Algebra 2 Module 5 Roots and Radicals

Section 5.1 Roots of Real Numbers

Practice Problems 5.1

For Problem 1-5, simplify the radicals and find the principal square roots.

1. $\sqrt{\frac{1}{4}}$

2. $\left(-\sqrt[3]{11}\right)^3$

3. $-\sqrt{\frac{1}{81}}$

4. $\sqrt[4]{(-5)^4}$

5. $\sqrt[5]{(-5)^5}$

For Problem 6-10, simplify the radicals and find all the real number solutions.

6. $x^2 = 36$

7. $x^2 - 5 = 0$

8. $4x^2 = 100$

9. $4x^2 + 36 = 100$

10. $x^2 = -9$

For Problem 11-20, tell whether each equation is true or false for all real values of the variable.

11. $\sqrt[9]{x^9} = |x|$

12. $\sqrt[4]{w^4} = |x|$

13. $\left(\sqrt[4]{x}\right)^4 = x$

14. $\sqrt{y^8} = y^4$

15. $\left(\sqrt[4]{x}\right)^4 = |x|$

16. $\sqrt[3]{y}$ is always a real number

17. $\sqrt[3]{y^3}$ is always a real number

18. $\sqrt[3]{y-4}$ is always a real number

19. $\sqrt{(x-2)^2}$ is always a real number

20. $\sqrt{x^2-1}$ is always a real number

Section 5.2 Roots of Non-Real Numbers

Practice Problems 5.2

For Problem 1-4, simplify the radicals and give the exact answer.

1. $\sqrt{13}$

2. $\sqrt{72}$

3. $\sqrt{20xyz}$

4. $\sqrt[3]{x^3y^5z^6}$

5. $2\sqrt{8}$

For Problem 6-15, simplify the radicals and give the non-real answers using imaginary numbers. Simplify any imaginary numbers if possible.

6. $\sqrt{-15}$

7. $\sqrt{-81}$

8. $3\sqrt{-169}$

9. $5i \cdot 4i$

10. $(i\sqrt{5})^2$

11. $-\frac{7}{i}$

12. $\sqrt[3]{-27}$

13. $\frac{9}{4i}$

14. $(5i)^2$

15. $(2i\sqrt{3})^2$

For Problem 16-19, solve the equation for the variable and simplify the non-real results.

16. $x^2 = -49$

17. $-3y^2 = 150$

18. $8 - w^2 = 10$

19. $4 + z^2 = 3$

For Problem 20, solve the word problem.

20. Find the reciprocal of i and simplify it.

Section 5.3 Radicals and nth Roots

Practice Problems 5.3

For Problem 1-3, rewrite the rational exponents using radical notation.

1. $x^{\frac{1}{m}}$

2. $x^{\frac{2}{3}}$

3. $3^{\frac{1}{3}}$

For Problem 4-6, rewrite the radicals using rational exponent notation.

4. $\sqrt[5]{y}$

5. $\sqrt[5]{x^2}$

6. $\sqrt[4]{41}$

For Problem 7-11, solve the word problem.

7. Show that $f(x) = x^2$ and $g(x) = x^{\frac{1}{2}}$ are inverses.

8. What is the inverse of $f(x) = x^6$? Use $f^{-1}(x)$ to solve.

9. What is the inverse of $f(x) = x^n$? Use $f^{-1}(x)$.

10. The tallest waterfall in the world is Angel Falls in Venezuela. It is 3,230 ft. high. The water falls with an uninterrupted drop for 2,647 feet. The structure at the top is completely flat. The equation that models the drop is $16t^2 = 2{,}647$ (t is time in seconds). Calculate how long it takes the water to fall from the top to the pool at the bottom.

11. The tallest falls in the USA is Yosemite Falls in California. It has a vertical drop of 2,425 feet. Use the free fall equation from Problem 10 to calculate how long it takes the water to fall.

For Problem 12-14, rewrite the rational exponents as radicals and simplify if possible.

12. $81^{\frac{1}{2}}$

13. $81^{\frac{1}{3}}$

14. $5^{\frac{2}{3}}$

For Problem 15-17, solve the word problem.

15. The volume of a marble has the spherical formula $V = \frac{4}{3}\pi r^3$. Solve for r so that the function for volume is in terms of the radius, $V(r)$.

