# Module 2 Introducing Linear Equations

## Section 2.1 Understanding Graphs

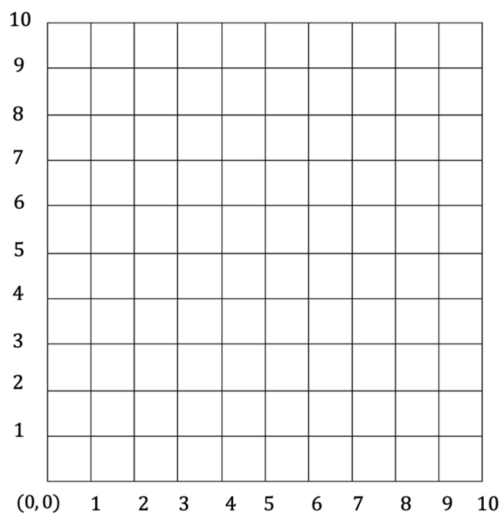
### Looking Back 2.1

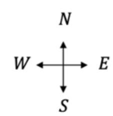
In Genesis 1:1, we read: "In the beginning God created the heavens and the earth." This verse laid the foundation for the world around us. Linear equations lay the foundation for higher order equations and model occurrences in God's natural world.

There is an endless list of amazing things created by God. If we look closely, we can see patterns of nature that have been evident from the beginning of time; these patterns are evidence of God's intelligent design. This module is about linear equations, which also have many identifiable patterns.

Previously, we have solved equations and graphed them on a number line. Now, we will graph linear equations on the coordinate plane. We will be introduced to this by looking at what is called Quadrant I of the coordinate plane and see where points lie. In this module points will connect to form a line.

We are going to play a game called "Where is High Hat Hiding?"
You will use a blank grid like the one show below and the cardinal coordinate system to the right below with the directions North, South, East, and West to play the game. North and South are followed by East and West when giving directions. So other directions may be Northeast or Southwest.




To begin, you will place two numbers on a card in parenthesis to represent where High Hat is hiding. For example, $(4, 2)$. This point is called an ordered pair. This means if you start at $(0, 0)$ and move $4$ spaces right and then $2$ spaces up, you will find where High Hat is hiding. So, the $(0, 0)$ is the start, the first number in parenthesis is the move number of spaces you move right, and the second number in parenthesis is the number of spaces you move up. The ordered pair is where the vertical line labeled $4$ meets the horizontal line labeled $2$.
Let us continue with our example of High Hat at $(4, 2)$. Do not let your partner see the numbers on your card. They will put a guess in the first box (the **Guess** box shown below). Let us say they guess High Hat is at $(1, 1)$. You would then give them a hint in the first hint box (the **Hint** box shown below) for the next direction, which would be NE for North and East (northeast). The hint tells you that High Hat is located somewhere northeast of that point.

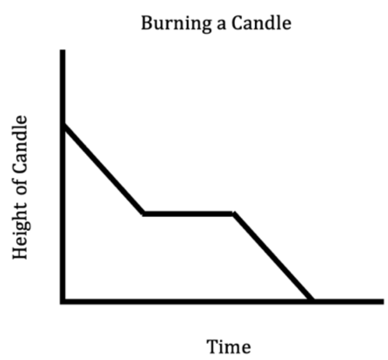
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Guess** | $(1, 1)$ | | | | | | | | | |
| **Hint** | NE | | | | | | | | | |

If your partner's second guess is $(2, 2)$, you would give the hint E for East. If they guess $(6, 2)$ on their third guess, you would give the hint W for West. Every time your partner guesses a point, a point is put on the board to mark it. You will keep playing until your partner finds High Hat. For the next game, you partner will hide High Hat by writing two numbers in parenthesis and you will guess while your partner gives hints.
You will find a game board for "Where is High Hat Hiding?" in the Practice Problems section. Play a few rounds with a partner and enjoy!

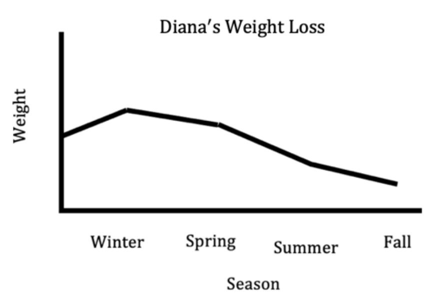
Looking Ahead 2.1

Now we are going to look at the graph with no gridlines or numbers. Instead of points, we will see connected lines which tell a story.

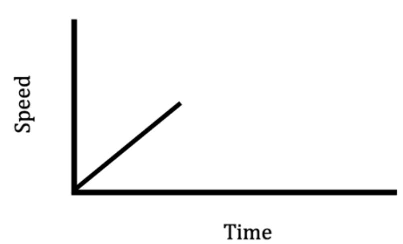
| |
|---|
| Example 1: What is the story of the graph below? The line shows the height of a candle over time. What is happening? |




| |
|---|
| Example 2: What is the story of the graph below? What can you say about Diana's weight loss throughout the year? |



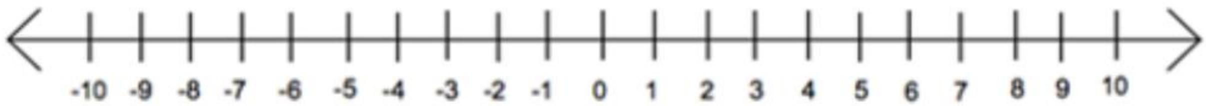

| |
|---|
| Example 3: What is the story of the graph below? |



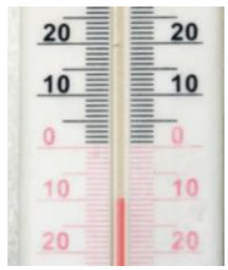
## Section 2.2 Graphing Ordered Pairs

### Looking Back 2.2

If we look at a number line, it goes right and left. The positive numbers go right of 0 and increase. The negative numbers go left of 0 and decrease.



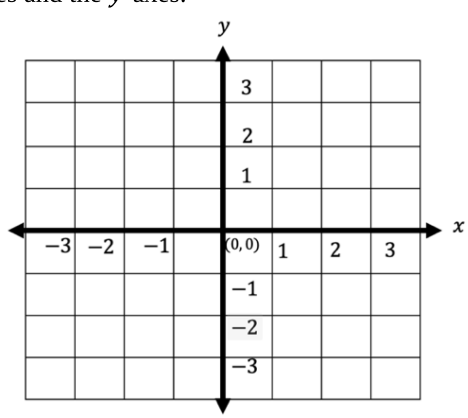

If we look at a thermometer (example shown below), the positive numbers go up from 0 and increase. The negative numbers go down from 0 and decrease.



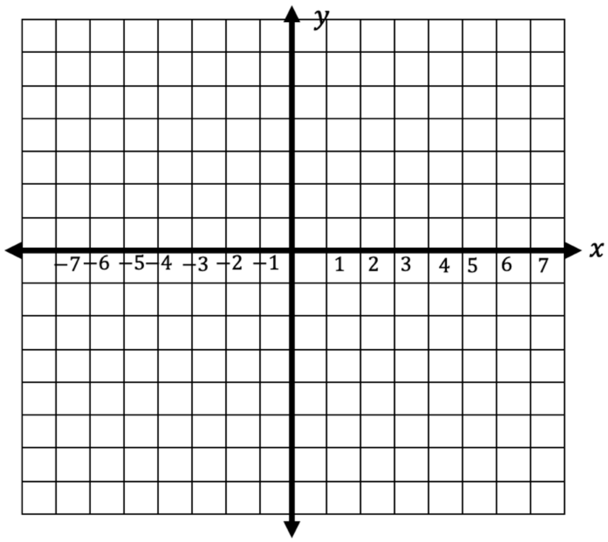

If we put the two together (the number line and the thermometer), we have the $xy$-coordinate plane. The line that is like the number line is called the $x$-axis. The line that is like the thermometer is called the $y$-axis . A story has been passed down that the mathematician René Descartes was in a hospital bed looking up at a ceiling with rectangular tiles and wondered how he could describe to someone far away in the world where a fly was sitting on the ceiling. This is how the Cartesian coordinate system, named after René Descartes, came about.

### Looking Ahead 2.2

The center of the $xy$-coordinate plane is the origin, point $(0, 0)$. The first number in the ordered pair $(x, y)$ is where we move on the $x$-axis (the number line) and the second number in the ordered pair is where we move on the $y$-axis (the thermometer). So, the first move ($x$) is only right or left from 0. A positive number is right on the $x$ axes, and a negative number is left on the $x$ axes. The second move ($y$) is only up or down from $x$. A postive number moves up on the $y$ axes and a negative number moves down on the $y$ axes. The confusing part is that the $x$-axis is right and left and is horizontal, but the $x$ grid lines actually go up and down vertically through the $x$-axis. The $y$-axis is up and down and is vertical, but the $y$ grid lines actually go right and left horizontally through the $y$-axis. That is why they are called the $x$-axes and the $y$-axes.



Example 1: Plot the points below on the $xy$-coordinate plane.

