Module 7 Cubic Equations

Section 7.1 The Parent Cubic Equation

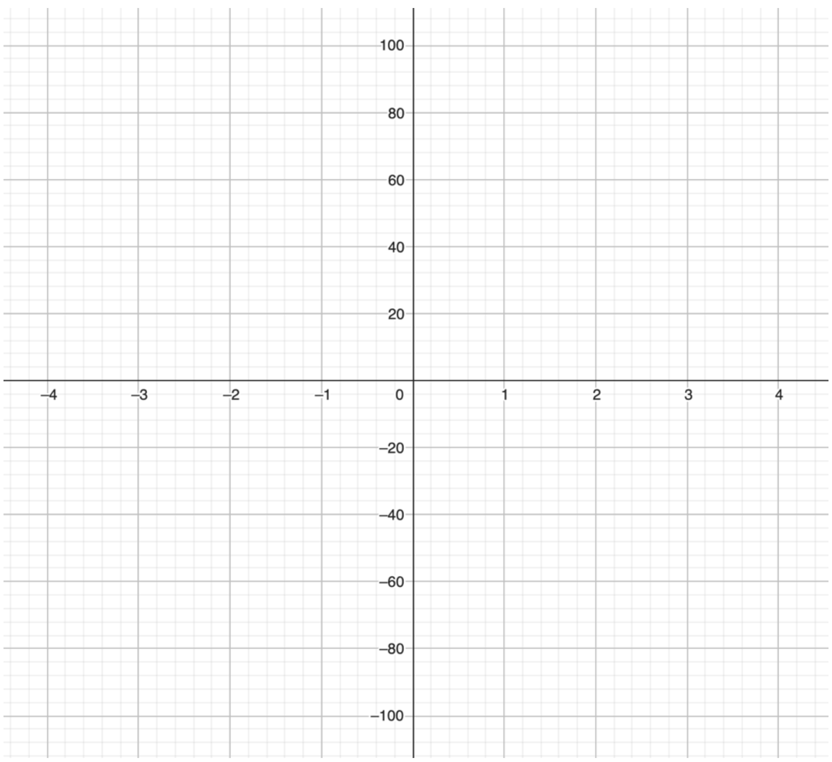
Practice Problems 7.1

For Problem 1-6, follow the instructions given to solve the problem.

1. Complete the table for $y = -x^4$. Remember, this means to multiply the value of x by itself four times and then take the opposite of the result. When $x = -2$, then $y = -(-2)^4 = -(-16) = 16$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

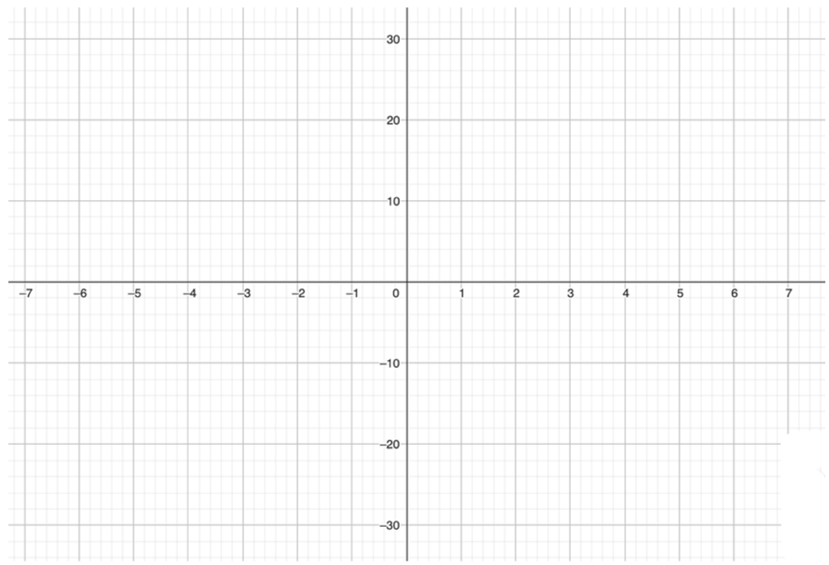
2. Graph the function $y = -x^4$.



3. How does the graph of $y = -x^4$ compare with the parent function $y = -x^2$?

4. Draw the graph of $y = -x^5$. How does it compare to $y = -x^3$?

x	y
-2	
-1	
0	
1	
2	



5. The equations $y = x^2$ and $y = x^4$ are called even functions and the equations $y = x^3$ and $y = x^5$ are called odd functions. Why are these equations called even functions or odd functions?

6. The coefficient in front of the quadratic and cubic term is a. What happens when a is not positive but negative?

For Problem 7-20, put the letter that matches the solution in the space given (below) to answer the question below: The farmer counted only 26 sheep but the talking sheep-dog said there were 30 sheep; why did the sheep-dog guess this number?

7. $x^5 \cdot y^4 \cdot x^3 \cdot y^2$ I

8. $-x^3 \cdot x^5 \cdot y^5 \cdot (-y^2)$ S

9 $(2x^3)(-6y)$ P

10. $(4xy^4)(-3y^7)$ E

11. $(6x^6)(3x^7)$ H

12. $(x^4y^3)(2x^3)(3x^3y^2)$ O

13. $(x^3) \cdot x^8$ U

14. $-6z^9 \cdot z^2$ C

15. $(4xy^8)(-3xy^5)$ N

16. $-2x^3y^3z^3 \cdot 5z^9 \cdot (-3y^3z^2)$ R

17. $(2x)(-3xy)(4xyz)$ D

18. $(-4abc)(a^2b)(2b^3c^2)$ T

19. $(3x^2)(-3yz)$ M

20. Rule: $x^m \cdot x^n = x$—— !

$\overline{x^8y^7}$ $\overline{x^8y^6}$ $\overline{-12x^2y^{13}}$ $\overline{-6z^{11}}$ $\overline{-12xy^{11}}$ $\overline{18x^{13}}$ $\overline{-12xy^{11}}$ $\overline{30x^3y^6z^{14}}$ $\overline{6x^{10}y^5}$ $\overline{x^{11}}$

$\overline{-12x^2y^{13}}$ $\overline{-24x^3y^2z}$ $\overline{-12xy^{11}}$ $\overline{-24x^3y^2z}$ $\overline{-8a^3b^5c^3}$ $\overline{18x^{13}}$ $\overline{-12xy^{11}}$ $\overline{-9xy^2}$

$\overline{x^{11}}$ $\overline{-12x^3y}$ $\overline{m+n}$

Section 7.2 Standard Form of the Cubic Equation

Practice Problems 7.2

For Problem 1-4, identify the values of a, b, c, and d in the cubic equation given.

1. $y = 5x^3 + 2.4x^2 + 0.5x$

2. $-4x^3 + 2x^2 + 6x - 3 = y$

3. $-2x^3 - 8 = y$

4. $22x^3 + 3x^2 - x + 1 = y$

For Problem 5-8, convert the factored cubic equation given to standard form.

5. $x(x+2)(x-2) = y$

6. $-2x(x+1)(x+1) = y$

7. $(x+1)(x+2)(x+3) = y$

8. $2x(x+1)(x-2) = y$

For Problem 9-20, simplify the division expression given.

9. $\frac{3^7}{3^5}$

10. $\frac{2^3}{2^5}$

11. $\frac{x^{19}}{x^9}$

12. $\frac{x^6y^3}{xy^2}$

13. $\frac{x^6y^8z}{y^4z}$

14. $\frac{5^{10}}{5^9}$

15. $\frac{5x^{10}}{5x^9}$

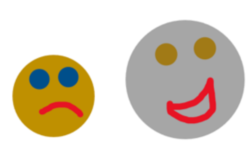
16. $\frac{x^9y^4z^2}{yx^3}$

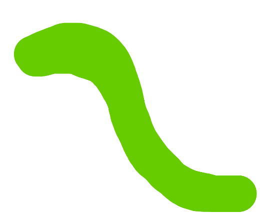
17. $\frac{-4a^6b}{16ab}$

18. $\frac{x^2y^3}{z^2x^4} \cdot \frac{z^5}{x^2}$

19. $\frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{x}$

20. Write the rule: $\frac{x^m}{x^n} = x$





Did you hear about the penny who would not roll down the hill with the nickel because it just didn't make cents!

Section 7.3 Revisiting the Zero-Product Property

Practice Problems 7.3

For Problem 1-6, find the zeroes of the factored cubic function given. Write the coordinate of the y-intercept.

1. $(x+2)(x-1)(x+4)=y$

2. $x(x+4)(x-3)=y$

3. $-2x(x+1)(x-1)=y$

4. $(2x-3)(x+1)(x-4)=y$

5. $y=3x(x-2)(x+1)$

6. $-2x(x-2)(3x+1)=y$

7. What does it mean that the ordered pairs are the x-intercepts? Where are these on the table of the equations? Where are they found on the graph of the equations?

For Problem 8-10, simplify the power to the power exponential given.

