Module 4 Radicals and Their Roots

Section 4.1 Exponents Revisited

Practice Problems 4.1

For Problem 1-4, simplify the expression given.

1. -7^2

2. $(-4)^2$

3. 18^0

4. 1^{12}

For Problem 5-8, tell if the standard form solution for the exponent given is positive or negative.

5. $(-3)^7$

6. $(-5)^{10}$

7. -8^3

8. -4^{12}

For Problem 9-20, simplify the exponent given.

9. $x^4 \cdot x^{12}$

10. $y^8 \cdot y^{-3}$

11. $z^{-2} \cdot z^5 \cdot z^{-9}$

12. $(q^2)^7$

13. $(r^3)^{-2}$

14. $t^{-3} \cdot t^5 \cdot t^{-2}$

15. $c^3 \cdot d^5$

16. $x^3 \cdot y^3 \cdot z^7 \cdot x^5 \cdot y^{-3}$

17. $(q^2r^2)^3$

18. $(z^4)^3$

19. $z^4 \cdot z^3$

20. $z^4 \cdot z^{-3}$

Section 4.2 Perfect Squares

Practice Problems 4.2

For Problem 1 and 2, follow the instructions to solve the problem given.

1. Define a perfect square.

2. Draw a square made up of 1×1 squares that has 64 square units. How many rows are there in your square?

For Problem 3-10, use the multiplication table from Example 2 (Lesson Notes) to find the length of the side of a square that has the given area in square units.

3. $A = 36$ square units

4. $A = 400$ square units

5. $A = 49$ square units

6. $A = 576$ square units

7. $A = 121$ square units

8. $A = 441$ square units

9. $A = 289$ square units

10. $A = 196$ square units

For Problem 11-14, use the multiplication table from Example 2 (Lesson Notes) to find the area of a square with the side lengths given.

11. 13 units

12. 15 units

13. 18 units

14. 23 units

For Problem 15-18, tell whether or not the number given is a perfect square.

15. 144

16. -64

17. y^{-2}

18. t^8

For Problem 19 and 20, solve the word problem given.

19. Is a geometric figure with an area of x^5 a square or a rectangle?

20. What are the possible factors of x^5 that represents the lengths of the sides of the rectangle with an area of x^5 square units?

Section 4.3 Perfect Squares and Their Square Roots

Practice Problems 4.3

For Problem 1-6, find the square root of the expression given.

1. $\sqrt{81}$

2. $\sqrt{144}$

3. $-\sqrt{169}$

4. $\pm\sqrt{64}$

5. $\sqrt{-196}$

6. $-\sqrt{100}$

For Problem 7-12, identify the index and radicand in the expression given.

7. $\sqrt{49}$

8. $\sqrt[4]{81}$

9. $\sqrt[8]{256}$

10. $\sqrt[2]{196}$

11. $\sqrt[6]{64}$

12. $\sqrt[4]{16}$

For Problem 13, use the given information to solve the problem.
Nikko is trying to find the length of the side of a square table. He tells his teacher that the side is either 18 inches or -18 inches.

13. Why does Nikko's solution seem unreasonable? What may have Nikko been thinking?

For Problem 14-18, name the number of identical factors for the radical given and name them.

14. $\sqrt[2]{441}$

15. $\sqrt[4]{81}$

16. $\sqrt[6]{64}$

17. $\sqrt{225}$

18. $\sqrt[100]{1}$

For Problem 19 and 20, use your understanding of exponents to simplify the expression given.

19. $\sqrt[3]{27}$

20. $\sqrt[3]{-27}$

Section 4.4 Non-Perfect Squares and Square Roots

Practice Problems 4.4

For Problem 1-6, tell which two square roots of perfect square number the non-perfect square root given lies between. Write the square roots of the perfect square number in simplified form as an integer.

1. $\sqrt{21}$

2. $\sqrt{75}$

3. $\sqrt{38}$

4. $\sqrt{101}$

5. $\sqrt{68}$

6. $\sqrt{72}$

For Problem 7-12, tell which two integers the square root given lies between.

7. $\sqrt{27}$

8. $\sqrt{61}$

9. $\sqrt{24}$

10. $\sqrt{87}$

11. $\sqrt{39}$

12. $\sqrt{14}$

For Problem 13-15, approximate the value of the square root given.

13. $\sqrt{19}$

14. $\sqrt{7}$

15. $\sqrt{53}$

For Problem 16-20, graph the expression on the number line given.

16. $\sqrt{22}$ 17. $\sqrt{7}$ 18. $\sqrt{38}$ 19. $\sqrt{34}$ 20. $\sqrt{15}$

Section 4.5 Simplifying Non-Perfect Squares

Practice Problems 4.5

For Problem 1-15, simplify the expression given if possible.

1. $\sqrt{72}$
2. $\sqrt{10}$
3. $\sqrt{300}$
4. $\sqrt{7}$
5. $\sqrt{600}$
6. $\sqrt{800}$
7. $-\sqrt{12}$
8. $\sqrt{m^5}$
9. $-\sqrt{n^6}$
10. $\pm\sqrt{x^3y^4}$
11. $\sqrt{14z^4}$
12. $\sqrt{4x^4}$
13. $\sqrt{8x^3}$
14. $\sqrt{12x^5y^4}$
15. $\sqrt{xy}$

Section 4.6 Ordering Numbers with Square Roots

Practice Problems 4.6

For Problem 1-8, put the numbers given in order from greatest to least.

