Module 5 Working with Exponents

Section 5.1 Prime Numbers

Practice Problems 5.1

For Problem 1-10, use the given 100s chart to solve the problem.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Create a “Sieve of Eratosthenes.”

a) Cross out 1 because it is not a prime number.

b) Circle 2, the first prime number, and draw a diagonal blue line through every second number (multiples of 2). Do you see a pattern?

c) Circle 3, the second prime number, and draw a diagonal red line through every third number (multiples of 3). Do you see a pattern?

d) Circle 5, the next prime number, and draw a diagonal green line through every fifth number (multiples of 5). Do you see a pattern?

e) Circle 7, the next prime number, and draw a diagonal purple line through every seventh number (multiples of 7). Do you see a pattern?

f) Circle all the remaining numbers that do not have diagonal lines through them. Do you see a pattern?

2. Why did you not cross out all the multiples of 4 and 6?

3. What is the next prime number after 7? Put a circle around it and every prime number after it.

4. Can you stop checking for multiples after 7? Do you need to cross out multiples of 8, 9, or 10?

5. To find factors of 150, what number must you check up to?

6. Demonstrate the next five sums in Goldbach's Conjecture by using pairs of prime numbers.

7. Why is 1 not prime?

8. Why is 1 not composite?

9. Is 47 prime or composite?

10. Is 93 prime or composite?

<u>Section 5.2 Prime Factors</u>

<u>Practice Problems 5.2</u>

For Problem 1 and 2, solve the problem given.

1. Which is the product of the primes (the prime factorization) of 162?

a) $2 \cdot 3^4$ b) $2^2 \cdot 11$

c) $2^2 \cdot 3^2 \cdot 6$ d) $2 \cdot 7 \cdot 13$

2. Is $2^3 \cdot 5 \cdot 7$ the prime factorization of 140?

For Problem 3-6, find the prime factorization (product of primes) for the number given.

3. 122
4. 169
5. 328
6. 95

For Problem 7-10, solve the word problem given.

7. Anna Maria wrote the prime factors of 42 as $2 \cdot 3 \cdot 7$. Is she correct?

8. What is the sum of the prime factors of 68? Is it a prime or composite number?

9. Solve the cryptarithm:

Use 0, 1, 2, 7, 8, and 9. Each number is only used once.

```
     NO
    GUN
+    NO
   HUNT
```

10. The first cryptarithm appeared in the Belgian Journal Sphinx in 1931, when the term crypta-arithmetic was first introduced. Cryptarithms carried secret codes. Use the numbers 1 through 8 for letters A through G to solve the cryptarithm below.

```
   ABC
×   DE

   FEC
  DEC
  HGBC
```

Section 5.3 Prime Factors and Probability

Practice Problems 5.3

For Problem 1-4, solve the problem given.

1. How many factors does 504 have?

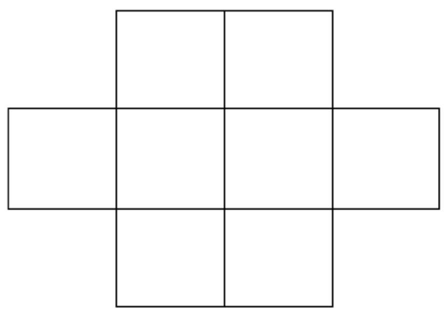
2. How many factors does 328 have?

3. What are the factors of 328?

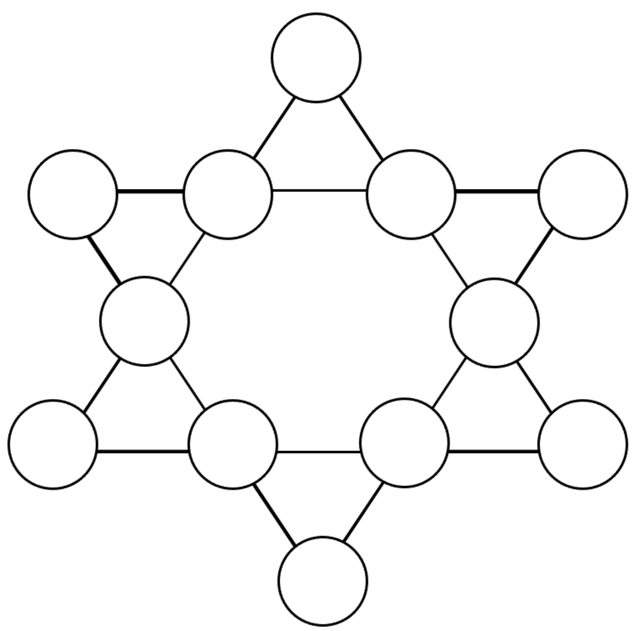
4. What are the prime factors of 504?