8. $[(2^2)^3]^4$

9. $(x^4 \cdot x^2)^6$

10. $(\frac{x^3}{y^4})^2$

For Problem 11-20, we are going to play a game called: "Who is the Real Dr. Math?" Answer each question given. The true statements give you hints about who the Real Dr. Math is.
The three possible identities for Dr. Math are described below:

<u>Dr. Jeom Etry</u>
- Born in Obtuseville
- Lives in a triangular house
- Drives an X-clusive Car

<u>Mearsure Ment</u>
- Born in Linearland
- Lives in an octagonal house
- Drives a Y-grid Mobile

<u>Al Gebra</u>
- Born in Extopia
- Lives in a square house
- Drives a Z-Gebra Cruiser

11. If $(x^4)^5$ is equal to x^{20}, then Dr. Math is born in Obtuseville.

12. If $(y^5)^4$ is equal to y^9, then Dr. Math drives a Z-Gebra Cruiser.

13. If $(x^2 \cdot x^4)^6 = x^{12}$, then Dr. Math lives in an octagonal house.

14. If $(\frac{x^2}{y^3})^2$ is equal to $\frac{x^4}{y^3}$, then Dr. Math lives in a square house.

15. If $(x^4 \cdot y^2 \cdot z^6)^3$ is equal to $x^{12}y^6z^{18}$, then Dr. Math drives an X-clusive Car.

16. If $(x^2y^2)^5$ is equal to x^7y^7, then Dr. Math is born in Extopia.

17. If $(\frac{3xy^2z^3}{4xyz})^2 = \frac{9x^2y^4z^6}{16xyz}$, then Dr. Math drives a Y-gird Mobile.

18. If $(a^5b^2)^4(-7a^3b^4)^2(ab)^3$ is equal to $-42a^9b^7$, then Dr. Math was born in Linearland.

19. If $(5a^2b)^3 \cdot [(ab^4)^2]^2$ is equal to $125a^{10}b^{19}$, then Dr. Math lives in a triangular house.

20. Who is Dr. Math?

Section 7.4 Graphing Cubic Equations

Practice Problems 7.4

For Problem 1-3, solve the word problem given.

1. Convert the equation $x(x+4)(x-3) = y$ to standard form.

2. Find the zeroes of $x(x+4)(x-3) = y$.

3. When you plug the zero values for x in Problem 2 in the standard form equation from Problem 1, what values should you get for y? Plug the zeroes into the equation and see if you get the values you should; why does it work?

For Problem 4-7, answer true or false given the equation $x(x+4)(x-3) = x^3 + x^2 - 12x = y$.

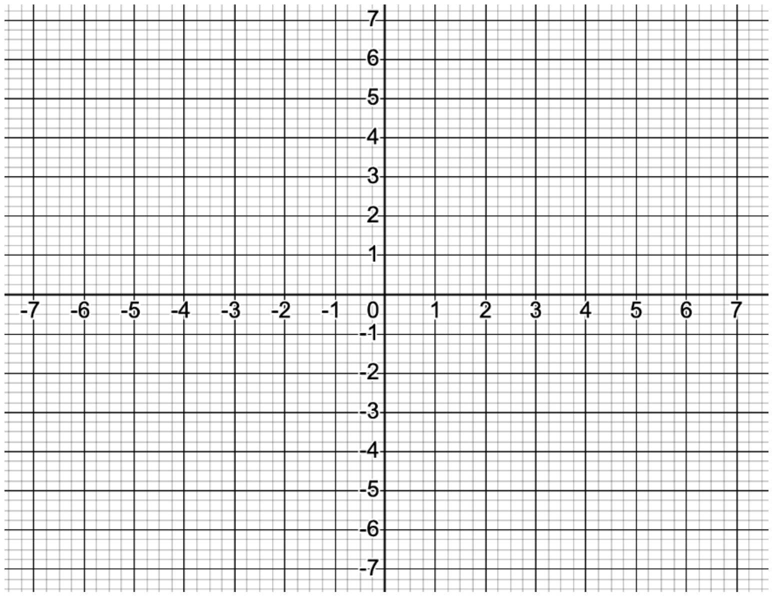
4. The equation $x^3 + x^2 - 12x = y$ has three real zeroes.

5. One of the zeroes of $x^3 + x^2 - 12x = y$ is $(4, 0)$.

6. The equation $x^3 + x^2 - 12x = y$ has two positive zeroes and one negative zero.

7. The graph of $x^3 + x^2 - 12x = y$ bounces at the negative zero.

8. Graph the cubic equation $x(x+4)(x-3)=y$.



For Problem 9-20, simplify the expression given so there are no negative exponents.

9. $3^5 \cdot 3^{-5}$

10. $y^7 \cdot y^0$

11. $\frac{x^4}{x^0}$

12. $\frac{x^{-4}}{x^0}$

13. 52^0

14. $a^0 \cdot a^3$

15. $\left(\frac{5s^2t^4u^0}{-9s^3ty^{22}}\right)^0$

16. $\frac{y^5}{y^8}$

17. $\frac{x^5}{x^{-2}}$

18. $\frac{x^{-2}}{x^5}$

19. $\frac{x^7 \cdot y^5}{x^{-2} \cdot y^3}$

20. $\frac{x^{-4} \cdot y^{-1}}{x^{-6} \cdot y^{-3}}$

Did you hear about the square who thought everything the circle said was pointless?!

Section 7.5 Finding the Cubic Equations from the Graph

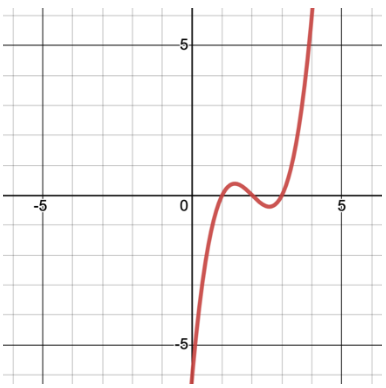
Practice Problems 7.5

For Problem 1-6, match the graph given to its equation and name its y-intercept. Let $a = 1$.

a) $(x-1)^2(x+3) = y$ b) $(x+3)(x-2)(x+1) = y$ c) $x(x+2)(x+3) = y$

d) $(x-1)(x-2)(x-3) = y$ e) $(x-2)(x+2)^2 = y$ f) $(x-1)^2(x+2) = y$

1.



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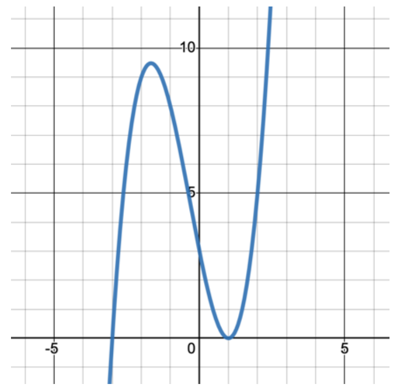

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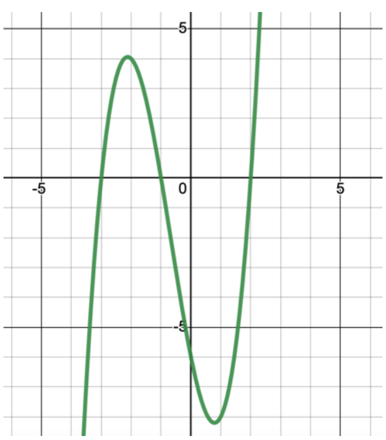

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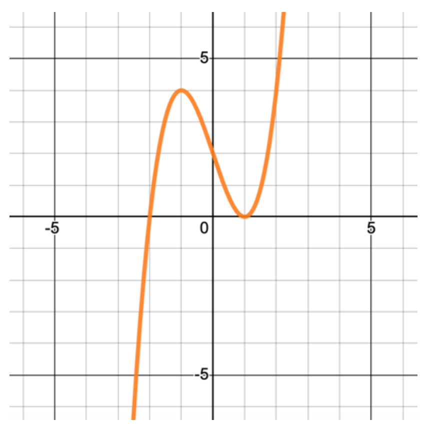
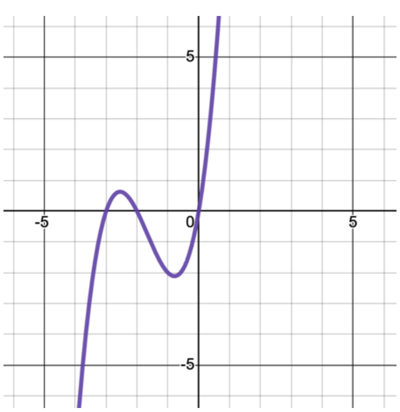
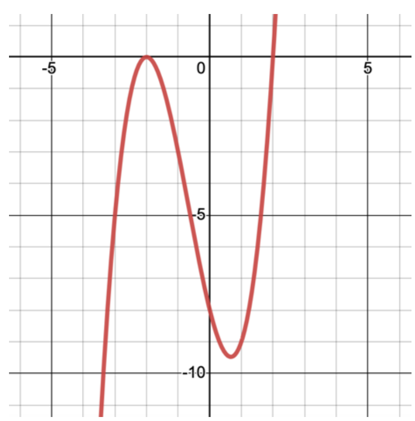


5.