1. $\sqrt{35}$ $\sqrt{32}$ $\sqrt{37}$ $\frac{20}{3}$

2. $\sqrt{10}$ 3.5 $\frac{133}{3}$ $\frac{14}{3}$

3. $-\sqrt{65}$ $-\sqrt{60}$ -8.4 $-\frac{35}{4}$

4. $\sqrt{39}$ $\sqrt{26}$ 5.2 $\sqrt{27}$

5. $\sqrt{12}$ $\sqrt{17}$ 4.4 $\frac{14}{5}$

6. $-\frac{1}{2}$ -0.7 $-\sqrt{2}$ 2%

7. 10 $\sqrt{5}$ -11 $\frac{1}{4}$

8. 0.75 $\sqrt{33}$ $\frac{7}{2}$ -3

For Problem 9-12, compare the expressions using inequality or equality symbols.

9. $\sqrt{9}$ ___________ $2\sqrt{2}$

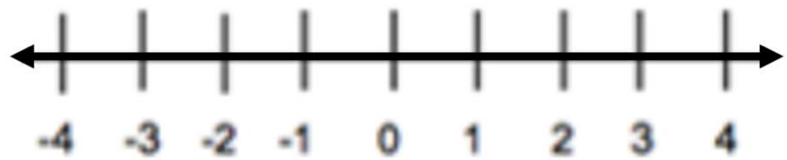
10. $\frac{9}{5}$ ___________ 1.8

11. $\sqrt{13}$ ___________ $\frac{25}{10}$

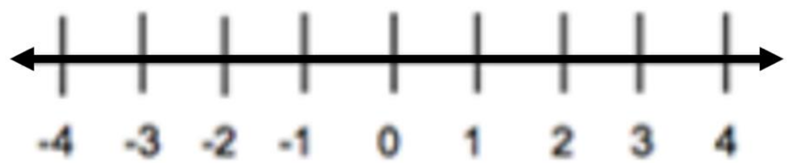
12. $-\sqrt{11}$ ___________ $-\sqrt{3}$

For Problem 13-15, place the numbers given on the number line in their approximate position.

13. $-\sqrt{2}$ $\frac{1}{5}$ 3.4 -2.1



14. $-\sqrt{7}$ $\sqrt{6}$ $-\frac{5}{3}$ 0.65



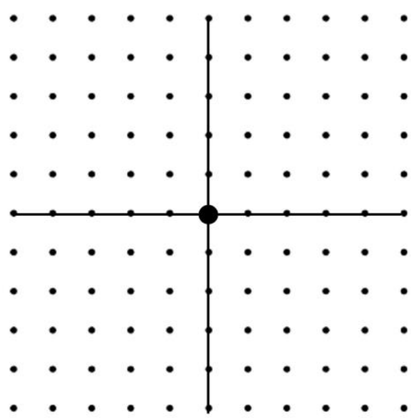
15. $-\sqrt{17}$ $\sqrt{10}$ 2.5 $\frac{3}{2}$

Section 4.7 Squares and the Coordinate Grid

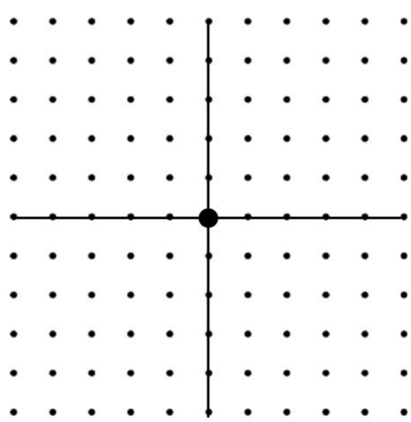
Practice Problems 4.7

For Problem 1-10, use the dot grids to complete the problem. Assume any horizontal or vertical distance between points is 1 unit.

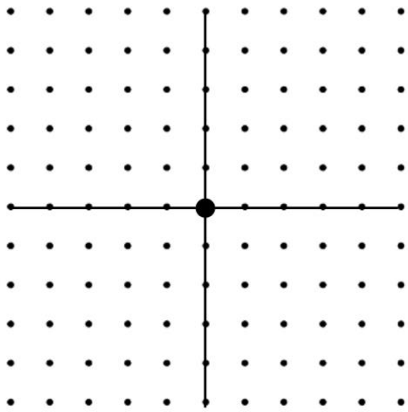
1. If High Hat ran from point $(1, 3)$ to point $(3, 5)$, what directions should he be given if he can only move horizontally and vertically on the game board? Draw two different routes High Hat could take.



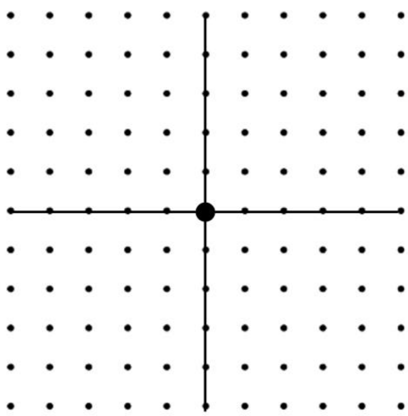
2. If the routes from Problem 1 were a city called Highhatville, how many square city blocks are inside those two routes?



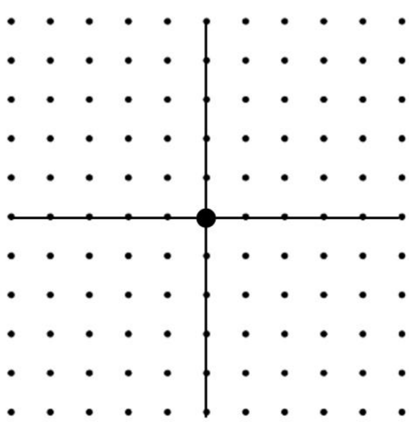
3. What is the name of the ratio of the vertical distance over the horizontal distance from Problem 2?