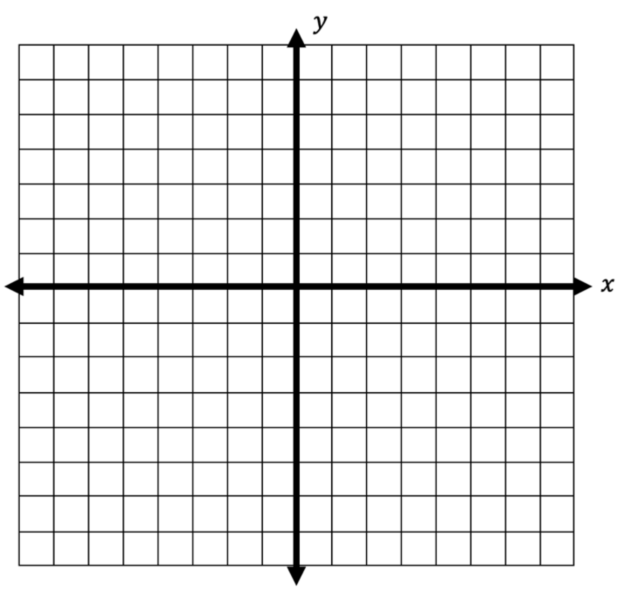



a) $(1, 6)$ b) $(-2, -6)$

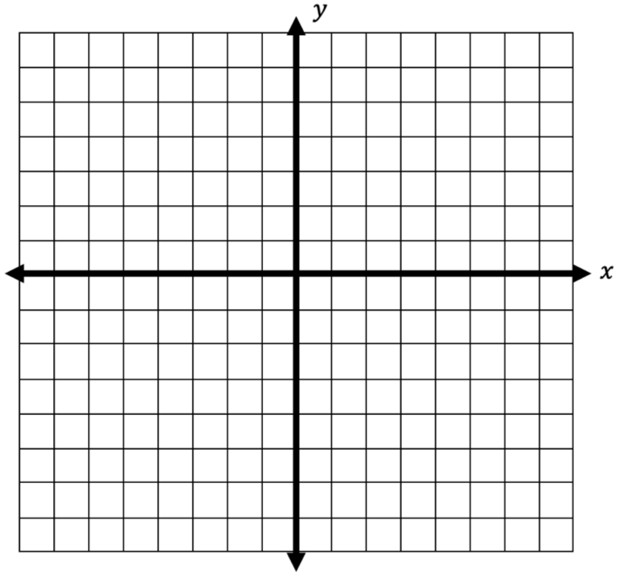
c) $(4, -3)$ d) $(-5, 7)$

The $xy$-coordinate plane has four different quadrants that are separated by the $x$ and $y$ axes. The upper right is Quadrant I in which $x$ is positive and $y$ is positive. The upper left is Quadrant II in which $x$ is negative and $y$ is positive. The lower left is Quadrant III in which $x$ is negative and $y$ is negative. The lower right is Quadrant IV in which $x$ is positive and $y$ is negative.

Example 2: Label the Quadrants on the $xy$-coordinate plane below. Name one point in each quadrant.



Example 3: Graph each ordered pair below and tell which quadrant it is in.

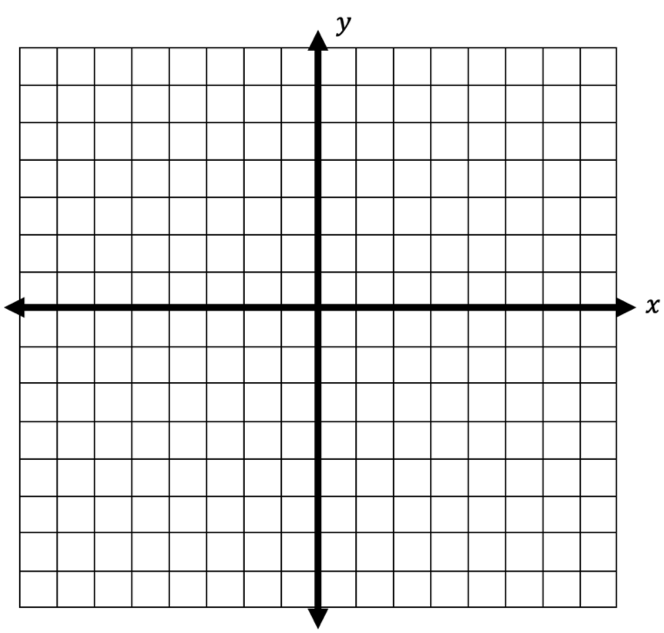



a) $(-5, -5)$ b) $(2, 3)$

c) $(-6, -1)$ d) $(4, -4)$

Some ordered pairs are not in a Quadrant but lie on the axes. For example, the point $(4, 0)$ lies on the $x$-axis and is called an $x$-intercept. There is confusion here again as an $x$-intercept has a $y$-coordinate that is $0$. The line that has all points $x = 4$ goes through that point $(4, 0)$ and is vertical. A $y$-intercept has an $x$-coordinate that is $0$. The line that has all points $y = 4$ goes through that point $(0, 4)$ and is horizontal. It is confusing because it is parallel to the $x$-axis which is also horizontal!

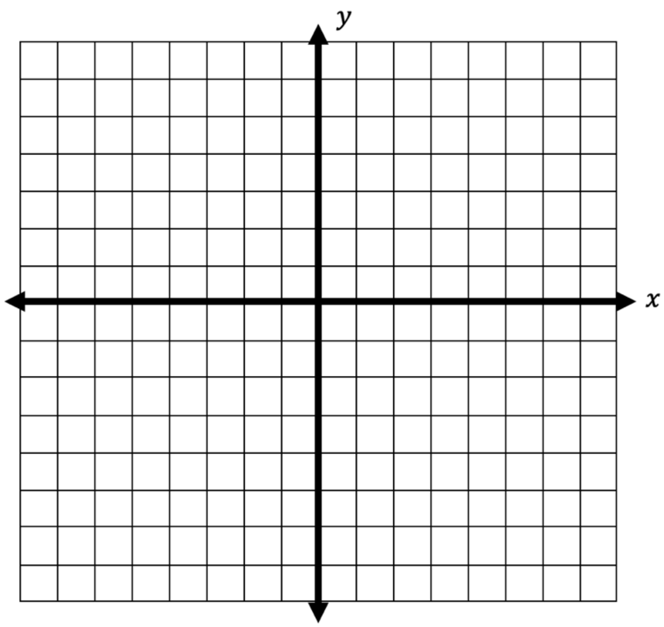
Example 4: Graph the points below on the $xy$-coordinate plane and tell which axis they lie on.




a) $(-5, 0)$ b) $(0, -5)$

c) $(-3, 0)$ d) $(0, 0)$

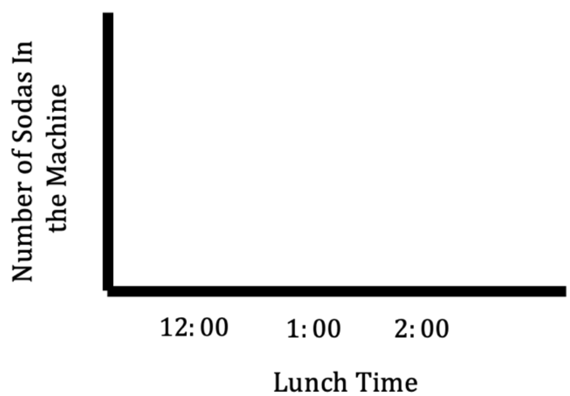
Example 5: Graph the lines below on the coordinate plane. Name at least three points on each line.




a) $x = -5$ b) $y = 6$

c) $x = 3$ d) $y = -1$

Example 6: There is a long line of employees at the soda machine waiting to buy pop at lunch. Draw the graph of the number of sodas in the machine ($y$-axis) over lunch time ($x$-axis).



## Section 2.3 Input-Output Tables

### Looking Back 2.3

We began this module by looking at the story a graph tells; now, we will take a deeper look at how graphs are constructed, and we will see they are built on a relationship. That relationship can be called a function. A function relates the independent and dependent variables so there is only one output for every input.

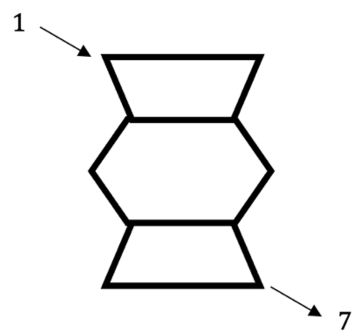
When we look at nature and see its wonders and beauty, we are amazed, but when we think about how God created it all, we are awestruck. Now, we will be amazed by how graphs are made. When we see what goes into the process, we will have a deeper understanding of functions. We will also see that functions can model all that God has created.

To begin this section, we will look at function machines. We will learn more about functions in Algebra 1, but for now we will look into Input-Output tables, which are the first step towards understanding functions.

### Looking Ahead 2.3

A function is the relationship between an input and output in which there is only one output for every input. The function machine creates the relationship between the input and the output. When some number is put into the function machine, something happens to it and another number comes out. What the function machine does to the number is called the rule. Let us look at the input and output values below and see if we can find the rule for the function machine.

| Input |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |




| Input | Output |
|---|---|
| 1 | 7 |
| 2 | 8 |
| 3 | 9 |
| 4 | 10 |
| 5 | 11 |

The function machine takes the input and adds 6 to it to get the output. The rule is: Input + 6.