For Problem 5-10, solve the number theory problem given.

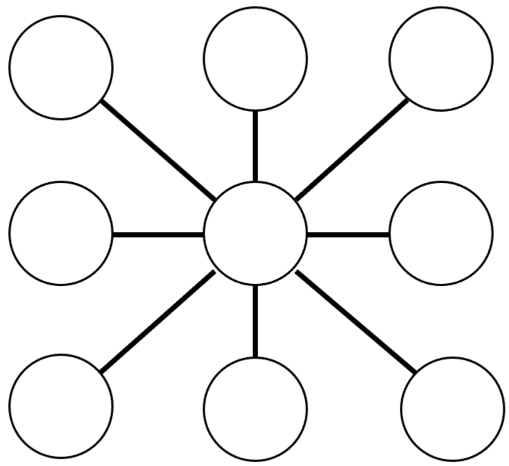
5. Use the numbers 1 through 8 in each of the given boxes so that a different number is in each box and no consecutive numbers are in touching boxes. For example, the number 4 cannot have the number 3 or 5 in the box above it, below it, or beside it.



6. There are twelve circles on the star below. Use the numbers 1 through 12 so that every line adds up to 26. Use each number once and once only.



7. Use the numbers 1 through 9 so that each number is in a circle on the number wheel and the sums of each spoke add up to 15.



8. Corey, Casey, and Tammy each choose one of the prime numbers 2, 5, or 11 (they can choose the same number). How many possible products are there for the number/numbers of their choosing?

9. If Corey, Casey, and Tammy each choose the same number from one of the prime numbers 2, 3, or 11 how many possible products are there?

10. Miles and Dakota are playing a game in which they start with one card and pick up two more. The odd sum wins; if both sums are odd, the cards go in the discard pile and they play again. At the beginning of the game, they can choose a 2 or a 3 for their first card. Which card should they choose to start with?

Section 5.4 Perfect Numbers

Practice Problems 5.4

For Problem 1-10, solve the problem given.

1. What is the next perfect number after 14 that is less than 30?

2. Euclid stated that if n is prime and $2^n - 1$ is prime, then $(2^{n-1})(2^n - 1)$ is perfect. The number 2 is a prime number, so let us see if it works when n is equal to 2.

For $n = 2\ldots$

$2^n - 1$

$2^2 - 1$

$4 - 1$

The number 3 is prime.

Because 3 is prime, then for $n = 2$, $(2^{n-1})(2^n - 1)$ must be perfect.

$= (2^{2-1})(2^2 - 1)$
$= (2^1)(4 - 1)$
$= (2)(3)$
$= 6$

The number 6 is perfect.

Now, see if Euclid's proof works using $n = 3$.

- Is $2^n - 1$ prime when $n = 3$?

- What is 2^{n-1}?

- When you multiply the products of $2^n - 1$ and 2^{n-1}, is the answer perfect?

3. The next two perfect numbers after 6 and 28 are 496 and 8,128. These two numbers were discovered by the Greeks before the time of Christ. What do you notice about these perfect numbers and the perfect numbers we have previously discovered?

4. Continue the pattern below. How long could this go on for?

$$41 + 2 = 43$$

$$43 + 4 = 47$$

$$47 + 6 = 53$$

______+______=______

______+______=______

______+______=______

5. Binary code is used in computers. Below is the binary code for the first three perfect numbers. If this pattern continued, what would be the binary code for the next perfect number?

IIO

IIIOO

IIIIIOOOO

IIIIIII____________________

6. Below is another interesting mathematics problem using number theory:

a) What is 12×30?

b) What is the LCM (Least Common Multiple) of 12 and 30?

c) What is the GCF (Greatest Common Factor) of 12 and 30?

d) What do you get when you multiply the LCM $(12, 30)$ by the GCF $(12, 30)$?

7. What is 0! (zero factorial)?

8. Euler's statement about primes is partially true, but we do know that prime numbers greater than 3 can be expressed as $6n + 1$ or $6n - 1$ when n is 1, 2, 3, 4, etc.

