# Module 7 Functions

## Section 7.1 Function or Relation

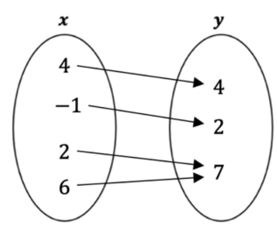
### Looking Back 7.1

The types of linear equations we have investigated in Module 2 and Module 3 are functions. Though it may appear functions are the same as equations, they are not; not all equations are functions but all functions are equations. There are types of tests that can be done to determine if an equation is a function or a relation only.

By now, we know what an equation is. A relation is a set of ordered pairs: $(0, 4)$, $(-0.2, -0.5)$, and $(-3, -2)$ are all ordered pairs. If the relation has a rule in which there is only one unique output for each input, then the relation is a function.

Therefore, a function is a relation in which each input has only one possible output. Ordered pairs can be represented by tables or mappings.

Mappings are another way to represent data. The arrow maps the input onto the output. We can see that each $x$-value maps to only one $y$-value.




A table can be made of the ordered pairs in the mapping. Once the table is made, we can see the mapping is a function because the input $(x)$ does not repeat.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
|---|---|
| $4$ | $4$ |
| $-1$ | $2$ |
| $2$ | $7$ |
| $6$ | $7$ |

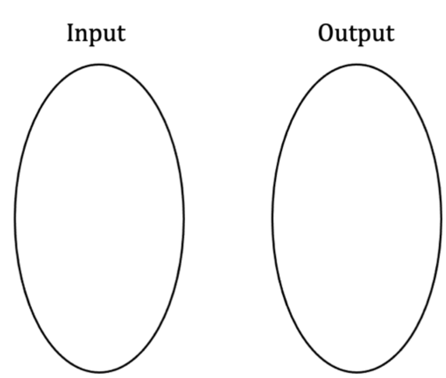
### Looking Ahead 7.1

Example 1: Determine if the ordered pairs below are functions or relations only.

$(10, -1); (6, 2); (13, -1); (5, 5)$

Put the ordered pairs into a table as sets of inputs with their outputs; or draw arrows to connect the inputs to the outputs. This is called a "mapping" in mathematics: 10 maps onto $-1$; 6 maps onto 2; etc. Using a graph would be a better way to find out if the equation is a function because the table represents only discrete data, but the graph could represent continuous data.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
|---|---|
| 10 | $-1$ |
| 6 | 2 |
| 13 | $-1$ |
| 5 | 5 |



Looking again at the ordered pairs: (10, $\underline{-1}$), (6, 2), (13, $\underline{-1}$), (5, 5), there is an output that is repeated in two different sets (underlined), but there is no $x$ repeated for the values of $y$. In the mapping above, there are two arrows coming to $-1$ (output), but they are coming from two different numbers (inputs), 10 and 13. So, 10 has a unique solution, and 13 has a unique solution.

Example 2: Determine if the ordered pairs below follow the rule defining them as a function.

| **Input ($x$)** | **Output ($y$)** |
|---|---|
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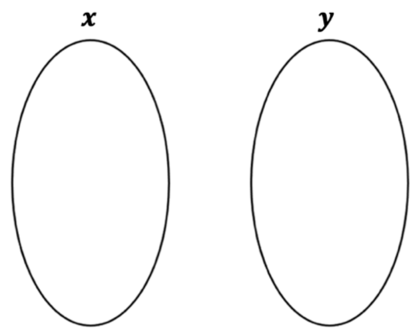
| | |
| | |




Notice that the inputs ($x$) repeat (underlined once) and the outputs ($y$) repeat (underlined twice):

$$(\underline{10}, \underline{\underline{-1}}); (\underline{6}, \underline{\underline{2}}); (\underline{10}, \underline{\underline{5}}); (\underline{5}, \underline{\underline{5}}).$$

This can be confusing, but if there is only one unique output for each input, then it is actually the input that cannot repeat. The outputs can repeat. So, just check the input values of the relation; if inputs repeat, the relation is not a function.

Example 3: Below is a table that shows Carbon Dioxide Levels at 15-year intervals for 120 years. Does the relationship represent a function?

| $x$ | $y$ |
|---|---|
| 0 | N/A |
| 15 | N/A |
| 30 | 290 |
| 45 | 295 |
| 60 | 300 |
| 75 | 320 |
| 90 | 330 |
| 105 | 355 |
| 120 | 375 |

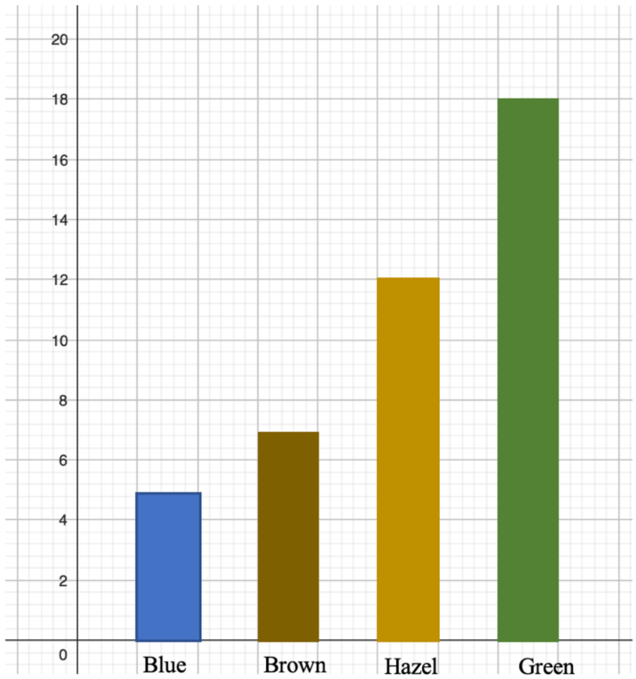
In this example, the input values ($x$) do not repeat, and neither do the output values ($y$). However, the output values may repeat in a function, just not the input values.

Section 7.2 Types of Data

Looking Back 7.2

Data can be qualitative (about qualities) or quantitative (about quantities).
Qualitative data is categorical. It is based on categories such as hair color or shampoo types.
Quantitative data is numerical. It is based on measurements such as height or weight.

| Example 1: | Is the graph below categorical or numerical? |
|---|---|




What are the numerical values assigned to each category?

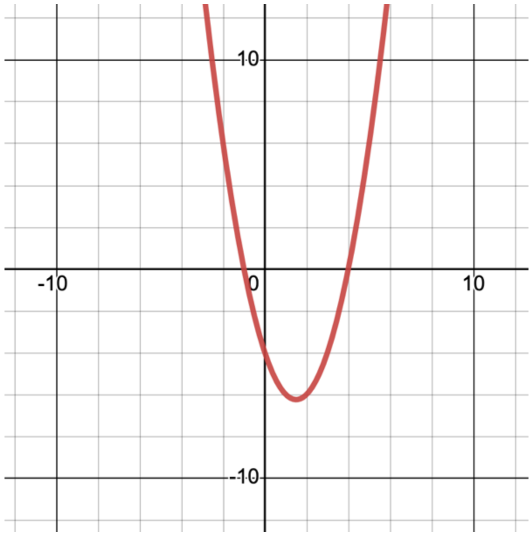
Looking Ahead 7.2

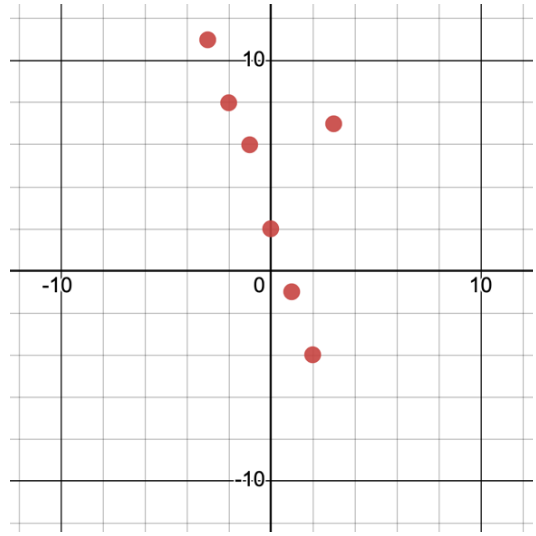
Numerical (measurable) data may be discrete data or continuous data. Discrete data is represented by specific limited values that are finite and can be counted. It is usually whole numbers or integers. These are made up of separate data points and your pencil must be lifted to shade each one.

