Remember, a polynomial has exponents that are non-negative integers $(0, 1, 2, \dots)$. Polynomials do not have fractional or negative exponents.

Looking Ahead 4.11

The degree of the polynomial is the degree of the monomial with the greatest exponent given there is only one variable. If a monomial has more than one variable, add the exponents in the monomial. The sum of the exponents that is the greatest is the degree of the polynomial. If a variable does not have an exponent, it is a degree of one.

The chart below gives the number of terms, the name using the number of terms, and the degree of the polynomial.

Polynomial	**Number of Terms**	**Name of Polynomial**	**Degree of Polynomial**
$-3x^5y$	1	Monomial	
$2x + 1.7$	2	Binomial	
$2x^2 - 4x + 2$	3	Trinomial	
$2x^2y^2 - 6x + 8y - 7$	4	Polynomial	

Special Names for Polynomials

A polynomial to the first degree is linear.

A polynomial to the second degree is called a quadratic.

A polynomial to the third degree is called a cubic.

These are special names we use to make it easier because these kinds of equations are the ones we use most often.

Polynomials are written in descending order of the exponents as we go from left to right. The exponent must be a non-negative (positive or 0) integer. The degrees of the exponents decrease in standard form. If there are more than one variable in the terms, the variables are written in alphabetical order. The lead term is the monomial with the greatest degree. The lead coefficient is the number in front of the variable that is the lead term.

Example 1: Write each polynomial in standard form. Name the polynomial by its terms and tell the degree of the polynomial.

a) $3m - 6 + 5m^3$

b) $-3x^2 + 4x^3y$

Polynomials can be added and subtracted just like monomials, binomials, and trinomials. You can only add or subtract the coefficients of the terms that have common bases and the same exponent, so they are called "like terms." Adding or subtracting polynomials is considered combining like terms.

Example 2:	Subtract the trinomials below.

$$(4p^2 + 6p + 5) - (2p^2 - 9p + 2)$$

Example 3:	Add the binomial and trinomial below.

$$(3x^2 + 5) + (10x^3 + 2x^2 - 3)$$

Example 4:	Add or subtract the polynomial below.

$$(13x^2 + 5x^3) - (2x^2 + x - 2x^3) + 4x$$

Section 4.12 Multiplying Polynomials

Looking Back 4.12

We have multiplied two monomials. We have multiplied a monomial and a binomial and a monomial and a trinomial. We have multiplied two binomials.

Now, we are going to multiply a binomial by a trinomial using all three methods we have learned.

Looking Ahead 4.12

Example 1:	Multiply the two polynomials below using the distributive property.

$$(x+7)(2x^2-4x+5)$$

Example 2:	Multiply the polynomials below using a geometric model (a rectangular array or rectangular diagram).

$$(x+7)(2x^2-4x+5)$$

Example 3:	Multiply the polynomials below using long multiplication.

$$(x+7)(2x^2-4x+5)$$

Example 4:	Multiply the two polynomials below together using all three methods.

$$(x-2)(x^2+4x-7)$$

Section 4.13 Factoring Trinomials Using the Greatest Monomial Factor

Looking Back 4.13

There are several ways to factor binomials. There are several ways to factor trinomials. There are many ways to factor polynomials. However, the only factoring we have done with binomials was to take out the Greatest Monomial Factor (GMF). This means we found the GMF, divided each term of the binomial by the GMF, and rewrote it as a monomial multiplied by a binomial. We have been multiplying monomials by binomials.

Factoring is using reverse thinking. In factoring, we start with the product and then find the factors. We will learn several methods to factor polynomials in *Module 5*, but the first step is always to see if there is a common factor to all the terms and if there is, "un-distribute" it. This is the only method we will be using for our trinomials in this section.

Looking Back 4.13

Example 1: Find the Greatest Monomial Factor of the trinomial below and factor it out.

$$3x^3 - 9x^2 + 27x$$

3: ________________ x^3: ________________

9: ________________ x^2: ________________

27: ________________ x: ________________

The Greatest Common Factor is ____________________.

The greatest common variable(s) is/are ____________________.

The Greatest Monomial Factor of $3x^3$, $-9x^2$, and $27x$ is ____________________.

Divide each monomial term by the GMF; the result is ______________________________.

$3x^3 - 9x^2 + 27x =$ ___________(___________ $-$ ___________ $+$ ___________)

Example 2: Divide (or factor out) the GMF in the trinomial below and rewrite it as a monomial multiplied by a trinomial.

$$2m^2 - 4m + 6$$

2: ________________ m^2: ________________

4: ________________ m^1: ________________

6: ________________ m^0: ________________

The Greatest Common Factor is ____________________.

The greatest common variable(s) is/are ____________________.

The GMF of $2m^2$, $-4m$, and 6 is ____________________.

Divide each monomial term by the GMF; the result is ______________________________.

$2m^2 - 4m + 6 =$ ___________(___________ $-$ ___________ $+$ ___________)

Example 3: Find the GMF of the monomial terms and factor the trinomial below.

$$4m^3 - 16m^2 + 20m$$

4: ____________ m^3: ____________

16: ____________ m^2: ____________

20: ____________ m: ____________

The Greatest Common Factor is ____________________.

The greatest common variable(s) is/are ____________________.

The GMF of $4m^3$, $16m^2$, and $20m$ is ____________________.

Divide each monomial term by the GMF; the result is ______________________________.

$4m^3 - 16m^2 + 20m =$ ____________(____________ $-$ ____________ $+$ ____________)

Example 4: Factor $2x^3 - 4x^2 + 10x$ and check your solution to make sure you are correct.