6.



For Problem 7-20, simplify the exponents, leaving no negative exponents.

7. $\frac{x}{(-y)^{-5}}$

8. $(\frac{m}{n})^{-3}$

9. $\frac{4x^{-3}y}{2^{-2}}$

10. $6^{-2}x^{3}y^{-4}z^{-1}$

11. $\frac{15r^{-8}s^{2}}{5r^{-3}}$

12. $\frac{6a^{-5}b^{2}c^{3}}{3d}$

13. $(\frac{a^{2}}{b})^{3}$

14. $-\frac{4x^{2}y^{5}}{8x^{4}y}$

15. $\frac{-12a^{-4}b^{2}}{15a^{3}b^{-7}}$

16. $\frac{7a^{-2}b^{4}c^{-7}d^{-11}}{84a^{0}b^{-6}c^{-10}d^{-5}}$

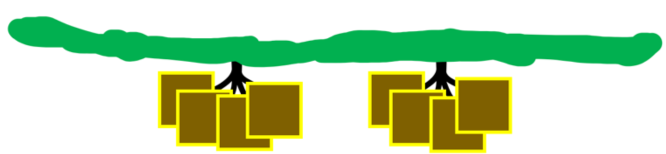
17. $\frac{32a^{3}b^{0}c^{-1}}{2bc}$

18. $\frac{(2m^{3}n^{-2})^{2}}{(m^{2}n^{0})^{-3}}$

19. $(\frac{5x^{3}y}{25xy^{4}})^{3}$

20. $\frac{xy}{x^{-1}y^{-1}}$

Did you hear about the plants who were afraid of math because it gave them square roots?!



Section 7.6 Comparing Quadratic and Cubic Equations

Practice Problems 7.6

For Problem 1-4, name the highest number of real zeroes possible in the function given. (Power functions can have the same number of zeroes as their highest power of x in standard form.)

1. $y = (x-3)(x+4)(x-2.5)$

2. $y = x + 5x - x^4$

3. $f(x) = 3x^2 + 1$

4. $f(x) = 2x^3 + 4x - 8$

For Problem 5 and 6, answer true or false.

5. Do you think the local high points and local low points on a cubic function graph occur halfway between the zeroes as does the vertex of a parabola? Look back at some examples to make your guess. One counterexample will make it false.

6. The greatest monomial factor of $12x^3 + 6x^2 + 3x$ is $3x$.

7. Convert the factored form of the cubic equation $y = (x+2)(x-3)(x+4)$ to standard form.

For Problem 8-12, simplify the expression given.

8. $\frac{7(ab)^{-2}}{c^0 d^0} \cdot \frac{3^{-1}a^3}{b^0 c^2}$

9. $(\frac{-2a^{-2}b^4c^6}{-4a^{-2}b^{-5}c^{-7}})^{-1}$

10. $(\frac{5^0c^2d^3e}{10c^{-4}d^{-2}})^{-3}$

11. $(\frac{-3x^{-5}y^{-1}z^{-2}}{6x^{-2}yz^{-4}})^{-2}$

12. $\frac{(4ad)^{-2}}{11b} \cdot \frac{16a}{(13c)^{-1}}$

We will end this section with a review of unit rates. Unit rates are used to find out the amount or price per item or ounce, etc. For example, if 12 ounces of beans sell for $\$1.19$, then the price per ounce is $\frac{\$1.19}{12\text{ oz.}} \approx \$0.10$ per ounce. We will go through the Unit Rate PowerPoint to solve these problems.

Section 7.7 Patterns in Power Functions

Practice Problems 7.7

For Problem 1-6, answer the questions a)-f) for the function given and then graph the function given using the method described below.

a) What are the x-intercepts?
b) Is it of an even or odd degree?
c) Is $a > 0$ or is $a < 0$?
d) Does it start up or down?
e) Does it end up or down?
f) What are the number of bounces?

Sketch the graphs using the following method:

1. Get a tray with a beveled edge or trim to make it a beveled edge.
2. Cover the edge about one-quarter inch deep with cool-whip using a spatula to smooth the surface of the cool-whip
3. Wash your hands and make a cross in the tray to represent the x-axis or y-axis.
4. Place red-hots at the x-intercepts of the function and sketch each of the functions below using your finger.
5. Use a spatula between functions to smooth the surface and start again.
6. Lick your fingers after sketching each function and eat the red-hots.

We call these *Yummy Functions!* When you are done sketching the graphs, be proud of yourself because, literally: "you licked it!"

1. $y = (x + 2)(x - 1)$

2. $f(x) = (x + 4)^2$

3. $f(x) = (x - 5)^2$

4. $y = (x + 3)(x - 2)(x + 1)$

5. $y = -x(x + 2)^2(x - 4)$

6. $f(x) = (x + 3)(x - 1)(x + 4)(x + 2)^2$

For Problem 7-16, simplify the expression given.

7. $\sqrt[3]{27}$

8. $\sqrt[4]{81}$

9. $\sqrt[3]{216}$

10. $\sqrt{x^5}$

11. $\sqrt[3]{8x^6}$

12. $\sqrt{\frac{4x^2}{64x^4}}$

13. $\sqrt{\frac{8}{25}}$

14. $\sqrt{\frac{4y^3}{25x^2}}$

15. $\sqrt{\frac{36y^2}{49z^8}}$

16. $\sqrt{-27}$

For Problem 17-20, answer true or false.

17. If a is -2 and the graph starts up and ends down, the degree of the function is odd.

18. An odd function always has end behavior that goes from negative infinity to positive infinity.

19. If a is less than 0, an even degree function has end behavior that starts and ends the same.

20. If a function has a double zero, then the graph changes direction at that point.

Did you hear about the bee who solved the math problem and shouted: “Hive got it!”
(Sound it out.)

Section 7.8 Multiple Representations of Cubic Functions

Practice Problems 7.8

For Problem 1-6, solve the word problem given.

1. What is the vertex (graphing) form for a quadratic equation?

2. What does (h, k) represent?

3. What is the graphing form for a cubic equation?

4. What does (h, k) represent in a cubic equation?

5. Let us say that two factors of a cubic equation are $x - 1$ and $x^2 + 3x + 1$. Use the area model to find the cubic equation that has these factors.

	x^2	$+3x$	$+1$
x			
-1			

6. The quadratic factor in Example 6 is $x^2 + 1$. It cannot be factored into two linear factors so the Zero-Product Property cannot be used to find the x-intercepts. However, the quadratic formula can be used to find the other two x-intercepts of the cubic equation found in Example 6. What are a, b, and c in the equation $x^3 - 2x^2 + x - 2 = 0$? Use the quadratic formula to find the other two x-intercepts. We know that one x-intercept is 2 from the linear factor $x - 2$. The other two are not real but imaginary so you will get a negative number under the radical sign. Find the other two x-intercepts using the Quadratic Formula shown below:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For Problem 7-9, tell whether each product is a perfect square, a perfect cube, or neither.

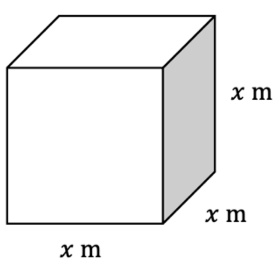
7. 676

8. 3,213

9. 1,331

For Problem 10-12, find the value of x in the cube given.

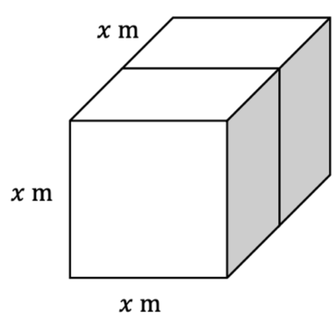
10.



The cube has a volume of 4,096m^3

$x =$ ____________ m

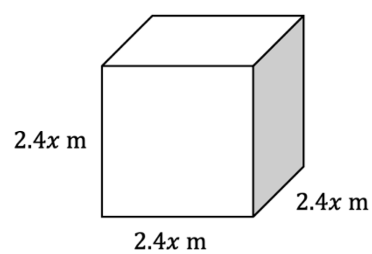
11.



The entire box has a volume of 11,664m^3

$x =$ ____________ m

12.



The volume of the cube is 2,985.98m^3

$x =$ ____________ m

We are going to end this section by playing “Matho” to review the point-slope form of equations. You will need to play with others! Cover the Matho Board with the solutions that are at the beginning of the Power Point that is next. There are 36 solutions: 12 points, 12 slopes, and 12 equations. Pick 8 of each and then one more. After you answer each question, cover the solution with a chip or penny. You may not have any solution on your gameboard since you did not use them all. Shout “Matho” when you complete a row, column, or diagonal with chips or pennies. Then clear the board and keep playing. Start now! You could play several times by creating new boards and changing the questions. You could find the point on the line, the slope, and the equation on each slide.