16. Let $V = 3\text{cm.}^3$ Find the length of the radius.

17. Use the radius in the original formula and see if it checks.

True or false.

18. The equations $f(x)$ and $g(x)$ are inverses if $f(x) = \sqrt[4]{x}$ and $g(x) = x^{\frac{1}{4}}$.

19. The ordered pair (1, 1) is a point that is a solution to every nth root function.

20. The decimal 1.58 is a sixth root of 15.625 if $1.58^{\frac{1}{6}} = 15.625$.

Section 5.4 Rational Exponents

Practice Problems 5.4

For Problem 1-9, simplify the radicals.

1. $(x^{\frac{3}{2}})(x^{\frac{2}{5}})$

2. $\frac{16x^5y^4}{-2x^{-4}y^3}$

3. $\frac{10abc^2}{0.5a^5b^2c}$

4. $(\frac{x^4}{xy})(\frac{xy}{x^3y^2})$

5. $(\frac{3x^3}{y})^2(\frac{1}{4x})^3$

6. $2^{-3} \cdot 2^4 \cdot \frac{1}{2}$

7. 5^{-2}

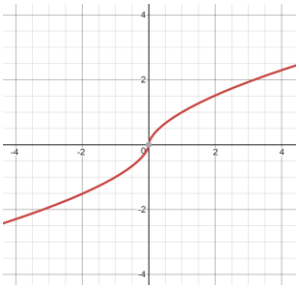
8. $125^{-\frac{2}{3}}$

9. $(\frac{xyz}{x^2y^5})^2$

For Problem 10 and 11, solve the word problem.

10. Let $f(x) = x^{\frac{3}{5}}$. Find the inverse $f^{-1}(x)$.

11. The graph of $x^{\frac{3}{5}}$ is shown below. Sketch the graph of the inverse. Is the inverse a function?



For Problem 12-16, simplify the expressions so there are no rational exponents.

12. $64^{-\frac{1}{3}}$

13. $64^{\frac{1}{3}}$

14. $64^{-\frac{2}{3}}$

15. $64^{-\frac{3}{2}}$

16. $64^{-\frac{1}{2}}$

For Problem 17-20, solve the word problem.

17. Write the following numbers in order from least to greatest:

$2^{\frac{1}{4}}$ 2^{-3} 2^{3} 2^{0}

18. Write the following numbers in order from greatest to least:

$3^{\frac{1}{3}}$ $3^{-\frac{2}{3}}$ $3^{\frac{1}{2}}$ 3^{-1}

19. What is 3^0, 2^0, 1^0, $(\frac{1}{2})^0$, $(\frac{1}{4})^0$, $(\frac{1}{8})^0$? From this sequence, what can you infer that 0^0 would be?

20. What is 0^3, 0^2, 0^1, $0^{\frac{1}{2}}$, $0^{\frac{1}{4}}$, $0^{\frac{1}{8}}$? From this sequence, what can you infer that 0^0 would be?

Section 5.5 Operations with Radicals

Practice Problems 5.5

For Problem 1-7, simplify the radicals (assuming they are positive real numbers) by combining like terms.

1. $2\sqrt{5} + 7\sqrt{5}$

2. $2\sqrt{5} - 7\sqrt{5}$

3. $\sqrt[3]{2} + \sqrt[3]{7} + 6\sqrt[3]{2} - 4\sqrt[3]{7}$

4. $\left(\sqrt[3]{3} + \sqrt[3]{4}\right) + (2\sqrt[3]{4} - 5\sqrt[3]{3})$

5. $\left(\sqrt[3]{5} + 7\sqrt[3]{3}\right) - \left(4\sqrt[3]{5} + 2\sqrt[3]{3}\right)$

6. $\sqrt[3]{2} + \sqrt[3]{3} + \sqrt[4]{5} + 3\sqrt{2} - 5\sqrt[3]{3} + \sqrt[3]{2}$

7. $\sqrt[2]{(x+2)^2} - 3\sqrt[2]{(x+2)^2} - 4\sqrt[2]{x^2+2}$

For Problem 8-10, use the instructions given to solve the problem (assuming the radicals are positive real numbers).

8. We have already seen that $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$. Is $\sqrt[4]{100}$ the same as $\sqrt{\sqrt{100}}$?