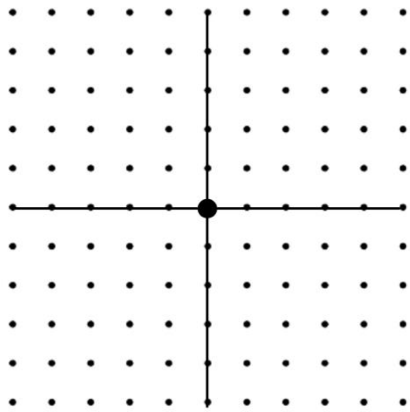
4. Does the ratio from Problem 3 tell us the shortest distance between the two points in Problem 1 if High Hat did not have to move first horizontally and vertically?



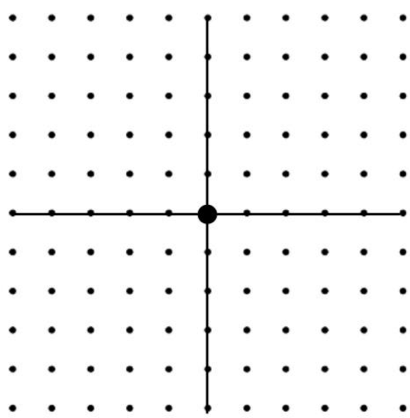
5. Using a ruler to measure the shortest distance between the two points from Problem 1 results in the diagonal line between the two points (the shortest distance between two points is a straight line).
If 1 cm $=$ 2.3 miles, how far would High Hat travel in Highhatsville to get from one point to the other given he took the shortest route and that diagonal line measured 2.8 units?



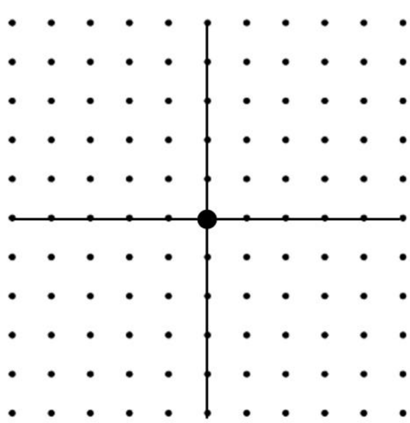
6. If High Hat were at point $(2, 3)$, how many square houses of 4 square units could be built with the point along the side of the house?



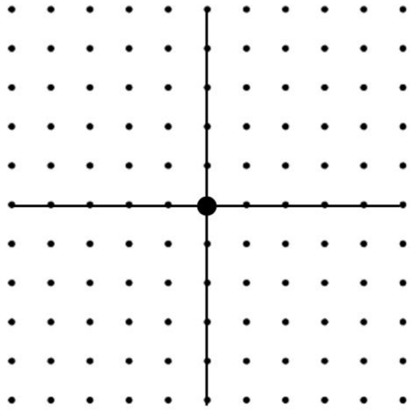
7. High Hat is hiding at $(3, 4)$. Your partner guessed $(5, 5)$. How many spaces would you tell your partner to move horizontally and vertically and in which direction to get to High Hat?



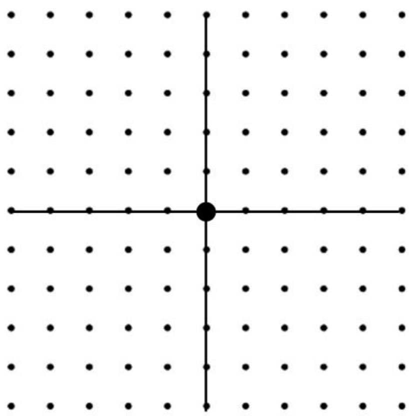
8. What if High Hat was hiding at $(5, 5)$ and you guessed $(3, 4)$; which direction and how many spaces would your partner tell you to move?



9. What is the distance in centimeters that High Hat covered if he walked from your guess to his space and back again?



10. What is the area in square units inside the rectangle made by High Hat moving from his space to your guess and back again along a different route that is horizontal and vertical as well?

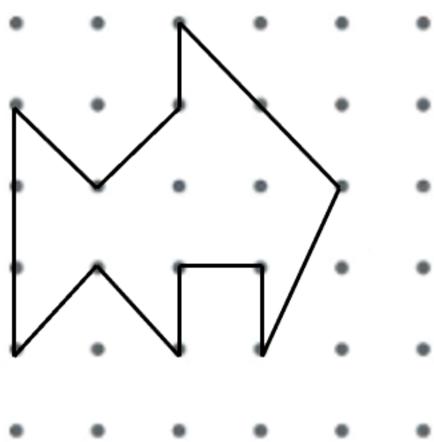


Section 4.8 Finding Area Using the Chop Strategy

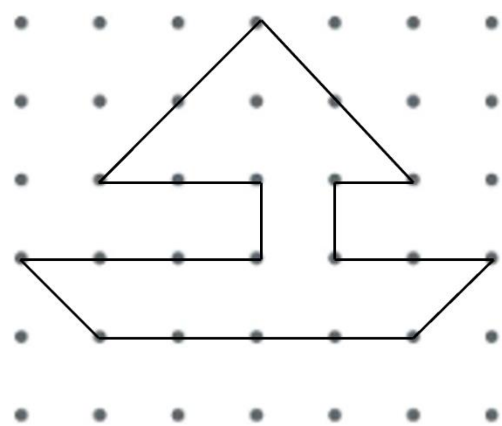
Practice Problems 4.8

For Problem 1-12, use the chop strategy to find the area of the shape given.

