Module 1 Solving Equations and Inequalities

Section 1.1 Solving Equations with Variables on Both Sides

Looking Back 1.1

In General Math, we learned about solving equations. We have seen how important it is to keep both sides of the equation balanced by performing the same operation on both sides of the balance scale. There are still things we need to cover so we can have a strong background in equations: this background will prepare us for Algebra 1.

Looking Ahead 1.1

When solving equations with variables on both sides, we use the same properties of numbers we learned in Module 1 of General Math. Look at those notes if you need a refresher on the rules.

To solve for the unknown variable, we need to isolate the variable by getting it on one side of the equation by itself. This means getting all variables on one side of the equation. All the numbers must be moved to the other side of the equation. The variable can be moved to either side by adding the opposite, which allows us to eliminate it on one side of the equation and combine it on the other side of the equation. Remember, like terms have the same variable. We can only combine like terms. In this section, like terms are all to the first power.

Example 1:   Solve the equation below. (Check your solution.)

$$5x - 16 = 2x - 4$$

Example 2:   Solve the equation below. (Check your solution.)

$$2b + 6 = -9 - 8b$$

Example 3:   Solve the equation below. (Simplify the equation first.) (Check your solution.)

$$3f + 4 = 2 + 5f - \frac{1}{2}$$

Section 1.2 Undoing Exponents
Looking Back 1.2

We learned about exponents in the previous book when we investigated problem-solving. We have been using inverse operations to solve equations and we will continue to do that in this section but for exponents. The inverse of squaring a number is finding the square root of a number. The inverse of a cube is a cube root and the inverse of x^n (read: "x to the nth power") is $\sqrt[n]{x}$ (read: "the nth root of x").

Looking Ahead 1.2

Example 1:	Solve for the variables in the radicands given.

a) $\sqrt[2]{m^2}$ b) $\sqrt[2]{10^2}$

c) $\sqrt[2]{x^2}$ d) $\sqrt[2]{3^2}$

e) $\sqrt[2]{(x+3)^2}$

In undoing exponents, we have to use reverse thinking and the radicand symbol ($\sqrt{}$). When we are trying to find the solution to $\sqrt[2]{4^2}$, we are asking what number when multiplied by itself two times results in 4^2. That number would be 4 because $4 \times 4 = 4^2$ so $\sqrt[2]{4^2} = 4$. The square root of anything squared is itself. When we are trying to find the solution to $\sqrt[3]{x^3}$, we are asking what letter results in x^3 when multiplied by itself three times. That letter would be x because $x \cdot x \cdot x = x^3$. The cubed root of anything cubed is itself.

Notice that $4 \cdot 4 = 16$ and $(-4)(-4) = 16$ so the radicand $\sqrt[2]{16}$ has two solutions: positive 4 and negative 4. Therefore, if $x^2 = 16$, we are asking what number(s) result(s) in 16 when multiplied by itself twice. It would be
4 and -4 because we undo the square root of both sides and put "$\pm$" in front of the radicand.

A cube root is different: $\sqrt[3]{27} = 3$ because $3 \cdot 3 \cdot 3 = 27$, but $\sqrt[3]{-27} = -3$ because $-3 \cdot -3 \cdot -3 = -27$. There is a positive solution given the cube root is positive and a negative solution given the cube root is negative.

Example 2:	Solve for x in the equations given. (Check your solutions.)

a) $x^2 = 16$ b) $x^2 = 4$

Example 3:	Solve the radicands given. (Check your solutions.)

a) $\sqrt[3]{8}$ b) $\sqrt[3]{-8}$

c) $\sqrt[3]{64}$ d) $\sqrt[3]{-64}$

We have been looking at perfect squares. Let us make a perfect square calculator from 1-100.

|	1	2	3	4	5	6	7	8	9	10
Exponents of 2	1^2		3^2		5^2					
Perfect Square	1		9		25					
Square Root	$\pm\sqrt{1}$		$\pm\sqrt{9}$		$\pm\sqrt{25}$					
Two Solutions	±1		±3		±5					

The numbers in between the perfect squares are not perfect squares. They are irrational numbers. We can leave them in exact form with the square root sign or write them as decimal approximations. Irrational numbers do not end or repeat so we approximate them when we do not leave them in exact form.

