Module 1 Solving Equations and Inequalities

Section 1.1 Solving Equations with Variables on Both Sides

Practice Problems 1.1

For Problem 1-9, solve and check the equation given. (Simplify each side first by combining like terms.)

1. $4t + 2 = 6t - 6$

2. $6n - 2 = 2n + 10$

3. $3p + 6 = -18 - 9p$

4. $1.6s - 1 = 5.6s + 3$

5. $2 - 3p - 5 = 6p + 4 + 5p$

6. $5 + \frac{2}{3}x = 2 + \frac{1}{3}x$

7. $5x + 2 = 3x - 38$

8. $6s + 3.5 + 65 = 26.5$

9. $m - 1 = 5m + 3m - 8$

For Problem 10, solve the word problem given.

10. When we solve equations, does it matter which side the variable ends up on? Why or why not?

Section 1.2 Undoing Exponents

Practice Problems 1.2

For Problem 1-6, solve for the variable in the equation given.

1. $y^2 = 25$

2. $m^2 = 100$

3. $x^3 = 64$

4. $x^3 = 125$

5. $y^3 = 1{,}000$

6. $z^2 = 81$

For Problem 7-15, solve for the variable in the equation given and check your solution(s).

7. $5m^2 = 500$

8. $p^2 - 6 = 115$

9. $19 = 4n^3 - 13$

10. $233 = q^3 + 17$

11. $2r - 24 = 14$

12. $-2m^2 + 60 = 10$

13. $(x + 6)^3 = 27$

14. $(y + 2)^2 = 49$

15. $2(y - 15)^2 = 98$

Section 1.3 Undoing Parenthesis

Practice Problems 1.3

For Problem 1-10, solve for the variable in the equation given and check your solution(s).

1. $(x-3)-5=22$

2. $14.9=16+(y-3.1)$

3. $(m+14)^3=64$

4. $144=(n-6)^2$

5. $14+(6+y)^2=210$

6. $21+(x-8)^2=37$

7. $87=(15-n)^2-13$

8. $30=(24+m)^3+3$

9. $-28+(5a-5)=-3$

10. $(4k+40)=(6+2k)$

Section 1.4 Simplifying and Solving

Practice Problems 1.4

For Problem 1-10, solve for the variable in the equation given and check your solution.

1. $3 + (4 - s)^2 + 11 = 23$

2. $6t^2 + 18 = 168$

3. $(3 + p)^2 - 14 = 11$

4. $2m + 3 - 6m = -13$

5. $7x + 21 = 6x$

6. $10 = 7x^3 + 17$

7. $-22 = -15 + 4q^3 - 7$

8. $(5 - t)^3 + 19 = 46$

9. $-4 + (x - 7)^2 = 32$

10. $(3x - 8)^3 = 64$

Section 1.5 Reverse Thinking

Practice Problems 1.5

For Problem 1-6, solve the word problem given.

1. A colt and a mare were eating apples. If the mare gave the colt one apple, they would have the same number of apples. If the colt gave the mare one of his apples, the mare would have twice as many apples as the colt. How many apples did each have to begin with?

2. Daniel pays one dollar to get into a basketball game. He loans his friends half of what money he has left and must pay another dollar to leave the game. Daniel goes to another high school where the same thing happens. At a third game, the same thing happens again. Finally, Daniel leaves the third game with only one dollar in his pocket (not what you would call a "thrifty spender!"). How much money did Daniel begin with? (Use reverse thinking to solve the problem.)

3. During school hours, Stephen, Kelly, and Danny were continually loaning and borrowing supplies. The first friend loaned the second friend and third friend as many pencils as they already had. Then, three months later, the second friend loaned the first friend and the third friend as many pencils as they already had. Finally, another three months later, the third friend loaned the first friend and second friend as many pencils as they already had. Now, each friend has 24 pencils. How many pencils did each friend begin with? (Work backwards to solve the problem.)