Example 1: Find the rule for the input and output values in the table below.

| Input | Output |
|---|---|
| D | e |
| F | g |
| H | i |
| J | k |
| L | ? |
| N | ? |
| P | ? |

Example 2: Find the rule for the input and output values in the table below.

| Input | Output |
|---|---|
| ○ | ○○ |
| ○ ○ | ○ ○ ○ |
| ○ ○ ○ | ○ ○ ○ ○ |
| ○ ○ ○ ○ | ? |
| ○ ○ ○ ○ ○ | ? |
| ? | ○ ○ ○ ○ ○ ○ ○ |
| ? | ○ ○ ○ ○ ○ ○ ○ ○ |

Example 3: Find the rule for input and output values in the table below.

| Input | Output |
|---|---|
| ↖ | ↘ |
| ← | → |
| ↑ | ↓ |
| ↘ | ? |
| → | ? |
| ↓ | ? |
| ↗ | ? |

Example 4: Use the rule to complete the function table below.

| Rule: $-$ Input $+ 4$ | |
|---|---|
| Input | Output |
| 2 | |
| 1 | |
| 0 | |
| $-1$ | |
| $-3$ | |

Example 5: Use the rule to complete the function table below.

| Rule: $4 - 2(\text{Input})$ | |
|---|---|
| Input | Output |
| $-8$ | |
| $-5$ | |
| $-2$ | |
| $1$ | |
| $4$ | |

Example 6: Use the rule to complete the function table below.

| Rule: $\frac{1}{2}(\text{Input}) + 2$ | |
|---|---|
| Input | Output |
| $-6$ | |
| $-4$ | |
| $-2$ | |
| $0$ | |
| $2$ | |

## Section 2.4 Introducing Linear Equations

### Looking Back 2.4

We continue talking about linear equations, but what exactly are they? That is the focus of this section. We will also investigate ways to identify linear equations.

The most fundamental equation is a linear equation. In a linear equation, each variable ($y$ and $x$) has an exponent of 1. The graph of a linear equation is always a straight line. The following are examples of linear equations:

- $y = x$
- $y = 2$
- $x = 2$
- $y = x + 2$

In a linear equation, the input value, which is often represented by $x$, is called the independent variable. We choose values to input into the equation. The output value, which is often represent by $y$, is called the dependent variable. This output value depends on the number we put in for the input value in the equation. The output changes as the input changes.

Another was to identify a linear equation that we will explore is the rate of change which is constant.

### Looking Ahead 2.4

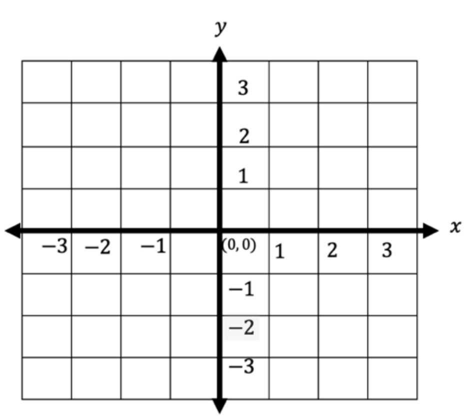
Example 1: Complete the Input-Output tables below and plot the ordered pairs on the graph. Connect the points to make a straight line.

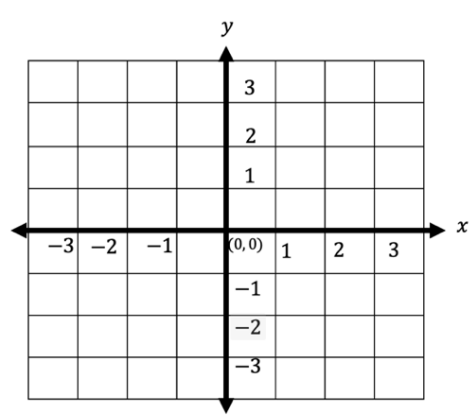
a) $y = x$

| $x$ | $y$ |
|---|---|
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

b) $y = 2$

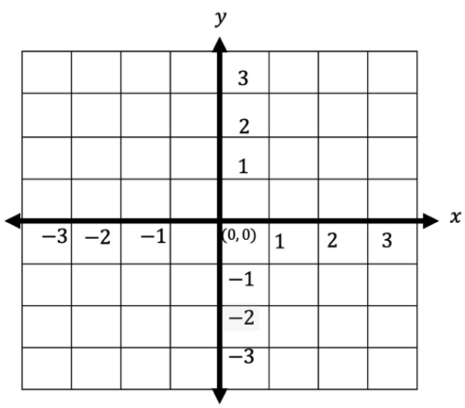
| $x$ | $y$ |
|---|---|
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |



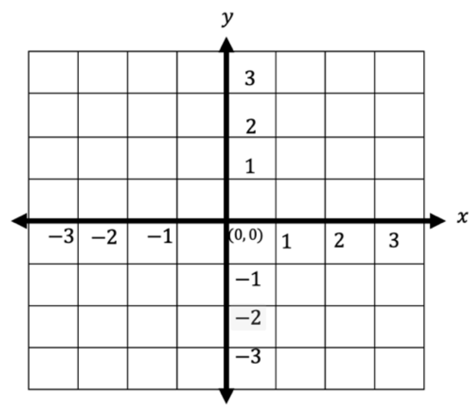
c) $x = 2$

| $x$ | $y$ |
|---|---|
| 2 | |
| 2 | |
| 2 | |
| 2 | |
| 2 | |




d) $y = x + 2$

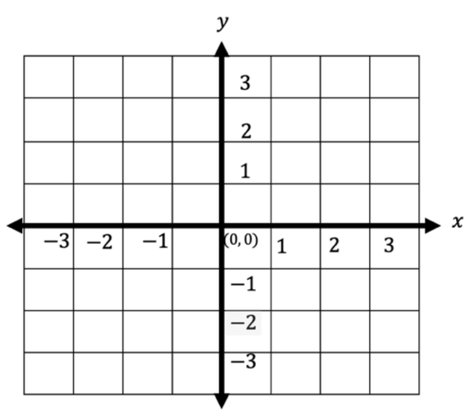
| $x$ | $y$ |
|---|---|
| −3 | |
| −2 | |
| −1 | |
| 0 | |
| 1 | |




The three types of straight lines in Example 1 are horizontal, vertical, and diagonal.

Example 2: Complete the Input-Output table below for $y = -x + 2$ and graph the equation. How is this different from the graph of $y = x + 2$? How is it the same?

| $x$ | $y$ |
|---|---|
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |



Suppose Isaac works at a restaurant and makes $7.50 an hour.

- In one hour, he makes $7.50
- In two hours, he makes $15.00
- In three hours, he makes $22.50
- In four hours, he makes $30.00

This is what we mean by a constant rate; Isaac's constant rate is $7.50 per hour. The input value, $x$, is the hours that he works. The output value, $y$, is the money that Isaac earns after he works a certain number of hours.

| Example 3: Use the above information to answer the questions below. |
|---|

a) What is the independent variable?

Hours Worked or Money Earned

b) What is the dependent variable?

Hours Worked or Money Earned

c) What is the unit rate of earnings per hour?

d) For every hour Isaac works, what do his earnings go up by?

e) What is the constant rate of change in this equation?

All linear equations have a constant rate of change. As the independent variable increases or decreases by one unit, the dependent variable increases or decreases by a constant rate: in this example with Isaac, it was $7.50.

In Isaac's situation concerning his hours worked and money earned, we can write an equation in which $y$ is money earned and $x$ is hours worked.

$$\$7.50 = \$7.50(1)$$
$$\$15.00 = \$7.50(2)$$
$$\$22.50 = \$7.50\ (3)$$
$$\$30.00 = \$7.50(4)$$

Using $x$ as the input and $y$ as the output (total earnings), $y = 7.50x$.

Example 4: Complete the table below, fill in the ordered pairs, and graph the equation.

| $(x)$ | $(y)$ |
|---|---|
| 1 | $7.50 |
| 2 | |
| 3 | |
| 4 | |

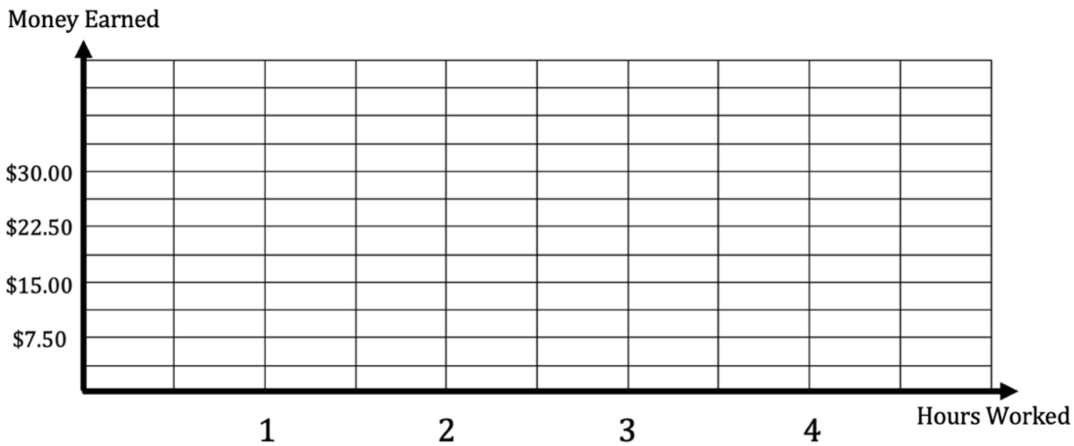
$(x, y)$

$(1. 7.5)$

$(2,$ ________$)$

$(3,$ ________$)$

$(4,$ ________$)$




You can see the constant rate of change in each of these. As $x$ increases by one hour, $y$ increases by $7.50. This constant rate of which we call a unit rate is a hallmark of linear equations.

