Module 3 Working with Probability

Section 3.1 Combinations of Two or More Items

Looking Back 3.1

This module is about probability. The next module is about statistics. Both topics have real-world applications in working with numbers.

For now, we will find mathematical shortcuts in working with combinations of two or more items. We will investigate combinations by studying Morse Code, which was invented by Samuel Morse.

Samuel Morse was an artist from Massachusetts born in 1791. Though he was talented and a few of his sculptures became famous, his parents did not think art was a good way to make money. Therefore, he eventually found success as an inventor.

Morse's work caused him to travel a lot. While away from home on business, his wife died. In addition to his grief, Morse was frustrated because he did not receive the message of her death until a week after her passing and missed the funeral.

While traveling home by boat, Morse had a revelation: he believed that messages transmitted electronically would travel much quicker than others. These electronic messages would not be delayed for extended periods of time like the letter of his wife's death. Therefore, he combined his art with electronics and invented the Morse Code, which uses a series of dots and dashes to communicate messages.

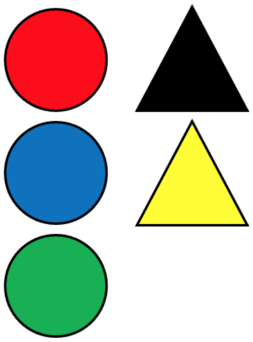
The first time Samuel Morse tried to demonstrate his invention through an insulator in New York's harbor, the boat's anchor cut into the insulator and Morse was laughed at. For the next eleven years, he worked to perfect his invention until it was the success it has become today.

Morse's income was often up and down but he always gave money to God's work and to the missionaries that carried it out. When his invention of the Morse Code became a success, he gave all the glory to God. Morse wrote, "I am perfectly satisfied that, mysterious as it may seem to me, it has all been ordered in view of my heavenly Father's guiding hand (J.H. Tinier, *Samuel F. B. Morse – Artist with a Message*, 1987, Mott Media, Milford, Michigan, VSA, p. 166)."

Looking Ahead 3.1

Example 1: In Morse Code, a message is composed of two components: a dash ($-$) or a dot ($\cdot$) followed by a dash ($-$) or a dot ($\cdot$). How many messages can be composed using a dash or dot followed by a dash or dot?

Example 2: A message is composed of a red, blue, or green circle followed by a black or yellow triangle. How many different messages can be composed using these figures?



If the first choice can be made in 3 ways and the second choice can be made in 2 ways, the total number of choices is $3 \cdot 2 = 6$ ways.

The Fundamental Principal of Counting says that if the first choice can be made in r ways, the second choice can be made in n ways, and the third choice can be made in z ways, then the total combination of choices can be made in $r \cdot n \cdot z$ ways, and so on.

Combinations are different ways that items can be combined numerically in which the order of the items does not matter.

Example 3: A password is made up of three one-digit numbers greater than 2 followed by three letters. How many possible passwords can be made?

Section 3.2 Permutations of Two or More Items

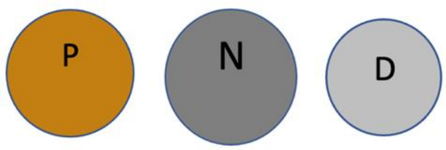
Looking Back 3.2

The Fundamental Principle of Counting was introduced in the previous section. Combinations refer to different groupings in which items are arranged in an order that does not matter. Permutations refer to different groupings in which items are arranged in an order that does matter.

Looking Ahead 3.2

Permutations are the different ways things can be arranged or ordered in which order does matter.

Example 1: How many ways can you arrange a penny, a nickel, and a dime?



Example 2: There are four children in a family. How many ways can they be arranged at a lunch counter with four stools? Three columns of combinations have been done for you starting with the letters. A, B, or C. Fill in the fourth column starting with the letter D.