4. Jonathan, Gaje, and Mauricio had an equal share of 24 technological magazines each. They split them up between them. Each ended up with a number equal to his/her age three years before. The youngest (Mauricio) kept half the magazines he had and divided the rest equally between the other two. The middle (Gaje) then kept half of the remaining magazines they had and split the rest between the other two. The oldest (Jonathan) then did the same. How old is each friend?

5. Michaela, Gabriel, Maddy, Graham, and Erin were writing algebra story problems for a class assignment. Michaela wrote problems while the others went to a movie. When the problems were due, Michaela felt sorry for her friends and gave them each an equal amount of her problems to turn in, leaving none for herself. The day they were to be handed in, Gabriel made up six more of his own, Maddy discovered two incorrect problems and threw them away, Graham tripled the number of problems he had, and Erin threw away two-thirds of her problems (the ones she did not review). When they handed them in, they had 36 problems all together. How many problems did Michaela give each of her friends to begin with?

6. Jared bought Twinkies® to share equally with his friends Christian and Ian. After each ate four Twinkies®, the total number of Twinkies® was equal to the number after the division. How many Twinkies® did Jared bring to begin with?

Section 1.6 Writing Algebraic Inequalities

Practice Problems 1.6

For Problem 1-6, write an algebraic inequality for the phrase given.

1. x is greater than or equal to 11

2. y is less than -5

3. v is at most 23

4. m is no more than -2

5. t is no less than 13

6. j is less than 15

For Problem 7-9, write the algebraic inequality as a verbal inequality.

7. $a > 4$

8. $d \leq -7$

9. $t \geq -3$

For Problem 10-12, write the verbal inequality as an algebraic inequality.

10. The classroom can hold at most 25 students. Let s be students in the classroom.

11. The high school play was attended by at least 450 people. Let p the number of people who attend the play.

12. Since starting his job, Timothy has saved more than \$200. Let s be how much Timothy has saved.

For Problem 13-15, solve the word problem given.

13. Can you write $t \neq 1$ as an inequality at least two different ways?

14. How would you use a clever form of an equal sign to write $f > 4$ and $f < 4$?

15. How would you use a clever form of an equal sign to write r is all numbers except -2?

Section 1.7 Inequalities on the Number Line

Practice Problems 1.7

For Problem 1-6, graph the inequality given.

1. $t > 3$

2. $p \leq -1$

3. $m > 6$

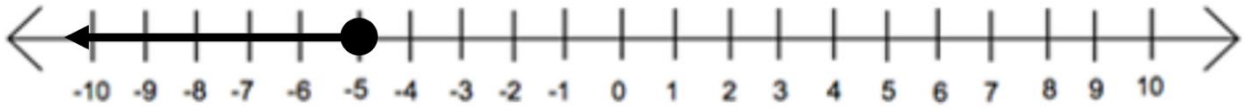
4. $1 < r$

5. $v \neq 4$

6. $-5 \geq y$

For Problem 7-12, write an inequality for the number line given. (Use any variable.)

7.



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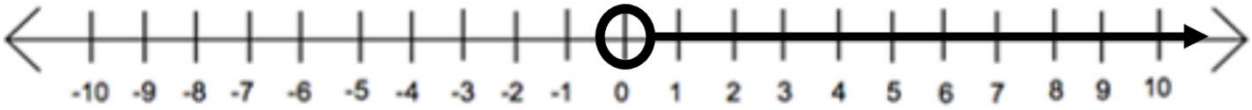

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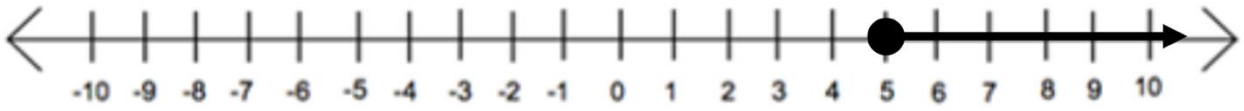


10.