Continuous data can take on two particular real values in an interval and all the real values between them. These values are not fixed and may be an infinite number of possible values. The measurements can be broken down into smaller individual parts. Your pencil does not need to be lifted at any point to shade a continuous graph from the beginning of the interval to the end of the interval.

As you may recall from Module 2 and Module 3, linear equations have graphs that are straight lines because the slope is a constant rate of change. Any line that is not straight in two dimensions is non-linear.

| Example 2: Tell whether the data below is linear or non-linear and whether it is discrete or continuous. |
|---|




| | | | |
|---|---|---|---|
| Linear | Non-Linear | Linear | Non-Linear |
| Discrete | Continuous | Discrete | Continuous |

## Section 7.3 Vertical Line Test

### Looking Back 7.3

Functions are relations in which each input ($x$-value) has only one output ($y$-value). Two different inputs can be mapped to the same output and the relation is still a function. Any letters, $m$ and $n$, $p$ and $q$, etc., can represent inputs and outputs. The letters $x$ and $y$ are used most often because functions are frequently graphed on a coordinate plane that includes the $x$-axis and $y$-axis. Remember, the input is the independent variable (represented on the $x$-axis), and the output is the dependent variable (represented on the $y$-axis). This is a universal convention used by mathematicians and scientists all over the world to share, interpret, and analyze data correctly.

Mapping is one method that can be used to tell if a table of values is a function. If one $x$-value maps to more than one $y$-value then the data does not represent a function. It is only a relation.

### Looking Ahead 7.3

There is a method that can be used to determine if the graph of an equation represents a function. This method is called the Vertical Line Test. If a vertical line is drawn through each graphed point on a graph and only passes through one point each line, then the graph represents a function. If the line crosses two or more points, then it is not a function.

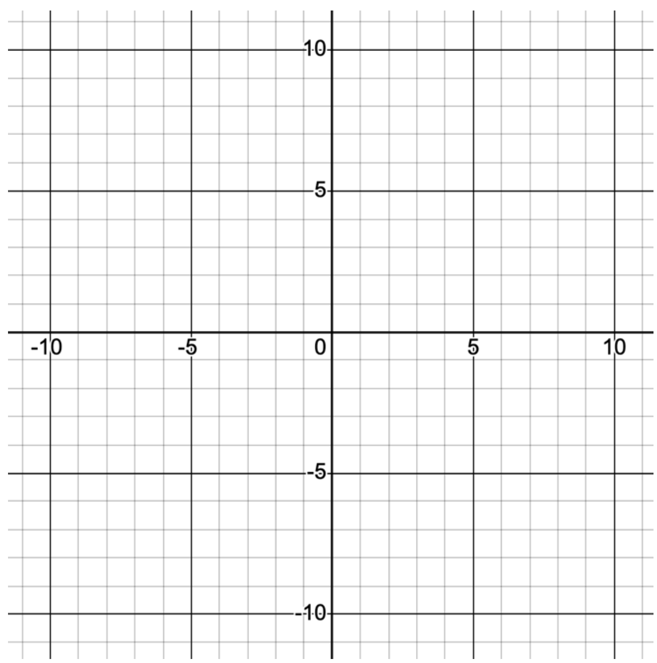
| Example 1: | Use the Vertical Line Test to see if the relation below is a function. |
|---|---|

$$\{(-3, 2), (-1, 1), (0, -4), (2, 2), (2, 4)\}$$

1. Graph the points

2. Draw a vertical line through each point

3. Does the line cross through one point only or more than one point?




This shows that because if one input maps to two different outputs, then the $x$-values repeat. This means two points would cross through the vertical line that crosses through the $x$-axis, and shows that the relation is not a function.

Example 2: Make a table for the values of the equation $y = x^2$ and then graph the ordered pairs. Use the Vertical Line Test to see if it a function.

| $x$ | $y$ |
|---|---|
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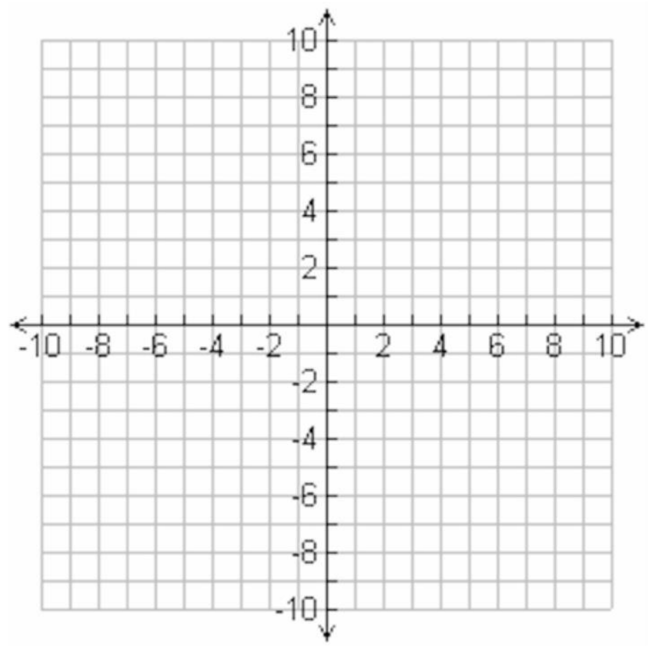
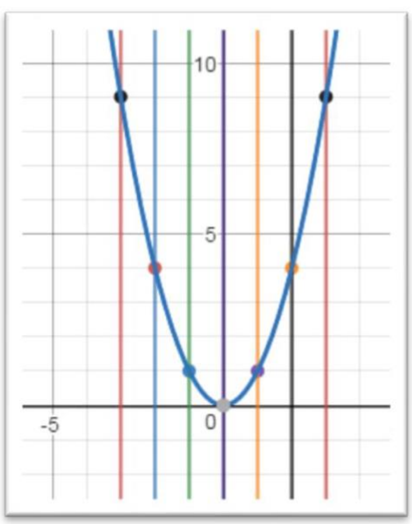
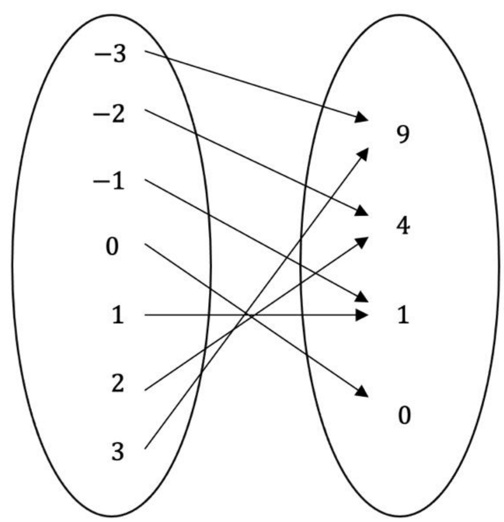



To the left is a table of perfect squares because $x$ is being multiplied by itself (sign included) to get $y$. Graph the points and connect them with a line. Draw a vertical line through each point. Each vertical line only goes through one point on the graph of the equation. To the right is a mapping of the table. Each input is mapped to only one output.



If the graph of an equation fails the Vertical Line Test, it is not a function. This follows the same principal as mappings but uses a graph instead of a table.

The parabola $y = x^2$ passes the Vertical Line Test, which makes it allowable, given the definition of a function. The points $(2, 4)$ and $(-2, 4)$ are both on the graph. The $x$ values are not repeated, but the $y$-values may be. There is only one output when the input is $2$, which is $4$. There is only one output when the input is $-2$, which is also $4$.