A-B-C-D	B-A-C-D	C-A-B-D	_____-_____-_____-_____
A-B-D-C	B-A-D-C	C-A-D-B	_____-_____-_____-_____
A-C-B-D	B-C-A-D	C-B-A-D	_____-_____-_____-_____
A-C-D-B	B-C-D-A	C-B-D-A	_____-_____-_____-_____
A-D-B-C	B-D-A-C	C-D-A-B	_____-_____-_____-_____
A-D-C-B	B-D-C-A	C-D-B-A	_____-_____-_____-_____

Formula $n!$

Once an item is chosen, it cannot be chosen again. This is why order matters! To find the total permutations (or arrangements) of n items, multiply:

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots\ 3 \cdot 2 \cdot 1$$

There will be one less item to choose from each time until all the items have been chosen. The notation for this is as follows:

$$n!$$

(Read: "n factorial")

From Example 1, there are three items to choose from: a penny, a nickel, and a dime. The number of arrangements is as follows:

$$3! = 3 \cdot 2 \cdot 1 = 6$$

There are six permutations of the three coins.

Example 3: Use factorial notation to find the number of permutations of four children seated at a lunch counter with four stools.

Notice that AB is in the same group as BA, but in a different order. There is one combination of A and B, but two arrangements, one with A first and B second, or the other with B first and A second. A permutation is an arrangement in which order matters. In a combination, order is not important.

Section 3.3 Theoretical Probability

Looking Back 3.3

In this section, we will continue to explore probabilities. If something is probable, it will probably occur! It is likely to happen.

Looking Ahead 3.3

There are many terms we need to understand in dealing with probability.

An *experiment* is an activity such as rolling a die or tossing a coin. An experiment could be rolling a die twenty times. An experiment could be tossing a coin six times.
A *trial* is one activity of an experiment. A trial would be one roll of a die. A trial would be one toss of a coin.
An *outcome* is the result from a trial of an experiment. If you roll a die and get a 2 then 2 is the outcome.

Example 1: Jenita tosses a coin ten times. On the fourth toss, she gets a Heads. Name the experiment, trial, and outcome.

A *sample space* is all the possible _____________________ of a(n) _____________________. When tossing a coin, the sample space is Heads or Tails.

Example 2: What is the sample space when rolling a die?

A(n) ________________ is a non-zero subset of the ________________ ___________.

When rolling the die, an event would be $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, or $\{6\}$.

A ________________ ____________ has only ___________ outcome.

The set of $\{1\}$ is a simple event but the set of $\{1, 2\}$ is a not a simple event.
A compound event has more than one outcome.

A _____________________ ________________ is when you get the ________________ that you are

______________________ or hope to get in an experiment.

Rolling a 3 would be a successful event if you were hoping to get an odd number.

Example 3:	What are the simple events when tossing a coin?

Example 4:	Taliq wants to roll an even number when rolling a die. What would be a successful event?

We can write probabilities as ____________________. It is the ________________ between the number of

____________________ ________________ to the total number of ____________________

____________________.

The probability is found by using the following formula:

$$P(E) = \frac{\text{number of successful \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}}{\text{total number of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ outcomes}}$$

Example 5:	What is the probability of Taliq rolling an even number when he rolls a die?

Theoretical probabilities are what we want to happen, but when we actually do an experiment, we may get different results. We call the actual results experimental probabilities. When we toss a coin 50 times, we expect to get Heads about 50% of the time and Tails about 50% of the time. However, when we actually do the experiment, we often find different results. We will learn about experimental probabilities in the next section.

Section 3.4 Experimental Probability

Looking Back 3.4

We said in the previous section that theoretical probabilities are mathematical probabilities that can be calculated. Isn't it wonderful to know that the theoretical probability of God doing what He says He will do is always CERTAIN! He will do what His Word says; He will do it 100% of the time!

A probability occurs on a scale from 0 to 1, with 0 being no chance and 1 being a certainty. With God, it is always 1. This is why we can place our trust in Him. It is not a bet; it is a sure thing!

In this section, we will look at experimental probabilities. The experimental probability is what actually happens when the experiment is performed a certain number of trials. The experimental probability of an event is as follows:

$$\mathrm{P(E)} = \frac{\text{number of times a successful outcome actually occured}}{\text{total number of trials}}$$

Example 1: Tia rolls a die six times and gets a 2 four of those times. What is her probability of rolling a 2?

$$\mathrm{P(2)} = \frac{\square}{\square} = \frac{\square}{\square}$$

Let us play a game in which we will look at the theoretical probability and the experimental probability of an event.

Example 2:

Multiplying Dice Game

Below are the possible products when we roll a pair of dice:

x	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

What is the sample space?