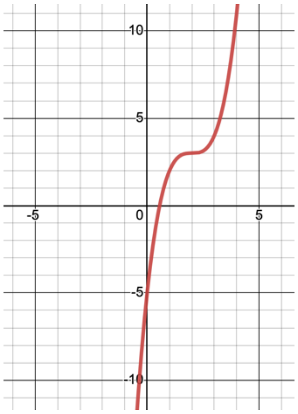
MATHO BOARD

Section 7.9 Horizontal and Vertical Transformations of Cubic Equations

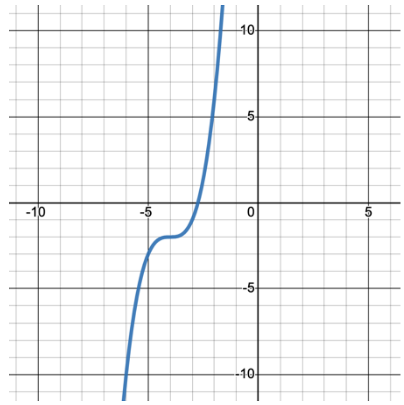
Practice Problems 7.9

For Problem 1-6, find the inflection point and the vertex form that goes with the graph given. Let $a = 1$ for Problem 1-5, and $a = -1$ for Problem 6.

1.



2.



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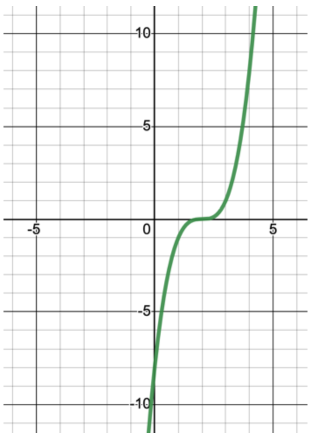
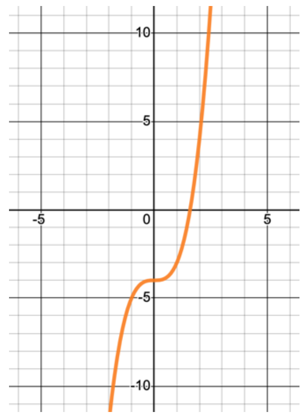
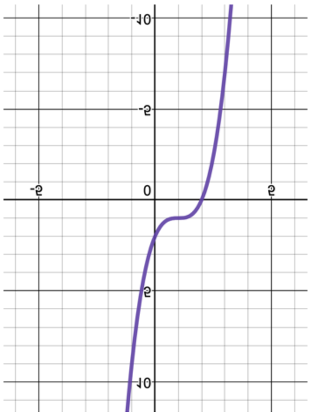


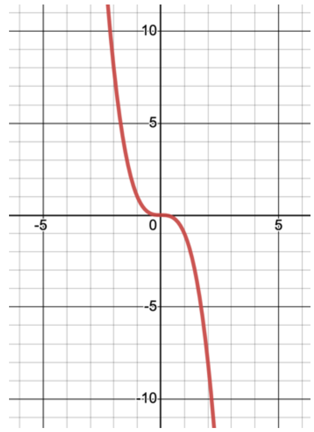
4.



5.

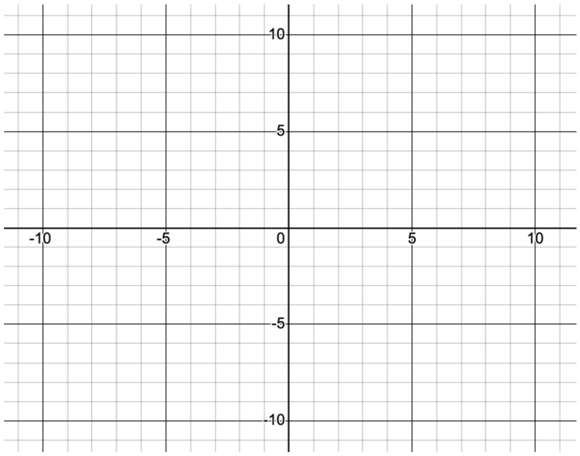


6.

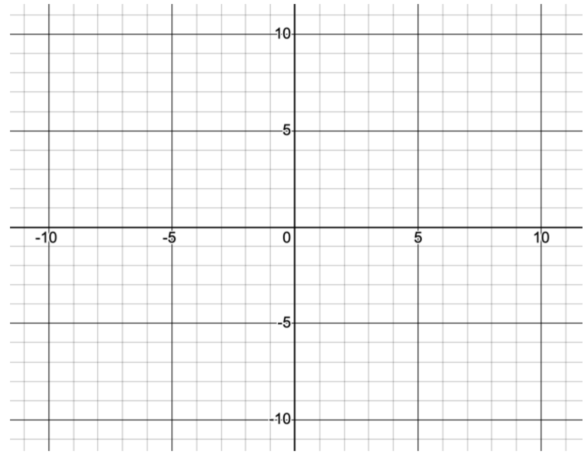


For Problem 7-12, sketch the cubic equation given. Find the point of inflection first.

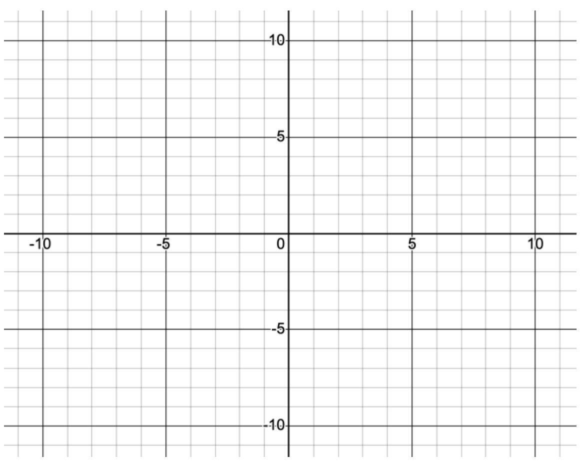
7. $y = (x - 1.5)^3 + 2$



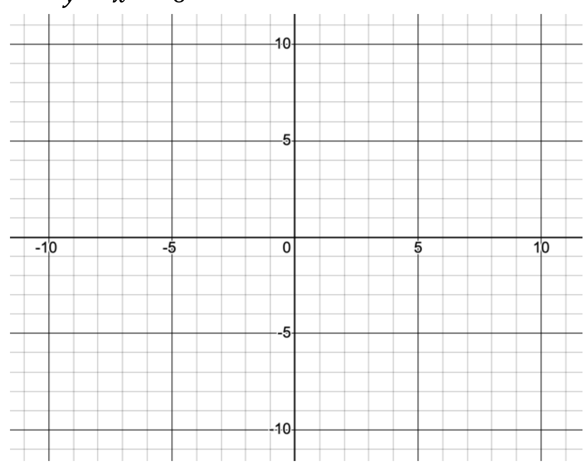
8. $y = (x + 4)^3 - 1$



9. $y = (x + 3)^3$

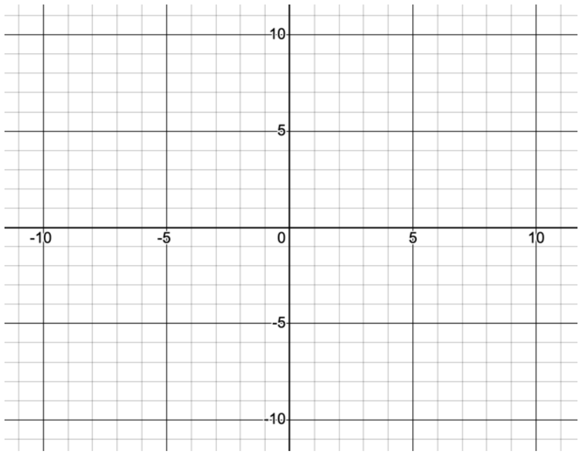


10. $y = x^3 - 6$

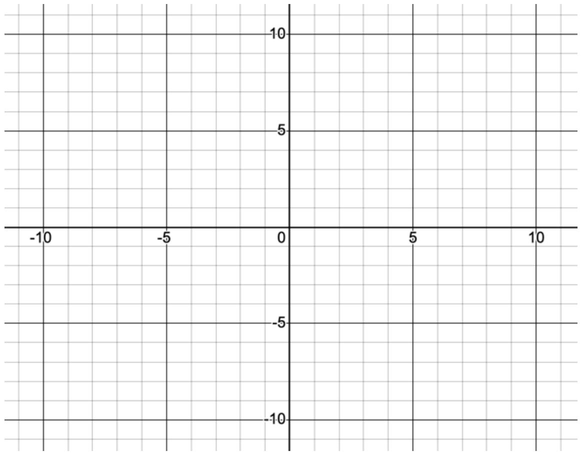


For Problem 11-20, simplify the radical given.

11. $y = -x^3 + 4$



12. $y = -(x + 5)^3$



Simplify the radicals for Problem 12-20.