Circle the prime numbers greater than 3 below. Find n using $6n - 1$ or $6n + 1$ to get each of those numbers. For example, when n is equal to 1, then $6(1) - 1 = 5$ and $6(1) + 1 = 7$.

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31

9. Find a partner to a play a game of Factor Blast with. Use the 100s chart below to play the game.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Pick a number and add up all its factors; that is your total number of points (this does not include the number itself). For example, if you pick 22, its factors are 1, 2, 11, and 22; therefore, you would have $1 + 2 + 11 = 14$ for a total of 14 points. Cross out the number after it has been picked and all of its factors once they are found. Then have your partner pick a number and add up its factors (not including the number itself) for their total number of points. Remember to cross out all of the factors after each play also. You and your partner will take turns picking numbers and adding up their factors (and crossing them out) until all one-hundred numbers have been crossed out. After they have all been crossed out, whoever has more points wins!

10. Find the sum of the reciprocal of all the factors of the perfect numbers 6 and 28. What do you notice?

<u>Section 5.5 Amicable, Abundant, and Deficient Numbers</u>

<u>Practice Problems 5.5</u>

For Problem 1-10, solve the problem given.

1. Consider the amicable pair of numbers $69,615$ and $87,633$. Add up the sum of the digits. What do you notice? How friendly is this pair of numbers?

2. On page 167 of *Mathematical Magic Show*, Martin Gardener states: "The Pythagorean Brotherhood regarded 220 and 284 as symbols of friendship. Biblical commentators spotted __________ in Genesis 32:14 as the number of goats given to Esau by Jacob. A wise choice, the commentators said, because __________ expressed Jacob's love for Esau." What friendly number do you find in Genesis 32:14 even though its amicable mate is missing?

3. List each number as deficient, abundant, or perfect.

14	28
16	13
12	22
24	

4. Euclid wrote: "If as many numbers as we please, beginning from a unit, be set out continuously in double proportion until the sums of all become prime, and if the sum multiplied by the last make some number, the product will be perfect."

A unit is 1…

Double it is 2…

Add them and $1 + 2 = 3$*…*

Because 3 is prime, multiply 3 by the number before it (referring to the number before the equal sign, which is 2) …

Then $3 \times 2 = 6$*…*

The number 6 is perfect.

Use this method to find the next perfect number after 6. It is not as complicated as it seems. Begin with $1 + 2 + 4$.

5. Use Euclid's method to find the next perfect number after the one you found in Problem 4.

6. Find the factors of the perfect number from Problem 5, then find the sum of those factors to make sure it is a perfect number.

7. Sometimes an integer forms an amicable set with itself. We call this integer perfect, not amicable. Why is a perfect number such as 6 not amicable?

8. Is 25 perfect, deficient, or abundant?

9. Is 32 deficient or abundant?

10. Why are all prime numbers deficient?

<u>Section 5.6 Figurate Numbers</u>

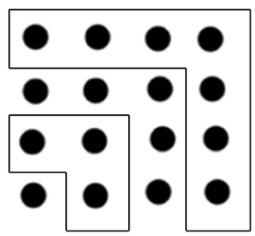
<u>Practice Problems 5.6</u>

For Problem 1-11, solve the problem given.

1. What is the sixth square number? Draw it below.

2. What is the seventh square number? Draw it below.

3. A nested model of square numbers can be drawn as well. Use the model below to answer the given questions.



a) What do you add to the first square number to get the second square number?

b) What do you add to the second square number to get the third square number?

c) What do you add to the third square number to get the fourth square number?

d) What do you add to the fourth square number to get the fifth square number?

e) What are these numbers called?

4. What is the sixth triangular number? Draw it below.

5. What is the sum of $1 + 2 + 3 + 4 + 5 + 6$ (the first six integers)? Use Gauss' method to show how you got your solution.

6. Use Gauss' method to find the two-hundredth triangular number.

7. Find the tenth square number.

8. Find the eleventh square number.

9. Find the twelfth square number.

10. Why is 50 not a perfect square?

11. Can you draw a square of 50 dots? What shape can you draw with 50 dots?

<u>Section 5.7 Fibonacci Numbers</u>

<u>Practice Problems 5.7</u>

For Problem 1-12, solve the problem given.

1. The number of petals on a flower are…

Rose: 5 Lily and Iris: 3 Buttercup: 5

What do you notice about these numbers?

2. How many legs does a starfish have? (Its likeness can be seen in the center of the sand dollar pictured to the right.

How many petals does a sand dollar have?