Example 4:   Solve for x in the equations given. (Check your solutions.)
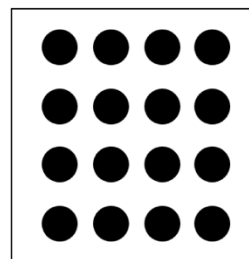
a) $x^2 = 5$ b) $x^2 = 2$

The perfect square is the area of a square and the square root is the length of the side of the square.
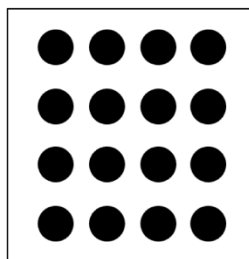
Area = 4 square units
Length = 2 units
Width = 2 units

Area = 2 square units
Length = $\sqrt{2}$
Width = $\sqrt{2}$

We can solve problems with x^2 (x squared) just like we solved problems with x, but the last step is undoing the root.

| Example 5: | Solve for x in the equation given. (Check your solution.) |

$$2x^2 = 32$$

| Example 6: | Solve for x in the equation given. (Check your solution.) |

$$x^2 - 5 = 11$$

| Example 7: | Solve for x in the equation given. (Check your solution.) |

$$3x^2 - 11 = 37$$

| Example 8: | Solve for x in the equation given. (Check your solution.) |

$$3(x + 4)^2 = 75$$

Section 1.3 Undoing Parenthesis

Looking Back 1.3

In previous modules, variables were combined with like variables and numbers were combined with numbers. We call this "combining like terms."

In this module, we will be solving algebraic equations to find the value for the variable that solves the equation given. We are not just simplifying and combining like terms, but finding a solution. All the equations in this module have only one common variable. When we get a numerical solution for an equation, it is called "solving for the unknown." The unknown is the variable.

Once the variable has been solved, it is known. We can check to make sure the value is the correct solution by substituting it in the equation for the variable and seeing if it works. If it does, the equation will be balanced.

When evaluating expressions, grouping symbols are evaluated firstly. When solving equations for the variable, grouping symbols are evaluated lastly. The operations that are farthest away from the variable (outermost) are undone firstly and those closest to the variable (innermost) are undone lastly.

In the previous section, we were "undoing" exponents by finding the roots. In this section, we will be "undoing" what is in parenthesis last. Because it is the first thing we do when evaluating expressions, it is the last thing we "undo" when working backwards to solve equations.

Looking Ahead 1.3

Example 1:   The last Practice Problem in Section 1.2 is shown below. The solutions are $y = 22$ and $y = 8$. Complete the DO and UNDO chart and check your solutions.

$$2(y - 15)^2 = 98$$

DO	UNDO

When we "undo" equations to solve for the variable, we work in the reverse order of the "DO" column and use the inverse operations.

Example 2:   Complete the DO and UNDO chart for the equation below and check your solutions.

$$6 + (m + 2)^3 = 131$$

DO	UNDO

Example 3:   Solve for p in the equation below by "undoing" operations outermost to innermost and check your solution.

$$5 + 2(p + 4)^2 = 55$$

Section 1.4 Simplifying and Solving

Looking Back 1.4

       In this section, we will be putting everything we have learned all together by using simplifying (combining like terms) and solving (finding the value of the variable). Remember, when we simplify expressions, we "do" the grouping symbols in order from left to right, but when solving equations, we "undo" the steps going in reverse order and using inverse operations. In this section, we must simplify the problems first before finding the solution.

Looking Ahead 1.4

Example 1:   Simplify first and then solve for m in the equation below. (Check your solution.)

$$6 + (m + 2)^3 - 21 = 110$$

Example 2:   Simplify first and then solve for m in the equation below. (Hint: Combine the two ms using addition first.) (Check your solution.)

$$m + 4 + m = 22$$

<u>Section 1.5 Reverse Thinking</u>
<u>Looking Back 1.5</u>

Matthew 7:12 says: "Therefore, however you want people to treat you, so treat them, for this is the Law and the Prophets."

Galatians 5:14 says: "For the whole Law is fulfilled in one word, in the statement, 'You shall love your neighbor as yourself."

This is often called the *Golden Rule*. In previous modules, we have learned the *Golden Rule of Algebra*, which is: "What you do to one side of an equation, do unto the other." Now, it is time to apply this rule and use it to solve equations.