Example 5: In each situation below, put one line under the independent variable and two lines under the dependent variable.

a) The cost of air conditioning used based on the temperature outside

b) How well you do in school and the amount of homework you do

c) The amount of fertilizer used and how tall your plants grow

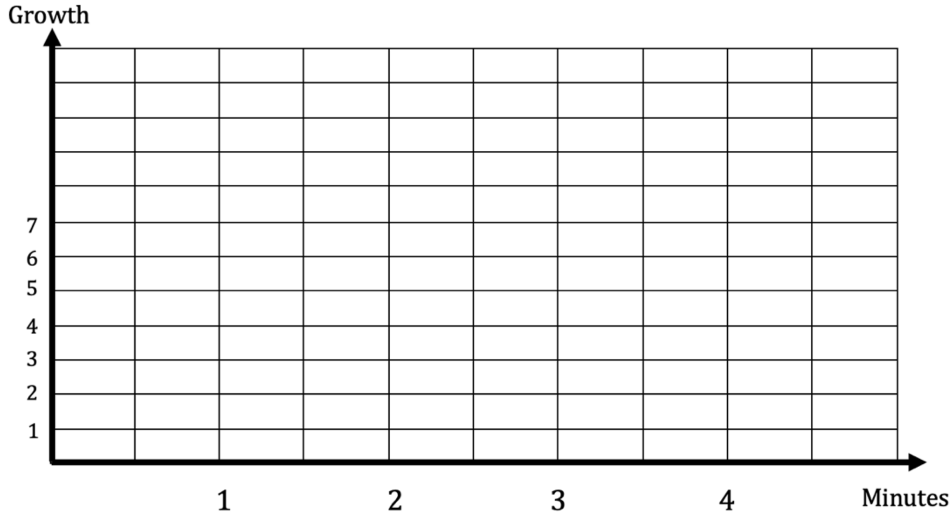
d) The amount of exercise you do and the calories you burn

Example 6: For each minute that passes in an experiment, the bacteria in a petri dish grows according to the table below. Graph the ordered pairs. Is the graph linear?

| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 7 |

$+1$, $+2$, $+3$

$(1, 1)$
$(2, 2)$
$(3, 4)$
$(4, 7)$




Example 7: Identify which of the equations and graphs below are linear and which are non-linear.

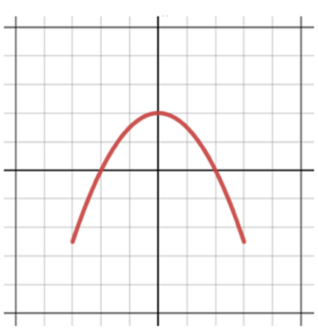
a) $y = x^2 + 1$

c) $y = x^3$

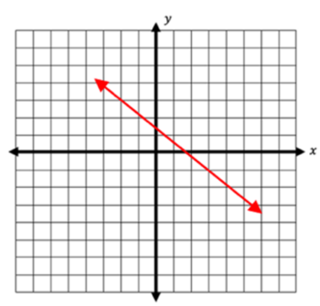
b) $y = 2x + 4$

d) $y = 2^x + 6$

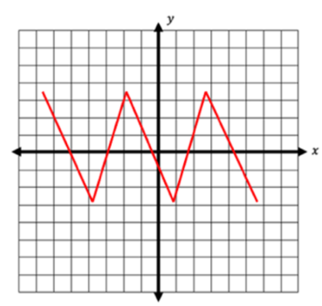
e)



f)




g)



## Section 2.5 Graphing an Equation Using a Table

### Looking Back 2.5

Now that we understand the importance of graphs, we want to pay particular attention to graphing equations. Even when graphing linear equations without knowing the situation or story behind them, we can still gather much information about the equation and tell a story of our own.

When graphing an equation on a coordinate plane, we plot the ordered pairs after making an Input-Output table. The input values for the table are not given, we must pick them. We should pick at least one or two negative input values, zero (0), and one or two positive input values. If we still need more information, we can try larger or smaller input values. After picking the input values, we find the output values.

Example 1: Graph $y = 3x - 4$.

We know this is a linear equation because $x$ and $y$ are both to the first power. They both have exponents of 1.

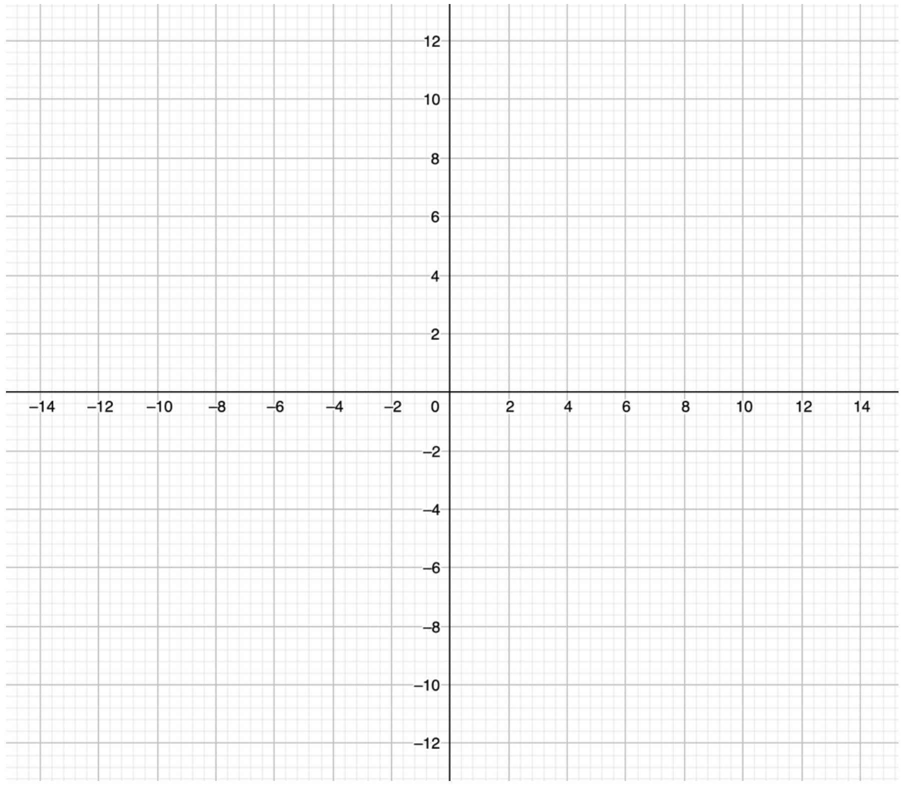
| $x$ | $3x - 4$ | $y$ |
|---|---|---|
| | | |
| | | |
| | | |

Example 2: All of the outputs in Example 1 are negative. We need to pick three more values to get some positives outputs. Complete the table below.

| $x$ | $3x - 4$ | $y$ |
|---|---|---|
| | | |
| | | |
| | | |

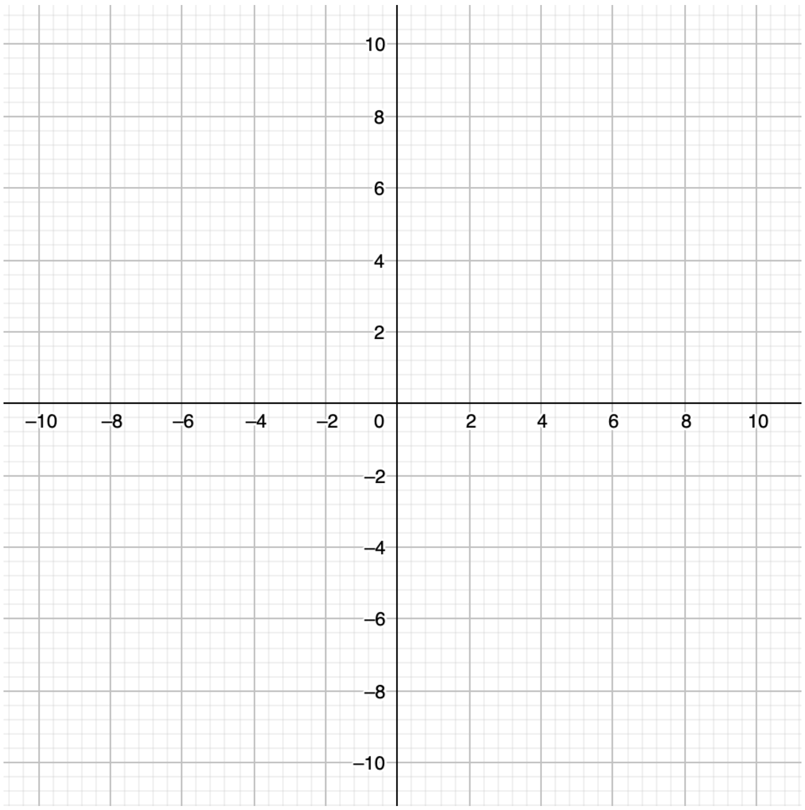
Why does $y$ increase by 3 as $x$ increases by 1?