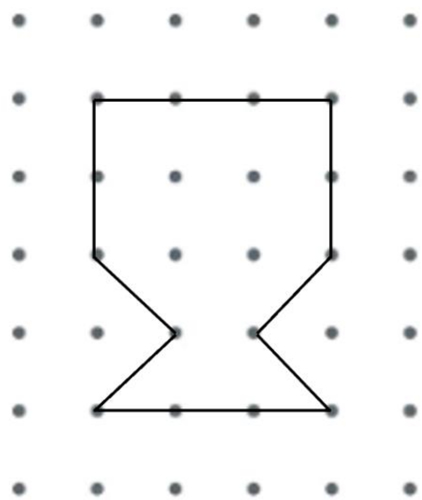
1.



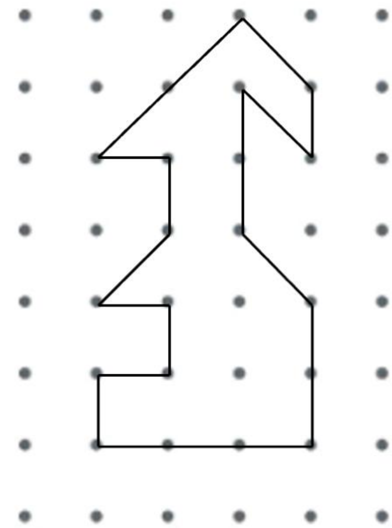
2.



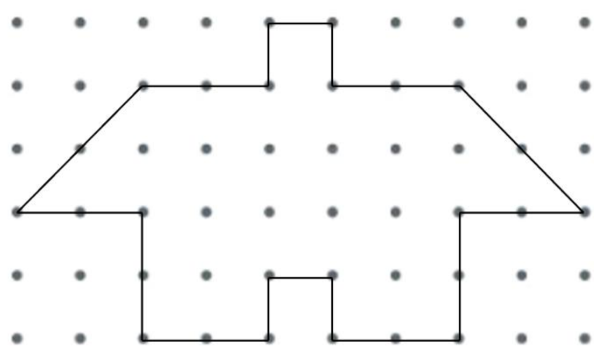
3.



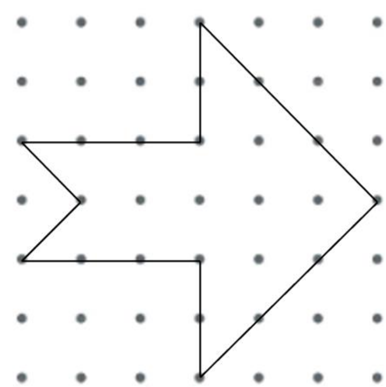
4.



5.

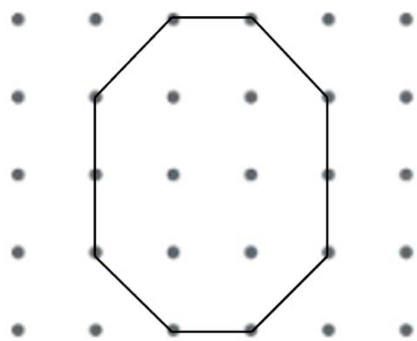


6.

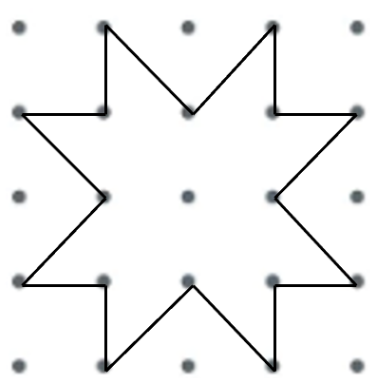


7.

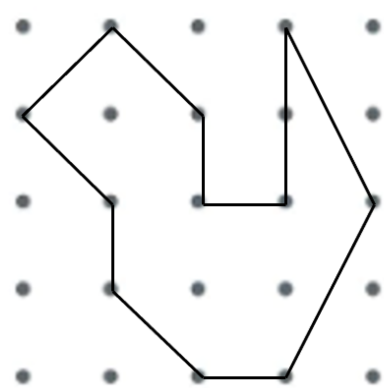
8.



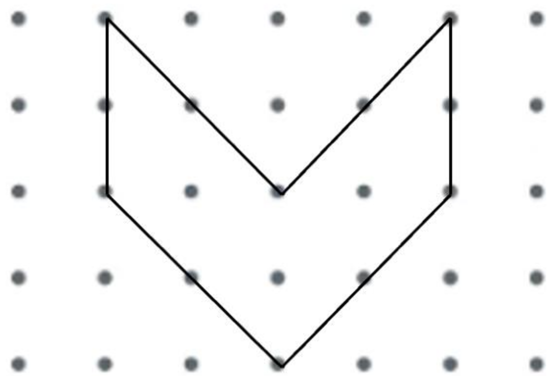
9.



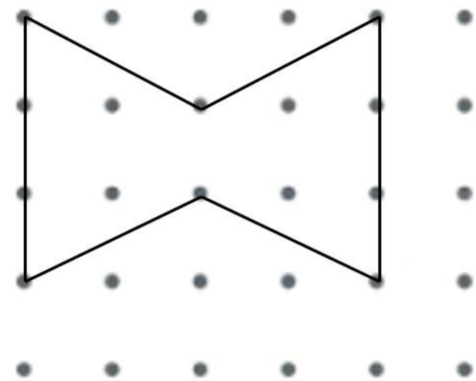
10.



11.



12.