What is the probability of rolling an even product?

$\mathrm{P(Even)} =$

What is the probability of rolling an odd product?

$\mathrm{P(Odd)} =$

Example 3:	Look at the game board to answer the questions below.

When do you get an odd product?

When do you get an even product?

What is the total probability of rolling an even *or* odd product?

Section 3.5 Simple and Compound Events

Looking Back 3.5

In the previous section, we talked about simple events. We said that the sample space for rolling a die is $\{1, 2, 3, 4, 5, 6\}$. A $\{3\}$ is only one outcome and is considered a simple event. An event is any non-empty and non-zero subset of a sample space. Therefore, an event must have at least one number or it becomes the empty set (written: "{ }" or "$\emptyset$"). If a subset includes only one member, it is called a simple event. The probability of rolling a 3 is 1 out of 6 (written: "$P(3) = \frac{1}{6}$").

Looking Ahead 3.5

A compound event has more than one outcome. Finding the probability means finding the sum of the probabilities of the individual events.

Example 1:	Use the given information to solve the problem.

An example of a compound event would be rolling a 4 or 5 when rolling a die. The event would be $\{4, 5\}$.

What is $P(E)$?

Example 2:	The results of an experiment that consists of rolling a six-sided die 100 times are given below.

Outcome	**Number of Occurrences**
1	32
2	20
3	10
4	18
5	8
6	12

a) Given the event is $\{1, 3\}$, then what is $P(E)$?

b) Given the event is $\{1, 5, 6\}$, then what is $P(E)$?

c) Given the event $\{1, 2, 3, 4, 5, 6\}$, then what is $P(E)$?

Section 3.6 Permutations of Some Objects

Looking Back 3.6

In the previous Practice Problems section, there was a coin collector arranging his Indian-head pennies. Let us suppose that although he has seven coins in his Indian-head penny collection, he can only display three at a time in his display case.

So, for the first frame of the display case, the collector chooses from all seven coins. For the second frame of the display case, the collector chooses from the six remaining coins. For the third frame of the display case, the collector chooses from the five remaining coins.

In all, there are $7 \cdot 6 \cdot 5 = 210$ different possible arrangements for the coins to be displayed in the three frames of the display case.

Before this section, the problems we looked at arranged all the objects in the problem together. Now, we are going to look at several objects, but only arrange a certain number of them at any time.

Looking Ahead 3.6

Example 1: If you have 6 books in your collection, but can only display 4 on your desk at a time, how many ways can they be arranged?

This is a ______________________________ of only 4 of the 6 books, so we do not include the last 2 books in the computation.

This is only a permutation of part of the objects, so we find the factorial of all the objects and divide by the objects we did not choose. This written:

$${}_nP_r$$

which is read: "n permute r," in the case there are n objects but only r are chosen. The formula to calculate permutations of part of the objects is as follows:

$${}_nP_r = \frac{n!}{(n-r)!}$$

Example 2: Use the permutation formula to find the arrangements for 4 of the 6 books in the collection.

Example 3: A grandmother has 10 pictures of her grandchildren but only has enough space on her shelf to arrange 5 of them. How many ways can she arrange the pictures on the shelf?

Section 3.7 Combinations of Some Objects

Looking Back 3.7

At the beginning of this module, we looked at combinations (groupings) of objects when all the objects were used. Then we looked at arrangements (order) of objects when all the objects were used. In the previous section, we looked at choosing an arrangement of some objects from several objects when not all the objects were arranged; the arrangement (order) was important. Those are called permutations. Think of position as important in permutations.

Looking Ahead 3.7

When choices are made and the order is not important, we call these combinations. We will now look at choosing a combination of some objects from a selection of several objects. Again, not all of the objects will be combined.

For example, if you have a salad with lettuce, tomatoes, and cheese, the order does not matter. However, if you have a locker combination of $24-14-6$, order does matter. If you have three numbers for your area code and they can repeat, the possible arrangements are $10 \cdot 10 \cdot 10$ or 10^3. The formula for n choices given you choose r is: n^r.