Example 3: Graph the ordered pairs from the Input-Output tables from Example 1 and Example 2 on the $xy$-coordinate plane.




Example 4: Complete the table below for $y = \frac{1}{2}x + 6$ and graph the equation. Then answer the questions that follow.

| $x$ | $y = \frac{1}{2}x + 6$ | $y$ |
|---|---|---|
| $-4$ | | |
| $-2$ | | |
| $0$ | | |
| $2$ | | |
| $4$ | | |



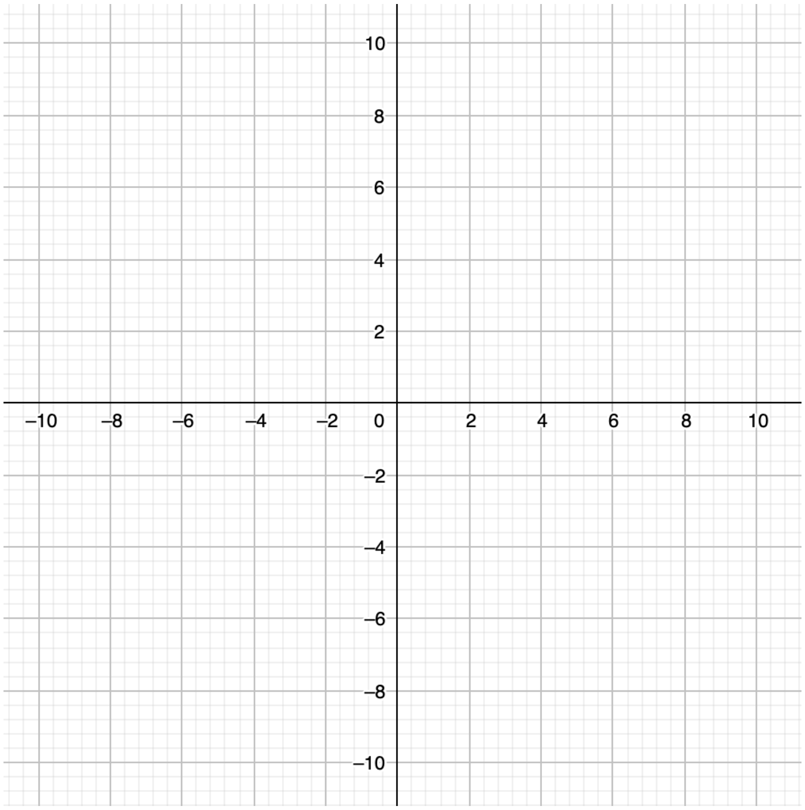

a) What is the $y$-intercept? Where do you see this in the equation?

b) What is the $x$-intercept? Let $y = 0$ and solve for $x$.

c) As $x$ increases by 2, what does $y$ increase by? As $x$ increases by 1, what does $y$ increase by and where do you see this in the equation?

Example 5: Complete the table below and graph the equation $y = 7$. (This can also be written: "$y = 0x + 7$.") Then answer the questions that follow.

| $\boldsymbol{x}$ | $\boldsymbol{7 + 0x}$ | $\boldsymbol{y}$ |
|---|---|---|
| $-2$ | | |
| $-1$ | | |
| 0 | | |
| 1 | | |
| 2 | | |



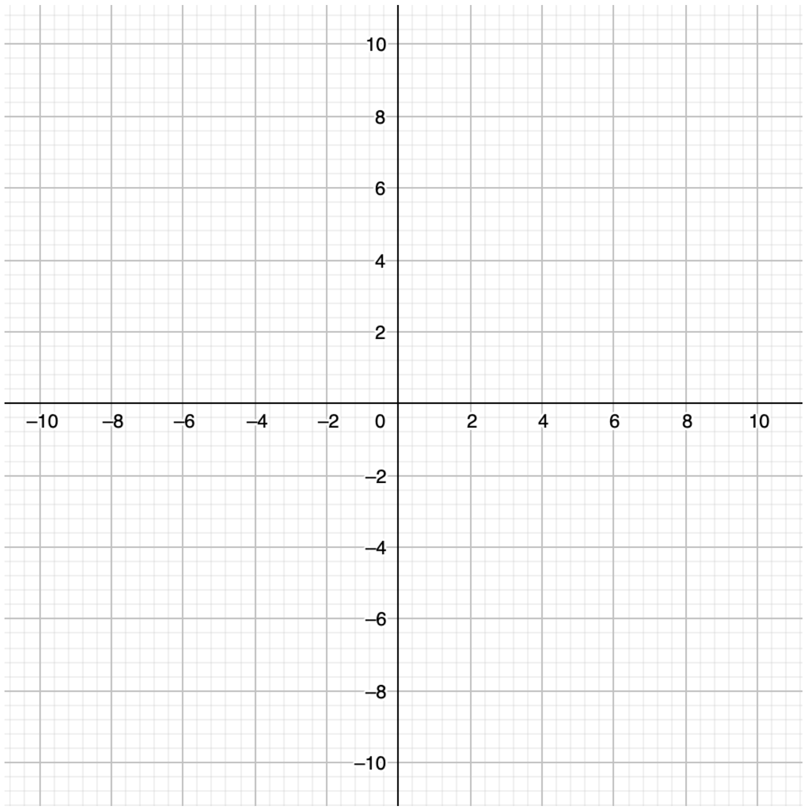
a) What is the $y$-intercept?

b) What is the $x$-intercept?

c) For any input value of $x$, what is the output value of $y$?

Example 6: Complete the table below and graph the equation $x = -4$. Then answer the questions that follow.

| $x$ | $x = -4$ | $y$ |
|---|---|---|
| | | $-2$ |
| | | $-1$ |
| | | 0 |
| | | 1 |
| | | 2 |




a) What is the $y$-intercept?

b) What is the $x$-intercept?

c) For any output value of $y$, what is the input value of $x$?

Section 2.6 The $x$-intercept

Looking Back 2.6

Crossroads are the point where two roads meet (cross). Have you ever thought about crossroads? When we reach a crossroad, we must make a choice. In your life, you will face many crossroads. At those points, you will make a decision. While the decision may seem as simple as which way to turn when driving a car to a destination, some crossroads are life changing. In the Bible, David faced many crossroads in his life; sometimes, his choices were "good;" sometimes, they were not so good. Each choice led him to different outcomes, some good, some not so good, but all with consequences.

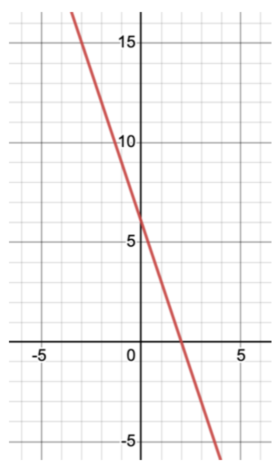
On the way to Damascus, Saul met Christ in a bright light. Christ asked why Saul was persecuting Him. That was certainly a crossroad in his life. Saul had been hurting Christ and His followers; at this crossroad, he chose to change, follow Christ, and stop harming those who followed the Lord's teachings.

When we graph equations, we have crossroads. They are certainly not life-changing, but they tell us information about a graph. Our first "linear crossroad" is the $x$-intercept of the graph. We have previously seen this, but now we will learn more about it.

Looking Ahead 2.6

The $x$-intercept is where a graph of an equation crosses the $x$-axis on a graph. The $x$-intercept can be written: "$x = a$" in which $a$ is some real number $(a, 0)$. The $x$-intercept has a $y$-value of 0 because we move right or left of the origin to get to it, not up or down. In other words, the $x$-intercept is on the $x$-axis where the $y$ value is 0.

| Example 1: | What is the $x$-intercept on the graph below? |
|---|---|




When you want to find the $x$-intercept given a graph, find the value of $x$ where the line crosses the $x$-axis.

| Example 2: | What is the $x$-intercept in the table below? |
|---|---|

| $x$ | $y$ |
|---|---|
| $-4$ | 0 |
| $-3$ | 2 |
| $-2$ | 4 |

Example 3: Find the $x$-intercept by completing the table below for a linear equation.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
|---|---|
| 3 | 3 |
| 4 | 2 |
| 5 | 1 |
| | |

When you want to find the $x$-intercept given a table, find the value of $x$ when $y = 0$.

Example 4: Find the $x$-intercept of the graph for the line that represents the equation $y = 2x - 4$ or $2x - 4 = y$.