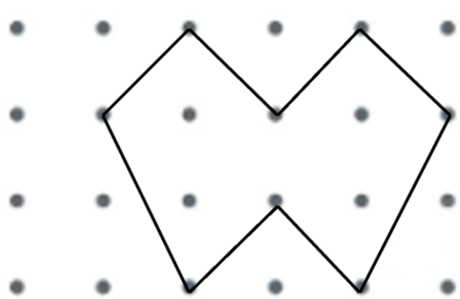


Section 4.9 Find Area Using the Subtraction Method

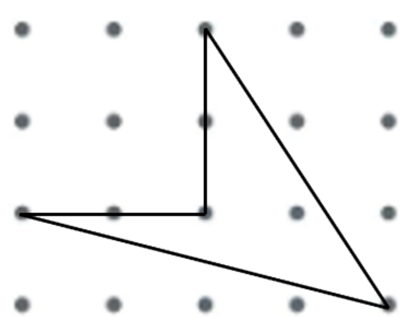
Looking Back 4.9

For Problem 1-10, use the chop strategy or subtraction method to find the area of the polygon given.

1.

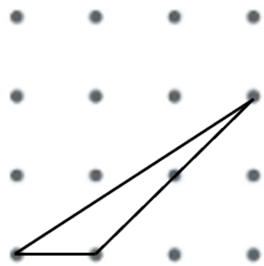


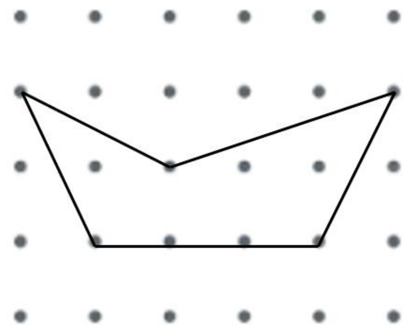
2.



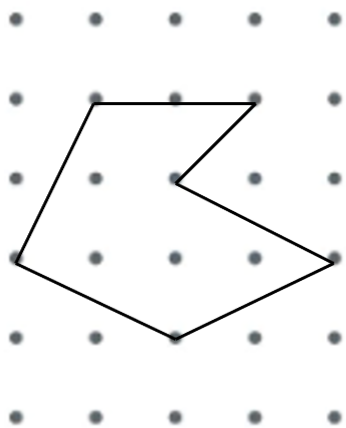
3.

4.

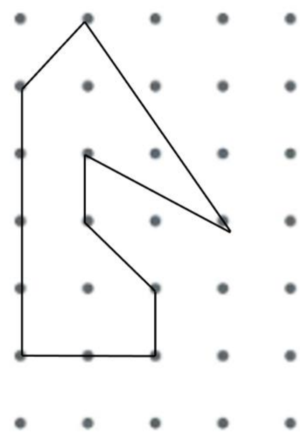




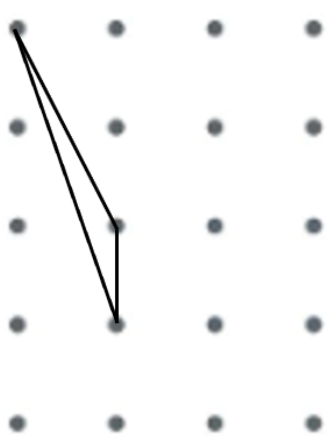
5.



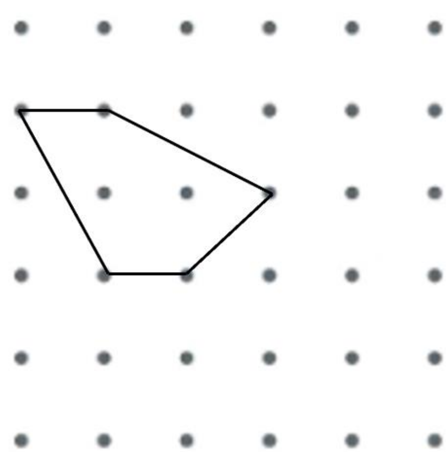
6.



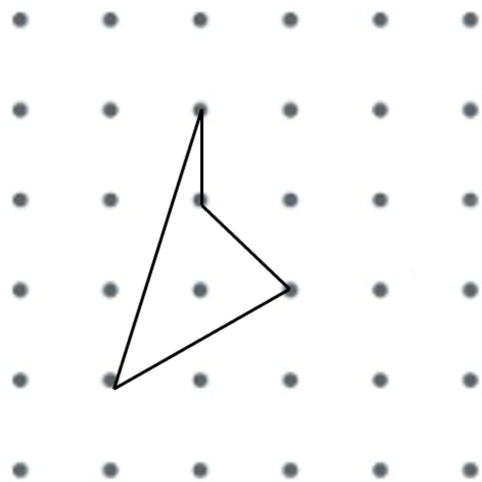
7.



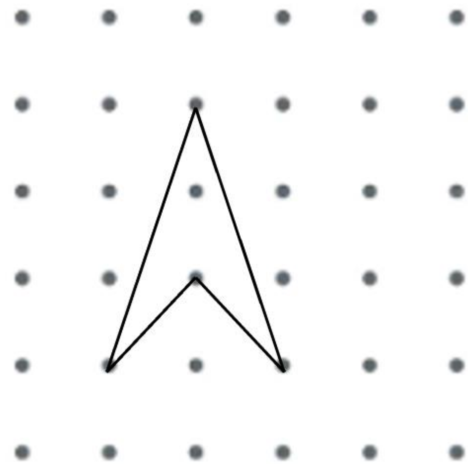
8.



9.