If there are eight lanes on a track and you want to arrange eight runners in them, you can put any one of the eight in the first lane, but then there are only seven remaining for the second lane, and so on ($8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, which is $8!$ or $n!$) given there are n objects and all of them are arranged without repeating. However, if you have eight runners and only choose to put three of them in lanes, then you have $8 \cdot 7 \cdot 6 = 336$ ways to choose them, or…

$$\ldots \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{5\cdot 4\cdot 3\cdot 2\cdot 1} \frac{\text{total choices}}{\text{the ones not chosen}} \ldots$$

… $\frac{8!}{5!}$, which is equal to $\frac{8!}{(8-3)!}$. This is $\frac{n!}{(n-r)!}$ for n objects when we choose r of them. There is repetition and order does matter.

Example 1: How many ways can first and second place be awarded to the 8 runners described above?

If we have n objects and choose only r of them, and there are no repeats and order does not matter, the formula gets adjusted. We use the formula:

$${}_nC_r = \frac{n!}{(n-r)!} \cdot \frac{1}{r!} = \frac{n!}{(n-r)!r!}$$

We reduce it by the ways the objects could be in order ($r!$) because we do not care about order in a combination.

Example 2: Let us look at a family with 6 children. If there are only 4 chairs at a table, they can be arranged in $6 \cdot 5 \cdot 4 \cdot 3$ ways, or 360 ways. Once 1 child sits down, there are only 5 children left to be seated, etc. Let us suppose they decide to play a game after dinner that requires 4 players at the table of 4 chairs. How many combinations are possible for children at the table of 4 chairs?

The general agreement is that $0! = 1$. There is only one way to arrange zero items. This helps solve equations.

Example 3: Let us look at the results from Example 2 given we have 0 children seated at the table to play the game. (We will go up from 0 to see if any patterns develop.)

Example 4: Let us look at the results from Example 2 given we have 1 child seated at the table to play the game.

Example 5: Let us look at the results from Example 2 given we have 2 children seated at the table to play the game.

Example 6: Let us look at the results from Example 2 given we have 3 children seated at the table to play the game.

Example 7: Let us look at the results from Example 2 given we have 4 children seated at the table to play the game.

Example 8: Let us look at the results from Example 2 given we have 5 children seated at the table to play the game. What do you think the result will be for 6 children seated at the table?

A very helpful triangle that displays this pattern is named after Blaise Pascal (Pascal's Triangle). It may have been discovered before Pascal's time in China, but for our purposes, the name remains Pascal's Triangle.

Row 0:					1					
Row 1:				1		1				
Row 2:			1		2		1			
Row 3:		1		3		3		1		
Row 4:	1		4		6		4		1	

Every number in a row is the sum of the diagonal numbers in the row above it (except for the 1s along the sides).

Example 9: Find the many patterns in Pascal's Triangle.

Example 10: Use Pascal's triangle to find the combinations in Example 2 through Example 6.

Blaise Pascal was a mathematician who spent time looking for mathematical probabilities in gambling after a gambler came to him with a specific problem (letters between Pascal and fellow mathematician Pierre de Fermat later confirmed they laid the foundation for probabilities). Pascal used his triangle to verify his conclusions about gambling. You can read more about this online if you are interested.

For now, we will examine Pascal's greatest wager. He thought it best to gamble on faith and not wager on unbelief. He said: "Belief is a wise wager. Granted faith cannot be proved, what harm will come to you if you gamble on its truth and it proves false? If you gain, you gain all; if you lose, you lose nothing. Wager then, without hesitation, that He (God) exists." How would you explain to a friend what Pascal means by this?

When one gambles, or places a wager, there is a risk of loss. God warns us against this. There is no guessing with God; faith in Him and what He will do for you is a sure thing!

We will end this section with another quote from Pascal: "There is a God-shaped vacuum in the heart of every man which cannot be filled with any created thing, but only by God, the Creator, made known through Jesus."

Section 3.8 The Law of Large Numbers

Looking Back 3.8

We are going to look at coin toss experiments and true-false problems again in this section. Let us first discuss what the exponential means.

Base: What is being multiplied → 3^8 ← Exponent: How many times the base is being multiplied by itself

The exponential 3^8 means 3 is being multiplied by itself 8 times: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 6{,}561$.

