

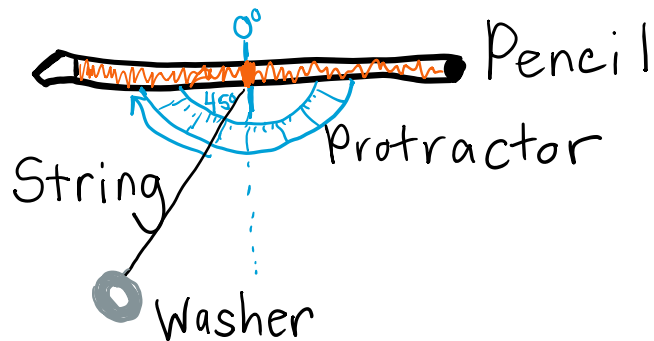
Algebra 2 Module 4 Quadratic EquationsSection 4.1 The Parts of the PendulumPractice Problems 4.1

In order to build a pendulum to model the experiment, use the following supplies:

- Pencil
- Protractor
- String
- Washers

To build the pendulum:

1. Tie the string to the middle of the pencil at one end.
2. Tie the bob to the string at the other end.
3. Place the protractor flat against the pendulum and measure the angle of release.



4. Follow the directions below to test each of the three variables.
5. Watch the video of the sample experiment. The students are using a manufactured pendulum.

Pendulum Unit

**Amplitude**

1. The variable to test is the amplitude or angle of release. You want to determine if changing the angle of release of the pendulum changes the time for the period of the pendulum. That means the length of the string and the weight of the bob or bob type must be controlled but the amplitude or angle of release will vary.
2. Choose the length of string and keep it the same for all three trials. This does not need to be measured, just keep it the same each time.
3. Choose the bob. It doesn't matter what the weight it is. What matters is that bob stays the same for each trial and does not change. This does not need to be weighed, just keep using the same one for each trial.
4. Tie the string to a T stand or pencil. Tie the bob to the end of the string. This will be the same for each trial. Release the bob from an angle of chosen degrees and record it in the first row of the table below. This will change for each trial. Release the bob and time the periods of the pendulum for 10 full swings. Record that in the table below. Then divide that number by 10 to get the average time for 1 period of the pendulum at that angle. Do three trials and divide the total time by 3 to get the average in the final column of the table.
5. Change the angle of release and record the angle used in the second row below and repeat step 4 above. Complete the second row of the table.
6. Choose a third and final angle of release that is different from the two already tried and complete the third row of the table below.

Note: You don't necessarily need to measure angle of release. You could just mark three separate angles on a protractor as *a*, *b*, and *c* and release the bob from those points each time to keep this variable controlled. Whether or not it affects the period is what is important. If the times change as the angle of release changes then you will know amplitude effects the period of the pendulum.

Angle of Release (Amplitude)	Trial 1 10 periods in seconds	Trial 1 1 period in seconds (Divide column A by 10)	Trial 2 10 periods in seconds	Trial 2 1 period in seconds (Divide column C by 10)	Trial 3 10 periods in seconds	Trial 3 1 period in seconds (Divide column E by 10)	Total time for all three trials of the pendulum (Add columns B, D and F)	Average time for three trials of the period of the pendulum (Divide column G by 3)
	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	

Pendulum Unit

**Length of the String**

1. The variable to test is the length of the string. You want to determine if changing the length of the string changes the time for the period of the pendulum. That means the weight of the bob or bob type and the angle of release must be controlled but the length of the string will vary.
2. Choose the bob and keep it the same for all trials. This does not need to be weighed, just keep it the same each time.
3. Choose the angle of release. It doesn't matter what the degree is. What matters is that the degree of release stays the same for each trial and does not change. Once you choose an angle, keep using the same one for each trial. You could just use halfway between 0 and 90 degrees.
4. Tie one unit of string to a T stand or pencil. This can be one foot or 20 centimeters or whatever you choose. Record this length in the first row of the table below.
5. Release the bob from an angle of chosen degrees. Release the bob and time the periods of the pendulum for 10 full swings. Record that in the table below. Then divide that number by 10 to get the average time for 1 period of the pendulum at that angle. Do three trials and divide the total time by 3 to get the average in the final column of the table.
6. Change the length of the string by making it longer or shorter and record that in the second row below and repeat step 5 above. Complete the second row of the table.
7. Choose a third and final string length that is different from the two already tried and complete the third row of the table below.

Note: You don't necessarily need to measure the length of the string. You could just mark three separate lengths as *a*, *b*, and *c* or double and triple the length of *a* to get *b* and *c*. Whether or not it affects the period is what is important. If the times change as the length of the string changes then you will know that string length effects the period of the pendulum.

Length of String	Trial 1 10 periods in seconds	Trial 1 1 period in seconds (Divide column A by 10)	Trial 2 10 periods in seconds	Trial 2 1 period in seconds (Divide column C by 10)	Trial 3 10 periods in seconds	Trial 3 1 period in seconds (Divide column E by 10)	Total time for all three trials of the pendulum (Add columns B, D and F)	Average time for three trials of the period of the pendulum (Divide column G by 3)
	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	

Pendulum Unit

**Weight of the Bob (Bob Type)**

1. The variable to test is the bob. You want to determine if changing the weight or type of bob changes the time for the period of the pendulum. That means the length of the string and the angle of release must be controlled but the bob weight and type will vary.
2. Choose the length of the string and keep it the same for all trials. This does not need to be measured, just keep it the same each time.
3. Choose the amplitude (or angle of release). It doesn't matter what the degree is. What matters is that the degree of release stays the same for each trial and does not change. Once you choose an angle, keep using the same one for each trial. You could just use halfway between 0 and 90 degrees.
4. Tie string to a T stand or pencil. Tie one bob to other end of the string. You can choose from washers, ping-pong balls, quarters, tennis balls, etc. These will change for each trial since this is the test variable. Or change the number of washers each time. Record the bob type in the first row of the table below.
5. Release the bob from an angle of chosen degrees. Release the bob and time the periods of the pendulum for 10 full swings. Record that in the table below. Then divide that number by 10 to get the average time for 1 period of the pendulum at that angle. Do three trials and divide the total time by 3 to get the average in the final column of the table.
6. Change the bob at the end of the string and record the type used in the second row below and repeat step 5 above. Complete the second row of the table.
7. Choose a third and final bob that is different from the two already tried and complete the third row of the table below.

Note: You don't necessarily need to measure the weight of the bob. You could just use three different materials that seem to be lighter or heavier than each other. Whether or not it affects the period is what is important. If the times change as the weight or type of bob changes then you will know that the bob effects the period of the pendulum.