What do you notice about these numbers?

3. Use Example 2 from the Lesson Notes to find the number of petals in a complete turn (revolution) of each of the plants below.

Elm plant: Cherry plant: Pear plant:

What do you notice about these numbers?

4. Find the missing number in the arithmetic series below and tell what number gets added each time.

a) 5, 12, __________, ___________, __________, 40, 47, __________, 61

b) 0.55, 0.60, 0.65, __________, __________, 0.8, __________, 0.9

5. What is the rule for the geometric series below?

a) 1, 4, 9, __________, __________, __________, 49, 64, __________

b) 3, 6, 12, 24, __________, __________, 192

Arithmetic series involve addition. Geometric series involve multiplication.

6. Continue the Fibonacci sequence up to the fifteenth number.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, __________, __________, __________, __________

7. Divide each Fibonacci number by the number before it in the sequence for the first twelve numbers. What does the quotient seem to be approaching? (Round to the nearest thousandths place.)

8. Below are pairs of Fibonacci numbers that are consecutive (one after another). What are the quotients of each?

a) $\frac{233}{144}$

b) $\frac{377}{233}$

c) $\frac{610}{377}$

9. Use your birthday (month and day) to make a Fibonacci sequence. For example, if you were born May 18th, it would be the fifth month on the eighteenth day: the start of your series would be 5, 18 and continue 5, 18, 23, 41, etc.

Take your birthday out to the fifteenth number of the Fibonacci sequence:

_________ , _________ , _________ , _________ , _________ , _________ , _________ , _________ ,

_________ , _________ , _________ , _________ , _________ , _________ , _________ ,

10. Divide each number in your birthday sequence from Problem 9 by the number before it and see what number it approaches. (Round to the nearest thousandths place.)

$\frac{18}{5} =$

$\frac{23}{18} \approx$

$\frac{41}{23} \approx$

It should be getting close to the same number as in Problem 7 and Problem 8.

11. Here is another interesting pattern in the Fibonacci numbers: the sum of the first n numbers is equal to one less than the $(n + 2)$ Fibonacci number. Mathematically, we would write the following:

$$\mathrm{F}_1 + \mathrm{F}_2 + \mathrm{F}_3 \ldots \mathrm{F}_\mathrm{n} = \mathrm{F}_{\mathrm{n+2}} - 1$$

The sum of the first five Fibonacci numbers is one less than the seventh Fibonacci number.

$$\underbrace{1+1+2+3+5}_{12}\quad \underbrace{8\quad 13}_{n+2=7}$$

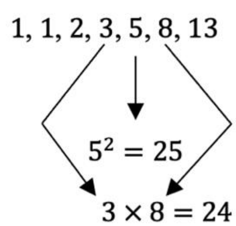
If n is equal to 5, the sum of the first n (first five) numbers is 12. If n is equal to 5, then $n + 2$ is equal to $5 + 2$, which is equal to 7, and the seventh Fibonacci number is 13.

One less than 13 is 12, which is the sum of the first five numbers!

$$1 + 1 + 2 + 3 + 5 = 12$$

Try it up to the sixth number in the sequence. Does it work?

12. Here is yet another interesting pattern in the Fibonacci numbers (there are many more!): square a number in the sequence and multiply the numbers on either side of the number you squared; the product and squared number will have a difference of one.



$$3 \times 8 = 24$$

$$25 - 1 = 24$$

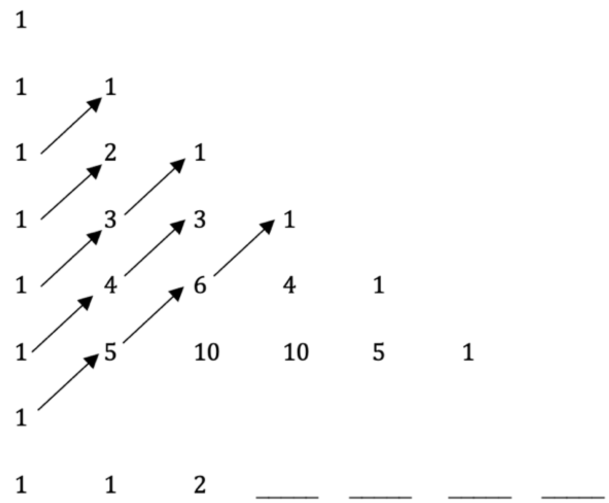
Try it for the sixth number in the sequence. Does it work?