13. $3\sqrt{20} \cdot 4\sqrt{10}$

14. $4\sqrt{xy} \cdot \sqrt{2x^2y}$

15. $\sqrt{2y} \cdot \sqrt{16y^5}$

16. $-2\sqrt[3]{a} \cdot 4\sqrt[3]{a^7}$

17. $-6\sqrt{3a} \cdot 2\sqrt{6a^3}$

18. $\sqrt[3]{20} \cdot 4\sqrt[3]{27} \cdot -2\sqrt[3]{5}$

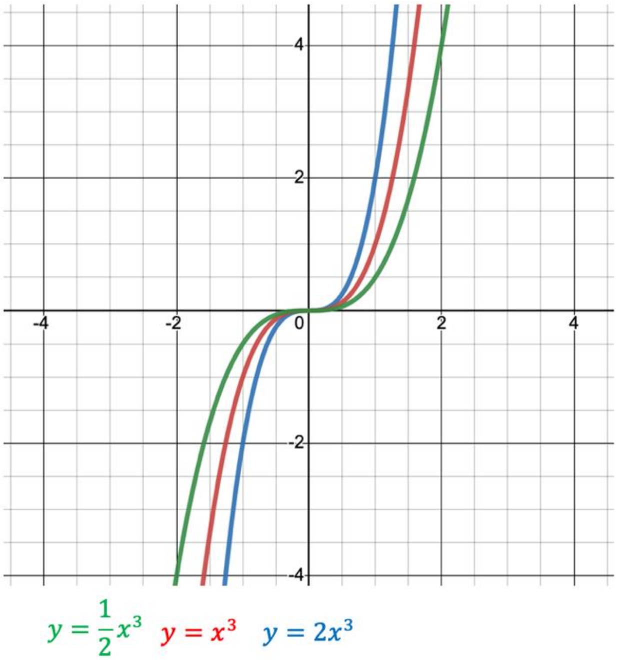
19. $2\sqrt{x} \cdot -5\sqrt{x^5}$

20. $\sqrt[3]{2} \cdot -\sqrt[3]{2x} \cdot \sqrt[3]{2x^2}$

Section 7.10 Sketches and Compressions of Cubic Graphs

Practice Problems 7.10

For Problem 1 and 2, use the given graph to solve the problem.



1.

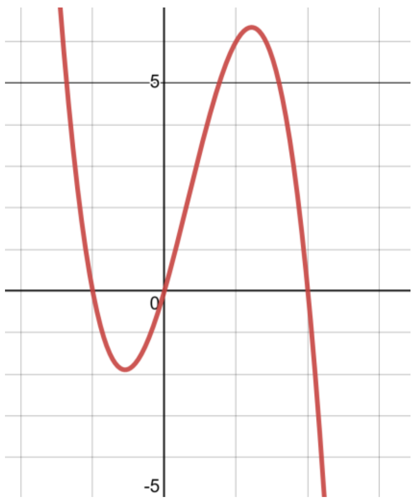
a) What happens to the graph when $a > 1$?

b) What happens to the graph when $0 < a < 1$?

2. Write the factored form of the graph that has the x-intercepts of -1, 0, and 2 when $a = 1$.

For Problem 3-5, use the given graph to solve the problem.

3. Below is the graph of the equation with three x-intercepts and one other point, $(1, 6)$. Is a positive or negative? How do you know?



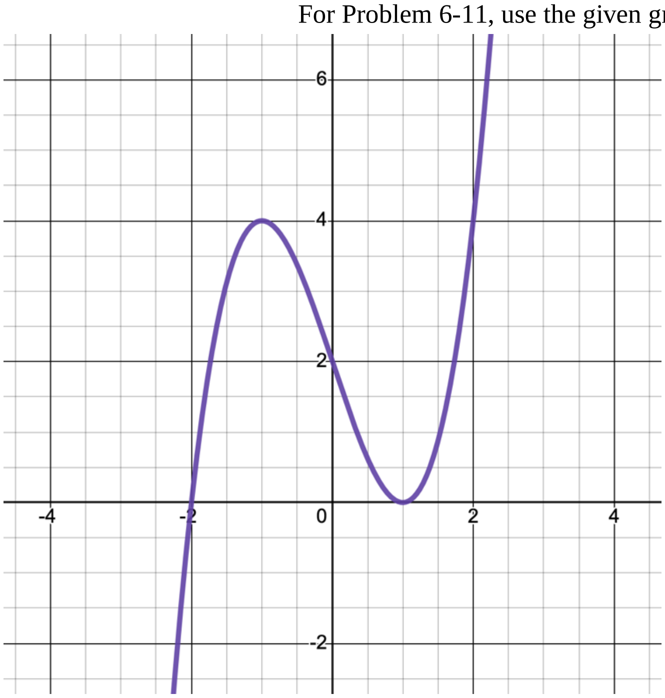
4. Using the x-intercepts and the one other point on the graph, $(1, 6)$, find the value of a.

$$y = ax(x + 1)(x - 2)$$

Notice that 0 is also written as an x-intercept. It is written as just x in the equation above.

5. Now substitute the value for a in the equation, $y = ax(x + 1)(x - 2)$, expand it (multiply through) and write the equation in standard form.

For Problem 6-11, use the given graph to solve the problem.



6. What are the x-intercepts of the graph?

7. Is there a double zero in the graph?

8. Write the factors of the cubic equation. (Remember the double zero is to the second power.)

9. What are the coordinates of the y-intercept?

10. Use the y-intercept as the other point on the graph. Substitute the values from the y-intercept for x and y and solve for a.

11. Substitute the value for a in the cubic equation and write it in standard form.

For Problem 12-20, simplify the division of the radicals given.

12. $\frac{9\sqrt{48x^2}}{2\sqrt{24x^2}}$

13. $\frac{8\sqrt{56}}{4\sqrt{10}}$

14. $\frac{5x\sqrt{6x}}{6\sqrt{x^5}}$

15. $\frac{3\sqrt{4x^3}}{9\sqrt{16x^2}}$

16. $\frac{\sqrt{16}}{\sqrt{64}}$

17. $\frac{\sqrt{9}}{\sqrt{3}}$

18. $\frac{\sqrt[3]{27}}{\sqrt[3]{8}}$

19. $\frac{\sqrt{x^2y^4}}{\sqrt{x^5y^6}}$

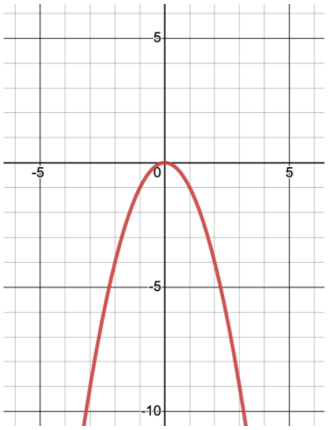
20. $\frac{2x\sqrt{6}}{\sqrt{4x^2}}$

Section 7.11 Even and Odd Functions

Practice Problems 7.11

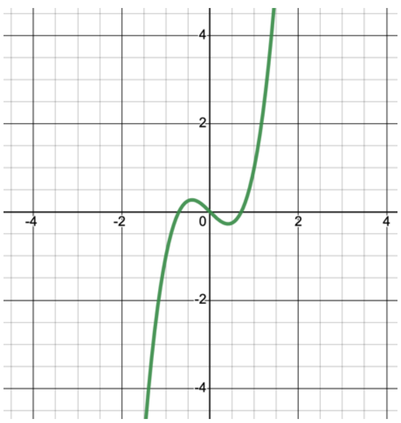
For Problem 1-6, use the graph given and/or solve the word problem given.

1. Look at the graph of the even function below. Is it symmetric over the x-axis or y-axis?



A hallmark of odd functions is mirroring over the __________________________.

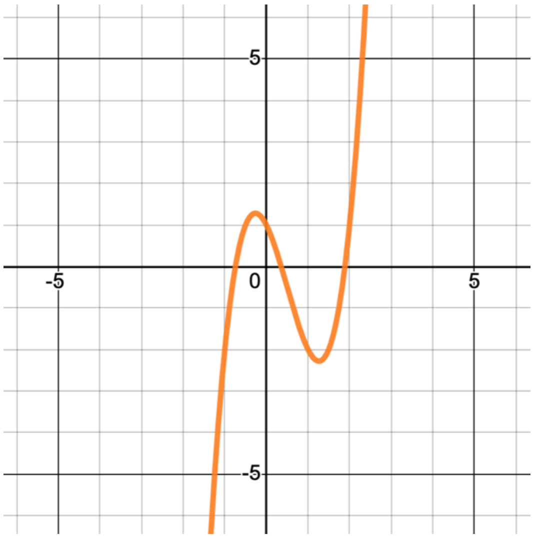
2. From the Looking Ahead section, the equation $f(x) = 2x^3 - x$ is symmetric about the origin.



A hallmark of odd functions is mirroring about the __________________________.

3. Determine algebraically whether $f(x) = 2x^3 - 3x^2 - 2x + 1$ is even, odd, or neither? Substitute $-x$ for x and simplify the equation. How can you tell if the function is even, odd, or neither?

4. Below is the graph of the equation in Problem 3. How can you tell from the graph if the function is even, odd, or neither?



5. Is $f(x) = x^3 + 2$ even, odd, or neither?

6. Is $f(x) = 3x + 1$ even, odd, or neither?

For Problem 7-20, simplify the radical using the Distributive Property.