Bob Type or Weight	Trial 1 10 periods in seconds	Trial 1 1 period in seconds (Divide column A by 10)	Trial 2 10 periods in seconds	Trial 2 1 period in seconds (Divide column C by 10)	Trial 3 10 periods in seconds	Trial 3 1 period in seconds (Divide column E by 10)	Total time for all three trials of the pendulum (Add columns B, D and F)	Average time for three trials of the period of the pendulum (Divide column G by 3)
	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	

Section 4.2 The Period of the Pendulum

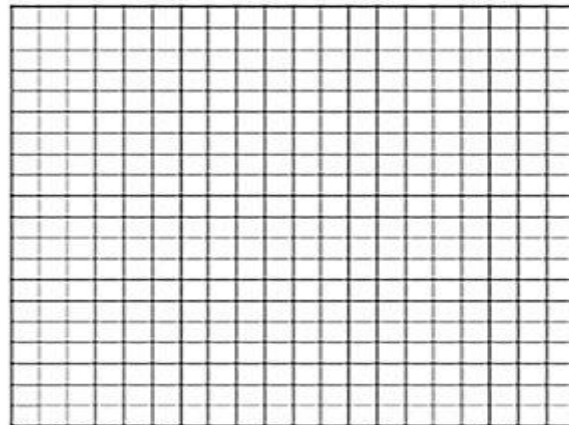
Practice Problems 4.2

Now that we know that the length of the string is the variable that effects the period of the pendulum, we can make a graph to see the pattern in these two relationships.

Set up your pendulum using the table below. Measure to the bottom of the bob. Use the same procedure you used when testing the pendulum variables using one bob at a 45° angle each time.

Length of String	Trial 1 10 periods in seconds	Trial 1 1 period in seconds (Divide column A by 10)	Trial 2 10 periods in seconds	Trial 2 1 period in seconds (Divide column C by 10)	Trial 3 10 periods in seconds	Trial 3 1 period in seconds (Divide column E by 10)	Total time for all three trials of the pendulum (Add columns B, D and F)	Average time for three trials of the period of the pendulum (Divide column G by 3)
	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	
10								
20								
30								
40								
50								
60								
70								

Graph the results using the *x*-axis for the length of the string (in cm.) and the *y*-axis for the period of the pendulum (in sec.). What type of graph below does this model?



Does the graph model:

A linear function:  $y = mx + b$

A quadratic function:  $y = x^2$

A cubic function:  $y = x^3$

An exponential function:  $y = ab^x$

An inverse variation function:  $y = 1/x$

A square root function:  $y = \sqrt{x}$ ?

The formula for the period of the pendulum is:

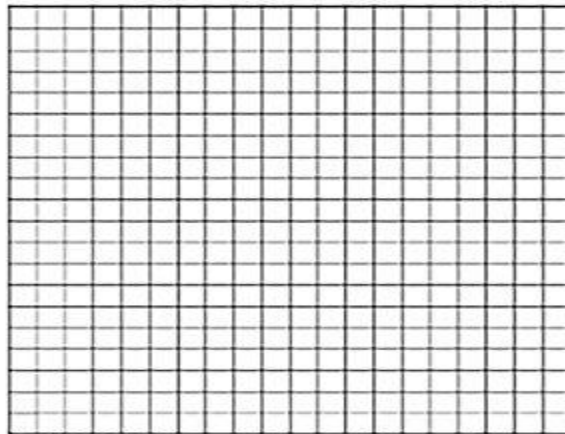
$$p = 2\pi\sqrt{\frac{L}{g}}$$

where  $p$  is the period of the pendulum,  $L$  is the length of the string and  $g$  is gravity  $9.8 \text{ m/s}^2$ . This is  $32.2 \text{ ft./s}^2$ .

Graph the inverse of the results below. Let the  $x$ -axis be the period of the pendulum and the  $y$ -axis be the length of the string. What function above does this model?

The formula for the period of the pendulum is also:

$$L = \left(\frac{p}{2\pi}\right)^2 g$$



Section 4.3 The Pit and the PendulumPractice Problems 4.3

In the actual story, *The Pit and the Pendulum*, Edgar Allan Poe writes that the pendulum is 30-40 feet long. He says that he thinks there are 10-12 swings left and that he thinks he has about 1 to 1-1/2 minutes to escape.

Use the formula for the period of the pendulum to complete the table below.

(Hint use  $32.2 \text{ ft/s}^2$  instead of  $9.8 \text{ m/s}^2$  for gravity).

Time -1 period	Time- 10 periods	Time- 12 periods	Length (ft)
			40
			35
			30
			25
			20
			15
			10
			5

Does the prisoner in “The Pit and the Pendulum” have time to escape? Given his assumptions, are there times when he will be able to escape and other times when he will not? Is Edgar Allen Poe really a mathematician? Based on the information extracted from the story (look for details) if he does escape, when do you think that happens? In other words, which assumptions are true? Brainstorm here. Use charts, tables, graphs, etc.

At 25 ft, is there time for a prisoner to escape?

At what length do you think a person would NOT have time to escape? Explain your reasoning.

Will he always be able to escape if the rats chew through the rope in 1 minute? Will he ever be able to escape if it takes the rats 1-1/2 minutes to chew through the rope? Explain your reasoning.

Section 4.4 Factoring Quadratic Equations without a Linear TermPractice Problems 4.4

For Problem 1-16, solve for  $x$  in the quadratic equation.

1.  $x^2 = 100$

2.  $5x^2 = 500$

3.  $x^2 + 3 = 39$

4.  $4x^2 - 2 = 22$

5.  $(x - 8)^2 = 64$

6.  $(x + 8)^2 - 3 = 22$

7.  $3x^2 = 75$

8.  $3x^2 = 14$

9.  $x^2 - 81 = 0$

10.  $3x^2 + 4 = 79$

11.  $x^2 - 49 = 0$

12.  $9x^2 - 81 = 0$

13.  $x^2 + 1 = 0$

14.  $6x^2 + 2 = 0$

15.  $9x^2 - 16 = 0$

16.  $16x^2 = 49$

17. The Eiffel Tower has a height of 984 feet! If a bird dropped a seed from the top, how much longer would it take to fall to the ground than if it fell from the Leaning Tower of Pisa? (Use the equation  $-16t^2 + 984 = 0$  to solve).

18. What equation would be used to find the time for a seed to drop from the clock tower Big Ben, which has a height of 315 feet?

19. Use the equation from Problem 18 to determine how much time it takes for the seed to fall from Big Ben.

20. How much longer would it take a seed to drop from the tip of the lightning rod on the Empire State Building (1,454 ft.) than from the top of the building (1,250 ft.)?