<u>Section 5.8 Fibonacci Numbers and the Golden Ratio</u>

<u>Practice Problems 5.8</u>

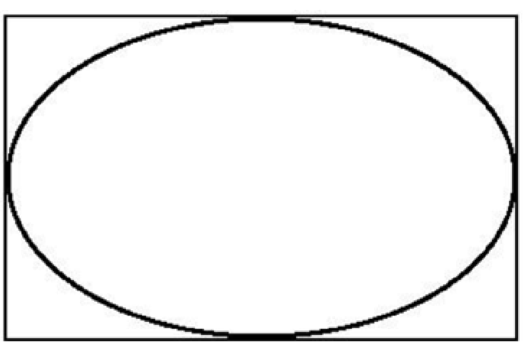
For Problem 1-4, solve the problem given.

1. Below is Pascal's Triangle written in rows and columns. Add the diagonals and see what you get.



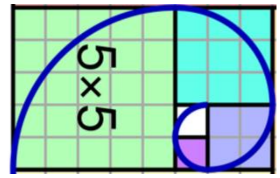
2. Measure the length and width of rectangle *b* from Activity 2 of the Lesson Notes in millimeters (this was the rectangle that was most pleasing to the eye). What is the ratio of its length to width?

3. Put an egg on a piece of paper and draw a rectangle around it. Measure the length and width in milliimeters. What are the dimensions of the egg? Find the ratio of the length to the width of the egg.



$\frac{\text{Length of Egg}}{\text{Width of Egg}} =$

4. Use the diagram to the right and directions below to make a Golden Spiral using Fibonacci squares.



a) In the middle of a piece of graph paper, color a 1×1 square (shown in white).

b) Below that square, color a 1×1 square using a second color (shown in violet).

c) To the right of the colored 1×2 rectangle, color a 2×2 square using a third color (shown in purple).

d) Above the colored 3×2 rectangle, color a 3×3 square using a fourth color (shown in teal).

e) To the left of the colored 3×5 rectangle, color a 5×5 square using a fifth color (shown in green).

f) To draw the spiral, begin at the top right corner of the first square you colored and make an arc to the opposite corner. From there, make an arc to opposite corner of the second square you colored. Next, make an arc to the opposite corner of the 2×2 colored square. Again, make an arc to the opposite corner of the 3×3 colored square. Lastly, make an arc to the opposite of the 5×5 colored square.

What are the dimensions of the final rectangle? What type of numbers are these?

For Problem 5-10, use Fibonacci numbers to solve the problem given.

5. Below are some interesting facts about Fibonacci numbers:

a) No two consecutive Fibonacci numbers have like factors.

b) The twelfth Fibonacci number is the square of 12 (this is the only perfect square in the Fibonacci numbers.)

c) There is only one perfect cube in the sequence, which is 2^3…

What Fibonacci number is 2^3?

6. Every third Fibonacci is divisible by 2.
Every fourth Fibonacci number is divisible by 3.

Do you recognize the pattern? Fill in the blanks below.

Every fifth Fibonacci number is divisible by __________.

Every sixth Fibonacci number is divisible by __________.

Every seventh Fibonacci number is divisible by ___________.

7. Demonstrate that your solutions in Problem 6 are true.

8. The number 2 multiplied by any Fibonacci number minus the number after the original number is equal to the second number before the original number. Test this with 8 in the sequence, which is the sixth Fibonacci number.

1, 1, 2, 3, 5, 8

9. The sum of the square of two consecutive Fibonacci numbers is also a Fibonacci number. Test this for 1 and 1, 1 and 2, 2 and 3, 3 and 5, and 5 and 8.

10. Find any other patterns you can in the Fibonacci sequence.

Section 5.9 Adding with Exponents

Practice Problems 5.9

For Problem 1-4, write the terms given as a base and exponent and then find the product of the terms.

1. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2. $4 \cdot 4$

3. $5 \cdot 5 \cdot 5$

4. $6 \times 6 \times 6$

For Problem 5-8, expand the product given as a base and exponent (power). Simplify that if possible.

5. 27

6. m^3

7. n^5

8. 144

For Problem 9-13, simplify the power(s) and add the terms given.

9. $2^3 + 3^2$

10. $4^2 + 3^2 + 2^2$

11. $(5 \times 5) + (3 \times 3)$

12. $(9 \times 9) + (10 \times 10)$

13. $10^3 + 10^4$

For Problem 14 and 15, tell which of the terms given is larger or smaller.