When you want to find the $x$-intercept given an equation, substitute $0$ for $y$ and solve for $x$.

## Section 2.7 The $y$-intercept

### Looking Back 2.7

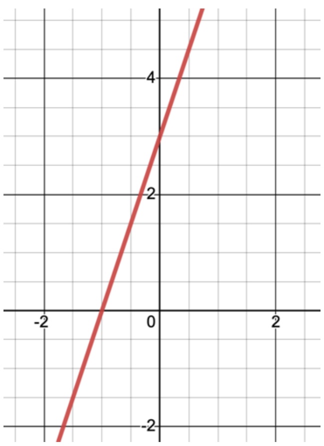
Another "crossroad" in a linear equation is the $y$-intercept. It is more widely used than the $x$-intercept. The $y$-intercept often determines the starting point in applied situations. For example, let us suppose you sell brownies for $3 each at a bake sale. If someone donates $5 before you even begin to sell anything, the equation for your profit would be $y = 3x + 5$ in which $y =$ money earned and $x =$ number of brownies. When $x = 0$, $y = \$5$. This is the $y$-intercept, $(0, 5)$. It means before you sell even one brownie, you start with $5 profit. The $y$-value is your initial (start) value.

### Looking Ahead 2.7

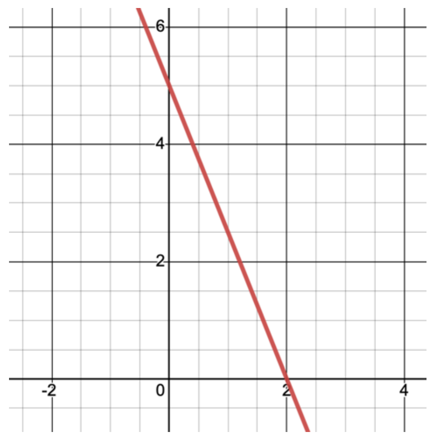
The $y$-intercept is where the graph of an equation crosses the $y$-axis on a graph. Although we only find some points to draw the graph of an equation, all the points in between are included in a linear graph that continues forever in two directions. The $y$-intercept can be written: "$y = b$" in which $b$ is a real number; or as an ordered pair: "$(0, b)$." The $y$-intercept has an $x$-value of 0 because we move up and down from the origin to get to it, not right or left. In other words, the $y$-intercept is on the $y$-axis where the $x$ value is 0.

| Example 1: | What is the $y$-intercept of each graph below? |
|---|---|

a)




b)




When you want to find the $y$-intercept given a graph, find the value of $y$ where the line crosses the $y$-axis.

| Example 2: | What is the $y$-intercept in the table below? |
|---|---|

| $x$ | $y$ |
|---|---|
| $-10$ | 20 |
| $-5$ | 15 |
| 0 | 10 |
| 5 | 5 |

Example 3: Find the $y$-intercept by completing the table below for a linear equation.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
|---|---|
| 8 | 2 |
| 6 | 3 |
| 4 | 4 |
| 2 | 5 |
| | |

When you want to find the $y$-intercept given a table, find the value of $y$ when $x = 0$.

Example 4: Find the $y$-intercept of the graph of the equation for the line given.

a) $y = 12x - 5$

b) $4x - 2y = 18$

c) $y = 12$

d) $3x = 3y$

When you want to find the $y$-intercept given an equation, substitute 0 in for $x$ and solve for $y$.

## Section 2.8 The Slope of a Line

### Looking Back 2.8

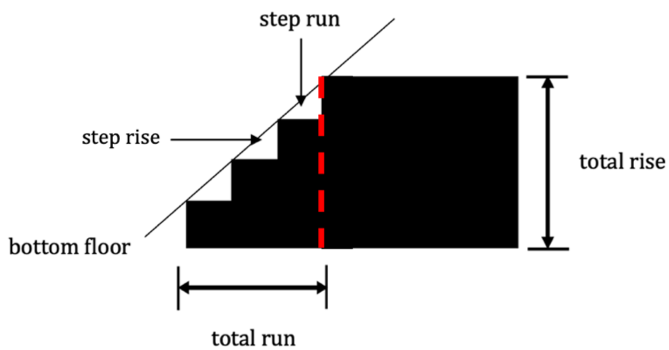
What do the following have in common: a wedge, a wheelchair ramp, a roof, an escalator, and a ski slope? The answer is they all have a slope. Stairs and escalators go up and down. The steepness of the plane is called the slope. Slope is very important in the real world. Skiing would not be much fun without steep slopes!

Slope is a part of your everyday life or routine from morning to night. There are many other places we see slope; however, we cannot possibly name them all.

Now, we have given examples of slope and described its affects, but we still have not answered the question: "What is slope?" Let us get started with understanding this very important concept and answering the question.

### Looking Ahead 2.8

In real-life, engineers use the word slope to describe the steepness, gradient, or incline of a plane. On a graph, the slope is the incline of a line. It can have no incline which is a zero slope, or it can go up from left to right or down from left to right. The steepness of a line is called the slope.




Each part of the step that goes up is called the rise. The flat part that is walked on is called the run. We define the slope of a line as rise over run "$\frac{\text{rise}}{\text{run}}$." Therefore, slope $= \frac{\text{rise}}{\text{run}}$. That is how we find the steepness of a line. We can call the triangles made by the steps under the line slope triangles.

### Activity 1

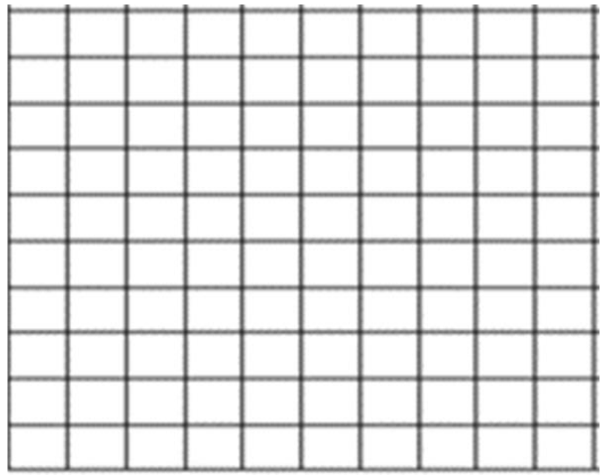
Find a set of stairs in your house or outside.

a) Measure the rise of one step (vertical part of the step).
b) Measure the run of one step (horizontal part of the step).
c) Write the measures of the step as the ratio of rise over run ($\frac{\text{rise}}{\text{run}}$). This is the slope of each step.
d) Multiply the measure of the rise by the number of steps. This is the total rise.
e) Multiply the measure of the run by the number of steps. This is the total run.
f) Write the total measures of the step as the total rise over the total run ($\frac{\text{total rise}}{\text{total run}}$). This is the slope of all the steps. When you simplify it, it should be the same as the ratio or slope of one step.

These ratios are the slope of the staircase. An easier way to find the slope is to use a graph.

Activity 2

Let us put our steps on a graph.



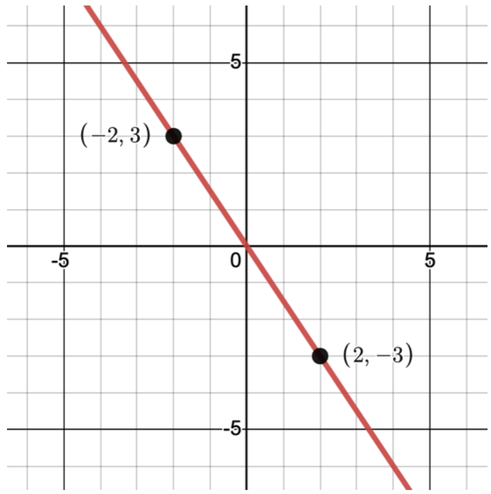
- On the graph, the rise is ________________ and the run is ________________. The slope would be ________________ or ________________.
- The ordered pair at the bottom is ________________. The ordered pair at the top is ________________. From the bottom point to the top point, the rise would be ________________ and the run would be ________________. The slope would be ________________ or ________________.
- On the graph, the run moves along the ________________ and the rise moves along the ________________ . The slope is the ratio of ________________ or ________________.
- In algebra, we use the letter ___________ to represent slope. We write it as $m =$________________. We say the ratio is the change in ___________ divided by the change in ___________. The Greek symbol for delta, $\Delta$, means "change." So, we often write slope as "$\frac{\Delta y}{\Delta x}$." Think of $\Delta$ as the slope triangles.

Section 2.9 Finding Slope from a Graph

Looking Back 2.9

Now that we understand slope, we are going to look at different ways to find slope. Slope is steepness. We cannot always directly measure steepness. For instance, it would be difficult to find the slope of a mountain or deep tunnel. It might be dangerous as well!

Looking Ahead 2.9




Draw the slope triangle between the two points. Remember, slope is $\frac{\Delta y}{\Delta x}$.

Count the change in $y$ from one point to the other. If you count up, it is 6 units. If you count down, it is $-6$ units.

Now, count the change in $x$ from one point to the other. If you count left, it is $-4$ units. If you count right, it is 4 units.