Example 3: Make a table for the values of $y = \pm\sqrt{x}$ and then graph the ordered pairs. Use the Vertical Line Test to see if it is a function. (Use perfect squares for the values of $x$ because the square roots can be found without using a calculator. Only use positive values for $x$ because you cannot get a real number when you take the square root of a negative number. The solution of any square root can be positive or negative; this accounts for the $\pm$ sign in front of the problem.)

| $x$ | $y$ |
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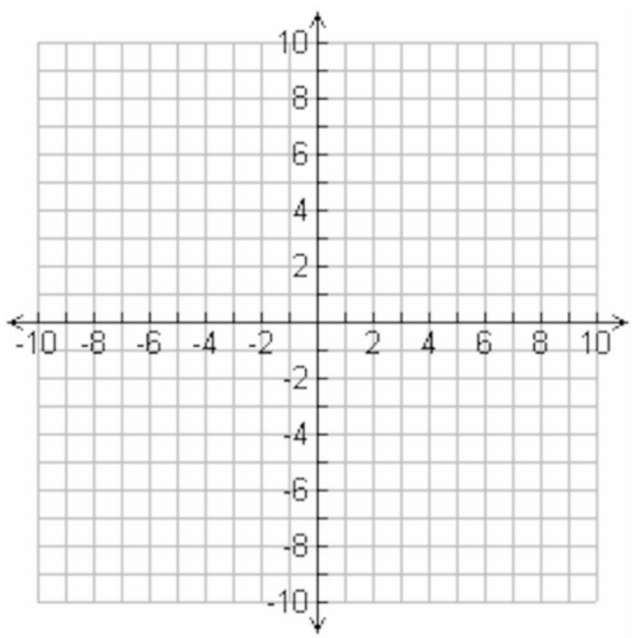
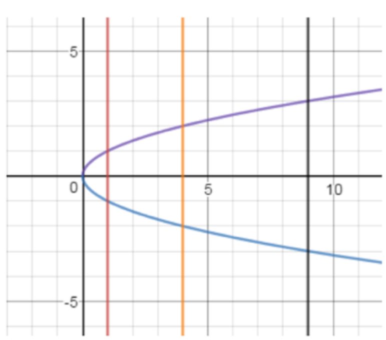
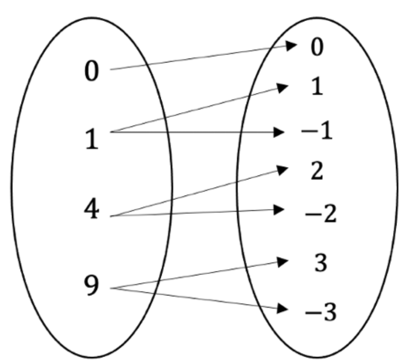



To the left is the graph of $y = \pm\sqrt{x}$. This is not a function; it fails the Vertical Line Test. We call this a lazy parabola (or a sleeping parabola). Looking at the mapping to the right for $y = \pm\sqrt{x}$, you can see three $x$-values: $1; 4; 9$, which each map onto two different $y$-values, one positive value and one negative value. The mapping confirms that the lazy parabola is not a function.



Now we can establish a more detailed definition of a relation and a function. A relation is a set of ordered pairs containing one $x$-value that corresponds to one or more $y$-values. A function is a relation in which each $x$-value corresponds to one unique $y$-value.

Example 4: Is the line $y = x$ a function? Does it pass the Vertical Line Test?

| $x$ | $y$ |
|---|---|
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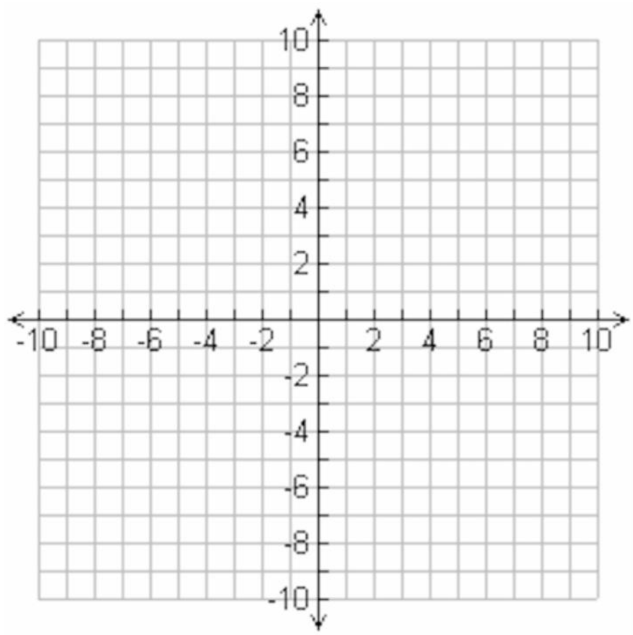
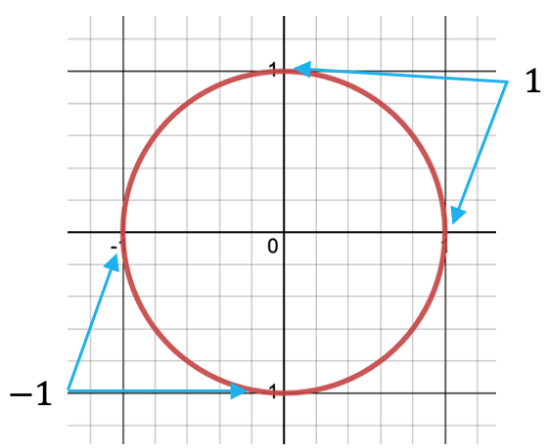
| | |

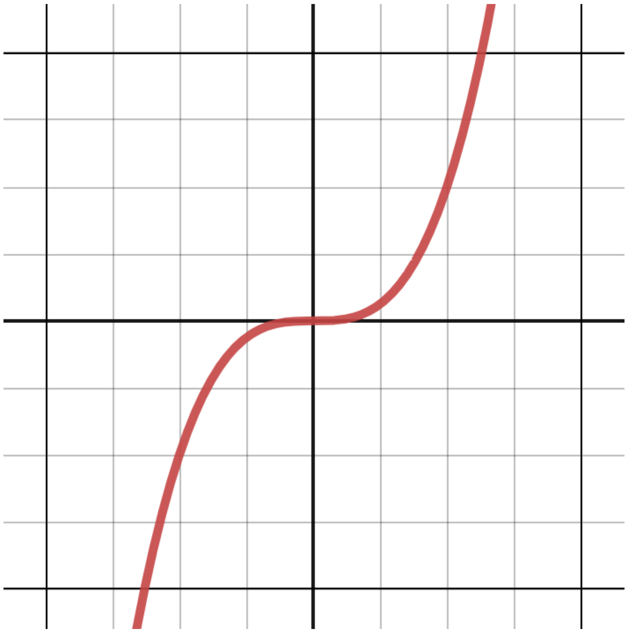



The equation $y = x$ does pass the Vertical Line Test. In fact, a horizontal line drawn at any point on the line would only pass through it once as well! (There is also a Horizontal Line Test that we will learn about in Algebra.) This special function is called a one-to-one function. For every $x$-value there is only one $y$-value that corresponds to it and for every $y$-value there only one $x$-value that corresponds to it.

Example 5: Is $x^2 + y^2 = 1$ a function? Does it pass the Vertical Line Test?



| Example 6: | Is the graph below a one-to-one function? |
|---|---|



Linear equations ($y = x$), quadratic equations ($y = x^2$), and cubic equations ($y = x^3$) are all functions because they pass the Vertical Line Test. Only $y = x$ and $y = x^3$ are one-to-one functions. Do you think that all odd exponents for parent power equations are one-to-one functions and all even exponents to parent power equations are not one-to-one functions? (Parent means there are no shifts right, left, up, or down and that the function is centered at the origin.)

## Section 7.4 Input and Output

### Looking Back 7.4

We know that we can use any letter for a variable and assign it to represent anything in a problem or situation. However, on the coordinate plane, the $x$-axis represents the independent variable and the $y$-axis represents the dependent variable. It is important to note that when we are dealing with problems involving time (seconds, minutes, hours), we usually let $x$ represent time because we cannot change or control it. However, as the $x$ (input) changes, the $y$ (output) changes with it. The reason $y$ is dependent on $x$ is because if we input values in for $x$, we get an output of values for $y$ in terms of $x$. There is a relationship between $x$ and $y$.

When we have an equation, we can use a table to substitute values for $x$ and solve for $y$. This is easiest if the equation is set up for $y$ in terms of $x$ because when a number is substituted for $x$ all numbers combine on one side of the equation and directly give the solution (output of $y$).

$$2x + 4y = 8$$

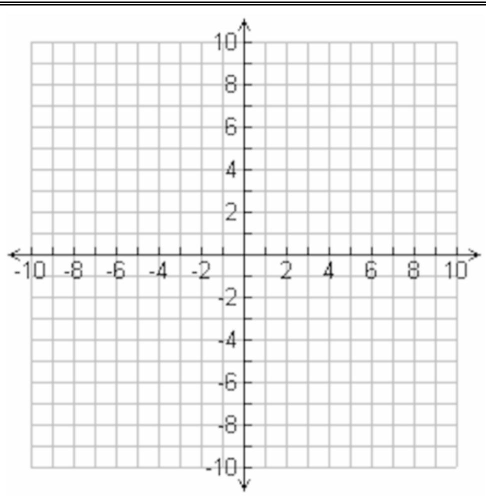
Solve for $x$ in terms of $y$ (substitute the values for $x$ and solve directly for $y$):

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |

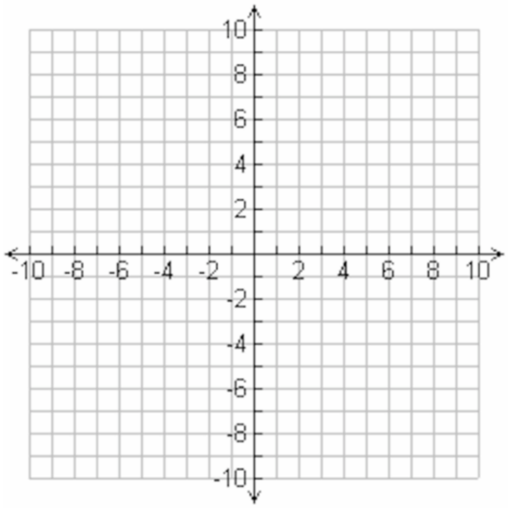
### Looking Ahead 7.4

If we leave the equation in standard form and substitute in values for $x$, we will have to isolate the $y$ every time, which is far more work. We have also learned along the way that certain input values are not allowable and certain output values are not possible. In the previous few sections, we discussed that $y = \pm\sqrt{x}$ works for zero and positive values of $x$. The variable $x$ cannot be a negative number because we cannot take the square root of a negative number and get a real number.

