Module 1: Linear Relationships

Section 1.1 Solving Multi-Step Equations

Practice Problems 1.1

For Problem 1-13, solve the equation given.

1. $6x - 14x + 14 = 5x - 9 + 3x + 7x$

2. $3(5n + 3) = 2(n - 7) + 4(3n - 2)$

3. $-(7 - 4x) = 9$

4. $2(4x - 3) - 8 = 4 + 2x$

5. $-6(3 + k) = 6(1 + 3k)$

6. $3n - 5 = -8(6 + 5n)$

7. $-(1 + 7x) - 6(-7 - x) = 36$

8. $24a - 21 = -3(1 - 6a)$

9. $-3(4x + 3) + 4(6x + 1) = 43$

10. $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

11. $\frac{2}{3}(1-2x)=-5$

12. $\frac{5}{4}x-2=4$

13. $\frac{2}{3}m-\frac{1}{6}=-2$

For Problem 14, name the properties used in the step given.

14.

$$2(x-4)=5x-2+3-4x$$

$2(x-4)=x-2+3$ ____________________

$2x-8=x-2+3$ ____________________

$2x-8=x+1$ ____________________

$x-8=1$ ____________________

$x=9$ ____________________

For Problem 15-18, write the equation to represent the phrase and then solve it.

15. Your test scores are 87, 82, and 87 on three tests. What do you have to score on the fourth test to get an average of 84?

16. You spend $\frac{1}{3}$ of your allowance on ice cream twice a week. You spend \$7.90 on ice cream. How much is your allowance?

17. Does the equation $-a + 3a = 7a - a$ have a solution of 0 or no solution?

18. Does the equation $-3(m + 2) = -3m - 4$ have a solution of 0 or no solution?

For Problem 19 and 20, state whether the equation given will have one solution, no solutions, or infinite solutions. If there is one solution, solve for it.

19. $m(m + 2m) + 4 = 4 - m^2$

20. $6n - 2 = -2 + 6n$

Section 1.2 Solving Equations with Exponents and Radicals

Practice Problems 1.2

For Problem 1-10, solve the equation given.

1. $x^5 = -243$

2. $\sqrt{v-4} = 3$

3. $y^2 - 14 = 6$

4. $\sqrt{v+3} - 1 = 7$

5. $(b+3)^2 = 4$

6. $-6 + \sqrt{5a-6} = -3$

7. $(4n+12)^3 = -64$

8. $10\sqrt{9x} = 60$

9. $(x-2)^4 - 5 = 11$

10. $\sqrt{3n+12} = \sqrt{n+8}$

For Problem 11-14, simplify the expression given.

11. $\frac{1}{x^2 \cdot x^{-4}}$

12. $x^4 x^1$

13. $b^3(\frac{1}{b})$

14. $m^4(\frac{1}{m^2})$

For Problem 15-17, solve for the variable given. Write the principal (positive) root in exact form.

15. $b^2 = 20$

16. $b^2 + 8 = 20$

17. $b^2 - 16 = 20$

For Problem 18-20, solve for the variable given.

18. $\sqrt{3a} = -3$

19. $\sqrt{2a-1} = \sqrt{7a+9}$

20. $a^2 + 4 = 8$

Section 1.3 Solving Distance-Rate and Mixture Problems

Practice Problems

For Problem 1-6, complete the table and write the equation for the situation given but do not solve it.

1. Sam left school and traveled toward Macy's house at an average speed of 40 mph. Macy left school one hour later and traveled in the opposite direction with an average speed of 50 mph. Find the number of hours Macy needs to travel before they are 140 miles apart.

	Rate (mph)	Time (hours)	Distance (miles)
Sam	40	t	
Macy	50		

2. Lexie mixed 9 gallons of Cherry Fruit Drink with 8 gallons of Lemon Fruit Drink. The Lemon Fruit Drink contains 48% fruit juice. Find the percent of fruit juice in the Cherry Fruit Drink if the resulting mixture contained 30% fruit juice.

	Amount (gal)	Percent of Juice	Gallons of Fruit Juice
Cherry Fruit Drink			
Lemon Fruit Drink			
Final Mixture			

3. Janice sold cards for $3.00 each and packages of gift wrap for $1.50 each. She has 140 items total. She wants to know how much of each to reorder. If she made a total of $270.00, how many of each item did she sell?

	Amount	Cost Per Item	Total Money Made
Card	x	$3.00	
Gift Wrap		$1.50	
Packages	140		$270.00

4. MaryAnn made punch for her daughter's birthday party. She read the recipe wrong and ended up making the punch way too sweet. To correct her mistake, she wants to add water to even out the sweetness. She made 3 gallons of sweet punch, which should contain 70% of the concentrate, but actually contains 90%. How much water should MaryAnn add to get the punch to 70% concentration?

	Amount (gal)	Percent of Juice	Total Gallons
Sweet Punch			
Water			
Concentrate			

5. Eight gallons of sugar solution was mixed with 15 gallons of unsweetened tea. If you want your sweetened tea to be 40% sweet, what must the percent of sugar be in the sugar solution?

	Amount (gal)	Percent of Sugar	Total Gallons
Sugar Solution	8		$8x$
Unsweetened Tea	15		$15(0)$
Sweet Tea		40%	$23(0.40)$

6. For a craft fair, Debbie is making her special tea. She mixes Orange Drink which costs \$1.25 a pound with 8 lbs. of black tea which costs \$3.00 a pound, and throws in some additional spices. How much of each must she combine if Debbie wants to sell her tea for \$2.00 a pound?

	Amount (lbs.)	Cost Per Pound	Total Cost
Orange Drink			
Black Tea			
Special Tea			

For Problem 7-12, write an equation for the situation given and solve it.

7. The amount of 2m^3 of soil which contains 45% sand is mixed with 8m^3 of soil which contains 25% sand. What is the percent of sand content of the mixture?

	Amount (m^3)	Percent of Sand	Total cubic meters
First Soil	2m^3	45%	0.45(2)
Second Soil	8m^3	25%	0.25(8)
Final Mixture	10m^3	x	$10x$

8. Mauricio is mixing 12 lbs. of Indonesian cinnamon which costs \$25 a pound, with 48 lbs. of Italian cinnamon which costs \$14 a pound. Find the total cost of the mixture.

	Amount (lbs.)	Cost Per Pound	Total Cost
Indonesian	12	\$25	12(25)
Italian	48	\$14	48(14)
Final Mixture	60	x	$60x$

9. A car and a bus leave at 8 AM from the same apartment complex headed in the same direction. The car travels at a rate of 60 mph. In 3 hours, the car is 60 miles from the bus. Find the rate of the bus.

	Rate (mph)	Time (hours)	Distance (miles)
Car	60	3	60(3)
Bus	r	3	$3r$

10. Michael and Jenny are meeting for lunch at noon. They live 540 miles apart. They leave their homes at the same time. Michael is traveling at 65 mph and Jenny is traveling 70 mph. What time did they leave their homes?

	Rate (mph)	Time (hours)	Distance (miles)
Michael	65	t	$65t$
Jenny	70	t	$70t$

11. Silver is 93% pure silver. How many grams of Sterling silver must be added to a mixture of 85% silver alloy to get 500 g of 90% alloy?

	Amount (g)	Percent of Purity	Total Grams
Sterling Silver	x	93%	$0.93x$
Silver Alloy	$500-x$	85%	$0.85(500-x)$
Alloy	500	90%	$0.90(500)$

12. Tickets for a school play were $6.00 for adults and $2.50 for children. The school sold 600 tickets and raised $3,001.50. However, they did not know how many tickets sold were to adults and how many were to children. Can you figure out how many tickets the school sold to adults and how many tickets the school sold to children?

	Tickets Sold	Cost	Total Cost
Adult	$600-x$	\$6	$\$6(600-x)$
Children	x	\$2.50	$\$2.50x$

For Problem 13 and 14, find the mistake(s) made in completing the table for the situation given.

13. Frederick and Garber were teamed up in a relay race. Frederick started by rowing a canoe for 15 miles. Then he tagged Garber, who rowed the remaining 10 miles of the race. Find Garber's speed given he rowed 4 mph slower than Frederick rowed and the entire race took 3 hours to complete.

	Rate (mph)	Time (hours)	Distance (miles)
Frederick	r	$15 \div r$	15
Garber	$r + 4$	$10 \div (r - 4)$	10
Total		3 hours	

14. Breakfast cereal is 30% sugar. If we add 6 grams of sugar to 40 grams of breakfast cereal, what is the percent of sugar in the resulting mixture?

	Amount (g)	Percent of Sugar	Total Grams
Cereal	40	30%	0.30(40)
Sugar	6	100%	6(0)
New Cereal	46	x%	$46x$

For Problem 15-17, use the given information to solve the problem.

Miriam and Joel are making a 250-mile trip. Miriam drives through town and goes 45 mph for t hours. Joel takes over on the open road at the end of Miriam's t hours and drives at a rate of 65 mph. They reach their destination in 4 hours.

15. Complete the table.

	Rate (mph)	Time (hours)	Distance (miles)
Miriam			
Joel			

16. Write an equation for the table in Problem 15.

17. Solve the equation from Problem 16 to find the time that Miriam drove and the time that Joel drove (in hours).

For Problem 18-20, use the given information to solve the problem.
Two hundred liters of Pineapple Punch that contains 35% of fruit juice is mixed with 300 liters of Strawberry Punch. The resulting blended fruit juice is 20% fruit juice.

18. Complete the table.

	Amount (liters)	Percent of Fruit Juice	Liters of Fruit Juice
Pineapple Punch	200	35%	
Strawberry Punch	300	x	
Resulting Blended Fruit Punch		20%	

19. Write an equation for the table in Problem 18.

20. Solve the equation from Problem 19 to find the percent of fruit juice in 300 liters of Strawberry Punch.

Section 1.4 Solving Literal Equations

Practice Problems 1.4

For Problem 1-14, solve the equation given for the variable given.

1. $A = 2(l + w)$ for w

2. $12x - 4y = 20$ for x

3. $\frac{x+y}{3} = 5$ for y

4. $A = \frac{1}{2}bh$ for b

5. $7x - y = 14$ for x

6. $A = P(\text{l} + rt)$ for r

7. $x = \frac{yz}{6}$ for z

8. $L = a + (n - 1)d$ for d

9. $A = \frac{r}{2L}$ for L

10. $\frac{t+m}{A} = N$ for A

11. $e = mc^2$ for c

12. $A = \frac{t+m}{n}$ for n

13. $V = \pi r^2 h$ for h

14. $\frac{1}{2}(10x + y) = 14$ for y

For Problem 15-20, use the equation $F = G\frac{m_1 m_2}{d_2}$ to solve the problem.