Section 4.5 Factoring and the Zero-Product PropertyPractice Problems 4.5

For Problem 1-3, factor the quadratic equations using the greatest monomial factor.

1.  $3x^2 - x = y$

2.  $5x^2 - 5x = y$

3.  $2x^2 - 12x - 2 = y$

For Problem 4-6, factor the quadratic equations using difference of squares.

4.  $x^2 - 81 = y$

5.  $16x^2 - 49 = y$

6.  $4x^2 + 36 = y$

For Problem 7-9, factor the perfect trinomial squares into binomial squares.

7.  $x^2 + 12x + 36 = y$

8.  $x^2 - 18x + 81 = y$

9.  $x^2 - 6x - 9 = y$

For Problem 10-12, use the zero-product property to solve for  $x$ .

10.  $(x - 3)(5x - 8) = 0$

11.  $x(3x + 4) = 0$

12.  $(2x - 4)(3x + 7) = 0$

For Problem 13-15, factor using any method and solve for  $x$  using the zero-product property.

13.  $x^2 - 3x - 28 = 0$

14.  $x^2 - 6x + 9 = 0$

15.  $3x^2 + 3x = y$

For Problem 16-18, factor the equations and find the  $x$ -intercepts, the axis of symmetry, and the vertex.

16.  $x^2 - 10x + 25 = y$

17.  $2x^2 + 12x + 16 = y$

18.  $5x^2 - 4x = y$

For Problem 19 and 20, solve the word problem.

19. How many  $x$ -intercepts does a binomial square have?

20. What is another name for the  $x$ -intercepts of a difference of squares?

Section 4.6 Completing the SquarePractice Problems 4.6

For Problem 1-5, find the vertex of each quadratic equation in vertex form.

1.  $(x - 3)^2 + 4 = y$                       2.  $(x + 3)^2 - 4 = y$                       3.  $2(x - 5)^2 - 2 = y$

4.  $-5(x + 2.4)^2 + 3.6 = y$                       5.  $(x - 1.2)^2 = -4.8$

For Problem 6-8, find the  $x$ -intercepts of the quadratic equation.

6.  $(x - 3)^2 - 4 = y$                       7.  $(x + 2)^2 - 36 = y$                       8.  $4(x - 1)^2 = 1$

For Problem 9-14, find the vertex and  $x$ -intercepts of the quadratic equation and sketch a graph.

9.  $(x - 6)^2 = 25$                       10.  $2(x + 9)^2 = 32$                       11.  $(x - 5)^2 = 4$

12.  $4(x + 5)^2 = 36$                       13.  $(x - 2)^2 - 49 = 0$                       14.  $(x + 11)^2 - 14 = 2$

For Problem 15-20, complete the square to find the vertex of the equation.

15.  $x^2 - 8x + 15 = 0$

16.  $x^2 + 6x = 72$

17.  $3x^2 + 12x + 5 = 0$

18.  $2x^2 + 36x - 18 = 20$

19.  $8x^2 + 16x - 90 = 0$

20.  $7x^2 + 14x = 56$

Section 4.7 The Vertex FormPractice Problems 4.7

For Problem 1-4, find the vertex of each quadratic equation and tell whether it is a minimum or a maximum.

1.  $y = -x^2 + 8x - 20$

2.  $y = -3x^2 + 12x - 10$

3.  $y = -\frac{1}{3}x^2 + \frac{4}{3}x - \frac{16}{3}$

4.  $y = 2x^2 - 4x - 2$

For Problem 5-8, find the vertex of the quadratic equation, the axis of symmetry, and the  $y$ -intercept.

5.  $y = -x^2 - 10x - 30$

6.  $y = x^2 + 2x - 1$

7.  $y = \frac{1}{4}x^2 - x + 9$

8.  $y = x^2 + 4x + 5$

For Problem 9-12, name the vertex in the quadratic equations and find the value of “ $a$ .”

9.  $f(x) = -2(x - 4)^2 + 3$

10.  $f(x) = \frac{4}{5}x^2 + 5$

11.  $f(x) = 2x^2 - 5x$

12.  $f(x) = \frac{1}{3}(x - 1)^2 - 8$

For Problem 13-17, find the vertex in the quadratic equation and convert the vertex form to standard form.

13.  $f(x) = (x + 5)^2 + 4$

14.  $f(x) = (x + 4)^2 - 7$

15.  $f(x) = (x - 2)^2 - 9$

16.  $f(x) = (x + 2)^2 + 11$

17.  $f(x) = (x + 3)^2 + 11$

For Problem 18-20, given the vertex and one point on the graph of the quadratic equation,

solve for “ $a$ ” in vertex form and convert it to standard form.

18. The vertex is  $(-5, 7)$  and the  $y$ -intercept is  $(0, -93)$ .

19. The vertex is  $(5, 2)$  and a point on the graph is  $(3, -2)$ .

20. The vertex is  $(-3, -1)$  and a point on the graph is  $(1, 1)$ .

Section 4.8 Imaginary NumbersPractice Problems 4.8

For Problem 1-6, simplify the radicals. Use imaginary numbers.

1.  $\sqrt{-16}$

2.  $-\sqrt{64}$

3.  $-\sqrt{-25}$

4.  $\sqrt{-18}$

5.  $\sqrt{-50}$

6.  $\sqrt{-1}$

For Problem 7-11, simplify the imaginary numbers to 1,  $i$ ,  $-1$ , or  $-i$ .

7.  $i^0$

8.  $i^{100}$

9.  $j^{37}$

10.  $j^{13}$

11.  $i^{54}$

12. Show that  $(i^2)^{10} = i^{20}$

For Problem 13-20, perform the operations on the imaginary numbers.

13.  $i + 3i - 14i$

14.  $\frac{5i}{25i}$

15.  $(3i)(6i)(2i)$

16.  $(5i + 3i) - (6i + 2i)$

17.  $(3i - i) + (-10i - 2i)$

18.  $(3i^2)(-5i^4)$

19.  $(-i)(-i)(-i)$

20.  $\frac{22i^4}{-11i^2}$

Section 4.9 Complex NumbersPractice Problems 4.9

For Problem 1-6, add or subtract the complex numbers.

1.  $5i - 10i$

2.  $3 + 6i - 4i$

3.  $4 + 2i - 8$

4.  $(3 - 6i) + (4 + i)$

5.  $(3 - 6i) - (4 + i)$

6.  $(11 + 11i) + (20 - 20i)$

For Problem 7-10, name the real component and imaginary component in each complex number.