Example 1: What are the possible input and output values for $y = \pm\sqrt{x} + 2$?



Example 2: What are the input and output values for $y = \frac{1}{x} + 1$?




In a function, there is a relationship between input and output in which for every input there is one unique output. This means Example 1 is not a function, but Example 2 is.

Our heart is much like a function machine. The rule is: if you put good things into your heart, good things come out. This is true for health as well: if you eat well, you are likely healthier than if you eat poorly. Therefore, negative thoughts produce negative actions.

Matthew 15:18 says: "But the things that proceed out of the mouth come from the heart."

Proverbs 23:7 says: "For as he (man) thinks within himself, so he is."

After King David had hurt another, he wrote these words in Psalm 51:10: "Create in me a clean heart, O God, and renew a steadfast spirit within me." David was sad for what he had done. He had acted upon selfish desires and hurt others in the process. When God sent the prophet Nathan to tell him this, he asked for God to cleanse his mind and change his heart so that good would pour forth and others would be blessed.

Psalm 119:11 (NIV) says: "Thy word I have treasured in my heart that I might not sin against you." What better input can we have in our lives that our output will be pleasing to God and helpful to others.

## Section 7.5 Function Machines

### Looking Back 7.5

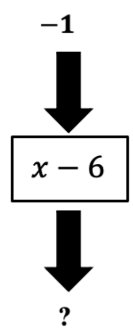
A function machine operates much like a soda pop machine. If one soda pop in a machine costs $1.25, I can insert 5 quarters or one dollar bill and one quarter in the machine to buy it. When I press the "Cola" button, it is because I want a Cola, or expect a Cola. Therefore, if I get one Cola, which is what I paid for and what I expect, the machine is functioning. It is a function because I can put in two different inputs and get the same output.

However, if I put 5 quarters in the machine and get one Dew or one Red Soda, the machine is not functioning. It is not a function because I put in one input and got two different outputs; neither of which is what I expected.

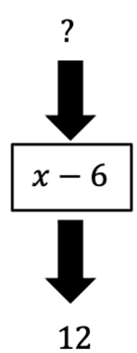
### Looking Ahead 7.5

Think of the function machine as the expression; when we substitute a value for $x$ we get an output for $y$.

| Example 1: | If $-1$ is put in for $x$ and the function machine has a rule of $x-6$, then what comes out? |
|---|---|




| Example 2: | Using the same function machine from Example 1, if the output is 12, what is the input? |
|---|---|



An equation is a number sentence in which two expressions are equal. In simple terms, a function shows the relationship between an input and an output; each input has a unique output.

| |
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| Example 3: If $y = 2x - 7$, what is $y$ when $x = -3$; $x = 0$; $x = 5$? |

| |
|---|
| Example 4: For Example 3, there were two negative solutions and one positive solution. Is it possible to get a solution of 0? |

## Section 7.6 Domain and Range

### Looking Back 7.6

We have been talking about input and output of equations. There are other names for the input and output. Domain is another term for all the possible input values. Range is another term for all the possible output values. So, now there are several representations for input: $x$, domain, and the first coordinate of an ordered pair. There are also several representations for output: $y$, range, and the second coordinate of an ordered pair.

| $x$ | $y$ |
|---|---|
| Input | Output |
| Domain | Range |
| First coordinate in the ordered pair | Second coordinate in the ordered pair |
| Independent variable | Dependent variable |

### Looking Ahead 7.6

We are going to play a game to help you understand how to determine the domain and range of a function. You will need the following items to play:

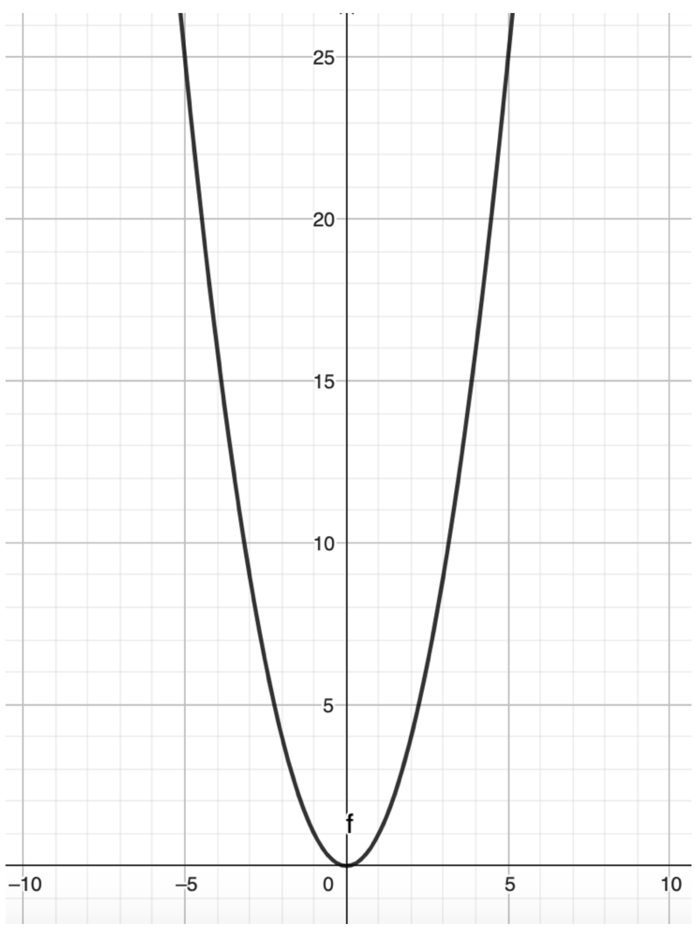
- A *posterboard* marked with the coordinate grid with the $x$-axis numbered from $-5$ to $5$, the $y$-axis numbered from $0$ to $25$, and integers one-inch apart.
- *Chips* (any color) or *coins* (any value)

In Game 1, you will be using the function $y = x^2$.

1. Use the numbers on the $x$-axis for input values. In other words, let $x$ be $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$.
2. Substitute the values for $x$ in the equation $y = x^2$ and find the output values of the $y$ for each of the input values.
3. Put each input and output together in ordered pairs $(x, y)$, and put a chip or coin on each of these ordered pairs on the coordinate grid.
4. Tape a piece of yarn to each chip or coin and connect the yarn to make the parabola of the equation.
5. Move all of the chips or coins with the yarn until they all lie on the $x$-axis. These are the possible values for the domain. Ask yourself: "If the $x$-axis continues indefinitely, how much of the axis will be covered?" Find the pattern given the $x$-axis continues indefinitely.
6. Put the chips or coins back on the ordered pairs as you did in Step 3. Now, move them directly right or left until they are squished onto the $y$-axis (chips or coins may lay on top of other chips or coins). These are the possible values for the range. Ask yourself: "If the $y$-axis continues indefinitely, how much of the axis will be covered?" Find the patten given the $y$-axis continues indefinitely.