11.

12.

For Problem 13-15, write an inequality for the situation given.

13. The number of people at the ball game was over 2,000. Let n be the number of people at the ball game.

14. The number of cars on the lot was less than 350. Let a be the number of cars on the lot.

15. Matthew 18:21-22 reads: "Then Peter came to Jesus and asked, 'Lord, how many times shall I forgive my brother when he sins against me? Up to seven times?' Jesus answered, 'I tell you, not seven times, but seventy-seven times. So, we should forgive people no less than seventy-seven times.'" Let f be the number of times we shall forgive people.

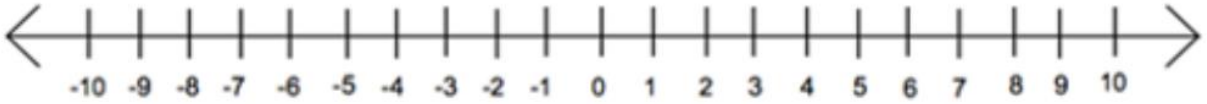
"Be kind and compassionate to one another, forgiving each other, just as in Christ God forgave you." -NIV. Ephesians 4:32

Section 1.8 Solving Inequalities with Addition and Subtraction

Practice Problems 1.8

For Problem 1-12, solve the inequality, graph the solutions, and check your final solution.

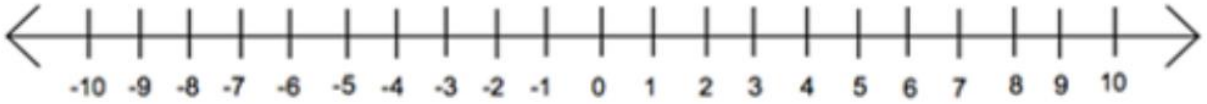
1. $-6 \geq x + 4$



2. $w + \frac{3}{5} \leq \frac{3}{5}$

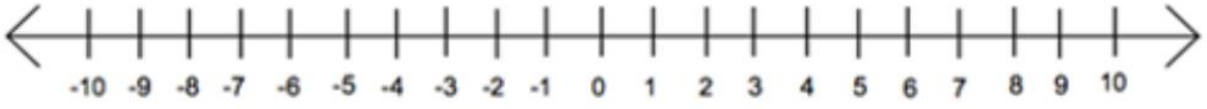
3. $-\frac{4}{9} < t + \frac{1}{18}$

4. $q - 12 + 7 > 4$



5. $5v - 2v + 5 < -10$

6. $m - 3 < 6$



7. $b - 8 \leq -16$

8. $k - 4 < -8$

9. $-7 \leq t - 2$

10. $-5.4 > p - 2.1$

11. $g - \frac{1}{5} \geq 7$

12. $14 \leq 10 + z$

Section 1.9 Solving Inequalities with Multiplication and Addition

Practice Problems 1.9

For Problem 1-5, solve the inequality and graph your solution. (Check your solution.)

1. $-4x > 24$

2. $\frac{3}{4}y < 3$

3. $9t > -18$

4. $63 \leq 9q$

5. $\frac{-5}{7}m \geq \frac{-3}{4}$

For Problem 6-10, solve the inequality and tell whether or not you need to change the inequality sign. Also tell what the final inequality sign will look like. (Keep the variable on the side it is on in the original problem.)

6. $12 < -\frac{x}{5}$

7. $3y \geq -72$

8. $\frac{-6}{7}r \leq -\frac{3}{4}$

9. $18 < -6w$

10. $13v - 9v > -15$

Section 1.10 Solving Two-Step Inequalities

Practice Problems 1.10

For Problem 1-6, solve and graph the inequality given.

1. $20 \leq 2x + 6$

2. $10 \geq -3x + 1$

3. $x < -2x + 3$

4. $-4y \geq 5y + 27$

5. $23 \leq 3x + 2$

6. $4 < \frac{1}{2}x + 5$

For Problem 7-13, solve the inequality given and check your solution.