7. $\sqrt{3}\left(-5 - 2\sqrt{6}\right) + 8\sqrt{3}$

8. $\sqrt[3]{5}(\sqrt[3]{5} - \sqrt[3]{25})$

9. $2\sqrt{7x}\left(4 + \sqrt{7x}\right) - 3\sqrt{7x}$

10. $\sqrt[3]{3} - 2\sqrt[3]{3}$

11. $\sqrt{2} + 4\sqrt{2}$

12. $9\sqrt{5} + 2\sqrt{5}$

13. $6\sqrt[4]{x} - 2\sqrt[4]{x}$

14. $\sqrt{8} + 5\sqrt{2}$

15. $10\sqrt{3y} + \sqrt{27y}$

16. $\sqrt[3]{z^5} + 2z\sqrt[3]{z^2}$

17. $\sqrt{100x} - \sqrt{49x}$

18. $3\sqrt{6} + \sqrt{7} + 4\sqrt{6} + 5\sqrt{7}$

19. $4\sqrt{x^3} - 3\sqrt{x^3}$

20. $\sqrt{81x^2} - \sqrt{36y^2}$

Section 7.12 Modeling a Cubic Relationship with "The Box Problem"

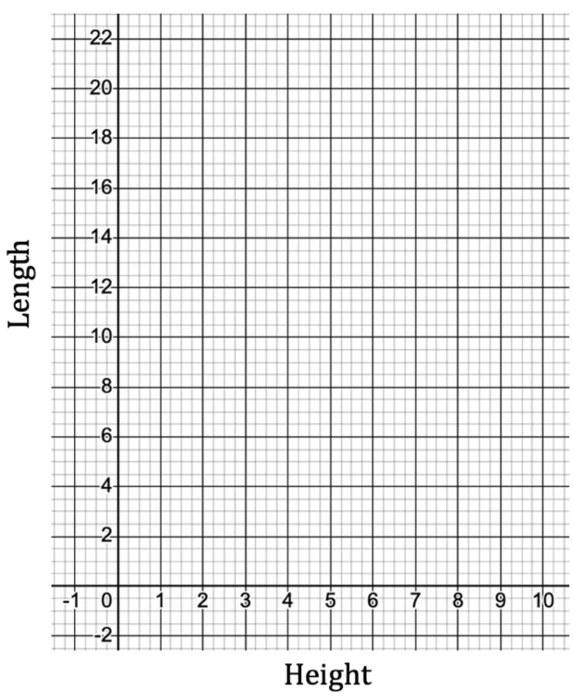
Practice Problems 7.12

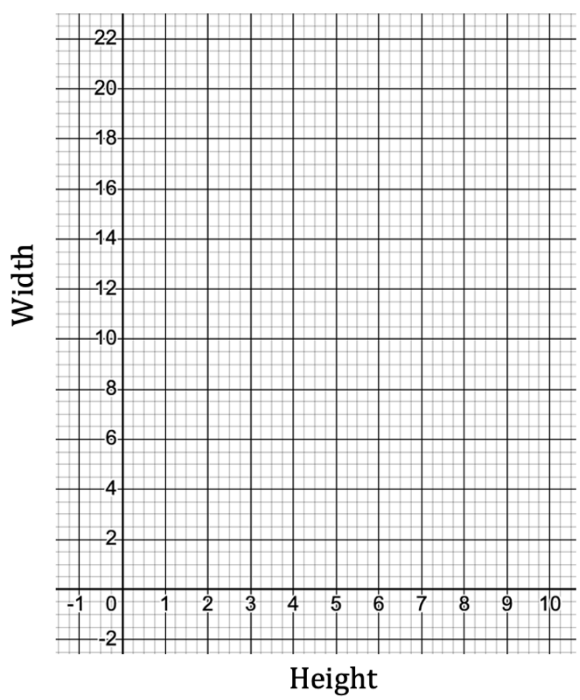
For Problem 1-8, use the given table from the box problem to solve the problem.

h	l	w	V
0	22	18	0
1	20	16	320
2	18	14	504
3	16	12	576
4	14	10	560
5	12	8	480
6	10	6	360
7	8	4	224
8	6	2	96
9	4	0	0

1. What is the answer to the box problem? What is the largest volume of the box needed to ship the unit cubes given their constraints? What are this box's dimensions?

2. Notice that as the height increases, the length and width decrease at a constant rate. What kind of graph would represent height to length or height to width if height were the independent variable (x) and length or width were the dependent variable (y)?





3. What happens to the box when the height is 9? What does this mean?

4. If you continued to increase the height of the box, what kind of numbers would you get for the length and width and why do these not make sense?

5. Looking at the relationship between height and volume, what happens to the volume as the height increases?

6. What type of graph and equation does this appear to be?

7. If you drew the graph of height versus volume, what are the x-intercepts and y-intercepts and what do they represent?

8. Given h is height, find an equation for the length and width of the box in terms of h. Find a formula for the changing volume of the box in terms of height. The amount subtracted from the length is double the height. The amount subtracted from the width is double the height. The starting length is 22cm. Let h be x (the height) and V be y (the volume). What is the degree of the equation? Is it what it appeared to be in Problem 6? Draw the graph of the equation on your graphing calculator.

9. Use the factors to find the $x-$intercepts of the equation in Problem 8.

10. Find the standard form equation in Problem 8.

For Problem 11-14, write the radical given as an exponential expression.

11. $\sqrt{4m}$

12. $\sqrt[2]{y^3}$

13. $\sqrt[4]{x}$

14. $\sqrt[3]{m^2}$

For Problem 15-20, rewrite the radical as an exponent expression and use the rules of exponents to simplify the expression. Then rewrite your solution for fractional exponents.

15. $\sqrt{y} \cdot \sqrt[3]{y}$

16. $\sqrt{p^5} \cdot (\sqrt{p})^3$

17. $\frac{\sqrt{x}}{\sqrt[3]{x}}$

18. $\frac{\sqrt{p^7}}{\sqrt{p^3}}$

19. $(\sqrt{m^2} \cdot \sqrt[3]{m})$

20. $(\sqrt{x^2} \cdot \sqrt{x^3} \cdot \sqrt{x^4})$

21. $\sqrt[2]{x^2} \cdot \sqrt[3]{y^3} \cdot \sqrt[4]{z^4}$

Section 7.13 Revisiting "The Dipped Cube"

Practice Problems 7.13

For Problem 1-5, answer the question/use the information given to solve the problem.

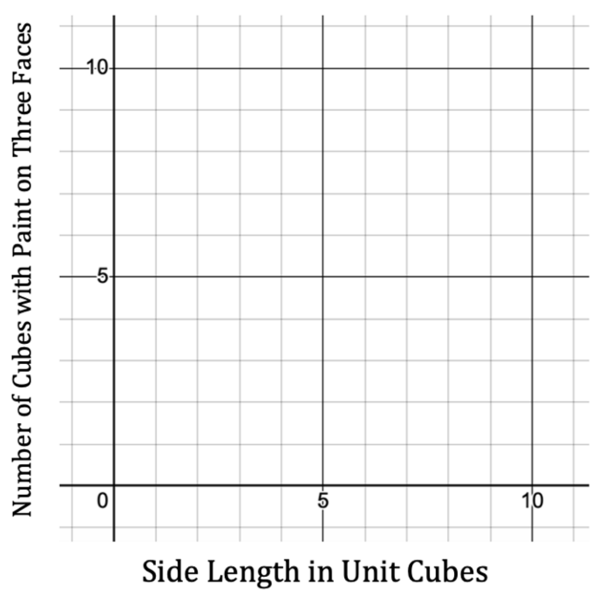
1. Why does a $2 \times 2 \times 2$ cube have zero cubes with zero faces painted when dipped in paint?

2. Record the type of relationship (for example, linear, quadratic, etc.) for the number of painted faces on a cube when dipped in paint.

Painted Faces	**Type of Relationship**
3	Constant
2	
1	
0	

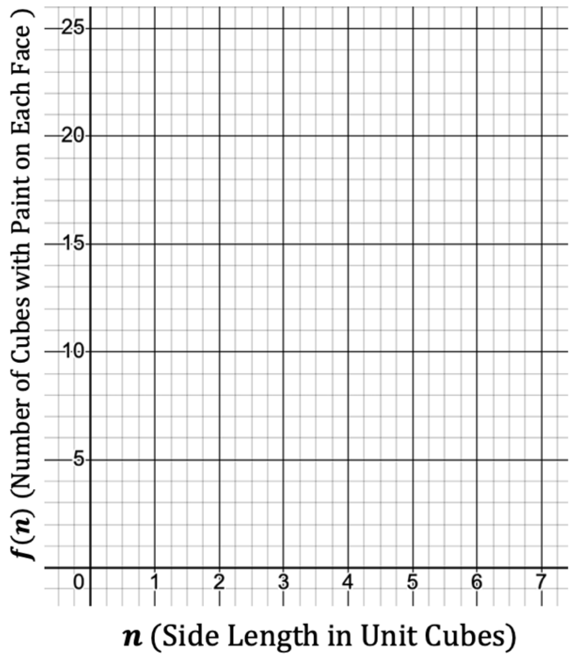
3. Paint on three faces when a cube is dipped in paint has yet to be explored. Complete the table, draw the graph, and find the equation. Why is it called "constant" for the type of relationship it shows?