15. Solve for G.

16. Solve for m_2.

17. Solve for m_1.

18. Solve for d_2.

19. Why can $F = G\frac{m_1 m_2}{d_2}$ also be written $F = \frac{G m_1 m_2}{d_2}$?

20. How are the formulas in Problem 16 and Problem 17 similar? How are they different?

<u>Section 1.5 Direct Variation</u>

<u>Practice Problems 1.5</u>

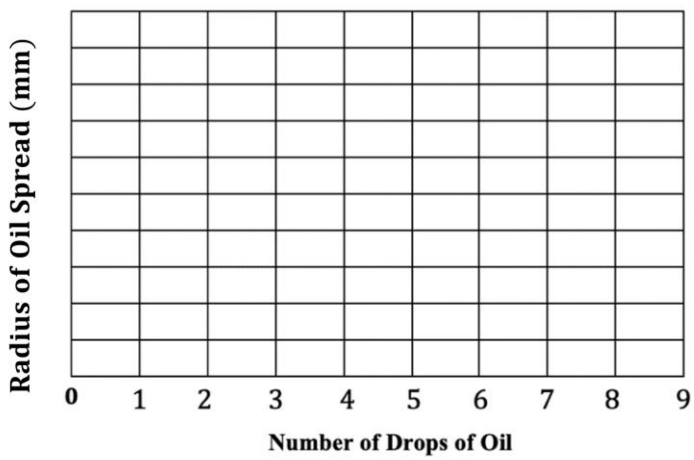
Experiment 1.5a

You will need:

- 6 sheet of paper towels
- Eye dropper
- Cooking oil

1. Number the paper towel sheets 1-6, placing the number in the center of each paper towel
2. Fill the eye dropper with cooking oil and hold it $\frac{1}{2}$-inch above the paper towel with the number 1 and drop a drop of oil on the number (the number should be in the center of the paper towel).
3. Drop two drops of oil on the paper towel numbered 2.
4. Drop three drops of oil on the number 3 paper towel and repeat the process for the rest of the paper towels (four drops on 4, five drops on 5, etc. ending with six drops on 6).
5. Measure the radius of the circles from the drops of oil in millimeters. The radius is the line from the center of the circle to the circumference. This will measure the spread of the oil. If the drop was placed in the center of the number from $\frac{1}{2}$-inch above it should spread out fairly regularly to form a circle.
6. Complete the table and graph for all six paper towels and solve problems a)-f).

Number of Drops of Oil	Radius of Oil Spread (mm)
1	
2	
3	
4	
5	
6	



a) How do you know the data is linear?

b) The formula for a direct variation is $y = kx$. This means $\frac{y}{x} = k$. Use these ratios, $\frac{\text{Radius of Oil Spread}}{\text{Number of Drops of Oil}}$, to get a good approximation for k; $k \approx$ ________.

c) Once it is an experiment the data can be messy. What can cause discrepancies (differences)? What variables were difficult to control?

d) Why is the number of drops of oil the independent variable and the radius of the oil spread the dependent variable?

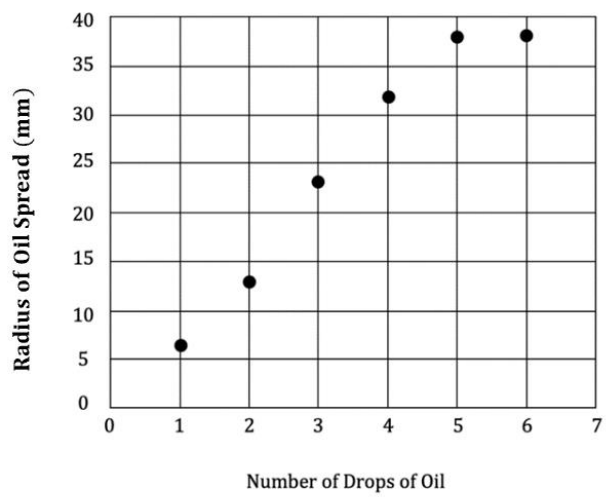
e) Using the constant of proportionality found in b) above, write the direct variation equation.

f) Direct variations are proportional. That means y varies directly with x. The ratios are constant. What else makes this equation a direct variation?

Experiment 1.5b

Below are the results from **Experiment 1.5a** performed by Margo, Miranda, and Kylee Ann in a group. Use the graph to complete the table below and solve problems a)-d).

Number of Drops of Oil	**Radius of Oil Spread (mm)**
1	
2	
3	
4	
5	
6	



a) How do you know the data is linear?

b) The formula for a direct variation is $y = kx$. This means $\frac{y}{x} = k$. Use these ratios, $\frac{\text{Radius of Oil Spread}}{\text{Number of Drops of Oil}}$, to get a good approximation for k; $k \approx$ ________.

c) Using the constant of proportionality found in b) above, write the direct variation equation.

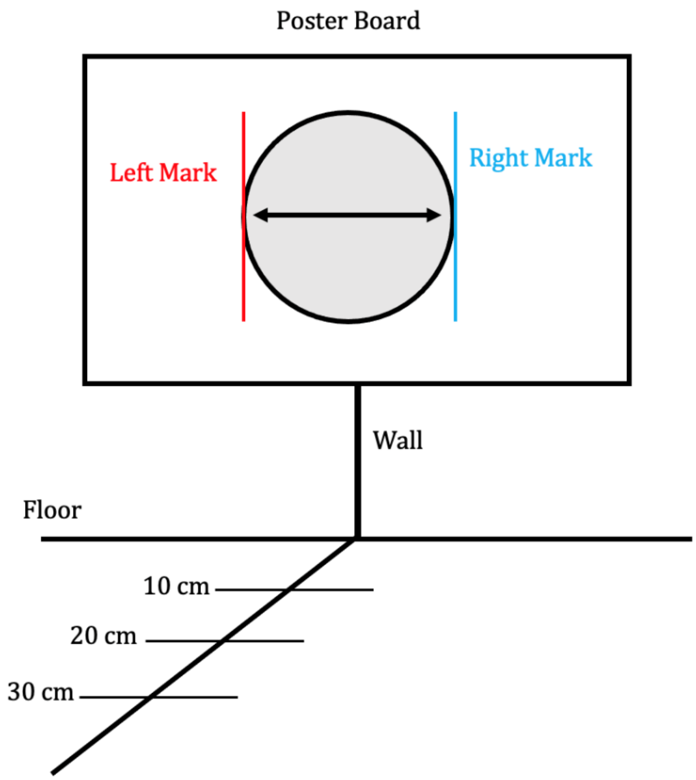
d) Determine whose paper towels have a higher rate of absorbency: the ones in your experiment or the ones in Margo, Miranda, and Kylee Anns' experiment. If you try the experiment with two different brands of paper towels and graph then on the same coordinate axes, how can you determine which is more absorbent?

Experiment 1.5c

You will need:

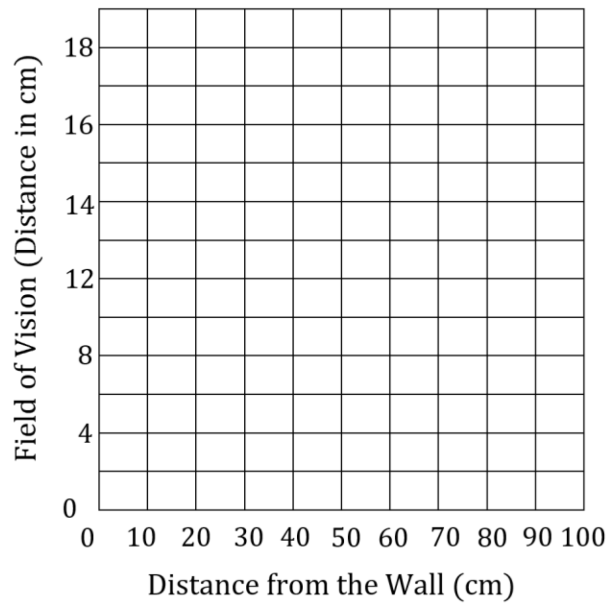
- A poster board
- A paper towel tube
- A tape measure, meter stick, or ruler
- A helper

1. Tape the poster board on a wall so you can look directly at the center.
2. Tape the tape measure to the floor beginning at the wall where the poster is hanging.
3. Move to the 10 cm mark on the poster board and hold the paper towel roll directly in front of one of your eyes.
4. Have a helper mark the circle that you see on the paper. Have the helper move their finger directly to right side of the poster board and make a mark with a pen. Have the helper move their finger directly to left side of the poster board and make a mark with a pen. This is your field of vision.
5. Your field of vision is marked by the two-sided arrow in the diameter of the circle. Use centimeters to measure it. Mark the diameter length on the table under "**Field of Vision**."



6. Move back to the 20 cm mark and repeat the process. Collect 8-10 data points by repeating the process every 10 cm as you move away from the poster board.
7. Complete the table and graph below and solve problems a)-f).

Distance from the Wall (cm)	Field of Vision (Diameter in cm)
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	



a) How do you know the data is linear?

b) The formula for a direct variation is $y = kx$. This means $\frac{y}{x} = k$. Use these ratios, $\frac{\text{Distance from the Wall}}{\text{Field of Vision}}$, to get a good approximation for k; $k \approx$ ________.

c) Once it is an experiment the data can be messy. What can cause discrepancies (differences)? What variables were difficult to control?

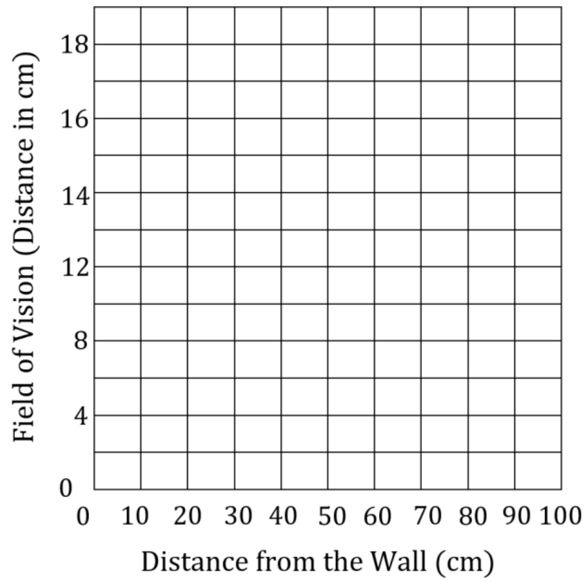
d) Why is the distance from the wall the independent variable and the field of vision the dependent variable?

e) Using the constant of proportionality found in b) above, write the direct variation equation.