7. $4x + 8 \geq 20$

8. $3x - 9 \leq 21$

9. $-5x + 14 \geq 94$

10. $-11x - 11 < 88$

11. $5x - 6 \leq 9$

12. $3x + 12 \geq 11$

13. $23 \geq 2x + 9$

Section 1.11 Multi-Step Inequalities

Practice Problems 1.11

For Problem 1-10, solve the multi-step inequality given and graph the solution on the number line. (Check your solution.)

1. $3(x-1) < 21$

2. $-14(x-2) \geq 42$

3. $16(x+6) - 12 < 34$

4. $4(x+6) \geq 24$

5. $-10x + 9 \leq 21$

6. $-6x - 15 > 3(x + 4)$

7. $-2(4 + n) \geq n + 1$

8. $4(t + 2) - 3 \geq 1$

9. $3u + 5u < 2(u - 6)$

10. $11 - 2t > 44 + 9t$

For Problem 11-14, write the inequality for the phrase or situation given.

11. 4 more than v less 6 is no less than -2

12. n is a non-negative integer

13. A parking garage sign says: "Maximum $15.00 per day." Let p represent parking fees.

14. A sign for a job says: "Wages are at least $15.00 an hour." Let w represent wages.

For Problem 15, solve the word problem.

15. What is the greatest integer that satisfies $u < -6$?

Section 1.12 Solving Inequalities

Practice Problems 1.12

For Problem 1-4, write an inequality to represent the situation given.

1. Bananas sell for between \$0.34 per pound and \$0.45 per pound. Let b be price per pound for bananas.

2. The price of silver is between \$13.50 per ounce and \$17.60 per ounce. Let s be price per ounce for silver.

3. Discounted rates at an amusement park are given for groups between 8 and 12 people or groups of more than twelve people. Let n be the number of people for whom discounted rates apply.

4. In any triangle, the lengths of two sides of the triangle are greater than the third side. Let a, b, and c be the sides of a triangle.

For Problem 5-9, solve the compound inequality given and graph the solution on the number line.

5. $0 < p - 5 \leq 2$

6. $-16 \leq 2m + 4 < 22$

7. $2 - 17h > 2h - 36$ or $-3h + 5 < -10$

8. $-6 < 2(x + 3) < 8$

9. $\frac{3p+1}{4} > 7$ or $\frac{3-2p}{5} < 1$

For Problem 10, solve the word problem.

10. A given spring is stretched d inches by applying a force F in pounds. The force (F) exerted on a spring is directly proportional to the distance (d) it is stretched from resting position. The ratio for this given spring is $\frac{d}{\mathrm{F}} = 0.6$. An exerted force is between 30 and 45 pounds including 30 and 45 pounds. Write an inequality describing the stretch and find the possible distances stretched.

Section 1.13 Solving Equations and Inequality Word Problems

Practice Problems 1.13

For Problem 1-5, identify the variable in the phrase given, write an algebraic equation for the phrase, and then solve the equation.

1. Timothy is making a garden for the local food pantry. He has 110 feet of fencing and the garden is 30 feet long. What are the possible dimensions for the width of the garden? Let w be width.

2. What is the width of a rectangle that has a length of 6.5 cm. and an area of 245 cm.?

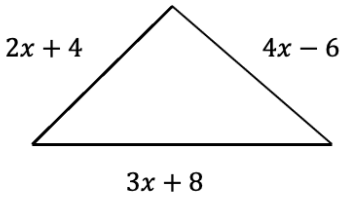
3. A car makes a trip at the rate of 50 mph. A plane makes the same trip at 140 mph and does it in 5 hours less than the car. How long did it take the car and the plane to make the trip? Use the formula: $\text{distance} = \text{rate} \times \text{time}$ to solve the problem.