<u>Looking Ahead 1.5</u>

Example 1:	Fill in the blanks to solve the problem of antiquity below (ancient problem).

The problem states: "Diophantus' boyhood lasted $\frac{1}{6}$ of his life, and his beard grew in $\frac{1}{12}$, and in $\frac{1}{7}$ more, he married. He had a son five years later; the son lived half of his father's age, and Diophantus died four years after his son. If we let x be Diophantus' age when died, then:

_____ is the number of years his boyhood lasted

_____ is the age at which he grew a beard

_____ is the age at which he was married

_____ is the age at which he had a son (five years after his marriage)

_____ is the age which his son lived to be (half Diophantus' age)

_____ is the age at which he died (four years after his son died)

Example 2:	Use reverse thinking to solve the equation below. (Check your solution.)

$$3x + 8 = 14$$

Example 3:	Use reverse thinking to solve the equation below. (Check your solution.)

$$\frac{3}{5}x + 2 = 5$$

Example 4:	Use reverse thinking to solve the equation below. Combine like terms first. (Check your solution.)

$$\frac{1}{2}x + \frac{3}{4}x - 7 = -2$$

Section 1.6 Writing Algebraic Inequalities

Looking Back 1.6

In Module 8 of General Math, we learned to write algebraic equations by looking for important words or key words in a situation that represented operations. We said 4 less than 6 could mean $6 - 4$ or $4 < 6$ depending on the situation. In General Math, we said these situations represented equations (equalities) only and we used the operations of addition, subtraction, multiplication, and division.

In this section, we are learning about inequalities. Therefore, we will be using the inequality symbols: less than ($<$); greater than ($>$); less than or equal to ($\leq$); greater than or equal to ($\geq$).

In General Math, we said that we must carefully read the situation, but now, in Pre-Algebra, we will see that we must also carefully read the instructions.

Looking Ahead 1.6

When two mathematical expressions are equal, we have an equation. If the two expressions are not equal, they be greater than ($>$) or less than ($<$), greater than or equal to ($\geq$), or less than or equal to ($\leq$). These are called inequalities.

Example 1:	Use inequalities to write algebraic expressions for the phrases below.

a)   A number less than 15

b)   Eight is less than or equal to a number

c) x is greater than or equal to -5

d)   A number is greater than 0.2

Fill in the blanks with key words that mean to use the inequalities greater than ($>$) or less than ($<$).

_____   _____

_____   _____

_____   _____

_____   _____

_____   _____

Example 2:	A band is selling candies to help pay for a trip. Their profit must be at least $900 to afford the trip. Write this situation as an inequality. Let p be profit.

Example 3:   In order to have enough money for a day at the amusement park, Reginald must spend no more than \$12 at the bowling alley. Let m be money Reginald may spend. Write this situation as an inequality.

Example 4:   In order to ride a roller coaster at the amusement park, Russell must be taller than 50 inches. Let h represent the height one must be to ride the roller coaster. Write this situation as an inequality.

Example 5:   The highest temperature ever recorded at the South Pole was 73°F. Let t be temperature recorded at the South Pole. Write this situation as an inequality.

Example 6:   Write verbal inequalities for the algebraic inequalities below.

a) $a \leq 14$ b) $t > -12$

c) $r < 5$ d) $m \geq 0.9$

Section 1.7 Inequalities on the Number Line
Looking Back 1.7

Inequalities are not equal (=); they are greater than (>) a given number, less than (<) a given number, greater than or equal to (≥) a given number (that number and all the numbers larger than it are solutions), or less than or equal to (≤) a given number (that number and all the numbers less than it are solutions). There are many solutions to an inequality.

The inequalities we wrote in Example 6 of Section 1.6 are actually the solutions to inequality problems. When there is only one variable in an inequality, we can graph our solutions on a number line to get a graphic representation of all the possible solutions.

Looking Ahead 1.7

An inequality has several solutions. The number line can represent fractions, decimals, whole numbers, integers, or other numbers. The solutions to the inequality $y > 4$ could be $4\frac{1}{2}$, 7.8, 20, or 1,010. There are an infinite number of solutions.

