Module 3: Inverse Variation and Rational Expressions

Section 3.1 Inverse Variation Experiments

Practice Problems 3.1

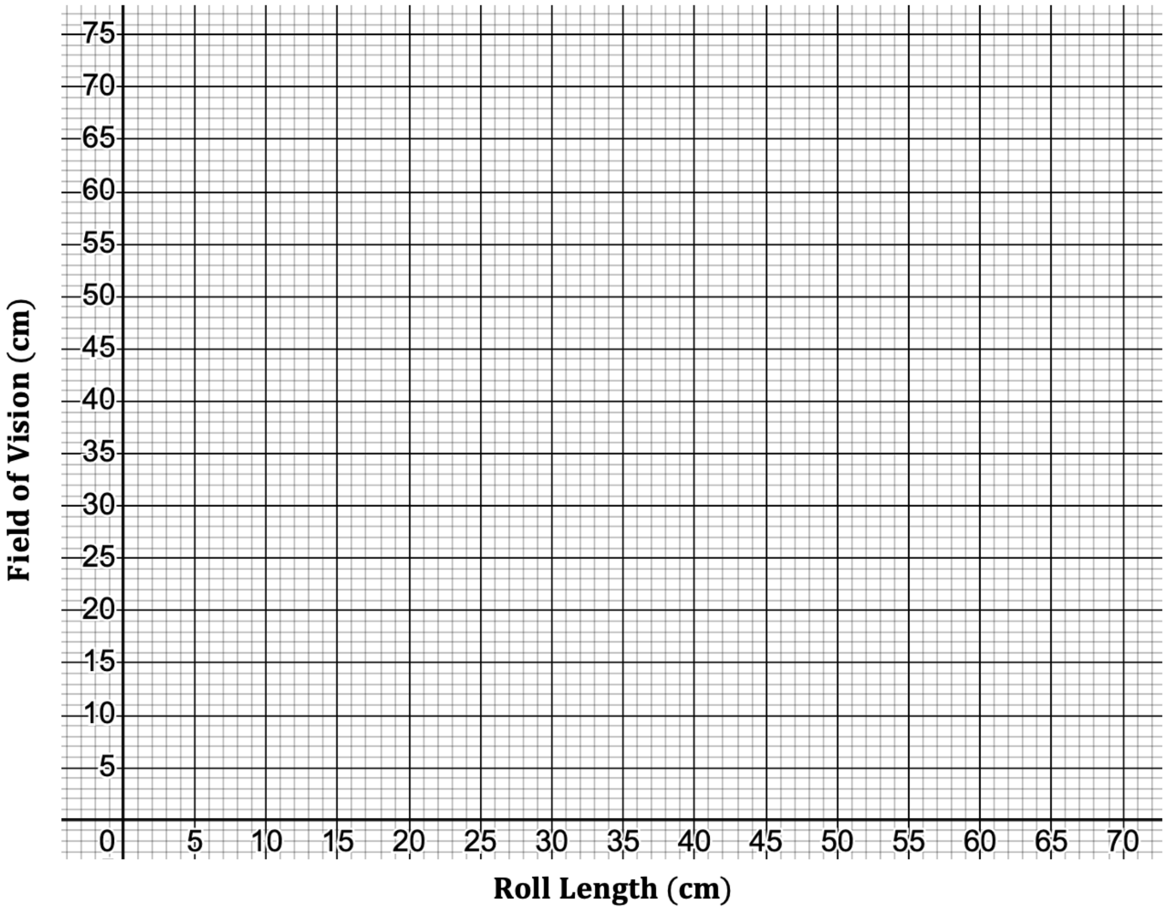
Experiment 3.1a

You will need the following materials:

- Poster board
- At least eight paper towel rolls with the same diameter but different lengths. Some of the paper towel rolls should be cut to be shorter than the original. Some of the paper towel rolls should be taped together to be longer than the original.
- A tape measure, meter stick, or ruler
- A helper

1. Tape the poster board on a wall so you can look directly at its center when 100 centimeters away from it.
2. Make a mark that is 100 centimeters back from the poster board by placing tape on the floor.
3. Measure the paper towel rolls and record their lengths in centimeters in the table.
4. Stand behind the 100-centimeter tape marking holding one of the rolls.
5. Hold the roll directly in front of one of your eyes and have your helper mark your field of vision on the right side and left side of the poster board (you may remember doing this in Section 1.5a; to see a diagram that can help, refer to Section 1.5c). Your partner will move their finger from the outside towards the center of the poster board until you tell them to stop and mark your field of vision.
6. Record the diameter of your field of vision in the table next to the roll length.
7. Repeat Steps 5 and 6 for all eight roll lengths.

Roll Length (cm)	**Field of Vision (cm)**	$x \cdot y$



a) What type of relationship is modeled by the length of the roll and the field of vision?

b) Between Field of Vision and Roll Length, which is the dependent variable, and which is the independent variable? Tell why.

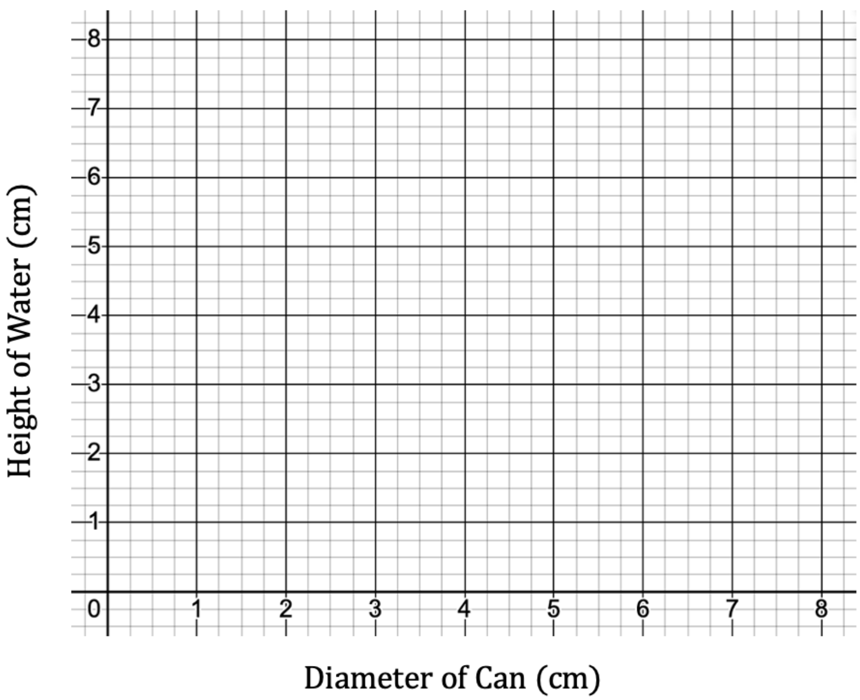
c) Because $y = \frac{k}{x}$, that means that $k = y \cdot x$ or $k = xy$ by the Commutative Property of Multiplication. Multiplying x and y gives us several different solutions for the constant of variation. Perhaps, as in the previous module, finding the average of all the constants of variation will give us a better estimation. Find k (the constant of variation).

d) Will the value of x and y ever be zero? Why or why not? What does this mean mathematically?

e) What might be an acceptable equation that could model the line?

Experiment 3.1b

a) Below is the table from the height of the water given the radius of the can from Looking Ahead Section 3.1. Complete the bottom of the table if the diameter were measured rather than the radius.
b) Draw the graph below using the data for the diameter of the can and the height of the water, which stays the same since it is the same can. Remember, the diameter is double the radius ($d = 2r$).



Radius of Can (cm)	**Height of Water (cm)**
4.2	1.5
3.6	1.7
2.5	2.8
2.0	4.2
Diameter of Can (cm)	**Height of Water (cm)**
	1.5
	1.7
	2.8
	4.2

c) In Section 3.1, you calculated the constant of variation: $k \approx 6.96$. Write the inverse variation equation using the radius of the can for x.

d) Use the equation to approximate the height of the water if the radius of the can is 3 cm. Does the height seem reasonable?

e) Now calculate the constant of variation, or k, using the diameter of the can. Multiply x and y. Take the average of the products to find k.

f) Write the inverse variation equation using the diameter of the can for x.

g) Use the equation to approximate the height of the water if the diameter of the can is 6 cm. Does the height seem reasonable?

Section 3.2 Direct and Inverse Variation Problems

Practice Problems 3.2

For Problem 1-4, use the given information to solve the problem.

In an inverse variation, the two quantities x and y are inversely proportional. If the two quantities x and y are inversely proportional when $x = 2$ and $y = 4$, find the missing value in the pair given.

1. $(2, y)$
2. $(6, y)$
3. $(x, 16)$
4. $(x, 8)$

For Problem 5-8, tell whether or not the data table has sets that are directly proportional, inversely proportional, or neither.

5.

$\boldsymbol{x}$	5	10	15	20	25	30
$\boldsymbol{y}$	2.5	5	7.5	10	12.5	15

6.

$\boldsymbol{x}$	5	10	15	20	25	30
$\boldsymbol{y}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$

7.

$\boldsymbol{x}$	-2	20	-10	2.5	-2.5
$\boldsymbol{y}$	-2.5	0.25	-0.5	2	10

8.

x	-3.1	2	4	-6	0.5
y	-6	1	4	-2.1	3.5

9. Find the direct or inverse variation equations in Problem 5-6.

For Problem 10 and 11, use the table and graph given to solve the problem.

10. On the same graph draw the graphs of the following tables:

$xy = 12$

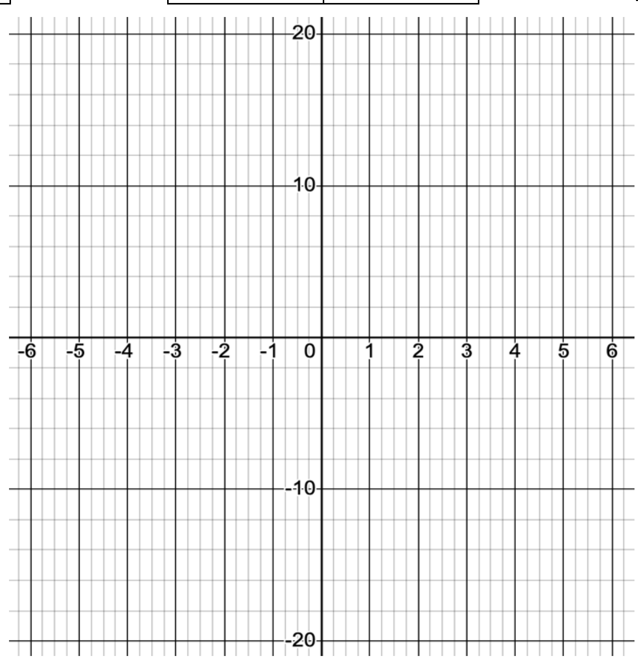
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$xy = 6$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

$xy = 3$

x	y
-3	
-2	
-1	
0	
1	
2	
3	

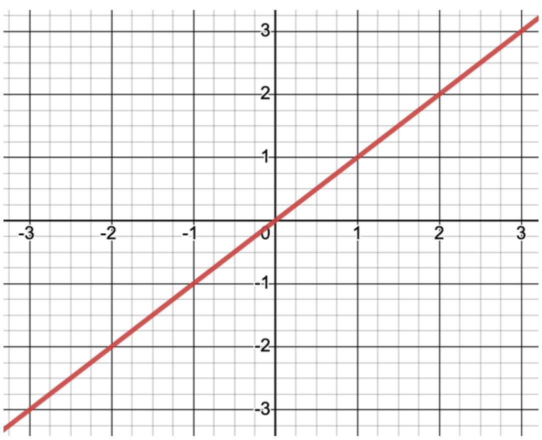


Why do you think the graphs have this shape?