14. Which is larger: 2^8 or 8^5?

15. Which is smaller: 2^4 or 4^2?

For Problem 16-21, add the like terms given.

16. $3x + 2x$

17. $3x^2 + 2x^2$

18. $x^2 + x^2 + x^2 + x^2$

19. $(x^2 + x^2) + (x^2 + x^2)$

20. $x + 3x + 2y + 2y$

21. $m + 2n + 6m + 14n$

For Problem 22-25, let $c = 4$ and $d = 2$ to answer the question given.

22. Which is smaller: 4^c or 4^d?

23. Which is larger: c^4 or d^4?

24. What is $3^c + 7^d$?

25. What is $3c + 5d$?

Section 5.10 Subtracting with Exponents

Practice Problems 5.10

For Problem 1-6, solve the word problem given.

1. Which of the expressions below is seven less than a number?

a) $n \times 7$ b) $n \div 7$

c) $n - 7$ d) $7 - n$

2. Which of the expressions below is five more than a number?

a) $n \times 5$ b) $5 \times n$

c) $n + 5$ d) $\frac{n}{5}$

3. What is another way to write $5 + 6$?

4. What is another way to write $x \div 1.2$?

5. Which of the expressions below is the product of three and a number?

a) $3 \times n$ b) $3 + n$

c) $n + 3$ d) $\frac{n}{3}$

6. Which of the expressions below is the quotient of a number and 42?

a) $\frac{m}{42}$ b) $42 \times m$

c) $m + 42$ d) $m - 42$

For Problem 7-14, subtract the like terms given. (Remember, like terms have the same variable and exponent.)

7. $5n - 15n$

8. $5m + 6m - 6n - 2n$

9. $13p - 2p - 5p$

10. $3x - 10x + 14y$

11. $8.5x^2 - 3.2x^2$

12. $4m^2 - \frac{1}{3}m^2$

13. $15y^2 + 2y^2 - 6y^2$

14. $2x + 9y + 5x - 7y$

For Problem 15-24, simplify the exponents given and complete the operations.

15. $3^2 - 2^5$

16. $5^2 - 2^4$

17. $(4^3 - 3) + (5^3 - 4^2)$

18. $5.32 + 2^2$

19. $5^3 - 0.27$

20. $5b^3 - 2^3$

21. $2^3 - 2^4 - 2^5$

22. $5^2 + 1^9 + 0^9$

23. $(7^2 - 5^2) + (8^2 - 4^2)$

24. $(6^2 - 8^2) + (3^4 - 5)$

For Problem 25, solve the word problem.

25. In the monomial term $-10.4q^4$…

a) …what is the coefficient?

b) …what is the base?

c) …what is the exponent?

d) …how do you say it?

Section 5.11 Multiplying with Exponents

Practice Problems 5.11

For Problem 1-25, multiply and simplify the expression given.

1. $n^3 \times n^6$

2. $m^4 \times m^{-3}$

3. $m^2 \times n^5 \times m^2 \times n^5$

4. $(3x^2)(4y^5)$

5. $(-2.2x^3)(-0.1x^3)$

6. $(3x)(4x^2)(-7x^4)$

7. $m^5 \cdot n^{-5} \cdot m^4 \cdot n^3$

8. $(5m^4)(-2n^2)$

9. $q^3 \cdot q^4 \cdot q^2$

10. $(r^5)(3r^7)$

11. $-p^5 \cdot -p^4$

12. $-q^6 \cdot p^4$

13. $x^3 \cdot y^2 \cdot p^3 \cdot q^4$

14. $y^3 \cdot y^5 \cdot y^2 \cdot y$

15. $x^4 \cdot x^7 \cdot x^5 \cdot x^6$

16. $p \cdot p \cdot p \cdot p$

17. $p \cdot p \cdot q \cdot q \cdot r \cdot r \cdot r$

18. $(x)(-x)(y)(-y)(y^2)(z)(-z)(-z)$

19. $x^3 \cdot x^5 + y^4 \cdot y^6$

20. $m^2 \cdot m^4 \cdot -m^3 \cdot m$

21. $p \cdot p \cdot q \cdot q^2 \cdot q^3 \cdot r^2 \cdot r^3$

22. $-r \cdot -r \cdot -s \cdot -s$

23. $t^2s^4 + t^2s^4$

24. $(-5xyz)(10xyz)$

25. $(-3x^2y)(2xy^2) + (-4xy)(3x^2y^2)$

Section 5.12 Dividing with Exponents

Practice Problems 5.12

For Problem 1-8, divide and simplify the expression given.