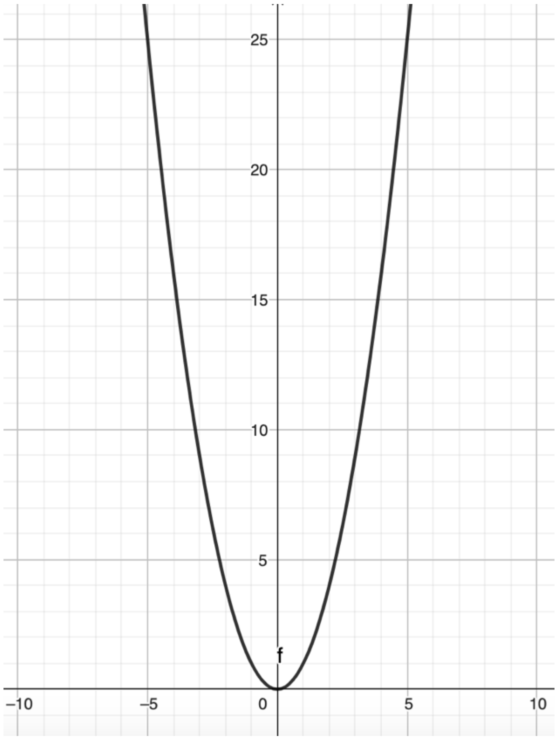
Remember, in Game 1, you will be using the function $y = x^2$.

| **For Step 1 and Step 2 above** | | **For Step 3** |
|---|---|---|
| **Input ($x$)** | **Output ($y$)** | **Ordered Pairs** |
| $-5$ | 25 | $(-5, 25)$ |
| $-4$ | 16 | $(-4, 16)$ |
| $-3$ | 9 | $(-3, 9)$ |
| $-2$ | 4 | $(-2, 4)$ |
| $-1$ | 1 | $(-1, 1)$ |
| 0 | 0 | $(0, 0)$ |
| 1 | 1 | $(1, 1)$ |
| 2 | 4 | $(2, 4)$ |
| 3 | 9 | $(3, 9)$ |
| 4 | 16 | $(4, 16)$ |
| 5 | 25 | $(5, 25)$ |

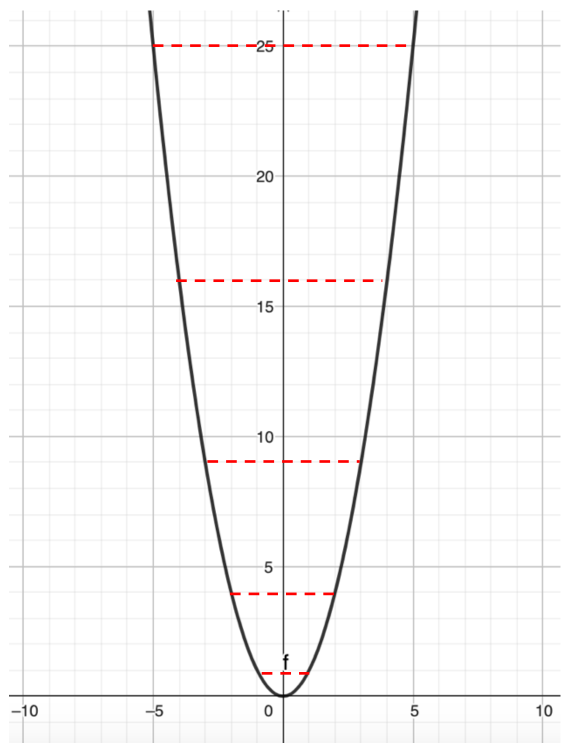


When the ordered pairs are moved to the $x$-axis, as you did in the Step 5, all of the chips will lie on the entire $x$-axis. The yarn represents all the points in between the chips that would lie on the $x$-axis as well. The parabola of the equation would go infinitely to the left and infinitely to the right on the axis given the axis continued.

The domain is real numbers. If you put any positive numbers in for $x$, you will get a solution that is positive. If you put 0 in, you will get a solution that is $0$. If you put any negative numbers in for $x$, you will get a solution that is positive.




When the ordered pairs are squished to the $y$-axis, as you did in Step 6, all points lie on the $y$-axis above $0$. There are no negative values below $0$. All points in between are also included on the yarn. If the $x$ inputs continued indefinitely, the $y$ values would continue so $y$ is 0 and all numbers greater than $0$. This is the range for the equation $y = x^2$. This is logical because when any number is squared, the solution is always positive. The positive values for $y$ are in the first and second quadrant when $y$ is greater than or equal to $0$. This is everything above the $x$-axis. Therefore, for the equation $y = x^2$, the domain is all real numbers, and the range is all numbers greater than or equal to $0$.



Section 7.7 Inequality Notation

Looking Back 7.7

Let us summarize what we have learned:

The set of all possible inputs, often referred to as the independent variable, is called the domain. The set of all possible outputs, often referred to as the dependent variable, is called the range. If there exists no more than one output for each input, then the relation is called a function.

Therefore, in $y = 2x - 3$, the expression $2x - 3$ is called the rule for the function because the input $(x)$ is doubled, and then three is subtracted from it to get the output $(y)$.

The input and output possibilities can be written using inequality notation. If a value is included in the domain or range, $\leq$ (less than or equal to) or $\geq$ (greater than or equal to) is used. If a value is not included in the domain or range, then $<$ (less than) or $>$ (greater than) is used.

Looking Ahead 7.7

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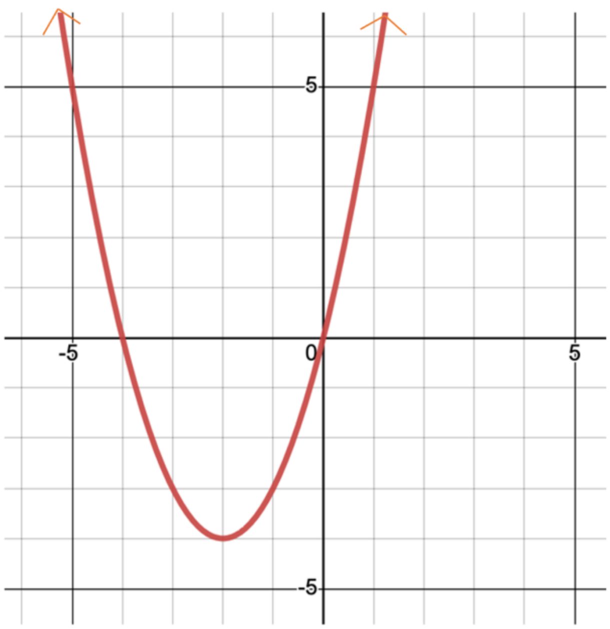
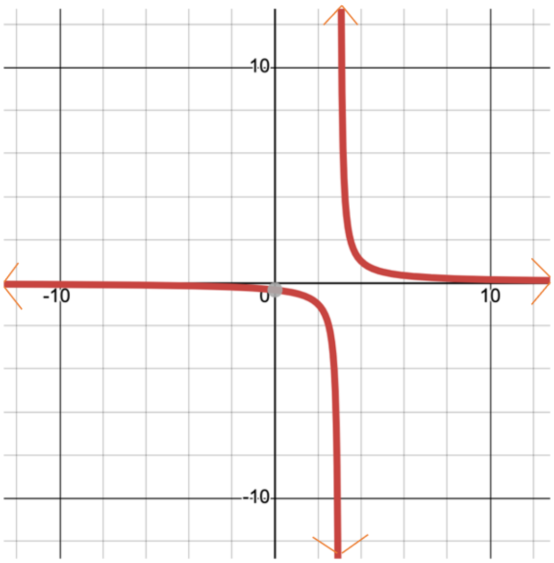
|---|
| Example 1: Find the domain and range of the equation $(x + 2)^2 - 4 = y$. Write the domain and range using inequality notation. |



| | |
|---|---|
| Example 2: | Find the domain and range of the function $y = \frac{1}{x-3}$ and write each using inequality notation. |




If we write "$-\infty < x < \infty$" for the domain, we must explain that $x \neq 3$. The variable $x$ cannot be 3 because it makes the function undefined. There is an asymptote there. There will never be an output of $0$. So, if we write "$-\infty < y < \infty$" for the range, we must explain that $y \neq 0$.

Notice that the arrows at the end of the lines demonstrate they continue on infinitely. If a dot were there instead of an arrow, the line would stop at that point. If the dot is open, the point is not included. If the dot is closed, the point is included.

## Section 7.8 Interval Notation

### Looking Back 7.8

The terms input and output are used for $x$ and $y$. The domain is the set of all possible input values and the range is the set of all possible output values. Domain can be expressed as an inequality. Notice that $x < 3$ is the same as $3 > x$. However, putting the variable first makes the inequality easier to understand. Values that work for this inequality would be 2, 1, 0, $-1.5$, $-3\frac{1}{8}$ and infinitely other values that could be shaded to the left of 3 on the number line. The inequality $x < 3$ looks like this on the one-dimensional number line:


The number 3 is not included so there is an open circle at 3 on the number line. Everything to the left of 3 is shaded. When we do this on the coordinate plane, there is a dashed line at $x = 3$ and everything to the left of this line is shaded.