9. If $\sqrt{25a + 25b} = \sqrt{25 \cdot (a+b)} = 5\sqrt{a+b}$, find $\sqrt{9a + 9b}$.

10. If $\sqrt{25a^2 + 25b^2} = \sqrt{25(a^2+b^2)} = 5\sqrt{a^2+b^2}$, find $\sqrt{9a^2 + 9b^2}$.

For Problem 11-16, simplify the quotient or product so there are no negative exponents and rationalize the denominator.

11. $\sqrt{3a} \cdot \sqrt{3a}$

12. $\sqrt[3]{64y^{-4}} \cdot \sqrt[3]{y^7}$

13. $\frac{\sqrt{27x}}{\sqrt{3x^{-1}}}$

14. $\sqrt{6x^{-3}} \cdot \sqrt{2x}$

15. $\sqrt[3]{8x} \cdot \sqrt[3]{6x^{-5}}$

16. $\frac{\sqrt{8x}}{\sqrt{2x^{-1}}}$

For Problem 17-20, solve the word problem.

17. Show that $(\sqrt[n]{x} \cdot \sqrt[n]{y})^n = xy$.

18. What is $\sqrt[n]{n}$ when $n = 2, 3, 4,$ etc. What does it seem to approach?

19. Rationalize the denominator to simplify.

a) $\sqrt{\frac{4}{10}}$

b) $\sqrt[3]{\frac{3}{10}}$

20. Why can't $\sqrt[3]{2x}$ and $\sqrt{5x}$ be multiplied?

Section 5.6 Products of Binomials with Radicals

Practice Problems 5.6

For Problem 1-4, simplify the radicals using the distributive property.

1. $(5+\sqrt{2})(5-\sqrt{2})$

2. $(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})$

3. $(2+2\sqrt{3})(3+4\sqrt{5})$

4. $(4+\sqrt{8})(3+\sqrt{8})$

For Problem 5-8, expand the radicals and simplify.

5. $(3-\sqrt{8})^2$

6. $(\sqrt{3}-\sqrt{6})^2$

7. $(\sqrt{2}+\sqrt{5})^2$

8. $(3\sqrt{7}-\sqrt{10})^2$

For Problem 9-12, multiply the radicals using long multiplication.

9. $(3-4\sqrt{3})(2+\sqrt{3})$

10. $(3-\sqrt{2})(5-2\sqrt{2})$

11. $(\sqrt{11}+1)^2$

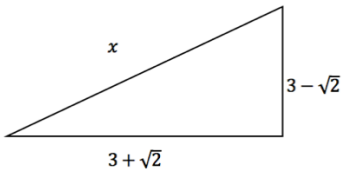
12. $(\sqrt{5}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

For Problem 13-20, solve the word problem.

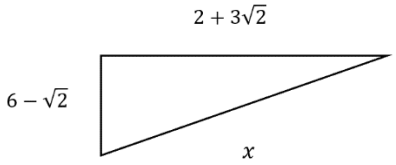
13. Show that $3-\sqrt{5}$ is a root of $x^2-6x+4=0$ by substitution.

14. Show that $1+\frac{\sqrt{2}}{2}$ is a root of $2x^2-4x+1=0$ by substitution.

15. Use the Pythagorean Theorem to solve for x in the triangle below.



16. Use the Pythagorean Theorem to solve the triangle below.



17. Simplify the radicals.

$$(\sqrt{a+1}+\sqrt{a})(\sqrt{a+1}-\sqrt{a})$$

18. Simplify $\sqrt{a-\sqrt{a}}\cdot\sqrt{a+\sqrt{a}}$. Use the distributive property under the radical sign.

19. If $f(x)=\frac{x}{x-1}$, find $f(1+\sqrt{3})$.

20. If $f(x)=\frac{x^2}{x+1}$, find $f(1-\sqrt{3})$.

Section 5.7 Quotients of Binomials with Radicals

Practice Problems 5.7

For Problem 1-12, simplify.