Experiment 1.5d

Below are the results for **Experiment 1.5c** performed by Matthew, Ryan, and Autumn in a group. Use the table to complete the graph below and solve problems a)-d) from the previous experiment.

Distance from the Wall (cm)	**Field of Vision (Diameter in cm)**
10	7
20	8.8
30	12
40	13.4
50	15
60	17
70	17.45
80	19.1



a) How do you know the data is linear?

b) The formula for a direct variation is $y = kx$. This means $\frac{y}{x} = k$. Use these ratios, $\frac{\text{Distance from the Wall}}{\text{Field of Vision}}$, to get a good approximation for k; $k \approx$ ________.

c) Using the constant of proportionality found in b), write the direct variation equation.

Experiment 1.5e

You will need:

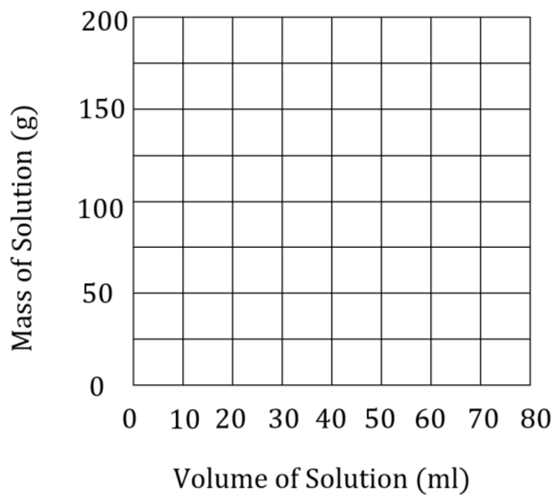
- A canned beverage that has sugar (a soda pop)
- A beaker (or graduated cylinder)
- A digital gram scale

1. Place your beaker on the gram scale and record the mass in the blank below.
2. Pour 10 ml. of your beverage into the beaker.
3. Place your beaker and solution (beverage) on the gram scale and record mass in the table below.
4. Subtract the mass of the beaker from the solution and record the mass in the table below.
5. The formula $d = \frac{m}{v}$ is $density = mass\ divided\ by\ volume$. Use this formula to complete the final column of the table.
6. Add 10 more ml. of solution to the beaker and repeat the steps above.
7. Complete the table and graph and answer questions a)-f).

Type of Beverage: ________________

Weight of Beaker_________________

Volume of Solution (ml)	**Mass of Beaker and Solution (g)**	**Mass of Solution (g)**	**Density of Solution**
10			
20			
30			
40			
50			
60			
70			
80			



a) What units are we using to measure density?

b) What is the average density of your solution?

c) Why do you think the density of your beverage is different than other beverages? What do you think could be a controlling factor of density for these beverages?

d) Find the equation of the line of best fit for the graph. What is the slope and what does slope represent?

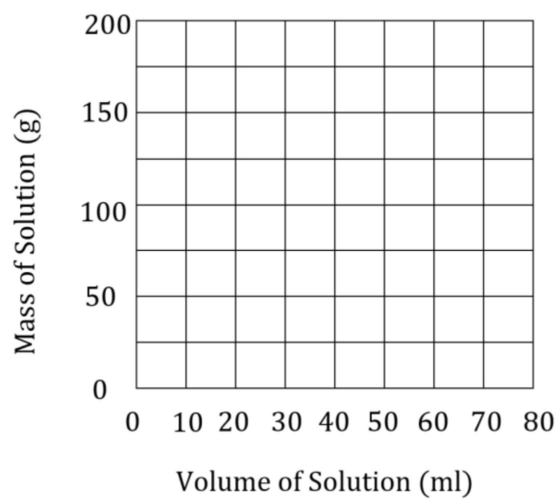
e) What would happen to your graph if you added more sugar to your beverage?

f) What percentage of your beverage is sugar?

Experiment 1.5f

Below are the results for (**Experiment 1.5d**) performed by Silas, Kendyl, and Moritz in a group. Complete the table and graph of the volume of sweet tea compared to the mass and answer questions a)-c). The solution in this experiment is sweet tea and the mass of the beaker is 95.58 g.

Volume of Sweet Tea (ml)	Mass of Beaker with Sweet Tea (g)	Mass of Sweet Tea (g)
10	106.13	
20	116.35	
30	126.88	
40	137.1	
50	147.22	
60	157.43	
70	168.47	
80	178.44	



a) Find the equation of the line of best fit for the graph.

b) What is the density of sweet tea (g/ml)? What represents density on the graph and in the equation of the line of best fit?

c) What is the percentage of sugar for sweet tea if the nutrition label says there is 40 g of sugar and 350 ml of beverage?

Below is a table of experimental results of the beverage Grape Juice. Use the data to answer questions d) and e).

Beverage	Density	Percentage of Sugar (Experimental)	Nutrition Label
Grape Juice	1.05 g/ml	13.0%	40 g/240 ml

d) Calculate the percentage of sugar for grape juice.

e) Find the percentage of error for grape juice:

$$\text{Percent of Error} = \frac{|\text{Calculated Value} - \text{Experimental Value}|}{\text{Calculated Value}} \cdot 10$$

Section 1.6 Direct Variation Equations In the Real World

Practice Problems 1.6

For Problem 1-4, use the given information and data to solve the problem.

Betsy and Scott did an experiment with sound waves. They measured the frequency and wavelengths of tuning forks in order to determine the speed of sound. The data is listed below:

$\boldsymbol{f}(\mathbf{Hz})$	$\boldsymbol{\lambda}(\mathbf{m})$
384	0.84
320	1.05
256	1.49

1. If in the formula $V = f\lambda$, f represents frequency, λ represents wavelength, and velocity represents the speed of sound, how would you calculate the speed of sound?

2. Complete the table for velocity.

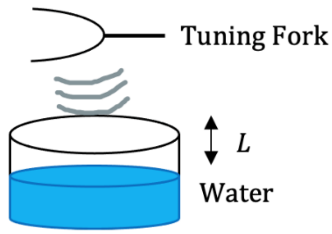
$\boldsymbol{f}(\mathbf{Hz})$	$\boldsymbol{\lambda}(\mathbf{m})$	$\boldsymbol{V}(\mathbf{m/sec})$
384	0.84	
320	1.05	
256	1.49	

3. Average the three velocities to approximate the speed of sound.

4. The speed of sound varies with the fluid dynamics, but at 0°C is $343.2\ \mathrm{m/sec}$. How close was the speed of sound calculated from Betsy and Scotts' experiment to the speed of sound at 0°C?

For Problem 5-15, solve the word problem given.

5. Philip is conducting an experiment with a tuning fork set into vibration above a vertical open tube filled with water. The water level is slowly dropping. As it drops, the air above the tube resonates loudly with the tuning fork when the distance from the tube opening to the water level is $L_1 = 0.100$ m and $L_2 = 0.450$ m. Assuming the speed of sound of air is 343 m/sec. and the distance between any two resonant lengths in a closed tube is ($L_2 - L_1 = \frac{1}{2}\lambda$), what is the frequency of the tuning fork? Because $L_2 - L_1 = \frac{1}{2}\lambda$, $\lambda = 2(L_2 - L_1)$. Substitute the latter formula ($\lambda = 2(L_2 - L_1)$) into the formula for wavelength and solve.



6. Frequency is the number of times an event occurs per unit of time and is measured in Hertz (Hz), which is the top of one wave to the top of another. Humans can hear from 20 Hz to 20,000 Hz. Dogs can hear up to 60,000 Hz. Bats can hear as low as 20 Hz. Sound is perceived in pitches. High frequencies sound like a whistle; low frequencies sound like a tuba.

A tuning fork resonates (vibrates) at a frequency and pitch that depends on the length of the prong. A tuning fork can be used to tune musical instruments. It can be adjusted. Smaller prongs have less distance, faster vibrations, and higher frequency. The largest tuning fork in the world is in Berkley, California; it is 45 ft. (13.7 m) tall. The very slow vibration is too low to hear. Changing the medium also changes the speed.

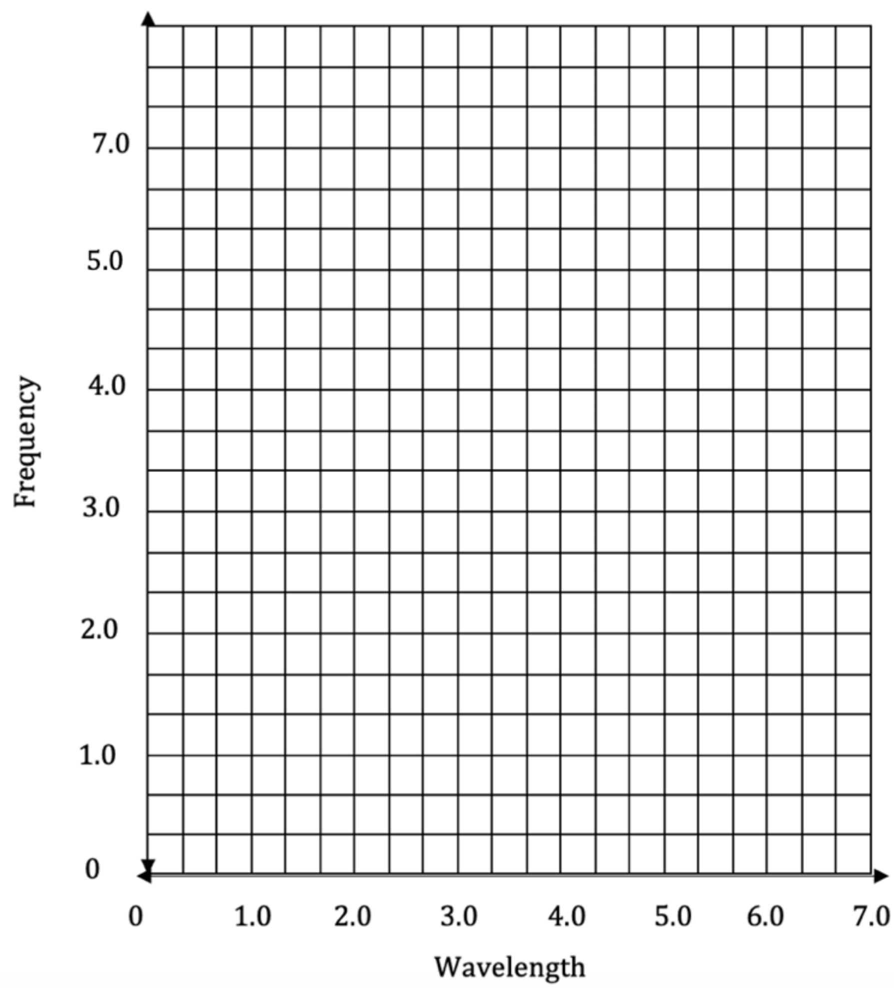
The speed of sound in water is 1,484 m/sec (more than four times the speed in air). This is why whales and dolphins can communicate quickly over long distances. If the speed of sound in air (343 m/sec) travels a mile in 5 seconds, how many seconds does it take to travel a mile in water?