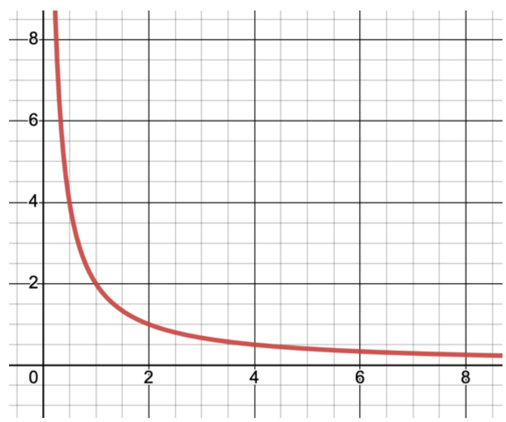
11. The points $(-6, 16)$ and $(2, y_2)$ are two points on the graph of an inverse variation. Find the value of y_2.

For Problem 12-15, tell whether the graph is directly proportional or inversely proportional.

12.



13.



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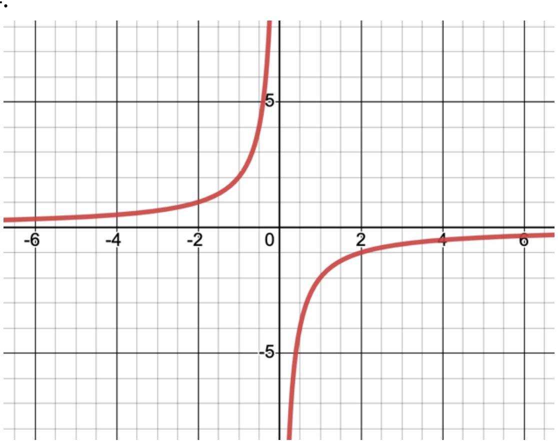
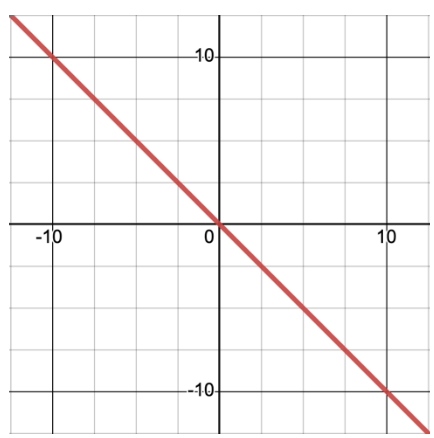


15.



For Problem 16 and 17, fill in the blanks.

16. y varies inversely with x

y is ____________________ proportional to x

the product of ________________ is constant

17. y varies directly with x

y is ____________________ proportional to x

the quotient of ________________is constant

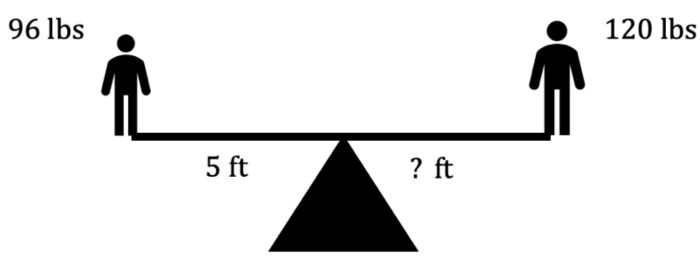
For Problem 18 and 19, find the constant of variation (k) for the inverse variations and write the inverse variation equations.

18. $y = 14$ $x = 4$

19. $y = 22$ $x = 3.4$

For Problem 20, use the given information to solve the problem.
A weight and distance vary inversely on a level with a fulcrum. A 96-pound person is on a seesaw 5 feet from the fulcrum.

20. How far does a 120-pound person have to sit from the fulcrum to balance it? Use $w_1 d_1 = w_2 d_2$.



Section 3.3 Inverse Variation in the Real World

Practice Problems 3.3

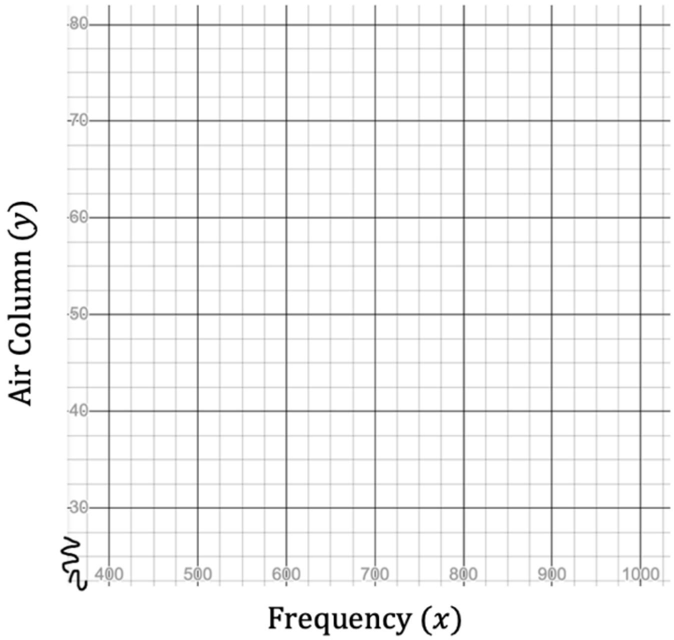
For Problem 1-10, use the given information and table to solve the problem.

Below are the results of a tuning fork experiment. Takiya and Darvis used tuning forks of different frequencies that corresponded with the frequencies of musical notes and measured the air columns during the experiment. After the experiment was finished, they realized they forgot to record two of the air column lengths.

Note	**Frequency (f)(Hz)**	**Air Column (cm)**	**Wavelength (λ)(cm)**
C_5	523.25	71.1	62.2
D_5	587.33	63.4	58.7
E_5	659.26		52.3
F_5	698.46	53.3	49.4
G_5	783.99	47.5	
A_5	880	42.3	39.2
B_5	987.77		34.9

1. With the formula for speed of sound in air, $v = f\lambda$, use two notes from above to calculate velocity. How close is it to the speed of sound in air which is $v = 343\frac{m}{s}$? (Remember, 100 cm $= 1$m.) Hertz (Hz) is the number of periods or cycles per second.

2. Make a graph with x representing frequency and y representing the air column. Is the relationship a direct variation or inverse variation?



3. Find k for C_5, D_5, F_5, G_5, and A_5. Calculate the average of k. Write an equation for the relationship between the frequency of the note and the air column.

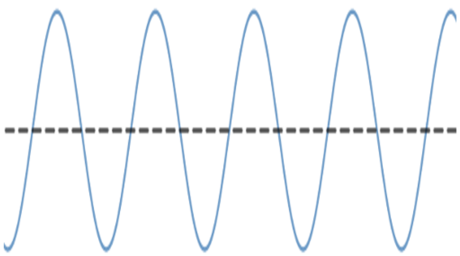
4. Using the equation, find the air column length for note E_5.

5. Using the equation, find the air column length for note B_5.

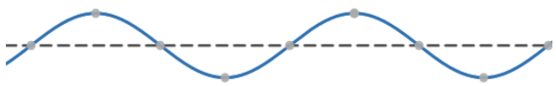
6. Looking at the air column lengths, do the solutions for E_5 and B_5 seem logical?

7. Fill in the blanks using the information from the graph and table above: Because there is an inverse relationship, as the frequencies get higher, the wavelengths get shorter. As the ___________________________ increases, the wavelength __________________________.

8. True or False: The graphs below represent the situation in Problem 7.



High Frequency - Short Wavelength



Low Frequency - Long Wavelength

9. Using the equation $v = \lambda f$ in which $v = 343$ m/s and $f = 783.99$ Hz for G_5, the wavelength is:

$783.99\lambda = 343 \ldots \lambda = \frac{343}{783.99} \ldots \lambda = 0.44$. Why is this solution not reasonable?

10. Correct the wavelength for Problem 9 so there is a reasonable solution given the table.

Section 3.4 Graphs of Inverse Variation

Practice Problems 3.4

For Problem 1 and 2, solve the word problem given.

1. How is the graph of $y = \frac{1}{x+2}$ similar to the graph of $y = \frac{1}{x}$?

2. How is the graph of $y = \frac{1}{x+2}$ different from the graph of $y = \frac{1}{x}$?

For Problem 3-5, use the given information to solve the problem.

A group of elderly residents at the retirement village decides to go to the Audubon Society for the day. The fee is $\$125.00$ for the group, which will be divided evenly by the total number of residents that attend.