### Looking Ahead 7.8

When a number is included in an inequality, a less than or equal to ($\leq$) or greater than or equal to ($\geq$) symbol is used to show inclusion. If a number is not included, a greater than ($>$) or less than ($<$) symbol is used because only numbers before or after that number are included.

In internal notation, brackets are used to show inclusion and parenthesis are used to show non-inclusion. Therefore, $x < 0$ could also be written as "$-\infty < x < 0$" and the interval notation for that is $(-\infty, 0)$. Parenthesis are used on both sides; they are on the right because it is the right most boundary ($x$ cannot be 0 but everything less than 0); they are on the left because $-\infty$ is the left-most boundary (unbounded). One can never include infinity. It is never reached. We say the equation is bounded on the right by 0 and unbounded on the left.

For $-2 \leq x \leq 4$, brackets would be used on both sides to show inclusion: $[-2, 4]$.

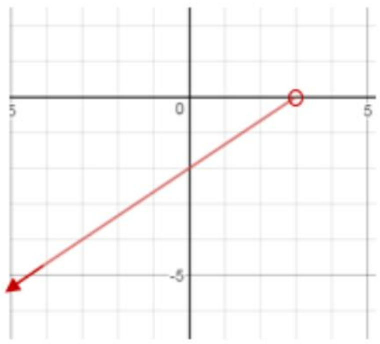
For $-2 < x \leq 4$, the bracket is only on the right side and parenthesis are on the left side: $(-2, 4]$.

For $-2 \leq x < 4$, parenthesis is on the right side and a bracket is on the left side $[-2, 4)$.

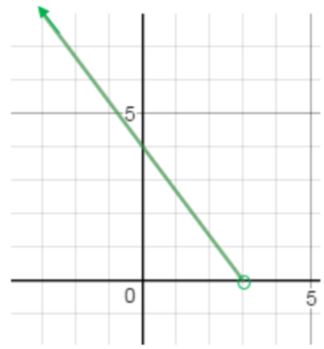
Think of our never-ending God. He alone is God. There is no other. He is not finite. He is infinite, omnipotent, omnipresent, and omniscient (all-powerful, all-present, and all-knowing). He is unlimited, immeasurable, and fathomless; we could even say boundless!

| Example 1: | Use interval notation to define the domain of the graphs below. |
|---|---|

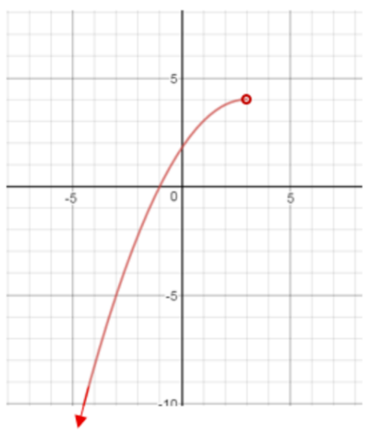
a)



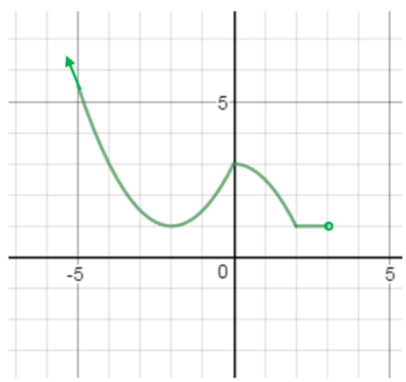

b)




c)




d)



| | |
|---|---|
| Example 2: | Use interval notation to define the range in the graphs from Example 1. |

a) The $y$-values are below the $x$-axis, but do not include $0$. We could write "$y < 0$." The inequality notation is $-\infty < y < 0$. The interval notation for the range is $(-\infty, 0)$.

b) The $y$-values are above $0$ so $y > 0$ and goes on infinitely. The inequality notation is $0 < y < \infty$. The interval notation for the range is $(0, \infty)$.

c) This time, the highest value of $y$ is $4$ but does not include $4$ so $y < 4$ because it goes to $-\infty$ as you can see by the arrow at the end of the graphed line. The inequality notation is $-\infty < y < 4$. The interval notation for the range is $(-\infty, 4)$.

d) Despite the crazy shape, you can see that $y$ goes up infinitely but begins at $1$ and includes $1$. Even though there is an open circle at $(1, 3)$, there is a line between $x = 2$ and $x = 3$ at $y = 1$. Therefore, the inequality notation for the range is $1 \leq y < \infty$. The interval notation for the range is $[1, \infty)$. The range may be written "$y \epsilon [1, \infty)$," which is read "y is a member of the set from one to infinity."

To summarize: Remember, if the line goes right from the $x$-value or up from the $y$-value, then it goes to positive infinity. If the line goes left from the $x$-value or down from the $y$-value, then it goes to negative infinity. When there is an open circle and the number is not included, we use parenthesis for the interval notation. When there is a closed circle and the number is included, then we use brackets for the interval notation.

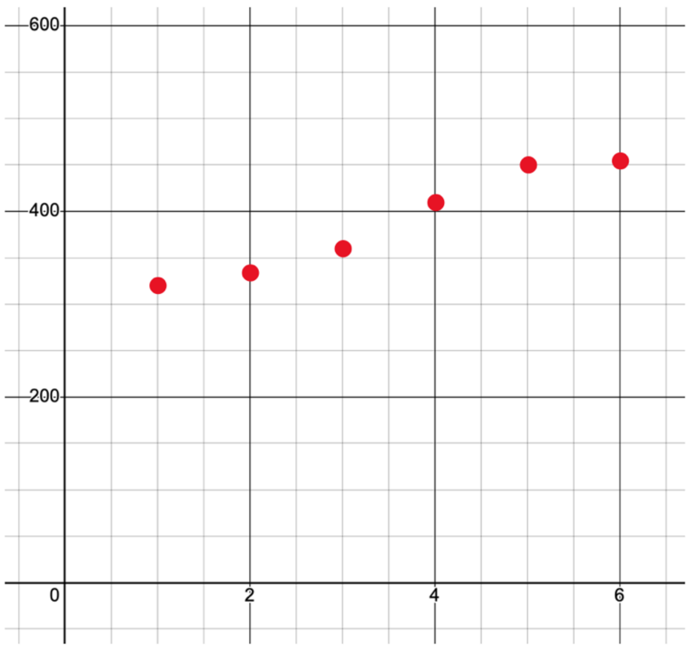
## Section 7.9 Lines of Best Fit

### Looking Back 7.9

A scatterplot is a graph that shows a relationship between two sets of data called bivariate data. When data is collected in real-life, the points may approximate a linear relationship, but do not actually form a straight line. Theoretically, data may form a straight line, but experimentally, it often does not. A line of best fit can be used to show or model a linear relationship. This line is very close to most of the data points. Most of the data lie on this line and an equal amount of data may lie above and below it.

### Looking Ahead 7.9

Example 1: The data below shows the amount spent on entertainment in billions of dollars at major United States cities over a six-year period.




Draw a line of best fit and make predictions as to how many billions of dollars will be spent in these cities in the seventh year.

Line of Best Fit:

Step 1: Select two points on the line and find the slope of the line.

Step 2: Find the $y$-intercept of the line.