8. Linda creates transverse waves of varying frequency by moving one end of a slinky. All the waves move along the slinky at a speed of 6.0 meters per second. Use the frequency formula $v = \lambda f$ (velocity = wavelength times frequency) to complete the table. Because $v = \lambda f$, then $\lambda = \frac{v}{f}$.

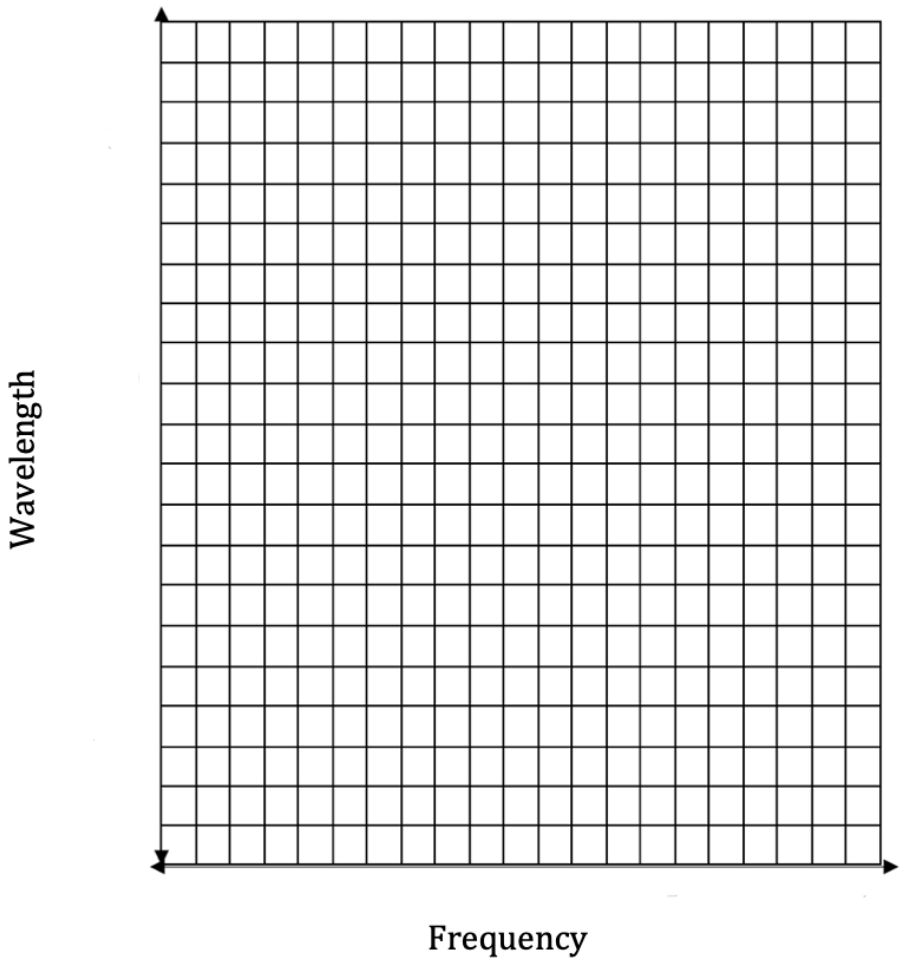
Frequency (Hz)	**Wavelength (m)**
1.0	
2.0	
3.0	
4.0	
5.0	
6.0	

8. How do you know the graph will not be linear?

9. Draw the graph of the wavelength versus frequency.



10. Draw the graph of frequency versus wavelength.



11. As the frequency is increasing, what is happening to the wavelength?

12. Do wavelength and frequency vary directly or inversely?

13. Use $v = \lambda f$ (velocity = wavelength times frequency) to complete the table.

$\boldsymbol{\lambda}$	$\boldsymbol{f}$	$\boldsymbol{v}$
1.0	6	
2.0	3	
3.0	2	
4.0	1.5	
5.0	1.2	
6.0	1	

14. Why is the velocity the same in the table?

15. If the table gave you wavelength and velocity and asked for frequency, what formula would you use to find the frequency?

Section 1.7 Arithmetic Sequences

Practice Problems 1.7

For Problem1-3, complete the next three numbers in the arithmetic sequence given.

1. $14, 20, 26, 32,$ ________ , ________ , ________

2. $54, 51, 48, 45, 42, 39,$ ________ , ________ , ________

3. $0.25, 0.5, 0.75, 1,$ ________ , ________ , ________

For Problem 4-6, find the common difference of each arithmetic sequence.

4. 202, 192, 182, 172, 162, 152

5. $1\frac{1}{4}, 2, 2\frac{3}{4}, 3\frac{1}{2}, 4\frac{1}{4}, 5$

6. $11, 15, 19, 23, 27, 3$

For Problem 7-12, use the given table to solve the problem.

Term Value ($a(n)$)	45	50	55	60	65	70
Term Number (n)	1	2	3	4	5	6

7. If $n = 1$, what is $a(n)$?

8. If $a(n) = 65$, what is n?

9. If $n = 7$, what is $a(n)$?

10. If $n = 3$, what is $n - 1$?

11. If $n = 4$, what is $a(n - 2)$?

12. If $a(n) = 60$, what is $a(n + 1)$?

For Problem 13-15, complete the table for the arithmetic sequence.

13.

n	$a(n)$
1	2
2	5
3	8
4	
5	
6	

14.

n	$a(n)$
1	10
2	8
3	6
4	
5	
6	

15.

n	$a(n)$
1	0.75
2	0.5
3	0.25
4	
5	
6	

For Problem 16 and 17, list the first six terms in the arithmetic sequence. Assume constant growth.

16. A puppy weighs 8 pounds at birth and gains 2 pounds a month.

17. A college loan is $3,500.00 and the monthly payment is $170.00. (Do not worry about interest.)

For Problem 18, solve the word problem given.

18. There are eight A keys on a piano. The frequency when the key is struck is the number of vibrations per second the key string makes. The four frequencies below middle C are: 27.5, 55, 110, 220. Is this an arithmetic sequence? Why or why not?

For Problem 19 and 20, find the value of the term given.

19. Find $a(3)$ for $a(n) = (n - 1)(-2) + 5$.

20. Find $a(6)$ for $a(n) = (n - 1) + \frac{1}{2}$.

Section 1.8 Recursive Formulas for Arithmetic Sequences

Practice Problems 1.8

For Problem 1-3, solve the word problem given.

1. A ferris wheel costs $3.00 per person to ride. The ride starts with twenty people on it. After each ride, the cars are unloaded and twenty new people are loaded in the cars. Show the sequence for the ticket money collected for four full rides on the ferris wheel.

2. Write a recursive formula for the ferris wheel ride from Problem 1. Use the formula to find the amount collected on tickets after five full rides.

3. A pond is stocked with 800 fish. Only 5 fish are allowed to be caught by any fisherman per day. A group of 20 Optimist Club members rent the pond for a week and fish everyday using this rule. Write a sequence to show how many fish will be caught at the end of the week. What term number will that be? How many fish will be left in the pond?

Day	1	2	3	4	5	6	7
Total Fish Caught							

For Problem 4-7, in the recursive formula given, find the common difference and explain the rule.

4. $a(n) = a(n-1) + 0.2$

5. $a(n) = a(n-1) + 6\frac{1}{2}$

6. $a(n) = a(n-1) - 1.4$

7. $a(n) = a(n-1) + 15$

For Problem 8-10, find the common difference, write a recursive formula, and then use the formula to find the fifth term in the sequence given.

8. $1.2, 1.5, 1.8, 2.1$

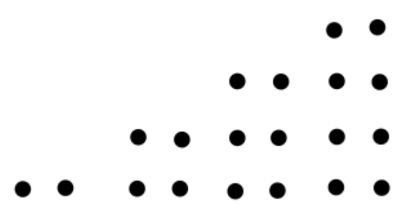
9. $480, 470, 460, 450$

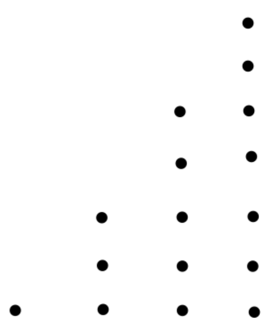
10. $22, 44, 66, 88$

11. How could you use a recursive formula to find the term before 22 in Problem 10?

For Problem 12-14, draw the next figure(s) for the sequence given.

12. 

13. 

14. 

For Problem 15 and 16, find the common difference and the next term in the sequence given.

15. $3, a+3, 2a+3, 3a+3$

16. $x+y+z, 3x+4y+z, 5x+7y+z$

17. Use the sequence $12, 9, 6$, ... to answer the following questions.

a) What is the first term?

b) What is the common difference?

c) What is the recursive formula?

d) Find the fourth term using the recursive formula.

For Problem 18-20, solve the word problem given.

18. For the recursive formula $a(n) = a(n-1) - 5$, what is the common difference?

19. Fill in the blanks: In an arithmetic sequence, the value of any term is the value of the previous term plus the

____________________ ____________________.

20. Paige earns a monthly salary of $\$1{,}200.00$ and gets a $\$300$ bonus for each month for the next year. Write a recursive formula for Paige's monthly income and list the first three months pay.

Section 1.9 Explicit Formula for Arithmetic Sequences

Practice Problems 1.9

For Problem1-3, find the common difference and first term of the sequence from the explicit formula given.

1. $a(n) = 15.4 + (n-1)1.8$

2. $a(n) = -4.5 + (n-1)0.5$

3. $a(n) = 16 + (n-1)$

For Problem 4 and 5, use the explicit formula given to find the third term.