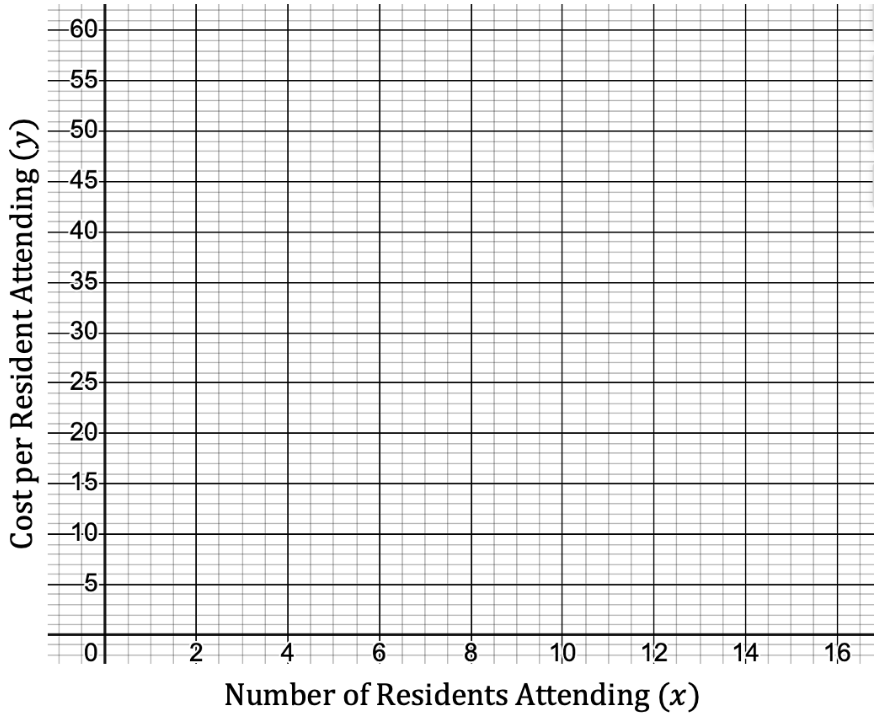
3. Write the equation that gives the cost per person going to the Audubon Society (y) as a function of the number of residents attending the Audubon Society (x).

4. If more and more residents go on the trip, how does that affect the cost for each resident?

5. If less and less residents go on the trip, how would that affect the cost for each resident?

For Problem 6-8, use the table and graph below to solve the problem.

x	y
16	
14	
12	
10	
8	
6	
4	
2	
0	



6. Complete the table and graph the function.

7. Why are there only positive values for x and why is the graph only in the first quadrant?

8. What type of relationship exists between the number of people attending and the cost of the trip?

For Problem 9 and 10, complete the table given.

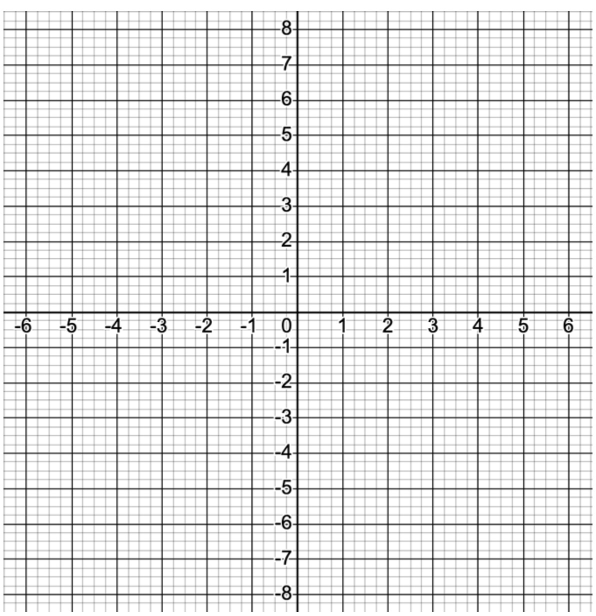
9. $y = \frac{2}{x}$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

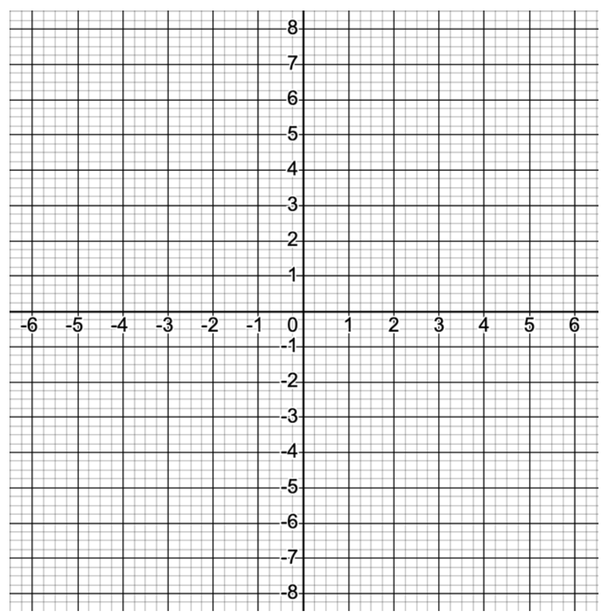
10. $y = \frac{1}{x-3}$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	

11. Graph the coordinates from Problem 9.

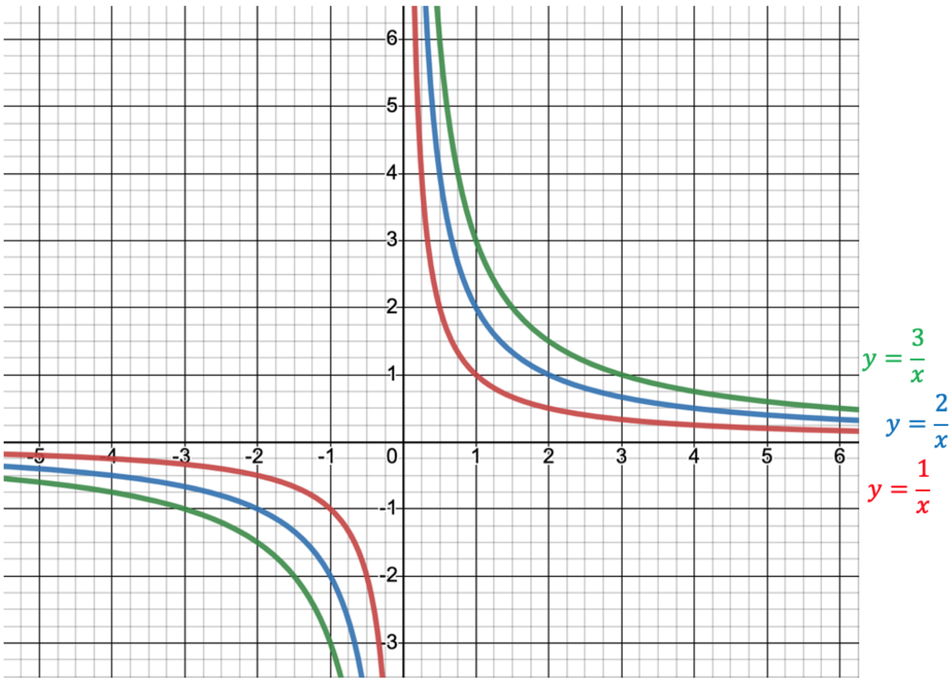


12. Graph the coordinates from Problem 10.

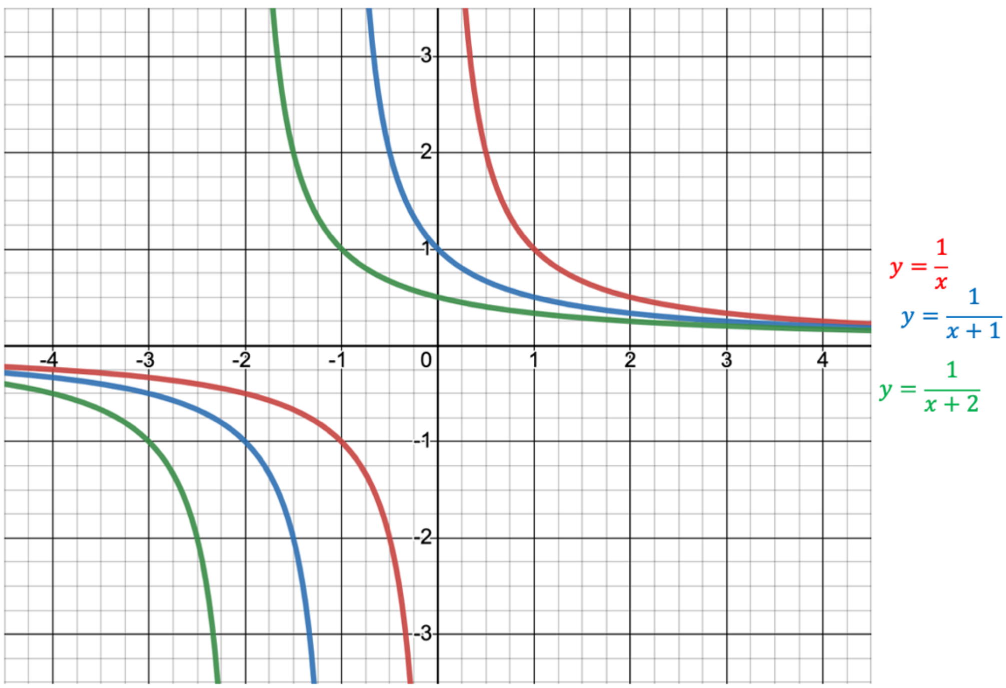


For Problem 13-16, use the graph given to solve the problem.

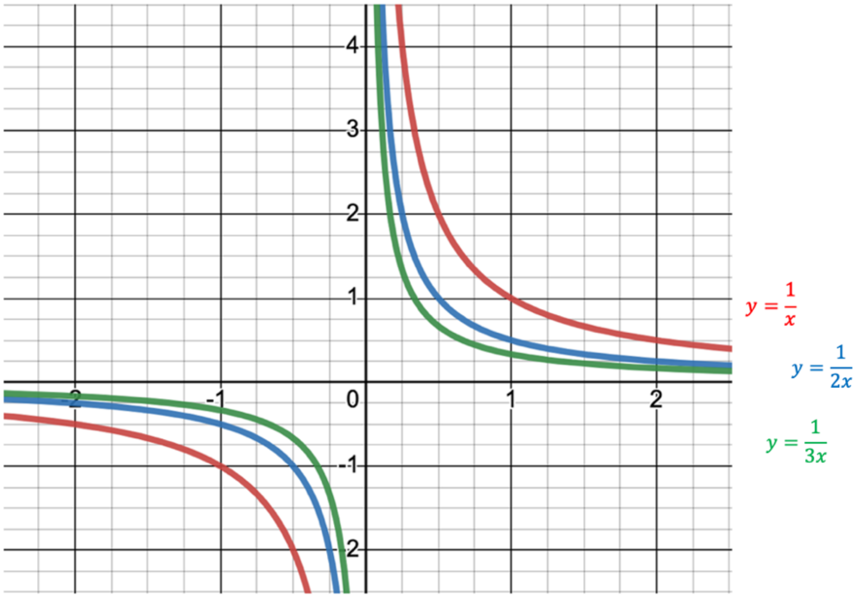
13. Below are the graphs of $y = \frac{1}{x}$, $y = \frac{2}{x}$, and $y = \frac{3}{x}$. What happens to the graph as k increases?