7.  $4 - 2i$

8.  $18 + 11i$

9.  $2 - 4 + 6i$

10.  $22i - 11i + 6$

For Problem 11-14, name the complex conjugate and multiply the conjugate pair.

11.  $(2 + 3i)$

12.  $6 - 2i$

13.  $4 + i$

14.  $2 - i\sqrt{3}$

For Problem 15-17, multiply the complex numbers.

15.  $(4 + i)(5 - i)$

16.  $(3 + 3i)(2 - 2i)$

17.  $5i(3 + i)$

For Problem 18-20, divide the complex numbers.

18.  $\frac{i}{3+i}$

19.  $\frac{4+i}{5-i}$

20.  $\frac{2+3i}{4-5i}$



Section 4.10 The Quadratic Formula and the DiscriminantPractice Problems 4.10

For Problem 1-5, use the discriminant to determine if the quadratic equation has one solution, two solutions, or complex solutions.

1.  $x^2 + 3x - 6 = y$

2.  $3x^2 - 18 = y$

3.  $x^2 - 4x + 5 = y$

4.  $x^2 - 4x + 4 = y$

5.  $y + 2x = -2x^2 + 5x - 1$

For Problem 6-9, find the vertex form of the quadratic equation and use square roots to solve for  $x$ .

6.  $x^2 - 2x + 1 = y$

7.  $x^2 - 12x + 27 = y$

8.  $x^2 - 13x = 48$

9.  $x^2 + 2x + 19 = 0$

For Problem 10-14, solve for  $x$  using the quadratic formula.

10.  $6x^2 - 8x - 18 = y$

11.  $-4x^2 + 12x + 70 = 2$

12.  $8x^2 - x + 15 = 5$

13.  $x^2 + 2x + 3 = 0$

14.  $3x^2 - 2x + 4 = 0$

For Problem 15-20, solve the word problem.

15. The product of two consecutive integers is 1,122. Write an equation to represent the product.

16. Use the quadratic formula to find the two integers for Problem 15.

17. A garden is 12 x 16 meters. A walkway is to be constructed around the garden so the total area of the garden and walkway will be 285 square meters. Write the equation that models this situation.
18. Use the quadratic formula to find width and length of the garden and the surrounding walkway.
19. A shell is fired at a target from a trench 4 feet in the ground and travels at a velocity of 100 feet per second. The equation that models the shell fired is of the form  $h(t) = -16t^2 + vt + s_0$ , where  $t$  is time,  $v$  is velocity, and  $h_0$  is the initial height. Write an equation for how many seconds it will take the shell to hit the target at ground level?
20. Use the quadratic formula and the information given in Problem 19 to solve for the time.

Section 4.11 Projectile MotionPractice Problems 4.11

For Problem 1-12, use the equation below, which models a rock launched from a cliff.

Let  $t$  be time in seconds and let  $r(t)$  be the height of the rock in feet.

$$r(t) = -16t^2 + 48t + 96$$

1. What is the initial velocity of the rock launched?
2. How high is the cliff the rock is launched from?
3. Use  $t_v = -\frac{b}{2a}$  to find the time the rock reaches its maximum height.
4. Substitute  $t_v$  in the equation for  $r(t)$  to find the maximum height of the rock launch.
5. If the rock was launched from the ground using a trebuchet, what is the initial height?
6. Write an equation for the rock launched from a trebuchet that starts at ground level at an initial velocity of 44.1 ft/sec.
7. Write an equation to find the time the rock launched from the trebuchet will have a height of 29.4 ft.
8. Use the quadratic formula to find the time in Problem 7.

9. If you drop a rock from a cliff that is 144 ft. high, what is the initial velocity?
10. Write an equation that models the height in terms of time for the drop in Problem 9.
11. Your friend throws the rock from the same cliff in Problem 9 with an initial velocity of 64 ft./sec. Write the equation that models the situation.
12. Whose rock will land sooner, yours or your friend's, and how much sooner? (Solve for  $t$  using any previously learned method.)

For Problem 13-16, solve the word problem.

13. Paige hits a foul ball in a softball game. It goes straight up into the air at a speed of 90 ft./sec. She contacts the ball 3.5 feet above the ground. Write the projectile motion function that models the height of the ball over time.
14. At what time after contact does the ball stop ascending and start descending?
15. How high in the air does the ball travel?
16. The catcher dives and catches the ball just as it is about to hit the ground. Paige is out! How many seconds has she been running before the catcher gets her out?

For Problem 17 and 18, solve the word problem.

17. Two water bottle rockets are launched straight up. The height of each rocket in meters is given by the following quadratic equations:

$$\text{Rocket 1: } H_1 = -16t^2 + 98t$$

$$\text{Rocket 2: } H_2 = -16t^2 + 62t$$

The height is  $H$  and the time is  $t$ . Write one expression that shows how much farther Rocket 1 travels than Rocket 2.

18. Write an expression to show how far both rockets travel together.

For Problem 19 and 20, solve the word problem.

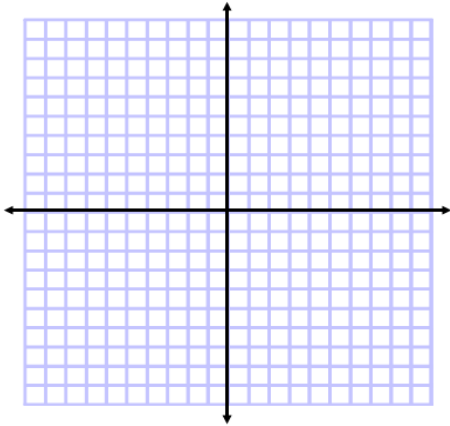
19. Professor Swiharts' Ace Flyer blasts off from the ground with a velocity of 56 ft./sec. The equation which models this is  $h(t) = -16t^2 + v_0t + s_0$ , where  $v_0$  is the initial velocity,  $t$  is the time, and  $s_0$  is the initial height. Find the time and height when the "Ace Flyer" is at its highest.

20. The vertex of the parabola occurs at the maximum height  $(t, h(t))$  found in Problem 19. Let the  $x$ -axis be time in seconds and the  $y$ -axis be height in feet. The maximum point occurs halfway through flight. Sketch the path of the Ace Flyer. What is the total length of the flight?