4. $a(n) = 2 + (n-1) \cdot 4$

5. $a(n) = -10 + (n-1) \cdot 3$

For Problem 6-9, find the explicit formula for the sequence given. Find $a(1)$ and d first.

6. $22, 24, 26, 28, 30$

7. $14, 12, 10, 8, 6$

8. Find the 10th term in Problem 6 using the explicit formula.

9. Find the 100th term in Problem 7 using the explicit formula.

For Problem 10, solve the word problem given.

10. A missionary challenges a youth group to save quarters for missions. A youth starts with 9 quarters and adds 5 each week. How many quarters will he have in 20 weeks and how much money is that for missions?

For Problem 11-20, use the given information to solve the problem.

Rayven wants to buy a bicycle that costs \$250. She pays \$22 a week towards the cost of the bike from money she earns at a summer job.

11. What is the first (start) value $a(1)$? Why is that $a(1)$?

12. What is the common difference, d? Why is that d?

13. Write the sequence representing the amount left after each weekly payment. List the first six terms. How much will Rayven owe after 3 weeks of payments?

14. Write the explicit formula for the bike balance at any given week in time.

15. How much will Rayven owe after 8 weeks?

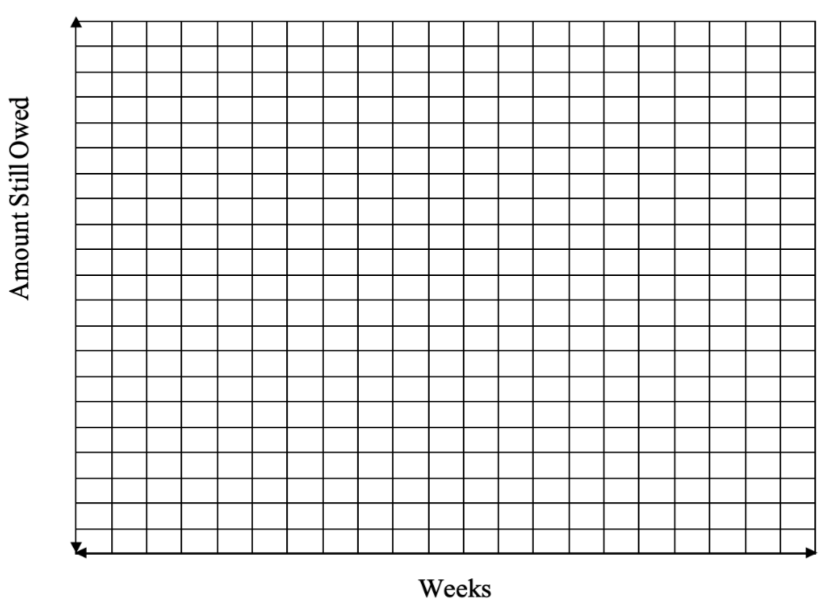
16. If n represents the term number and $a(n)$ represents the term value, what do n and $a(n)$ represent in this problem?

17. Will Rayven have the bike paid off by the 13^{th} week?

18. Complete the table for n and $a(n)$.

$\boldsymbol{n}$	$\boldsymbol{a(n)}$
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

19. Graph the problem. Let the x-axis be weeks and the y-axis be amount still owed.



20. How can you tell from the table and graph the sequence is linear? Are all arithmetic sequences linear? Why or why not?

Section 1.10 Connecting Recursive and Explicit for Arithmetic Sequences

Practice Problems 1.10

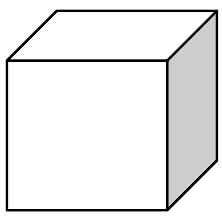
Make a table and graph for each game. Find an explicit formula and linear equation for each game. Use the formulas to answer the questions.

1. Rod Stamping

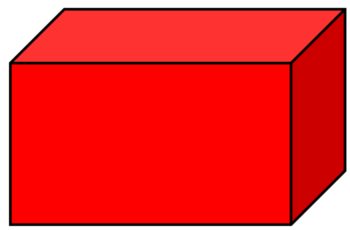
a) Imagine the white Cuisenaire Rod® is a rubber stamp that stamps 1-centimeter squares. Each consecutive colored rod on the table is one white stamp more than the previous rod. How many stamps would it take to cover each of the other rods? What is the pattern? Complete the table to find the pattern.

A red rod is equal to two white rods. A light green rod is equal to three white rods. The number next to the color is equal to the number of white rods it is equal to.

This white rod takes six stamps



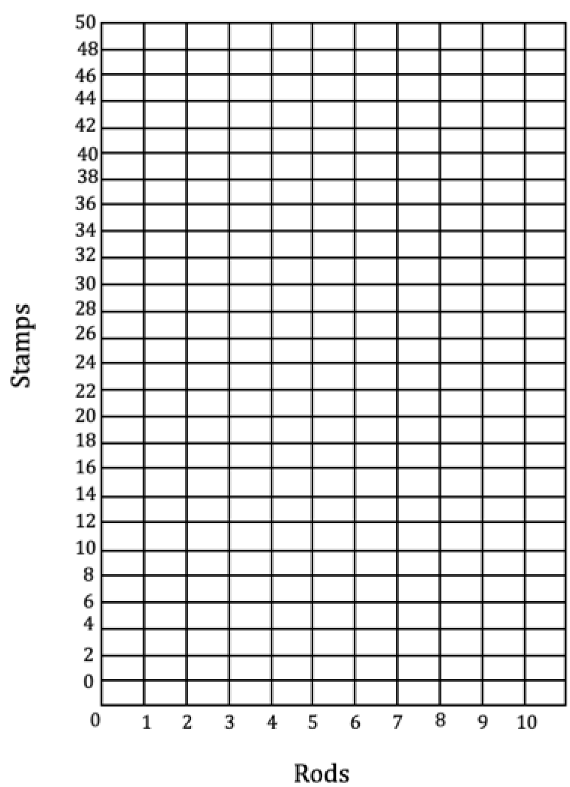
The red rod takes ten stamps



Continue the Pattern:

Rod	**Stamps**
1 white	6
2 red	10
3 light green	
4 purple	
5 yellow	
6 dark green	
7 black	
8 brown	
9 blue	
10 orange	

b) Graph the rod stamping data in the table below.



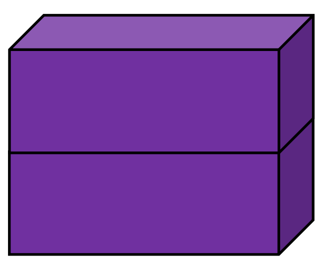
c) What is the explicit formula for Rod Stamping?

d) Use the explicit formula to find the number of stamps needed to cover an 11 rod?

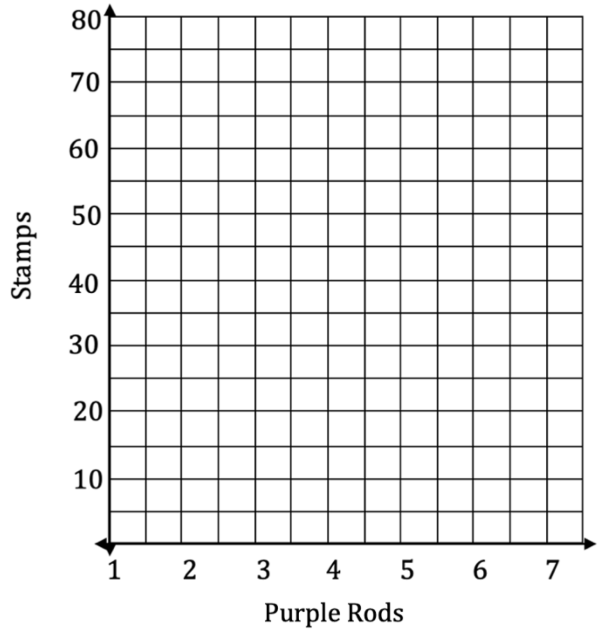
e) Use the distributive property to simplify the explicit formula. Substitute x for n and y for $a(n)$. Write the linear equation for the Rod Stamping game and use it to find the stamps needed to cover an 11 rod.

2. More Rod Stamping

a) Suppose you take rods of one color and pretend to glue them together as the picture shows. Figure out how many stamps are needed for each rod and look for the pattern. Can you determine the number of stamps needed to cover 100 purple rods glued together? Try this with the different colors. You will get different tables, graphs, and equations. Complete the table below to answer the following questions. (A purple rod is made up of 4 white rods.)

<table>
<tr>
<td>The purple rod (as a sample) takes 18 stamps
</td>
<td>Two purple rods glued together take 28 stamps
</td>
<td>Continue the Pattern. How many stamps would three rods glued together take? Four rods?
<table>
<tr><th>Rod</th><th>Stamps</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
</table>
</td>
</tr>
</table>

b) Graph the More Rod Stamping data in the graph below.



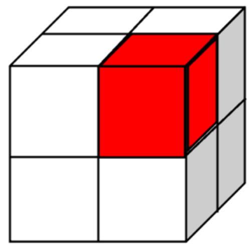
c) What is the explicit formula for More Rod Stamping?

d) Use the explicit formula find the number of stamps needed to cover 100 purple rods.

e) Use the distributive property to simplify the explicit formula. Substitute x for n and y for $a(n)$. Write the linear equation for the Rod Stamping game and use it to find the stamps needed to cover 100 purple rods.

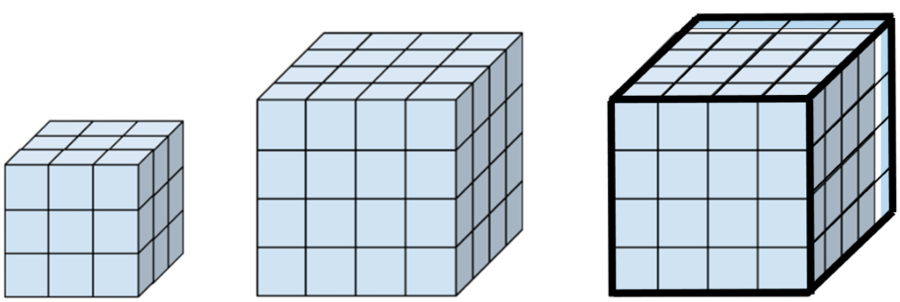
3. Dipping Cube

a) Suppose you have a cube that is made of smaller unit cubes glued together, say two on each side (as in the figure below), and you dipped it in a can of paint; how many unit cubes would have paint on two of the faces? What about a $3 \times 3 \times 3$ cube, a $4 \times 4 \times 4$ cube, or a $5 \times 5 \times 5$ cube? How many unit cubes would have paint on two faces if the cube dipped was a $10 \times 10 \times 10$ cube?