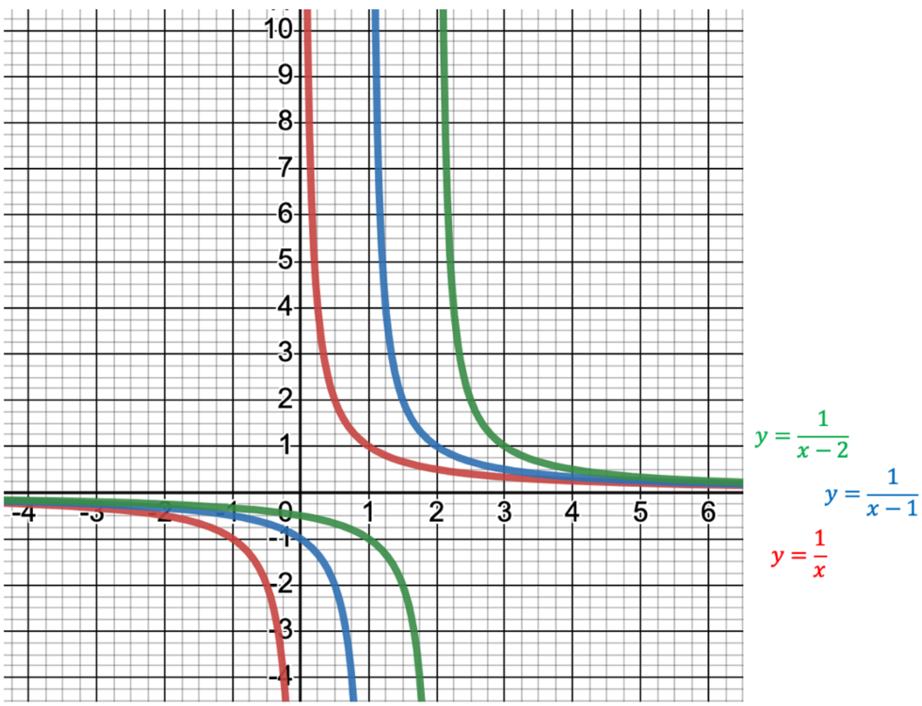
14. Below are the graphs of $y = \frac{1}{x}$, $y = \frac{1}{x+1}$, and $y = \frac{1}{x+2}$. What appears to happen each time 1 is added to x in the denominator?



15. Below are the graphs of $y = \frac{1}{x}$, $y = \frac{1}{2x}$, and $y = \frac{1}{3x}$. What happens to the graph as the coefficient of x in the denominator increases?



16. Below are the graphs of $y = \frac{1}{x}$, $y = \frac{1}{x-1}$, and $y = \frac{1}{x-2}$. What appears to happen each time 1 is subtracted from x in the denominator?



For Problem 17-20, fill in the blanks.

17. Increasing k in the equation $y = \frac{k}{x}$ vertically ______________________________ the graph.

18. Decreasing k in the equation $y = \frac{k}{x}$ vertically _______________________________ the graph.

19. In the graph of $y = \frac{1}{x-4}$, the graph moves ____________________ four spaces from the parent function $y = \frac{1}{x}$.

20. In the graph of $y = \frac{1}{x+4}$ the graph moves ______________________ four spaces from the parent function $y = \frac{1}{x}$.

Section 3.5 Horizontal and Vertical Asymptotes

Practice Problems 3.5

For Problem 1, use the given information to solve the problem.

In the equation $y = \frac{125}{x} + 6$, the horizontal asymptote is $y = \$6$.

1. Where did this horizontal asymptote come from?

For Problem 2-4, find the horizontal and vertical asymptotes in the equation given.

2. $\frac{1}{x+1} = y$

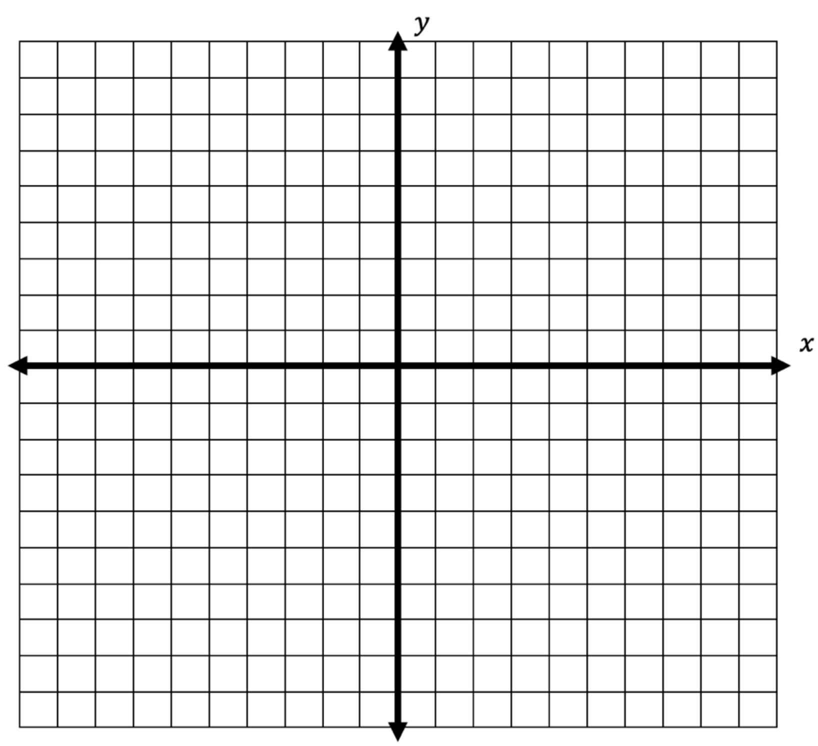
3. $\frac{1}{x} - 3 = y$

4. $\frac{1}{x+4} + 2 = y$

For Problem 5-7, complete the table and graph to see if your asymptotes from Problem 2-4 are correct.