1. $\frac{50x^5}{2x^5}$

2. $\frac{2x^3}{12x^3}$

3. $\frac{x^{12}}{x^9}$

4. $\frac{x^4}{x^{10}}$

5. $\frac{y^{10}}{y^{10}}$

6. $\frac{m^6n^2}{m^4n}$

7. $\frac{-b^7}{b^4}$

8. $\frac{3^5}{3^2}$

For Problem 9-14, rewrite the expression given as a fraction.

9. 7^{-2}

10. $5x^{-2}$

11. 11^{-1}

12. $3m^{-4}$

13. y^{-6}

14. $8z^{-1}$

For Problem 15 and 16, solve the word problem given.

15. If $x = 2$, what is $\frac{3^x}{4^x}$?

16. If $x = 2$, what is $\frac{x^3}{x^4}$?

For Problem 17-20, divide and simplify the expression given.

17. $\frac{5m^9}{5m^9}$

18. $\frac{10p^8}{10p^6}$

19. $\frac{1.3q^4}{q}$

20. $\frac{2mn}{6mn}$

Section 5.13 Power of a Power

Practice Problems 5.13

For Problem 1-6, simplify the expression given.

1. m^0
2. z^0
3. x^0y^0
4. -14^0
5. -0.36^0
6. $(4^0)(-3^0)$

For Problem 7-20, simplify the expression given and do not leave any negative exponents.

7. $(4^2)^3$
8. $(5)^4$
9. $(3^3)^2(2)^2$
10. $(3n)^2$
11. $(-3m)^3$
12. $(xy)^8$
13. $(x^2y^3)^4$
14. $-(m^2n^2)^3$
15. 4^{-6}
16. 4^{-x}
17. $\frac{12x^2}{6}$
18. $\frac{13}{39y^4}$
19. $(6^{x+y})y^0$
20. z^{-7}

Section 5.14 Module Review

For Problem 1-14, solve the problem given.

1. Find the prime factors of 51.

2. Find the prime factorization of 82.

3. Use combinations to find the number of possible factors of 194.

4. Tell whether the given number is perfect, deficient, or abundant.

a) 22 b) 28

c) 44 d) 6

e) 24

5. Match the given mathematician with their discovery and/or description.

1) Eratosthenes	a) Figurate Numbers
2) Fibonacci	b) 1, 1, 2, 3, 5, 8 ...
3) Goldbach	c) Italian youth; found the amicable pair 1,184 and 1,210
4) Marin Mersenne	d) A way to find the prime numbers from 1 – 100 using the Sieve
5) Euler	e) Every even number greater than 2 is the sum of two primes
6) Pythagoras	f) French monk; name is attached to a prime number algorithm
7) Niccolo Paganini	g) Showed that there are an infinite number of prime numbers

6. How are amicable numbers different than perfect numbers?

7. Why are square numbers and triangular numbers called figurate numbers?

8. What is the eighth square number?

9. Using Gauss' method, $\frac{n(n+1)}{2}$, what is the eleventh triangular number given n is the number of the series?

10. Using the Fibonacci sequence, how many pairs of rabbits would there be after nine months given the description of the rabbit problem in Section 5.7?

11. What number does the quotient of any pair of Fibonacci numbers approach?

12. How does the square of a Fibonacci number compare to the product of the numbers on either side of it?

13. Use four examples to demonstrate that every third Fibonacci number is divisible by 2.

For Problem 14-25, simplify the expression given.

14. $m^2 + n^3 + n^3$

15. $3^0 + 14m + 21n - 2n$

16. $x + 4y - x - 2y$

17. $14.2p - 2.4q + 1.4p + q^2$

18. $b \times b \times b \times a \times a \times a$

19. $(p \cdot p)(q \cdot p^2 \cdot q^3)$

20. $(2x^3)(3x)$

21. $(-3m)(-16n)$

22. $(-2.1m)(-3.2n)$

23. $\frac{12x^4}{4x^2}$

24. $\frac{2x^3}{6x^3}$

25. $\frac{(3x)^2}{(4x)^2}$

<u>Section 5.15 Module Test</u>

For Problem 1-17, solve the problem given.