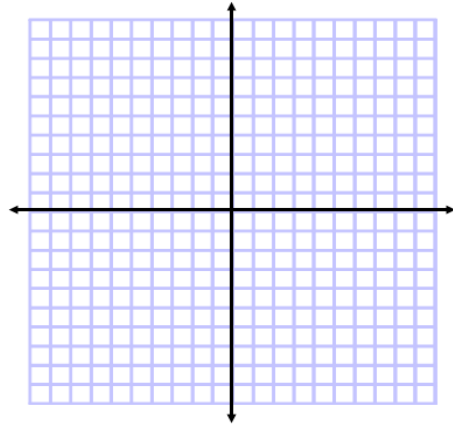
Section 4.12 Quadratic InequalitiesPractice Problems 4.12

For Problem 1-20, sketch the graph of each inequality. Find the scale to fit as much of the graph as possible.

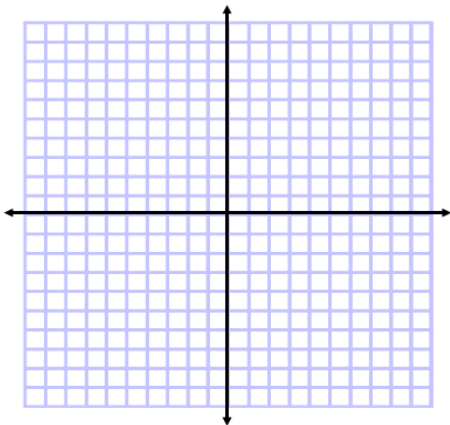
1.  $x^2 - 8x + 12 < y$



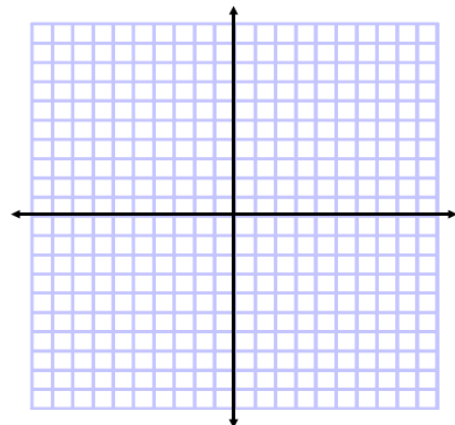
2.  $x^2 - 2x - 8 \leq y$



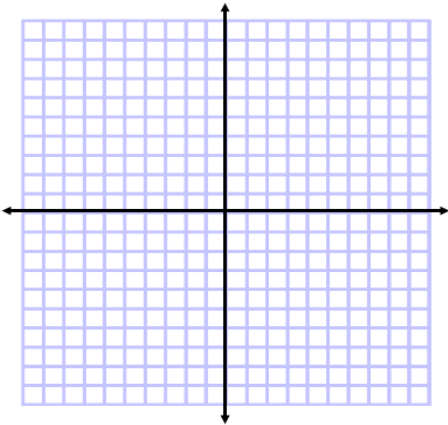
3.  $y > x^2 - 4x$



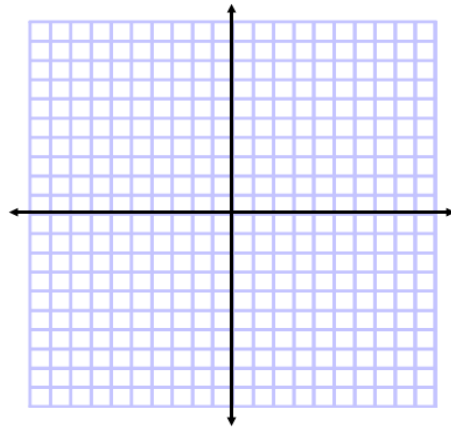
4.  $x^2 - 9x + 20 \leq y$



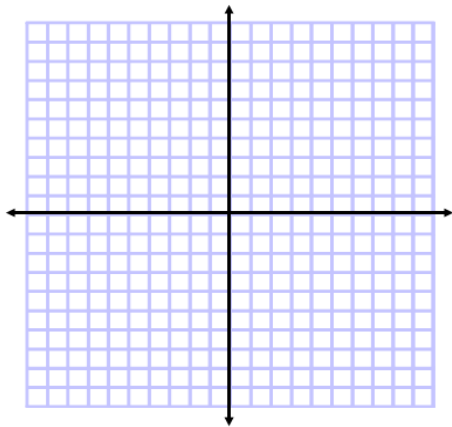
5.  $x^2 - 4x + 3 \geq y$



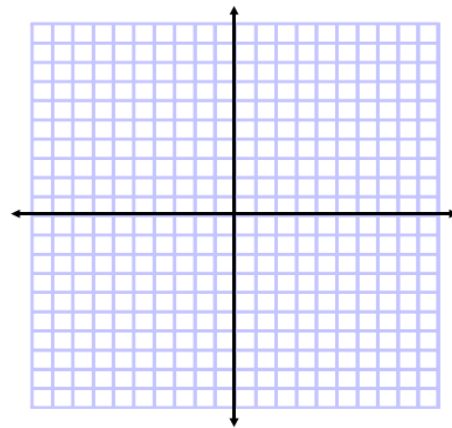
6.  $x^2 + 6x + 7 \geq y$



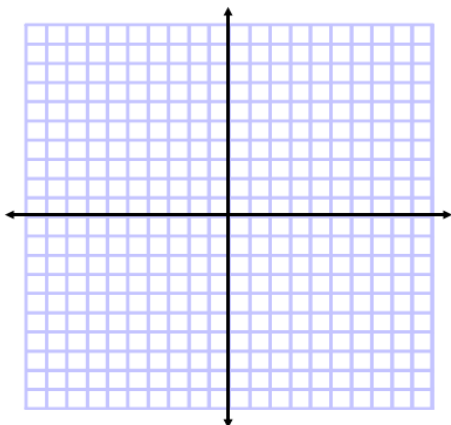
7.  $y \geq x^2 + 2x + 3$



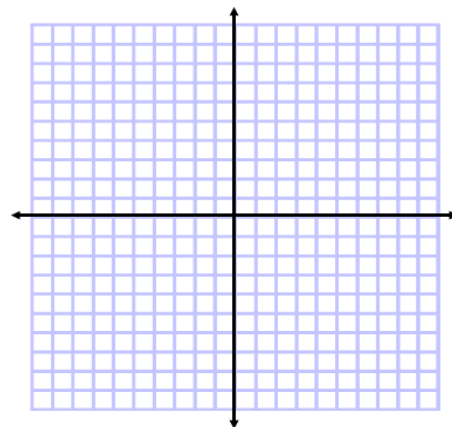
8.  $x^2 + 6x + 12 < y$



9.  $y \geq x^2 - 6x + 11$

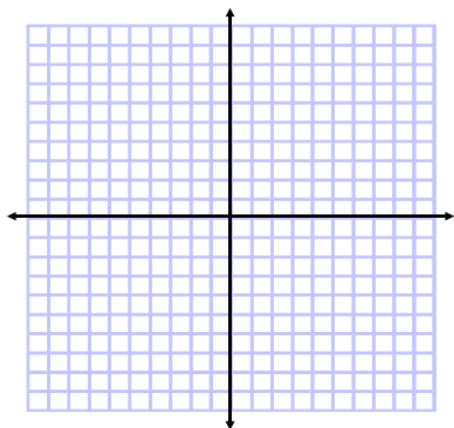


10.  $x^2 + 2x + 5 \leq y$

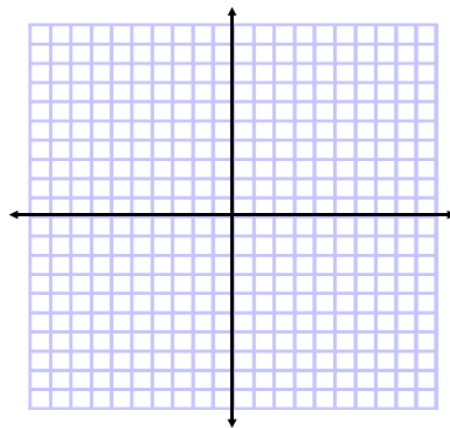