1. $\sqrt{6x}(\sqrt{3y}+\sqrt{3x^3})$

2. $\frac{2\sqrt{20}+3\sqrt{10}}{\sqrt{2}}$

3. $\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{5}}$

4. $\sqrt{\frac{1}{5}}-\sqrt{\frac{2}{5}}$

5. $\sqrt[3]{4}-\sqrt{\frac{1}{2}}$

6. $\sqrt{\frac{2}{3}}+\sqrt{8}$

7. $\sqrt{5x^3}-x\sqrt{3x}$

8. $\sqrt{2w^5}+5\sqrt{w}$

9. $4t(\sqrt{2t^5}+\sqrt{3t})$

10. $\sqrt{\frac{45y^2}{32x^4}}$

11. $\frac{14\sqrt{2}-5}{\sqrt{7}}$

12. $\frac{3}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

For Problem 13-15, name the conjugates of the binomial.

13. $5+\sqrt{6}$

14. $\sqrt{2}+7$

15. $\sqrt{w}+\sqrt{z}$

For Problem 16-20, simplify by multiplying by the conjugate of the denominator.

16. $\frac{1}{\sqrt{x}-\sqrt{y}}$

17. $\frac{1}{\sqrt{x}+1}$

18. $\frac{1}{3-\sqrt{y}}$

19. $\frac{\sqrt{7}+1}{3-\sqrt{5}}$

20. $\frac{4\sqrt{7}-\sqrt{2}}{\sqrt{3}+\sqrt{7}}$

Section 5.8 Solving Equations Containing Radicals

Practice Problems 5.8

For Problem 1-6, solve the radical equation and check the solution.

1. $\sqrt{x} = 13$

2. $\sqrt{y} = 2.2$

3. $\sqrt{w} + 4 = 10$

4. $\sqrt{x} - 5 = 0$

5. $4\sqrt{x} - 3 = 2$

6. $\sqrt[3]{x} + 4 = 9$

For Problem 7-10, tell whether each equation is linear or radical.

7. $x\sqrt{5} = 7$

8. $5\sqrt{x} = 3$

9. $x\sqrt{2} + x\sqrt{7} = 14$

10. $2\sqrt{x} + 7\sqrt{x} = 12$

For Problem 11-14, solve the linear equation without squaring both sides. Check by graphing each side of the equation and finding the point of intersection.

11. $5 + x\sqrt{3} = 10$

12. $x = 9 - x\sqrt{5}$

13. $4x + 2 = x\sqrt{7}$

14. $x\sqrt{20} = 15$

For Problem 15-18, solve the radical equation by squaring both sides in the final step. Check the solution.

15. $10\sqrt{x} = 20$

16. $1 + \sqrt{x} = 11$

17. $5\sqrt{x} + 4 = 24$

18. $2\sqrt{x} - 7 = -5$

19. Detectives investigating a crime can determine the speed of a car based upon the length of its skid marks on dry pavement using the formula $s = \sqrt{22d}$, where s is speed in mph and d is the distance the car traveled after applying the brakes. How fast was a car traveling if the skid marks measured were 110 feet long?

20. Researchers who design cars can also determine the braking distance required for a car's speed using the same formula used in Problem 19 ($s = \sqrt{22d}$). If a car is traveling at 60 mph, what is the required braking distance (given the car will leave skid marks)?

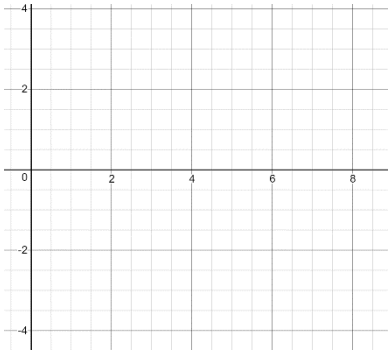
Section 5.9 Graphs of Radicals

Practice Problems 5.9

Complete the tables and graph the radicals for Problem 1-6. Explain any transformation from the given function.