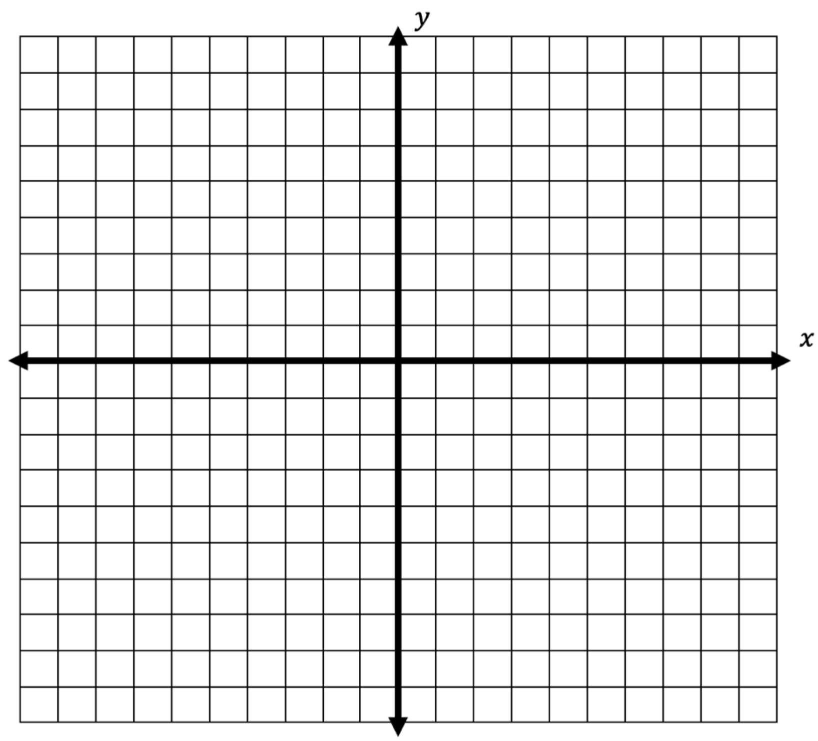
5. For Problem 2: $\frac{1}{x+1} = y$

x	y
-5	
-3	
-1	
0	
1	
3	
5	



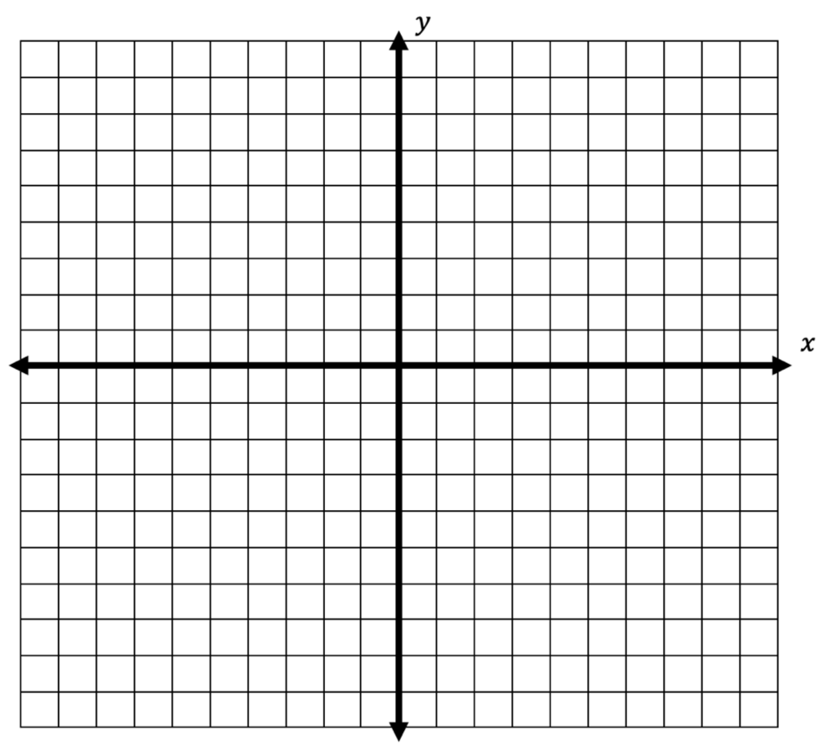
6. For Problem 3: $\frac{1}{x} - 3 = y$

x	y
-5	
-3	
-1	
0	
1	
3	
5	



7. For Problem 4: $\frac{1}{x+4} + 2 = y$

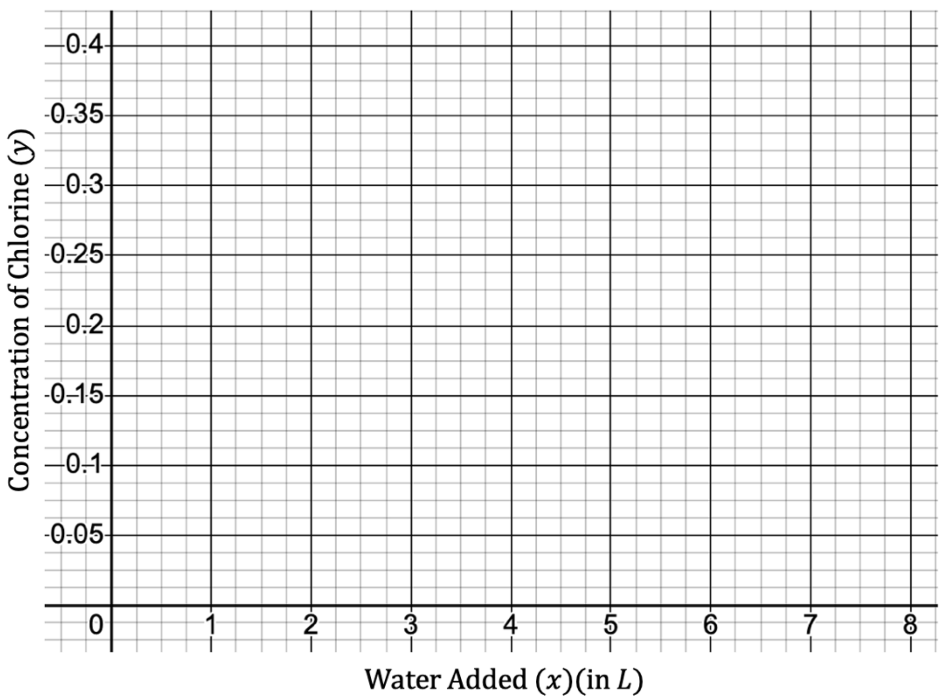
x	y
-10	
-8	
-6	
-4	
-2	
0	
2	



For Problem 8, use the given information to solve the problem.
A chlorine solution is made from chlorine and water. A bottle contains 1 liter of a 30% chlorine solution. This means the chlorine solution is 30% (0.3) of the whole solution.

8. a) Keep adding 1 liter of water to the solution. The 30% solution stays the same as you add water, but the amount of whole solution (water + chlorine) increases. Each time you add water, calculate the concentration of chlorine as the ratio of amount of chlorine to the whole solution (for example, $\frac{0.3}{1} = 1$, $\frac{0.3}{2} = 0.15$, $\frac{0.3}{3} = 0.1$). Complete the table for the rest below and draw the graph letting x be water added and y be concentration of chlorine.

Amount of Chlorine	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Amount of Water Added $(\boldsymbol{x})$(in $\boldsymbol{L}$)	0	1	2	3	4	5	6	7
Whole Solution (in $\boldsymbol{L}$)	1	2	3	4	5	6	7	8
Concentration of Chlorine $(\boldsymbol{y})$	0.3	0.15	0.1					



b) What happens to the concentration of chlorine as water is added to the solution? What kind of relationship is this?

$$\text{Concentration of Chlorine} = \frac{\text{Constant Amount of Chlorine}}{\text{Starting Amount of Solution} + \text{Added Amount of Water}}$$

c) Using the formula above, substitute in x and y and the constants (corresponding numbers) to write the equation that models the science experiment.

d) Using the equation above, how much water must you add to get a solution of 2% chlorine?

For Problem 9-12, name the vertical and horizontal asymptote in the equation given.

9. $y = \frac{1}{x-3}$

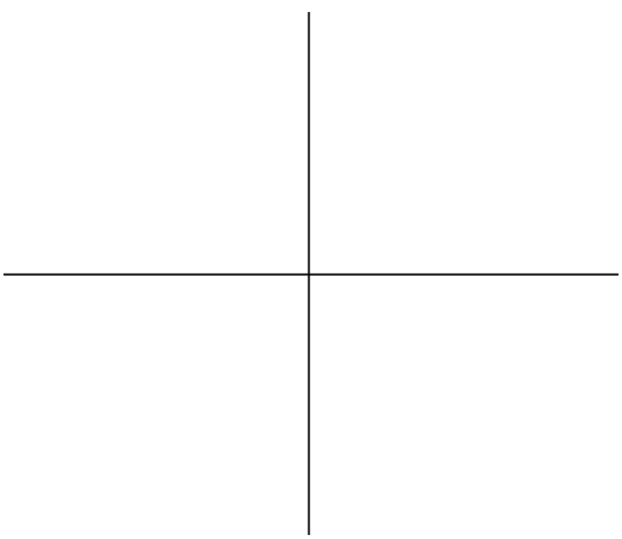
10. $y = \frac{1}{x}$

11. $y = \frac{1}{x+2} + 4$

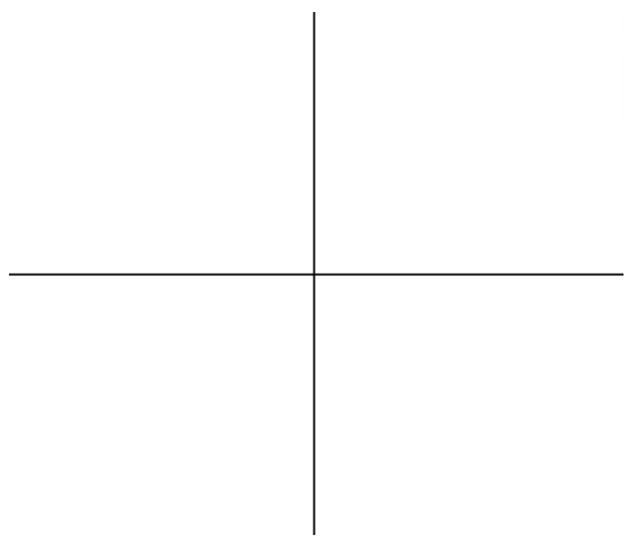
12. $y = \frac{1}{x-5} - 2$

For Problem 13-16, sketch the graphs of Problem 9-12 and use dashed lines to represent asymptotes.

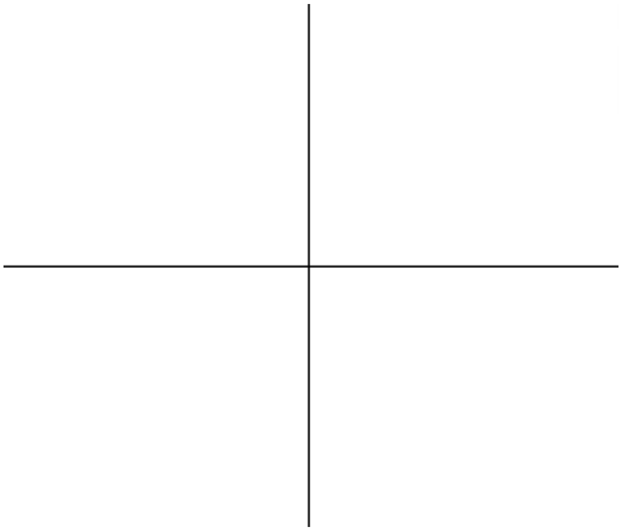
13. For Problem 9: $y = \frac{1}{x-3}$



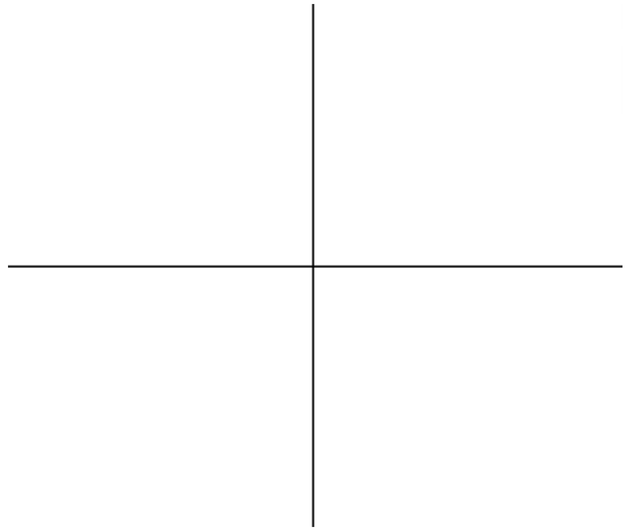
14. For Problem 10: $y = \frac{1}{x}$



15. For Problem 11: $y = \frac{1}{x+2} + 4$



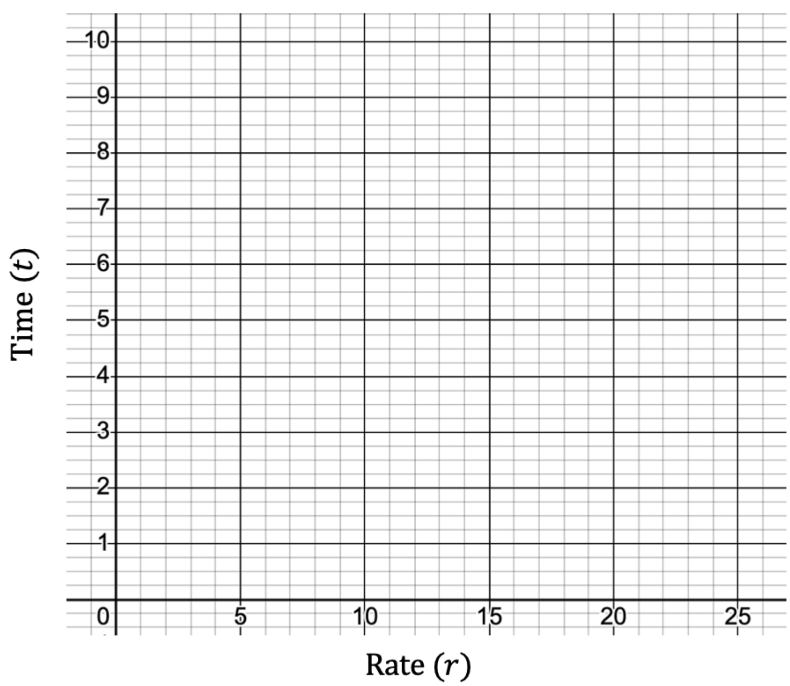
16. For Problem 12: $y = \frac{1}{x-5} - 2$



For Problem 17-20, use the given information to solve the problem.
The time you travel in a car varies inversely with your average speed. The equation $t = \frac{50}{r}$ is a model of the time (t) for a trip of 50 miles at different rates of speed (r).