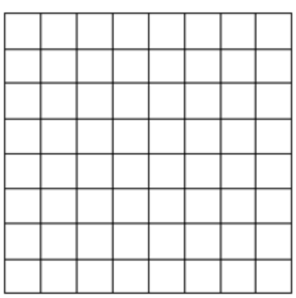
Some events in an experiment are dependent on one another: when one event occurs, it influences the probability of another event occurring. These are called *dependent* events.

Some events in an experiment are not dependent on one another: when one event occurs, it does not influence the probability of another event occurring. These are called *independent* events.

Is a true-false test made up of dependent events or independent events? Each problem can be true *or* false, which means each event is independent of the other event.

Example 1: If you take a 10-problem true-false test, how many trials are there? How many possible outcomes are there?

Example 2: How many pieces of rice will be on a checkerboard if 2 pieces of rice are placed in Square One, and it is doubled in Square Two, and so on for all sixty-four squares on the 8" by 8" checkerboard shown below?



The Law of Large Numbers says that the more trials of an experiment we perform, the better (more accurate) the results. In mathematical terms, the experimental probability gets closer to the theoretical probability, which means what we actually want to happen gets closer to what think should probably happen in the experiment.

A genome is an organism's complete set of deoxyribonucleic acid (DNA); DNA molecules are composed of two strands of nucleotide bases paired together. There are four nucleotide bases: Adenine (**A**), Guanine (**G**), Cytosine (**C**), and Thymine (**T**); **A** always pairs with **T** and **G** always pairs with **C**. These matches are referred to as base pairs. Each human is made in the image of God and has a unique genome that contains 23 pairs of chromosomes and approximately 3.2-billion base pairs (3.2-billion nucleotide bases on each strand). Scientists estimate that all humans share 90% to 99.9% of the same DNA code.

Example 3: If humans have 23 pairs of chromosomes, how many possible combinations are there in human DNA?

Example 4: There are 3.2-billion nucleotides and 4 bases in genomes. How many possible combinations are there in genomes?

Example 5: What is the probability the nucleotides and bases will combine to make one unique you?

Section 3.9 Area Models

Looking Back 3.9

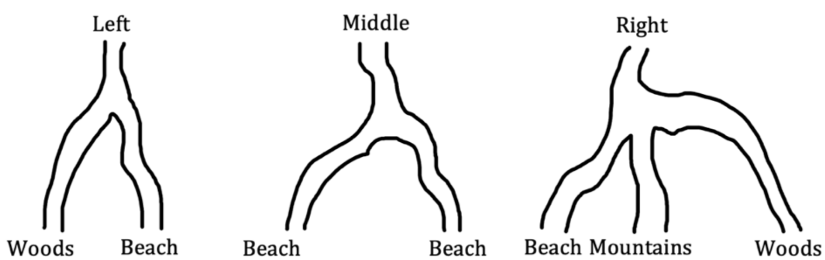
Tree diagrams help us understand probabilities. We can also look at probabilities using a diagram that is called an area model.

In using models to find the probability of one outcome *and* another outcome, we multiply. In using models to find the probability of one outcome *or* another outcome, we add.

Example 1: Suppose you have a penny and a dime in your left pocket and two nickels and two quarters in your right pocket. Given you pull one coin out of each pocket, make a diagram to represent the different probabilities involved in the scenario. Write all the possible sums of money that can be pulled from the two pockets.

Example 3: Suppose you are walking through the woods and want to go to the beach. At the start, you can choose from three paths with the end goal of getting to the beach.

The left path leads to more woods or the beach. Both middle paths lead to the beach. The right path leads to the beach or the mountains or the woods. What is the probability of reaching the woods, the mountains, or the beach through each of the paths?



Section 3.10 Expected Outcomes

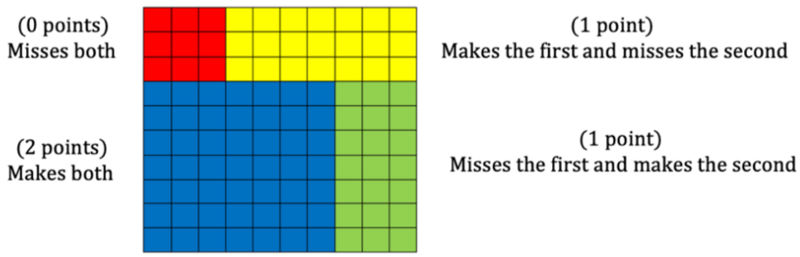
Looking Back 3.10

In this section, we will look into experiments that have expected outcomes. We will see if we can look at given situations and predict what would be expected to happen.