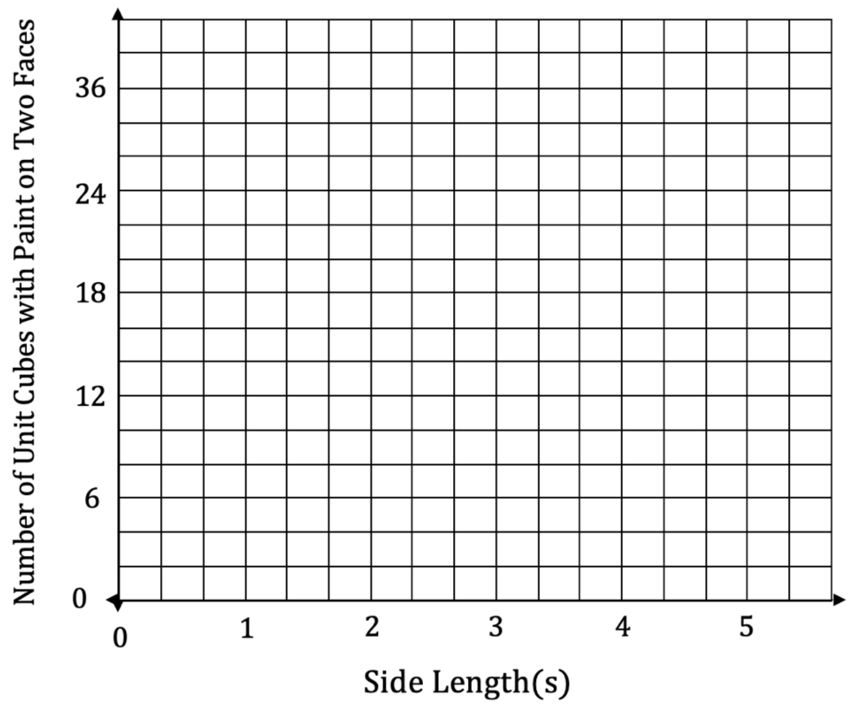


For example: If a $2 \times 2 \times 2$ cube were dipped into paint, the corner would have paint on 3 faces. The other 3 faces of each corner cube are on the inside of the cube. Only the outside faces get covered in paint. Therefore, all the cubes in a $2 \times 2 \times 2$ cube are corners, and each have 3 faces painted; none of the unit cubes will have paint on 2 faces.

S (side length of cube in units)	**Number of Unit Cubes with Paint on Two Faces**
2	
3	
4	
5	



b) Graph the Dipping Cube data in the graph below.



c) What is the explicit formula for the Dipping Cube? Notice that the table starts with 2 so $a(1) = -12$ because it is 12 before 0.

d) Use the explicit formula to find how many unit cubes would have paint on two faces if the cube was a $10 \times 10 \times 10$ cube?

e) Use the distributive property to simplify the explicit formula. Substitute x for n and y for $a(n)$. Write the linear equation for the Dipping Cube and use it to find the number of unit cubes that would have paint on two faces if the cube was a $10 \times 10 \times 10$ cube.

Section 1.11 Linearity and Calorie Burning

Practice Problems 1.11

For Problem 1-10, use the given table to solve the problem.

Activity	$\frac{\textbf{Calories}}{\textbf{min} \cdot \textbf{kg}}$
Dancing	0.08
Running (5 mph) (12 min/mile)	0.12
Running (6 mph) (10 min/mile)	0.13
Mowing Lawn	0.80
Raking Leaves	0.70
Walking (3 mph) (20 min/mile)	0.06
Walking (4 mph) (15 min/mile)	0.08
Vacuuming	0.05
Cleaning House	0.06

1. How many calories does a 140-pound person burn raking leaves for 40 minutes?

2. How many calories does the same person burn vacuuming for 25 minutes at 4 mph (15 min/mile pace)?

3. If the same person ran 5 mph (12 min/mile pace) for 10 minutes, how many calories would they burn compared to walking for 10 minutes at 4 mph (as we found in Example 2 of the Lesson Notes)?

4. Would a 140-pound person burn more calories walking a 3 mph pace for 30 minutes or running at a 5 mph pace for 15 minutes?

5. How much does a 165-pound person weigh in kilograms given $1 \text{ lb} = 0.453392$ kg?

6. Would a 165-pound person expend more energy vacuuming for 20 minutes or cleaning house for 15 minutes?

7. How many kilograms does a 200-pound person weigh?

8. How many kilograms does a 220-pound person weigh?

9. How many more calories does a 220-pound person burn per minute vacuuming than a 200-pound person?

10. Who burns more calories: a 220-pound person who vacuums for 20 minutes and mows the lawn for 30 minutes, or a 200-pound person who cleans house for 30 minutes and rakes leaves for 40 minutes?

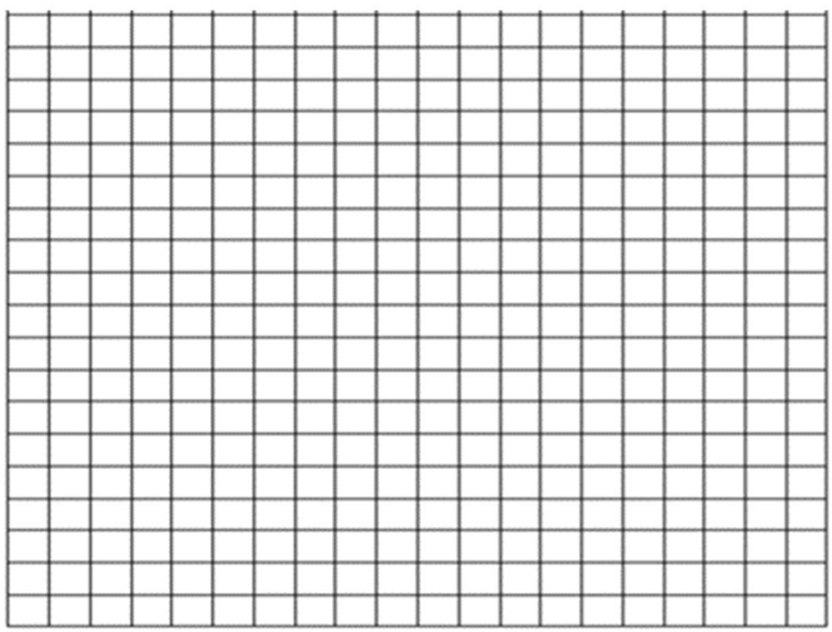
Section 1.12 Trend Lines

Practice Problems 1.12

For Problem 1-20, use the given table of fast-food sandwiches to solve the problem.

Sandwich	**Saturated Fat (g)**	**Total Fat (g)**	**Calories**
Arty's Classic Roast Beef	5	14	360
Burger Queen	10	35	315
McChicken's	1	3.5	310
5 Gals Hamburger	19.5	43	700
Jill's Hamburger	5	18	340
Marcus and DeMarco's Burger	13	34	520
McDee's Big Burger	10	29	550
Kris & Kieras' Burger		100	1,354
Smushburger	20	54	780

1. Make a graph of saturated fat to total fat (saturated fat (x), total fat (y)).



2. Draw the trend line for total fat depending on the amount of saturated fat. (Skip Kris & Kieras' Burger because no amount of saturated fat is listed.)

3. Pick two points that are on the line and calculate the slope of the trend line. Use the points $(5,14)$ and $(10,29)$.

4. Estimate the y-intercept of the trend line. Look at the graph to see where the line crosses the y-axis.

5. Using the slope and y-intercept, find the line of best fit for the total fat compared to saturated fat in each sandwich.

6. The calculator estimates a line of best fit to be $y = 2.3x + 4.7$. What is the slope and y-intercept of the line of best fit? Is your line close?

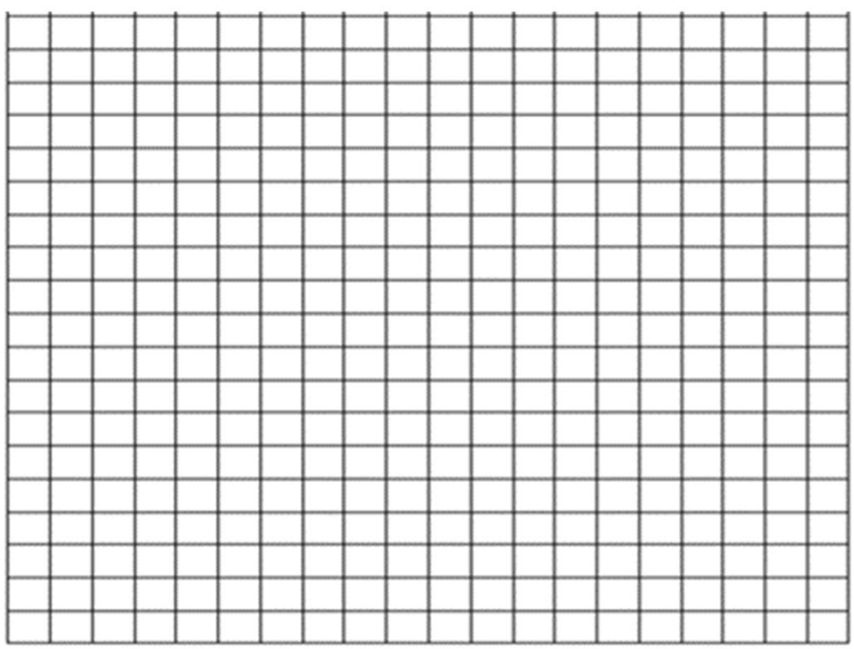
7. Using the calculator equation $y = 2.3x + 4.7$, approximate the total fat in a sandwich that has 28 grams of saturated fat.

8. The total fat in Kris & Kieras' Burger is 100 grams. Approximate the saturated fat in this burger using the equation for the trend line made by the calculator estimation.

9. What would you predict the total amount of fat to be in a sandwich that has 25 grams of saturated fat?

10. Is there some relationship between saturated fat and total fat? How would you describe it?

11. Make a graph of total fat to calories (total fat (x), calories (y)).



12. Draw the trend line for the number of calories depending on the total amount of fat. Why would you assume there would be a correlation?