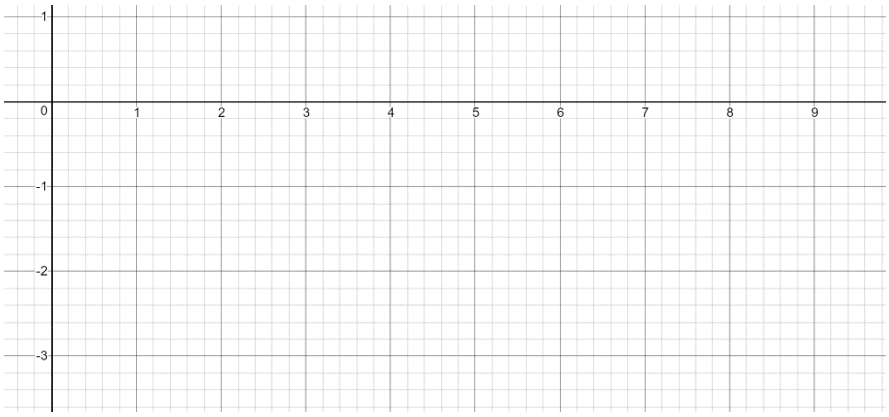
1. $f(x) = \sqrt{x} - 3$

x	y
1	
2	
4	
5	
9	



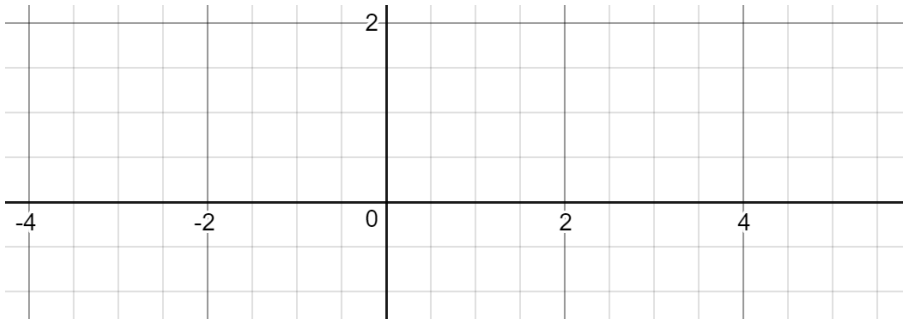
2. $f(x) = -\sqrt{x}$

x	y
1	
2	
3	
4	
5	
9	



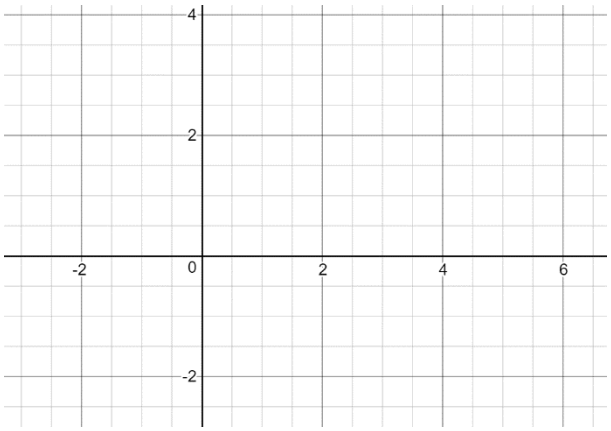
3. $f(x) = \sqrt{x+3}$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



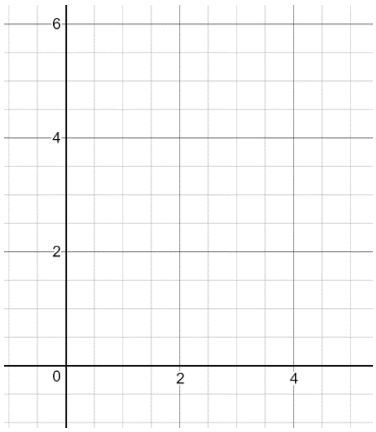
4. $f(x) = \sqrt[3]{x} + 2$

x	y
-3	
-1	
0	
1	
3	
5	



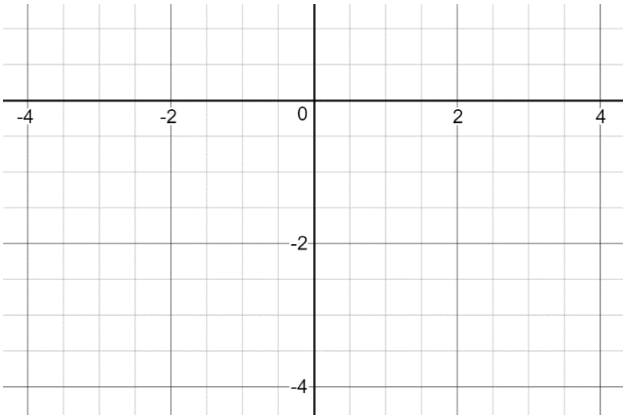
5. $f(x) = 2\sqrt{x} + 2$

x	y
0	
1	
2	
3	
4	



6. $f(x) = \frac{1}{3}\sqrt[3]{x} - 3$

x	y
-3	
-1	
0	
1	
3	



Section 5.10 Rational and Irrational Numbers

Practice Problems 5.10

For Problem 1-6, fill in the blanks to solve the radical for y.