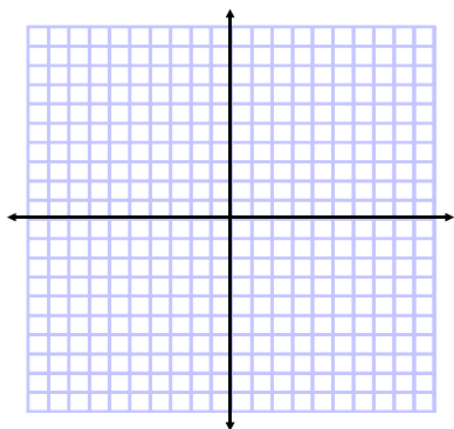
11.  $x^2 - 4 \geq y$



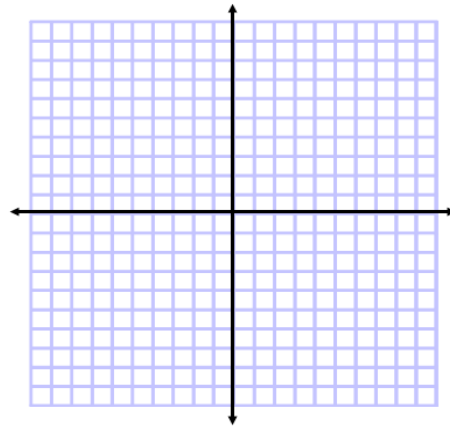
12.  $x^2 - 7x - 18 \leq y$



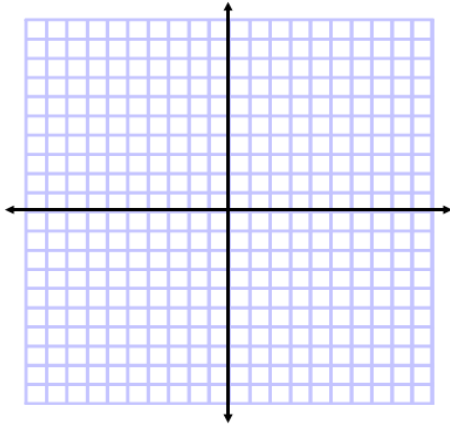
13.  $y < x^2 - 3x$



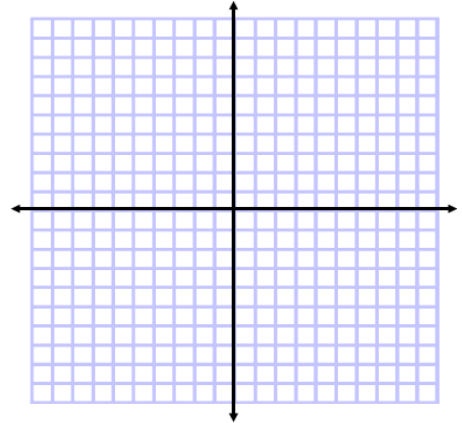
14.  $y > x^2 + 3x + 5$



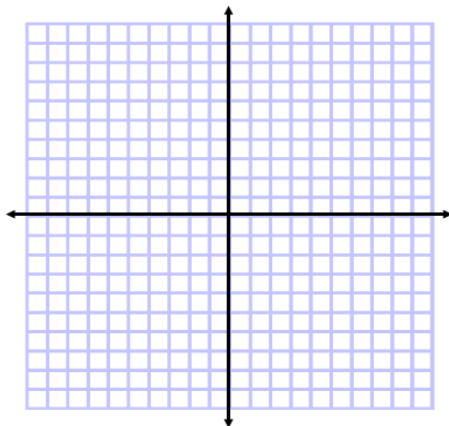
15.  $x^2 - 6x + 13 > y$



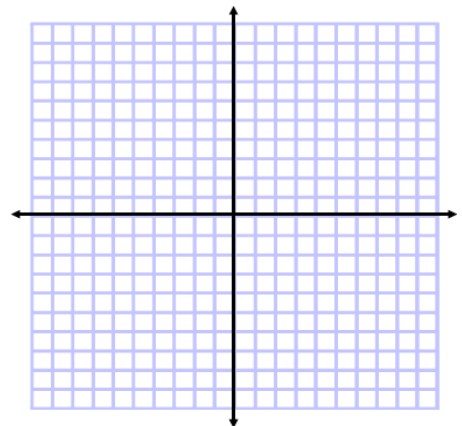
16.  $x^2 + 2x + 2 \leq y$



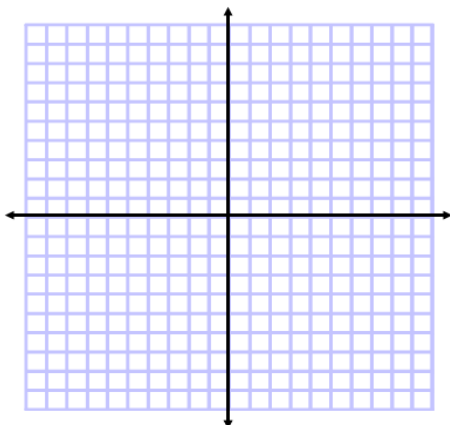
17.  $y \geq x^2 - 8x + 17$



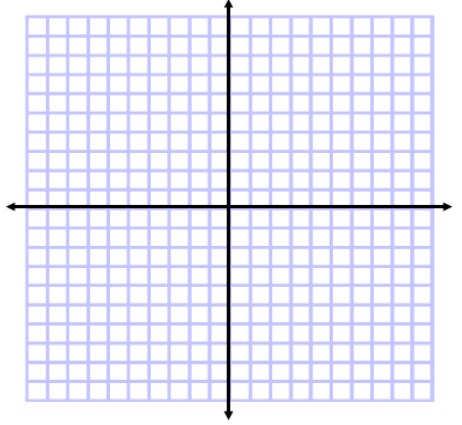
18.  $x^2 + 2x - 15 \geq y$



19.  $x^2 - 6x + 5 > y$



20.  $x^2 + 8x + 15 \leq y$



Section 4.13 Quadratic Math HandsPractice Problems 4.13

## Finding the Quadratic Equation of Your Hand

1. Use your hand that is drawn on the graph. Locate the coordinates of the vertex. \_\_\_\_\_

(This is the point where the parabola changes directions and is the pair  $(h, k)$  in the graphing equation.)