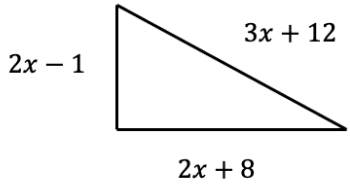
4. Tickets for the concert are $24 each. There is also a $5.95 transaction fee per ticket. If Jordan spent $599.00 on tickets, how many tickets did he buy?

5. At a concert, posters are on sale for $2.50 each and t-shirts are also on sale. Miriam wants a t-shirt and 3 posters. Madison wants 2 t-shirts and 2 posters. Emily wants a poster and Lexie wants 5 posters. If Mom spends $75.50 for all of these items, how much does a t-shirt cost?

For Problem 6, use the diagram given to find the length of each side of the triangles and the perimeter of each triangle. Both triangles have the same perimeter.

6.





For Problem 7-10, solve the word problem given.

7. Mariah has $130 to spend on DVDs. Because of the sale at the store she goes to, each DVD costs only $8.99. Write an inequality for how many DVDs Mariah can buy at that price and then solve the inequality to find out just how many DVDs she can buy.

8. Judy earns $400 a week plus 16% commission on all her sales. She needs to earn at least $875 this week to be able to go on a trip she has planned for next week. What is the minimum number of sales Judy must make this week to earn enough money for her trip?

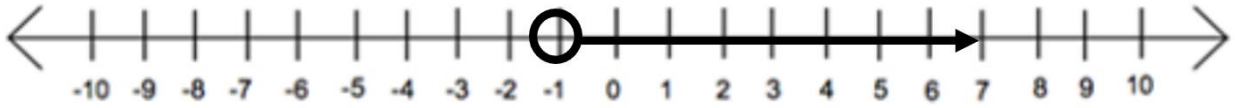
9. A company has $2,000 in their budget to purchase toner for their copier. Each cartridge of toner costs $58.00. Suppose there is a 5% discount on each cartridge and a flat shipping rate of $13.99 if they place a bulk order. How many cartridges can the company purchase in a bulk order without going over budget?

10. The cost of renting a car for a trip is $25 a day plus $2 a mile. Sandra is going on a six-day trip and she cannot spend more than $2,500 on travel. How many miles can Sandra drive on the trip without going over budget but spending all of her budget?

Section 1.14 Module Review

For Problem 1 and 2, write an inequality for the number line given.

1.



2.

For Problem 3-7, graph the given inequality on the number line.

3. $x < -5$

4. $h \geq -3$

5. $8 \geq y$

6. $t > 3\frac{1}{2}$

7. m is all real numbers but $m \neq 5$

For Problem 8, solve the word problem.

8. Write Problem 7 as a compound inequality.

For Problem 9-11, graph the inequality given on the number line.

9. $2x > -2$

10. $-3y \geq -12$

11. $7 > 3x - 2$

For Problem 12-15, solve the equation given.

12. $-5(q + 20) = 35$

13. $-2(p + 4) = 12$

14. $3(x-4)=-51$

15. $6+x=12+4x$

For Problem 16 and 17, solve the inequality given.

16. $3y+2y-16\leq 49$

17. $2x+5-6x\leq -4(3x-2)+5$

For Problem 18-20, solve the word problem given.

18. Helen wants to make a table runner for her dining room table. It needs to be 3 ft. wide. She has 20 square feet of material. How long can Helen make the table runner?

19. A number is at least 3 less 6 times the number. Write an inequality and solve the inequality.

20. Tess wants to go the Olympics. Travel would cost her: $1,832 round trip for plane tickets; $1,250 for food and tickets to events; $800 for souvenirs. She cannot spend more than $6,500. How much can Tess spend on a motel?

Section 1.15 Module Test

For Problem 1 and 2, write an inequality for the number line given.