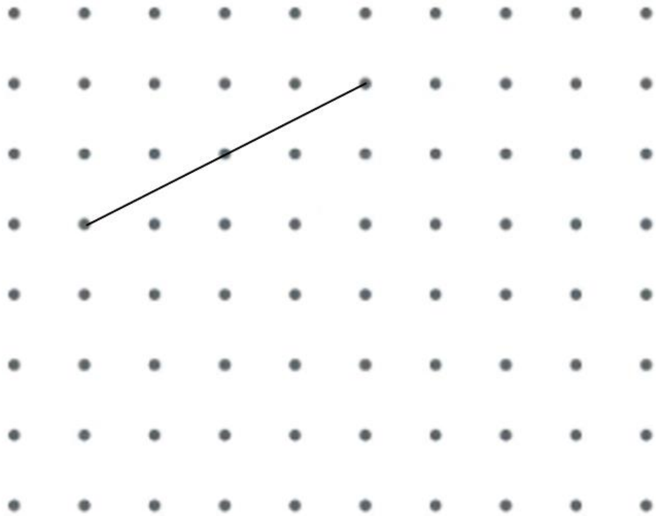
10.



<u>Section 4.10 Irrational Square Roots</u>

<u>Practice Problems 4.10</u>

For Problem 1-4, use the diagram to solve the problem.



1. To the left is a side length of a square. On the grid, draw the square so that all sides are equal.

2. Opposite sides of a square are parallel. Parallel lines have the same slope. How can you prove that opposite sides of the square are parallel?

3. What is the area of the square?

4. Without measuring, what is the exact length of the first line given?

For Problem 5-10, use grid paper on the next page to solve the problem.

5. Draw all the squares, both standard (upright) and non-standard (titled) that can be drawn in a 5×5 array.

6. What is the area and the length of one side of the largest standard square in your array?

7. What is the area and length of one side of the largest non-standard square in your array?

8. What is the area and length of one side of the smallest standard square in your array?

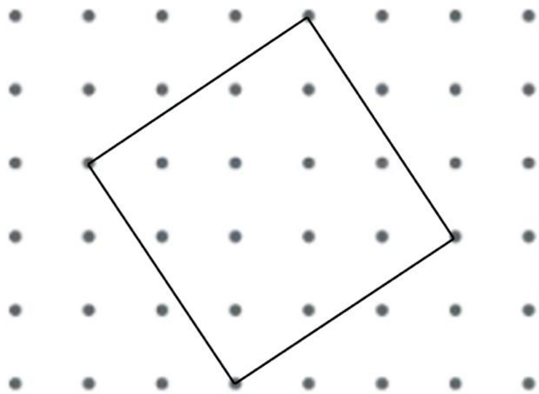
9. What is the area and length of one side of the smallest non-standard square in your array?

10. What is the slope of the opposite sides of the largest non-standard square in your array? Are the opposite sides parallel? Are the opposite sides equal?

Section 4.11 Approximating Square Roots

Practice Problems 4.11

For Problem 1-6, use the diagram below to solve the problem.



1. Find the area of the square. What method did you use?

2. What is the exact length of the side of the square?

3. What perfect square is less than this, but close? What perfect square is greater than this, but close? Which one is it closer to?

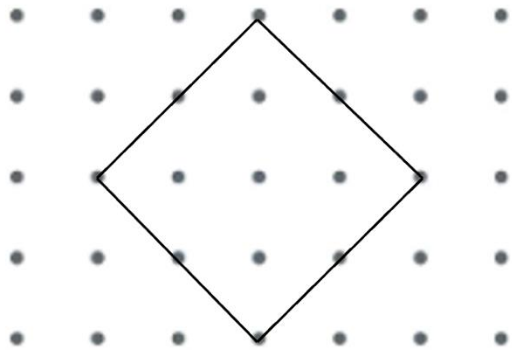
4. a) What two whole numbers on the number line does the exact side length in Problem 3 lie between?

   b) Which of those two whole numbers is the side length closer to?

5. Guess the decimal approximation of this exact solution (from 4.b)) for the side length. Multiply this by itself and see how close you get to the exact side length.

6. Try to make your next guess smaller or larger to see if it gets closer to the exact length when you multiply it by itself. Try two or three other guesses to see how close to the exact solution you can get.

For Problem 7-12, use the diagram below to solve the problem.



7. Find the area of the square. What method did you use?

8. What is the exact length of the side of the square?

9. What perfect square is less than this, but close? What perfect square is greater than this, but close? Which one is it closer to?

10. a) What two whole numbers on the number line does the exact length from Problem 9 lie between?

    b) Which whole number is it closer to?

11. Guess the decimal approximation of this exact solution (from 10.b)) for the side length. Multiply this by itself and see how close you get to the exact side length.

12. What is the exact perimeter of the square?

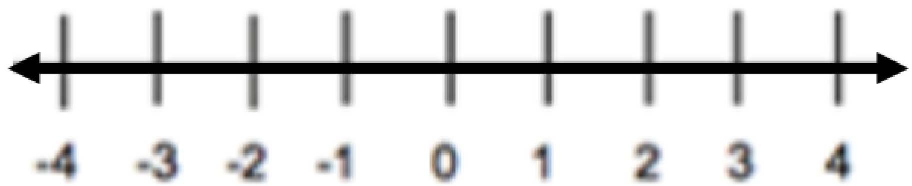
Section 4.12 Finding Square Roots Using the Elimination Method

Practice Problems 4.12

For Problem 1-6, solve the problem given.

1. Use a calculator to tell the decimal approximation for the length of a side for a square that has an area of 11 square units?

2. Show $\sqrt{11}$ on the number line.



3. If $\sqrt{6} \approx 2.4494$ and $\sqrt{8} \approx 2.82843$, what do you guess $\sqrt{7}$ to be?

4. Try at least six guesses for $\sqrt{7}$ to see how close you can come with the decimal approximation.

Guess	$(\textbf{Guess})^2$	**Too High/ Too Low**

5. Are all square roots that are not perfect squares irrational?

6. When you divide two even integers will you always get a rational number?

For Problem 7-10, estimate the decimal approximation for the given square root and check your solution using a calculator.