2. Locate three other points on the parabola. Draw them in colored pencil and list them below.

Point 1 \_\_\_\_\_ Point 2 \_\_\_\_\_ Point 3 \_\_\_\_\_

3. Is the parabola in your hand opening upward or downward? \_\_\_\_\_

Is the value of  $a$  positive or negative? \_\_\_\_\_

4. The graphing form of a parabola is  $y = a(x - h)^2 + k$ . Substitute the coordinates of your vertex in the equation and write it here. \_\_\_\_\_

5. Using one of the points above and substitute that in the equation for  $(x, y)$  and solve for  $a$ .

Show work here:

$a =$  \_\_\_\_\_

6. Using the graphing form of the equation, substitute the values of  $a$ ,  $h$ , and  $k$  in the equation and leave  $x$  and  $y$  as the variables  $x$  and  $y$ . This is the quadratic equation of the parabola in your hand.

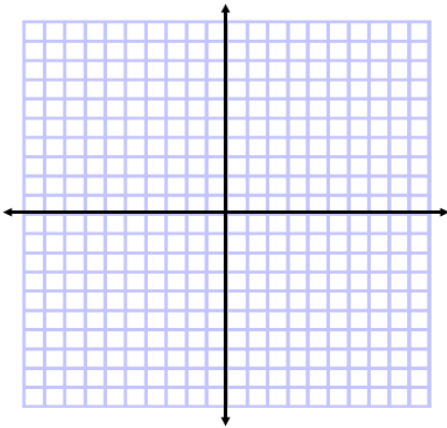
7. Rewrite the equation in standard form and then check it on the DESMOS® calculator. Put in the domain restrictions to see if the line of the equation covers the line of your hand.

Section 4.14 Module Review

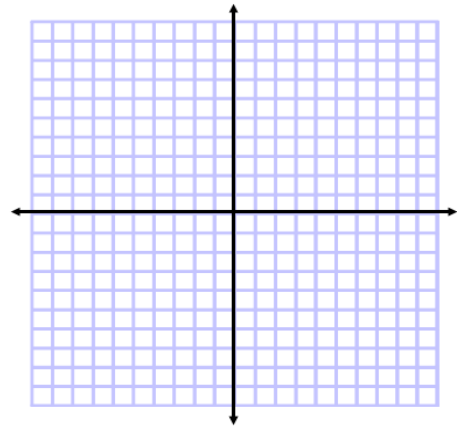
1. Use the formula  $p = 2\pi \sqrt{\frac{L}{g}}$  to find the formula for the length of the chord ( $L$ ) in terms of the period ( $p$ ) of the pendulum.

For Problem 2-3, find the zeroes of the equation and the vertex and sketch the graph.

2.  $y = x(x + 8)$



3.  $y = -x(x + 6)$

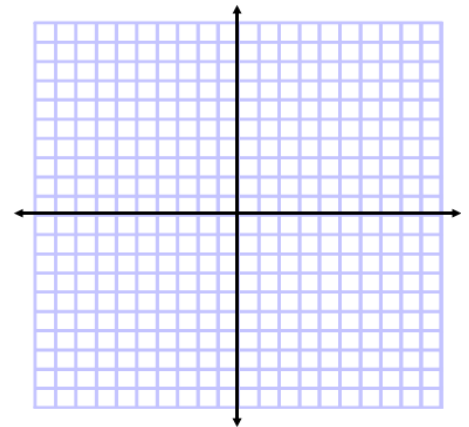
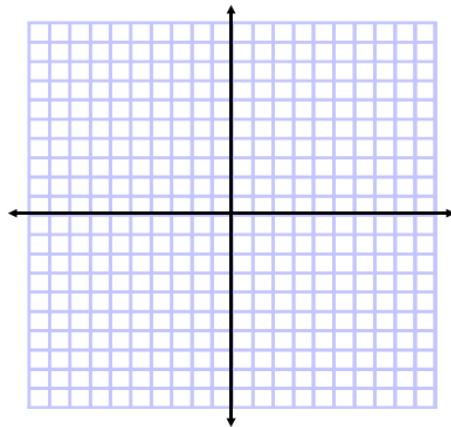
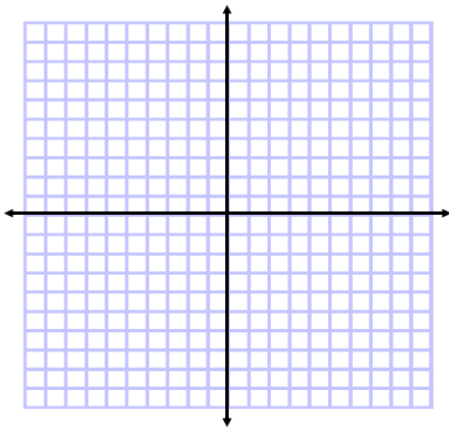


For Problem 4-6, factor the quadratic equations and use the zero-product property to find the  $x$ -intercepts. Find the vertex and sketch the graph.

4.  $x^2 - 7x + 10 = y$

5.  $x^2 + 2x - 35 = y$

6.  $2x^2 + 11x + 12 = y$



For Problem 7 and 8, use the formula  $x_v = \frac{-b}{2a}$  to find the value of  $x$  at the vertex, then substitute  $x_v$  in the equation, and solve for  $y_v$  to find the vertex.

7.  $5x^2 - 2x - 16 = y$

8.  $3x^2 + 3x - 18 = y$

For Problem 9 and 10, find the vertex by completing the square.

9.  $x^2 - 6x + 10 = y$

10.  $x^2 - 10x + 29 = y$

For Problem 11-13, use the discriminant to tell if there are one or two solution(s), or no real solution. Write the equation in standard form first.