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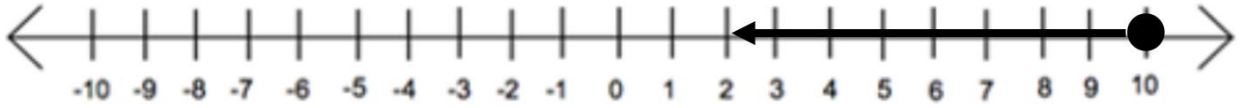

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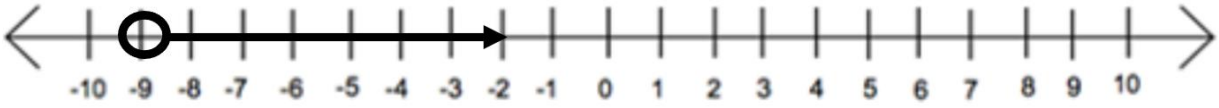


For Problem 3-6, graph the inequality given on the number line.

3. $x > 3$

4. $h \geq 7$

5. $-2 \geq y$

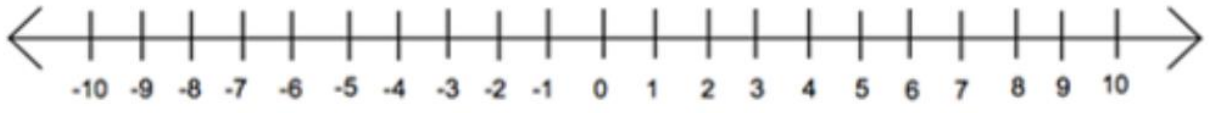
6. $t < 3.75$

For Problem 7, solve the word problem given.

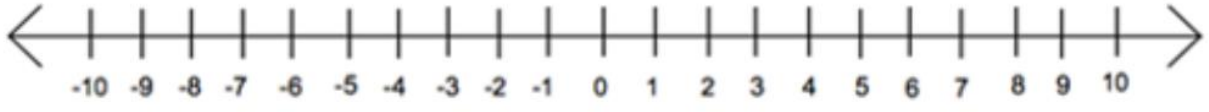
7. Write two different inequalities to represent the graph on the number line below.

For Problem 8 and 9, graph the inequality given on the number line.

8. $-9y \geq -27$



9. $11 < -2x + 3$



For Problem 10 and 11, solve the equation given.

10. $-4(x - 3) = -40$

11. $(\sqrt{x + 2})^2 = 25$

For Problem 12-15, solve the inequality given.

12. $-15 \leq -22 - 14x$

13. $\frac{1}{9} \geq \frac{2}{9}x - \frac{4}{9}$

14. $-2(x + 4.2) < 5.6$

15. $5x + 8 > 4(3x + 4)$

For Problem 16-21, solve the word problem given.

16. Ceilidh must score an 80% or higher in her after school program to be eligible for the placement exam. Her current grades are 63%, 79%, 84%, and 88%. What is the minimum score Ceilidh can get on her final exam to be eligible to take the placement exam?

17. The sum of a number and 36 is greater than the product of -3 and that number. What are the possible values for the number?

18. Marilou works in Chicago at the Sears Tower. She makes \$57 an hour but work requires she wear a suit every day and it must be in perfect shape: no spots, no wrinkles, etc. Marilou only owns one suit, so every day after work she gets it cleaned. To clean the suit costs \$10. In order to pay all her bills and still have money left to enjoy the museums and activities in the city, Marilou needs to make at least \$450 a day plus the money for dry cleaning. How many hours must she work a day?

19. The student council collected \$414.00 from fundraisers. They want to buy t-shirts for all 27 members of the student council. What is the most they can spend on each t-shirt?

20. Riddhi solved $-10t \leq 135$ by adding $+10$ to both sides of the equation and wrote $t \leq 145$ for her solution. When she substituted 145 for t, she got a solution that worked. Why did her solution work for t? What is the correct solution for $-10t \leq 135$?