13. How many data points are above your trend line? How many data points are below your trend line?

14. Pick two points on the line (or close to the line) to calculate the slope. Use $(14, 360)$ and $(34, 520)$.

15. Estimate the y-intercept from the trend line on the graph.

16. Find the equation of the line of best fit for total calories compared to total fat in each fast-food sandwich.

17. If a sandwich has 900 calories, what would you predict the amount of grams of total fat would be in the sandwich?

18. The calculator estimates the trend line to be $y = 9.3x + 216.32$. What are the slope and y-intercept of the trend line? Is it close to yours?

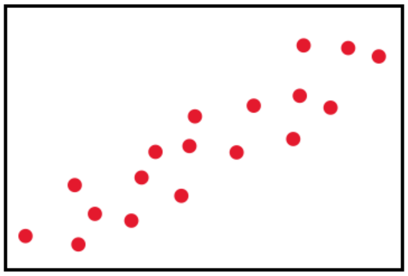
19. Estimate the total fat in a sandwich that has 900 calories using the equation for the trend line estimated by the calculator.

20. Kris and Kieras' burger has 54 grams of total fat and 780 total calories; does this fit with the trend?

Section 1.13 Coefficient of Correlation

Practice Problems 1.13

For Problem 1-3, use the given diagram to solve the problem.

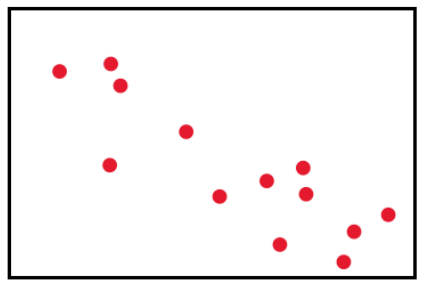


1. Draw a trend line for the data. Try to get as many data points above the line as below the line.

2. Which real-world situation could be modeled by the graph in Problem 1?
   a) As the time of day increases, the temperature increases
   b) The distance that seeds are blown as soon as a plant pod is opened
   c) The number of calves born each month of the year
   d) Interest earned in a bank account as money is deposited

3. Name the correlation modeled by the graph.
   a) Weak positive
   b) Weak negative
   c) Strong positive
   d) Strong negative

For Problem 4-6, use the given diagram to solve the problem.



4. Draw a trend line for the data. Try to get as many data points above the line as below the line.

5. Which real-world situation could be modeled by the graph in Problem 4?
a) Temperatures taken in the United States from December to February
b) The distance of a hummingbird from a feeder as it approaches to eat in a quarter of a minute
c) Days left in a week as the week progresses
d) Money in a bank when weekly withdrawals are made

6. Name the correlation modeled by the graph.
   a) Weak positive
   b) Weak negative
   c) Strong positive
   d) Strong negative

For Problem 7 and 8, tell which kind of correlation is given.

7. As more cars on a lot are sold, more money is deposited in the bank. Is there a positive, negative, or no correlation?

8. As more cars are sold, there are less cars on the lot until a new shipment comes in. Is there a positive, negative, or no correlation?

For Problem 9-14, use the given information to solve the problem.

Shoe sales for a company for the past five years are as follows (in billions of dollars):

Year	Sales
2011	10.2
2012	11.4
2013	11.3
2014	12.5
2015	14.9

9. Does there seem to be a trend in shoe sales for the last five years? What is it?

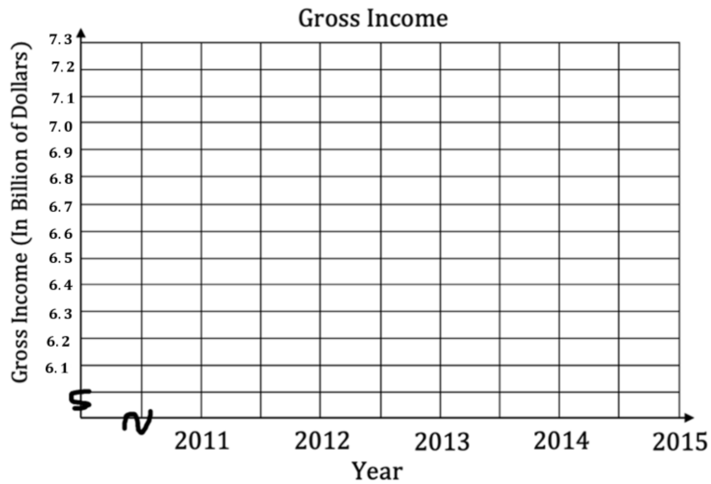
10. Draw a graph of the shoe sales for the five years as represented in the table. Draw the trend line.

The gross income for the shoe company for the past five years is in the following table. Again, the gross income is in billions of dollars. Use the following table for Problem 11 and 12.

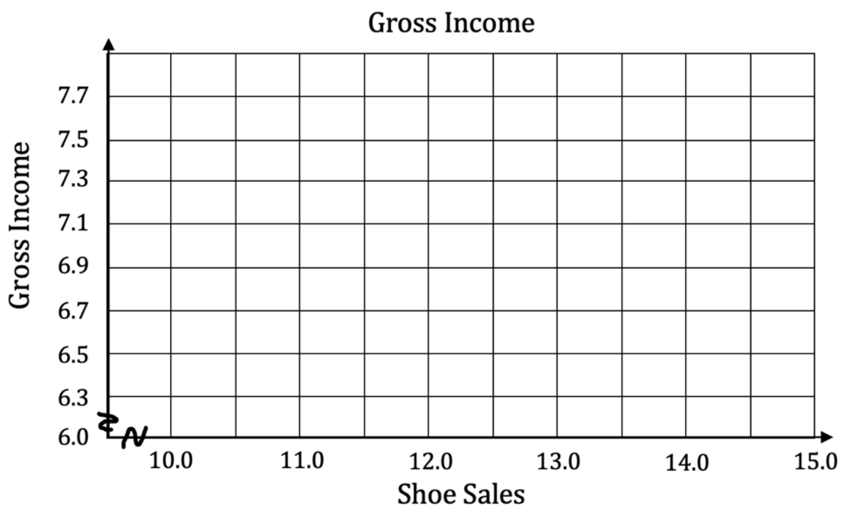
Year	Gross Income
2011	6.05
2012	6.85
2013	6.82
2014	6.9
2015	7.6

11. Does there seem to be a trend in gross income of the company for the past five years? What is it?

12. Draw the graph of the shoe sales and gross income of the shoe company and draw the trend line.

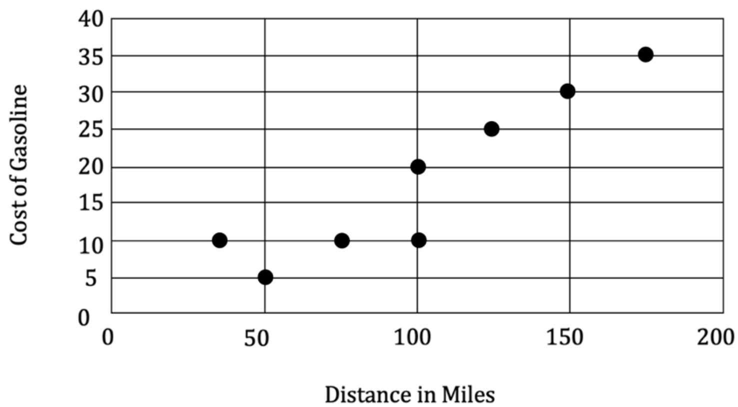


13. Draw the graph of the shoe sales and gross income. Does there seem to be a correlation; if so, what is it?



14. If there was a negative correlation between shoes sales and gross income, what does that mean?

For Problem 15-20, use the given graph to solve the problem.



15. Did the cars that traveled the same distance spend the same amount of money on gas?

16. Did any two cars spend the same amount on gas?

17. Does the graph give us information on miles per gallon used by the cars?

18. Is there a correlation between distance traveled and money spent on gas?

19. Is there a strong or weak correlation?

20. Draw the trend line for the data. How many data points are above the trend line, below the trend line, and on the trend line?

Section 1.14 Module Review

For Problem 1-3, tell whether or not the equation is a direct variation. If it is, identify the constant of variation (k).

1. $-4x = y$

2. $-3y = 2x$

3. $5x - 2y = 8$

For Problem 4-10, write a direct variation for the situation given and find the constant of variation. Use the constant of variation to find the missing value.

4. Let y vary directly with x. When $y = 25$, $x = 5$. What is the value of y when $x = 7$?

5. If someone weighs 140 lbs. on Earth, their weight on Mars is 5.32 lbs. If weight on Mars (y) varies directly with the weight on Earth (x), how much will 55 lbs. of space equipment from Earth weigh on Mars?

6. Complete the table using the equation relating Earth weight to Mars weight. Use the values up to 20 lbs. in 5 lbs. increments for Earth weight. Use interpolation to complete the table and find how much the weight of a radio on Mars is if it weighs 6 lbs. on Earth.

Earth Weight (x)	**Mars Weight (y)**

7. If a car weighs $2{,}000$ lbs. on Earth, how much does it weigh on Mars?

8. Why is it better to use the equation than use the table to find the solution for Problem 7?

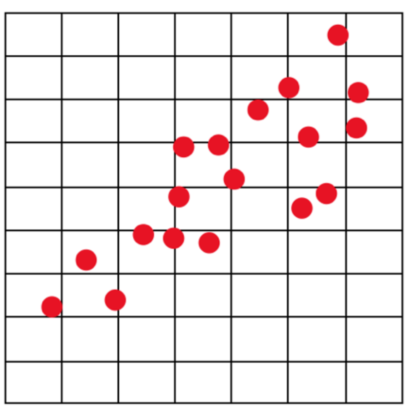
9. Natasha gets \$5.50 each week for allowance. She gives \$1.50 to church and saves the rest. Write an explicit formula (for how much Natasha saves each week) from the arithmetic sequence. How much will Natasha have saved after twelve weeks? When will she have \$88.00 saved (after how many weeks)?

10. Continue the arithmetic sequence for four more terms and write the recursive formula:

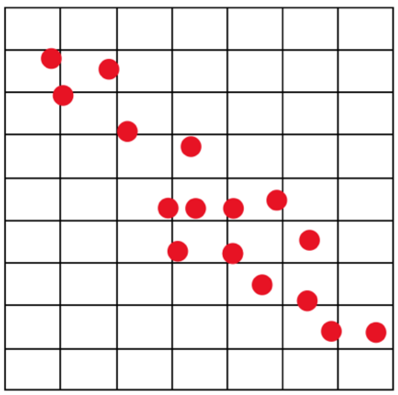
$$52, 49, 46, 43, 40, 37 \ldots$$

For Problem 11-13, tell whether or not each graph has a strong correlation, weak correlation, or no correlation. If there is a correlation, is it positive or negative?