Once you have these numbers, put them in the slope formula:

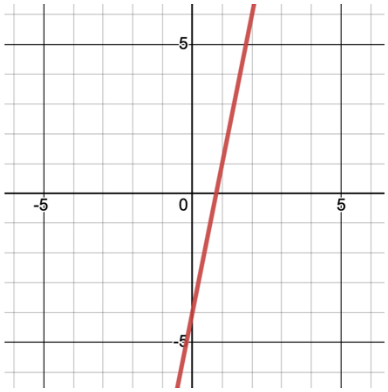
$$m = \frac{6}{-4} \text{ or } m = -\frac{3}{2} \text{ and } \frac{-6}{4} \text{ or } -\frac{3}{2}$$

$\frac{\text{up}}{\text{left}}$ $\frac{\text{down}}{\text{right}}$

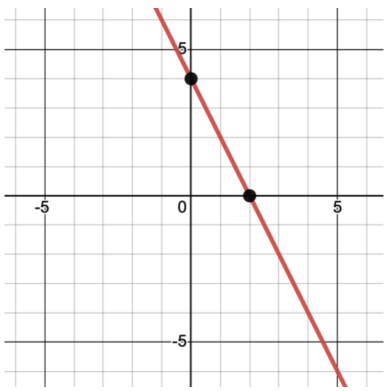
The ratio of $\Delta y$ to $\Delta x$ is ___________________. We can write a ratio for the entire line as $\frac{y}{x} =$___________________.

If we solve for $y$ in terms of $x$, we have the equation for the line:

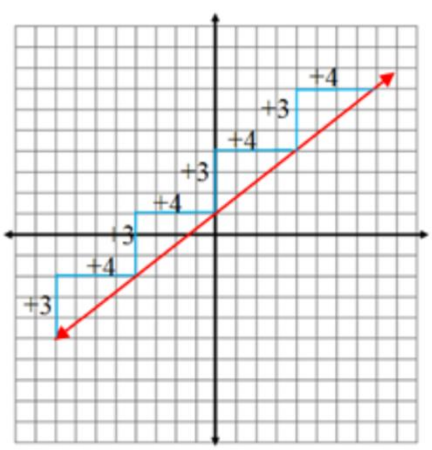
| | |
|---|---|
| Example 1: | Find the slope of the graph below. |




| | |
|---|---|
| Example 2: | Find the slope of the graph below in two different ways. (Use the slope triangle above the line and the slope triangle below the line.) |




| | |
|---|---|
| Example 3: | Find the slope of the graph below using several different sets of points. |



## Section 2.10 Finding Slope from Two Points

### Looking Back 2.10

Now that you have a better understanding of slope, how do you think you would find the slope of a line if you were only given two points on the line, but they were not shown on a graph? You could graph them and count out the slope, but in this section, we will look at another way to find slope given two points without having to draw a graph.

### Looking Ahead 2.10

When we find the rise of a line, we are counting the vertical unit spaces between the two $y$-coordinates; we will call them $y_1$ and $y_2$. When we find the run, we are counting the horizontal unit spaces between the two $x$-coordinates; we will call them $x_1$ and $x_2$. The distance between the $y$-coordinates is $y_2 - y_1$, which is the change in $y$ ($\Delta y$). The distance between the $x$-coordinates is $x_2 - x_1$, which is the change in $x$ ($\Delta x$). Substituting these expressions in for $\Delta y$ and $\Delta x$ in our slope gives us a slope formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

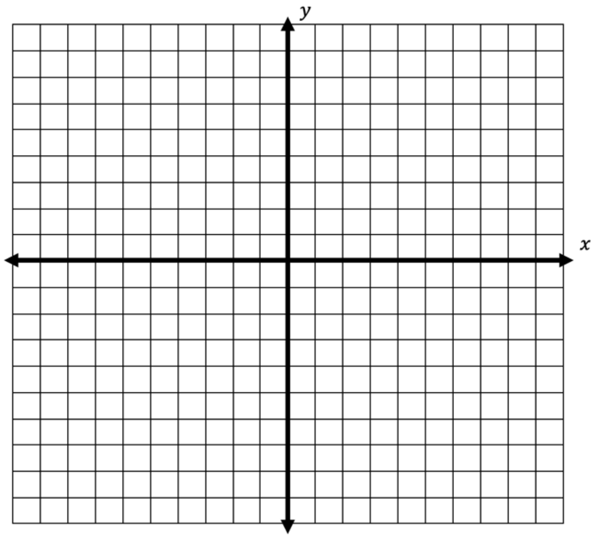
Now, if we know the two points $(x_1, y_1)$ and $(x_2, y_2)$, we can substitute them in the slope formula and find the slope of the line without having to graph them first.

| |
|---|
| Example 1: Find the slope of the line that crosses the two points $(0, 8)$ and $(1, 5)$. Let $(x_1, y_1) = (0, 8)$ and $(x_2, y_2) = (1, 5)$. |

| |
|---|
| Example 2: Find the slope of the line that crosses the two points $(0, 8)$ and $(1, 5)$. This time, let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (0, 8)$. |

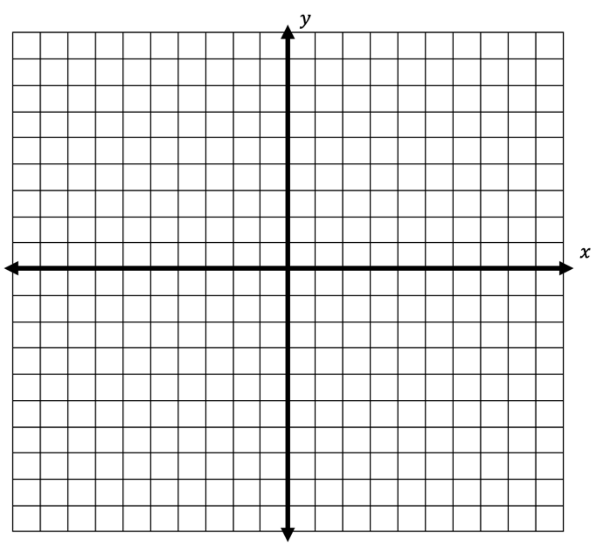
| |
|---|
| Example 3: Find the slope of the line that crosses the two points $(3, 7)$ and $(9, 4)$. |

Example 4: Find the slope of the line that crosses the two points $(7, -4)$ and $(7, 3)$. Draw the line on the graph. Is it vertical or horizontal?




This is an impossible slope to walk on. It is only possible to climb in cartoons! If you were to climb it, you would have to use a rope and propel up the mountain with your feet against the surface. So, for a vertical line, just remember "rope" means "no slope."

Example 5: Find the slope of the line that crosses the two points $(4, -3)$ and $(-2, -3)$. Draw the line on the graph below. Is it vertical or horizontal?




Think of the horizontal line as the horizon. When the sun peaks over the horizontal it is round like the number "0." So, for a horizontal line, just remember "horizon" means "0 slope."

## Section 2.11 Slope-Intercept Form of an Equation

### Looking Back 2.11

There are three forms of a linear equation that we use often in algebra: slope-intercept form; point-slope form; standard form. In this section, we will take a look at slope-intercept form. Slope-intercept form is used most often of the three and is the most direct. Standard form must be changed to slope-intercept form so the slope and $y$-intercept can be found.

### Looking Ahead 2.11

The equation $y = mx$ is called a direct variation because $y$ varies directly with $x$. It comes from the equation $m = \frac{y}{x}$. (We will learn more about this in Algebra 1.) The equation $y = mx$ goes directly through the origin on a graph because the start value ($y$-intercept) is $0$. This is also because $y = mx$ is also $y = mx + 0$.

Example 1: Complete the three tables below for the linear equations. What is the $y$-intercept for each? Where do you see the $y$-intercept in the equation?

| $y = 2x$ | |
|---|---|
| $x$ | $y$ |
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

| $y = 2x + 3$ | |
|---|---|
| $x$ | $y$ |
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

| $y = 2x + 5$ | |
|---|---|
| $x$ | $y$ |
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

The $y$-intercept is the constant in the equation. The $y$-intercept is found when $x = 0$ for the equation $y = mx + b$ in which $m$ and $b$ are real numbers. Then $y = m \cdot 0 + b$, and $y = b$ in which $b$ is the constant. Notice the variable it is equal to is $y$ and the coefficient of $y$ is $1$.