7. $\sqrt{13}$

8. $\sqrt{15}$

9. $\sqrt{5}$

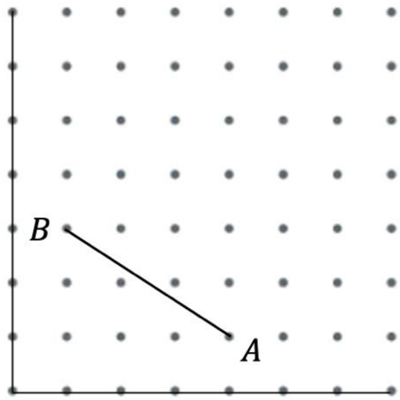
10. $\sqrt{3}$

Section 4.13 Finding Square Roots Geometrically and Algebraically

Practice Problems 4.13

For Problem 1-12, solve the problem given.

1. Without measuring, find the length of AB. Draw a square with this side. Find the area of the square and take the square root.



2. Draw a square diagram using tens and ones to represent 13^2. Find the area of the square.

3. Use the square root division algorithm to check that 13 is the side of the square with the area in Problem 1.

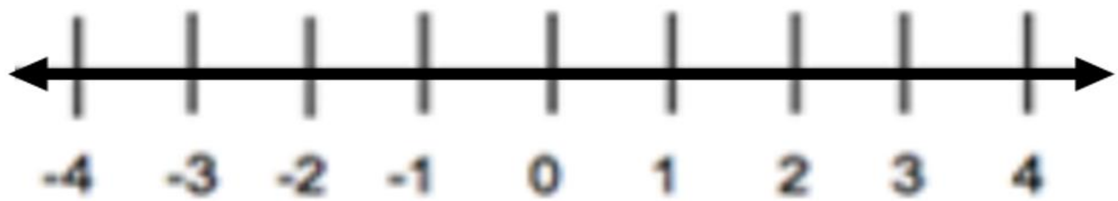
4. Use repeated subtraction to show the square root of 169.

5. Find the decimal approximation for the square root of 17.

6. Find the decimal approximation for the square root of 2.

7. Locate the numbers below on the number line.

$1.7 \qquad \sqrt{17} \qquad \frac{13}{6} \qquad -\sqrt{2} \qquad -0.3$



8. Find 13^2, 14^2, and 15^2. Find a pattern between the three. Without using a paper and pencil or calculator, find 16^2. Why does this work?

9. Determine which numbers are rational and which numbers are irrational below. Put an “R” under rational numbers and an “I” under irrational numbers.

$\sqrt{9} \qquad \frac{2}{3} \qquad 0.444 \qquad \sqrt{15} \qquad \frac{2}{5} \qquad 6.2 \qquad -11 \qquad \frac{5}{4} \qquad \sqrt{21} \qquad 2\sqrt{3}$

10. Let us try to prove why the square root of 2 is irrational.

a) $\sqrt{2} = \frac{a}{b}$ (The fraction is in simplest terms)

b) $2 = \frac{a^2}{b^2}$ (From squaring both sides)

c) $2b^2 = a^2$ (Multiply both sides by b^2)

d) $2b^2 = (2n)^2$ (a is a multiple of 2, so it is an even number; we will call it $2n$)

e) $2b^2 = 4n^2$ (Power to a Power Rule)

f) $b^2 = 2n^2$ (Divide both sides by 2)

Fill in the blanks:

In Step c) above, the square of an even number is ____________________. The only way to get an odd number when you square a number is if the number is ____________________. If a and b are both even, then they are divisible by 2 and the quotient $\frac{a}{b}$ is not in simplest terms, but Step __________ says $\frac{a}{b}$ is in simplest terms; therefore, this is a contradiction and $\sqrt{2}$ is not rational but ________________________________.

Section 4.14 Module Review

For Problem 1 and 2, tell whether the number given is a perfect square.

1. 18
2. 25

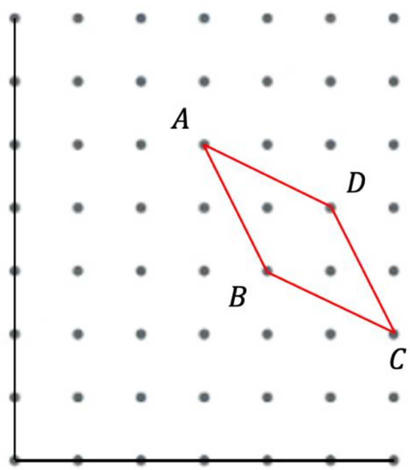
For Problem 3 and 4, decide which two numbers the square root of the term given lie between and then approximate the solution.

3. $\sqrt{28}$
4. $\sqrt{44}$

For Problem 5-12, find the square root of the term given in exact form. Your solution should be a whole number or a simplified radical.

5. $\sqrt{64}$
6. $-\sqrt{100}$
7. $\pm\sqrt{75}$
8. $\sqrt{96}$
9. $\sqrt{400}$
10. $\sqrt{x^8}$
11. $\sqrt{x^5}$
12. $\sqrt{x^4y^6}$

For Problem 13-15, use the diagram below to solve the problem.