11.



12.



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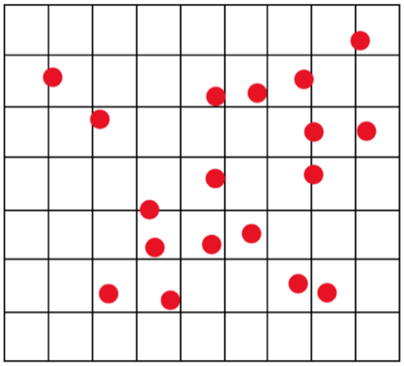


For Problem 14 and 15, given two points on a line use the point-slope form to find the standard form equation of the line.

14. $(0, 10)$ and $(4, -6)$

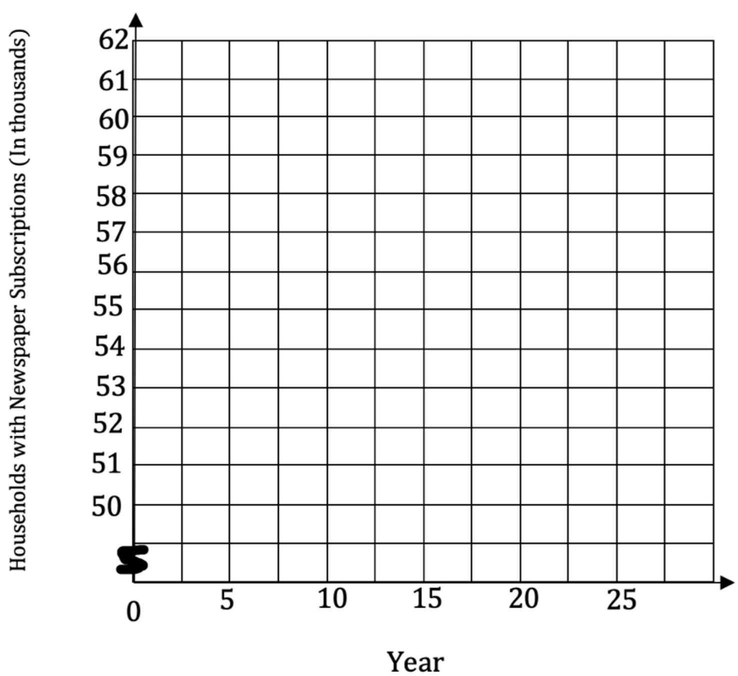
15. $(-3, -8)$ and $(4, 20)$

For Problem 16-18, use the given table to solve the problem.

Year	**Number of Households with Newspaper Subscriptions**	***In Thousands**
1985	62.1	
1990	62.2	
1995	58.2	
2000	55.3	
2005	53.2	
2010	51	

*The data given is in thousands, so 62.1 represents 62,100.

16. Graph the data and draw a trend line. Let 1985 be year 0, 1990 be year 5, 1995 be year 10, etc.



17. Determine the equation of the trend line of best fit. Use the points $(10, 58.2)$ and $(20, 53.2)$.

18. Is there a strong positive or strong negative correlation?

For Problem 19-25, solve the equation given.

19. $2(4a + 6) = -3(a - 1) + 31$

20. $x^2 - 45 = 36$

21. $2\sqrt{x - 4} + 6 = 22$

22. Solve for y in terms of x: $4x - 12y = 7$

23. Solve for l: $V = lwh$

24. Suzy's Sweet Shop sells imported chocolate for $18.00 a pound. The shop sells domestic chocolate for $9.00 a pound. How much of each would have to combine to get 20 pounds that sell for $15.00 a pound? Complete the table to solve the problem.

	Amount	**Cost Per Pound**	**Total Value**
Imported			
Domestic			
Final			

25. Her doctor told Miriam to use a rubbing alcohol solution that is 80% to cleanse a wound. Miriam had some rubbing alcohol in her medicine cabinet that was 95% rubbing alcohol. She ran to an all-night grocery store and bought some rubbing alcohol that was 70% rubbing alcohol. How much of each should Miriam mix to make 16 ounces of the 80% solution that she needs? Complete the table to solve the problem.

	Amount (Ounces)	**Percent of Solution**	**Total Solution**
95%			
70%			
Final			

Section 1.15 Module Test

For Problem 1-3, tell whether or not the equation is a direct variation. If it is, identify the constant of variation (k).

1. $\frac{1}{3}x = y$

2. $2x = -6y$

3. $y = -3x + 4.2$

4. If y varies directly with x, and $y = 40$, when $x = 4$, write a direct variation equation and find k (the constant of variation).

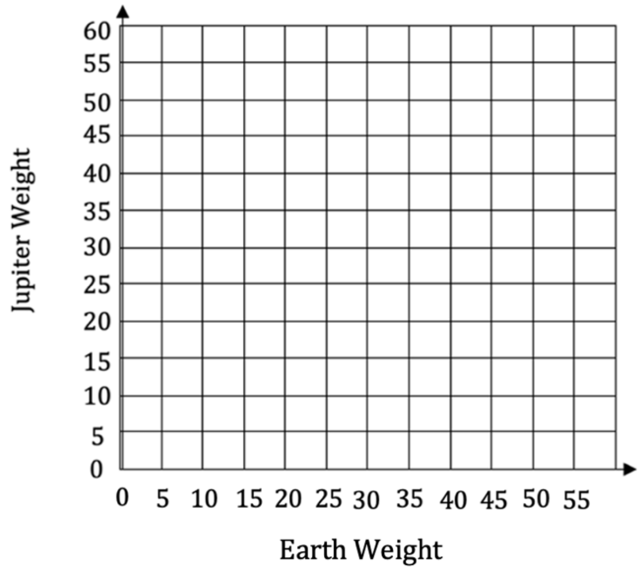
For Problem 5-8, use the given information and table to solve the problem.

Weight on Jupiter (y) varies directly with weight on Earth (x). Eighty-five pounds of radar equipment on Earth would weigh 199 pounds on Jupiter.

Earth Weight (x)	**Jupiter Weight (y)**
0	
5	
10	
15	
20	
25	
30	

5. What is the constant of variation (k)? Write the equation for the weight on Jupiter given the weight on earth and complete the table.

6. Draw the graph of the equation relating Jupiter weight to Earth weight (complete the table if you have not already). What does the constant of variation represent on the graph?



7. How could you use interpolation with the graph to find your dog's weight on Jupiter given he weighs 27 pounds on Earth?

8. If space equipment weighs 464 pounds on Jupiter, how much would it weigh on Earth. Why is it better to use the equation than a graph?

For Problem 9 and 10, solve the word problem given.

9. A work lunch card is \$70.00 for one month. After one lunch, \$66.50 remains on the card. After the second lunch, \$63.00 remains on the card. After the third lunch, \$59.50 remains on the card. Write an explicit formula for the value remaining on the card as an arithmetic sequence. How much will be left after 13 lunches are purchased?

10. Continue the arithmetic sequence for three more terms and write the recursive formula:

$$4, 9, 14, 19, 24, 29, \ldots$$

For Problem 11-13, tell whether the situation is likely to be closer to $r = -1.0$, $r = 0$, or $r = 1.0$.

11. The price of gas and the temperature outside

12. A person's shoe size and a person's height

13. A person's birthdate and the city in which they live

For Problem 14 and 15, given two points on a line, use the point-slope form to find the standard form equation of the line.

14. $(2, -6)$ and $(4, 10)$

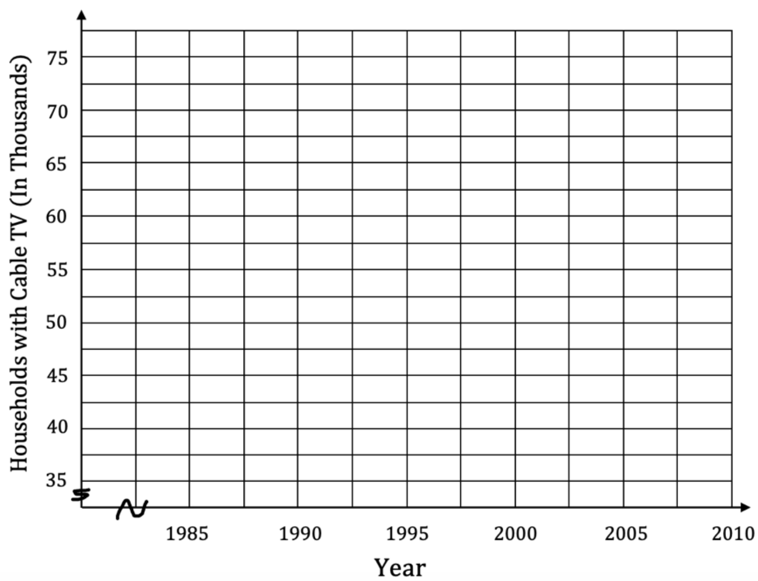
15. $(-2, -9)$ and $(-1, 9)$

For Problem 16 and 17, use the given table to solve the problem.

Year	**Number of Households with Cable Television***
1985	35
1990	49.3
1995	56.5
2000	62.7
2005	72.5
2010	82

*The data given is in thousands, so 15.3 is 15,300.

16. Graph the data from the table and draw the trend line.



17. Using the line of best fit (trend line), what year would you expect the number of households that have cable television to be 52,000? Use interpolation to solve the problem.

18. Using the line of best fit, what would you expect the number of households that have cable television to be in 2015? Use extrapolation to solve the problem.

For Problem 19-25, solve the equation given.

19. $8(x-4)+2x=4x-20$

20. $6+x^3=-21$

21. $(\sqrt{2x-4})^2=(\sqrt{5x+5})^2$

22. Solve for z: $x=\frac{2x-z}{4}$

23. Solve for x: $3x-3y=15$

24. Ethan and Byron left home at the same time, one heading east and one heading west. Ethan was traveling at a rate of 55 mph. Byron was traveling at a rate of 65 mph. How long will they travel before they are 600 miles apart?

25. Michael had a 3-hour drive to the nearest airport so he left at 9 AM traveling at a speed of 50 mph to be sure to make it on time for his 2 PM flight. One hour later, his brother Tom realized Michael had forgotten one of his suitcases so he took off for the airport, hoping to catch Michael in time. If Tom wants to reach Michael in 2 hours, how fast does he need to drive to catch up with him?