17. Complete the table and draw the graph of the given model.

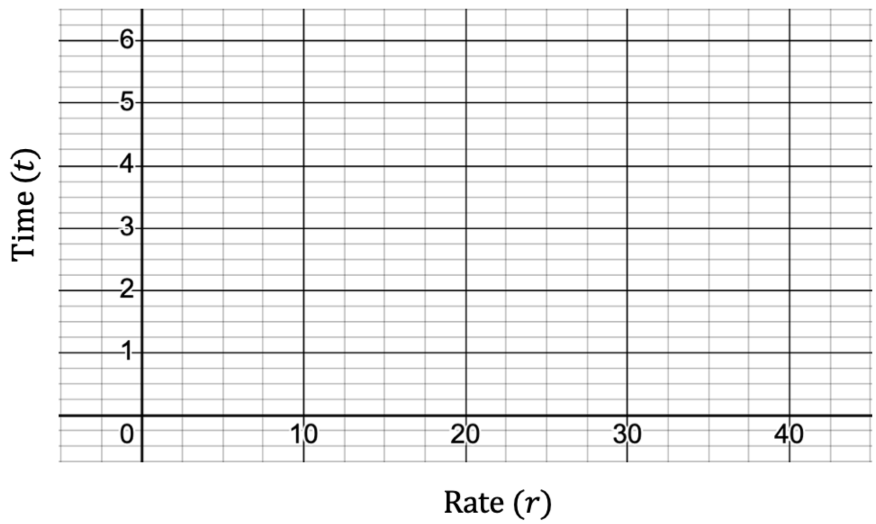
r	t
5	
10	
15	
20	
25	



18. What is the equation that models the time for a trip that travels 40 miles at different rates of speed? The time is in hours and the rate of speed is in miles per hour.

19. Complete the table and graph for Problem 18.

r	t
10	
20	
30	
40	



20. What are the two asymptotes in the equation in Problem 18. Why?

Section 3.6 Rational Expressions

Practice Problems 3.6

For Problem 1-6, evaluate the rational expression when $x = -3$, $y = 4$, and $z = 2$.

1. $\frac{x^2-2x+4}{x-3}$

2. $\frac{2xy}{6z+4x}$

3. $\frac{xyz-3x^2}{2xz+4yz}$

4. $\frac{xy}{xz}$

5. $\frac{x+y+z}{x+z}$

6. $\frac{x-z}{xy-xz}$

For Problem 7, use the given information to solve the problem.

Jo needs to clean his contact lenses. He asks Emmanuel to buy some saline (salt) solution for him at the pharmacy. Emmanuel brings home a bottle of 10% saline solution, but only 1% solution is needed to clean contacts.

7. How many liters of water should Jo add to the dilute the solution? The bottle is 1 liter of solution ($1L$).

For Problem 8-11, simplify the rational expression given.

8. $\frac{12x^3}{24y^2}$

9. $\frac{10x^4}{20x^6}$

10. $\frac{9x^5}{3x}$

11. $\frac{5x^3}{10x^5}$

For Problem 12-17, find the values that make the problem undefined.

12. $\frac{x^2+3x-4}{2x-8}$

13. $\frac{x^2-25}{x^2-4}$

14. $\frac{2y^2-3y+4}{y^3+8}$

15. $\frac{x^2+5x}{x}$

16. $\frac{y^2+2y-6}{2y-10}$

17. $\frac{10y}{2x^7}$

For Problem 18, Match the parts of the graph with the parts of the story. Assume the walking or running rates are constant.

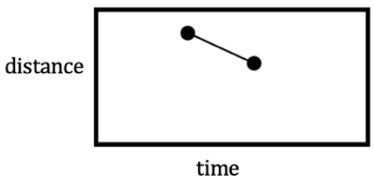
18. I. Red Riding Hood leaves her house to take cookies to Grandma.

II. Red Riding Hood sees a wolf in the forest so she throws some cookies to him and runs the rest of the way to Grandma's.

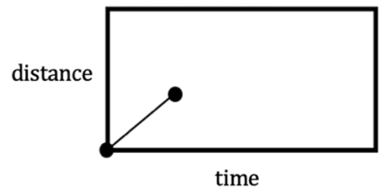
III. They have a nice visit and eat cookies and then Red Riding Hood starts walking home but she stops when she gets to the point where she saw the wolf.

IV. At that point, Red Riding has no cookies to throw so she runs all the way home.

a)



b)



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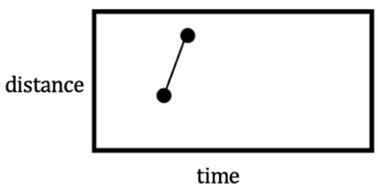

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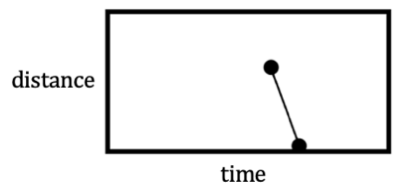


For Problem 19-22, tell whether the rational expression is sometimes true, always true, or never true given a, b, and c are real numbers.

19. $\frac{4c}{c} = 4$

20. $\frac{xy^2}{y^4} = \frac{x}{y}$

21. $\frac{a+3}{2a+6} = 2$

22. $\frac{n-3}{3-n} = -1$

For Problem 23, circle the expression that is equivalent to 1.

23. $\frac{x+y}{y-x}$ $\frac{x+y}{y+x}$ $\frac{x-y}{-y+x}$

Section 3.7 Simplifying Rational Expressions

Practice Problems 3.7

For Problem 1-4, simplify the rational expression.

1. $\frac{4xy^2}{10x^2y}$

2. $\frac{12-5t}{5}$

3. $\frac{-2mn}{18m^4n^4}$

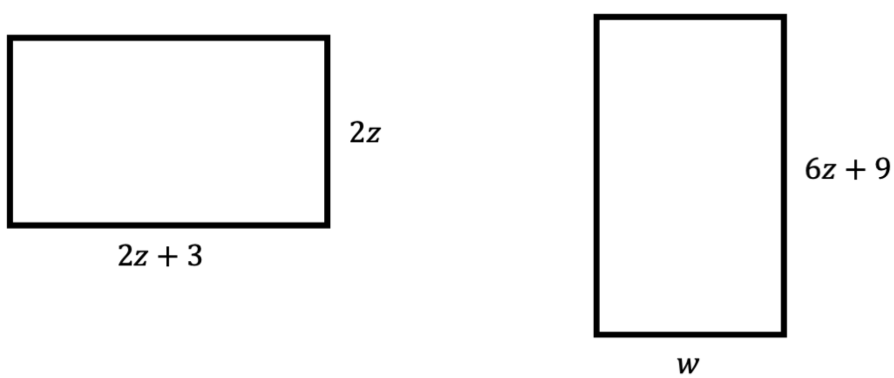
4. $\frac{5n-10}{-5}$

For Problem 5, use Problem 1 and 3 to solve the problem.

5. In Problem 1, $x \neq 0$ and $y \neq 0$. In Problem 3, $m \neq 0$ and $n \neq 0$. Why?

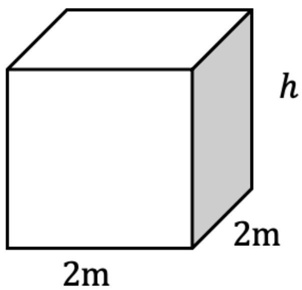
For Problem 6-8, solve the word problem given.

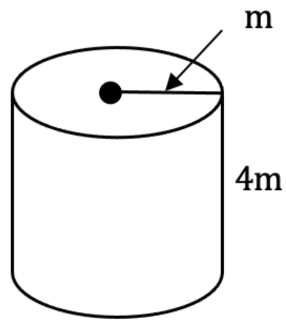
6. The two rectangles below have the same area.



Find the width (w) in terms of z so all the lengths and widths are in terms of the same variable ($A = lw$) for a rectangle.

7. Two wastebaskets hold the same amount of waste, which means they have the same volume. The volume of a cylinder is $v = \pi r^2 h$. The volume of a rectangular prism is $V = l \cdot w \cdot h$. The dimensions are given in terms of m. Find the height of the rectangular prism (h) wastebasket in terms of m. Let $r =$ radius.





8. If $m = 3$ in in Problem 7, will both wastebaskets or only one fit under a toilet paper roll holder that is 10 in above the floor?

For Problem 9-14, find common factors in the numerator and denominator to simplify the rational expression.

9. $\frac{3x+6}{x+2}$

10. $\frac{25+5t}{t+5}$

11. $\frac{4x-8}{-3x+6}$

12. $\frac{3}{9-3m^2}$

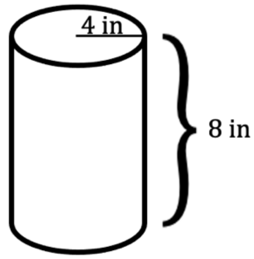
13. $\frac{16c^2}{4c^2-12c}$

14. $\frac{9-3m^2}{3}$

For Problem 15 and 16, find the volume of the cylinders given the formula $V = \pi r^2 h$ and given the radius and height of the cylinder.

15. $r = 6$ ft $h = 10$ ft

16.



For Problem 17-20, solve the word problem given.

17. Find the error in simplifying the rational expression given:

$$\frac{\overset{1}{\cancel{x^2}} - \overset{1}{2\cancel{x}}}{\underset{1}{\cancel{x^2}} - \underset{1}{\cancel{x}}} = \frac{1-1}{1-1} = \frac{0}{0}$$

18. Simplify the expression given:

$$\frac{x^3 - 2x}{3x^3 - x}$$

19. Find the error in the expression given:

$$\frac{4c}{2c+4}$$

$$\frac{\overset{2c}{\cancel{4c}}}{\underset{1}{\cancel{2c}} + 4}$$

$$\frac{2c}{5}$$

20. Simplify the expression given and check the expressions are equal by substituting $c = 4$:

$$\frac{4c}{2c+4}$$

Section 3.8 Adding Rational Expressions

Practice Problems 3.8

For Problem 1-4, find the least common denominator of the two rational expressions given.

1. $\frac{4x}{2n}$ and $\frac{-7}{10n^2}$

2. $\frac{xy^3}{xy}$ and $\frac{-6}{xy^2}$

3. $\frac{b}{7}$ and $\frac{b^2}{14}$

4. $\frac{7}{11x}$ and $\frac{13}{22y^3}$

For Problem 5-8, add the rational expressions given.

5. $\frac{12}{19xy^2}+\frac{4}{x^3y^3}$

6. $-\frac{5}{9}+\frac{2}{x^2y^2}$

7. $\frac{7d-3}{2xy}+\frac{3d+5}{4xy}$

8. $\frac{4}{ab}+\frac{3}{ab}+\frac{2}{ab^2}$

For Problem 9-11, let $f(x)=\frac{1}{x}$ and $g(x)=\frac{x+2}{x}$.

9. Add: $f(x)+g(x)$

10. Add: $g(x)+f(x)$

11. Subtract: $f(x)-g(x)$

For Problem 12 and 13, simplify the expressions given.