When graphing inequalities on a number line, use a small circle for the start of the graph. When the starting number is not included in the set, use an open circle to represent it. When the starting number is included in the set, use a closed circle to represent it.

- Greater than (>) is represented by a(n) _____ _____.

-

- $x > 4$ has an open circle at _____

-

- Less than (<) is represented by a(n) _____ _____.

-

- $y < -2$ has an open circle at _____

-

- Greater than or equal to (≥) is represented by a(n) _____ _____.

-

- $m \geq -0.3$ has a closed circle at _____

-

- Less than or equal to (≤) is represented by a(n) _____ _____.

-

- $n \leq 11\frac{1}{2}$ has a closed circle at _____

If the variable is first, followed by the < sign, everything to the left of the number is shaded on the number line.

If the variable is first, followed by the > sign, everything to the right of the number is shaded on the number line.

Example 1:   Graph the inequality $x > 1$ on the number line below.

Example 2:   Graph the inequality $x > -4$ on the number line below.

Example 3:   Graph the inequality $x \leq -3$ on the number line below.

Example 4:   Graph the inequality $x \geq 5$ on the number line below.

If the variable is after the inequality sign, you may flip the whole problem and the inequality sign to rewrite it so the variable is first. For example, $5 < x$ could become $x > 5$ and $-2 > y$ could become $y < -2$. Then you could graph it as you have done before.

Example 5:   Graph the inequality $-7 \leq x$ on the number line below.

Example 6:   Graph the inequality $3 \geq x$ on the number line below.
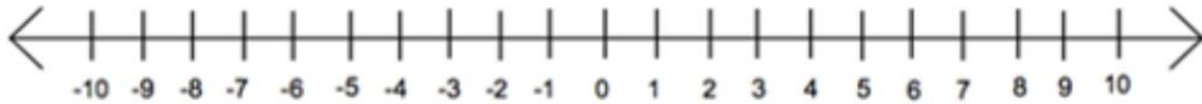
- When the variable is first, the line gets shaded in the direction the inequality is pointing.

- Remember that if the variable follows the inequality sign, you draw the line in the opposite direction the inequality is pointing.

- In $2 < x$, the less than sign ($<$) ordinarily means draw the line to the left. However, because the variable is behind the inequality sign, you would do opposite and draw the line to the right.

It is easier to rewrite an inequality so the variable is first.
If 2 is less than x, that means x is greater than 2; therefore, rewrite $2 < x$ as $x > 2$ and then graph it.

Example 7:   Graph the inequalities below.

a) $p < -\frac{1}{4}$

b) $7 > t$

c) $s > 2.1$

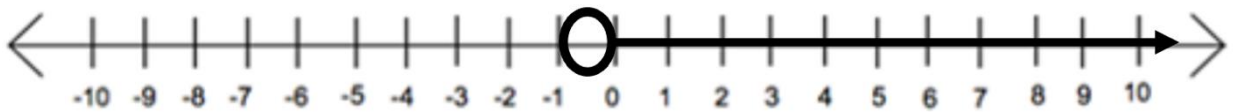Example 8:   Write an inequality for the graphs below.

a)

b)

c)

Section 1.8 Solving Inequalities with Addition and Subtraction

Looking Back 1.8

Now that we have learned how to write inequalities and how to graph the solutions of an inequality on the number line, we will begin to learn how to solve inequalities. We will find that inequalities look like equations, except that instead of an equal sign in our problem, we will have a greater than ($>$), less than ($<$), greater than or equal to ($\geq$), or less than or equal to ($\leq$) sign. Once we have our equation simplified to an inequality such as $x < 2$ or $y \geq -3$, we will graph the solutions on a number line.

An equation may have one solution or no solution, but an inequality has infinite solutions. Still, we solve inequalities just as we would equations. We treat the inequality symbol as an equal sign by considering it the fulcrum of the balance scale.

Looking Ahead 1.8

Example 1:	Solve the inequality below.

$$x + 4 < -3$$

What are some possible solutions that "satisfy" this inequality?
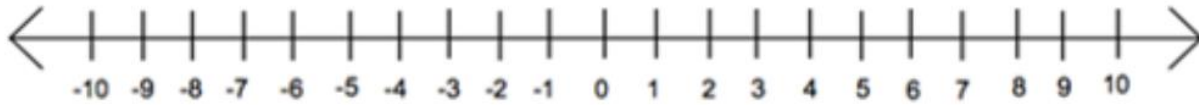
Graph the solutions to the inequality on the number line.