Side Length in Unit Cubes	**Number of Cubes with Paint on Three Faces**
3	
2	
1	
0	



4. When a cube is dipped in paint, how many cubes are painted for each face of the entire cube? Complete the table, draw a graph, and write an equation for a cube with n cubes on a side. Try 2, 3, 4, and 5 cubes on a side. Then find the expression for n cubes on a side. One is done for you, 2.

$\boldsymbol{n}$ Side Length in Unit Cubes	$\boldsymbol{f(n)}$ Number of Cubes with Paint on Each Face
2	4
3	
4	
5	
n	



5. How many total unit cubes are in a cube that has a length of a side of n cubes? Again, try 2, 3, 4, and 5 side lengths before finding an expression for n cubes.

$\boldsymbol{n}$ Side Length in Unit Cubes	$\boldsymbol{f(n)}$ Number of Unit Cubes in the Cube
2	
3	
4	
5	
n	

For Problem 6-10, solve the word problem given.

6. In a $6 \times 6 \times 6$ cube, what is the total number of cubes with paint on 2 faces when it is dipped in paint?

7. If a cube is dipped in paint and has a total number of 72 cubes with paint on 2 faces, what is the side length of the cube? What are the dimensions of the cube?

8. In a $6 \times 6 \times 6$ cube, what is the total number of cubes with paint on one face when it is dipped in paint?

9. If a cube is dipped in paint and has a total number of 750 cubes with paint on one face, what is the side length of the cube? What are the dimensions of the cube?

10. In a $6 \times 6 \times 6$ cube, how many cubes would have paint on 3 faces when it is dipped in paint? Complete the table below for cubes with edges of 7 units, 8 units, and 9 units.

$\boldsymbol{n}$ **(Side length in unit cubes)**	$\boldsymbol{f(n)}$ **(Number of cubes with paint on 3 faces)**
6	
7	
8	
9	

What is the function that represents this relationship?

For Problem 11-20, solve the equation given using exponents and radicals.

11. $y^2 = 64$

12. $\sqrt{x} - 4 = 10$

13. $(2x + 1)^3 = 729$

14. $\sqrt{2m + 1} = \sqrt{4m - 1}$

15. $(2y - 12)^2 = (9 - y)^2$

16. $(3y + 8)^3 = 125$

17. $\sqrt{2x + 7} = 13$

18. $m^2 = 49$

19. $x^3 = 8$

20. $x^3 + 8 = 72$

Section 7.14 Module Review

For Problem 1-3, name the inflection point for the cubic equation given.

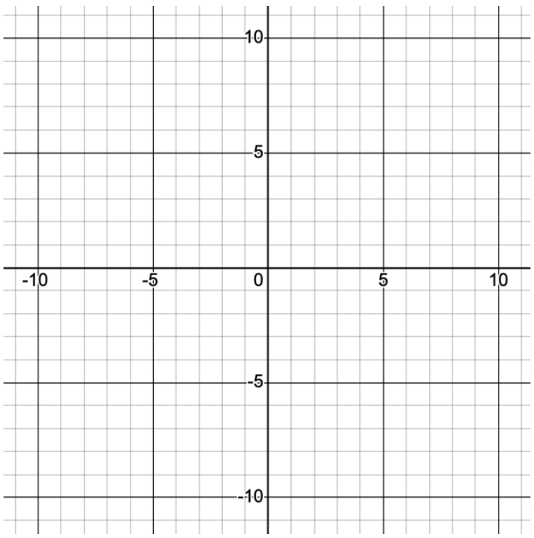
1. $y = x^3 + 8$

2. $y = (x - 3)^3 + 4$

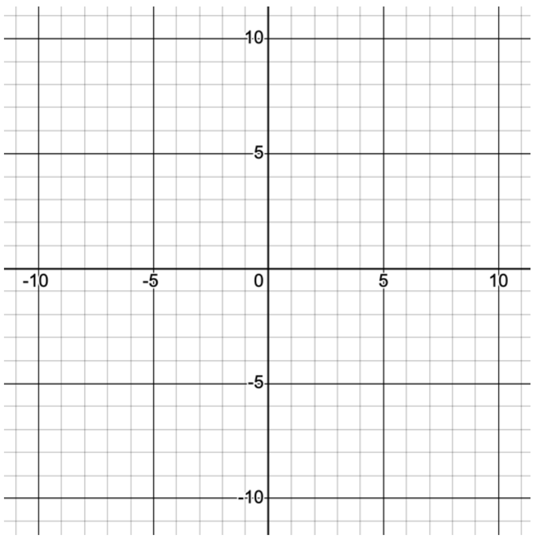
3. $y = (x + 2)^3$

For Problem 4-6, sketch the graph of the cubic equation given.

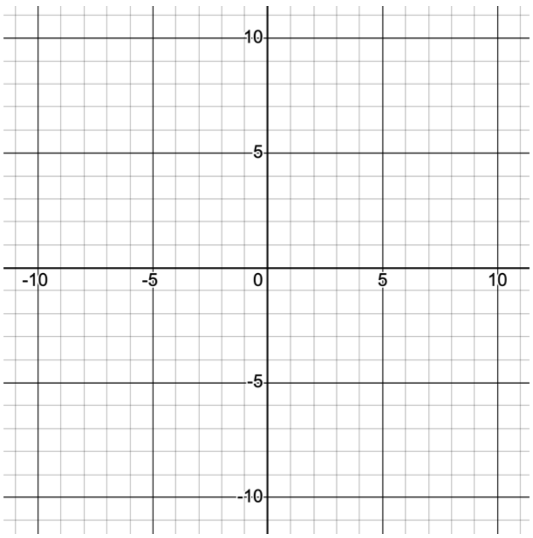
4. $y = x^3 + 8$



5. $y = (x - 3)^3 + 4$



6. $y = (x + 2)^3$



For Problem 7-9, expand the cubic equation given and then covert it to from the vertex form to the standard form.

7. $y = (x - 3)^3 + 5$

8. $y = (x + 4)^3$

9. $y = (x - 1)^3 - 3$

For Problem 10-12, answer the question given using the cubic equation $y = 5x(x + 3)(x - 2)$.

10. What is the value of a?

11. Is this a vertical stretch or vertical compression from the parent function for a cubic equation?

12. Does the graph of the equation start up $(+\infty)$ or down $(-\infty)$?

For Problem 13-16, fill in the blank(s).

13. The __________-intercepts are the zeroes of the equation.

14. The equation $y = (x + 5.2)(x - 1.4)(x + 3)$ has __________ linear zeroes.

15. A cubic equation has at most __________ zeroes.

16. The volume of a cube is $y = (x + 1)(x^2 + 4x + 3)$. Use the array to find the expanded form representing this area.

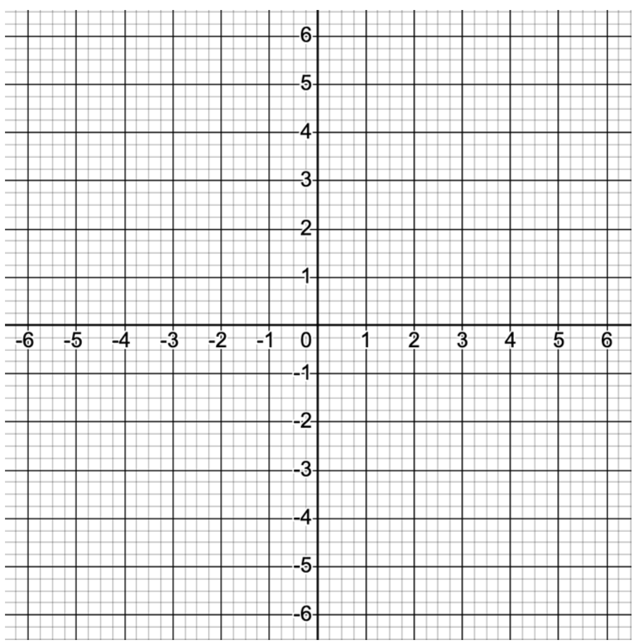
	x^2	$+4x$	$+3$
x			
$+1$			

For Problem17-20, solve the word problem given.