11.  $10x^2 - 10x + 5 = y$

12.  $5x^2 + 38 = 9x$

13.  $3x^2 - 6x + 3 = 0$

For Problem 14-17, solve using the quadratic formula. Write the equation in standard form first.

14.  $-15x^2 - 8x - 1 = 0$

15.  $4x^2 - 6x = -2$

16.  $x^2 + 1 = -x$

17.  $3x^2 - 6x = -1$

For Problem 18-20, solve the word problem.

18. The length of a fence is three more than twice the width. What length and width give a total area of  $27m^2$ ?

19. A golf ball is chipped from near a sand dune that is 3 feet below the green with an initial velocity of 49 feet per second. How long is the golf ball in the air before it lands on the green?

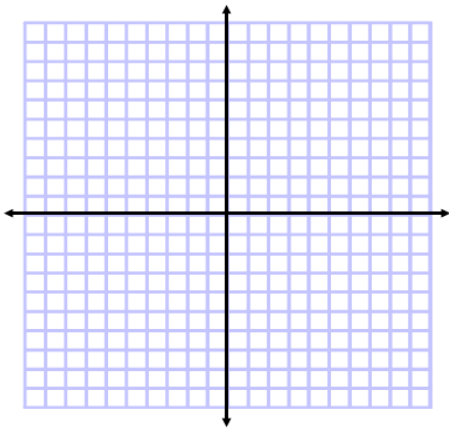
20. A tennis ball is launched from a mini catapult 506 feet up on a hill with an initial velocity of 48 feet per second. It lands on a cliff 218 feet above the ground. How many seconds is the tennis ball in the air?

Section 4.15 Module Test

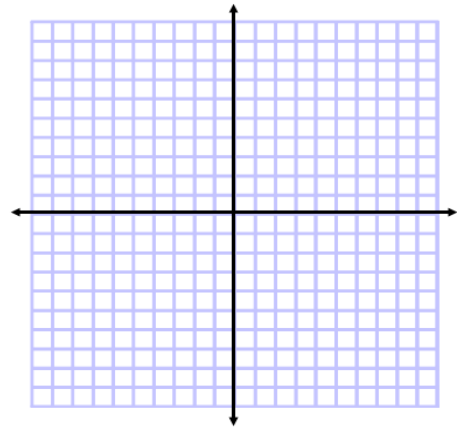
1. Use the formula  $(\frac{p}{2\pi})^2 g = L$  to find the formula for the time of the period ( $p$ ) of the pendulum in terms of the length ( $L$ ).

For Problem 2-3, find the zeroes of the equation and the vertex and sketch the graph.

2.  $y = 2x(x - 3)$

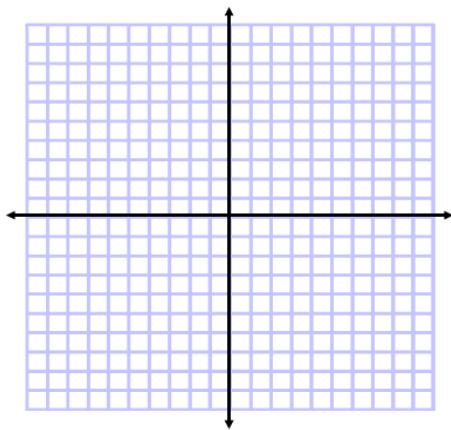


3.  $y = -2x(x + 4)$

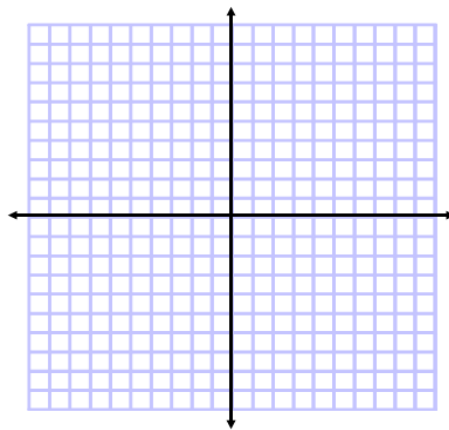


For Problem 4-6, factor the quadratic equations and use the zero-product property to find the  $x$ -intercepts. Find the axis of symmetry and the vertex and sketch the graph.

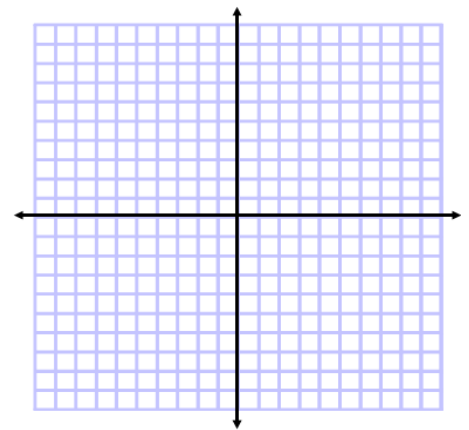
4.  $x^2 + 6x + 5 = y$



5.  $x^2 - 8x + 12 = y$



6.  $3x^2 - 7x + 2 = y$



For Problem 7 and 8, use the formula  $x_v = \frac{-b}{2a}$  to find the value of  $x$  at the vertex, then substitute  $x_v$  in the equation, and solve for  $y_v$  to find the vertex.

7.  $10x^2 - 20x - 15 = y$

8.  $7x^2 + 2x - 10 = y$

For Problem 9 and 10, find the vertex by completing the square.

9.  $x^2 - 60x + 25 = y$

10.  $x^2 + x - 2 = y$

For Problem 11-13, use the discriminant to tell if there are one or two solution(s), or no real solution. Write the equation in standard form first.

11.  $3x^2 - 6x + 1 = y$

12.  $3x^2 - 6x = -3$

13.  $x^2 + 36 = -12x$

For Problem 14-17, solve using the quadratic formula. Write the equation in standard form first.

14.  $x^2 + 3x - 6 = y$

15.  $3x^2 - 18 = y$



16.  $y + 2x = -2x^2 + 5x - 1$

17.  $3x^2 + 4x = 3x + 2$

18.  $x^2 - 81 = 0$

For Problem 19 and 20, solve the word problem.

19. Makenzie throws the discuss at the school track meet. It is modeled by the equation  $h(t) = -16t^2 + 60t + 5$ .

- a) What is the initial velocity of the discuss in feet per second?
- b) What is the initial height of the discuss in feet?
- c) How many seconds after release will the discuss hit the ground?

20. Professor Swiharts' "Ace Flyer" cleared a 30-foot flagpole when launched from a pad 4 feet above the ground with an initial velocity of 60 feet per second. The rocket landed on a garage 10 feet above the ground. How long was the rocket in flight?