Check to see if your possible solutions are included in the shaded region of the number line.

Example 2:	Solve the inequality below.

$$-2 \geq -5 + y$$

What are some possible solutions that satisfy this inequality?

Graph the solutions to the inequality on the number line.

Check to see if your possible solutions are included in the shaded region of the number line.

Example 3:	Solve the inequality below.

$$\frac{4}{7} < d - \frac{3}{7}$$

What are some possible solutions that satisfy this inequality?

Graph the solutions to the inequality on the number line.

Check to see if your possible solutions are included in the shaded region of the number line.

The properties that apply to inequalities are summarized below:

If $a > b$, then $a + c > b + c$

If $a < b$, then $a + c < b + c$

If $a > b$, then $a - c > b - c$

If $a < b$, then $a - c < b - c$

Adding or subtracting the same number to or from both sides of an inequality does not change the direction of inequality sign. The original inequality still holds true.

Section 1.9 Solving Inequalities with Multiplication and Division

Looking Back 1.9

Solving inequalities with addition and subtraction is just like solving equations with addition and subtraction. The only difference is there is more than one solution. We use the number line to show these infinite solutions in solving inequalities with addition and subtraction.

In this section, we will see that multiplying and dividing in inequalities is a little different.

Looking Ahead 1.9

Example 1:	Look at each inequality below and tell whether it is true or false.

$$-3 > -5 \qquad 12 < 15 \qquad -6 < 7 \qquad 8 > -9 \qquad 5 > 0$$

Example 2:	Now, let us multiply both sides of each inequality by -1. Find the product of each side of the inequality and tell whether it is true or false.

$$-3(-1) > -5(-1) \qquad 12(-1) < 15(-1) \qquad -6(-1) < 7(-1)$$

$$8(-1) > -9(-1) \qquad 5(-1) > 0(-1)$$

Are all of the inequalities still true?

Example 3:	Now, let us divide both sides of each inequality by -1. Find the quotient of each side of the inequality and tell whether it is true or false.

$$-3 \div (-1) > -5 \div (-1) \qquad 12 \div (-1) < 15 \div (-1) \qquad -6 \div (-1) < 7 \div (-1)$$

$$8 \div (-1) > -9 \div (-1) \qquad 5 \div (-1) > 0 \div (-1)$$

Are all of the inequalities still true?

Multiplying by a negative number on both sides of the inequality or dividing by a negative number on both sides of the inequality makes the inequality false. Therefore, the inequality sign must also change to its opposite to make the statement true.

From the above examples, we see that when we multiply or divide each side of the inequality by a negative number, then the inequality sign must become its opposite (change direction). The less than sign ($<$) must become a greater than sign ($>$) and the greater than sign ($>$) must become the less than sign ($<$).

The Multiplication Property of Inequalities states:

When $c > 0$	When $c < 0$
If $a > b$ then $ac > bc$	If $a > b$ then $ac < bc$
If $a < b$ then $ac < bc$	If $a < b$ then $ac > bc$

The Division Property of Inequalities states:

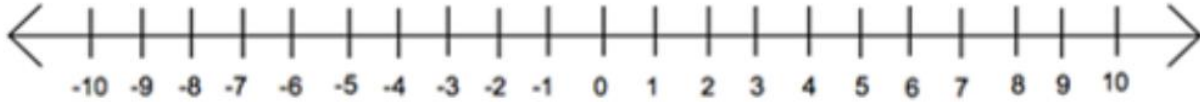| When $c > 0$	When $c < 0$
If $a > b$ then $\frac{a}{c} > \frac{b}{c}$	If $a > b$ then $\frac{a}{c} < \frac{b}{c}$
If $a < b$ then $\frac{a}{c} < \frac{b}{c}$	If $a < b$ then $\frac{a}{c} > \frac{b}{c}$

Example 4:   Solve the inequality below and graph your solution. (Check your solution.)

$$-5x \leq 15$$

Example 5:   Solve the inequality below and graph your solution. (Check your solution.)

$$14x > -56$$

Example 6:   Solve the inequality below and graph your solution. (Check your solution.)

$$-\frac{n}{4} \geq 2$$

Section 1.10 Solving Two-Step Inequalities
Looking Back 1.10

Now, we are going to put what we have learned in the previous two sections together. When we are solving an equation or inequality and we have to undo addition *and* subtraction *or* multiplication *and* division, we are undoing two things. These are called two-step equations or two-step inequalities.