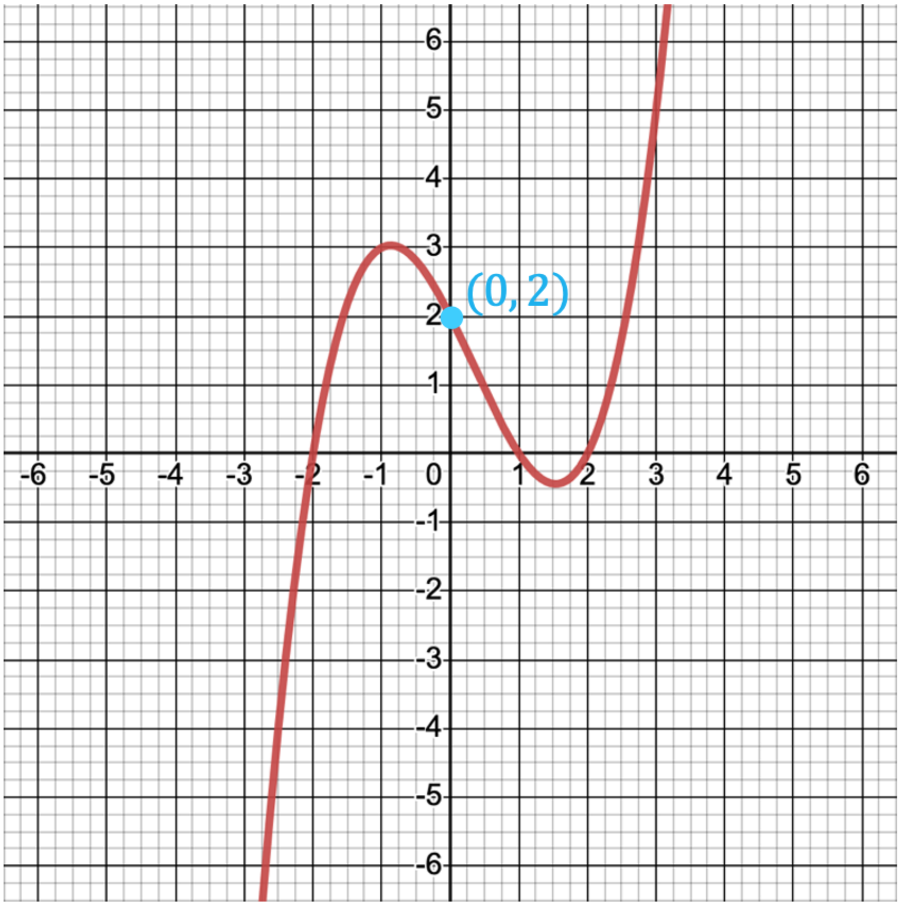
17. Factor the quadratic part of the equation $y = (x+1)(x^2+4x+3)$ to find the two other linear factors that go with the equation.

18. Use the Distributive Property to multiply the three linear factors and get the standard form of the equation in Problem 17; does it match the equation in Problem 16?

19. Sketch the graph of the equation $y = (x+1)(x^2+4x+3)$.



20. Find the equation for the graph below. Find the x-intercepts and the value of a first.



<u>Section 7.15 Module Test</u>

For Problem 1-3, name the inflection point for the cubic equation given.

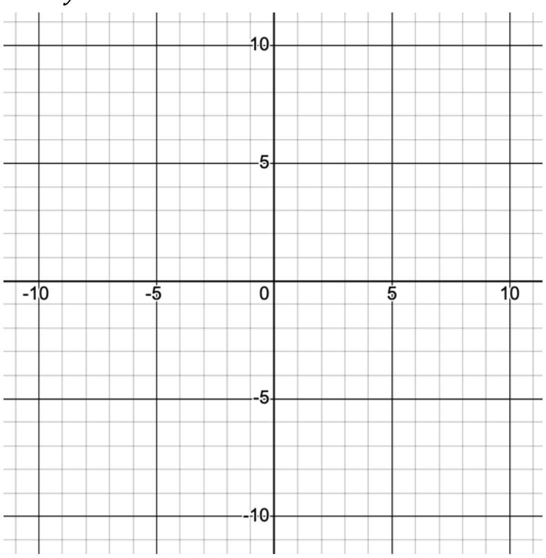
1. $y = x^3 - 4.5$

2. $y = (x - 1.5)^3$

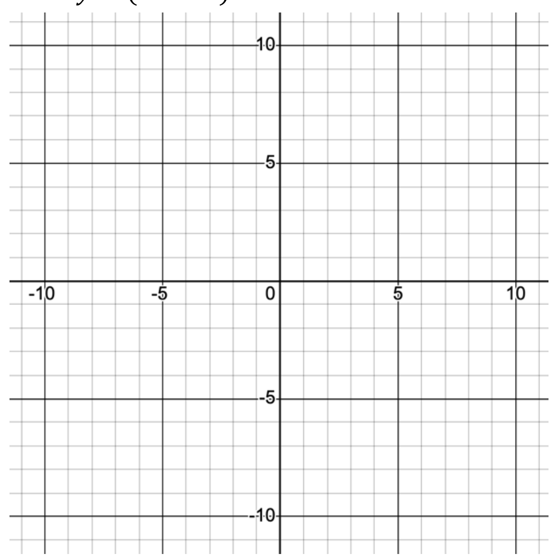
3. $y = (x - 4)^3 + 2.5$

For Problem 4-6, sketch the graph of the cubic equation given.

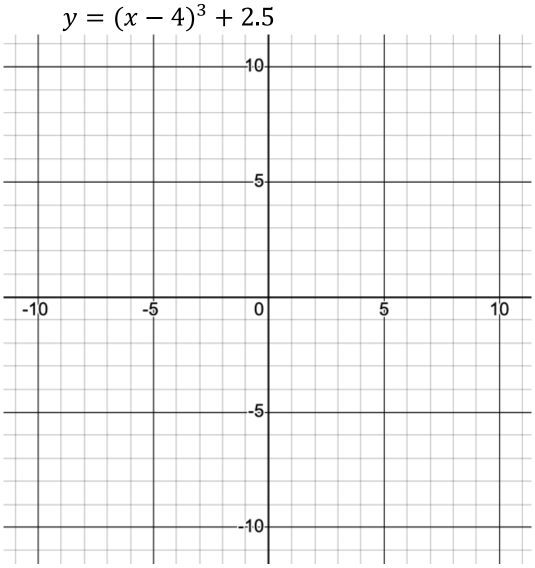
4. $y = x^3 - 4.5$



5. $y = (x - 1.5)^3$



6. $y = (x - 4)^3 + 2.5$



For Problem 7-9, convert the cubic equation given from vertex form to standard form.

7. $y = (x + 1)^3 - 1$

8. $y = (x + 3)^3 + 2$

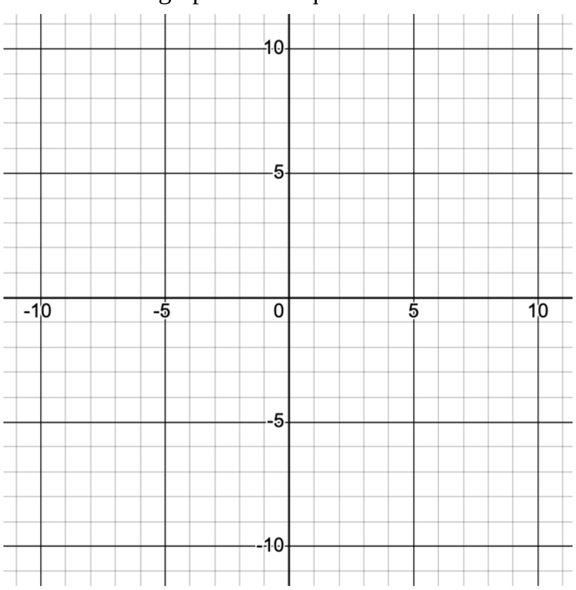
9. $y = (x - 2)^3$

For Problem 10-12, solve the problem given using the cubic equation $y = -2x(x + 1)(x - 3)$.

10. What is the value of a?

11. Looking at the end behavior, does the graph end going up to positive infinity or going down to negative infinity?

12. Sketch a graph of the equation below. Find the zeroes first.



For Problem 13-16, fill in the blank(s).

13. If a is negative, the cubic equation is reflected in the __________-axis.

14. If $a = 0.5$, a vertical ______________________________ by a factor of 0.5 is applied to the equation.

15. The graph of $y = (x-2)^2(x+4)$ has one linear zero and a ____________________ zero.

16. The equation $y = (x-3)(x^2+2x+1)$ is a cubic equation. Use the area model below to find the cubic equation in standard form.

	x^2	$+2x$	$+1$
x			
-3			

For Problem 17-20, solve the word problem given.

17. Use the Distributive Property to convert $y = (x-3)(x^2+2x+1)$ to standard form.

18. The equation (x^2+2x+1) has two linear factors. What are they? Using that information, what are the three linear factors of $y = (x-3)(x^2+2x+1)$?

19. What does a double zero at the x-intercept of the graph mean?

20. A box of 1 cm unit cubes is to be shipped to a classroom in Nigeria. A rope tied around the length of the box is 110 cm. A rope tied around the width is 80 cm. The height of the box is 10 cm.

a)What is the width of the box?

b) What is the length of the box?

c) How many centimeter cubes are in the box? What is the volume of the box?

d) If the height of the box is x, find an expression that represents the width in terms of x?

e) Find the length of the box in terms of x.

f) Find the volume of the box in terms of x.

g) What is the volume of the box which has a height of 10cm? Use the factored form equation. Write the equation in standard form also and solve to see if you get the same answer.