$$5 + \sqrt{y + 7} = y$$

1. $\sqrt{y + 7} = y - \_\_\_$
2. $(\sqrt{\_\_\_\_\_})^2 = (y - 5)^2$
3. $y + 7 = y^2 - \_\_\_ + \_\_\_$
4. $0 = y^2 - \_\_\_ + \_\_\_$
5. $0 = (y - 9)(y - \_\_\_)$
6. $y = 9 \qquad y = \_\_\_$

7. Check both solutions in the original equation. Which one is the extraneous solution?

For Problem 8-16, tell which numbers are rational and which are irrational.

8. $\sqrt{3}$

9. $\sqrt{\frac{49}{100}}$

10. 1.787878 ...

11. 1.78788788 ...

12. $\frac{5}{9}$

13. Φ

14. $\sqrt{13}$

15. 3.79126

16. $\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{3}}$

For Problem 17-20, write each decimal as a proper or improper fraction in lowest terms.

17. $0.\overline{83}$

18. 3.04

19. $1.\overline{27}$

20. $0.\overline{4}$

Section 5.11 Complex Numbers

Practice Problems 5.11

For Problem 1-4, tell whether the solution is real or non-real.

1. $x^2 = 5$

2. $x^2 = -5$

3. $x^2 + 14 = 11$

4. $5x^2 + 5 = 20$

For Problem 5-12, simplify the radical to the form *a* + *bi*.

5. $\sqrt{-100}$

6. $-14\sqrt{-5}$

7. $-22\sqrt{-81}$

8. $(4i\sqrt{5})^2$

9. $(-i)^2$

10. $-\frac{13}{i}$

11. $\frac{3}{\sqrt{-3}}$

12. $\frac{\sqrt{22}}{2i\sqrt{7}}$

For Problem 13-17, tell whether each number is real, imaginary, or complex.

13. $\sqrt[3]{5}$

14. $i\sqrt[3]{5}$

15. $-17 + 2i$

16. $-\frac{2}{7}i$

17. $5.\overline{67} + \pi$

True or false.

18. $-2\sqrt{-144} = -4\sqrt{-36}$

19. $2i^2 = (i\sqrt{2})^2$

20. $(-3i\sqrt{6})^2 = (7i)^2$

Section 5.12 Sums of Complex Numbers

Practice Problems 5.12

For Problem 1-5, find the solution of the complex numbers.

1. $(9 + 2i) + (3 + 5i)$

2. $(7 - 2i) + (5 - 7i)$

3. $(9 + 2i) - (3 - 6i)$

4. $(11 + 9i) - (1 - 11i)$

5. $(6 - 4i) - (-3 + i)$

For Problem 6-10, find the reciprocals of the complex numbers and simplify to the form *a* + *bi*.

6. $3 + 4i$

7. $1 - 3i$

8. $-\sqrt{5} + i\sqrt{2}$

9. $-\sqrt{5} - i\sqrt{2}$

10. i^2

For Problem 11-16, solve the problem.

11. Is $1 - i\sqrt{7}$ a solution of $x^2 - 3x + 5 = 0$?

12. Is $1 + i\sqrt{5}$ a solution of $x^2 - 2x + 6 = 0$?

13. If $f(x) = \frac{x}{x+1}$, find $f(i)$.

14. If $g(x) = \frac{1}{x-1}$, find $g(i^2)$.

15. If $h(x) = \frac{1}{x^2}$, find $h(i+3)$.

16. If $k(x) = x^2$, find $k(i-\sqrt{2})$.

For Problem 17-20, simplify.

17. $\frac{1}{i^2}$

18. $\frac{2}{3-i}$

19. $\frac{\sqrt{5}}{i}$

20. $\frac{\sqrt{3}}{i-4}$

Section 5.13 Products of Complex Numbers

Practice Problems 5.13

For Problem 1-5, find the conjugates of the complex numbers.