The very steps we have learned here are being used all over the world. If we travel around the world, we will find many people speak many different languages and have many different customs; however, when it comes to mathematics, the processes and symbols mean the same thing wherever we go. Mathematics is a universal language. Galileo, a famous mathematician and astronomer of the seventeenth century, said: "Mathematics is the language with which God has written the universe."

Algorithms are used to solve equations and inequalities. There are a series of steps used to solve problems. When we learn to follow the steps, solving the problems becomes easier. Remember to use reverse thinking to "undo" operations to solve for the unknown variable. It is always good to check your solution and make sure it makes sense.

Looking Ahead 1.10

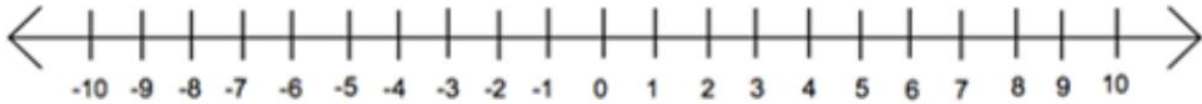| Example 1:	Find the solutions for x below and graph them on the number line.

$$3x + 4 \geq 10$$

Example 2:	Find the solutions for x below and graph them on the number line.

$$-4 \leq 5x - 19$$

Example 3:	Find the solutions for x below and graph them on the number line.

$$-3x + 4 \leq 16$$

Section 1.11 Multi-Step Inequalities

Looking Back 1.11

In the previous section, we graphed inequalities after finding the solutions to the inequalities. In order to find these solutions, we used the same process we did for equations. Multi-step inequalities may have more than one or two steps and involve parenthesis. In this section, we will practice multi-step problems.

We must make sure to simplify each side of the inequality by combining like terms before we begin "undoing" operations.

Looking Ahead 1.11

Example 1:	Solve for x below and graph your solution. (Check your solution.)

$$-6x + 14 \leq 50$$

Example 2:	Solve for x below and graph you solution. (Check your solution.)

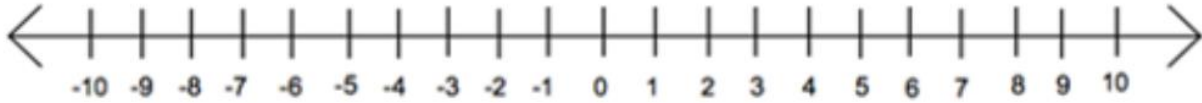$$3v - 2v + 8 \geq 4(2 + v)$$

Example 3:	Solve for x below and graph your solution. Check your solution.

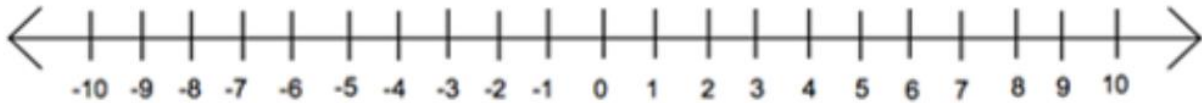$$-5(x + 4) > -70$$

Example 4:   Solve for u below and graph your solution. (Check your solution.)

$$-4(2u + 3u) \leq -2(5u + 50)$$

Example 5:   Solve for p below and graph your solution. (Check your solution.)

$$2(3p + 7) > (-2 - 2p)$$

Section 1.12 Solving Inequalities
Looking Back 1.12

Inequalities can be solved much like equations by isolating the variable on one side and seeing what is greater than or less than the other side. If a number is less than some numbers, then it is greater than other numbers; every number is between two other numbers. If we graph $-4 < m < 3$, then m is between -4 and 3, but not equal to either -4 or 3.

This can be split into two inequalities: $-4 < m$ and $m < 3$. The intersection of these two graphs is
$$-4 < m < -3.$$

This is called a compound inequality because it is one thing *and* another at the same time. Both parts are true.