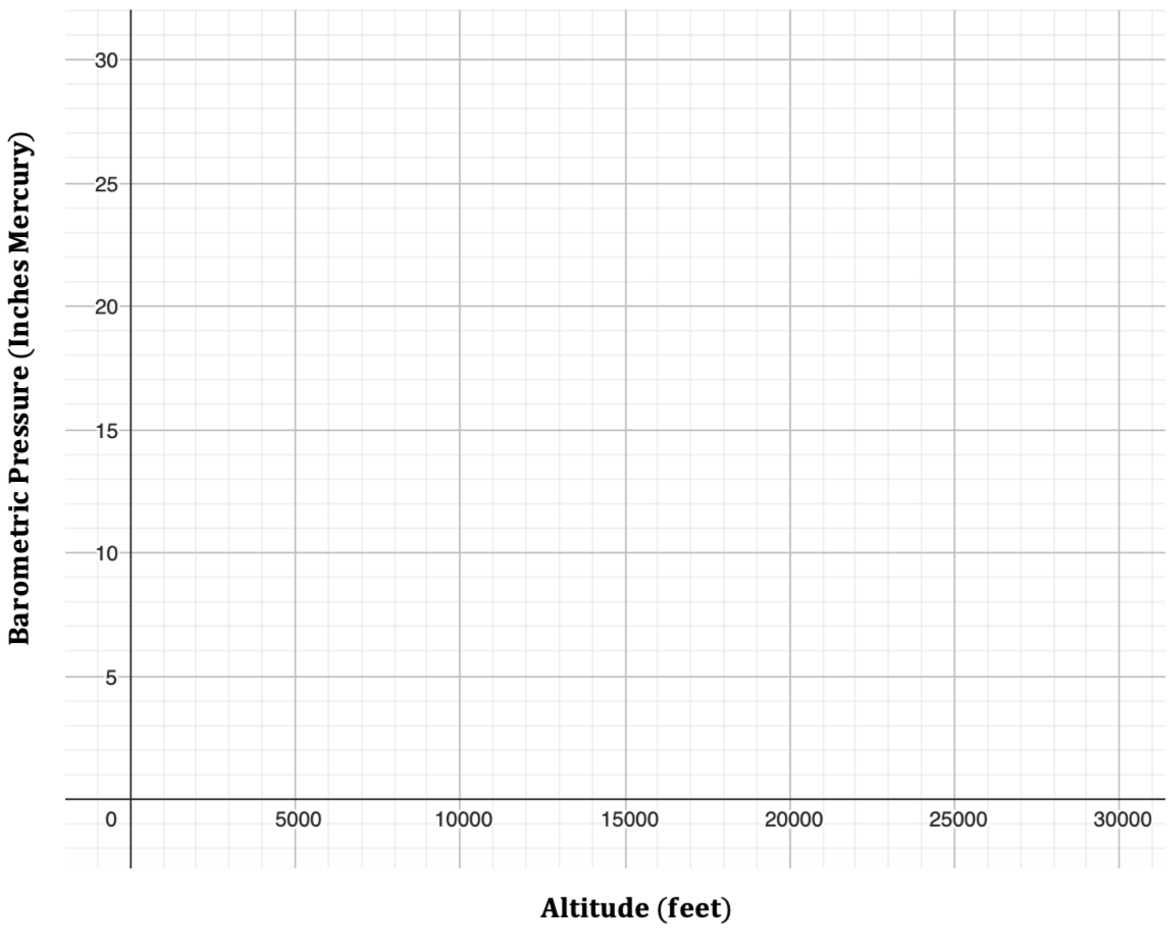
Step 3: Write the equation of the line of best fit.

Step 4: Predict the billions of dollars spent on entertainment for seven years in major United States cities.

Example 2: The table below shows the approximate barometric pressure (inches mercury) given the altitudes in feet.

| **Altitude (ft.)** | 0 | 5,000 | 10,000 | 15,000 | 20,000 | 25,000 | 30,000 |
|---|---|---|---|---|---|---|---|
| **Barometric Pressure (in. mercury)** | 25 | 24 | 22 | 19 | 15 | 13 | 9 |

Draw the line of best fit. Predict what you think the barometric pressure will be at 35,000 feet?



## Section 7.10 Correlation Coefficient

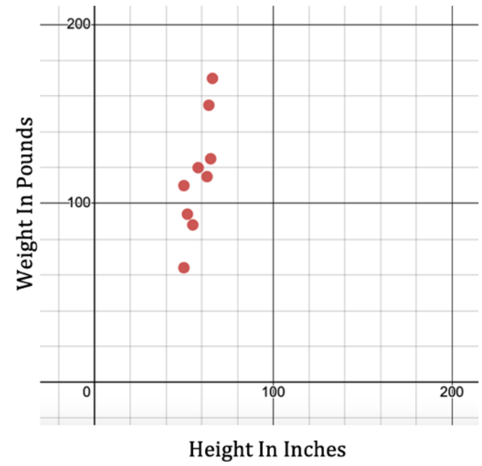
### Looking Back 7.10

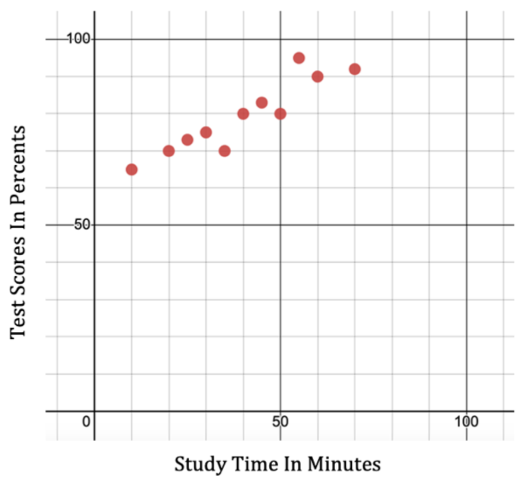
A graph that shows a relationship between two sets of data is called a scatterplot. The bivariate data is two sets of data that are graphed as ordered pairs on a coordinate plane or system.

There are patterns that can be detected in various sets of data.

### No Correlation

There is no obvious trend or pattern. A person's weight is not necessarily affected by their height. There does not seem to be a relationship.



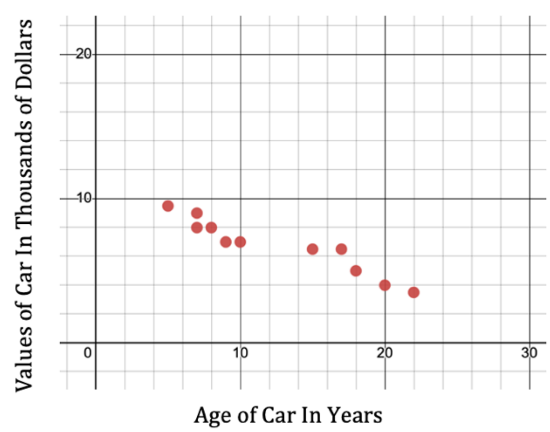




### Positive Correlation

As $x$ increases, $y$ increases. The points can be approximated by a line of best fit that has a positive slope.

### Negative Correlation

As $x$ increases, $y$ decreases. The points can be approximated by a line of best fit that has a negative slope.



Looking Ahead 7.10

The coefficient of correlation denoted by the letter $r$ measures how well a collection of data points can be modeled by a line of best fit. The measure ranges from $-1$ to $1$.

The closer the $r$-value is to $1$, the stronger the positive correlation. The line stays close to the positive slope. A correlation coefficient of $1$ is a perfect positive correlation.

The closer the $r$-value is to $-1$, the stronger the negative correlation. The line stays close to the negative slope. A correlation coefficient of $-1$ is a perfect negative correlation.

All the data fits perfectly on the line for a correlation coefficient that has an $r$-value of $1$ or $-1$.

Example 1: Is there a relationship between the grams of fat in a fast-food sandwich and the total calories in a fast-food sandwich? Use the graphing calculator to get a line of best fit and find the coefficient of correlation for the data below.

| **Sandwich** | **Total Fat (Grams)** | **Total Calories** |
|---|---|---|
| Hamburger | 8 | 250 |
| Cheeseburger | 11 | 313 |
| Half-Pounder | 20 | 440 |
| Half-Pounder with Cheese | 30 | 522 |
| Burger with Special Sauce | 32 | 555 |
| Grilled Chicken | 18 | 420 |
| Fried Chicken | 23 | 485 |
| Fish Fillet | 26 | 538 |

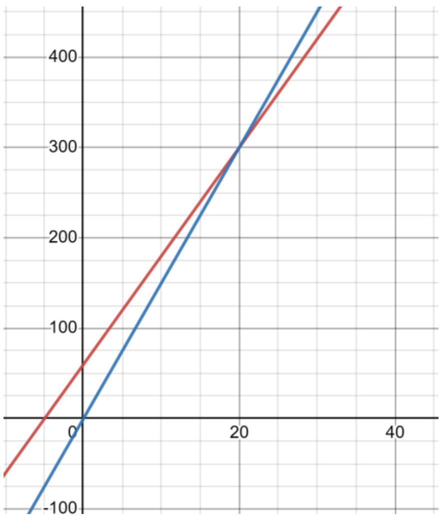
Section 7.11 Solving Systems by Graphing

Looking Back 7.11

Sometimes, there are two functions and where they meet is the solution to a problem. For example, Silas wants to get a job to save money for college. One restaurant pays $12.00 per hour with a $60 sign on bonus. A second job pays $15.00 per hour but with no sign on bonus.