Example 2: Identify the slope and the $y$-intercept of the equations in the table below.

| | $m =$ | $b =$ |
|---|---|---|
| a) $y = 3x + 4$ | | |
| b) $y = \frac{2}{3}x - 3$ | | |
| c) $y = -5x + 3$ | | |
| d) $y = -x + \frac{2}{5}$ | | |
| e) $y = 8x$ | | |
| f) $y = -4$ | | |
| g) $x = -4$ | | |

When an equation is in the form $ax + by = c$ in which $a$, $b$, and $c$ are all real numbers, it is called standard form. In this equation, $b$ is the coefficient of the variable $y$. In the slope-intercept form $y = mx + b$ in which $m$ and $b$ are real numbers, $b$ is the $y$-intercept.

| Example 3: | Given the slope and $y$-intercept, write an equation in slope-intercept form. |
|---|---|

a) $m = 6$ and $b = 4$

b) $m = 3$ and $b = -7$

c) $m = -2$ and $b = 3$

d) $m = \frac{1}{2}$ and $b = -6$

| Example 4: | Given the equations, change them to slope-intercept form to find the slope and $y$-intercept. |
|---|---|

a) $2x + y = 4$

b) $y - 3 = 5x$

c) $3y = 5x + 12$

d) $2x + 3y = 9$

e) $-y = -2x + 4$

f) $3x = y + 4$

## Section 2.12 Using Slope-Intercept Form to Graph Equations

### Looking Back 2.12

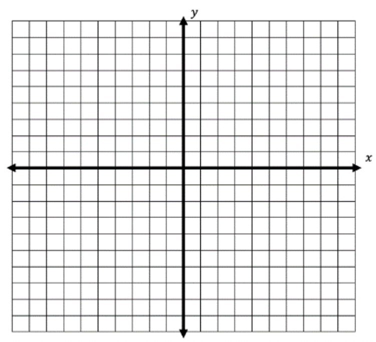
We have previously found the slope of a line using the graph of an equation. In this section, we are going to use an equation of the line to draw a graph.

We know how to draw the graph of a line using a table of the input and output from an equation. Here, we are going to graph linear equations using the slope and $y$-intercept of a line.

### Looking Ahead 2.12

We know the slope-intercept form of an equation is $y = mx + b$ in which $m$ is the slope and $b$ is the $y$-intercept. The slope is the incline (or gradient) of the line and the $y$-intercept is the point where the line crosses the $y$-axis. When we know the slope and the $y$-intercept of a specific line, we can substitute the values in the slope-intercept form of the equation to get the equation of the line. We can keep it in slope-intercept form or change it to standard form.

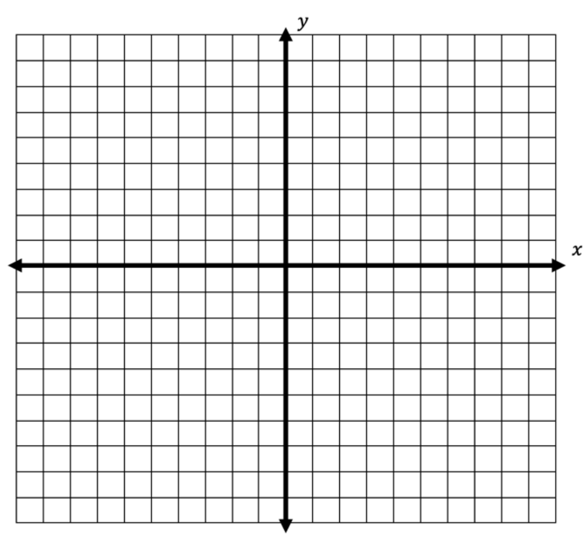
In the slope-intercept form of the linear equation $y = 3x - 2$, the slope is 3 and the $y$-intercept is $-2$. To graph this line, we first put a dot on the $y$-axis for the point that is the $y$-intercept. Next, we take the slope of 3 and make it a fraction by putting it over 1. The slope is $\frac{3}{1}$. The numerator is the change in the vertical distance (rise) of $+3$. The denominator is the change in the horizontal distance (run) of $+1$. Start from the $y$-intercept as the initial point of $-2$ and move up 3 units and right 1 unit. We put another dot at the point where we finish moving, which is $(1, 1)$. Notice that because the line is positive, it goes up from left to right.




| Example 1: Graph $y = -2x + 4$ using slope-intercept form. |
|---|

$y$-intercept = ___________

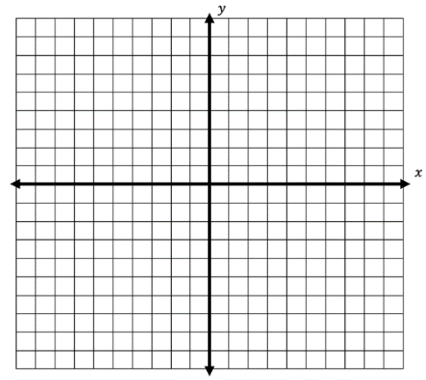
$m$ = ___________ or ___________



Example 2: Graph $y = -\frac{2}{3}x - 2$ using slope-intercept form.

$y$-intercept = __________

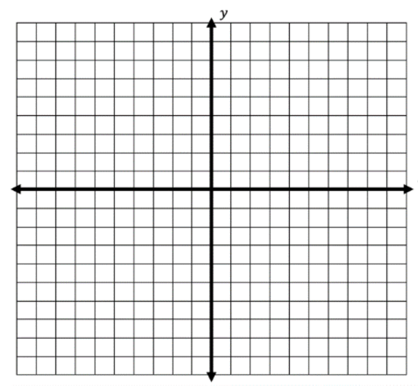
$m$ = __________ or __________




Example 3: Graph $5y = 3x$ using slope-intercept form. (Change it to slope-intercept form first.)

$y$-intercept = __________

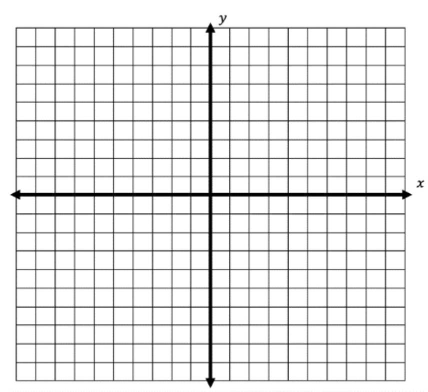
$m$ = __________




Example 4: Graph $9x - 3y = 6$ using slope-intercept form. (It is in standard form; change it to slope-intercept form.)

$y$-intercept = __________

$m$ = ___________ or ___________



Section 2.13 Writing Equations from a Graph

Looking Back 2.13

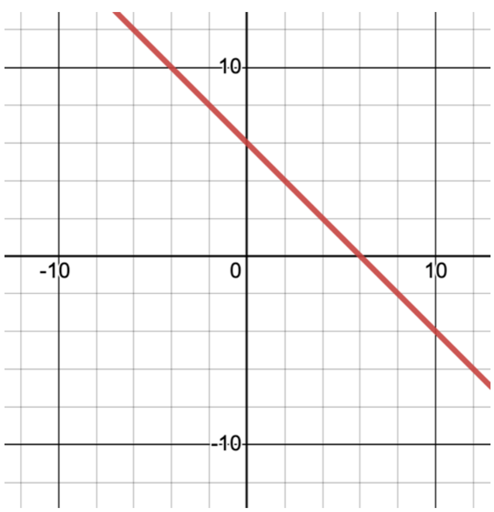
Now, in this final section, we will use the concepts of slope and $y$-intercept to look at a graph and write the equation of the line. We will also use the given slope and $y$-intercept to substitute values into the slope-intercept form of an equation and convert it to standard form. Slope-intercept form is $y = mx + b$ in which $m$ is the slope and $b$ is the $y$-intercept of the line. Standard form of a linear equation is $ax + by = c$ in which $a$ is the coefficient of $x$, $b$ is the coefficient of $y$, and $c$ is the constant.

Looking Ahead 2.13

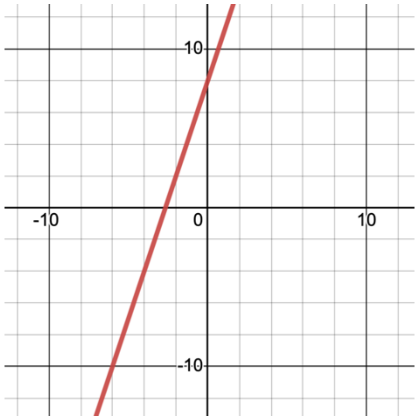
Example 1: Write the equation of a line given $m = 5$ and $b = -6$. (This is a review.)

Example 2: Write the equation of a line given $m = -\frac{2}{3}$ and $b = 3$. (This is a review.)

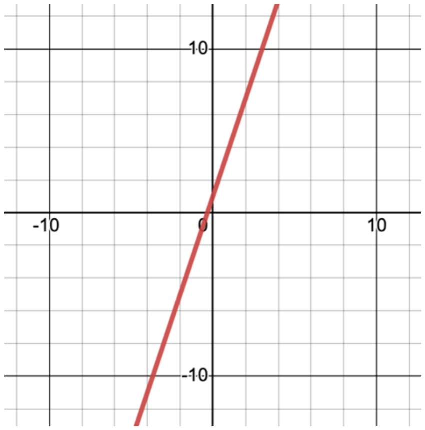
Example 3: Write an equation for the line shown on the graph below.



Example 4: Write an equation for the line that goes through the point $(-1, 5)$ and has a slope of 3 using the graph below and the slope-intercept equation. Find the y-intercept first.




Example 5: Write an equation for the line that goes through the point $(0, 1)$ and has a slope of 3 using the graph and the slope-intercept equation.




Example 6: Write the equation for the line below given the two points $(0, 6)$ and $(4, 0)$ using the graph below and the slope-intercept equation. Find the slope first and then the y-intercept.