Looking Ahead 3.10

Example 1: Mosley's basketball team is down by two points at the end of the game. Mosley has just been fouled, so he gets to shoot two free-throws because his team is in the double bonus. Given his average free-throw percentage is 70%, what is the probability he will tie the game and put his team into overtime?

The geometric model below will help us analyze this situation. It is a 10 by 10 square.



This experiment is called a binomial experiment (or Bernoulli trial) for the following reasons:

1) The experiment consists of repeated trials (shooting free-throws).
2) Each trial has two possible outcomes that consist of a success or a failure (success is a make; failure is a miss).
3) The probability of success (P(E)) is the same for each successive trial (70% for a make; 30% for a miss).
4) The trials are independent. In other words, the outcome of one trial does not affect the outcome of another trial (whether the first free-throw is made or missed, the second free-throw could be made or missed).

This is named a Bernoulli trial after the Swiss mathematician Jacob Bernoulli, who worked specifically with probability and statistics. Bernoulli used "1" for a success and "0" for a failure.

Jacob Bernoulli first studied theology until his younger brother (a fellow mathematician, Johann) asked him to teach at Basel, Switzerland in 1687. At first, they worked together on Calculus, but would later go on to rival one another in the field.

Jacob Bernoulli would stay head of the mathematics department at his university until his death, leaving his brother Johann to take over the position.

Section 3.11 Expected Value

Looking Back 3.11

Let us revisit the basketball problem in which Mosley had the chance to tie the game and put his team into overtime by making two free-throws.

We discussed expected outcomes, which are what we think should happen. If all goes as expected, hoped for, or as planned, then we get an *expected value*.

Example 1: When Marcus was a sophomore, his free-throw average was 50%. That year, Marcus went to the foul line fifty times for a total of 100 free-throws (given each time he goes to the line he gets two shots). What percent of the time would you expect a score of 0, 1, or 2?

Looking Ahead 3.11

The expected value can also be determined in the lottery or for games at a carnival, and so on.

Example 2: A school has a fundraiser carnival to make money for Anne-Marie, who has Rhett's Syndrome. If the game wins, all the money goes to the charity, and the student has fun playing the game. If the student wins, the student keeps the money and donates it to a charity of their choosing.

Suppose your friend is at the fundraiser carnival playing a game. One game is called "Match Up." Three cards with numbers on them are pulled from a pile and put on a board. Your friend picks up 3 cards from the pile and turns them over. If he has one match with any of the cards on the board, he gets $1.00. If he has 2 matches with any of the cards on the board, he gets $2.00. If your friend has no matches with the cards on the board, he gets $0.00. Below (on the next page) are the experimental wins after ten rounds of this game are played. What are your friend's expected winnings per game?

Experimental Results for Ten Rounds of "Match Up"

Event	**Prize**	**Number of Wins**	**Probability of a Win**	**Winnings ($)**
0 matches	$0	5		
1 match	$1	4		
2 matches	$2	1		
	Total			

In this case, the expected value is the ___________________________ amount you expect to win per game. This actually means you won $0.60 per round for ten rounds of the game, or $6.00 in total. The game won $0.40 per round for ten rounds of the game, or $4.00 in total. You made $2.00 more than the game. Does this mean you won?

Section 3.12 "Dice Sums" Game

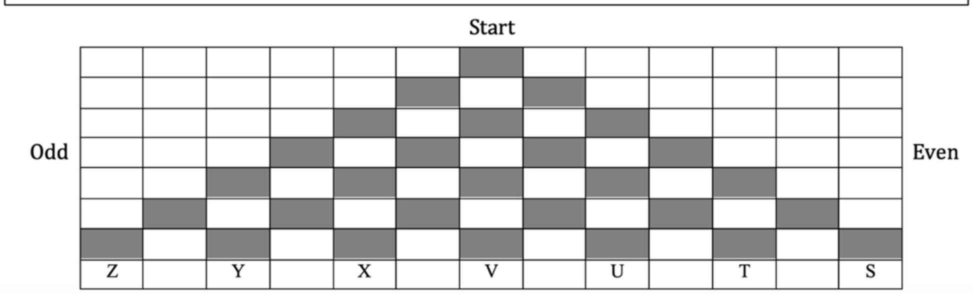
Looking Back 3.12

In Problem 1 of Section 3.3 Theoretical Probability: Practice Problems 3.3, you wrote all the possible outcomes of sums that could occur when you roll two dice. There were eighteen possible outcomes. You discovered the sample space was all numbers 2 through 12. The probability was $\frac{1}{2}$ for both even and odd sums occurring, so each were equally likely to occur. The sum of 2 or 12 had equally likely outcomes with a probability of $\frac{1}{18}$ for each.