1. $3 + 2i$

2. $5 - 6i$

3. $\sqrt{5} + 3i$

4. $-8i - \sqrt{3}$

5. $-2i + \sqrt{6}$

For Problem 6-8, multiply the conjugates.

6. $(2 + 3i)(2 - 3i)$

7. $(\sqrt{2} + i)(\sqrt{2} - i)$

8. $(3 - i)(3 + i)$

For Problem 9-12, multiply the complex numbers.

9. $3i(2 + 3i)$

10. $i(5 - 7i)$

11. $(4 + 3i)(2 - i)$

12. $(i + 7)(i - 4)$

For Problem 13-16, simplify.

13. $(i+\sqrt{2})^2$

14. $(\sqrt{7}-2i)^2$

15. $\frac{5+i}{5-i}$

16. $\frac{2}{4+i}$

For Problem 17-20, solve the word problem.

17. Is $2-i$ a solution of $x^2-4x+5=0$?

18. Is $1+3i$ a solution of $x^2-2x+10=0$?

19. The equations $\frac{1+i\sqrt{3}}{2}$ and $\frac{1-i\sqrt{3}}{2}$ are conjugates. Are they reciprocals? (If the product of the equations is equal to 1 then they are reciprocals.)

20. Find the reciprocal of $\frac{3+4i}{5}$. Is the reciprocal also the complex conjugate?

Section 5.14 Module Review

For Problems 1-3, simplify the radicals and find all real number solutions.

1. $\sqrt{\frac{4}{81}}$

2. $-\sqrt{\frac{1}{36}}$

3. $\sqrt[4]{(-3)^4}$

For Problem 4-10, simplify the radicals and give the exact answer. Use imaginary numbers for non-real solutions.

4. $3\sqrt{9}$

5. $2x\sqrt[3]{8}$

6. $\sqrt{108}$

7. $\sqrt{144x^2y^2}$

8. $-\frac{8}{i}$

9. $\frac{7}{5i}$

10. $(\frac{abc}{a^3b^3c^3})^2$

For Problem 11-18, perform the indicated operations on the radicals and simplify the answer using real and non-real solutions.

11. $5\sqrt{5}+9\sqrt{5}$

12. $3x(12x\sqrt{10}-4x\sqrt{10})$

13. $(7+\sqrt{13})^2$

14. $(4+2\sqrt{3})(19-5\sqrt{3})$

15. $\frac{\sqrt{7}-\sqrt{2}}{\sqrt{3}}$

16. $(3+4i)+(2-6i)$

17. $10i(5i+\sqrt{3})$

18. $\frac{1}{2i+1}+\frac{4}{2i-1}$

For Problem 19-20, solve the radical equations and simplify the answer.

19. $\sqrt{6x-1}=5$

20. $x=2-x\sqrt{11}$

Section 5.15 Module Test

For Problem 1-3, simplify and find the real number solution.

1. $\sqrt{\frac{6}{49}}$

2. $-\sqrt{\frac{9}{64}}$

3. $\sqrt[5]{(-2)^5}$

For Problem 4-10, simplify the radicals and give the exact answer. Use imaginary numbers for non-real solutions.

4. $5x\sqrt{81}$

5. $4\sqrt[3]{32}$

6. $xy\sqrt{16x^2y}$

7. $\frac{7}{i+1}$

8. $(\frac{3ab}{a^2b})^{-1}$

9. $\frac{2}{i}$

10. $(xy^2\sqrt{2x})^2$

For Problem 11-18, perform the operations and simplify.

11. $6\sqrt{x}-\sqrt{x}$

12. $-2(6x\sqrt{2}+5x\sqrt{2})$

13. $(5-\sqrt{5})^2$

14. $(x+4\sqrt{2})(y-5\sqrt{5})$

15. $\frac{3+\sqrt{7}}{\sqrt{2}}$

16. $-i(3i+i\sqrt{5})$

17. $(3+6i)-(18+4i)$

18. $\frac{7}{2i}-\frac{7}{5i}$

For Problem 19-20, solve the radical equation.

19. $\sqrt{4x+1}-8=13$

20. $4x+7=1-x\sqrt{2}$