12. $\frac{4x}{7}+\frac{3x}{7}$

13. $\frac{-2x}{3}+\frac{x}{3}$

For Problem 14 and 15, add the rational expressions given.

14. $\frac{2}{3y^4}+\frac{3}{5y^2}$

15. $\frac{1}{20x}+\frac{14}{100x}$

For Problem 16 and 17, solve the word problem given.

16. What is the LCD (Least Common Denominator) of $\frac{2}{x+3}$ and $\frac{1}{x-2}$?

17. What is the LCD of $\frac{1}{2x^2}+\frac{1}{x-1}$?

For Problem 18 and 19, add the rational functions given.

18. $\frac{2}{x+3}+\frac{1}{x-2}$

19. $\frac{1}{2x^2}+\frac{1}{x-1}$

20. Which is greater: $\frac{1}{x^2}$ or $\frac{2}{x^2}$?

<u>Section 3.9 Subtracting Rational Expressions</u>

<u>Practice Problems 3.9</u>

For Problem 1-3, find the error(s) in the problem given.

1. $\frac{2}{x+1} + \frac{5}{x+2} = \frac{7}{2x+3}$

2. $\frac{-5}{x} + \frac{7}{y} = \frac{2}{xy}$

3. $8y + \frac{1}{y} = \frac{8y}{y}$

For Problem 4-10, subtract the rational expression given.

4. $\frac{x}{2x} - \frac{5y}{10xy}$

5. $\frac{6-x}{xy^2z} - \frac{3+z}{xy^2z}$

6. $\frac{4}{m} - \frac{3}{m+1}$

7. $\frac{1}{(a+1)} - \frac{3}{(a-1)}$

8. $\frac{2m+1}{m-1} - \frac{m+2}{m-1}$

9. $\frac{3p+2}{p+4} - \frac{p-6}{p+4}$

10. $\frac{27}{s^3} - \frac{9}{7s^2}$

For Problem 11-15, find the LCD of the two monomials given.

11. $\frac{3t}{2}$ and $\frac{4}{t}$

12. $\frac{5}{2n}$ and $\frac{-1}{6n}$

13. $\frac{2}{6x}$ and $\frac{-3}{x^2}$

14. $\frac{b}{9}$ and $\frac{b}{3}$

15. $\frac{1}{2}$ and $\frac{3}{4x}$

For Problem 16-20, subtract the rational expressions using the common denominator given the common denominator from Problem 11-15.

16. From Problem 11:

$$\frac{3t}{2} - \frac{4}{t}$$

17. From Problem 12:

$$\frac{5}{2n} - \frac{-1}{6n}$$

18. From Problem 13:

$$\frac{2}{6x} - \frac{-3}{x^2}$$

19. From Problem 14:

$$\frac{b}{9} - \frac{b}{3}$$

20. From Problem 15:

$$\frac{1}{2} - \frac{3}{4x}$$

Section 3.10 Multiplying Rational Expressions

Practice Problems 3.10

For Problem 1-4, solve the word problem given.

1. Rational expressions can be multiplied and divided using the same properties by which numerical fractions can be multiplied and divided. The rule follows:

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ given a, b, c, and d represent polynomials in which $b \neq 0$ and $d \neq 0$

Why is 0 not included?

In the factored product below, what are the common factors in the numerator and the denominator that can be divided out? Factor and simplify:

$$\frac{-4p}{(q-1)} \cdot \frac{(q+1)(q-1)}{p \cdot p \cdot p}$$

2. a) Multiply the rational expressions below without doing any dividing first. What do you get?

$$\frac{2m+4}{n^2m} \cdot \frac{3}{2m+4}$$

b) Now simplify that product by factoring first. What do you get?

c) Do you get the same solution for a) and b)? Which process is easier?

3. a) Remembering what you learned in *Pre-Algebra*, what is the greatest common factor of each term in $2n^2m^2 + 4n^2m$?

b) If you divide each term in $2m^2n^2 + 4n^2m$ by the greatest common factor, what two terms will be left inside parenthesis as the other factor?

c) Now multiply the common factor of a by what is in parenthesis from b. Do you get the binomial in b?

4. a) Can the rational expressions below be simplified?

$$\frac{m}{m+1} \cdot \frac{n}{n+4}$$

b) What values for m and n are not included?

For Problem 5-8, multiply and simplify the rational expression.

5. $\frac{2m}{n^2m} \cdot \frac{3nm}{8m}$

6. $\frac{m}{n} \cdot \frac{n}{m}$

7. $\frac{m^2}{n^2} \cdot \frac{n^3}{m}$

8. $\frac{(n+2)}{m^2} \cdot \frac{4m-m^2}{(n+2)}$

For Problem 9-12, solve the word problem given.

9. Andra's walking rate is two times as fast as Nick's walking rate. If Andra's walking rate is r, what is the expression for Nick's walking rate?

10. If Nick's walking rate is r, what is Andra's walking rate?

11. Andra walks two miles to the bank. On her walk home she walks 70% of her walking rate (r) as she is so tired. Write an expression for the total time Andra spent walking using the equation $t = \frac{d}{r}$ for the time there and the time back and then add them together.

12. If Andra's walking rate is 2.5 miles per hour in Problem 11, how long did she spend walking?

For Problem 13-20, multiply the rational expressions given.

13. $\frac{m-3}{m+4} \cdot \frac{m}{2m}$

14. $\frac{s^2 t}{(t+1)} \cdot \frac{(t+1)}{s^3 t^4}$

15. $\frac{8x^2-12x}{x} \cdot \frac{x^5}{2x-3}$

16. $\frac{7m-6}{m+4} \cdot \frac{2m}{-3m}$

17. $\frac{5x^3}{x^2} \cdot \frac{2x^4}{x}$

18. $\frac{3x}{x-2} \cdot \frac{3(x-2)}{x}$

19. $\frac{m-2}{m-4} \cdot \frac{2m-4}{m-2}$

20. $\frac{2t+1}{2t} \cdot 4t$

Section 3.11 Dividing Rational Expressions

Practice Problems

For Problem 1-4, solve the word problem given.

1. Find the error in the calculations below.

$$\frac{3s}{s+2} \div \frac{s-2}{(s+2)^2}$$

$$\frac{3s}{s+2} \cdot \frac{(s+2)^2}{s-2}$$

$$\frac{3s}{\cancel{s}+\cancel{2}} \cdot \frac{(\cancel{s}+\cancel{2})^2}{s-2}$$

$$\frac{3s}{1} \cdot \frac{1^2}{s-2}$$

$$\frac{3s}{s-2}$$

2. Find the correct solution to Problem 1. Leave the solution in factored form.

3. What are the values for m that are not included in the expressions below? (This would be where the problem is undefined.)

$$\frac{m^2-3m}{6m} \div \frac{3m-9}{m-6}$$

4. Solve Problem 3 and check your solution.

For Problem 5-10, simplify the rational expressions given.

5. $\frac{-10x^2}{5+5x} \div \frac{1}{5}$

6. $\frac{x-1}{x-3} \div \frac{x-1}{x-3}$

7. $\frac{7x+14}{-2} \div \frac{7x}{22x^2}$

8. $(6t - 12) \div \frac{t-2}{3}$

9. $\frac{4x+16}{x^3} \div (x + 4)$

10. $\frac{4x+1}{5x-10} \div \frac{2x+2}{15x-30}$

For Problem 11-14, answer true or false to whether the rational expressions given are equal or not.

11.

$$\frac{\frac{x+1}{2}}{\frac{x^2+1}{3}} = \frac{x+1}{2} \div \frac{x^2+1}{3}$$

12.

$$\frac{x-3}{\frac{x}{3x^2}} = \frac{x-3}{x} \div \frac{3x^2}{1}$$

13.

$$\frac{\frac{10}{x}}{-22x} = \frac{10}{1} \div \frac{x}{-22x}$$

14.

$$\frac{\frac{1}{1+x}}{\frac{1}{x-1}} = \frac{1}{x+1} \div \frac{x-1}{1}$$

For Problem 15-18, solve Problem 11-14.

15. Solve Problem 11.

16. Solve Problem 12.

17. Solve Problem 13.

18. Solve Problem 14.

For Problem 19 and 20, solve the problem given.

19. Find the error.

$$\frac{3x}{x-4} \div \frac{(x+2)^2}{x+2}$$

$$\frac{3\cancel{x}^1}{\cancel{x}-\cancel{4}_2} \cdot \frac{\cancel{x}+\cancel{2}^1}{(\cancel{x}+2)^2}$$

$$\frac{3}{2} \cdot \frac{1}{2^2}$$

$$\frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8}$$

20. Solve Problem 15 correctly.

Section 3.12 Simplifying Multi-Step Expressions

Practice Problems 3.12

Note: You may want to just do problems 1-10 for practice and then check them using the video as Problem 11-20 are very complex and use function notation and are more Algebra 2 type problems. If you would like a challenge then try all twenty problems.

For Problem 1-10, simplify the expressions given.

1. $\frac{b^2}{b}+\frac{b+2}{b^2}-\frac{3}{b^3}$

2. $\frac{c-1}{c+1}\cdot\frac{2}{c}+\frac{4}{c^2+c}$

3. $\frac{x-2}{4}\div\frac{2-x}{2}$

4. $\frac{3a^2}{3}\div\frac{3a^2}{3}+\frac{a-2}{a+2}$

5. $\frac{3}{2a^2}\cdot\frac{4}{a^3}\div\frac{12}{10a^5}$

6. $\frac{\frac{5z^2}{10}}{z^2+1}-\frac{5z}{z^2+1}$

7. $4+\frac{x-3}{x+1}$

8. $\frac{c^2}{ab}-\frac{a^2}{bc}\div\frac{a}{b}$

9. $\frac{12}{b}-\frac{4}{a}\div\frac{b}{c}$

10. $\frac{k}{2k^4}+\frac{5}{2k}\div\frac{2}{k}$

For Problem 11-20, let $g(x)=\frac{1}{x}$, $h(x)=\frac{1}{x+1}$, and $j(x)=\frac{2}{-x^3}$ and use them to evaluate the problem given.

11. $g(x)+h(x)$

12. $h(x)-j(x)$

13. $g(x) \cdot h(x) \cdot j(x)$

14. $g(x) + j(x) \div h(x)$

15. $h(x) + g(x)$

16. $g(x) + h(x)$

17. Does $g(x) + h(x)$ equal $h(x) + g(x)$?

18. $g(x) - j(x)$

19. $j(x) - g(x)$

20. Does $g(x) - j(x)$ equal $j(x) - g(x)$?

Section 3.13 Solving Rational Equations

Practice Problems 3.13

For Problem 1-4, solve the rational expressions given and check your solution.