Now, if we have $m \leq -4$ or $m \geq 3$, then one *or* the other is true at any given time. Any number that is equal to -4 *or* less than it, or equal to 3 *or* greater than it, is a solution.

If one part of the inequality is true then the entire inequality is true. This is also a compound inequality because it may be one thing *or* the other thing.
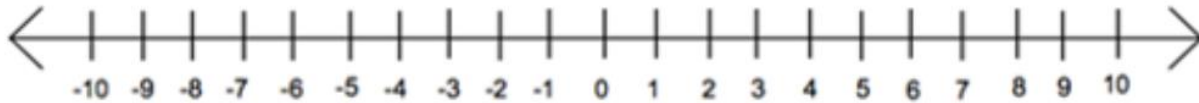
Example 1:	Graph the inequality below on the number line.

$$3 > m \geq -2$$

Example 2:   Graph the inequality below on the number line.

$$m > -2 \text{ or } m > 5$$

Looking Ahead 1.12

To solve an inequality using the word *and*, write the compound inequality as two inequalities. The inequality $-2 \leq r - 3 \leq 5$ can be written as $-2 \leq r - 3$ and $r - 3 \leq 5$. The variable r is then solved for in each inequality, then put back together with the variable in the middle.

$$-2 \leq r - 3 \qquad \text{and} \qquad r - 3 \leq 5$$

Example 3:   The best swimming temperature for a pool is when the average is between 78°F and 80°F. The temperature of a particular pool was taken and read: "81.2°F" but three hours later read: "76.3F." What must the final temperature reading be for the average temperature of this pool to be between 78°F and 80°F?

Example 4:   In science, you will learn how to find the total force of an object using the following equation: Total Force = (mass)(acceleration). If a man is pushing his 1,500 kg. car to the nearest gas station to the east, what amount of force (in Newtons) must he push the car to get it to move between 0.03 meters per second squared and 0.05 meters per second squared. Use the following formula: $\frac{\text{Total Force}}{\text{mass}} = \text{acceleration}$ to solve the problem.

Section 1.13 Solving Equations and Inequality Word Problems

Looking Back 1.13

We have learned to write and solve equations and inequalities. Now, we want to use all we have learned to solve word problems with equations and inequalities.

This section is more about the whole process, not just one part of it. Remember, we have all the skills and tools we need; we just need to apply them to the correct situation. Lastly, we must make sure our solutions make sense and seem reasonable. So, let us begin putting it all together.

Looking Ahead 1.13

When solving word problems, we use the following steps:
1) Identify the problem
2) Set up the equation or inequality
3) Solve the equation or inequality
4) Check the solution
5) Substitute values back in for the variables

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Example 1:   The Blazer Volleyball Team is making trail mix for the volleyball tournament. They need 30 lbs. of cashews for every 5 lbs. of raisins. Someone donated 24 lbs. of cashews to the team. How many pounds of raisins do they need to buy to make their trail mix?

Does your solution seem reasonable?

Example 2:   Tunes-R-Us charges a monthly membership fee of $14.50 and then $2 for each download. MyTunes charges $25 a month and then $1.25 for each download. After how many downloads will MyTunes be a better deal than Tunes-R-Us?

Example 3:   Cyndi and Sue are going to Wyoming on vacation with a budget of $2,000. They have $300 for admissions and souvenirs. They want to spend $\frac{1}{4}$ of what they spend on motel rooms for food. How much can they spend on food and motels if they use their whole budget?

Example 4:   A triangle that is to be used for a window has a base of 3 inches. The area of the triangle must be less than 48 square inches, but more than 24 square inches. What are some possible heights for the triangle?

3 inches
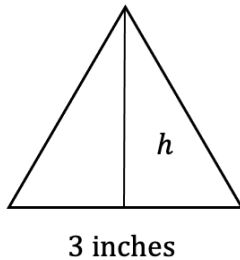
Example 5:   Your cell phone plan costs $34.00 a month plus $0.15 for each message sent or received. You have $70.00 to pay your bill each month. How many texts can you send or receive each month?

Example 6:   Jet skis are rented for $25.00 an hour for the first two hours and $20 for each additional hour. Hailee can spend no more than $150 to rent jet skis. How many hours can Hailee rent jet skis?