1. Find the prime factors of 32.

2. Find the prime factorization of 32.

3. Use combinations to find the number of possible factors of 32.

4. Fill in the blank.

a) A positive integer is __________________________ if it is equal to the sum of its proper divisors (not including the number itself).

b) Two numbers are __________________________ if each is the sum of the proper divisors of the other (not including the numbers themselves).

c) Numbers are __________________________ if the sum of their proper divisors is less than the numbers themselves.

d) Numbers are __________________________ if the sum of their proper divisors is greater than the numbers themselves.

5. Use the numbers 5 and 6 to create a Fibonacci sequence out to ten numbers.

6. Find the quotients of the sequence of the numbers in Problem 5. What do they seem to approach?

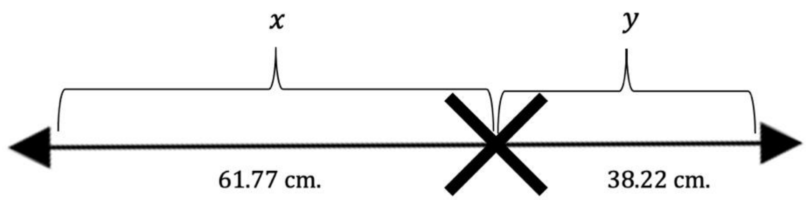
7. If n is the term number in the Fibonacci sequence, what is the previous term number in terms of n?

8. If n is the term number in the Fibonacci sequence, what is the next term in terms of n?

9. What is the sixth triangular number? Draw it.

10. Use the given information and diagram to answer the questions given.

Let 61.77 cm. be equal to x. Let 38.22 cm. be equal to y.



a) What is $\frac{x}{y}$?

b) What is $\frac{x+y}{x}$?

c) What do these numbers represent?

11. Which is true of Problem 10?

a) The whole length divided by the shorter length is equal to the longer length divided by the shorter length.

b) The whole length divided by the longer length is equal to the longer length divided by the shorter length.

12. The actual number for the Golden Ratio is 1.6180339887498948 ... The Greek letter phi (φ) approximates to 1.618. It does not repeat or terminate. Is phi rational or irrational?

13. Pick a number between 1 and 10. Divide 1 by the number you picked. Add 1. The number you get is your new number. Divide 1 by your new number. Add 1. Do this seven times and tell what it approaches.

14. Which of the following always approaches the Golden Ratio?

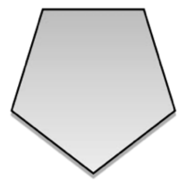
a) The ratio of the length of a rectangle to its width

b) The ratio of the whole length of a line divided by the shorter length to the shorter length divided by the longer length

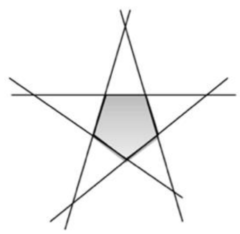
c) The ratio of the height of a triangle to its base

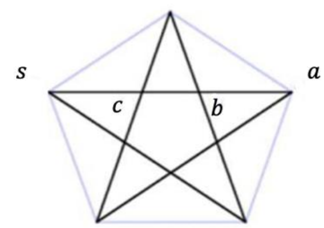
d) The ratio of one Fibonacci number to the previous number as the sequence continues

15. Beginning with a pentagon with all sides equal, draw a rectangular pentagram (a star with all sides equal).



You can also extend the sides of a pentagon until the shape is a star polygon. Connect the vertices of the star polygon to make a pentagon with equal sides (regular pentagon).





a) Measure from s to a (the entire length across).

b) Measure from s to b (the right-side diagonal intersection with the entire length).

c) Measure from s to c (the left side diagonal intersection with the entire length).

d) Divide the answer from part a) by the answer from part b).

e) Divide the answer from part b) by the answer from part c).

f) What special ratio do you get?

16. Let n be the term of the Fibonacci sequence. If n is equal to 4, what number in the sequence represents the $n+2$ term? What number represents the $n-3$ term?

For Problem 17-24, simplify the expression given.

17. $t^5 + 2t^2 - 3t^2$

18. $t^4 + s^3 - 2t^4 - 5s^3$

19. $(4x^2)^3 + (2x^3)^2$

20. $(2m \cdot n \cdot o)(-6m^2 \cdot n^3 \cdot o^4)$

21. $3x^0 - 16x^2$

22. $\frac{15m^4}{3n^3}$

23. $-\frac{10m^8}{50m^6}$

24. $\frac{p^2}{p^3q}$