1. $\frac{4}{p+1} = \frac{p+6}{p+1}$

2. $\frac{1}{3m} + \frac{1}{6m} = \frac{1}{2m}$

3. $\frac{5}{b} - \frac{3}{b} = 2$

4. $\frac{2t}{t-3} = \frac{t}{t-3} + 1$

For Problem 5-7, use the given information to solve the word problem given.

To solve a work problem, find the fraction of the job each person does in one unit of time. The sum of fractions for those working is the fraction of the job completed in one unit of time. For example, the tub in Example 2 of the Lesson Notes was filled in hours so 1 hour was the unit of time.

5. If the first pipe from Example 2 could fill the tub in 30 minutes and the second pipe could fill the tub in 15 minutes, the unit of time is 1 minute. How long would it take to fill the tub using both pipes?

6. What if the first pipe from Problem 5 filled the tub in $1\frac{1}{4}$ hours and the second pipe filled the tub in 20 minutes; how long would it take to fill the tub with both pipes?

7. Maximillian can polish the silver in 45 minutes and Meredith can polish the silver in 30 minutes. How long will it take them to polish the silver together?

For Problem 8-10, use the given information to solve the problem.

A 1-liter bottle of water contains 30% salt, which is 0.3 liters. Adding water increases the solution. The amount of salt stays the same.

The ratio for the concentration is $\frac{\text{salt}}{\text{whole solution}}$.

The equation for the amount of water to salt concentration is as follows:

$$C = \frac{0.3}{1 + W}$$

Concentration of Solution → C

Amount of Salt Stays the Same → 0.3

Starting Liter of Water → 1

Added Water → W

8. Suppose a salt solution has a concentration of 3.5%. Find the amount of water added to get this solution. (Hint: $3.5\% = 0.035$, which is C; solve for W)

9. If you add 2 liters of water to the salt solution, what will the concentration of salt be?

10. How much water is in the salt solution in Problem 9? Find the percent of water in the solution.

Section 3.14 Module Review

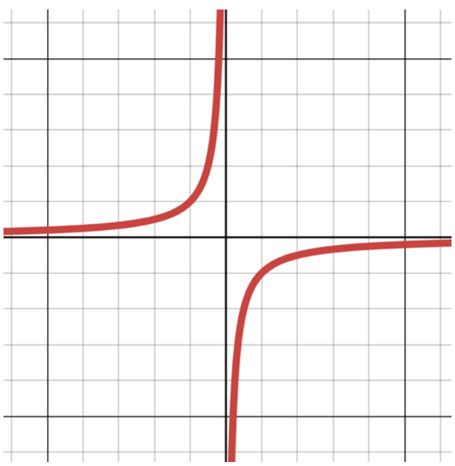
For Problem 1-4, identify the problem given as a direct variation or inverse variation and explain why.

1.

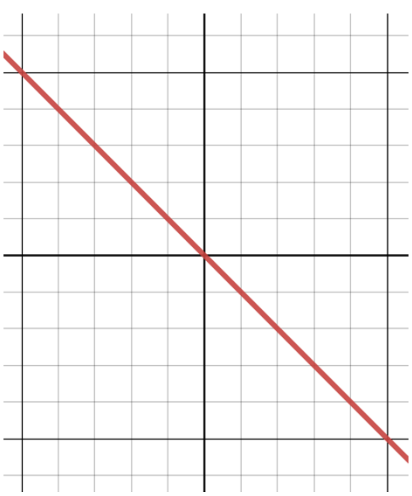
x	y
2	-6
4	-12
6	-18
8	-24
10	-30

2. The cost of supporting a child overseas is split among several family members.

3.



4.



For Problem 5-11, follow the instructions given to solve the problem.

5. Find the constant of variation (k) for Problem 1 and write the equation for the table.

6. True or False:

a) An inverse variation always has the asymptotes at $x = 0$ and $y = 0$. ____________________

b) Inverse variation equations have two branches that are symmetrical. ____________________

c) Inverse variation equations have asymptotes where the denominator is 0 and the equation is undefined.

____

7. The parent reciprocal function is $y = \frac{1}{x}$. Complete the table below for the transformation from the parent function for the rational functions and list any shifts or reflections. Find the parameters (h and k) to help determine the shifts.

Rational Function	**Parameters h and k**	**Description of Shift(s) or Flip(s)**
$y = \frac{1}{x+4} - 2$	$h = -4$ $k = -2$	Left 4 Down 2
$y = \frac{1}{x-1}$		
$y = \frac{-1}{x}$		
$y = \frac{1}{x+1} + 5$		

8. Suppose y varies inversely with x and $y = 3$ when $x = 7$. Find k, the constant of variation. What is the equation for the inverse variation?

9. If $(2, 7)$ is a pair of points of an inverse variation, find the y-value of a point on the same graph if $x = 7$.

10. Before attending camp, a runner can run around a lake in $1\frac{1}{2}$ hours at 8 miles/hour. At the end of the camp, she can run around the lake in 1 hour. What is her rate of speed at the end of camp if the time to travel around the lake varies inversely with her speed?

11. The time (t) to travel a distance of 8 miles at a rate of r is $t = \frac{8}{r}$. Is this an inverse variation or a direct variation?

For Problem 12 and 13, simplify the rational expressions given.

12. $\frac{2m^3+12m}{10m}$

13. $\frac{t-4}{8-2t}$

For Problem 14-17, add or subtract the rational expressions given.

14. $\frac{17}{x+3}+\frac{21}{x+3}$

15. $\frac{5}{2}+\frac{x}{x-2}$

16. $\frac{4}{9}-\frac{2x}{x+3}$

17. $\frac{5}{7x}-\frac{x}{7}$

For Problem 18 and 19, multiply or divide the rational expressions given.

18. $\frac{4n+12}{n^2-2n}\cdot\frac{3n}{6n+18}$

19. $\frac{a}{a+2}\div\frac{6a^2}{a+2}$

For Problem 20 and 21, solve the problem given.

20. Solve the rational equation: $\frac{1}{2} + \frac{3}{p} = \frac{7}{8}$

21. If 3 people could paint an entire fence given they each work 4 days, how many days would it take 5 people to paint the fence? Answer the questions below to find out.

a) Is this a direct variation or inverse variation?

b) What is the constant of variation?

c) What equation could help you solve the problem?

d) How many days does it take 5 people to paint the fence?

Section 3.15 Module Test

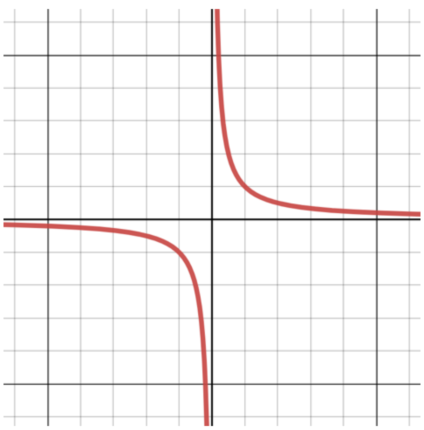
For Problem 1-4, identify the problem as a direct variation, inverse variation, or neither and explain why.

1.

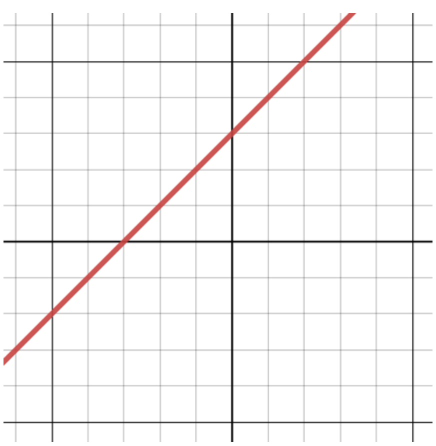
x	y
2	6
3	4
4	3
5	2.4
6	2

2. The total cost when you buy several goats for a family overseas at $25.00 per goat.

3.



4.



For Problem 5-11, follow the instructions given to solve the problem.

5. Identify the constant of variation (k) for each of the equations.

a) $y = -4x$

b) $y = \frac{1}{4}x$

c) $y = \frac{-2.5}{x}$

d) $y = 0.3x$

6. Fill in the blanks:

In a direct variation equation y ________________ directly with x. There is a value for k, which is the constant of ______________________. The line of a direct variation always passes through the origin so the coordinates of the x-intercept and y-intercept are always (___, ___).

7. Find the vertical and horizontal asymptotes for the rational functions below that are shifts of the parent reciprocal function $y = \frac{1}{x}$.

a) $y = \frac{1}{x-2} + 1.5$

b) $y = \frac{1}{x+0.4} - 6$

c) $y = \frac{1}{x} + 4$

d) $y = \frac{1}{x+7}$

8. Suppose y varies directly with x and $y = 10$ when $x = 5$. What is the equation for the direct variation?

9. If $(14, 7)$ is a pair of points on a direct variation equation, find the x-value of a point on the same graph if y is 6.

10. Buddy's family goes to visit relatives 2 hours from their house traveling 65 miles/hour. The travel time varies inversely with the speed of the car. If Buddy takes the back roads home traveling at 50 miles/hour, how long will it take him to get home?

11. The circumference (c) of a circle with the diameter (d) is $C = \pi d$. What is the constant of variation? If x represents the diameter and the y represents the circumference, what is the relationship between the two?

For Problem 12 and 13, simplify the expressions given.

12. $\frac{m-6}{4m-24}$

13. $\frac{z^3}{5z^2-z}$

For Problem 14-17, add or subtract the rational expressions given.

14. $\frac{4x}{x+1} - \frac{3x}{x+1}$

15. $\frac{3}{4} + \frac{2x}{x+2}$

16. $\frac{x+3}{15x} - \frac{x-2}{3x}$

17. $\frac{4}{m+2} + \frac{2m^2}{m}$

For Problem 18 and 19, multiply or divide the rational expressions given.

18. $\frac{2t^2}{t+3} \cdot \frac{t}{t-2}$

19. $\frac{p^2+3p-8}{p^3} \div \frac{p^2+3p-8}{p^2-2p}$

For Problem 20 and 21, solve the problem given.

20. Solve the rational expressions: $\frac{2}{n-4}+\frac{n}{3(n-4)}=\frac{6}{2(n-4)}$

21. Howard, Fran, Lance, Janet, and Trisha were trying to write story problems for an algebra book. Her four friends were busy for the weekend, so Trisha wrote 32 problems and shared them with her friends, but not equally. Howard made up 2 more of his own. Fran threw away 2 problems that were incorrect, Lance tripled the number he received, and Janet added a third more. When they got together to edit the problems, they each realized they had a different number of problems. How many problems did each friend get from Trisha?