13. What is the area of parallelogram $ABCD$?

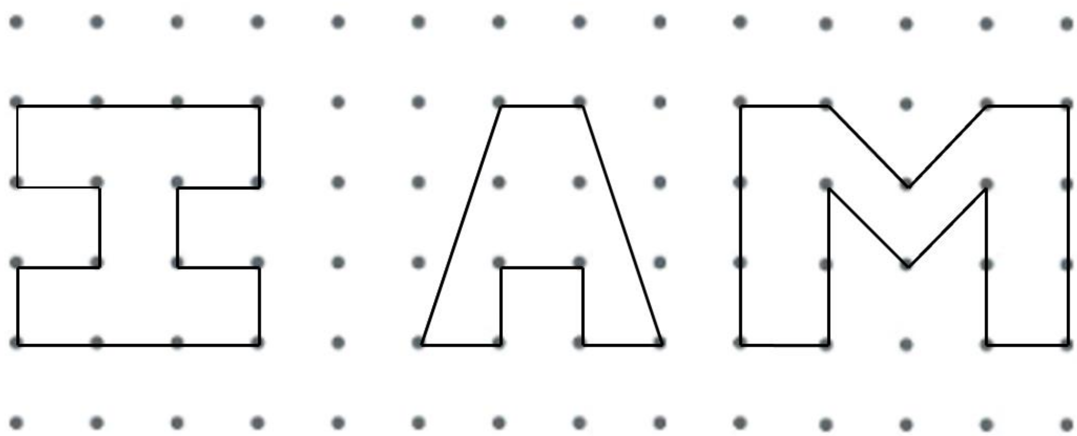
14. Draw a square on side AB.

    a) What is the area of the square?

    b) What is the length of side AB?

15. What other segments are equal to side AB?

For Problem 16-20, use the diagram below to find the area and perimeter of the figures given. The area is square units inside the figure. The perimeter is the length in units around the figure.



Exodus 3:16

16. I Area =
    Perimeter =

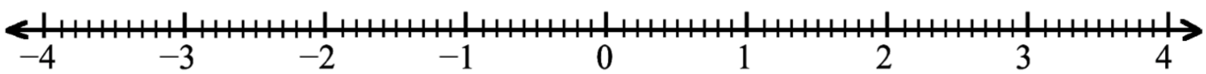
17. A Area =
    Perimeter =

18. M Area =
    Perimeter =

19. Which is smaller: $\sqrt{3} + \sqrt{3}$ or $\sqrt{4}$? How do you know?

20. Locate the numbers below on the number line?

0.3 $-\frac{1}{2}$ $-\sqrt{2}$ 3.6 $\sqrt{15}$



Section 4.15 Module Test

For Problem 1 and 2, tell whether the number given is a perfect square.

1. 64

2. 55

For Problem 3 and 4, decide which two numbers the square root of the term given lies between and then approximate the solution.

3. $\sqrt{32}$

4. $\sqrt{60}$

For Problem 5-12, find the square root of the term given in exact form. You solution should be a whole number or a simplified radical.

5. $\sqrt{27}$

6. $-\sqrt{50}$

7. $\pm\sqrt{800}$

8. $\sqrt{56}$

9. $\sqrt{88}$

10. $\sqrt{x^{12}}$

11. $\sqrt{x^9}$

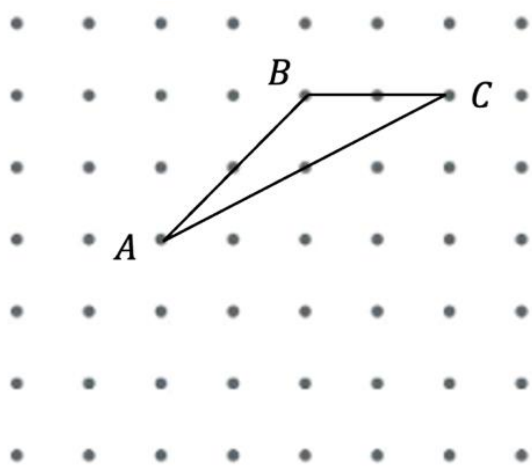
12. $\sqrt{x^2y^8}$

For Problem 13 and 14, solve the problem given.

13. Draw a line segment that is $\sqrt{5}$ on the grid below.

14. Which is bigger: $\sqrt{5} + \sqrt{5} + \sqrt{5}$ or $2\sqrt{5}$?

For Problem 15-17, use the diagram below to solve the problem.



15. What is the area of triangle ABC?

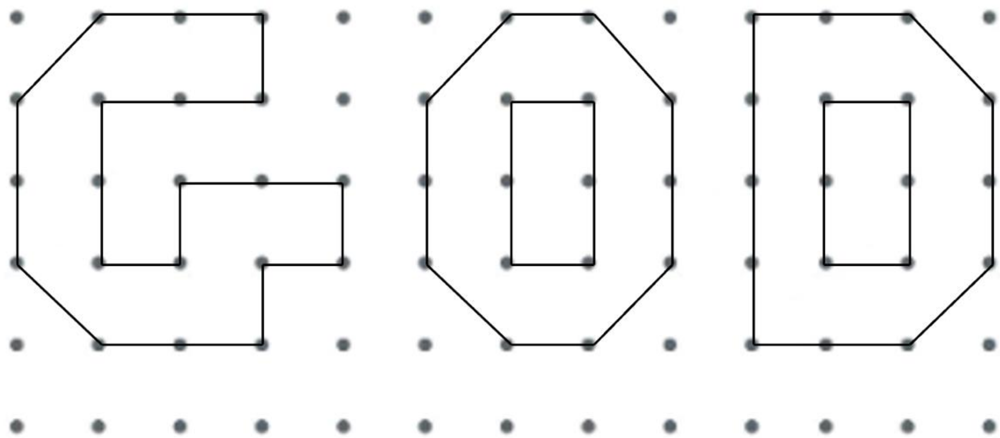
16. Which is longer: side AB or side BC?

17. Draw a square on side AB.

a) What is the area of the square?

b) What is the length of side AB?

For Problem 18-20, find the area and perimeter of the letters below.



18. G Area =

Perimeter =

19. O Area =

Perimeter =

20. D Area =

Perimeter =