Now, we are going to play a game called "Dice Sums" to see if the experimental probability is the same as the theoretical probability. We want to find out if what really happens is what we thought would happen.

"Dice Sums" Game

1. Put a penny on the top square marked "Start."
2. Roll two dice. If the sum is odd, move diagonally down the game board to the left one space. If the sum is even, move diagonally down the game board to the right one space.
3. Roll the two dice six times and move the penny accordingly; it will end on one of the letters. Put a tally mark below the letter if the penny ends there.
4. Play ten rounds of the game (a game being six rolls of the two dice). The game is over when there are ten tally marks below the last row of letters.
5. Find the experimental probability of landing on each of the letters: (Z, Y, X, V, U, T, S)
6. Remember, the more the game is played the closer the experimental probability will be to the theoretical probability.



Before you start the "Dice Sums" game, pick the letter you think you will end on the most. Explain why. Try to find how many ways there are to get to each of the possible letters. For example, there is only one way to get to Z; it is to roll an odd sum six times in a row. Which letter besides Z has only one way to get to it?

Section 3.13 "Dice Products" Game

Looking Back 3.13

In Section 3.4, we learned about experimental probability and wrote the sample space for the product of the roll of two dice. We discovered there are eighteen different numbers in the sample space when multiplying two dice. The probability of an even number is $\frac{3}{4}$, so $\mathrm{P(Even)} = \frac{3}{4}$. The probability of an odd number is $\frac{1}{4}$, so $\mathrm{P(Odd)} = \frac{1}{4}$.

In the previous section, we learned some strategies for playing the "Dice Sums" game. In this section, we will be playing the "Dice Products" game using the same game board but with different methods. Please solve Problem 1 through Problem 5 in the Practice Problems section before beginning this game.

Looking Ahead 3.13

"Dice Products" Game

1. Put a penny on the top square marked "Start."

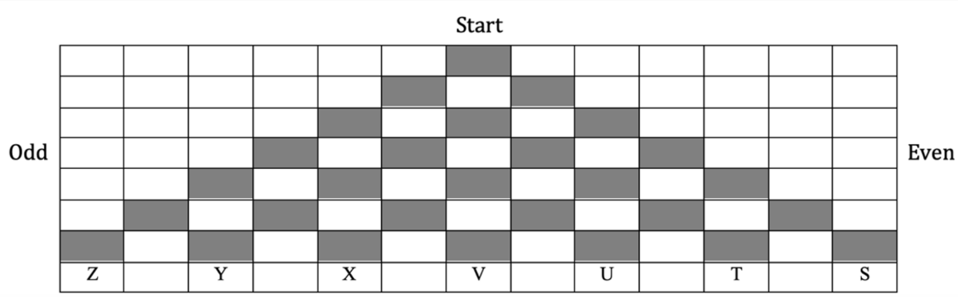
2. Roll two dice. If the product is odd, move diagonally down the left side of the game board one space. If the product is even, move diagonally down the right side of the game board one space.

3. Roll the two dice six times. The penny will end on one of the letters. Put a tally mark below the letter if the penny ends there.

4. Play ten rounds of the game (a game being six rolls of the two dice). The game is over when there are ten tally marks below the last row of letters.

5. Find the experimental probability of landing on each of the letters:
(Z, Y, X, V, U, T, S)

6. Does the "Dice Products" game have the same theoretical probabilities as the "Dice Sums" game? Can an area model be made for landing on each of the letters?



Before you start the "Dice Products" game, pick the letter you think you will land on the most. Explain why. Pick the letter you think you will land on the least. Explain why. Is it equally likely to get an even product as an odd product along the way? How does this affect the game?