Restaurant Job: $y = \$12.00x + \$60.00$

Second Job: $y = \$15.00x$




These equations assume $x$ represents the number of hours worked and $y$ represents the total pay. The two equations together are called a system of equations. The solution to the two is the one point that tells the number of hours in which the pay is the same. Graph the two equations on one coordinate-plane to find the ordered pair for $(x, y)$ in which $x$ is hours worked and $y$ is the amount paid.

The point of intersection of the two functions is $(20, 300)$ or $(20, \$300)$. That means after $20$ hours of working, the pay will be $\$300.00$ no matter which job Silas works. This can be reached in one week of work as that is half of a full-time $40$-hour work week. It is better to take the second job, which is $\$15.00$ per hour with no sign on bonus as Silas will make more money for college in the long run.

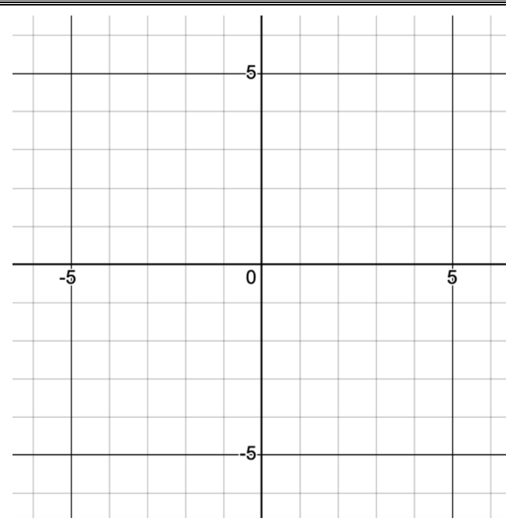
Looking Ahead 7.11

| | |
|---|---|
| Example 1: | Solve the system of equations by graphing. |

$y = 3x + 2$

$y = 3x - 1$

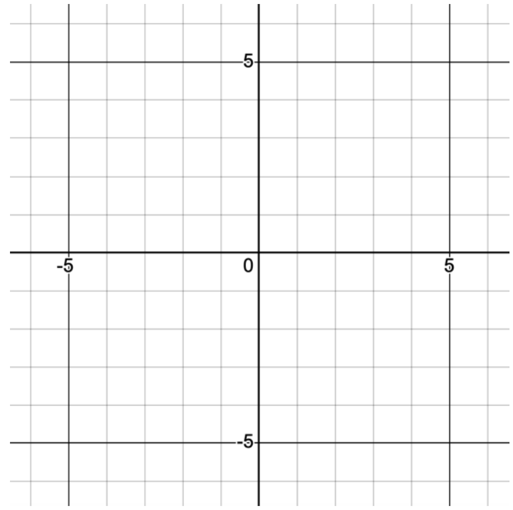
What do you notice?




The lines for these linear functions are parallel, which means all points are equidistant (the same distance apart). They will never meet or intersect. There is no solution that satisfies both equations.

Example 2: Solve the system of equations by graphing.

$$3y = x + 6$$
$$y = \frac{1}{3}x + 2$$




What do you notice?

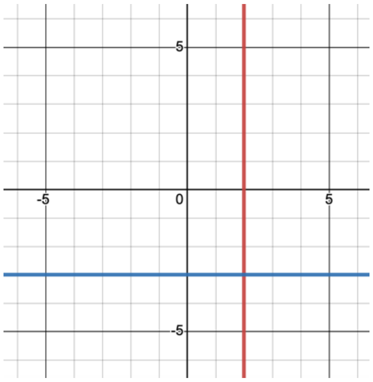
These are the same equation if you solve for $y$. They lie on the same line. There are infinite points of intersection and infinite solutions.

When done manually, graphing to find the point of intersection can be very difficult. A graphing calculator can graph the point of intersection to find the specific solution in which the two or more systems intersect given there is a point at which they meet.

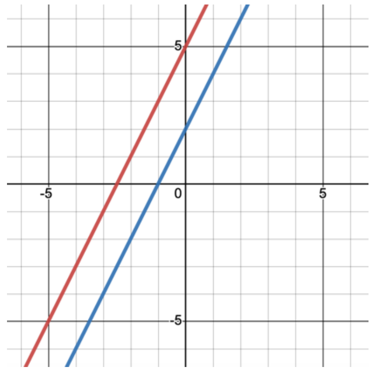
Section 7.12 Solving Systems by Substitution

Looking Back 7.12

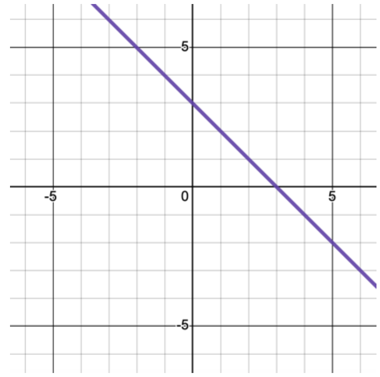
Now we know there are three possible solutions to systems of equations.




One Solution




No Solutions




Infinite Solutions

It may be difficult to find precise solutions when graphing by hand as the points of intersection may include fractions or decimals. A more accurate way to solve systems of equations is by using substitution to find the solution. This method involves setting both the equations equal to the same variable and then solving for that variable. Once that solution is found, it can be substituted in either equation to solve for the other variable.

Looking Ahead 7.12

| Example 1: | Solve the system of equations below by substitution. |
|---|---|

$$y = x + 8$$
$$y = 2x - 4$$

Because $y$ is the same number for both equations, set $y$ equal to $y$ ($y = y$) by setting the expressions on the right equal to one another and solving for $x$:

$$x + 8 = 2x - 4$$

Once you solve $x = 12$, substitute 12 in for $x$ in either equation and find $y$:

$$y = 12 + 8$$

Or

$$y = 2(12) - 4$$

The solution (point of intersection) is $(12, 20)$.

| Example 2: Solve the system of equations below by substitution. |
|---|

$$y = x + 10$$
$$y = x - 4$$

When setting $y = y$ or $x + 10 = x - 4$, the $x$ is eliminated and what is left is $10 = -4$, However, $10 \neq -4$, so there is no solution. These are parallel lines when we graph them.

| Example 3: Solve the system of equations below by substitution. |
|---|

$$2x + y = 4$$
$$y = -2x + 4$$

Solve for $y$ in the first equation:

$$2x + y = 4$$
$$y = -2x + 4$$

Now we can see the first equation is the same as the second equation.

$$y = -2x + 4$$

The lines are the same. There are infinite solutions.

Another way to solve this is to set $y = y$ and use the two expression that are equal to $y$ set equal to one another.

$$-2x + 4 = -2x + 4$$

Because any $x$ on the left side is the same as any $x$ on the right side, there are infinite solutions.

## Section 7.13 Solving Systems by Elimination

### Looking Back 7.13

There is one final method to solving systems of equations. This method involves eliminating one of the two variables given so there is only one variable to solve for. Once that value is known it is substituted into one of the two equations to solve for the other variable (substitution). Therefore, when we solve systems of equations by elimination, when we get one variable isolated, we then use substitution to solve for it.

The Golden Rule of Algebra tells us that what we do to one side of the equation we must do to the other. Anything can be multiplied, divided, added, or subtracted from an equation as long as it is done to both sides; the equation stays balanced as both sides stay equal to one another.

$$4+8=4+8$$
$$12=12$$

Or

$$2(4+8)=2(4+8)$$
$$2\cdot 12=2\cdot 12$$
$$24=24$$

Or

$$\frac{(4+8)}{2}=\frac{(4+8)}{2}$$
$$\frac{12}{2}=\frac{12}{2}$$
$$6=6$$

Or

$$3+(4+8)=3+(4+8)$$
$$3+12=3+12$$
$$15=15$$

Or

$$(4+8)-5=(4+8)-5$$
$$12-5=12-5$$
$$7=7$$

We can also add two equations or subtract two equations as long as we add both sides or subtract both sides.

### Looking Ahead 7.13

| | |
|---|---|
| Example 1: | Use elimination to solve the system of equations below. |

$$x+y=15$$
$$4x-y=10$$

| | |
|---|---|
| Example 2: | Use elimination to solve the system of equations below. |

$$2x + 4 = y$$
$$2x - 6 = y$$

If the two equations have a variable with the same coefficient but opposite signs, they can simply be added and that variable will be eliminated.

If the two equations have the same coefficient and the same sign, they can simply be subtracted and that variable will be eliminated.

Sometimes, nothing can be eliminated by simply adding or subtracting the two equations. In that case, something must be multiplied or divided to make the coefficients the same or opposites.

| | |
|---|---|
| Example 3: | Use elimination to solve the system of equations below. |

$$2x + 3y = 12$$
$$-4x + y = -10$$